The effects of possible policy withdrawal on investment timing and investment size

R.L.G. Nagy[*,1] and P.M. Kort[2]

[*] *Corresponding author*

[1] *Norwegian University of Science and Technology; roelnagy@hotmail.com*

[2] *Tilburg University and University of Antwerp; kort@tilburguniversity.edu*

January 31, 2018

Abstract

This paper analyzes the effect of a possible withdrawal of a tax credit policy on investment timing and investment size, and the interaction between investment timing and investment size. If the policy maker can only withdraw a policy once and not enact it in the future, we find that increasing the probability of withdrawal of a tax credit policy, increases the incentive to invest now and decreases the optimal investment size. Huisman and Kort [2015] show that investing later means that the investor invests at a larger capacity, which is confirmed in this paper. It is found that a firm that invests at the timing threshold value invests at larger scale when the policy is not in effect than when it is in effect. This results from the fact that subsidy speeds up investment and earlier investment is done at a lower capacity.

Unlike the price premium in Chronopoulos et al. [2016], these conclusions do not hold only for low withdrawal probabilities, but for all withdrawal probability values, as the tax credit policy is only relevant at the time of investment. Therefore, increasing the withdrawal probability to a large value speeds up investment more.

When the investor is a social planner who aims to the maximize social welfare, it is found that the social planner has the same timing as the profit-maximizing monopolist, but invests at twice the investment size. The monopolist seems to keep the price up by producing less.

1 Introduction

In an ever-changing world, also the economical and political landscape are continuously changing. These observations can be made just by looking into our newspapers. In February 2017, the World Bank warned that political developments could harm international trade as these political developments give rise to protectionism and threats to unwind trade agreements (Donnan [2017]). Another example is the Brexit later in 2017, which lead to even more political uncertainty. Mark Carney, the Governor of the Bank of England, warned that the unpredictable nature of Brexit was weighing on supply and demand, with some companies already delaying decisions about entering new markets (Jackson et al. [2017]). Most recently, South Korea proposed a change in regulations regarding the bitcoin, which caused some unrest among investors and a drop in the price (Jung-a and Dunkley [2017]).

For firms and investors it is important to be aware of risks in order to recognize and seize profitable investments. In the past, the Net Present Value criteria has been used, which says that it is optimal to invest whenever the expected payoff exceeds the investment costs. However, this criteria does not take into account that investment is usually irreversible and one can delay investment, i.e. it is not a now-or-never opportunity. As a typical investment opportunity is similar to a financial option, real options proposes an analysis of investments using knowledge of financial options. In this paper, a real options approach is taken to analyze investments (Dixit and Pindyck [1994]).

Apart from a proper analysis by the investors, it is also important that a government is aware of the economical consequences of their policies. Usually, the government's policies aim at a certain change, for example on the green energy market. Finjord et al. [2017] analyzes green certificate subsidy schemes currently employed on the energy markets in Norway and Sweden. These subsidies aim to increase the share of green/renewable energy, which is a target shared by countries worldwide. As the REN21 [2016] states: "As of year-end 2015, at least 173 countries had renewable energy targets ... and an estimated 146 countries had renewable energy support policies, at the national or state/provincial level." The European Union targets to cover 20% of the energy demand from renewable sources by 2020 (European Commission [2017]).

However, not only changing policies influences firms and investors, but even just debating about new policies or ending existing policies can influence the behavior of firms and investors. An example of this is the aforementioned drop in the price of the bitcoin as a result of *just saying* to plan on implementing regulations. Another example, related to the green energy market, is the delay in investment in wind farms as a result of uncertainty over government policy to boost investment in the industry (Bloomberg [2013]). This paper develops a framework to explain how uncertainty in the withdrawal of a tax credit policy affects invest timing and investment size.

Previous examples already imply there is some relation between policy uncertainty and investment timing. This paper focuses on the question what consequences policy uncertainty has on investment timing and investment size. The analyzed policy is a discount on investment cost, a so-called tax credit policy, which an investor only receives when the policy is in effect. In other words, it is not possible to invest at another time and collect the subsidy when investing while the policy is in effect. Therefore, the investor's optimal decision when to invest is subject to the policy uncertainty. As the investment size and timing are decisions that depend on each other (Huisman and Kort [2015]), also investment size is subject to the policy uncertainty.

Apart from deriving optimal decisions from the investors perspective, this paper also analyzes social welfare outcomes. An investor does not take the consumer surplus into account and solely looks at its own profit (the producer surplus), while a government aims to - depending on the sector - maximize total surplus or consumer surplus by use of its policies. In order to do some policy recommendations, we analyze the maximization problem of the total surplus, which is the problem of the social planner. The focus of this analysis is on the effect of policy uncertainty on the total surplus and socially optimal timing and capacity. Furthermore, these socially optimal decisions are compared with the firm's optimal timing and size decisions.

In the next section, the relevant literature is discussed. Section 3 discusses the investment under policy uncertainty model by Dixit and Pindyck [1994] with two extensions. The first extension is that the government can only intervene once; after policy withdrawal, there is no policy uncertainty anymore. Secondly, apart from a timing decision, the investor also has the choice to set investment size. The main conclusions of section 3 are compared with Chronopoulos et al. [2016] in section 4. Chronopoulos et al. [2016] have done a similar analysis on a different type of subsidy. 5 performs a robustness analysis on the results; this section is not complete yet, and will be finalized in the future. Finally, the main conclusions and recommendations for future research are listed in section 6.

2 Literature review

First we look into the relevant literature on the topic of investments and policy uncertainty. Dixit and Pindyck [1994] is one of the pioneering works on the topic of analyzing policy uncertainty using a real options approach. The authors analyze a situation in which one possible investor can choose the time to invest, and the government can retract/enact a subsidy. The subsidy is a discount on the (one-time) sunk investment cost, which the investor can receive when he/she invests at a time in which the policy is active. However, the time when the government changes the policy is uncertain. Uncertainty such as price uncertainty is modeled as a Brownian motion as the price changes continuously in a beforehand unknown direction. Dixit and Pindyck [1994] note that policy uncertainty is different, as the effect of implementing/withdrawing a policy is a sudden jump and not a continuous walk. Therefore, the policy is uncertainty is modeled as a Poisson jump. Unfortunately, Dixit and Pindyck [1994] are unable to analytically solve their model and derive their conclusions from a numerical example.

When there is no policy in effect initially, increasing the enactment probability leads to an increment in the timing threshold. This is the result of the fact that increasing the implementation parameter makes it more likely that investing in the future is cheaper, hence it increases the value of waiting. Dixit and Pindyck [1994] mention that this effect is rather large. While the policy is not in effect, increasing the removal probability decreases the timing threshold, but this effect is negligible.

While the policy is in effect, an increase in the withdrawal probability leads to a decrease in the timing threshold, as the risk of losing the tax credit increases and it gives value to investing now. Dixit and Pindyck [1994] note that the effect of the withdrawal probability on the timing threshold while the policy is in effect is smaller in magnitude that the effect of the implementation probability on the timing threshold while the policy is not in effect. A larger enactment probability increases the timing threshold while the policy is active, as it makes it more likely that the tax credit returns after withdrawal, reducing the incentive to invest as quickly as possible. This effect is especially relevant for situations in which the withdrawal probability is large.

The numerical example in Dixit and Pindyck [1994] shows that uncertainty in whether or not the tax credit policy is implemented increases the value of waiting, hence it decreases investment. Dixit and Pindyck [1994] conclude that a government aiming to increase investment can better start with the tax credit policy and swear to remove it soon with hardly any probability of implementing it again in the future. The authors note that "[t]he credibility of such a policy is open to doubt".

In the model by Dixit and Pindyck [1994], the investor is a monopolist and the only decision that has to be made is when to invest. However, apart from the timing of an investment, also the scale can be a crucial factor, especially in markets where multiple, competitive, firms may enter. This competition element can be captured by a game-theoretic model, which can be analyzed as done by Huisman and Kort (2015). Huisman and Kort (2015) analyze a two player game in which there is an incumbent firm (first investor) and an entrant (second investor). They find that the incumbent may overinvest in capacity to both delay the investment of the entrant and to reduce the chosen capacity of the entrant. It is also concluded that more uncertainty encourages the incumbent to deter entry for the entrant. This paper's main contribution is combining the policy uncertainty model by Dixit and Pindyck [1994] and the capacity choice model by Huisman and Kort [2015].

In reality, there are more types of subsidies than discounts on sunk investment costs. One can for example consider a fixed price premium that the investor receives on top of the output price, as done in Chronopoulos et al. [2016]. When it is not likely a new price premium policy is implemented, increasing the probability of implementing such policy leads postponement of investment by the firm. This is the result of the fact that it becomes more attractive for a firm to postpone investment to a time in which the policy is active. Chronopoulos et al. [2016] note that also the installed capacity increases, which is a result supported by Huisman and Kort [2015], as the latter concluded later investment leads to a increase in installed capacity. Similarly, when the policy is in effect and the probability of withdrawing the price subsidy is low, an increment in the probability of withdrawal leads to earlier investment and a lower installed capacity. When the probability of changing the current situation is relatively large, aforementioned conclusions no longer hold. When a price subsidy is (not) in effect and the probability of withdrawal (implementation) is large, increasing this probability leads to firms postponing (speeding up) investment and increasing (decreasing) the installed capacity. In the model of Chronopoulos et al. [2016], if it becomes very likely that the subsidy is withdrawn, the firm postpones investment to avoid being active on the market under detrimental conditions and as it invests later, the installed capacity is larger. If it becomes very likely that a new subsidy is implemented, the firm invests sooner to take the benefits from the subsidy as soon as the subsidy is implemented and as the firm invests earlier, the installed capacity is smaller. A more extensive discussion on the analysis by Chronopoulos et al. [2016] and a comparison of their assumptions and results with those in this paper is done in section 4.

