Debt financing with heterogeneous beliefs

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Abstract

We develop a real options model for a firm borrowing using a loan commitment and derive the optimal timing of investment and optimal capital structure in the presence of heterogeneous beliefs about the volatility of assets between debt and equity holders, equity financing costs and loan commitment fees. We show that unfavorable beliefs from debt holders about the volatility causes a delay in investment, higher credit spreads and a decrease in debt capacity and the option value to invest. On the positive side, unfavorable debt holders' beliefs reduce the agency costs between debt and equity holders associated with the timing of investment. High equity investment financing costs result in an increase in credit spreads and agency costs, while high loan commitment fees accelerate investment, however, they reduce debt financing and result in larger agency costs. We also present a multi-stage model with partial drawdowns and provide implications relating to the effect of expected time to new investments on commitment levels and fees.

1. Introduction

Differences in opinion about the prospects and risk of a firm by its claimholders can have important implications about a firm's investment and financing decisions (e.g., see early work of Stiglitz, 1972 and Thakor and Whited, 2010). Many small size or private firms face high equity financing costs or lack access to equity markets altogether forcing them to rely heavily on bank financing. Besides exasperating problems arising due to differences in opinion, resorting to bank financing also exposes these firms to agency conflicts between equity and debt holders over the optimal timing of investment (e.g., Leland, 1998 and Mauer and Sarkar, 2005). The purpose of this paper is the development of a real options framework with heterogeneous beliefs between debt and equity holders, external investment equity financing costs (resulting in equity financing constraints) and loan commitment fees. We use this framework to analyse the effect of heterogenous beliefs on firm value, optimal capital structure, investment and default timing, credit spreads and the level of agency costs.

Our focus is on loan commitments for three main reasons. First, Ergungor (2001), Saunders and Steffen (2016) and Chava and Jarrow (2008) show that more than 80% of all commercial and industrial lending is in the form of commitments. Secondly, in a loan commitment a bank pre-commits on the terms of a loan at origination where discrepancies in beliefs about firms' prospects may be high. Thirdly, we provide a direct measure of total commitment fees in order to contribute to studies that empirically attempt to estimate the total costs of loan financing including the fees for embedded options (see Berg et al., 2016).

Early attempts to price loan commitments using option pricing theory include Thakor et al. (1981) who value loan commitments as European put options. A more recent option pricing approach to value loan commitments is provided in Mauer and Sarkar (2005). Egami (2009) extends this framework to include the risk of default prior to exercising an expansion option, as well as, time-to-build while Sarkar and Zhang (2016) analyze performance-sensitive debt. Our model extends these frameworks to include heterogeneous beliefs between debt and equity holders about the volatility of assets, commitment fees for unused debt commitment and equity financing costs. In an extended version of our basic model (presented in Section 4 of our paper) we allow for default risk prior to exercising new investments and present a multi-stage model with partial drawdowns. Using this multistage framework we examine the impact of expected passage time until new investments on commitment levels and fees and further explore the impact of heterogenous beliefs. Our paper is related to a large literature on optimal capital structure using a contingent claims approach (see for example, Leland, 1994).[1] Our modeling of heterogeneous beliefs is conceptually related to Thakor and Whited (2010) (see also Dittmar and Thakor, 2007) who analyze manager-shareholder disagreement. In their context, managers and shareholders draw from different but potentially correlated priors and the measure of disagreement is determined by the correlation of priors. In our context, on the other hand, differences in beliefs is more directly determined by disagreement about the volatility of assets. Furthermore, our focus is on differences in beliefs between equity and debt holders while their focus is differences in beliefs between managers and shareholders. Other methodologically related work is Hackbarth (2008) (see also Hackbarth, 2009) who studies managerial traits and their impact on capital structure and

[1] Other related context is Hirth and Uhrig-Homburg (2010(a)) who highlight the important role of liquidity in mitigating agency conflicts of over and under investment and Hackbarth and Mauer (2012) who explain the important role of priority rules in the case of multiple debt issues.

agency costs using a contingent claims model. However, his frameworks explores managerial optimism while our model (similarly to Thakor and Whited, 2010) does not consider which group of investors holds “correct” beliefs. [2] Dumas et al. (2017) have analyzed differences in opinion between investors supporting the view that differences in opinion may enhance our understanding of financial phenomena. Besides the methodological contributions outlined above, we provide a number of new findings which are summarized below.

Firstly, we show that for more unfavourable debtholders beliefs (i.e., the higher their perceived volatility of assets), the lower firm value and optimal leverage. Unfavourable debt holder beliefs also induce a delay in firm investment and an increase in credit spreads. *Secondly*, high equity investment financing costs do not significantly alter investment policy because the firm resorts to debt financing. However, the need to use more debt leads to an increase in leverage, earlier default following investment and an increase in credit spreads. *Thirdly*, higher loan commitment fees induce firms to invest earlier to avoid incurring fees on unused debt capacity, except for the case where debt holders beliefs are highly unfavourable (in which case there is a delay in investment). Higher loan commitment fees also result in lower leverage ratios and credit spreads with this effect intensified for more unfavourable debt holder beliefs. Furthermore, unless the investment option is out of the money, higher commitment fees increase the probability of investment and reduce the probability of default prior to investment. In the extended model, we show that accounting for default prior to investment lowers the total expected

[2] Our work is thus also different from papers analyzing asymmetric information and signaling as an approach to reveal information. For example, Flannery (1985) discusses the use of debt maturity by equity holders to signal their credit quality and Brealey et al. (1977) discuss the use of equity to signal better prospects for the firm’s investments.

commitment fees incurred by firms. However, when the expected passage time of investment increases (e.g., due to an industry environment with limited timing) the loan commitment levels and fees increase significantly. *Fourthly*, we provide new implications about the agency conflicts between equity and debt holders showing that more unfavourable debt holders beliefs result in equity holders delaying investment and default, thus making policies more aligned with first-best optimal policies (i.e., reducing overinvestment and early default). For high equity investment financing costs, agency costs exhibit an increase when debt holders have unfavourable beliefs since the firm will then need to resort to expensive debt. Loan commitment fees cause a substantial increase in the agency costs of debt due to distortion on firms' optimal policies.

The paper presents models with increasing complexity by gradually incorporating realistic features. The rest of the paper is organized as follows. Section 2 describes our basic setup and Section 3 provides numerical sensitivity results using this basic setup. Section 4 extends our model, first to include default risk prior to investment, and then to present a multi-stage model with partial drawdowns. The last section concludes.

2. The basic setup

In this section we present an extension of Mauer and Sarkar (2005) to account for heterogeneous beliefs between debt and equity holders, equity financing costs and loan commitment fees. In this setup we assume away default risk prior to investment which is studied in Section 4 of the paper.

2.1. Contingent claims differential equation with heterogeneous beliefs

We assume that the value of firm's unlevered assets V follows the following stochastic process:

$$\frac{dV}{V} = \mu dt + \sigma_i dZ, i = E, D \tag{1}$$

where μ is the real drift (expected rate of change or capital gains) of the project, σ_i is the volatility of the project value and dZ is a standard Weiner process. Due to heterogeneous beliefs, the volatility perceived by debt holders (σ_D) may be different than that of equity holders (σ_E). Furthermore, the project pays constant cash flows $\delta V dt$ per interval dt once the project is initiated (following investment).[3]

Let $H(V)$ denote the value of a contingent claim on the value of the project V. For given beliefs, one can follow standard arguments in real options pricing literature (see for example, Dixit and Pindyck, 1994) to show that the contingent claim satisfies the following differential equation:

$$\frac{1}{2} H_{VV} \sigma_i^2 V^2 + (r - \delta) V H_V - rH = 0, \quad i = E, D \tag{2}$$

Note that the differential equation depends on which group of investors (equity or debt holders) beliefs is used which results in alternative *perceived* values for different claims. Similar to Thakor and Whited (2010) our model does not consider which group of investors has correct estimates. We assume however that each groups estimates is common knowledge (i.e., investors share information about their estimates when negotiating a new loan). Therefore, each claim holder will use their own estimates to value their claims accounting for the effect of the other group beliefs.

The general solution of the above claim $H(V)$ can be expressed as linear combination of two independent solutions of the form AV^β as follows (see Dixit and Pindyck, 1994 p.142):

$$H(V) = A_1^H V^{\beta_1^i} + A_2^H V^{\beta_2^i}, \qquad i = E, D \tag{3}$$

Solutions for $\beta_1^i, i = E, D$ are obtained by trying AV^β in the differential equation which results in the following fundamental quadratic equation:

$$Q = \frac{1}{2} \sigma_i^2 \beta(\beta - 1) + (r - \delta)\beta - r = 0, \quad i = E, D \tag{4}$$

[3] One can also examine heterogenous beliefs regarding the project cash flows δ. Most implications are similar as the case with volatility and are thus not presented for brevity of exposition.

The two roots of the quadratic (for equity and debt holders' beliefs) are:

$$\beta_1^i = \frac{1}{2} - \frac{(r-\delta)}{\sigma_i^2} + \sqrt{\left(\frac{(r-\delta)}{\sigma_i^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_i^2}} > 1, \quad i = E, D \tag{5}$$

$$\beta_2^i = \frac{1}{2} - \frac{(r-\delta)}{\sigma_i^2} - \sqrt{\left(\frac{(r-\delta)}{\sigma_i^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_i^2}} < 0, \quad i = E, D$$

We also derive the following relationships[4]:

$$\frac{\partial \beta_1^i}{\partial \sigma_i} < 0, \frac{\partial \beta_2^i}{\partial \sigma_i} > 0, \ i = E, D \tag{6}$$

Parameters A_1^H and A_2^H are constants to be determined by relevant boundary conditions alongside particular solutions depending on the contingent claim (equity, debt or firm value).

2.2. The value of equity after investment

Equity value $E(V)$ after capital investment X satisfies differential equation (2). Equity holders gain cash inflows δV and pay the tax deductible at a corporate tax rate τ coupon R to debt holders. Thus, following investment, equity value $E(V)$ satisfies the following partial differential equation:

$$\frac{1}{2} E_{VV} \sigma_E^2 V^2 + (r-\delta) V E_V - rE + \delta V - (1-\tau) R = 0 \tag{7}$$

Note that the differential equation uses equity holders' estimate of volatility. The solution for $E(V)$ is of the following form:

$$E(V) = V - (1-\tau)\frac{R}{r} + A_1^E V^{\beta_1^E} + A_2^E V^{\beta_2^E} \tag{8}$$

where $\beta_i^E, i = 1,2$ are defined in equations (5) above and the constants $A_i^E, i = 1,2$ are determined below by applying the following boundary and smooth-pasting conditions:

[4] To prove the above relationships we follow Dixit and Pindyck (1994, pp.142-144).Proofs available upon request.

$$E(V_B) = 0 \tag{9}$$

$$\frac{\partial E}{\partial V}|_{V=V_B} = 0 \tag{10}$$

Applying condition (9) results in $A_1^E = 0$ and $A_2^E = \left((1-\tau)\frac{R}{r} - V_B\right)\left(\frac{1}{V_B}\right)^{\beta_2^E}$.

Using these results and applying the smooth-pasting condition defined in equation (10) results in the optimal default trigger:

$$V_B = \frac{-\beta_2^E(1-\tau)}{1-\beta_2^E}\frac{R}{r} \tag{11}$$

The value of equity can then be written as follows:

$$E(V) = V - (1-\tau)\frac{R}{r} + \left((1-\tau)\frac{R}{r} - V_B\right)\left(\frac{V}{V_B}\right)^{\beta_2^E} \tag{12}$$

2.3. Debt after investment

Following investment, the value of debt $D(V)$ satisfies the differential equation (2) and includes the coupon R received each period:

$$\frac{1}{2}D_{VV}\sigma_D^2V^2 + (r-\delta)VD_V - rD + R = 0 \tag{13}$$

The general solution for debt is of the following form:

$$D(V) = \frac{R}{r} + A_1^D V^{\beta_1^D} + A_2^D V^{\beta_2^D} \tag{14}$$

Debt value satisfies the following boundary conditions:

$$\lim_{V\to\infty} D(V) = \frac{R}{r} \tag{15}$$

$$D(V_B) = (1-b)V_B \tag{16}$$

Applying equation (15) to the general solution of equation (14) implies that $A_1^D = 0$. From equation (16) we also obtain that $A_2^D = \left((1-b)V_B - \frac{R}{r}\right)\left(\frac{1}{V_B}\right)^{\beta_2^D}$.

Replacing these results into the general solution we obtain:

$$D(V) = \frac{R}{r} + \left((1-b)V_B - \frac{R}{r}\right)\left(\frac{V}{V_B}\right)^{\beta_2^D} \tag{17}$$

We note that debt holders use their perceived volatility which affects the probability of bankruptcy (through the auxiliary parameter β_2^D), as well as, equity holders beliefs regarding the determination of optimal default trigger V_B (see equation 11).

