The Option Value of Mortgage Interest Tax Deductibility (Preliminary Draft)

Abstract

It is well known that the true tax shield value of the mortgage interest deductibility (MID) can be significantly lower than what it may seem in the presence of standard deductions. We define the *effective tax deductible rate (ETDR)* for different households. Our pay-off model identifies convexities and concavities in the relationship between the effective mortgage interest tax deduction and a set of the underlying variables including the level and volatilities of household income, house price, and local and state tax rates. To quantify the expected value of the MID, a dynamic option-pricing type model is proposed and solved. The model is simulated for a range of realistic household characteristics. Using the quantitative model we compare the relative attractiveness of an Adjustable Rate Mortgage (ARM) versus a Fixed Rate Mortgage (FRM), and demonstrate an inverse U-shape relation between the tax shield of the two types of mortgages and the household income. We find several inverse-U relationships between the volatility of underlying variables and the present value of the MID. Using a large set of mortgage loans we test the prediction of the model that, ceteris paribus, a higher state and property tax rate will increase the incentive to take larger loans. Our model can provide useful insight to households, investors, and policy-makers especially in the period of transition to a new code.

Keywords: Mortgage Interest, Tax Shield, Option Value, Housing Policies, Dynamic Valuation

1. Introduction

This paper offers a novel dynamic option pricing type view of the mortgage interest tax deductibility (MID). Residential mortgage markets are one of the largest segments of asset markets in many developed countries (Campbell (2013)). Home equity and mortgage debt are among the largest asset classes in the US households' balance sheets. The tax shield value associated with the MID can possibly be sizable and is a significant factor in household's decisions related to home ownership.

Several papers (e.g. Gyourko and Sinai (2003)) have estimated the "static" tax value of the MID provisions. However, to the best of our knowledge, the tax shield value of the the MID has never been modeled as an asset pricing problem over the life of a loan. The current paper aims to fill the gap by first justifying the need for such as an approach and then offering the model.

We focus on the specific case of US to better highlight the problem. However, MID exists in many other advanced economies including major European countries.

This optimistic picture of mortgage-friendly tax regulations, however, has some limitations in the real world. Under the current US tax code, the tax benefit of a mortgage interest payment is fully capitalized only when the sum of the itemized expenses exceeds the standard deduction allowed by the government. As an extreme case, if the sum of itemized expenses (i.e. state tax, medical expenses, contributions to charities, property taxes, and last but not least the mortgage interest) is smaller than the standard deduction, the benefits of tax deductibility vanish completely. The existence of a standard deduction feature in the tax code creates a base *opportunity cost* for itemizing to include mortgage interest in the tax return form [1]. In the event that the sum of the itemized expenses is above the standard deduction limit, the marginal value of the tax shield only applies to the proportion of tax deductible interest above the standard deduction. As a result, the average (or effective) tax deductibility is smaller than the marginal one. The bottom-line is that despite the marketing hype, there are significant numbers of homeowners who receive zero or only a partial benefit from the MID provision either because their income is too low, or because they live in states with low or zero state tax rates[2].

The problem becomes more interesting in a dynamic environment. In a multi-period

[1]The US code also does not allow one year's deductible items to be carried forward or backwards against a taxable income in the past or future; thus, if the sum of itemized expenses is smaller than the standard deduction the household loses the total tax shield value of the mortgage interest.

[2]From a public economy perspective there is also a third group: households with no mortgage to benefit from the tax provision. Our focus in this paper is on a household with a significant mortgage, who is partially benefiting from the MID.

stochastic environment, the marginal and average tax deductibility may change from one year to another, depending on the realized values of the itemized expenses. If the household experiences a positive shock to income, the state tax component may increase; hence, pushing a larger fraction of mortgage interest to the deductible region. On the other hand, if the household suffers a negative income shock (e.g. losing job for a few months), the taxable income base may go down, resulting in a reduced mortgage interest deductibility. In the extreme case, if the household is unemployed for a large fraction of the year, it may completely lose the mortgage interest tax deductibility benefit[3].

Following this formulation, we offer a dynamic contingent claim valuation model of the mortgage interest tax shield by identifying the embedded options of the MID in a multi-year (and also a perpetual) payment structure. We set up both theoretical and simulation exercises for a realistic environment, in which the agent faces possible randomness in the mortgage rate, labor income, and house prices.

The extent of benefiting from the MID also depends on the size of the state-level income tax. In states with low or zero income tax rates (e.g. Texas) almost all households will have an effective MID smaller than one (i.e. a proportion of the MID will be lost). On the other hand, in high tax locations (e.g. California or NYC) a large number of households will already exceed the standard deduction by their local income taxes, thus having a full benefit of the MID. Another contribution of this paper is to focus on the details of effective tax deductions, in particular for households with low and medium income levels.

Mortgage contracts are known to have embedded options (including options for prepaying, defaulting, and equity-sharing); however, the options-type feature of the MID is typically not considered in this list. Unlike other mortgage options, which require a decision by the mortgage owner and are typically a one-time irreversible decision (such as defaulting on the

[3]We will further discuss this pro-cyclical feature of the MID in the conclusion section.

mortgage or prepaying), the option to gain from the MID is exercised automatically and possibly multiple times, whenever the option is in the money.

We consider multiple deterministic and stochastic dynamics in the model. As the mortgage payment continues over time, the remaining balance decreases and the mortgage interest may go below the standard deductions. Therefore, a household, which initially had a positive marginal tax deductibility, may lose after a few periods. However, this will happen in future and thus the missing cash-flows associated with it will be discounted. The higher the discount rate the weaker this effect will be.

Such a behavior has implications for the redistribution aspects of the MID. It is a well-known fact that due to higher marginal tax rates, wealthier individual receive a higher subsidy. We highlight another channel by emphasizing they are also more *likely* to have a mortgage interest beyond the standard deductions. Thus, the tax benefit of the MID is the product of *two* convex functions. Consider a middle income household with an annual income of $80K that may only get 40% of their mortgage interest outside of the standard deduction region and will get get back that 40% multiplied by a lower marginal tax rate, compared to wealthier households.