In the analysis by Chronopoulos et al. [2016], policy recommendations are done on basis of optimal decisions for the investor. However, one can also take the consumer surplus into account and analyze the social planner's maximization problem in order to give policy recommendations, as done by Wen et al. [2017]. Wen et al. [2017] study how the subsidy support, for example price support and reimbursed investment cost support, affects the investment decision of a monopolist under uncertainty and analyzes the implications for social welfare. It is found that unconditional support for a subsidy leads to earlier investment, an outcome also found by Chronopoulos et al. [2016]. Wen et al. [2017] find that unconditional subsidy regulation cannot align the profit and social planner's investment decisions under linear demand structure. In order to align the firm's profit and social welfare, a conditional subsidy regulation that aligns the firm's investment decision to the social optimal decision can be introduced. Furthermore, for non-linear iso-elastic demand, it is possible to implement an unconditional subsidy to align profit maximizing and social optimal investment decisions, depending on the form of subsidy regulations.

3 Model

The basis from the model comes from Dixit and Pindyck [1994]. Both in this paper's model and the model in Dixit and Pindyck [1994], there is an investor who can enter an empty market by paying sunk cost I. There is no existing production nor any competitor. At a certain time t, once the investor decides to enter the market, it sells Q products at price $P(t)$. The production size Q is a decision variable chosen by the investor, and the investor is unable to change this after investment. The price at time t is dependent on the chosen capacity as the law of supply and demand suggest[1]. The following set of equations is used to model this:

$$P(t) = X(t)(1 - \eta Q), \tag{3.1}$$

$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t). \tag{3.2}$$

In the above, μ is the trend parameter, σ the uncertainty parameter and $W(t)$ a Wiener process. Using the above set of equations to model the price implies that certain assumptions are made. Firstly, it gives a positive probability to observing a negative price. In reality, a negative price will never occur. Secondly, in certain markets it may perhaps even be the case that the price tends to revert to its mean. In other words, after an upward (downward) trend, it may be more like to see a decrease (increase) in price due to external factors, e.g. policies that aim to keep the price at a certain level. As there are multiple factors that influence the price, the above set of equations may not capture the way prices on the market behave in reality.

As (more complex) alternatives for modelling the price, Dias and Rocha [1999] and Cartea and Figueroa [2005] may be used. Dias and Rocha [1999] model changes in oil prices as a sum of a mean reverting part, Brownian motion and poison jump process with random jump sizes. The first part of the sum expresses the tendency of prices to stick to a certain mean, the second part models certain upward/downwards trends and the final part models external shocks. According to Dias and Rocha [1999], their model has "more economic logic than previous models used in real options literature, considering that normal news causes continuous small mean-reverting adjustment in oil prices, whereas abnormal news causes abnormal movements in these prices (jumps)". Cartea and Figueroa [2005] use a similar model to analyse electricity spot prices, but

[1]Note that the law of supply and demand is not always active on every market. For example, one may argue that on the energy market, one individual producer is unable to influence the market price

also take into account seasonality. The authors mention that the model "succeeds in capturing changing convexities, which is a serious flaw in models that fail to incorporate seasonality or enough factors".

To capture the effect of policy change and its uncertainty, the government can withdraw an investment tax credit at rate θ. When the policy is in effect, an investor pays $(1-\theta)I$ to enter the market, against I when the policy is not in effect. At time zero, there is a policy active, and it is uncertain when the policy will be withdrawn. Policy uncertainty is modeled as a Poison jump and assumed to be independent of the output and price. In (short) time interval dt there is a $\lambda_0 dt$ probability that the policy will become inactive, given the policy is in effect. Once the investment tax credit has been withdrawn, it will never be re-implemented, and there is no government uncertainty. The way policy uncertainty is modelled, is similar to Dixit and Pindyck [1994]. Just as in Dixit and Pindyck [1994], a subscript 0 is associated with no policy in effect/withdrawal of the policy, and a subscript 1 is associated with the investment tax credit being active. However, in Dixit and Pindyck [1994] the government is able to implement and withdraw a subsidy multiple times. If a government only withdraws a tax credit policy or not, but never re-implements it, the policy uncertainty decreases. In other words, there is more certainty in the future investment costs, which is beneficial for potential investors. Two practical examples similar to a setting in which an active subsidy is only withdrawn once are the green certificate subsidy schemes currently employed on the energy markets in Norway and Sweden. These subsidy schemes are ended in the near future, and this situation is analyzed by Finjord et al. [2017].

Despite it is assumed that the policy uncertainty is independent of the produced output and the price, in reality, politicians are not oblivious to the present and past affairs of the economy. One can argue that politicians are more likely to suggest an investment tax credit when the market performs poorly. Pawlina and Kort [2005] provide a model with consistent authority behaviour. As a result, assuming that a government will implement a policy detrimental to firms at high output prices, the investors know that the policy will not occur as long as the current price remains equal to or below the highest price observed in the past; the policy will only be implemented when a certain trigger value is reached. Pawlina and Kort [2005] analyze the model in which the government may implement a tax which increases the investment cost when the output price is large. They find that the impact of the trigger value uncertainty on the investment threshold is non-monotonic. Increasing the uncertainty decreases the investment threshold (i.e. it is optimal to invest at a lower output price) when uncertainty is low. However, when uncertainty is high, increasing the uncertainty increases the investment threshold.

Obviously, at a given value X (or given price P), the situation in which the policy is in effect is more beneficial to an investor compared to the situation in which the policy is not in effect. As Dixit and Pindyck [1994] discuss, it is intuitively clear that when the price is below a threshold X_1, the investor will not enter, independently of whether the policy is active or not. Furthermore, when the price is high enough, i.e. above a threshold X_0, the investor will always enter independently of he/she gets an investment tax credit. For a price in the interval $[X_1, X_0]$, the investor will only enter the market when the policy is in effect, and he/she will not enter when the policy is not in effect. These decisions can be visualized as done in figure 1.

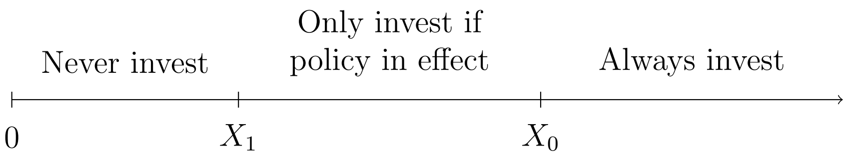

Figure 1: Optimal investment strategy under fixed capacity at different output prices

The value of the investment option is determined for every output price in order to determine the values of the thresholds X_1 and X_0. When the price is sufficiently high, the investor invests independently of whether the policy is implemented or not. The value of the investment option can be expressed as in equation (3.3) when the tax credit policy is not in effect, and as in equation (3.4) when the policy is in effect. In the equations below, δ is the cost of one unit of production, hence implementing a production capacity of size Q yields an investment cost of δQ when no policy is in effect, or $(1-\theta)\delta Q$ when the policy is in effect.

$$V_0(X,Q) = \frac{X(1-\eta Q)Q}{r-\mu} - \delta Q \tag{3.3}$$

$$V_1(X,Q) = \frac{X(1-\eta Q)Q}{r-\mu} - (1-\theta)\delta Q \tag{3.4}$$

Let X_1 be the value of the geometric Brownian motion at which the firm is indifferent between investing and not investing, while the tax credit policy is in effect. $Q_1^*(X)$ is defined as the chosen quantity by the investor at $X > X_1$ while the policy is in effect. Similarly, X_0 is the value of the geometric Brownian motion at which the firm is indifferent between investing and not investing, while the tax credit policy is not active. The chosen quantity at $X > X_0$ while the policy is not active is equal to $Q_0^*(X)$.