2.4. Investment option and agency conflicts with heterogeneous beliefs

We assume that equity holders hold an investment option and arrange for a loan commitment with terms defined at $t = 0$ to obtain debt financing K at the time of investment. In this setting, agency conflicts arise since the loan commitment "pre-arranges" debt financing terms allowing the firm to borrow on a future date. Thus, as in Mauer and Sarkar (2005), once the debt commitment is in place, equity holders have an incentive to exercise the investment option early (i.e., overinvest). We extend Mauer and Sarkar (2005) to include proportional per period costs c for unused loan commitment (see Berg et al., 2016) and equity financing investment costs. In this section we assume that there is no risk of default prior to exercising the investment option. We relax this assumption in Section 4.

The value of the firm's option to invest $F(V)$ which is owned by equity holders follows the following differential equation:

$$\frac{1}{2}F_{VV}\sigma_F^2V^2 + (r-\delta)VF_V - rF - cK = 0 \tag{18}$$

In equation (18) we have included a flow of proportional costs c incurred on unused debt capacity. At the investment trigger the fees cease to apply and thus we will make the necessary adjustment using appropriate boundary conditions at the investment trigger. Also, note that equity holders use their estimate of volatility (σ_E) when determining the value of the option. However, as we will shortly see, the value of this option depends on debt financing which in turn is affected by debt holders' beliefs.

The boundary conditions that need to be satisfied for the option to invest are the following:

$$F(0) = 0 \tag{19}$$

$$F(V_I) = E(V_I) + K - X - \varphi(X-K)1_{X>K} + \frac{cK}{r} \tag{20}$$

φ denotes proportional equity investment financing costs that need to be incurred by equity holders to finance the shortage arising when debt is not adequate to cover the full cost of the capital investment. Hence, equity financing costs are paid only when the investment cost exceeds debt financing ($X > K$), otherwise they are zero.[5] In equation (20) the last term captures equity holders savings in loan commitment fees when the firm invests (which is a perpetuity of costs that would be paid if debt capacity remained unused thereon).

The general solution for the option (firm) value $F(V)$ including the particular solution arising from the flow of payments due to unused loan commitment is:

$$F(V) = -\frac{cK}{r} + A_1^F V^{\beta_1^E} + A_2^F V^{\beta_2^E} \tag{21}$$

Using the boundary condition in equation (19) we obtain that $A_2^F = 0$. Using the boundary condition in equation (20) we also obtain that $A_1^F = \left(E(V_I) + K - X - \varphi(X-K)1_{X>K} + \frac{cK}{r}\right)\left(\frac{1}{V_I}\right)^{\beta_1^E}$

Thus, firm value is:

$$F(V) = -\frac{cK}{r} + \left(E(V_I) + K - X - \varphi(X-K)1_{X>K} + \frac{cK}{r}\right)\left(\frac{V}{V_I}\right)^{\beta_1^E} \tag{22}$$

The smooth-pasting condition for optimization of the investment trigger is $\frac{\partial F}{\partial V}|_{V=V_I} = \frac{\partial E}{\partial V}|_{V=V_I}$ leading to the following equation for determining the optimal investment threshold V_I:

$$1 + \beta_2^E\left((1-\tau)\frac{R}{r} - V_B\right)\left(\frac{V_I}{V_B}\right)^{\beta_2^E}\left(\frac{1}{V_I}\right) - \beta_1^E\left(\frac{1}{V_I}\right)\left(E(V_I) + K - X - \varphi(X-K)1_{X>K} + \frac{cK}{r}\right) = 0 \tag{23}$$

[5] As in Mauer and Sarkar (2005) the level of debt financing may actually exceed the investment cost. For better comparability of our results with this model we do not impose a constraint on the level of debt financing. For an analysis of the impact of debt constraints in this context see Koussis and Martzoukos (2012).

Rational debt holders determine K by using equation (17), thus debt value is obtained as follows:

$$K = D(V_I) = \frac{R}{r} + \left((1-b)V_B - \frac{R}{r}\right)\left(\frac{V_I}{V_B}\right)^{\beta_2^D} \tag{24}$$

To optimize the capital structure, we run a search for the optimal coupon (based on a wide and dense coupon grid), where for each coupon we find V_I that satisfies the smooth-pasting condition in equation (23). We then pick the coupon that maximizes overall firm (option) value $F(V)$ (equation (22)). Similarly to Mauer and Sarkar (2005) we refer to this as the second-best solution since the optimization condition only caters for the interests of equity holders.

In order to obtain a measure of agency costs we also calculate the first-best solution. For the first-best solution we set $c = 0$ since in this case debt is determined at the investment trigger (i.e., there is no loan commitment and hence no unused capacity). Following similar steps as the ones above, we obtain first-best firm value as follows:

$$F(V) = \left(E(V_I) + D(V_I) - X - \varphi\left(X - D(V_I)\right)1_{X>K}\right)\left(\frac{V}{V_I}\right)^{\beta_1^E} \tag{25}$$

In the first-best solution equity holders' cater for the interests of debt holders, hence maximizing total value of the firm by applying the smooth-pasting condition $\frac{\partial F}{\partial V}|_{V=V_I} = \frac{\partial V^L}{\partial V}|_{V=V_I}$, where $V^L = E + D$. By applying this condition we get the following equation for the optimal investment trigger:

$$1 + \beta_2^E\left((1-\tau)\frac{R}{r} - V_B\right)\left(\frac{V_I}{V_B}\right)^{\beta_2^E}\left(\frac{1}{V_I}\right) + \beta_2^D\left((1-b)V_B - \frac{R}{r}\right)\left(\frac{V_I}{V_B}\right)^{\beta_2^D}\left(\frac{1}{V_I}\right)(1 + \varphi 1_{X>K}) - \beta_1^E\left(\frac{1}{V_I}\right)(E(V_I) + K - X - \varphi(X-K)1_{X>K}) = 0 \tag{26}$$

The first-best solution is found by running a search for the optimal coupon that satisfies the smooth-pasting condition equation (26) for V_I and maximizes overall firm (option) value $F(V)$ in equation (25). With equal beliefs between debt and equity holders $\beta_2^E = \beta_2^D$ and when loan commitment and equity financing costs are zero ($\varphi = c = 0$) our solutions contain the solution of Mauer and Sarkar (2005) as a special case.

2.5. Commitment fees, breakdown of firm value and agency costs

In this section we provide an estimate of the total commitment fees and also further insights about the equity holders' optimal decisions using a breakdown of firm value. The total expected present value of costs $T(V)$ paid for commitment fees for unused loan are calculated as follows:

$$T(V) = \frac{cD(V_I)}{r} - \frac{cD(V_I)}{r}\left(\frac{V}{V_I}\right)^{\beta_1^E} \tag{27}$$

Equation (27) can be derived by noticing that loan commitment fees $T(V)$ is a perpetuity of costs which is interrupted (stopped being paid) once V hits the investment trigger. Note that total loan commitment fees are subtracted from firm value presented in equation (22).

We now proceed to provide a breakdown of firm value into the sum of the option on unlevered assets $(E(V-X))$, the extra debt financing benefits (or losses) arising from heterogeneous beliefs $(E(F_B))$ and the expected net benefits of debt $(E(NB))$ which captures the anticipated tax benefits net of bankruptcy costs. This decomposition is obtained by replacing equity value and debt value at investment into firm value functions (equation (25) for first-best and equation (22) for second-best). The value of the firm can then be written as follows:

$$F(V) = E(V-X) + E(F_B) + E(NB) \tag{28}$$

where under the second-best solution:

$$E(V-X) = -T(V) + (V_I - X - \varphi(X-K)1_{X>K})\left(\frac{V}{V_I}\right)^{\beta_1^E} - V_B\left(\left(\frac{V_I}{V_B}\right)^{\beta_2^E} - \left(\frac{V_I}{V_B}\right)^{\beta_2^D}\right)\left(\frac{V}{V_I}\right)^{\beta_1^E}) \tag{29}$$

$$E(F_B) = \left(\frac{R}{r}\right)\left(\left(\frac{V_I}{V_B}\right)^{\beta_2^E} - \left(\frac{V_I}{V_B}\right)^{\beta_2^D}\right)\left(\frac{V}{V_I}\right)^{\beta_1^E} \tag{30}$$

$$E(NB) = \left(\frac{\tau R}{r} - \frac{\tau R}{r}\left(\frac{V_I}{V_B}\right)^{\beta_2^E} - bV_B\left(\frac{V_I}{V_B}\right)^{\beta_2^D}\right)\left(\frac{V}{V_I}\right)^{\beta_1^E} \tag{31}$$

Under the first-best solution $T(V) = 0$ in equation (29). The optimal V_I is determined by equation (25) for second-best and equation (26) for first-best solutions.

With equal beliefs between debt and equity holders $\beta_2^E = \beta_2^D$. When additionally $\varphi = c = 0$, our decomposition nests the one in Mauer and Sarkar (2005) since $E(V - X) = (V_I - X)\left(\frac{V}{V_I}\right)^{\beta_1^E}$, $E(F_B) = 0$ and $E(NB)$ remains as in Mauer and Sarkar (2005), i.e., not affected by debt holders' beliefs.

Interestingly, $\left(\frac{V_I}{V_B}\right)^{\beta_2^i}$, $i = E, D$ captures the present value of one dollar expected to be received at investment based on equity or debt holders beliefs. Similarly, $\left(\frac{V}{V_I}\right)^{\beta_1^E}$ captures the expected present value of one dollar received from today's perspective by holding the option to invest (based on equity holders' beliefs).

When debt holders have favorable beliefs ($\sigma_D < \sigma_E$) we can use equations (6) to infer that $\beta_2^D < \beta_2^E$ and thus $\left(\frac{V_I}{V_B}\right)^{\beta_2^D} < \left(\frac{V_I}{V_B}\right)^{\beta_2^E}$. This means that debt holders anticipate that default will be triggered with a delay compared to equity holders and thus the expected present value of a dollar at default anticipated by equity holders is lower than that anticipated by debt holders. Using this result, we can infer from equation (30) that with favorable debt holders beliefs there is an additional value created for equity holders arising due to "cheap" (as perceived by equity holders) debt financing (i.e., $E(F_B) > 0$) . Similarly, the expected net benefits of debt ($E(NB)$) (see equation (31)) with more favorable debt holder beliefs increases because debt holders anticipated losses from bankruptcy costs are expected with delay (hence impose lower perceived costs of financing for shareholders). On the other hand, debt holders anticipate the value of assets at default with a delay compared to equity holders beliefs which creates a negative impact for equity holders via costlier financing reflected in the last term in equation for $E(V - X)$) (when $\sigma_D < \sigma_E$). To summarize, when σ_D increases (decreases) there is an increase (decrease) in $E(V - X)$) and a decrease (increase) in $E(F_B)$ and ($E(NB)$). These directional effects are confirmed with numerical simulations presented in Appendix I.

Finally, we also break down the agency costs of debt into different components as follows:

$$AC = AC_U + AC_F + AC_{NB} \quad (32)$$

where

$$AC_U = \frac{E^1(V-X)-(E^2(V-X)+T(V))}{(F^2+T(V))}$$

$$AC_F = \frac{E^1(F_B)-E^2(F_B)}{(F^2+T(V))}$$

$$AC_{NB} = \frac{E^1(NB)-E^2(NB)}{(F^2+T(V))}$$

Note that in the above equations "1" denotes first-best and "2" denotes second-best. Also, note that to provide a better comparison between first-best and second-best solutions and the corresponding effect of agency conflicts on each component of value we add back the costs paid on commitment fees under the second-best solution.

3. Numerical results and discussion

In Section 3.1. our numerical results focus on the impact of heterogeneous beliefs with respect to the volatility of assets. Section 3.2 shows analyses the agency cost implications of heterogeneous beliefs. Section 3.3. focuses on the impact of equity financing costs and loan commitment fees.[6]

3.1. The impact of heterogeneous beliefs about the volatility of assets

Table 1 presents numerical results with varying degree of heterogeneous beliefs regarding the volatility of assets. The base case parameter values of Leland (1994) are used with an additional assumption of a positive opportunity cost δ = 6%. Other parameters values are as follows: value of assets V =100, risk-free rate r = 0.06, investment cost X =100, bankruptcy costs b = 0.5 and tax rate τ = 0.35. In this section we assume away the

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[6] We revisit the effect of loan commitment fees in Section 4 since they may introduce default risk prior to exercising the investment option and discuss additional insights.

presence of equity financing costs ($\varphi = 0$) and loan commitment fees ($c = 0$). For the symmetric beliefs case we use a volatility $\sigma_E = \sigma_D = 0.25$. For the sensitivity analysis, we fix the estimates for equity holders and vary debt holders' beliefs. Therefore, for $\sigma_D < 0.25$ equity holders face favorable beliefs and by increasing σ_D we study more unfavorable debt holders beliefs. Panel A focuses on the first-best solution based on pre-investment total firm value maximization while Panel B investigates the second-best case which assumes equity value maximization.