We also focus on comparing the tax shields of the FRM and the ARM mortgage contracts. ARM borrowers are subject to interest payment shocks, when interest rates hike. However, the mortgage borrower faces asymmetric real payments at low and high realizations of the interest rate. The higher likelihood of tax deductibility for ARM provides a partial hedging against positive interest rate shocks. However, the hedge component has a positive correlation with the house price and income, which reduces its attractiveness.

The results of our paper are relevant for both household and policy level decisions. There is a substantial body of literature on the empirical documentation of the magnitude of the MID; however, to the best of our knowledge, very little has been written on the modeling of the dynamics of the MID under different circumstances. Thus, this paper contributes to the literature of options pricing and real estate economics by explicitly modeling the tax shield value of mortgage interest and producing model outcomes under a wide range of parameter values.

Moreover, the literature usually overlooks the fact that even if a household decides to itemize, the full benefits of the the MID may not accrue.

The current paper is organized as follows. Section 2 provides a review of the relevant literature. The institutional background of the MID is discussed in Section 3. Section 4 introduces the theoretical option pricing model and offers closed-form solutions to the model. In Section 5 we numerically simulate the model and generate a range of mortgage tax shield values for different values of underlying parameters. Section 7 discusses the interpretation of the results and also provides empirical evidence on the presentation of the MID in real estate advising websites. Finally, we offer suggestions for future research in Section 9.

2. Relevant Literature

In a broad sense this paper contributes to a growing body of literature in household finance, in particular to the growing area of possible miscalculation and forgone values in financial decision making.

Our paper is also related to the literature on optimal dynamic mortgage decisions and also the design of contracts with dynamic features. Agarwal et al. (2013) derive a closed-form formula for the optimal refinancing decisions. Campbell and Cocco (2015) propose a dynamic model of households' mortgage decisions to study the effect of structural variables on mortgage defaults and the risk premiums of mortgage contracts. Their model covers key underlying variables including labor income, house price, and interest rate risk. We add to this literature by explicitly modeling the dynamics of tax deductibility and valuing the tax shield of the embedded MID features.

Gervais and Pandey (2008) consider the endogenous response of households' balance sheet to the removal of the MID. Authors argue that households will reshuffle their debt/equity composition toward more equity and this will affect the tax revenue of government under the new regime. The key conclusion is that the regressive benefits of the MID to richer people are lower than what it may seem in the first place.

Several papers (e.g. Sinai and Gyourko (2004)) have shown that higher income households (in particular those living in regions with high house prices) benefit the most from the federal MID policy. For example, Cole et al. (2011) show that while the rate of claiming mortgage interest in the tax form is close to 100% for high-income households, there is a gap between the percentage of low-income households actually having a mortgage and the percentage claiming it in their tax files. Hilber and Turner (2014) also review the impact of the MID on homeownership and find that the MID boosts homeownership only for higher income households and in less tightly regulated housing markets.

Our paper is also related to the literature that documents behavioral biases in the mortgage market. Amromin et al. (2007) identify an arbitrage strategy between mortgage payment and investing in tax-deferred retirement accounts (e.g. 401K). However, the authors find that a large number of households give up this tax arbitrage by accelerating their mortgages payback. Keys et al. (2016) show that 20% of unconstrained households decide not to refinance when the rates become low. For the median household the forgone savings of this suboptimal behavior is a sizable amount of $11,500. Agarwal et al. (2017) document behavioral biases in choosing the right type of mortgage contracts that offer mortgage points. They document that points takers lose about $700 on average. Bajo and Barbi (2015) use a natural experiment in the Italian market to support sluggish behavior of FRM holders to refinance at lower rates.

The effect of the MID on house prices has been studied by several authors. Martin and Hanson (2016) simulate changes to metropolitan area home prices from reforming the Mortgage Interest Deduction (MID). They provide a range of estimates between 3.5%-14.5%

for a drop in the value of houses if the MID feature is removed. Damen et al. (2016) offer international evidence that the borrower's ability to pay (ATP) through a mortgage is a long-run house price fundamental.

Alpanda and Zubairy (2016) consider the distortionary effect of taxes related to housing. They use a DSGE model to compare the macroeconomic and welfare impact of various housing policies. Among other findings, they also show that eliminating the mortgage interest deduction would be the most effective in raising government tax revenue, and in reducing household debt, per unit of output lost. Additionally, Hanson and Martin (2014) use IRS data to estimate the elasticity of mortgage to marginal tax rates and also to quantify the magnitude of the MID's distortionary effect. Yagan et al. (2013) uses differences in state-level MIDs to quantify the role of MID as an insurance against local labor market shocks.

3. Institutional Background and Stylized Facts

Based on their tax policies toward imputed rent and mortgage interests, countries can be classified into three major groups:

The MID has been part of the US the federal tax code since 1913[4]. The US tax code not only allows for the tax deductibility of mortgage interest, but also excludes the implicit rental value of owner-occupied houses from the taxable income[5]. As a result, from a tax perspective homeownership is a double-dividend investment: the cost of capital to invest in a residential property is subsidized through the MID provisions; whereas, the dividends of the asset (i.e. the imputed rental value of the housing service) are not considered as taxable

[4]To make the discussion more concrete, we focus on the case of the US; however, the MID feature is not limited to the US and is offered in many other countries. Therefore, the general insights of the paper extend to other contexts too.

[5]To see the impact of this policy, compare two investors, where one buys a house and resides there, and another one invests in another type of asset with a return on investment (ROI) equal to the ROI of the house, and uses its dividend to pay for rents. While the latter investor has to pay an income tax on her dividends, the former one pays no taxes. Thus, renting a house using the investment income is more expensive than owning the same house.

Policy	Examples of Countries	Implications
No tax on imputed rent, allow for mortgage interest deductibility	USA	Preferential cost of capital for owner-occupied houses, preferential after-tax dividend for housing assets
No tax on imputed rent, no mortgage interest deductibility	Canada	Equal cost of capital for owner-occupied and rental houses, preferential after-tax dividend for housing assets
Tax on imputed rent, allow for mortgage interest deductibility	The Netherlands	Equal cost of capital for owner-occupied and rental houses, equal after-tax dividend for housing and other asset assets

Table 1: Taxonomy of Housing Tax Policy

income. US tax code allows households to deduct their mortgage interest (on their primary and secondary residents) from the taxable income. The cap for the eligible total mortgage debt is \$1 million. Households can also use the interest deductibility on a maxim of \$100,000 of the home equity loan.