Proposition 1. *When it is optimal to invest, the following expressions give the optimal investment capacities:*

$$Q_0^*(X) = \frac{1}{2\eta}(1 - \frac{\delta(r-\mu)}{X}), \tag{3.5}$$

$$Q_1^*(X) = \frac{1}{2\eta}(1 - \frac{(1-\theta)\delta(r-\mu)}{X}). \tag{3.6}$$

Proof. See subsection A.1. □

Next the value of the investment opportunity is determined for $X(t)$ being sufficiently large that it is optimal to invest when the policy is active, but not when the policy is withdrawn (i.e. $X_1 < X(t) \leq X_0$). In this scenario, the above expressions for V_1 and the optimal Q_1 still hold, as it is optimal to invest when the policy is in effect. When the policy is not in effect, it has been withdrawn and will not return in the future. It can be shown that the following holds when the policy has been withdrawn:

$$\frac{1}{2}\sigma^2 X^2 V_0''(X) + \mu X V_0'(X) - rV_0(X) = 0. \tag{3.7}$$

Solving this ordinary differential equation yields $V_0(X) = A_0 X^{\beta_{01}} + B_0 X^{\beta_{02}}$. In this expression, A_0 and B_0 are constants. β_{01} (β_{02}) is the positive (negative) solution to $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0$. Since $V_0(0) = 0$ and $\beta_{02} < 0$, it follows that $B_0 = 0$, hence:

$$V_0(X) = A_0 X^{\beta_{01}}. \tag{3.8}$$

For $X(t) < X_1$, it holds that it is best to wait irrelevant of whether the policy is active or not. When the policy has been withdrawn, the value of the investment opportunity is still expressed by (3.8). The investment option while the policy is active satisfies the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 X^2 V_1''(X) + \mu X V_1'(X) - rV_1(X) + \lambda_0(V_0(X) - V_1(X)) = 0. \tag{3.9}$$

Solving the homogeneous part of the above ordinary differential equation yields solution $V_1^H(X) = A_1 X^{\beta_{11}} + B_1 X^{\beta_{12}}$. β_{11} (β_{12}) is the positive (negative) solution to $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - (r + \lambda_0) = 0$.

To find a particular solution to the ordinary differential equation in (3.9), one can try $V_1^P(X) = C_1 X^{\beta_{01}}$, as the in-homogeneous part is $A_0 X^{\beta_{01}}$. From this it follows that $C_1 = A_0$. Combining the homogeneous and particular solution gives $V_1(X) = A_1 X^{\beta_{11}} + B_1 X^{\beta_{12}} + A_0 X^{\beta_{01}}$. However, as $V_1(0) = 0$ and $\beta_{12} < 0$, it follows that $B_1 = 0$. This results in the following expression for $V_1(X)$:

$$V_1(X) = A_1 X^{\beta_{11}} + A_0 X^{\beta_{01}}. \tag{3.10}$$

In the above expression, A_1 and A_0 are constants that needs be determined. As before, β_{01} is the positive solution to $\frac{1}{2}\sigma^2\beta^2 + (\mu - \frac{1}{2}\sigma^2)\beta - r = 0$.

In conclusion:

$$V_0(X,Q) = \begin{cases} \frac{X(1-\eta \cdot Q)Q}{r-\mu} - \delta Q & \text{if } X \in [X_0, \infty) \\ A_0 X^{\beta_{01}} & \text{otherwise} \end{cases}$$

$$V_1(X,Q) = \begin{cases} \frac{X(1-\eta \cdot Q)Q}{r-\mu} - (1-\theta)\delta Q & \text{if } X \in [X_1, \infty) \\ A_1 X^{\beta_{11}} + A_0 X^{\beta_{01}} & \text{otherwise} \end{cases}$$

The constants A_0 and A_1 and thresholds X_0 and X_1 satisfy the value matching and smooth pasting conditions. The two value matching equations are (3.11) and (3.12). Equation (3.11) guarantees that the value for $V_0(X_0, Q_0^*(X_0))$ is uniquely defined, while (3.12) does the same for $V_1(X_1, Q_1^*(X_1))$.

$$A_0 X_0^{\beta_{01}} = \frac{X_0(1 - \eta Q_0^*(X_0))Q_0^*(X_0)}{r - \mu} - \delta Q_0^*(X_0) \tag{3.11}$$

$$A_1 X_1^{\beta_{11}} + A_0 X_1^{\beta_{01}} = \frac{X_1(1 - \eta Q_1^*(X_1))Q_1^*(X_1)}{r - \mu} - (1-\theta)\delta Q_1^*(X_1) \tag{3.12}$$

Apart from value matching conditions, there are also two smooth pasting conditions. Equation (3.13) guarantees that $V_0'(X_0, Q_0^*(X_0))$ has a unique value and equation (3.14) states that the two possible expressions for $V_1'(X_1, Q_1^*(X_1))$ match.

$$A_0 \beta_{01} X_0^{\beta_{01}-1} = \frac{(1 - \eta Q_0^*(X_0))Q_0^*(X_0)}{r - \mu} \tag{3.13}$$

$$A_1 \beta_{11} X_1^{\beta_{11}-1} + A_0 \beta_{01} X_1^{\beta_{01}-1} = \frac{(1 - \eta Q_1^*(X_1))Q_1^*(X_1)}{r - \mu} \tag{3.14}$$

A_0 can be determined by using the value matching condition (3.11) and smooth pasting condition (3.13) as stated in proposition 2. The result of the proposition and proof in the appendix can also be found in Huisman and Kort [2015].

Proposition 2. *Using conditions* (3.11) *and* (3.13), *it follows that*

$$X_0 = \frac{\beta_{01}+1}{\beta_{01}-1} \cdot \delta(r-\mu). \tag{3.15}$$

Furthermore, the following holds:

$$A_0 = \frac{\delta}{\eta(\beta_{01}^2-1)} \cdot \left(\frac{\beta_{01}+1}{\beta_{01}-1}\delta(r-\mu)\right)^{-\beta_{01}}. \tag{3.16}$$

Proof. See subsection A.2. □

Equations (3.11) - (3.14) cannot be solved analytically completely, as it is not possible to find expressions for constant A_1 and threshold X_1. Nevertheless, it is possible to derive some conditions for both, as shown in proposition 3.

Proposition 3. *Using conditions* (3.12) *and* (3.14), *it can be shown that* X_1 *satisfies the following equation:*

$$\frac{\beta_{11}-\beta_{01}}{\beta_{11}} \cdot A_0 X_1^{\beta_{01}} - \frac{\beta_{11}-1}{\beta_{11}} \cdot \frac{X_1(1-\eta Q_1^*(X_1))Q_1^*(X_1)}{r-\mu} + (1-\theta)\delta Q_1^*(X_1) = 0. \tag{3.17}$$

Proof. See subsection A.3. □

In order to gain some intuition on the effect of the withdrawal parameter λ_0, a numerical example is used. The parameter values follow from Dixit and Pindyck [1994], but for the parameters which are not present in their model, the values from Huisman and Kort [2015] are used. Therefore, $\mu = 0$, $\sigma = 0.1$, $r = 0.05$, $\eta = 0.05$, $\delta = 0.1$ and $\theta = 0.1$. The value of the thresholds X_0 and X_1 are shown in figure 2.

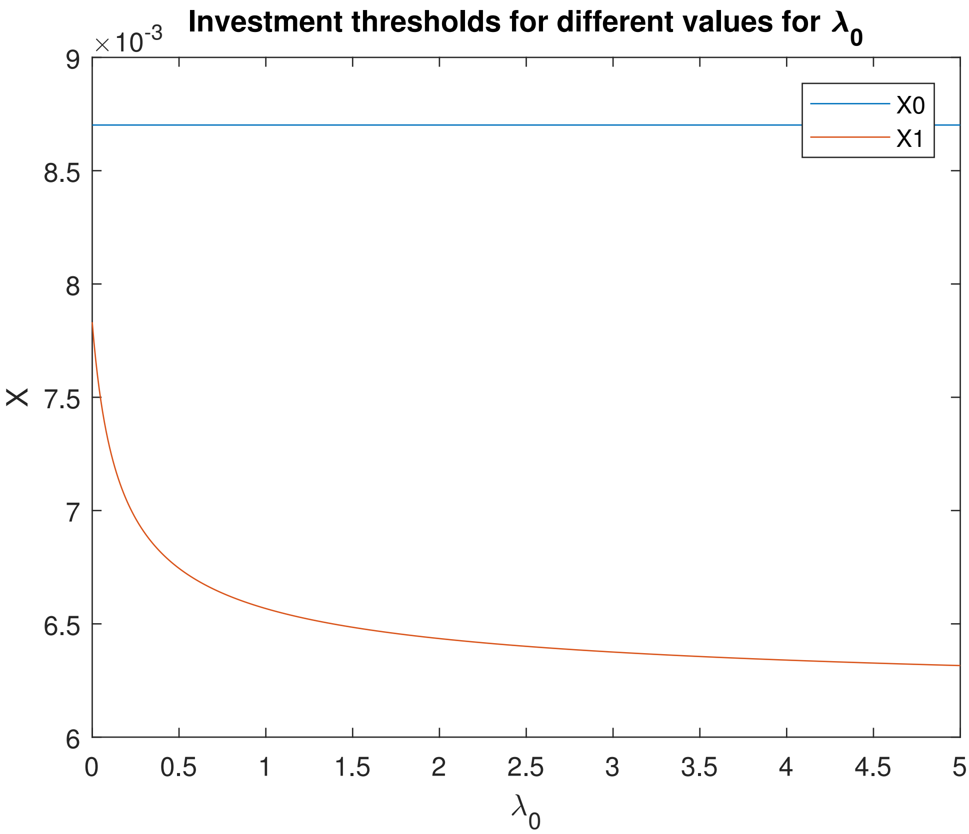

Figure 2: X_0 and X_1 for different withdrawal parameter values λ_0

From figure 2, it follows that increasing the withdrawal parameter λ_0 decreases the threshold X_1. In economic terms, increasing the probability of abandonment of the policy results in investors speeding up investment. Furthermore, as X_0 is the timing threshold after abandonment of the tax credit policy, it is not subject to policy uncertainty and does not depend on λ_0.