[Insert Table 1 here]

Based on our extensive sensitivity results of Table 1 we summarize our first result.

Result 1: *The effect of heterogeneous beliefs between equity and debt holders regarding the volatility of assets.* [7]

The more unfavourable debt-holders beliefs become (higher σ_D):

a) The lower firm value, debt and leverage ratios
b) The higher the investment trigger (i.e., there is a delay in investment)
c) The lower the default trigger (i.e., there is a delay in bankruptcy after investment)
d) Credit spreads increase

It is interesting to note that unfavourable debt holders beliefs create an indirect debt capacity constraint (lowering leverage). Our results regarding low leverage for unfavourable debt holders beliefs are broadly in line with Devos et al. (2012) which shows that the extremely low leverage ratios observed in practice is mostly a result of the presence

[7] The predictions are based on second-best solutions. The directional effects are similar for the first-best case except that we observe that credit spreads follow an inverse U-shape with respect to debt holders beliefs.

of financing constraints and frictions. In our model, low leverage is driven by lenders having unfavourable beliefs which makes debt (as perceived by equity holders) costly. On the other hand, Yang (2013) offers an alternative explanation where optimistic managers may prefer equity (which they consider undervalued) thus driving down leverage ratios.[8]

3.2. Agency cost implications

From Table 1 results we observe that in the second-best case (see panel B) investment is triggered earlier investment and result in lower firm values (reflecting the agency costs of debt) compared to the first-best case. This result is in line with Mauer and Sarkar (2005) and reflects the agency costs of debt. We observe the same increasing pattern of investment trigger with respect to debt holders beliefs regarding volatility for the second-best case as in the first-best case. However, bankruptcy triggers under second-best are lower compared to first-best (i.e., there is a delay in default compared to first-best). Similarly to the first-best case, leverage decreases, however, in the second-best solutions we observe that credit spreads are increasing when debt holders beliefs become unfavourable regarding volatility (while for first-best follow an inverse U-shape).

In Figure 1 we calculate the agency costs (see equation (32)). In panel (a) we observe that the more unfavourable debt holders beliefs become the lower the agency costs of debt (see also Table 1 where value differences between first-best and second-best become smaller as debt holder beliefs become more unfavourable). In fact, for very unfavourable beliefs by debt holders agency costs tend to zero.

[8] According to Trester (1998) information asymmetry may lead to a preference of equity over debt and Ascioglu et al. (2008) shows that firms facing higher information asymmetry invest less and rely more on internal capital to fund investment (see also, Claus 2011).

[Insert Figure 1 here]

In panel (b) we see that the agency component relating to the option value on unlevered assets ($E(V - X)$) is high for favorable debt holder beliefs and is reduced the more unfavorable debt holders beliefs become. Indeed, as one can see from our earlier detailed results in Table 1, the first-best and second-best investment and default thresholds are becoming more aligned the more unfavorable the debt holders beliefs become. On the other hand, the agency costs component relating to $E(F_B)$ and $E(NB)$ are negative when debt holders beliefs about volatility are low because as we have pointed out self-interested equity holders place more emphasis on exploiting additional financial benefits under the second-best solution when faced with favorable beliefs. However, the overall contribution of $E(F_B)$ and $E(NB)$ on the total agency costs becomes insignificant the more unfavorable debt holders beliefs become. We summarize the following main result regarding the relationship of agency costs with heterogeneous beliefs.

Result 2: *The effect of heterogeneous beliefs on agency costs*

The more unfavourable debt-holders beliefs become (higher σ_D) the lower the total agency costs of debt.

The above result is related to Egami (2009) who finds that agency costs are lower when leverage ratios are low. In line with this, Result 2 shows that in the presence of unfavourable debt holders beliefs (which reduce leverage) agency costs are reduced. Our analysis provides further insights about the relationship of leverage with agency costs. Specifically, we show that for favourable debt holder beliefs, equity holders deviate from first-best policies in order to exploit financial benefits using higher leverage at the expense of suboptimal exercising their investment and default options. On the other hand, when faced with unfavourable debt holders beliefs they anticipate little financial benefits arising from debt. In this case their focus is on optimizing the timing of investment and default (and thus their policies are more aligned with first-best solutions).

3.3. Equity financing costs and loan commitment fees

In our earlier numerical analysis we have assumed away the presence of equity financing costs and loan commitment fees. We start our analysis of this section by exploring the effect of equity financing costs. Equity financing costs may become particularly important especially for small size firms in which equity holders may be constrained in obtaining access to additional equity financing thus forcing them to rely on debt financing. [9] Therefore, equity financing costs may exasperate the impact of heterogeneous beliefs between equity and debt holders. Figure 2 explores the impact of equity financing costs on firm value, the investment and default triggers, debt level, leverage ratios and credit spreads. The figure explores the case of zero equity financing costs vis a. vis. prohibitively high equity financing costs (φ = 100% is used for the high φ case which creates significant equity constraints).

[Insert Figure 2 here]

Our results show that when debt holders beliefs are lower or at par with those of equity holders, equity financing costs has no effect on firm value, the investment and default policy, debt, leverage ratios and credit spreads (for both first-best and second-best solutions). This result is expected since for this range of debt holders beliefs equity holders fully finance investment with debt (debt financing in this range of beliefs often exceeds the investment cost level of 100). However, with unfavorable beliefs by debt holders, equity

[9] Other papers consider the effect of (exogenous) debt financing constraints in a contingent claim framework and on investment timing (see Shibata and Nishihara, 2012 and Koussis and Martzoukos, 2012).

holders would be prompted to partly finance investments with equity if not facing any equity financing costs. To see this, observe in panels (d) (of both the first-best and second-best solutions) that when equity holder are unconstrained ($\varphi = 0$) and face unfavourable beliefs (σ of debt higher than 0.25) their optimal level of debt financing would drop below the level of investment (which is at the level of 100). This implies that they would optimally finance part of the investment with equity. Under a second-best solution, the downward adjustment in debt would have been even more significant than the first-best case for unfavorable debt holder beliefs. Therefore, for this range of debt holders beliefs the constraint arising due to equity financing costs becomes binding. In this case we find that when faced with equity financing costs (high φ) equity holders resort to just enough debt to cover the level of investment. Due to the more severe adjustments taking place under the second-best solution, firm value drops more significantly compared to the first-best case.

In panel (b) of the second-best solutions we observe that there is no significant change for an equity constrained firm relative to an unconstrained regarding their optimal investment policy. Under the first-best solution, on the other hand, we observe a delay in investment when the firm faces equity constraints for very unfavorable beliefs. Regarding the bankruptcy trigger (see panels (c)), we observe that for a firm facing equity constraints, the bankruptcy trigger is at higher levels (leading to much earlier default following investment) and results in higher leverage ratios and credit spreads for more unfavorable debt holders beliefs. These effects appear more pronounced for the second-best solution compared to first-best. We summarize the following result regarding the effect of equity financing costs:

Result 3: *The effect of equity financing costs*[10]

When equity holders face higher equity financing costs:

a) Firm value is reduced when debt holders have unfavorable beliefs (otherwise it remains unaltered)

b) The investment trigger is not significantly different compared to the case with no equity financing costs

c) The default trigger after investment exhibits an increase (i.e, default is triggered earlier); this becomes more pronounced when debt holders beliefs become more unfavorable

d) Debt levels remain fixed at the level of investment when debt holders have unfavorable beliefs which results in an increase in leverage ratios

e) Credit spreads exhibit an increase which is more pronounced when debt holders have unfavorable beliefs

The more pronounced adjustments taking place in the presence of equity financing constraints for the second-best solution compared to first-best shown is reflected in the agency costs (shown in Figure 3). The figure shows that agency costs differences between the case with equity financing constraints versus no constraints are only prevalent for unfavorable debt holders' beliefs.

[Insert Figure 3]

[10] The summary result provides predictions under a second-best solution. Similar directional effects are observed for first-best solutions except that we observe a delay in investment when debt holders have unfavorable beliefs under the first-best solutions.

Agency costs for the case of equity constraints are about 5 to 6% when debt holders' beliefs are $\sigma_D > 0.3$. In contrast, in the absence of equity constraints, agency costs are only about 2% for $\sigma_D = 0.3$ and approach zero for even higher σ_D levels. We summarize the following result regarding the impact of equity financing costs on agency costs.

Result 4: *The effect of equity financing costs on agency costs*

Equity financing costs result in higher agency costs when debt holders have unfavorable beliefs in which case equity financing constraints become more binding.

In a related context, Hirth and Homburg (2010(b)) show that firms delay investment when facing external financing costs and have low internal liquidity. On the other hand, we show that the investment policy is not substantially changed in the presence of equity financing constraints. This is because in our context equity holders facing external equity financing costs still have access to a bank loan even if this is expensive due to unfavorable debt holder beliefs. Hirth and Uhrig-Homburg (2010(a)) higher levels of a firm's liquidity reduces investment distortions and agency costs. In line with this, our results show that lower levels of equity financing constraints reduce deviations from first-best investment policies and the agency costs of debt. Our results also suggest that agency costs may remain small when debt holders beliefs are not significantly unfavorable.

Our subsequent analysis focuses on the impact of loan commitment fees. In this section we assume that there is no default risk prior to investment (we add this feature in the following section). Berg et al. (2016) explain the importance of these fees in loan pricing since they reflect options embedded in loan contracts. Our framework provides a direct estimate by incorporating endogenous adjustments in firm's policies. Figure 4 shows the impact of varying loan commitment fees. We use a debt commitment fee of 0.5% which is in line with the median debt commitment fee reported in Berg et al. (2016).

[Insert Figure 4 here]

Panel (a) shows that the differences in firm values compared to the case of no commitment fees are more significant when the firm faces favourable beliefs by debt holders at $\sigma_D = 0.2$.[11] This is expected since for favourable debt holders beliefs the firm uses higher debt levels. Panel (b) shows that for a wide range of debt holders' beliefs, a higher loan commitment fee triggers earlier investment. This adjustment also makes intuitive sense because when equity holders face higher loan commitment fees they invest earlier in order avoid commitment fees for unused debt capacity. However, when debt holders' beliefs become highly unfavourable we observe that equity holders optimally delay investment despite incurring higher loan commitment fees.

In panel (c) we observe that higher loan commitment fees results in a delay in default following investment. Delay in default becomes even more significant for more unfavourable debt holders' beliefs. Furthermore, debt and leverage ratios (see panels (d) and (e)) are lower when the firm faces higher commitment fees with the adjustment being more significant the more unfavourable debt holders' beliefs become. Interestingly, a higher commitment fee reduces credit spreads. This result is driven by the lower debt levels used and the delay in default occurring at higher commitment fees. However, the reported spreads do not include the expected present value of costs incurred for commitment fees. In our context, this is easy to calculate using equation (27). Our analysis (not tabulated)

[11] Note that for $\sigma_D = 0.15$ solutions are the same between c = 0 and c > 0 because of immediate exercise of the investment option.

reveals that the expected present value of debt commitment fees follows an inverse U-shape with respect to debt holders beliefs. The expected present value of debt commitment fees are zero for $\sigma_D < 0.2$ (since equity holders exercise the investment immediately) but they reach to 2.93 (9.90% of firm value) when $\sigma_D = 0.3$ and then drop to 1.66 (6% of firm value) for $\sigma_D = 0.35$. We summarize the following result.

Result 5: *The effect of loan commitment fees (with no default risk prior to investment)*

When equity holders face higher loan commitment fees:

a) Firm value is reduced and the reduction is more significant when debt holders have favourable beliefs.
b) Investment is triggered earlier unless debt holders beliefs are highly unfavourable (in which case there is a delay in investment).
c) Debt and leverage ratios are reduced; the reduction is more significant the more unfavourable debt holders beliefs become.
d) Credit spreads are reduced compared to the case with no commitment fee; the reduction is more pronounced the more unfavourable debt holders beliefs become.
e) The total expected value of commitment fees follows an inverse U-shape with respect to debt holders beliefs

Figure 5 shows the agency cost of debt for different levels of loan commitment fees. Agency costs with loan commitment fees (c = 0.5%) range between 4.7% (for unfavourable debt holders' beliefs) to more than 20% (for favourable debt holders' beliefs at $\sigma_D = 0.2$). In comparison, in the absence of loan commitment fees, agency costs range from 0.25%

to less than 20%. Even for symmetric beliefs the agency costs are about 11.21% for the case with c = 0.5% versus only 6% when there are no commitment fees. Our results thus illustrate the economic significance of loan commitment fees on agency costs.