Recently, several major proposals to US tax code have been put forward. Some proposals include discussions of removal or at least curtailing the MID. Our paper is also relevant to the current policy debate by offering a more realistic view of the MID benefits for different households, in particle for low and medium income ones. Though the general equilibrium effects of MID removal need to be studied too, the partial equilibrium effect, discussed in this paper, provides a sharper picture of overlooked aspects of the MID.

However, the code also has an important caveat that the applicant should switch from the *standard deduction* to *itemized deduction* , in order to be able to include mortgage interest in the tax form. Any household with the sum of itemized expenses smaller than the standard deduction will naturally continue filing under the standard deduction. For such a household,

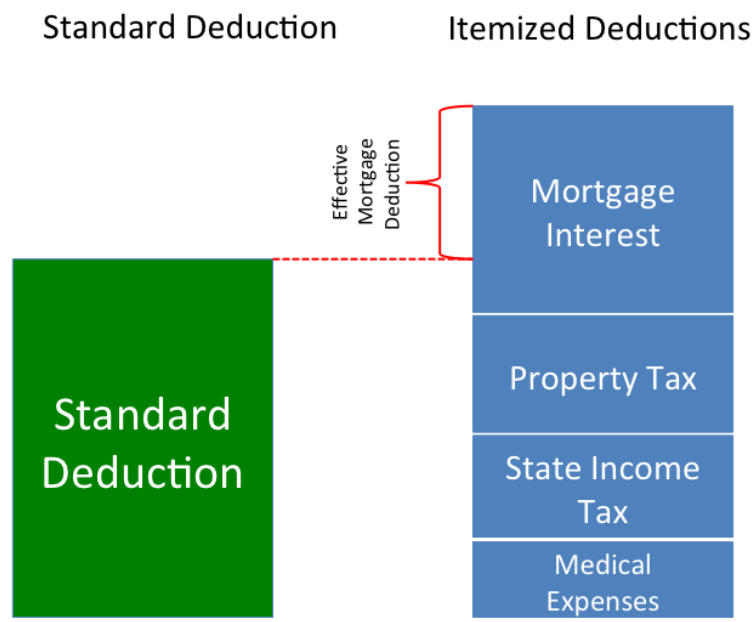

Figure 1: Standard versus Itemized Deductions. The figure shows main components of itemized deduction. The household effectively deduces only the fraction of the mortgage interest that after being cumulated by other itemized expenses exceeds the standard deduction.

the effective MID is equal to zero. Figure 1 provides a schematic view of the partial tax deductibility, when the standard deduction feature creates an *opportunity cost* for using the MID feature.

Over time the regulations governing the MID have changed. A key example is the US Tax Reform Act of 1986 that raised the standard deductions and as a result reduced the percentage of households specifically benefiting from the MID. See Ventry (2010) for a detailed historical account of mortgage-related tax code changes.

Subsidies to home ownership, through MID and tax exemptions for the imputed rent, have been the subject of criticism for a long time. The Economist magazine calls it a 'senseless subsidy'. Opponents argue that those subsidies distort housing markets through reducing the user cost of ownership, by giving incentives to purchase suboptimal larger homes, and by diverting capital from more productive sectors to the housing sector. Glaeser and Shapiro (2002) refer to the stable homeownership rates over several decades, despite

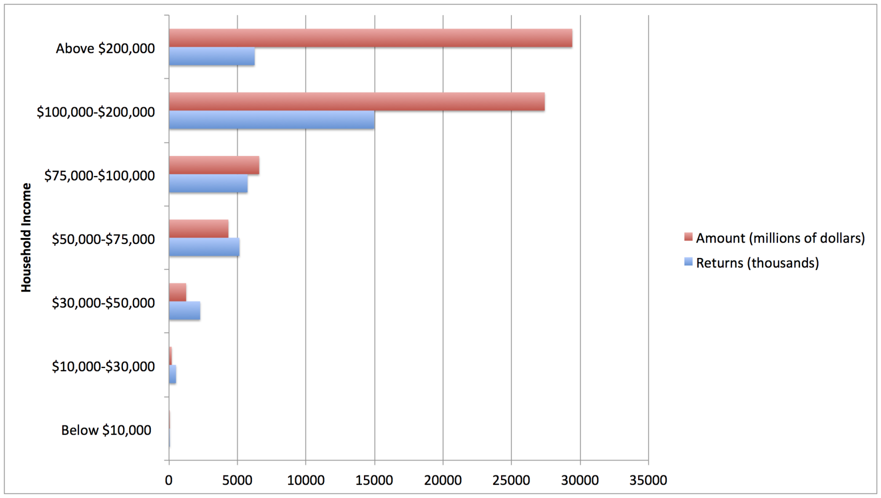

Figure 2: Number of MID Filed and Total Amount of Tax Return for Income Levels. Source: The Joint Committee on Taxation

significant changes in the tax code, as an evidence that those favorable tax policies do not necessarily influence homeownership decisions. Gervais (2002) uses a general equilibrium model to argue that individuals, regardless of their income level, will prefer a world without those subsidies.

Reforming the current tax system, however, faces major barriers. Removing or diluting the preferential tax treatment of home ownership will not only affect future home buyers but also has a substantial impact on 'current' home owners. The demand for purchasing homes will drop following those reforms causing the price of current houses to decline. One can imagine that a reform today will take away a proportion of the benefits paid in the past.

To track the phenomena in the data, Figure 2 shows the number of MID requests included in tax return files and the amount of relief households of different income level received (data source: Congress (2014)). The graph clearly shows that the largest benefits of the MID goes to households with an income level higher than $100,000. Given that the median US household income is around $52,000, which is a clear bias of the MID toward higher income households.

In 2017, the standard deductions for a married couple filing jointly are $12,600. Consider a typical household with an annual income of $100,000, house value of $250,000, loan-to-value of 80%, mortgage interest rate of 3%, state tax rate of 4%, and the property tax of 1%. The sum of state and property taxes of the household will be $6000 and the mortgage interest will be $6000. Thus, considering the standard deduction option the household is better off not itemizing because the sum of itemized expenses $6500 + $6000 = $12500, is smaller than the standard deduction[6].