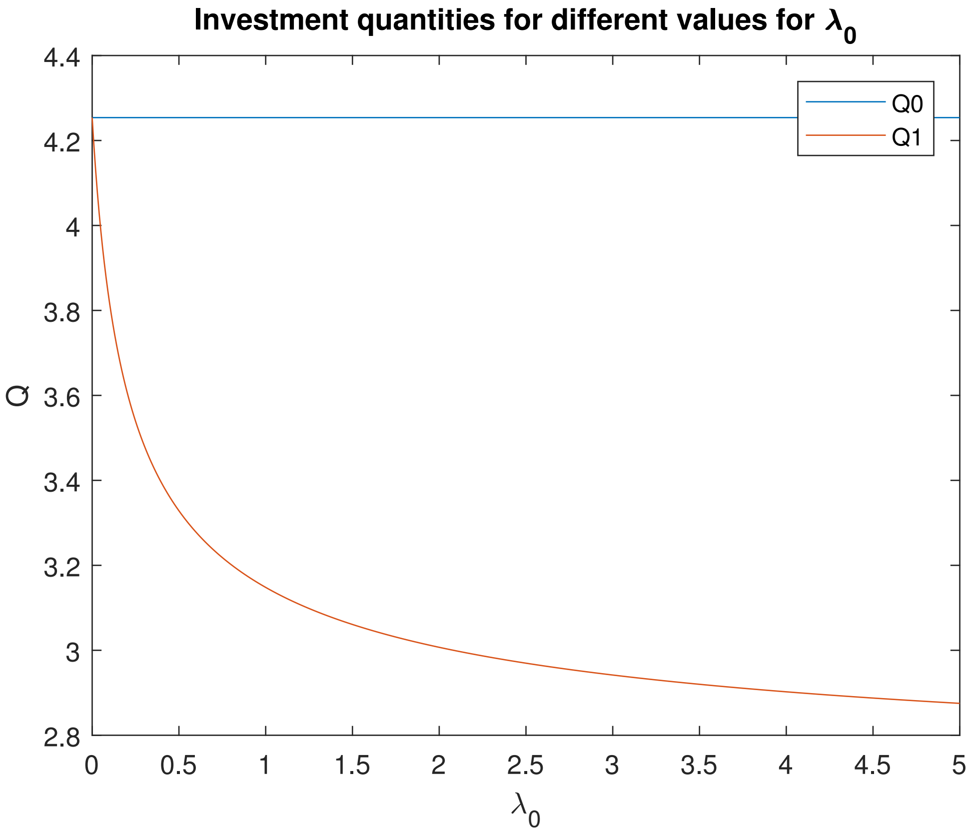

Figure 3: Q_0 and Q_1 for different withdrawal parameter values λ_0

The optimal quantity Q_1 for different values of λ_0 is shown in figure 3. Huisman and Kort [2015] note that investing sooner leads to investing at a lower capacity. This result can also be seen in both figures, since increasing λ_0 decreases both X_1 and Q_1. Therefore, a government aiming to decrease investment by retracting the tax credit policy may cause an increase in small investments due to policy uncertainty. The policy uncertainty has an effect contrary to the policy's aim. If the government aims to decrease the number of investments and wants to retract the tax credit policy, it is best to make firms believe the policy will not be withdrawn and withdraw it so sudden investors are unable to invest and set up firms/projects while the policy is active. In this case, investors cannot do the investment quickly before the termination of the policy and as the investment is more expensive after withdrawal, the incentive to invest now decreases after termination of the policy. However, this kind of policy making may frustrate firms and investors as the government needs to hide information from them and as a result, such kind of decision making may not be realistic for policy makers.

Nevertheless, policy uncertainty can be a (realistic) tool for policy makers to influence investors. The policy makers or government can speed up investment by promising to withdraw the tax credit policy in the near future. As the firms want to take advantage of the still available subsidy, they do investment now. As result of investing sooner, they invest on a smaller scale. This can be beneficial for the government if it prefers having smaller projects on short-term over having large-scale investments in the long run.

Apart from the firm's optimal timing and capacity decision to maximize the producer surplus, one can also consider the problem of the social planner. The social planner has the goal to maximize the total surplus, which consists of the sum of the consumer and producer surplus. The following discussion on the consumer, producer and total surplus can also be found in Huisman and Kort [2015].

The instantaneous consumer surplus is calculated by the following integral:

$$\int_{P(Q)}^{X} D(P)dP. \tag{3.18}$$

As it is assumed that all production is sold, the demand D and installed capacity Q are the same. Therefore, $D(P)$ is the inverse function of (3.1):

$$D(P) = \frac{1}{\eta}(1 - \frac{P}{X}). \tag{3.19}$$

Using (3.19) yields the following derivation for the instantaneous consumer surplus:

$$\begin{aligned}
\int_{P(Q)}^{X} D(P)dP &= \int_{X(1-\eta Q)}^{X} \frac{1}{\eta}(1 - \frac{P}{X})dP \\
&= \frac{1}{\eta}\left[P - \frac{P^2}{2X}\right]_{P=X(1-\eta Q)}^{P=X} \\
&= \frac{1}{2}XQ^2\eta
\end{aligned}$$

Before calculating the total expected consumer surplus (CS), lemma 1 is discussed as it is used in the derivation of the total expected consumer surplus.

Lemma 1. *When $dX(t) = \mu X(t)dt + \sigma X(t)dW$ with $W(t)$ as a standard Brownian motion, it can be shown that $X(s) = X(t) \cdot e^{(\mu - \frac{1}{2}\sigma^2)(s-t) + \sigma(W(s)-W(t))}$ for $s \geq t$.*

Proof. Let $dX = \mu X(t)dt + \sigma X(t)dW$ as defined in the lemma and let $Y(t) = \ln(X(t))$. By Itô's Lemma the following holds:

$$\begin{aligned}
dY &= (\mu X(t)\frac{dY}{dX} + \frac{dY}{dt} + \frac{1}{2}\frac{d^2Y}{dX^2}(\sigma X(t))^2)dt + \frac{dY}{dX}\sigma X(t)dW \\
&= (\mu X(t)\frac{1}{X(t)} + 0 + \frac{1}{2} \cdot -\frac{1}{X^2}(\sigma X(t))^2)dt + \frac{1}{X(t)}\sigma X(t)dW \\
&= (\mu - \frac{1}{2}\sigma^2)dt + \sigma dW
\end{aligned}$$

This can be rewritten as follows:

$$\begin{aligned}
Y(s) - Y(t) &= \int_t^s (\mu - \frac{1}{2}\sigma^2)du + \int_t^s \sigma dW \\
\iff Y(s) &= Y(t) + \int_t^s (\mu - \frac{1}{2}\sigma^2)du + \int_t^s \sigma dW \\
\iff \ln(X(s)) &= \ln(X(t)) + \int_t^s (\mu - \frac{1}{2}\sigma^2)du + \int_t^s \sigma dW \\
\iff X(s) &= X(t) + e^{\int_t^s (\mu - \frac{1}{2}\sigma^2)du + \int_t^s \sigma dW} \\
\iff X(s) &= X(t) + e^{(\mu - \frac{1}{2}\sigma^2)(s-t) + \sigma(W(s)-W(t))}
\end{aligned}$$

□

The total expected consumer surplus is the expectation of the integral over the discounted instantaneous consumer surplus. Using lemma 1 the following can be derived:

$$\begin{aligned}
\text{CS} &= \mathbb{E}[\int_{t=0}^{\infty} \frac{1}{2} X(t) Q^2 \eta e^{-rt} dt \mid X(0) = X] \\
&= \frac{1}{2} Q^2 \eta \cdot \mathbb{E}[\int_{t=0}^{\infty} X(0) \cdot e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W(t)} e^{-rt} dt \mid X(0) = X] \\
&= \frac{XQ^2\eta}{2} \cdot \mathbb{E}[\int_{t=0}^{\infty} e^{(\mu - \frac{1}{2}\sigma^2 - r)t + \sigma W(t)} dt \mid X(0) = X] \\
&= \frac{XQ^2\eta}{2} \cdot \int_{t=0}^{\infty} e^{(\mu - \frac{1}{2}\sigma^2 - r)t} \int_{-\infty}^{\infty} \frac{1}{\sqrt{t}\sqrt{2\pi}} e^{-\frac{(W(t))^2}{2t}} \cdot e^{\sigma W(t)} dW dt \\
&= \frac{XQ^2\eta}{2} \cdot \int_{t=0}^{\infty} e^{(\mu - \frac{1}{2}\sigma^2 - r)t} \int_{-\infty}^{\infty} \frac{1}{\sqrt{t}\sqrt{2\pi}} e^{-\frac{(W(t) - \sigma t))^2}{2t}} e^{\frac{1}{2}\sigma^2 t} dW dt \\
&= \frac{XQ^2\eta}{2} \cdot \int_{t=0}^{\infty} e^{(\mu - r)t} dt \\
&= \frac{XQ^2\eta}{2(r - \mu)}
\end{aligned}$$

In the derivation above it is used that $\int_{-\infty}^{\infty} \frac{1}{\sqrt{t}\sqrt{2\pi}} e^{-\frac{(W(t)-\sigma t)^2}{2t}} dW$ is an integral of the pdf of R where $R \sim N(\sigma t, t)$, hence this integral has value one.