[Insert Figure 5]

We summarize the result regarding the effect of loan commitment fees on agency costs.

Result 6: *The effect of loan commitment fees on agency costs.*

For high loan commitment fees agency costs increase with the increase becoming more pronounced for favourable debt holder beliefs.

4. Extensions of the basic setup

4.1. Definition of basic claims and expected passage times

So far we have assumed that there is no risk of bankruptcy prior to exercising an investment option. In this section we present a more general framework where equity holders have the option to default at an optimal default trigger V_B^i prior to exercising their investment option V_I^i , where i denotes the investment stage. In the next section we solve for a single stage investment ($i = 0$) and then in the subsequent subsection we extend the model adding another stage of possible investment $V_I^1$and default V_B^1 which allows for partial drawdown of the loan commitment in stages 0 and 1. After the final investment (stage i+1, depending on *i*), the Leland (1994) framework applies assuming no more investment stages and only an optimal timing of default at V_B .

Between each investment stages we now need to solve a double boundary optimal stopping problem. For the solution we follow Hirth and Uhrig-Homburg (2010) and Hackbarth and Mauer (2011) and define the following basic claims. Conditional that the current project value V is between a lower bound of V_B^i and an upper threshold V_I^i we denote by $J\left(V; V_B^i, V_I^i, k\right)$ the value of a claim that pays 1 when V reaches V_I^i and becomes worthless at V_B^i. This claim involves no intermediate payments and since it is a contingent claim on

V it satisfies the ordinary differential equation (2). This claim depends on the investment stage i and beliefs k which can be either based on equity holders ($k = E$) or debt holders $k = D$. In the general case beliefs may change depending on the stage i in which case we will write $k(i)$ to denote the beliefs of claimholder k in stage i.

Applying equation (2) and solving for $J\left(V; V_B^i, V_I^i, k\right)$ subject to boundary conditions $J\left(V; V_B^i, V_I^i, k\right) = 0$ and $J\left(V; V_B^i, V_I^i, k\right) = 1$ results in the following solution:

$$J\left(V; V_B^i, V_I^i, k\right) = \frac{(V_B^i)^{\beta_2^k} V^{\beta_1^k} - (V_B^i)^{\beta_1^k} V^{\beta_2^k}}{(V_I^i)^{\beta_1^k} (V_B^i)^{\beta_2^k} - (V_I^i)^{\beta_2^k} (V_B^i)^{\beta_1^k}} \quad \text{for } V_B^i < V < V_I^i \tag{33}$$

Similarly, we denote by $L\left(V; V_B^i, V_I^i, k\right)$ the value of a claim that pays 1 when V reaches V_B^i and becomes worthless at V_I^i. This claim satisfies the ordinary differential equation (2) subject to boundary conditions $L\left(V_B^i; V_B^i, V_I^i, k\right) = 1$ and $L\left(V_I^i; V_B^i, V_I^i, k\right) = 0$ which results in the following solution:

$$L\left(V; V_B^i, V_I^i, k\right) = \frac{(V_I^i)^{\beta_1^k} V^{\beta_2^k} - (V_I^i)^{\beta_2^k} V^{\beta_1^k}}{(V_I^i)^{\beta_1^k} (V_B^i)^{\beta_2^k} - (V_I^i)^{\beta_2^k} (V_B^i)^{\beta_1^k}}, \text{ for } V_B^i < V < V_I^i \tag{34}$$

We also provide the probability of investment $\Pi_J\left(V; V_B^i, V_I^i, k\right)$ and the probability of default $\Pi_L\left(V; V_B^i, V_I^i, k\right)$ at each stage (see Hackbarth and Mauer, 2011).

$$\Pi_L\left(V; V_B^i, V_I^i, k\right) = \frac{(V_I^i)^{2\lambda_k / \sigma_k^2} - (V)^{2\lambda_k / \sigma_k^2}}{(V_I^i)^{2\lambda_j / \sigma_k^2} - (V_B^i)^{2\lambda_j / \sigma_k^2}} \tag{35}$$

$$\Pi_J\left(V; V_B^i, V_I^i, k\right) = 1 - \Pi_L\left(V; V_B^i, V_I^i, k\right)$$

where $\lambda_k = -(\mu - \frac{\sigma_k^2}{2})$. Note that in many planning applications or when estimating probabilities using real data the real drift μ is commonly used (for risk-neutral probabilities one can replace with $\mu = r - \delta$). Furthermore, it will prove useful to calculate the expected exit time (see Hackbarth and Mauer, 2011) between thresholds V_B^i , V_I^i as follows:

$$\bar{T}_e^i(V; V_B^i, V_I^i, k) = \frac{1}{\lambda_k}\left(\ln(\frac{V}{V_B^i})\right) + \frac{1}{\lambda}\left(\ln(\frac{V_B^i}{V_I^i})\right)\left(1 - \Pi_L(V; V_B^i, V_I^i, k)\right) \tag{36}$$

4.2. Default risk prior to investment in a single stage investment model

We now present a solution with a single stage investment (and single drawdown) with default risk prior to investment. Using the basic claims defined in the previous subsection we can now define firm value at t = 0 as follows:

$$F(V) = -\frac{cK}{r} + \frac{cK}{r}L(V; V_B^0, V_I^0) + \left(E(V_I^0) + K - X - \varphi(X-K)1_{X>K} + \frac{cK}{r}\right)J(V; V_B^0, V_I^0)$$

(37)

Note that $E(V_I^0)$ and loan commitment K in the stage following investment are determined by equations (12) and (17). Default trigger following investment is determined by equation (11).

The optimal investment V_I^0 (second-best) and default trigger V_B^0 are found by solving a system of two equations resulting from applying the following smooth-pasting conditions:

$$\frac{\partial F}{\partial V}|_{V=V_I^0} = \frac{\partial E}{\partial V}|_{V=V_I^0} \tag{38a}$$

$$\frac{\partial F}{\partial V}|_{V=V_B^0} = 0 \tag{38b}$$

The analytic expressions for the two equations in (38) are provided in Appendix II. We obtain the optimal coupon that maximizes firm value (equation (37)) using a dense coupon grid search. We also calculate the total expected loan commitment fees as follows:

$$T(V) = \frac{cK}{r} - \frac{cK}{r}L(V; V_B^0, V_I^0) - \frac{cK}{r}J(V; V_B^0, V_I^0) \tag{39}$$

Figure 6 presents sensitivity results for this extended model using a low (c = 0.1%) and a high (c = 0.5%) loan commitment fee.

[Insert Figure 6]

We obtain similar insights obtained from our basic model of the previous section, albeit, we now provide some new implications regarding the effect of heterogeneous beliefs on the default boundary prior to investment. First, just like in our basic setup we find that a higher level of loan commitment fee accelerates investment, unless equity holders face highly unfavorable beliefs by debt holders (in which case there is a delay in investment). Furthermore, default after investment is triggered with a delay at higher loan commitment fees and the more unfavorable debt holders beliefs become. Panel (d) presents new implications regarding the default trigger prior to investment. The results show that higher loan commitment fees result in earlier default prior to investment (higher default trigger) and that for more unfavorable debt holder beliefs default prior to investment is triggered earlier. Other results largely corroborate with insights of our basic setup. Confirming the insights of our basic model, Panel (e) shows that leverage ratios are lower for high commitment fees and for more unfavorable debt holder beliefs. Panel (f) shows an inverse U-shape of credit spreads with debt holders beliefs (and that credit spreads are lower for high commitment fees).

In Figure 7 we investigate the total expected present value of loan commitment fees (panel (a)) and also take a look at the effect of heterogeneous beliefs on the probability of default prior to investment, the probability of initiating investment (panels (b) and (c)) and the effect on agency costs (panel (d)).

[Insert Figure 7]

We first observe that the expected present value of loan commitment fees in this extended model is low. Thus, even when banks charge high loan commitment fees, it is anticipated that optimal firm investment and default policies limit the expected present value commitment fees actually paid. We have verified that this result holds (untabulated results) even when the investment option is out of the money (low V relative to the investment costs) in which case one would anticipate a larger delay in investment and hence more unused debt capacity. Panel (b) shows that higher commitment fees increases the probability of investment except for very favorable or unfavorable beliefs where the probability of investment remains unaltered. This is in line with our earlier result of Figure 4 of our basic model which showed that investment is triggered earlier for higher commitment fees except for very unfavorable beliefs. Panel (c) shows that the probability of default is lower for higher commitment fees (except for very favorable and unfavorable beliefs which is not affected by commitment fees).[12] Additional sensitivity results (not shown for brevity) show that when the investment option is out of the money, higher commitment fees increase the probability of investment only for favorable beliefs whereas they result in a decrease in the probability of investment for unfavorable beliefs. These sensitivity results for out of money options also show that the probability of default is lower when loan commitment are high for favorable beliefs and increases for unfavorable beliefs. Therefore, the effect of loan commitment fees on the probability of investment and default depends on the moneyness of the investment option. Panel (d) shows that agency costs are higher for higher loan commitment fees, however, we observe that maximum agency costs is not necessarily at the most favorable debt holders beliefs (as was the case when not accounting for default prior to investment).

[12] We have shown earlier that the default trigger is higher at higher commitment fees, however, investment is also accelerated at higher commitment fees. Thus, the overall effect of a less likelihood of default results because investment is triggered sooner before the firm defaults.

Although more complete, our extended model of this section confirms most insights gained using our basic setup. We thus summarize some *new* insights gained from this extended model in Result 7.

Result 7: The effect of loan commitment fees (with default risk prior to investment)

In the presence of default risk prior to investment an increase in loan commitment fees results in:

a) An increase in the default trigger prior to investment that decreases for more unfavourable debt holder beliefs

b) May increase the probability of investment and decrease the probability of default when options are not out of the money (for out of the money options this relationship changes)

c) Agency costs increase, however, maximum agency costs may not necessarily exist at most favourable debt holder beliefs.

4.3. Multiple investment stages with partial drawdown

In order to solve for a partial drawdown model we start from the final stage and move backwards. From a methodological perspective this section extends Hackbarth and Mauer (2011) with the addition of another stage of investment and default decisions.[13] We illustrate the solution for a two stage investment and financing problem with partial drawdown in each stage of the loan commitment. We use the notation X_0, for the first stage

[13] We emphasize that our model focuses on multiple drawdowns of a loan commitment from a single borrower whereas the Hackbarth and Mauer (2011) focuses on borrowing from different borrowers, hence, their focus relates to priority rules.

investment cost. We assume that second stage investment (triggered at V_I^1) expands the value of assets V by $e_G > 1$ at a cost X_1. Following standard arguments the value of equity $E_2(V)$, following the second investment evaluated at V is[14]:

$$E_2(V) = e_G V - (1-\tau)\frac{R}{r} + \left((1-\tau)\frac{R}{r} - e_G V_B\right)\left(\frac{V}{V_B}\right)^{\beta_2^{E(2)}} \tag{40}$$

where $R = R_1 + R_2$, $V_B = \frac{-\beta_2^E(1-\tau)}{(1-\beta_2^E)}\frac{R}{e_G r}$, and $V > V_B$.[15]

Following investment in the second stage the full amount of the loan commitment has been drawn and hence there are no more debt commitment fees incurred. Note also that equity in stage 2 depends on equity holder beliefs about volatility holding in stage 2.

The value of second-stage (final) drawdown D_2^2 in period 2 and the first drawdown in period 2, D_1^2 can be easily derived as follows:

$$D_i^2(V) = \frac{R_i}{r} + \left((1-b)\psi_i(e_G V_B) - \frac{R_i}{r}\right)\left(\frac{V}{V_B}\right)^{\beta_2^{D(2)}} \quad , \quad V > V_B \tag{41}$$

where $\psi_i = \frac{R_i}{R}$ corresponds to fraction of assets which corresponds to a drawdown in the event of default.[16] Note that the evaluation of each drawdown's value as of stage 2 depends on debt holder beliefs about volatility holding in stage 2.

Moving one step back at the first investment stage triggered at V_I^0 and using the basic claims derived in the subsection 4.1. we now derive the value of equity at stage 1, $E_1(V)$ as follows:

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[14] With expanded revenues due to the exercise of the growth option asset value following investment becomes $V' = e_G V$ which follows a geometric Brownian motion like eq.(2). Thus, one can then follow standard arguments like the ones used in Section 2.2. to derive eq. (40).

[15] Note that the bankruptcy trigger is defined in terms of V. The actual default trigger is $e_G V_B$.