Now consider the following two scenarios discussed in Table 2. As long as the house price is below $260,000 the effective mortgage interest deductibility remains zero. However, when house price appreciates and goes above $260,000 the household partially benefits from the MID. When the house price exceeds $860,000 the household starts fully benefiting from the MID.

House Price	Property Tax	Sum of Itemized Expenses	Mortgage Interest Deduction
$100,000	$1000	$11500	$0
$200,000	$2000	$12500	$0
$300,000	$3000	$13000	$400
$860,000	$8600	$18600	$6000
$1,000,000	$10,000	$22600	$6000

Table 2: Effect of House Price Dynamics of Effective MID.

The effective mortgage interest deduction reported under various scenarios in Table 2 resemble a long position on a call option, when the price of a house is low and a short position on a put option, when the house price is high. The overall behavior of the mortgage tax deductibility is plotted in Figure 3.

At the high levels of income two new features kick in and reduce the effective MID. The first one is the cap on the MID, which limits the size of the mortgage to $1,000,000 for a married couple, filing jointly. The second provision is the alternative minimum tax (ATM),

[6]The new US tax law is likely to change the threshold for the standard deduction as well as the limit for state and local taxes. The current version of the paper is written based on the existing US tax law.

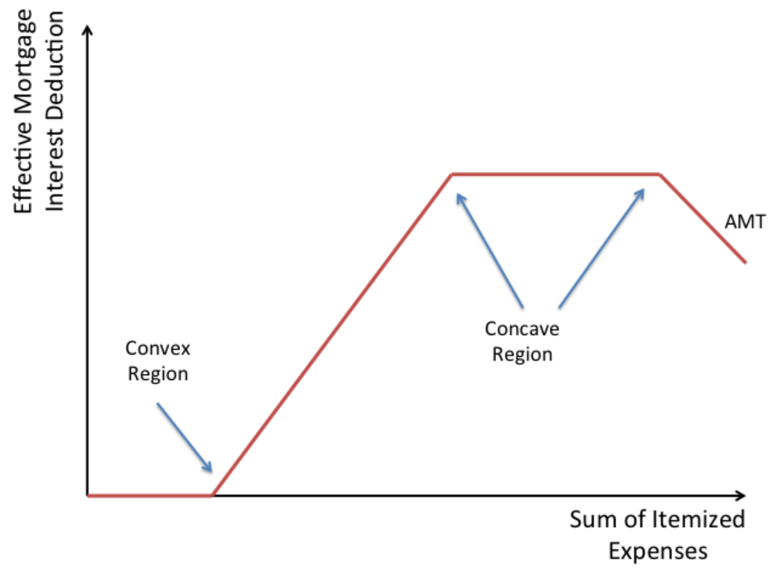

Figure 3: Effectives Mortgage Interest Deduction versus Sum of Itemizes Expenses. The curve contains convex, linear, and concave regions.

which limits the total size of eligible deductions from the taxable income. Though ATM does allow for the inclusion of mortgage interest under the alternative tax, it may still adversely affect the marginal value of the MID (Feenberg and Poterba (2003)).

4. Dynamic Option Pricing Model

In this section a dynamic contingent claim model of the tax shield of the mortgage interest is presented. The major goal of this model is to elaborate the qualitative behavior of the tax shield (as a non-transferable asset attached to the mortgage contract) in response to changes in the major underlying parameters. In order to preserve the elegance of the model to highlight the major economic force, we make a few simplifying assumptions. Those assumptions will be relaxed when in the next section, when we solve the model numerically, using real world basic parameters.

4.1. Benchmark Case: Full Marginal Deductibility

If homeowners did not have to give up their standard deductions from the taxable income and could directly subtract mortgage and property taxes from the taxable income the effective cost of homeownership would have been given by:

$$C_t = (1-\tau)iK + (1-\tau)\tau_P P_t + (1-\tau_i)r_m(P_t - K) + M \tag{1}$$

where, τ is the marginal tax rate, i is the mortgage interest, P_t is the price of house, K is the debt in the household capital structure, τ_P is the property tax rate, τ_i is the tax on investment revenue, r_m is the opportunity cost of investment, and finally M is the maintenance costs.

The valuation of the tax shield of the mortgage interest in this case is very straightforward:

$$V = \frac{(1-\tau)iK}{r} \tag{2}$$

where r is the discount rate. If the mortgage interest rate and discount rates are equal ($r = i$), the tax shield value simply reduces to $(1-\tau)(P_t - K)$, meaning that the agent pays only the $1-\tau$ fraction of the debt price.

Equation 2 is a standard way of representing the present value of the tax shield associated with a perpetual mortgage. However, under the realistic setting not all households can write off their mortgage interest $i(P_t - K)$ from their taxable income. Therefore, the magnitude of the tax saving will be a piece-wise linear function of underlying variables. Moreover, due to the stochasticity in income, property taxes, and possible mortgage interests (in the case of ARMs) a household may experience different regimes of the MID over the lifetime of the mortgage.

4.2. Value Function

The value of interest tax shield with the existence of standard deductions is setup as follows. A homeowner has to pay a loan with a finite remaining life expressed as n years. By the end of year j, the owner decides to either use the MID or use the standard deduction ($\bar{T}_j$); in other words, exercising or not exercising option j. The value of this option j (which expires at time Y_j with no early exercise possibility) can be formulated as:

$$V_j = \mathbb{E}\{(iK_j + t_j(P, I) - \bar{T}_j)^+\} \tag{3}$$

where $t_j(P, I)$ is the total state and property tax associated with income I and house price P at time j. Applying the same logic to all remaining years at year t, leads to expressing the total value of the tax shield for a loan that has n years to mature as:

$$V_j = \sum_{j=t}^{n} \mathbb{E}\{(iK_j + t_j(P, I) - \bar{T}_j)^+\} \tag{4}$$

Which is equivalent to representing the value of the interest tax shield as $n - j$ European basket options.

Further Assumptions. The representation of equation 4 can be further simplified with a few assumptions. When the interest rate is large enough and the mortgage is of a long-term one (e.g., 30-year) it is plausible to approximate the horizon payment as infinite. i.e., assuming that the mortgage is perpetual. This assumption allows us to eliminate time derivatives of the value function and focus on other state variables. There are two major stochastic processes that the agent faces: 1) the sum of labor income and house price[7]; 2) the interest rate on the mortgage.