The producer surplus when the investor decides to invest when the policy is withdrawn (PS_0) is equal to:

$$\text{PS}_0 = \frac{X(1 - \eta Q)Q}{r - \mu} - \delta Q. \tag{3.20}$$

When the policy is active, the investor gains the following by investing:

$$\text{PS}_1 = \frac{X(1 - \eta Q)Q}{r - \mu} - (1 - \theta)\delta Q. \tag{3.21}$$

The total surplus when the policy is withdrawn (TS_0) is equal to

$$\text{TS}_0 = \text{CS} + \text{PS}_1 = \frac{X(2 - \eta Q)Q}{2(r - \mu)} - \delta Q, \tag{3.22}$$

while in case the policy is still active (TS_1), the total surplus is:

$$\text{TS}_1 = \text{CS} + \text{PS}_0 = \frac{X(2 - \eta Q)Q}{2(r - \mu)} - (1 - \theta)\delta Q. \tag{3.23}$$

Taking the first order condition of TS_0 with respect to Q yields Q_0^S, the optimal social welfare quantity after policy withdrawal. Similarly, the optimal social welfare quantity when the policy is still in effect, Q_1^S, follows from taking the first order condition of TS_1 with respect to Q. Note that similar steps were taken to derive the optimal quantities chosen by the producer in proposition 1.

$$Q_1^S(X) = \frac{1}{\eta}(1 - \frac{(1 - \theta)\delta(r - \mu)}{X}) \tag{3.24}$$

$$Q_0^S(X) = \frac{1}{\eta}(1 - \frac{\delta(r - \mu)}{X}) \tag{3.25}$$

Since $Q_1^S(X) = 2Q_1(X)$ and $Q_0^S(X) = 2Q_0(X)$, it follows that a profit-maximizing firm should invest twice as much to achieve the social optimum.

Up to this point, expressions for the total surplus were derived - expressions (3.22) and (3.23) - given the social planner decides to invest. Given the timing of investment, also the optimal quantity is derived as shown by (3.24) and (3.25). When the social planner decides to wait, the same steps can be repeated as in the beginning of this section to determine expressions for the total surplus, with only TS_0 and TS_1 instead of V_0 and V_1. The total surplus when the social planner decides to wait while the policy has been withdrawn can be derived by solving (3.7), yielding (3.8). When the policy is still in effect and the social planner decides to wait, the total surplus is equal to (3.10), which follows from solving (3.9).

In the remainder of this section X_0^S is used to denote the threshold at which the social planner is indifferent between investing and not investing after the policy has been withdrawn, while X_1^S denotes the value for which the social planner is indifferent while the policy is active. Using this notation and previous derivations, we find the following expressions for the total surplus:

$$\text{TS}_0(X,Q) = \begin{cases} \frac{X(2-\eta\cdot Q)Q}{2(r-\mu)} - \delta Q & \text{if } X \in [X_0^S, \infty) \\ A_0 X^{\beta_{01}} & \text{otherwise} \end{cases}$$

$$\text{TS}_1(X,Q) = \begin{cases} \frac{X(2-\eta\cdot Q)Q}{2(r-\mu)} - (1-\theta)\delta Q & \text{if } X \in [X_1^S, \infty) \\ A_1 X^{\beta_{11}} + A_0 X^{\beta_{01}} & \text{otherwise} \end{cases}$$

The constants A_0 and A_1 and thresholds for the social planner X_0^S and X_1^S satisfy the value matching and smooth pasting conditions. The two value matching equations are:

$$\frac{X_0^S(2-\eta Q_0^S(X_0^S))Q_0^S(X_0^S)}{2(r-\mu)} - \delta Q_0^S(X_0^S) = A_0(X_0^S)^{\beta_{01}}, \tag{3.26}$$

$$\frac{X_1^S(2-\eta Q_1^S(X_1^S))Q_1^S(X_1^S)}{2(r-\mu)} - (1-\theta)\delta Q_1^S(X_1^S) = A_1(X_1^S)^{\beta_{11}} + A_0(X_1^S)^{\beta_{01}}, \tag{3.27}$$

and the two smooth pasting conditions are:

$$\frac{(2-\eta Q_0^S(X_0^S))Q_0^S(X_0^S)}{2(r-\mu)} = A_0\beta_{01}(X_0^S)^{\beta_{01}-1}, \tag{3.28}$$

$$\frac{(2-\eta Q_1^S(X_1^S))Q_1^S(X_1^S)}{2(r-\mu)} = A_1\beta_{11}(X_1^S)^{\beta_{11}-1} + A_0\beta_{01}(X_1^S)^{\beta_{01}-1}. \tag{3.29}$$

The interpretation of the value matching and smooth pasting conditions is similar to the value matching and smooth pasting conditions discussed earlier in this section.

The threshold X_0^S can be derived using the same steps as in the proof of proposition 2 and even yields the same expression for the threshold as derived in (3.15):

$$X_0^S = \frac{\beta_{01}+1}{\beta_{01}-1}\delta(r-\mu). \tag{3.30}$$

The optimal timing of the social planner and the optimal timing of the profit-maximizing firm are the same when the policy has been withdrawn, i.e. $X_0^S = X_0$.

As discussed in this section, an expression for X_1^S cannot be determined. However, similar to proposition 3, it can be shown that X_1^S satisfies the following:

$$\frac{\beta_{11}-\beta_{01}}{\beta_{11}} \cdot A_0(X_1^S)^{\beta_{01}} - \frac{\beta_{11}-1}{\beta_{11}} \cdot \frac{X_1^S(2-\eta Q_1(X_1^S))Q_1^S(X_1^S)}{2(r-\mu)} + (1-\theta)\delta Q_1^S(X_1^S) = 0. \tag{3.31}$$

Despite the above expression may seem different than (3.17) (the expression for X_1) at first sight, the timing thresholds for the social planner and the monopolist are the same. After substitution of (3.24) into the above, and substitution of (3.6), the firm's optimal quantity while the policy is active into (3.17), it follows that both equations are the same. As both equations are the same, $X_1^S = X_1$ does hold. Therefore, any differences in the total surplus created by the social planner and the monopolist are the result of the difference in investment size.

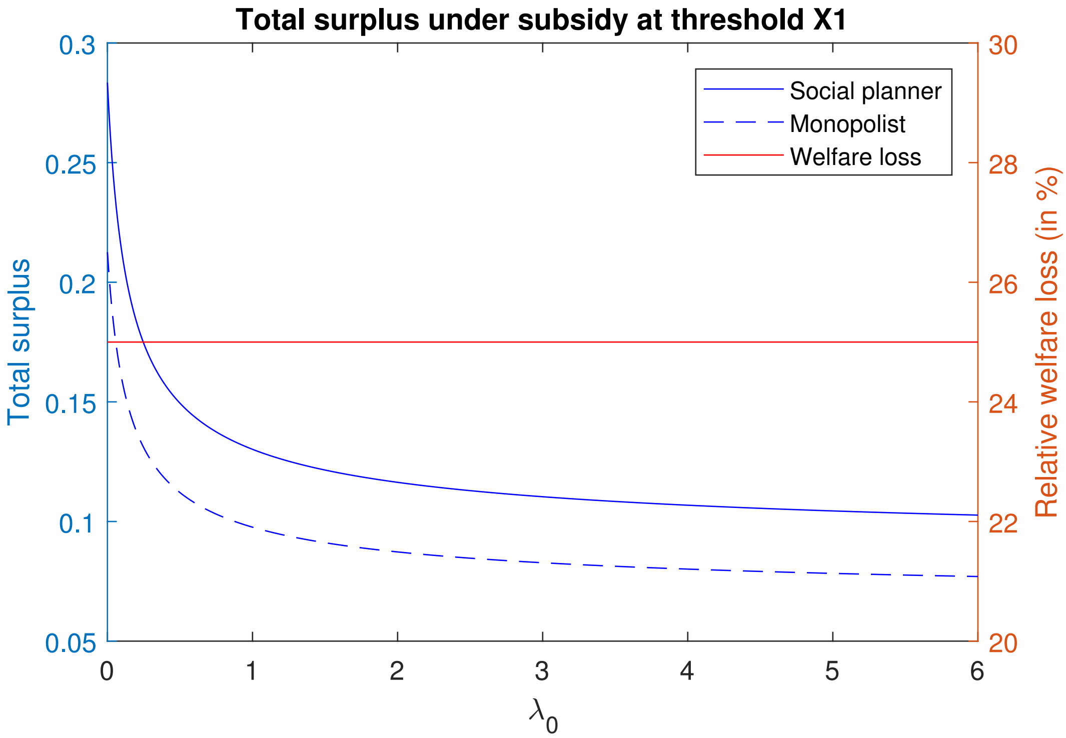

Figure 4: Total surplus under monopolist and social planner for different λ_0

A numerical example is used to compare the total surplus in the situation of a social planner and in the situation of a monopolist[2]. The parameter values are the same as before: $\mu = 0$, $\sigma = 0.1$, $r = 0.05$, $\eta = 0.05$, $\delta = 0.1$ and $\theta = 0.1$. It should be noted that the costs of the tax credit policy are not taken into account, hence the discussion of the remainder of this section focuses on the effects of the policy, and does not do a cost-benefit analysis.