[16] This is similar to the pari-passu rule used in Hackbarth and Mauer (2011) to assign equal footing to value of assets at default to different lenders. In our case, we have a single lender and so it claims 100% of the net of bankruptcy cost asset value (each drawdown can be thought to claim part of that value depending on its level).

$$E_1(V) = V - \frac{R_1(1-\tau)}{r} - \frac{cD_2^2(V_I^1)}{r} + \left(\frac{cD_2^2(V_I^1)}{r} + \frac{R_1(1-\tau)}{r} - V_B^1\right) L(V; V_B^1, V_I^1, j = E(1)) + \left(E_2(V_I^1) + D_2^2(V_I^1) - X_1 - \varphi\big(X_1 - D_2^2(V_I^1)\big)1_{X_1 > D_2^2} + \frac{cD_2^2(V_I^1)}{r} + \frac{R_1(1-\tau)}{r} - V_I^1\right) J(V; V_B^1, V_I^1, j = E(1)) \tag{42}$$

where $V_B^1 < V < V_I^1$.

Equity value in stage 1 has an intuitive interpretation. The first three terms capture the value of assets net of after tax coupons and commitment fees on the yet to be drawn second stage drawdown. The subsequent term reduces previous mentioned values (of assets, after tax coupon and commitment fees) in the event of default while the third term captures the anticipated additional values received (or paid) in the event of exercise of the investment option at V_I^1. Note that at investment V_I^1 the value of assets is replaced by a scaled version equal to $e_G V_I^1$. To see that note that at investment V_I^1 we have that $J(V_I^1; V_B^1, V_I^1, j) = 1$ and $L(V_I^1; V_B^1, V_I^1, j) = 0$, so $E_1(V_I^1) = \big(E_2(V_I^1) + D_2^2(V_I^1) - X_1 - \varphi\big(X_1 - D_2^2(V_I^1)\big)1_{X_1 > D_2^2}\big)$.

In stage 1 the value of the first drawdown $D_1^1(V)$ is as follows:

$$D_1^1(V) = \frac{R_1}{r} + \left((1-b)V_B^1 - \frac{R_1}{r}\right) L(V; V_B^1, V_I^1, j = D(1)) \ \left(D_1^2(V_I^1) - \frac{R_1}{r}\right) J(V; V_B^1, V_I^1, j = D(1)) \tag{43}$$

for $V_B^1 < V < V_I^1$.

The value of the first drawdown involves the coupon payment received, an adjustment in value in the event of default (second term) and the anticipated value of the first drawdown expected to be received in the event of subsequent investment option being exercised.

The value of commitment fees in stage 1, $T_1(V)$ is the following:

$$T_1(V) = \frac{cD_2^2(V_I^1)}{r} - \frac{cD_2^2(V_I^1)}{r} L(V; V_B^1, V_I^1, j) - \frac{cD_2^2(V_I^1)}{r} J(V; V_B^1, V_I^1, j) \tag{44}$$

For optimizations of investment and default triggers by equity holders we shall use equity holders beliefs, hence j = E(1).

Next, we move one step backwards to derive values at $t = 0$. Using the basic claims values firm value in stage 0 (received by equity holders), denoted by $F(V)$, is the following:

$$F(V) = -\frac{cK}{r} + \frac{cK}{r} L(V; V_B^0, V_I^0, j = E(0)) + \Big(E_1(V_I^0) + D_1^1(V_I^0) - X_0 - \varphi\big(X_0 - D_1^1(V_I^0)\big)1_{X_0 > D_1^1} + \frac{cK}{r}\Big) J(V; V_B^0, V_I^0, j = E(0)) \tag{45}$$

where $K = D_1^1(V_I^0) + D_2^2(V_I^1)$ and $V_B^0 < V < V_I^0$.

Finally, the value of total commitment fees at t = 0 is given by:

$$T(V) = \frac{cK}{r} - \frac{cK}{r} L(V; V_B^0, V_I^0, j) + \left(T_1(V_I^0) - \frac{cK}{r}\right) J(V; V_B^0, V_I^0, j)$$

The optimization conditions for solving for the optimal boundaries $V_I^0, V_B^0, V_I^1, V_B^1$ are the following:

$$\frac{\partial F}{\partial V}\Big|_{V=V_I^0} = \frac{\partial E_1}{\partial V}\Big|_{V=V_I^0} \tag{46a}$$

$$\frac{\partial F}{\partial V}\Big|_{V=V_B^0} = 0 \tag{46b}$$

$$\frac{\partial E_1}{\partial V}\Big|_{V=V_I^1} = \frac{\partial E_2}{\partial V}\Big|_{V=V_I^1} \tag{46c}$$

$$\frac{\partial E_1}{\partial V}\Big|_{V=V_B^1} = 0 \tag{46d}$$

We note that the above optimization for the timing of the investment options exercise take into account only equity values (corresponding to second-best as pointed in Mauer and

Sarkar, 2005 and Hackbarth and Mauer, 2011) since our analysis is based on loan commitment: equity holders once they agree on loan commitment will act opportunistically by maximizing equity value and not overall firm (equity plus debt) values. Debt holders internalize this risk in the valuation of the loan commitment.

Despite the sequential setup, for the model to generate partial drawdowns it must result in solutions where $V_I^0 < V_I^1$, i.e, the investment of first stage is triggered first (otherwise one will obtain a solution where investment in both stages occurs simultaneously and the full value of the commitment is drawn in a single stage). Our extensive numerical simulations reveal that $V_I^0 > V_I^1$ which implies that the problem collapses to a single stage. Dixit and Pindyck (1994) (pp. 322-328) show a similar result on a simplified setup of a perpetual horizon sequential investment problem (although no analysis on financing issues was considered in their setting). To retain the sequential nature of the problem they introduce time-to-build. Instead, in our analysis we focus on expected passage time (see equation 36) as a proxy for the average time it takes to wait until the next stage of investment (or default) is triggered. Expected exit times may depend on the level of competition in different industries with more competitive or technological intensive industries exhibiting shorter cycles (earlier investments). To introduce this feature we replace optimality conditions (46a) and (46c) with respect to the investment timing with targets relating to expected exit time set to a specific level t_e (measured in years). For simplicity we shall assume equal (stationary) expected time for the first and second stage investment in our simulations varying t_e between 1 and 5 years between investment stages. Hence, we replace conditions (46a) and (46c) with the following:

$$\bar{T}_e^0(V; V_B^0, V_I^0) = t_e \tag{47a}$$

$$\bar{T}_e^1(V_I^0; V_B^1, V_I^1) = t_e \tag{47b}$$

Equations 46(c) and (d) together with 47(a) and (b) describe a constrained optimization problem where the firm's equity holders optimize default and consider expected exit times as constraints. These constraints in essence pin down investment triggers thus implicitly determining the maturity of investment. We next present some simulation results based on the following parameters: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost $\delta =$ 0.06, volatility σ_E = 0.25, investment costs $X_0 = X_1$ =100, expansion factor for second stage investment e_G equal to 2, bankruptcy costs b = 0.5 and tax rate τ = 0.35, equity financing costs φ = 0 and loan commitment fees c = 0.5%. Figure 8 presents sensitivity results with respect to expected passage time t_e = 5 (solid line) and t_e = 1 (dotted line) using real drift μ equal to 0.06 for the calculation of expected hitting time. The figure also produces sensitivity analysis with respect to debt holders perceived estimate of volatility σ_D.

[Insert Figure 8 here]

With equal beliefs ($\sigma_D = \sigma_E$ 0.25) the results of show that the longer the expected passage time the higher the firm value. This behavior of firm value is similar to a call option with which increases with maturity (unless it is very long-term). Furthermore, the more unfavorable debt holders beliefs become the more important the value differences between long Vs short passage times. We observe that default decisions for different passage times under similar beliefs are roughly the same whereas the investment timing is delayed the longer the passage time. This should come as no surprise since, since as pointed out earlier the default decision is optimized, hence the expected passage time constraints in equations (47a) and (47b) effectively introduce a maturity effect for new investments. The different panels show that more unfavorable debt holder beliefs induce early investments. In general, we also observe a delay in default the more unfavorable debt holders beliefs become (except for the case of initial default trigger for short-passage time). The last three panels (g)-(i) show, as expected, that the longer the expected passage time the more the loan commitment (since investment is triggered at higher value levels) and the more the total commitment fees. Panel (h) shows that the shorter the passage time the higher the fraction of initial drawdown over the total. Finally, unfavorable debt holder beliefs reduce total commitment debt levels and fees and increase the fraction of the subsequent drawdown. We summarize some important implications relating to expected passage time with heterogenous beliefs.

Result 8: a) The longer the expected passage time for new investments (or default) the higher the firm value, loan commitments (and fees) and the lower the initial drawdown fraction relative to the total b) More unfavorable debt holders beliefs reduce debt commitments and fees and increase the fraction of future relative to early drawdowns of the loan commitment.

5. Summary

In this paper we have developed a framework with heterogeneous beliefs between equity and debt holders and studied their impact on firm value, optimal capital structure, investment and default timing, credit spreads and the level of agency costs. Our analysis showed that unfavourable beliefs by debt holders reduce firm value, optimal leverage and result in delayed investment and an increase in credit spreads. With equity financing costs, equity holders may resort to debt financing even when faced with unfavourable beliefs by debt holders which results in an increase in leverage and credit spreads. We show that higher loan commitment fees result in earlier investment except when debt holders beliefs are highly unfavourable. Furthermore, debt, leverage ratios and credit spreads are reduced when the firm faces higher loan commitment fees and this reduction is more significant the more unfavourable debt holder beliefs become. We show that the expected present value of loan commitment fees costs may not be economically significant when accounting for default risk prior to investment.

Our analysis also provides implications regarding the agency costs associated with conflicts between equity and debt holders over the optimal timing of investment. We show that agency costs of debt are lower when equity holders face unfavourable beliefs by debt holders. Higher equity financing costs increase agency costs when debt holders have unfavourable beliefs. Loan commitment fees cause a significant increase in the agency costs of debt. We generalize our framework to multiple stages with partial drawdown of the loan commitment and expected exit times for new investments.

Appendix I: Decomposition of the impact of heterogeneous beliefs

Figure A1 shows sensitivity results relating to the components of firm value analysed in equations (28)-(31). As predicted by our theoretical analysis, the more unfavorable debt holders beliefs become the higher $E(V-X)$) and the lower $E(F_B)$ and $E(NB)$). These directional effects indeed hold for both first-best (solid lines) and second-best (dotted lines). The figure shows that $E(V-X)$) increases when debt holders' beliefs become unfavorable since the higher σ_D the earlier the anticipated default and thus the higher the value of assets expected by debt holders value at default. On the other hand, with an anticipated acceleration of default expected by debt holders, the expected financing benefits obtained by equity holders are reduced (thus ($E(F_B)$ drops) and the anticipated bankruptcy costs expected to be incurred increase (thus $E(NB)$ is also reduced). It is interesting to note that for favorable debt holder beliefs, under a second-best solution, self-interested equity holders deviate from first-best policies in order to obtain (what they believe) additional financing benefits (see dotted lines representing second-best solutions in panels (b) and (c)). On the other hand, for unfavorable debt holders beliefs equity holders appear to align their policies closer to the first-best solution (shown by the solid lines in the different panels).

[Insert Figure A1 here]

Appendix II: Analytic expressions for optimization conditions for Section 4.2.