[7] One can discuss separate stochastic processes for income and house price. However, this just adds to the complexity of the model without providing any major new insights.

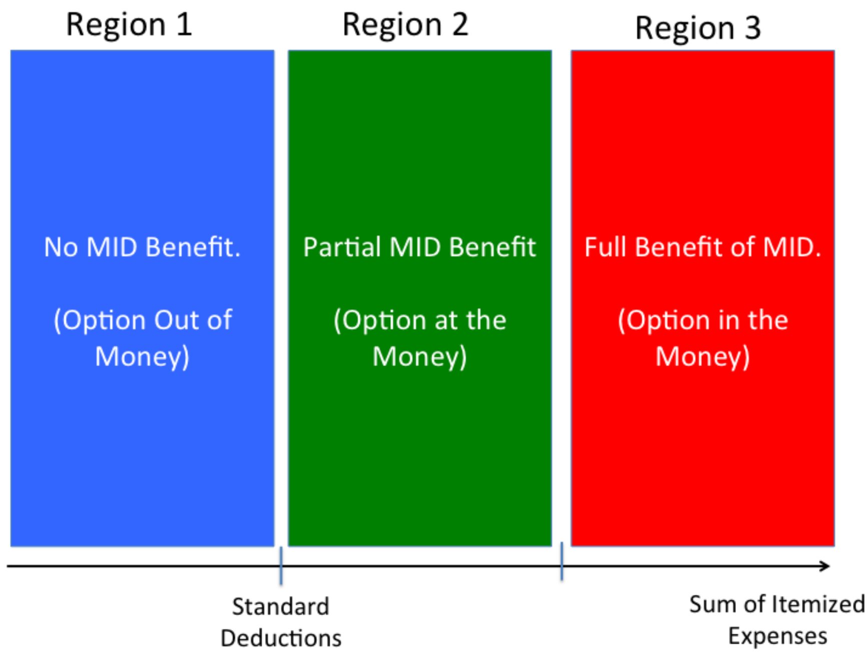

Figure 4: Value Function over Three Major Regions. The transition from one region to another is costless and the process can freely move between them.

We assume that the income and house price follow the geometric Brownian motion (GBM) processes and the mortgage interest rates is a mean-reverting process. These stochastic differential equations have been reported in other papers in the field as well (e.g. Wallace (2011)).

Consequently, the value of the interest tax shield, defined on the two state variables i, and P in a continuous time-frame would be given by:

$$V_t(i, P, I) = \mathbb{E}_t \int_t^{\infty} (iK + t(P, I) - \overline{T})^+ \tau e^{-rs} ds \tag{5}$$

Using the Feynman-Kac representation, the stochastic integral can be written in a partial differential equation (PDE) form. Based on the state variables there will be three regions with different PDEs associated with the value of the integral. Figure 4 provides a graphical illustration of the there regions.

The no-arbitrage Bellman equations associated with each region are:

1. When $iK + t(P,I) < \overline{T}$, the option is out of the money: $rV_1 = \mathbb{E}(\frac{dV_1}{dt})$

2. When $iK + t(P,I) > \overline{T}$ and $t(P,I) < \overline{T}$, the option is only partially in the money: $rV_2 = \underbrace{\tau(iK + t(P,I) - \overline{T})}_{\text{Current Cash-Flow}} + \mathbb{E}(\frac{dV_2}{dt})$

3. When $t(P,I) > \overline{T}$, the option is fully in the money: $rV_3 = \underbrace{\tau iK}_{\text{Current Cash-Flow}} + \mathbb{E}(\frac{dV_3}{dt})$

The solution to each PDE will contain a component that represents the present value of the current tax benefit plus terms adjusting for the possibility of visiting other regions.

Boundary Conditions.

1. Smooth pasting and value matching conditions should hold when V_1 and V_2 meet each other at the boundary point of $iK + t(P,I) = \overline{T}$

2. Smooth pasting and value matching conditions should hold when V_2 and V_3 meet each other at the boundary point of $t(P,I) = \overline{T}$

3. When $iK \to \infty$ or $t(P,I) \to \infty$ the likelihood of returning to region 2 becomes zero and the discount factor component of V_3 disappears and $V_3 = \frac{\tau iK}{r}$.

4. If $t(P,I)$ is approximated by a geometric Brownian motion (GBM) process, the boundary point of $t(P,I) = 0$ becomes an absorbing state. At that point the option value of visiting region 2 becomes zero and $V_1 = 0$.

4.3. Time-Invariant Interest Rates

The first case is when interest rates are constant but the sum of labor income and house price is volatile. Following other papers we assume that the P follows a geometric Brownian motion (GBM) process.

$$dP = \mu_P P dt + \sigma_P P dW_P \tag{6}$$

where μ_P is the growth rate, σ_P is the volatility parameters of the P process, and dW_P is the Brownian shock.

Applying the general framework of the previous subsection, we get the specific partial differential equations governing the value function in each region:

1. When $iK + t(P, I) < \overline{T}$: $rV_1 = \mu P V_1' + \frac{1}{2}\sigma_P^2 P^2 V_1''$

2. When $iK+t(P, I) > \overline{T}$ and $t(P, I) < \overline{T}$: $rV_2 = \underbrace{\tau(iK + t(P, I) - \overline{T})}_{\text{Current Cash-Flow}} + \mu P V_2' + \frac{1}{2}\sigma_P^2 P^2 V_2''$

3. When $t(P, I) > \overline{T}$: $rV_3 = \underbrace{\tau iK}_{\text{Current Cash-Flow}} + \mu P V_3' + \frac{1}{2}\sigma_P^2 P^2 V_3''$

The general solution for ODEs is known: $V = A_1 P^{\beta_1} + A_2 P^{\beta_2}$, where A_1 and A_2 are coefficients to be found using proper boundary conditions and β_1 and β_2 are the roots of the characteristic function $r - \beta\mu_P - \frac{1}{2}\beta(\beta - 1)\sigma_P^2 = 0$, $\beta_1 = \frac{1}{2} - \frac{\mu_P}{\sigma_P^2} + \sqrt{(\frac{1}{2} - \frac{\mu_P}{\sigma_P^2})^2 + \frac{2r}{\sigma_P^2}} > 0$, $\beta_2 = \frac{1}{2} - \frac{\mu_P}{\sigma_P^2} - \sqrt{(\frac{1}{2} - \frac{\mu_P}{\sigma_P^2})^2 + \frac{2r}{\sigma_P^2}} < 0$. The full solution would be the sum of specific and general solutions of ODEs.