In figure 4, the blue solid line is the total surplus obtained by the social planner when the policy is still in effect and $X = X_1$, but will be withdrawn with probability $\lambda_0 dt$ during time interval dt. Similarly, the dashed blue line is the total surplus under

[2]It has been pointed out the social welfare analysis in standard literature is done a bit differently. Therefore, this analysis will be adapted in a future version.

a monopolistic firm, with the policy still in effect and $X = X_1$. Finally, the solid red line is the relative difference in the social welfare between the social planner and the monopolist.

Looking at figure 4, it is immediately clear that the total surplus under the social planner is larger than the total surplus under the monopolist. This is the result of the difference in goals of the social planner and monopolist, as the former aims to maximize total surplus, and the latter aims to only maximize the producer surplus. The difference in total surplus is completely explained by the difference in installed capacity, as the outcomes for X_1 and X_1^S are the same. Thus, the firm seems to invest too little from a social welfare perspective to keep the price up and make more profit despite it maintains a lower capacity/sells fewer products by producing less. This means that 25 percent of the total surplus is lost as a result of a competitive market. This relative welfare loss does not change when increasing the policy uncertainty, as can be seen by the straight red solid line in figure 4. However, the total surplus decreases when increasing the withdrawal parameter λ_0. Intuitively, increasing λ_0, increases the policy uncertainty and thus harms social welfare.

4 Comparison with stepwise green investment under policy uncertainty (Chronopoulos, Hagspiel and Fleten, 2016)

In this paper the effect of policy uncertainty on invest timing and investment size has been analyzed. Chronopoulos et al. [2016] performs a similar analysis and analyse (sudden) provision and retraction of a subsidy. Despite the main topic of the paper and this paper are the same, before mentioning and comparing conclusions, several differences in the models and assumptions should be discussed.

First and foremost, there is a difference in the type of subsidy that is analyzed. Chronopoulos et al. [2016] analyze a subsidy that provides a fixed premium on top of the *price*, while in this paper the subsidy gives a fixed percentage discount of the sunk investment *cost*. This means the firm's decisions are no longer subject to policy uncertainty after entering the market in this paper' model. Simply put, the firm does not care whether or not the policy is active after it has invested. However, in case of the model by Chronopoulos et al. [2016], the firm's benefits of the subsidy depend on the price. Therefore, the firm's benefits are subject to policy uncertainty even after investing.

Secondly, there is a difference in the way production costs and prices are modeled. Both Chronopoulos et al. [2016] and this paper assumes that all produced demand is sold. This paper assumes the production costs are linear, similar to Huisman and Kort [2015]. Therefore, the marginal production costs are fixed. However, Chronopoulos et al. [2016] use strictly convex production costs. The price in this paper is modeled in such a way that an increase in production leads to a decrease in price. In other words, the firm has some market power. In Chronopoulos et al. [2016], the price follows a geometric Brownian motion and is independent of the size of the production. The firm has no market power and cannot influence the price. This assumption is relaxed in this paper, as equation (3.1) captures the effect of production on price. This effect may have large significance in Chronopoulos et al. [2016]'s model, as the larger the installed production, the lower the price, the *less* the firm's revenue per sold product. However, as the production increases, it does not mean that the firm's total revenue drops. Furthermore, the consumers may profit from the lower price, hence the price subsidy for the firm may lead to a larger consumer surplus. As the effect of relaxing the no market power assumption on the producer surplus is unknown beforehand, the effect of the policy on total welfare is unclear at this point. Therefore, relaxing the no market power assumption in the setting of Chronopoulos et al. [2016] may give new insights on the role of price subsidies. Furthermore, the question how the social welfare is influenced by policy uncertainty in a price subsidy is an interesting question.

This question also shows differences in the scope of Chronopoulos et al. [2016] and this paper. The question regarding the effects of policy uncertainty on social welfare is not addressed in Chronopoulos et al. [2016], and policy recommendations seem to follow from maximizing producer surplus. Of course, for national governments it may be more important to focus on firms in order to attract investment, but the goal of maximizing social welfare may not be accomplished by merely maximizing the investments of firms.

Apart from social welfare, there is also a second difference in the focus/scope of the two analyses. This paper focuses on the investment problem in which the investor can invest at most once, the so-called single or lumpy investment problem. Apart from the single investment problem, Chronopoulos et al. [2016] also analyze a multiple stage/stepwise investment problem. This problem is not analyzed in this paper, hence it is not possible to compare these results. Intuitively, it is expected that if the firm is able to increase production after investing, the flexibility of the firm will increase, as the future investment opportunity will not be (entirely) lost after investing. This suggests that the investment threshold will decrease, i.e. investing sooner will become more attractive, and usually this is combined with installing a smaller capacity. The latter is also rather intuitive as production can also be scaled upwards if market conditions become more beneficial later, but not downwards, as it is not possible to sell production capacity. However, Kort et al. [2010] find that it is not conclusive how uncertainty in market development influences the decision between choosing a single investment strategy or a multiple-stage investment strategy. They find the counter-intuitive result that "higher [market] uncertainty makes the single-stage investment more attractive relative to the more flexible stepwise investment strategy". Therefore, it is also an interesting question whether the outcomes of this paper still are the same when the investor can choose to either do the single investment or implement a stepwise investment strategy.

As the differences between the models and assumptions have been discussed, the conclusions of Chronopoulos et al. [2016] can be discussed and compared with the results of this paper. In the situation in which the policy is active and the probability of withdrawing a price subsidy is low, the authors find an increment in the probability of withdrawing such policy leads to an decrement in the investment threshold. In other words, it becomes more attractive for firms to speed up investment as the policy is still in effect. As shown by Huisman and Kort [2015], earlier investment leads to a decrease in quantity, which is a conclusion also Chronopoulos et al. [2016] draw. This conclusion for small λ is also found in this paper.

Before looking into the conclusions where the probability of change is large, a remark about the analysis by Chronopoulos et al. [2016] should be made. Similarly to this paper, Chronopoulos et al. [2016] let the probability of change follow from a Poisson process with parameter λ. However, the authors seem to imply that $\lambda = 1$ means that change will immediately occur with probability 1, but this is not the case. When $\lambda = x$, it is expected that the next change will occur after $\frac{1}{x}$ time intervals, thus for $\lambda = 1$, the (mathematical) expectation is that the next change will occur after the next time interval. Chronopoulos et al. [2016] mention that - for example - the investment timing threshold in a situation with the subsidy being in effect, at most one change (similar setting as section 3 in this paper) and $\lambda = 1$, is equal to the investment threshold in case of no subsidy and no policy uncertainty. With the correct interpretation of λ, it seems just a coincidence that this result occurs. In fact, what happens for $\lambda = 1$, is expected to happen for $\lambda \to \infty$, as then the probability of change during the next small time interval converges to one. Apart from the interpretation of the results at $\lambda = 1$, their results do not seem incorrect.

When the probability of changing the current situation is relatively large, aforementioned conclusions no longer hold. In case of an active price subsidy and the probability of withdrawal being large, an increment in this probability *increases* the investment threshold. As the subsidy discussed in Chronopoulos et al. [2016] influences the price, the firm's benefits from the policy also depend on the situation after investing. Therefore, if it becomes very likely that the subsidy is withdrawn, the firm postpones investment to avoid getting stuck in a not beneficial situation and as it invests later, the installed capacity is larger. This "postponement effect" does not occur in this paper, as the subsidy in this paper influences the investment cost, thus the firm's benefits after investing are not dependent on whether or not the policy is in effect. In fact, the larger the probability that the tax credit policy will be withdrawn soon, the stronger the incentive to invest now, to take advantage of the discount on the investment costs. As expected, the early investment is done with a small capacity.

In conclusion, from comparing Chronopoulos et al. [2016] and this paper, it follows that the two different subsidies have different effects when the probability that the policy regime changes is high. In case of a price premium subsidy, likely withdrawal leads to a decrease of the incentive to invest now, while the chosen capacity is increased. On the other hand, increasing the withdrawal probability of a tax credit policy when it is relatively high already, increases the incentive to invest now and decreases the chosen capacity. The mentioned effect for the tax credit policy holds irrelevant of the probability of withdrawal, or to put it in terms of the used parameters, it holds for both high and low values of λ_0. For low values of λ_0, i.e. in situations in which policy change is unlikely, the effect of policy uncertainty on investment timing and investment size is similar for the price subsidy and the tax credit.

5 Robustness analysis

In this section, a robustness analysis will be performed. This section will be added in the future and any input from others is encouraged. So far, this section will discuss the following topics:

- In this paper, it is assumed that demand is equal to the chosen production. It may be interesting to see if/how results change is demand is uncertain, e.g. additive or iso-elastic demand. In section 6 this is also mentioned as a future research recommendation and Boonman and Hagspiel [2014] and Wen et al. [2017] are mentioned.
- This paper focuses its analysis on a subsidy on investment cost, similar to Dixit

and Pindyck [1994]. However, also different types of subsidies can be analyzed, such as the price premium analyzed in Chronopoulos et al. [2016].