The smooth-pasting conditions (second-best) for optimization is $\frac{\partial F}{\partial V}|_{V=V_I^0} = \frac{\partial E}{\partial V}|_{V=V_I^0}$ leads to the following equation for determining the optimal investment threshold V_I^0 :

$$\frac{cK}{r}\frac{\partial L}{\partial V}|_{V=V_I^0} + \left(E(V_I^0) + K - X - \varphi(X-K)1_{X>K} + \frac{cK}{r}\right)\frac{\partial J}{\partial V}|_{V=V_I^0} = \frac{\partial E}{\partial V}|_{V=V_I^0} \quad \text{(A1)}$$

where

$$\frac{\partial L}{\partial V}|_{V=V_I^0} = \frac{\frac{1}{V_I}\left(\beta_2 (V_I^0)^{\beta_1} (V_I^0)^{\beta_2} - \beta_1 (V_I^0)^{\beta_2} (V_I^0)^{\beta_1}\right)}{\left[(V_I^0)^{\beta_1} (V_B^0)^{\beta_2} - (V_I^0)^{\beta_2} (V_B^0)^{\beta_1}\right]}$$

$$\frac{\partial J}{\partial V}|_{V=V_I^0} = \frac{\frac{1}{V_I}\left(\beta_1 (V_B^0)^{\beta_2} (V_I^0)^{\beta_1} - \beta_2 (V_B^0)^{\beta_1} (V_I^0)^{\beta_2}\right)}{\left[(V_I^0)^{\beta_1} (V_B^0)^{\beta_2} - (V_I^0)^{\beta_2} (V_B^0)^{\beta_1}\right]}$$

$$\frac{\partial E}{\partial V}|_{V=V_I^0} = 1 + \beta_2 \left((1-\tau)\frac{R}{r} - V_B\right)\left(\frac{V_I^0}{V_B}\right)^{\beta_2}\left(\frac{1}{V_I^0}\right)$$

To obtain the optimal timing of default V_B^0 prior to investment, we apply the following condition $\frac{\partial F}{\partial V}|_{V=V_B^0} = 0$ which results in the following equation:

$$\frac{cK}{r}\frac{\partial L}{\partial V}|_{V=V_B^0} + \left(E(V_I) + K - X - \varphi(X-K)1_{X>K} + \frac{cK}{r}\right)\frac{\partial J}{\partial V}|_{V=V_B^0} = 0 \qquad \text{(A2)}$$

where

$$\frac{\partial L}{\partial V}|_{V=V_B^0} = \frac{\frac{1}{V_B^0}\left(\beta_2 (V_I^0)^{\beta_1} (V_B^0)^{\beta_2} - \beta_1 (V_I^0)^{\beta_2} (V_B^0)^{\beta_1}\right)}{\left[(V_I^0)^{\beta_1} (V_B^0)^{\beta_2} - (V_I^0)^{\beta_2} (V_B^0)^{\beta_1}\right]}$$

$$\frac{\partial J}{\partial V}|_{V=V_B^0} = \frac{\frac{1}{V_B^0}\left(\beta_1 (V_B^0)^{\beta_2} (V_B^0)^{\beta_1} - \beta_2 (V_B^0)^{\beta_1} (V_B^0)^{\beta_2}\right)}{\left[(V_I^0)^{\beta_1} (V_B^0)^{\beta_2} - (V_I^0)^{\beta_2} (V_B^0)^{\beta_1}\right]}$$

In order to determine the optimal investment threshold V_I^0 and the optimal default trigger prior to investment we need to solve the system of equations described in equation (A1) and (A2). We obtain the optimal coupon that maximizes firm value (equation (37)) using a dense coupon grid search.

References

Ascioglu, A., Shantaram P. H., McDermott, J.B., 2008. Information Asymmetry and Investment-Cash Flow Sensitivity. Journal of Banking and Finance 32, 1036-1048.

Berg, T., Saunders, A., and Steffen, S. 2016. The total cost of corporate borrowing in the loan market: Don't ignore the fees. Journal of Finance, 71(3), 1357-1392.

Brealey, R., Leland, H.E., Pyle, D.H., 1977. Informational asymmetries, financial structure, and financial intermediation. Journal of Finance 32, 371-387.

Chava, S., and Jarrow, R. 2008. Modeling loan commitments. Finance Research Letters, 5(1), 11-20.

Claus, I., 2011. The Effects of Asymmetric Information Between Borrowers and Lenders in an Open Economy. Journal of International Money and Finance 30, 796-816.

Devos, E., Dhillon,U., Jagannathan, M., Krishnamurthy, S., 2012. Why are firms unlevered?. Journal of Corporate Finance 18, 664-682.

Dittmar, A., & Thakor, A. 2007. Why do firms issue equity?. The Journal of Finance, 62(1), 1-54.

Dumas, B., Lewis, K. K., and Osambela, E. 2017. Differences of opinion and international equity markets. The Review of Financial Studies, 30(3), 750-800.

Egami, M. 2009. A framework for the study of expansion options, loan commitments and agency costs. Journal of Corporate Finance, 15(3), 345-357.

Ergungor, O. E. 2001. Theories of bank loan commitments. Economic Review-Federal Reserve Bank of Cleveland, 37(3), 2.

Flannery, M., 1985. Asymmetric Information and Risky Debt Maturity Choice. Journal of Finance 41, 19-37.

Hackbarth, D., 2008. Managerial Traits and Capital Structure Decisions. Journal of Financial and Quantitative Analysis 43, 843-882.

Hackbarth, D., 2009. Determinants of Corporate Borrowing: a Behavioral Perspective. Journal of Corporate Finance 15, 389-411.

Hackbarth, D., and Mauer, D. C. 2011. Optimal priority structure, capital structure, and investment. The Review of Financial Studies, 25(3), 747-796.

Hirth, S., and Uhrig-Homburg, M. 2010(a). Investment timing, liquidity, and agency costs of debt. Journal of Corporate Finance, 16(2), 243-258.

Hirth, S., and Uhrig-Homburg, M. 2010(b). Investment timing when external financing is costly. Journal of Business Finance & Accounting, 37(7-8), 929-949.

Koussis, N., and Martzoukos, S. H. 2012. Investment options with debt-financing constraints. The European Journal of Finance, 18(7), 619-637.

Leland, H., 1994. Corporate Debt Value, Bond Covenants, and Optimal Capital Structure. Journal of Finance 49, 1213-1252.

Leland, H., 1998. Agency Costs, Risk Management, and Capital Structure. 1998. Journal of Finance 53, 1213-1243.

Mauer, D.C., Sarkar, S., 2005. Real Options, Agency Conflicts, and Optimal Capital Structure. Journal of Banking and Finance 26, 1405-1428.

Sarkar, S., and Zhang, C. 2016. Loan-commitment borrowing and performance-sensitive debt. Review of Quantitative Finance and Accounting, 47(4), 973-986.

Stiglitz, J., 1972. Some Aspects of the Pure Theory of Corporate Finance: Bankruptcies and Take-Overs. The Bell Journal of Economics 3,458-482.

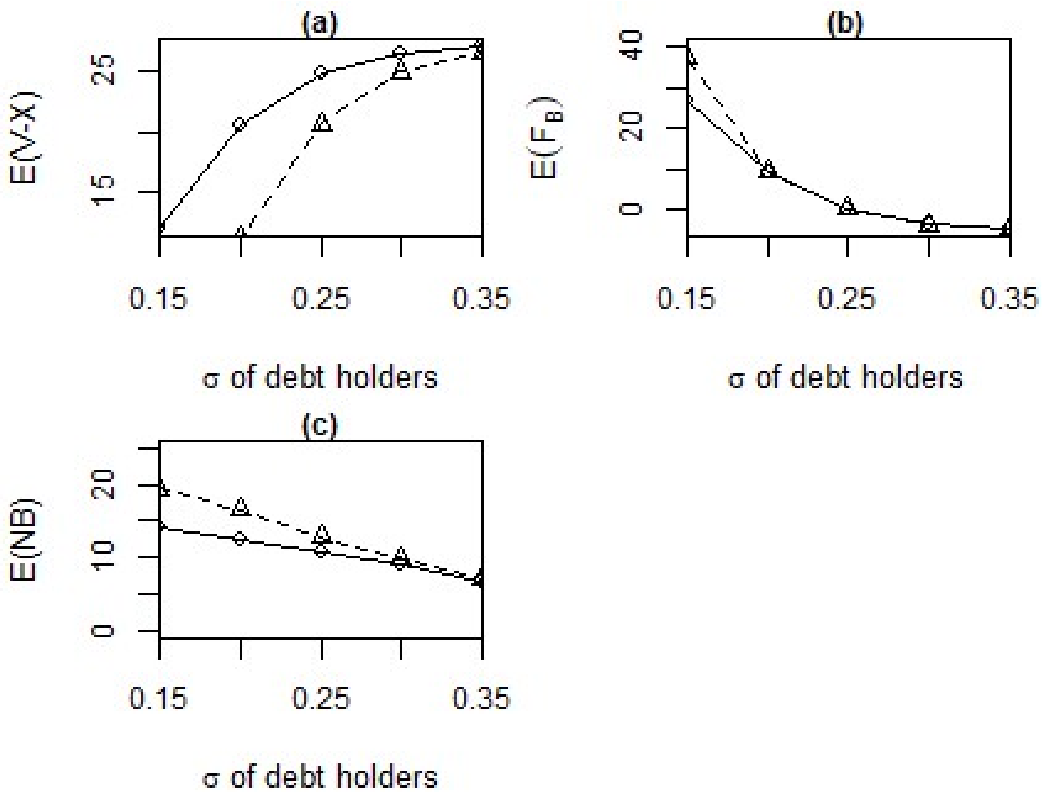
Shibata, Takashi, and Michi Nishihara, 2012. Investment timing under debt issuance constraint. Journal of Banking & Finance 36.4, 981-991.

Thakor, A. V., and Whited, T. M. 2010. Shareholder-manager disagreement and corporate investment. Review of Finance, 15(2), 277-300.

Trester, Jeffrey J., 1998. Venture capital contracting under asymmetric information. Journal of Banking & Finance 22.6, 675-699.

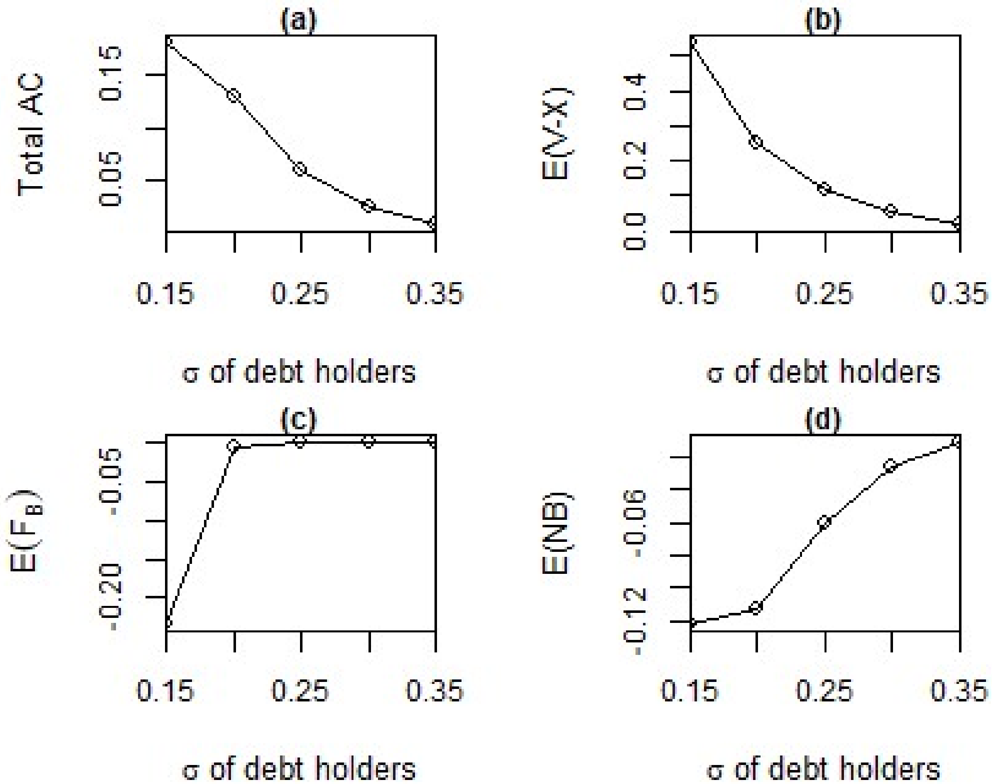
Yang, B. 2013., Dynamic capital structure with heterogeneous beliefs and market timing. Journal of Corporate Finance 22, 254-277.

Figure A1. Decomposition of firm value and the impact of heterogeneous beliefs



Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ= 0.06, volatility σ_E = 0.25, investment cost X = 100, bankruptcy costs b = 0.5 and tax rate τ = 0.35,equity financing costs φ = 0, loan commitment fees c = 0. Sensitivity analysis is with respect to debt holders perceived estimate of volatility σ_D. The figure shows the different components of firm value as analyzed in equations (29)-(31). Solid lines depict first-best solutions and dotted lines second-best solutions.