1. When $iK + t(P, I) < \overline{T}$: $V_1 = A_1^1 P^{\beta_1} + A_2^1 P^{\beta_2}$

2. When $iK + t(P, I) > \overline{T}$ and $t(P, I) < \overline{T}$: $V_2 = \frac{\tau(iK+t(P,I)-\overline{T})}{r} + A_1^2 P^{\beta_1} + A_2^2 P^{\beta_2}$

3. When $t(P, I) > \overline{T}$: $V_3 = \frac{\tau iK}{r} + A_1^3 P^{\beta_1} + A_2^3 P^{\beta_2}$

We need to find the value of six unknown coefficients $\{A_1^1, A_2^1, A_1^2, A_2^2, A_1^3, A_2^3\}$

Boundary conditions will eliminate some implausible solutions. In particular, when $P \to 0$ the option value of having a mortgage interest above the standard deduction is zero. Therefore, $A_2^1 = 0$. Moreover, if $P \to \infty$ the mortgage interest option is fully in the money and the likelihood of returning to region 2 or region 1 is zero. This immediately implies that $A_1^3 = 0$. We still need to find the four unknown $\{A_1^1, A_1^2, A_2^2, A_2^3\}$ by imposing value matching and smooth pasting conditions[8] at the two boundaries between regions 1,2 and regions 2,3.

$$\begin{cases} V_1'(\overline{T} - iK) = V_2'(\overline{T} - iK) \Rightarrow A_1^1 \beta_1 P^{\beta_1 - 1} = A_1^2 \beta_1 P^{\beta_1 - 1} + A_2^2 \beta_2 P^{\beta_2 - 1} + 1 \\ V_1(\overline{T} - iK) = V_2(\overline{T} - iK) \Rightarrow A_1^1 P^{\beta_1} = A_1^2 P^{\beta_1} + A_2^2 P^{\beta_2} \\ V_2'(\overline{T}) = V_3'(\overline{T}) \Rightarrow A_2^3 \beta_2 P^{\beta_2 - 1} = A_1^2 \beta_1 P^{\beta_1 - 1} + A_2^2 \beta_2 P^{\beta_2 - 1} + 1 \\ V_2(\overline{T}) = V_3(\overline{T}) \Rightarrow A_2^3 P^{\beta_1} = A_1^2 P^{\beta_1} + A_2^2 P^{\beta_2} \end{cases} \tag{7}$$

Details of solution are provided in the Appendix.

Theorem 4.1. *A higher volatility of underlying variables increases the MID option value in region I, reduce its value in region III. The effect of volatility on the value of the MID option for the second region is ambiguous.*

Proof. Appendix □

Corollary 4.2. *1) When $iK \to \infty$, region 1 vanishes.*
2) When $iK \to 0$, region 2 vanishes. In other words, the agent either gets zero benefit from the MID or gets a full benefit.
3) When $\tau \to 0$, region 3 vanishes.

4.4. Time-Variant Interest Rates

The second case, which lends itself to a closed-form analytical solution, is when the process P is assumed to be constant and the interest rate on mortgage is stochastic. This

[8]The reader should note that this class of problem, in which the underlying processes freely move between regions, is different than optimal stopping problems in the real options literature. However, smooth pasting and value matching conditions are applied in both classes of problems (with a different interpretation of the smooth pasting condition.)

problem resembles an adjustable rate mortgage (ARM) for a household with a stable income and house price.

Following the standard literature we assume that the interest rates dynamics is given by a mean-reverting model:

$$di = \mu_i(\bar{i} - i)idt + \sigma_i idW_i \tag{8}$$

The general approach to find the value function is similar to the previous case; however, the ODEs associated with the value functions will be different.

1. When $iK + t(P, I) < \overline{T}$: $rV_1 = \mu_i i(\bar{i} - i)V_1' + \frac{1}{2}\sigma_i^2 i^2 V_1''$

2. When $iK + t(P, I) > \overline{T}$ and $t(P, I) < \overline{T}$: $rV_2 = \underbrace{\tau(iK + t(P, I) - \overline{T})}_{\text{Current Cash-Flow}} + \mu_i i(\bar{i} - i)V_1' + \frac{1}{2}\sigma_i^2 i^2 V_1''$

3. When $t(P, I) > \overline{T}$: $rV_3 = \underbrace{\tau iK}_{\text{Current Cash-Flow}} + \mu_i i(\bar{i} - i)V_1' + \frac{1}{2}\sigma_i^2 i^2 V_1''$

The general solution to the valuation ODE is given by $Ai^\theta H(\frac{2\mu_i}{\sigma_i^2}, \theta, b)$, where θ is the roots of the characteristics function $\frac{1}{2}\sigma_i^2\theta(\theta - 1) + \mu_i\bar{i}\theta - r = 0$ and $b = 2\theta + \frac{2\mu_i\bar{i}}{\sigma_i^2}$, and H is the confluent hypergeometric function $H(X, \theta, b) = 1 + \frac{\theta}{b}X + \frac{\theta(\theta+1)}{b(b+1)}\frac{X^2}{2!} + ...$

$i = 0$ is an absorbing state for the process. Moreover, $i \to \infty$ means that it will take a very long-time for the interest rate to return to its long-run mean.

5. Quantitative Analysis

The theoretical model provides qualitative insights on the behavior of the MID option; however, we need to impose some restrictive assumptions in order to generate analytical results. In this section we relax several assumptions to produce quantitative model outputs, based on real-world parameter values. The model cannot be solved anymore; however, numerical solutions are applicable for our purpose.

5.1. Relaxed Assumptions

We relax several restrictive assumptions, which were necessary for producing closed-form solutions. First, we allow the mortgage to have a finite life. This new assumption introduces a major impact from the discount rates on the option value. If discount rates are higher the tax saving happening in far future periods will be heavily discounted. If the agent starts with an initial condition below the standard deduction and considers the option value of moving above the standard deduction in future, due to a higher level of income or house price, a high discount rate will significantly dilute the value of such option.