- This paper focuses on the situation in which an active policy can be withdrawn. However, it may also be relevant to analyze the situation in which no policy is in effect and possibly a new policy can be introduced. Furthermore, it is possible to analyze the situation when the government is able to implement and withdraw a policy infinite times, i.e. the model in Dixit and Pindyck [1994] with capacity choice. Despite this possibly cannot be solved analytically, a numerical example can be used to gain some insights.

6 Conclusions and future research

In the final section, the most important conclusions of this paper are summarized and also policy recommendations are given. This paper concludes with recommendations for future research.

When a policy maker can retract an existing tax credit policy, increasing the likelihood of policy withdrawal gives the investor an incentive to invest sooner in order still receive the discount on investment. As shown in section 3, investing earlier is done with a smaller capacity. At first sight it may be surprising, but the firm's optimal investment size under subsidy with the price at the threshold value X_1 is *smaller* than the firm's optimal investment size without subsidy with the price at the threshold value X_0. Since investment is done earlier with subsidy than without, i.e. $X_1 < X_0$, it results in $Q_1(X_1) < Q_0(X_0)$ and the firm optimally produces less without subsidy. Apart from the optimal decisions for the firm, this paper also analyzed the social welfare maximization problem of the social planner. It turns out that the optimal timing of the profit-maximizing firm and the welfare maximizing social planner are equal. However, the firm underinvests from a social perspective, as the optimal investment size for the social planner is twice the optimal investment size for the firm. As a result, welfare is lost. In the numerical example the relative welfare loss is equal to 25%, independently of the policy uncertainty (expressed by the withdrawal probability parameter λ_0).

The surprising result that the firm's optimal investment size under subsidy at the threshold price is *smaller* than the firm's optimal investment size without subsidy at the threshold price implies an important policy recommendations. For example consider a government aiming to have more green energy projects to reach environmental targets. Green energy projects usually have long-term goals and high investment costs. If a government prefers to have large-scale investments on the long-run over small investments on short-term, it is best never to withdraw an existing policy (i.e. $\lambda_0 = 0$). In the scenario in which the policy is active and has unconditional support, the firm not given any incentive to speed up investment, which would be the case if there is some chance the policy will be withdrawn in the future. If the social planner aims to have more investments now, but hardly any investments later, it is best to threat to withdraw the subsidy immediately. As a result of this kind of policy making, firms that want to make use of the beneficial market conditions need to invest within this small time frame.

One of the objections for the policy maker to withdraw a tax credit policy to delay investment, is that the uncertainty in withdrawal actually speeds up investment. Therefore, the policy maker has reason to act fast without uncertainty, as in this case, investment is not delayed too much as the policy is implemented early. However, in the political landscape of today, in which decisions and laws take time to be finalized, this may not be an option. An alternative solution to avoid this delay in investment as result of policy uncertainty is to use a price premium (like in the model by Chronopoulos et al. [2016]) instead of a tax credit. As the promise of withdrawal of a price premium discourages firms to invest now, even before the actual withdrawal of the policy, this seems to be in accordance with the goals of a government when retracting a policy that dissuades firms to do certain investments.

Even though this paper gives some insights on the effect of policy uncertainty on the investment timing and investment size and performs a social welfare analysis, this work has several interesting extensions for future research. A first example is finding optimal values for θ and λ_0 such that the optimal decisions for firm and social planner align. In other words, the possible subsidy withdrawal is chosen such that the firm itself optimizes social welfare by just trying to maximize its profits. This is the actual approach of a government, as it cannot takeover firms and take decisions for them in order to maximize social welfare, but it can only influence the profit-maximizing firms' decisions by implementing a subsidy and in this way encourage decisions that improve social welfare. However, as mentioned in section 3, the costs of the subsidy should also be taken into account in order to have the complete analysis of a policy implementation.

Secondly, the (implicit) assumption that there is no competition on the market can be relaxed. In this paper, the possible investor is a monopolist that does not face the threat of other entrants. Allowing for competition, by for example implementing a duopoly setting and doing a game-theoretic analysis, will alter the decisions made by the firms. As Huisman and Kort [2015] show, the fear of another entrant gives incentive to invest at a larger scale. The first entrant, or incumbent, can apply an entry deterrence strategy. By overinvesting, the incumbent delays the investment of the second investor (entrant) and the entrant will invest in a smaller capacity. Huisman and Kort [2015] also find that greater uncertainty makes entry deterrence more likely. This implies that entry deterrence strategies are even more likely to occur in case of situations of policy uncertainty. This analysis can be extended to a situation in which competitors are asymmetric. In that case certain policy measures may increase or decrease the competitive advantage of one firm over the other (Chronopoulos et al. [2016]).

Apart from introducing competition, it may also be interesting to give the investor the option to choose between a single lumpy investment and multiple stage investment. Kort et al. [2010] find that lumpy investment has a lower total cost, but stepwise investment gives more flexibility by letting the firm choose the timing individually for each stage. However, they also find the counter-intuitive result that the higher market uncertainty, the more attractive the single-stage investment is compared to the stepwise investment strategy. As flexibility of the firm is important in cases with policy uncertainty, analyzing the models in this paper with the option for the investor to implement a multiple stage investment strategy can give new insights. Chronopoulos et al. [2016] is an example of an analysis of the multiple stage investment strategy under policy uncertainty and they conclude that it is preferred by the firm over the lumpy investment strategy, as result of the increased flexibility with the multiple stage investment strategy.

A fourth example of future research consists of relaxing two (implicit) assumptions made when modeling the demand. In this paper, it is assumed that the demand is equal to the capacity and that the producer always produces up to capacity to simplify the analysis. In reality, the producer can keep stock and the consumer's demand is not equal to the production, but follows some function dependent on the price. Boonman and Hagspiel [2014] find that different inverse demand functions lead to different investment decisions, hence the assumption on the structural form of the demand function may change certain outcomes, also in the models discussed in this paper. In fact, Wen et al. [2017] study how the subsidy support affects the investment decision of a monopolist under uncertainty and analyzes the implications for social welfare. They also analyze differences when changing the demand function, and draw different conclusions for different demand functions. They find that the unconditional subsidy regulation cannot align the profit and social planner's investment decisions under linear demand structure. However, for non-linear iso-elastic demand, it is possible to implement an unconditional subsidy to align profit maximizing and social optimal investment decisions, depending on the form of subsidy regulations.

Fifthly, the assumptions regarding the modeling of the price can be relaxed. In this paper, the price follows a geometric Brownian motion, which means that the price can take negative values with a positive probability. Furthermore, the output price in reality is not necessarily a geometric Brownian motion and may depend on other information we have, such as prices of substitutes, or the price of relevant resources. Possibly, the price may be a different stochastic function such as a mean-reverting process or arithmetic Brownian motion as Chronopoulos et al. [2016] suggest. Dias and Rocha [1999] and Cartea and Figueroa [2005] have alternative approaches to model the price to take into account certain economic logic. Dias and Rocha [1999] model changes in oil prices as a sum of a mean reverting part (prices tend to stick to a certain mean), Brownian motion (upward/downward trend in the price) and poison jump process with random jump sizes (external shocks). Cartea and Figueroa [2005] use a similar model to analyse electricity spot prices, but also take into account seasonality.

Apart from taking other effects that (may) change the price into account, one can also change the demand uncertainty parameter σ as suggested by Huisman and Kort [2015] or change the way the the policy uncertainty is modelled. In case of the latter suggestion, it seems realistic that the policy uncertainty decreases over time as certain regulations are rejected or supported by certain politicians or political parties. As a result, the λ_0 converges to either zero or infinity over time. Therefore, over time, the situation is either identical to the situation without policy uncertainty, or the firms know for certain that a change will occur very soon. The firm will invest immediately if the government is planning to withdraw a subsidy in the very near future.

The final recommendation for future research is related to the fact that the lifetime of a firm after investment is assumed to be infinite in this paper. Gryglewicz et al. [2008] relax this assumption as in nowadays fast-changing environment, investment projects are more likely to have a finite lifetime. The authors find that investments may be accelerated when uncertainty increases if the project has a finite lifetime. Gryglewicz et al. [2008] show that this particularly happens at low levels of uncertainty and when project life is short. The conclusion of Gryglewicz et al. [2008] that the investment-uncertainty relationship is positive at low levels of uncertainty and negative at high levels, is surprising compared to other research. Also in this paper, as the lifespan of the project is assumed to be infinite, uncertainty delays investment and thus it is an interesting question whether the conclusions of this paper still hold for projects with a finite lifespan.

A Proofs of theorems and propositions

A.1 Proof of proposition 1

Proof of proposition (1). This proof shows that the expression for $Q_1^*(X)$ (expression (3.6)) holds for $X > X_1$. The proof that equation (3.5) is correct for $X > X_0$ follows the same steps.