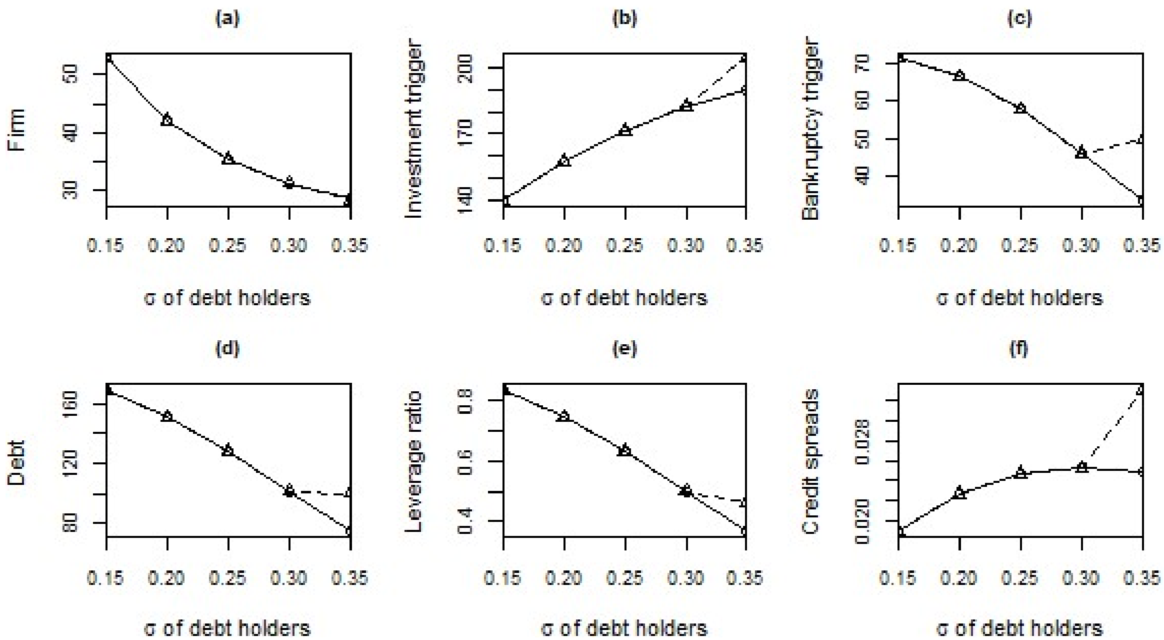
Figure 1. Analysis of agency costs of debt with heterogeneous beliefs



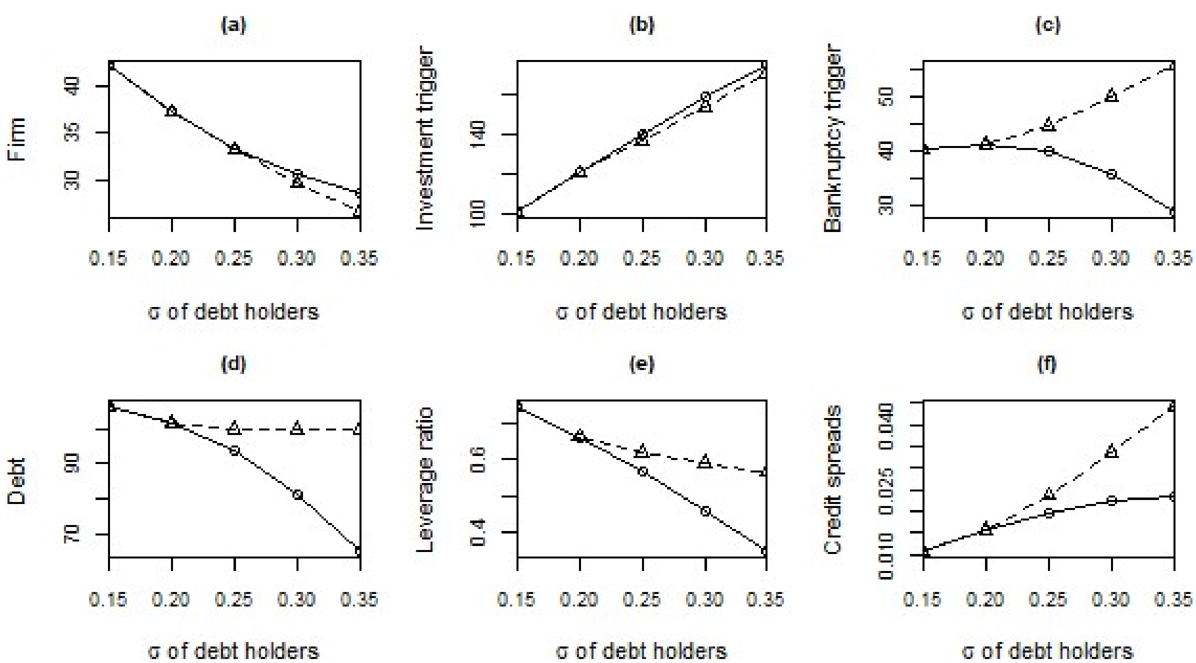
Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ= 0.06, volatility σ_E = 0.25, investment cost X = 100, bankruptcy costs b = 0.5 and tax rate τ = 0.35,equity financing costs φ = 0, loan commitment fees c = 0. Sensitivity analysis is with respect to debt holders perceived estimate of volatility σ_D. Sensitivity analysis is with respect to debt holders perceived estimate of volatility σ_D. The figure decomposes the total agency costs ("Total AC" in panel (a)) which is defined in equation (32).

Figure 2. The effect of equity financing costs

A. First-best

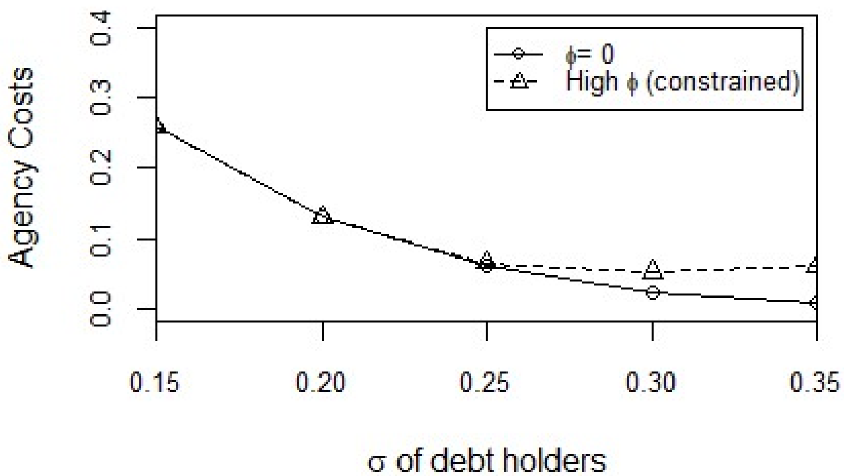


B. Second-best



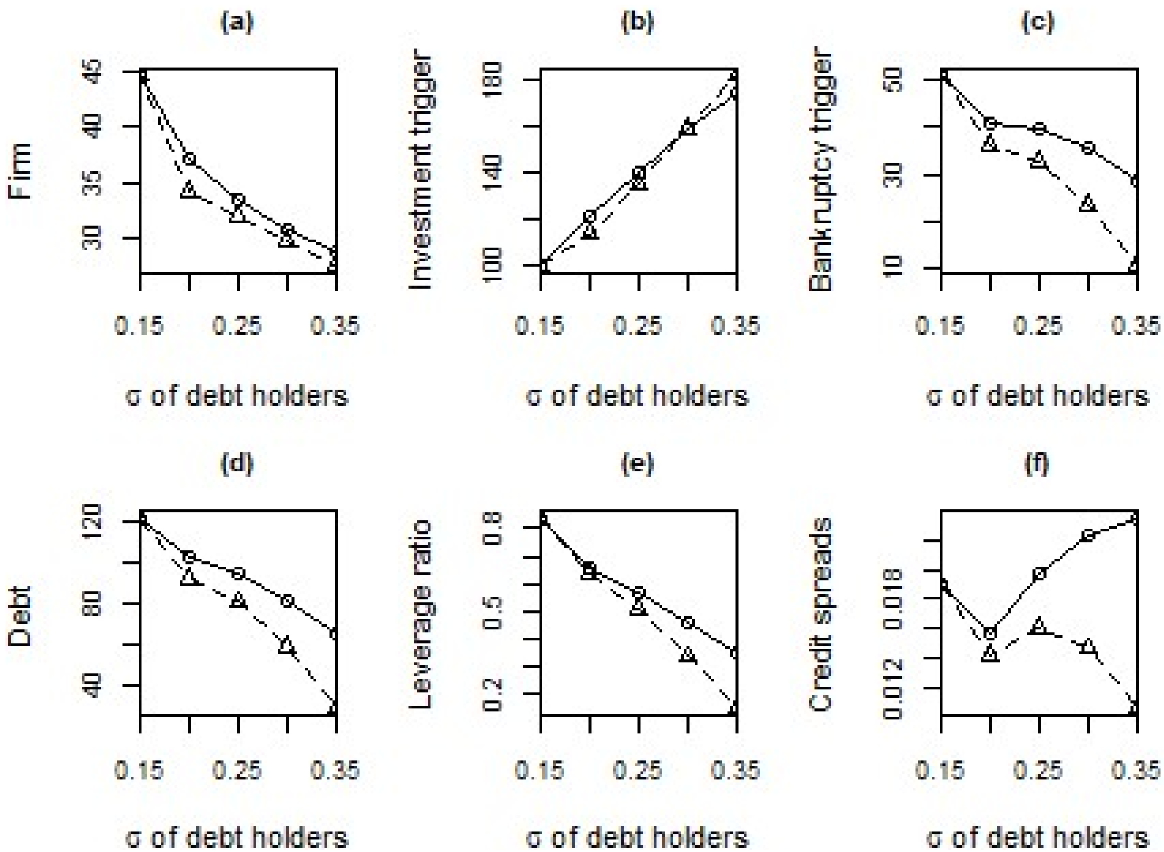
Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ= 0.06, volatility σ_E = 0.25, investment cost X = 100, bankruptcy costs b = 0.5 and tax rate τ = 0.35, loan commitment fees c = 0. Equity financing costs φ = 0 (solid line) or for φ = 1 (dotted line). Sensitivity analysis is with respect to debt holders perceived estimate of volatility σ_D.

Figure 3. The effect of equity financing costs on agency costs



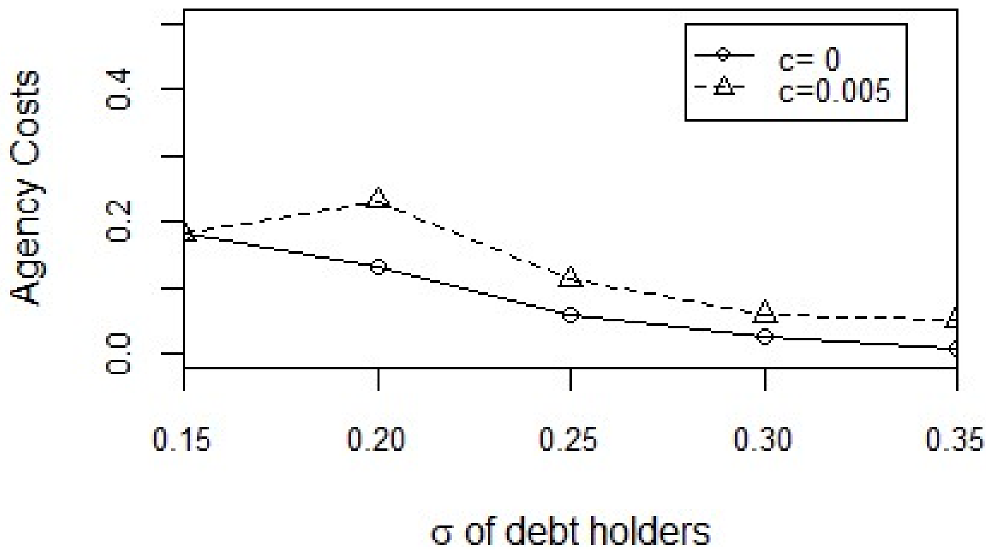
Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ= 0.06, volatility σ_E = 0.25, investment cost X = 100, bankruptcy costs b = 0.5 and tax rate τ = 0.35, loan commitment fees c = 0. Equity financing costs φ = 0 or for φ = 1 ("High φ (constrained)"). Sensitivity analysis is with respect to debt holders perceived estimate of volatility σ_D. "Agency costs" refer to total agency costs as calculated in equation (32).