Second, we assume three separate stochastic processes for the income, house price, and mortgage interest rates and allow all three to be random. In order to make the comparisons over different levels of income consistent, we keep the house price fixed [9].

The baseline parameters of the model are presented in Table 6. However, the results are plotted for a range of parameter values around the baseline values. The reader should note that the volatilities of aggregate house price and household income would be much lower than the values reported in the table. However, the aggregate volatility hides the idiosyncratic volatility by smoothing individual shocks. Our unit of analysis is a single household and thus we need to use estimates of individual volatility, rather than aggregate volatility.

In the following analysis we report both the absolute value of the tax shied as well as a measure, which is called the effective tax shield. Since the present value of the MID will depend on the size of the mortgage, we use a normalized measure defined as the ratio of the present value of MID under the current tax code to the present value of MID, if the tax code allowed for directly deducing mortgage interest. The measure varies between zero and one. The zero value means that for the given level of income, house price, and interest

[9] As an extension one can follow the standard practice, which recommends a reasonable ratio between the house price and the annual income. By fixing an value to income ratio, for every level of income the initial house price is a constant multiple of the income.

Variable	Source of Data	Estimated or Reported Value
House price growth rate	US House Price Index	4%
House price volatility	...	10%
Household income growth rate	Census Bureau	1.0%
Household income volatility (std)	Census Bureau	20%
Long-run mean of interest rates	FRED	4%
Mean-reversion rate of interest rates	FRED	0.3
Volatility of interest rates	FRED	2.5%
Standard deductions (married couple filing jointly)	IRS	12600
Ratio of House Price to Annual Income	Industry recommendations	2.5
Marginal Income Tax Rate	IRS	28%
Average State's Income Tax	-	4%
Property Tax	Industry recommendations	1.0%
Loan to Value (LTV)	Industry practice	80%
Median Household Income		$55000
Median House Price		$180000

Table 3: Key Baseline Parameters.

rates the household benefit nothing from MID. On the other hand, a measure close to one signifies that the sum of itemized expenses is high enough to let the household fully benefit from MID.

5.2. Stochastic Processes

We pick two forms of stochastic processes for the interest rate. The first form is the usual mean-reverting rate (using a Vasicek process), used by other papers. The second one is a geometric Brownian motion model, used by Li et al. (2004).

5.3. Effect of Income

Figure 6 shows the impact of household income on the effectiveness of the MID. The shape of the income-tax shield curve is convex for low levels of income and concave for high levels of income. The switching curvature implies that uncertainty will increase the value of the tax shield in the low levels of income and will *decrease* the tax shield value for high levels of income.

When the income is low the tax shield of the MID is close to zero. Higher uncertainty, will increase the likelihood of passing the standard deduction level. On the other hand, when the income is sufficiently high, the sum of itemized expenses is well above the standard deduction cutoff and the household is fully benefiting from the MID. In that case, a higher uncertainty, only increases the likelihood of going below the standard deduction line. Thus, it reduces the value of the MID. The full deductibility region behaves like a digital option, where the pay-off of the option is constant (equal to one in our case) and will be triggered if the underlying process hits an upper bound. Ghoddusi and Fahim (2016) provide insights on how an increased volatility of the underlying process may actually reduce the value of digital options.

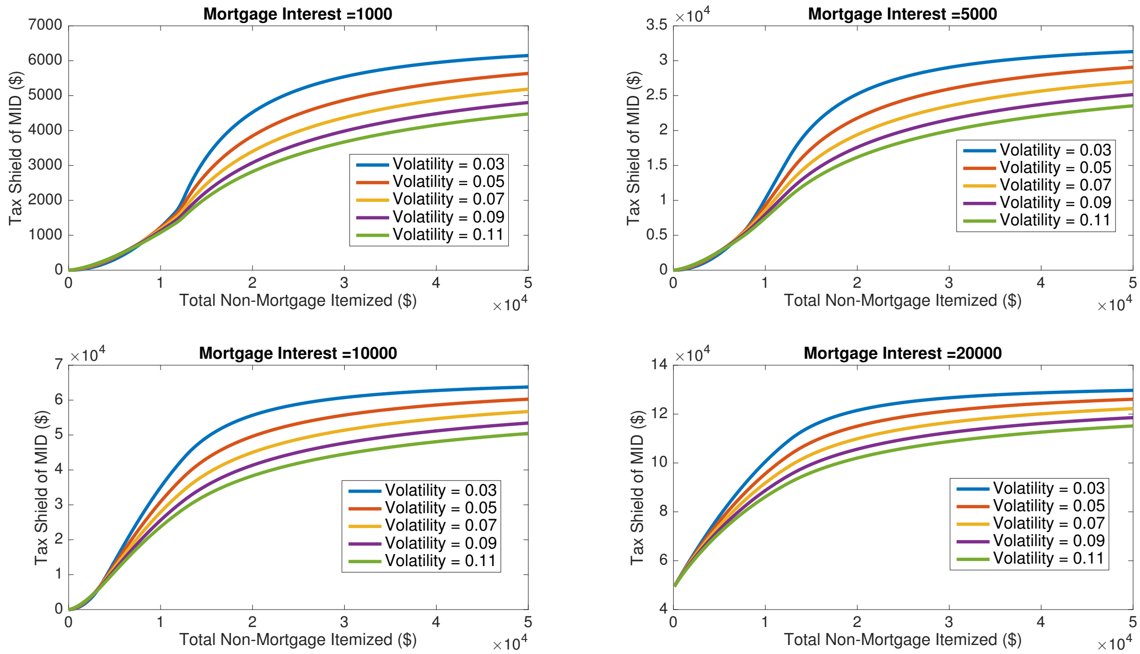

Figure 5: Value of MID Option as a Function of Current Income and Volatility

5.4. Effect of Mortgage Interest Volatility

If the household finances the house purchase using adjustable rate mortgage (ARM), the volatility in the mortgage interest rate affects the value of embedded tax shield options and the effectiveness MID. Figure 7 plots the effectiveness of MID for different levels of income and mortgage interest volatility. Consistent with the theoretical predictions, we observe that for low levels of income a higher volatility of mortgage interest increases the effectiveness of the MID.