The optimal quantity Q_1^* maximizes $V_1(Q_1, X)$ for $X > X_1$. Since $\frac{d^2V_1}{dQ^2} = -\frac{2\eta X}{r-\mu} < 0$ for $X > 0$, it holds that $V_1(Q_1, X)$ is concave in Q_1 as $X > X_1 > 0$. Therefore the first order condition, $\frac{dV_1}{dQ} = 0$, can be applied here.

$$\begin{aligned} \frac{dV_1}{dQ} = 0 \Leftrightarrow & \frac{X(1-2\eta Q)}{r-\mu} - (1-\theta)\delta = 0 \\ \Leftrightarrow & Q_1^*(X) = \frac{1}{2\eta}(1 - \frac{(1-\theta)\delta(r-\mu)}{X}). \end{aligned}$$

□

A.2 Proof of proposition 2

Proof of proposition (2). Substituting (3.11) into (3.13) yields:

$$\begin{aligned} & \frac{\beta_{01}}{X_0}\left(\frac{X_0(1-\eta Q_0^*(X_0))Q_0^*(X_0)}{r-\mu} - \delta Q_0^*(X_0)\right) = \frac{(1-\eta Q_0^*(X_0))Q_0^*(X_0)}{r-\mu} \\ \Leftrightarrow & (\beta_{01}-1)\cdot\frac{(1-\eta Q_0^*(X_0))Q_0^*(X_0)}{r-\mu} = \frac{\beta_{01}}{X_0}\delta Q_0^*(X_0) \\ \Leftrightarrow & \frac{\beta_{01}-1}{\beta_{01}}\cdot\frac{1-\eta Q_0^*(X_0)}{\delta(r-\mu)} = \frac{1}{X_0} \\ \Leftrightarrow & X_0 = \frac{\beta_{01}}{\beta_{01}-1}\cdot\frac{\delta(r-\mu)}{1-\eta Q_0^*(X_0)}. \end{aligned}$$

Using equation (3.5) for $Q_0^*(X_0)$ in the above expression yields:

$$\begin{aligned} X_0 = \frac{\beta_{01}}{\beta_{01}-1}\cdot\frac{\delta(r-\mu)}{1-\eta Q_0^*(X_0)} \Leftrightarrow & X_0 = \frac{\beta_{01}}{\beta_{01}-1}\cdot\frac{\delta(r-\mu)}{1-\frac{1}{2}(1-\frac{\delta(r-\mu)}{X_0})} \\ \Leftrightarrow & X_0 = \frac{\beta_{01}}{\beta_{01}-1}\cdot\delta(r-\mu)\frac{2X_0}{X_0+\delta(r-\mu))} \\ \Leftrightarrow & \frac{1}{2}(X_0+\delta(r-\mu)) = \frac{\beta_{01}}{\beta_{01}-1}\cdot\delta(r-\mu) \\ \Leftrightarrow & X_0 = \frac{2\beta_{01}}{\beta_{01}-1}\cdot\delta(r-\mu) - \delta(r-\mu) \\ \Leftrightarrow & X_0 = \frac{\beta_{01}+1}{\beta_{01}-1}\cdot\delta(r-\mu). \end{aligned}$$

Substituting the latter expression for X_0 into (3.5) yields an expression for the optimal capacity when the policy is not in effect.

$$\begin{aligned} Q_0^*(X_0) &= \frac{1}{2\eta}(1 - \frac{\delta(r-\mu)}{\frac{\beta_{01}+1}{\beta_{01}-1}\cdot\delta(r-\mu)}) \\ &= \frac{1}{2\eta}(1 - \frac{\beta_{01}-1}{\beta_{01}+1}) \\ &= [\eta(\beta_{01}+1)]^{-1}. \end{aligned}$$

Rewriting equation (3.13) and subsequently substituting the derived expressions for X_0 and $Q_0(X_0)$ gives:

$$\begin{aligned} A_0 &= \frac{X_0^{1-\beta_{01}}}{\beta_{01}} \cdot \frac{(1-\eta Q_0^*(X_0))Q_0^*(X_0)}{r-\mu} \\ &= \frac{1}{\beta_{01}}(\frac{\beta_{01}+1}{\beta_{01}-1}\cdot\delta(r-\mu))^{1-\beta_{01}} \cdot \frac{1}{r-\mu}(1-(\beta_{01}+1)^{-1})\cdot[\eta(\beta_{01}+1)]^{-1}) \\ &= \frac{1}{\beta_{01}}(\frac{\beta_{01}+1}{\beta_{01}-1}\cdot\delta(r-\mu))^{-\beta_{01}}(\frac{\beta_{01}+1}{\beta_{01}-1}\cdot\delta(r-\mu))\cdot\frac{1}{r-\mu}\cdot\frac{\beta_{01}}{\beta_{01}+1}\cdot\frac{1}{\eta(\beta_{01}+1)}) \\ &= (\frac{\beta_{01}+1}{\beta_{01}-1}\cdot\delta(r-\mu))^{-\beta_{01}}\frac{\delta}{\beta_{01}-1}\cdot\frac{1}{\eta(\beta_{01}+1)}) \\ &= \frac{\delta}{\eta(\beta_{01}^2-1)}\cdot\left(\frac{\beta_{01}+1}{\beta_{01}-1}\delta(r-\mu)\right)^{-\beta_{01}}. \end{aligned}$$

□

A.3 Proof of proposition 3

Proof of proposition (3). Subtracting $\frac{X_1}{\beta_{11}}$ times equation (3.12) from (3.14) yields:

$$\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot A_0 X_1^{\beta_{01}} = \frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1(1-\eta Q_1^*(X_1))Q_1^*(X_1)}{r-\mu} - (1-\theta)\delta Q_1^*(X_1).$$

Rearranging terms in the above leads to:

$$\frac{\beta_{11}-\beta_{01}}{\beta_{11}}\cdot A_0 X_1^{\beta_{01}} - \frac{\beta_{11}-1}{\beta_{11}}\cdot\frac{X_1(1-\eta Q_1^*(X_1))Q_1^*(X_1)}{r-\mu} + (1-\theta)\delta Q_1^*(X_1) = 0.$$

□

References

Bloomberg. Agl delays australian wind farm on government policy concern, 2013. URL `https://www.bloomberg.com/news/articles/2013-07-11/agl-delays-australian-wind-farm-on-government-policy-uncertainty`.

Hettie J. Boonman and Verena Hagspiel. Sensitivity of demand function choice in a strategic real options context. *Working paper*, 2014.

Álvaro Cartea and Marcelo G. Figueroa. Pricing in electricity markets: A mean reverting jump diffusion model with seasonality. *Applied Mathematical Finance*, 12(4):313–335, 2005. doi: http://dx.doi.org/10.1080/13504860500117503.

Michail Chronopoulos, Verena Hagspiel, and Stein-Erik Fleten. Stepwise green investment under policy uncertainty. *International Association for Energy Economics*, 37 (4):87–108, 2016. doi: http://dx.doi.org/10.1016/j.ejor.2012.01.017.

Marco Antônio Guimarães Dias and Katia Maria Carlos Rocha. Petroleum concessions with extendible options using mean reversion with jumps to model oil prices. Technical report, Institute for Applied Economic Research Working Paper 620, 1999.

Avinash K. Dixit and Robert S. Pindyck. *Investment under uncertainty.* Princeton university press, 1994.

Shawn Donnan. Policy uncertainty threatens trade growth, says world bank, 2017. URL `https://www.ft.com/content/9d49b092-f859-11e6-9516-2d969e0d3b65`.

European Commission. Renewable energy directive, 2017. URL `https:/ec.europa.eu/energy/en/topics/renewable-energy/renewable-energy-directive`.

Fredrik Finjord, Marius Tangen, Maria Lavrutich, and Verena Hagspiel. Real options valuation of wind energy investments in norway and sweden: A case study. Working paper, 2017.

Sebastian Gryglewicz, Kuno J.M. Huisman, and Peter M. Kort. Finite project life and uncertainty effects on investment. *Journal of Economic Dynamics and Control*, 32 (7), 2008. doi: https://doi.org/10.1016/j.jedc.2007.10.003.

Kuno J.M. Huisman and Peter M. Kort. Strategic capacity investment under uncertainty. *RAND Journal of Economics*, 46(2):376–408, 2015. doi: http://dx.doi.org/10.1111/1756-2171.12089.

Gavin Jackson, Gemma Tetlow, and Mehreen Khan. Brexit uncertainty is holding back investment, mark carney warns, 2017. URL `https://www.ft.com/content/23fd12b2-783b-11e7-90c0-90a9d1bc9691`.

Song Jung-a and Emma Dunkley. Bitcoin slips as south korea threatens to shut exchanges, 2017. URL `https://www.ft.com/content/75e13894-eba7-11e7-bd17-521324c81e23`.

Peter M. Kort, Pauli Murto, and Grzegorz and Pawlina. Uncertainty and stepwise investment. *European Journal of Operational Research*, 202(1):196–203, 2010. doi: https://doi.org/10.1016/j.ejor.2009.05.027.

Grzegorz Pawlina and Peter M. Kort. Investment under uncertainty and policy change. *Journal of Economic Dynamics and Control*, 29(7):1193–1209, 2005. doi: http://dx.doi.org/10.1016/j.jedc.2004.07.002.

REN21. Renewables 2016 global status report, 2016. URL `http://www.ren21.net/wp-content/uploads/2016/10/REN21_GSR2016_KeyFindings_en_10.pdf`.

Xingang Wen, Verena Hagspiel, and Peter M. Kort. Subsidized capacity investment under uncertainty. Working paper, 2017.