Figure 4. The effect of loan commitment fees



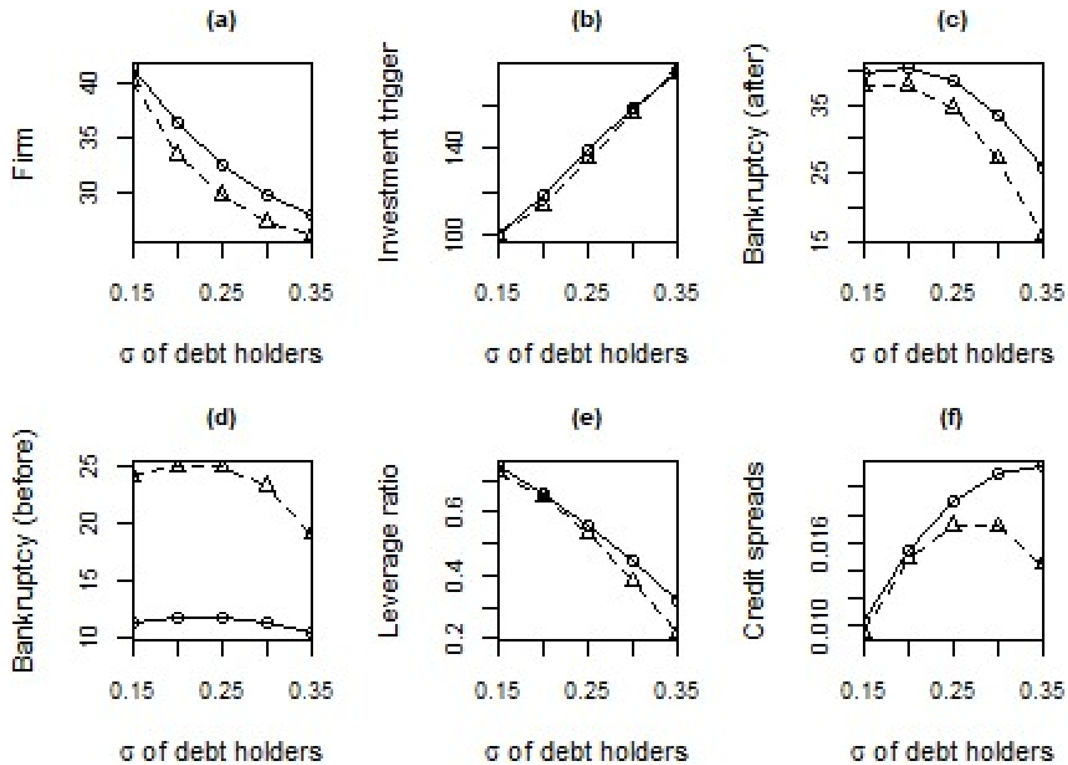
Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ= 0.06, volatility σ_E = 0.25, investment cost X = 100, bankruptcy costs b = 0.5 and tax rate τ = 0.35, equity financing costs φ = 0. Loan commitment fees c = 0 (solid line) or c = 0.005 (dotted line). Sensitivity analysis is with respect to debt holders perceived estimate of volatility σ_D. Results are based on the second-best solution (see equations (22)-(24)).

Figure 5. The effect of loan commitment fees on agency costs



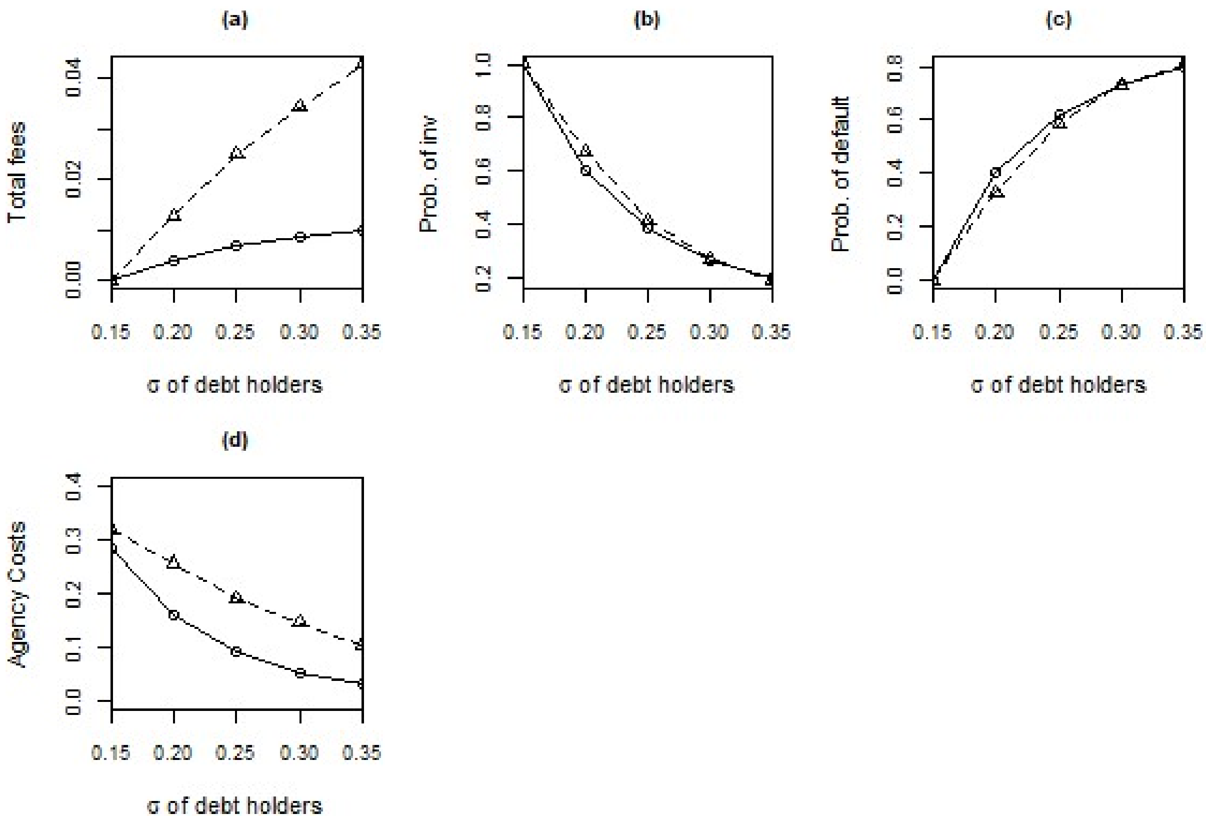
Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ= 0.06, volatility σ_E = 0.25, investment cost X = 100, bankruptcy costs b = 0.5 and tax rate τ = 0.35, equity financing costs φ = 0. Loan commitment fees c = 0 or c = 0.005. Sensitivity analysis is with respect to debt holders perceived estimate of volatility σ_D. "Agency costs" refer to total agency costs as calculated in equation (32).

Figure 6. The effect of loan commitment fees with default risk prior to investment



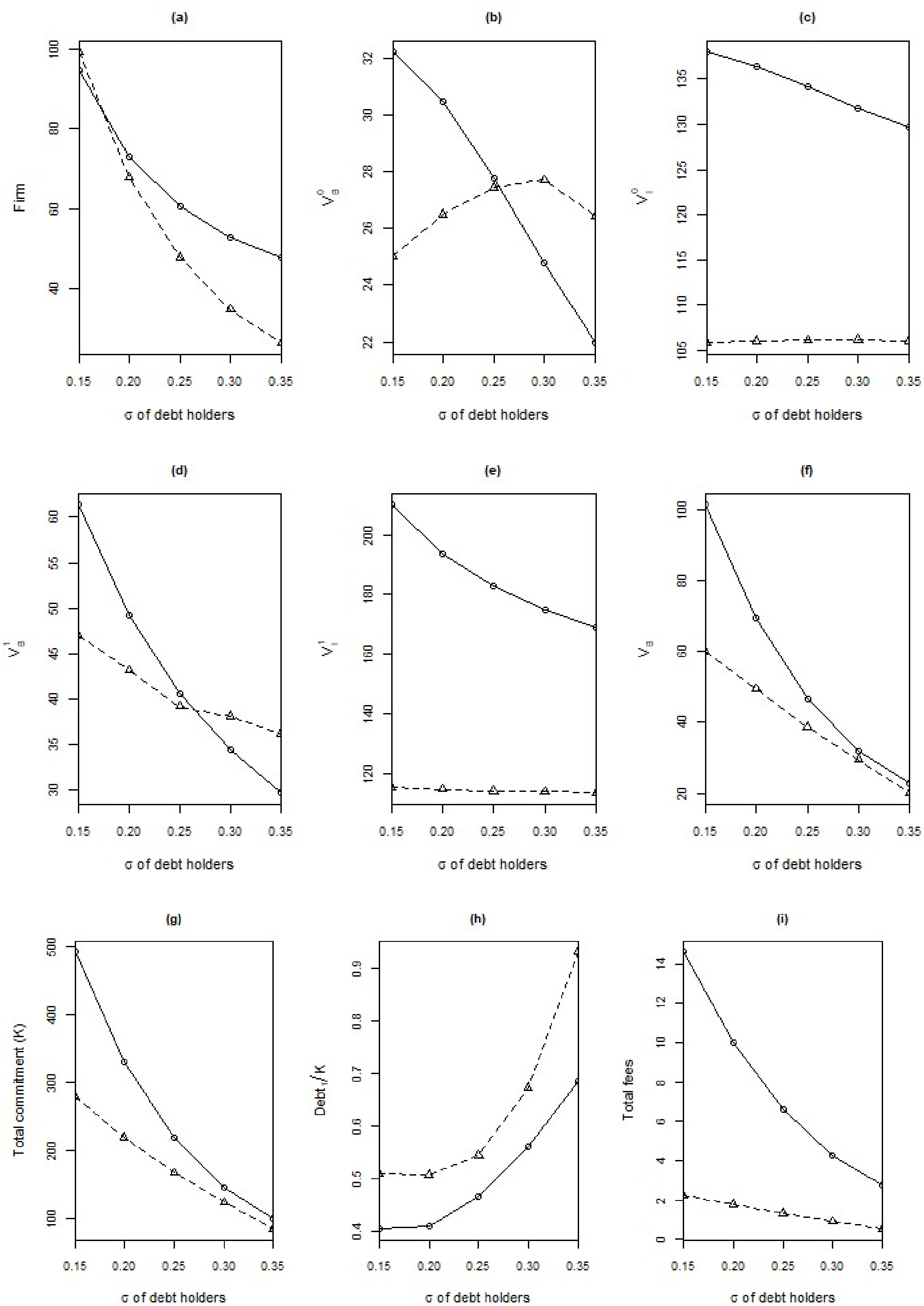
Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ= 0.06, volatility σ_E = 0.25, investment cost X = 100, bankruptcy costs b = 0.5 and tax rate τ = 0.35, equity financing costs φ = 0. Loan commitment fees c = 0.1% (solid line) or c = 0.5% (dotted line). Sensitivity analysis is with respect to debt holders perceived estimate of volatility σ_D. Results are based on the extended model of section 4.

Figure 7. The effect of loan commitment fees on total fees, probability of investment, default trigger and agency costs



Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ= 0.06, volatility σ_E = 0.25, investment cost X = 100, bankruptcy costs b = 0.5 and tax rate τ = 0.35, equity financing costs φ = 0. Loan commitment fees c = 0.1% (solid line) or c = 0.5% (dotted line). Sensitivity analysis is with respect to debt holders perceived estimate of volatility σ_D. Results are based on the extended model of section 4.2.

Figure 8. The effect of expected passage time of investment with heterogenous beliefs



Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, volatility σ_E = 0.25, investment costs $X_0 = X_1$ =100, expansion factor e_G = 2, bankruptcy costs b = 0.5 and tax rate τ = 0.35, equity financing costs φ = 0 and loan commitment fees c = 0.5% . Expected passage time t_e = 5 (solid line) or t_e = 1 (dotted line) using real drift μ = 0.06. Sensitivity analysis is with respect to debt holders perceived estimate of volatility σ_D. Results are based on the extended model of section 4.3.

Table 1: Heterogeneous beliefs between debt and equity holders with respect to volatility (σ)

A. First-best

				Optimal Capital Structure at Investment Trigger V_I				
Volatility	Firm value	Inv. Trigger (V_I)	Bankruptcy Trigger (V_B)	Equity	Debt	Leverage	Coupon	Credit Spread
σ_D =0.15	52.88	140.02	71.54	33.21	169.53	0.84	13.39	0.0190
σ_D =0.20	42.03	157.33	66.68	51.85	150.92	0.74	12.48	0.0227
σ_D =0.25 = σ_E	**35.42**	**171.57**	**57.92**	**74.82**	**127.94**	**0.63**	**10.84**	**0.0247**
σ_D =0.30	31.34	182.54	46.16	101.39	101.37	0.50	8.64	0.0252
σ_D =0.35	28.87	190.32	33.66	128.48	74.29	0.37	6.30	0.0248

B. Second-best:

				Optimal Capital Structure at Investment Trigger V_I				
Volatility	Firm value	Inv. Trigger (V_I)	Bankruptcy Trigger (V_B)	Equity	Debt	Leverage	Coupon	Credit Spread
σ_D =0.15	44.79	100.00	51.08	23.73	121.06	0.84	9.56	0.0190
σ_D =0.20	37.18	120.63	41.09	52.12	101.71	0.66	7.69	0.0156
σ_D =0.25 = σ_E	**33.41**	**140.30**	**39.86**	**71.52**	**93.65**	**0.57**	**7.46**	**0.0197**
σ_D =0.30	30.60	158.88	35.69	95.09	81.21	0.46	6.68	0.0223
σ_D =0.35	28.63	174.51	28.91	121.07	64.83	0.35	5.41	0.0234

Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, volatility σ_E = 0.25, investment cost X = 100, bankruptcy costs b = 0.5 and tax rate τ = 0.35,equity financing costs φ = 0, loan commitment fees c = 0. Sensitivity analysis is with respect to debt holders perceived estimate of volatility σ_D. For first-best equation (26) is used to derive the optimal investment trigger using total firm maximization while for second-best equation (23) is used which assumes equity-only maximization.