Moreover, we notice a monotonic effect from the length of the mortgage on the level of the MID effectiveness as well as on the sensitivity of the MID effectiveness to interest volatility.

5.5. Fixed versus Adjustable Mortgage

A major question related to our analysis is whether there is a significant difference between borrowers of fixed versus adjustable-rate mortgages. Johnson and Li (2014) report

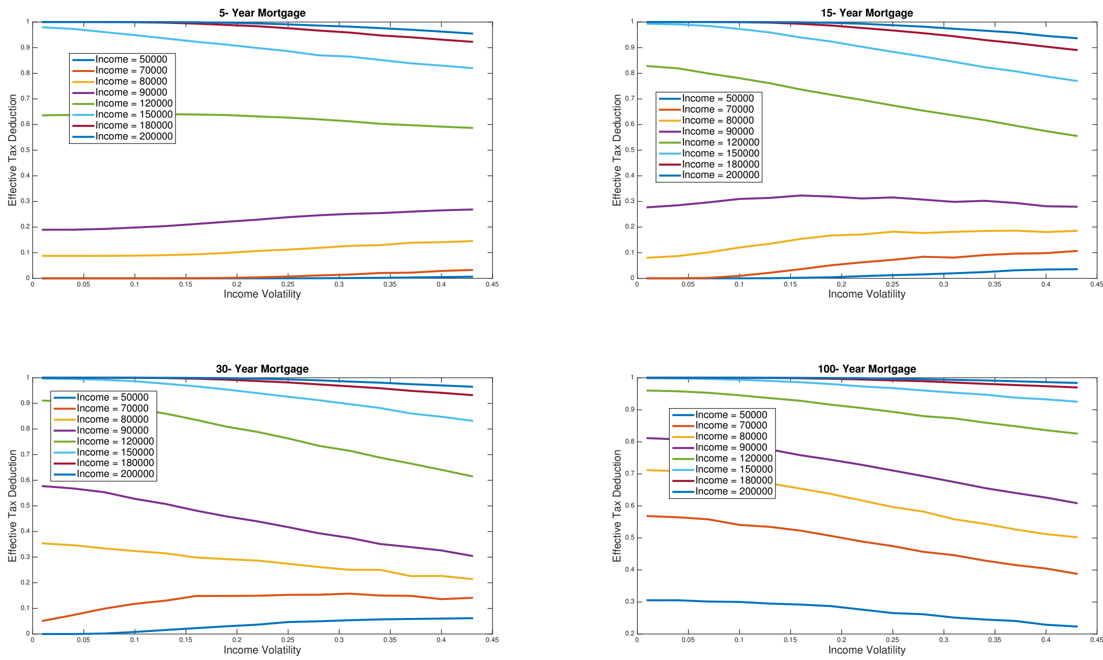

Figure 6: Effective Tax Rate as a Function of Income Volatility

that the demographic and financial characteristics of the community are not very different; however, ARM borrower are more likely to be credit constrained. Badarinza et al. (2014) study the dynamics of the ARM versus the FRM over time and across countries. They find that the short-term gap between FRM and ARM rates matter for the choice of one versus another.

In practice, ARM rates are typically lower than FRM contracts with similar characteristics. There are two reasons for such a difference. First, a FRM contact with the same rate as the expected rate of ARM is more attractive because it provides a valuable embedded option to refinance when rates drop. Second, risk-averse agents dislike volatile interest rates and are willing to pay a premium to obtain a fixed-rate mortgage.

However, by bringing the MID feature back to the picture we show that the relative attractiveness of the ARM versus the FRM may change. Our model suggests that ARM

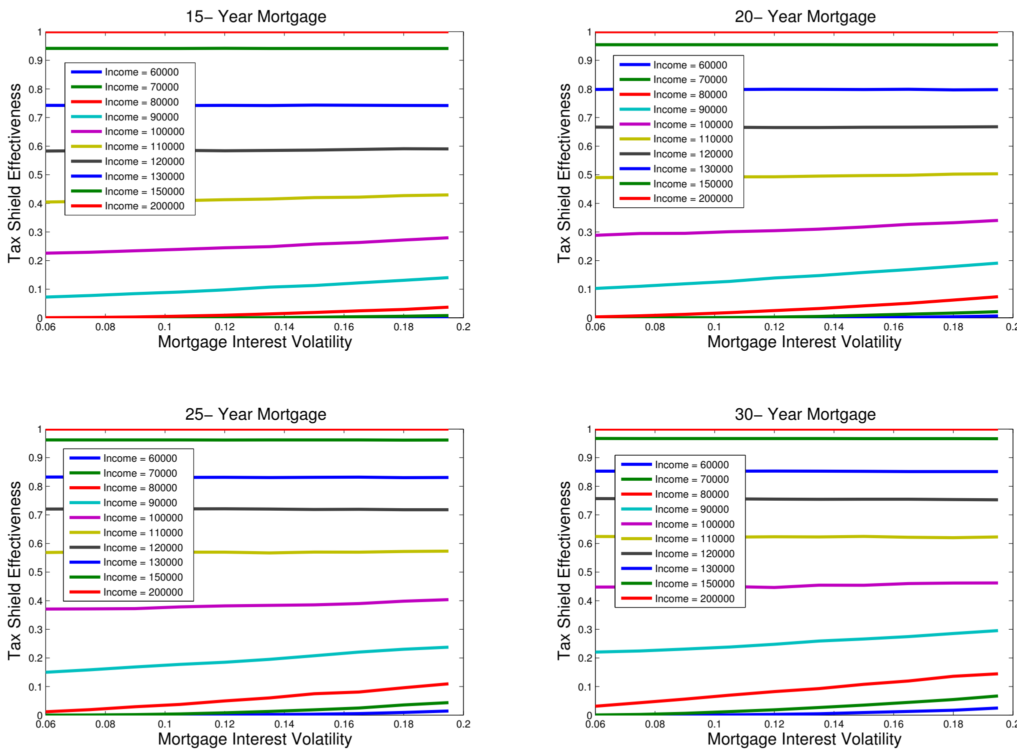

Figure 7: Impact of Mortgage Interest Volatility on MID Effectiveness

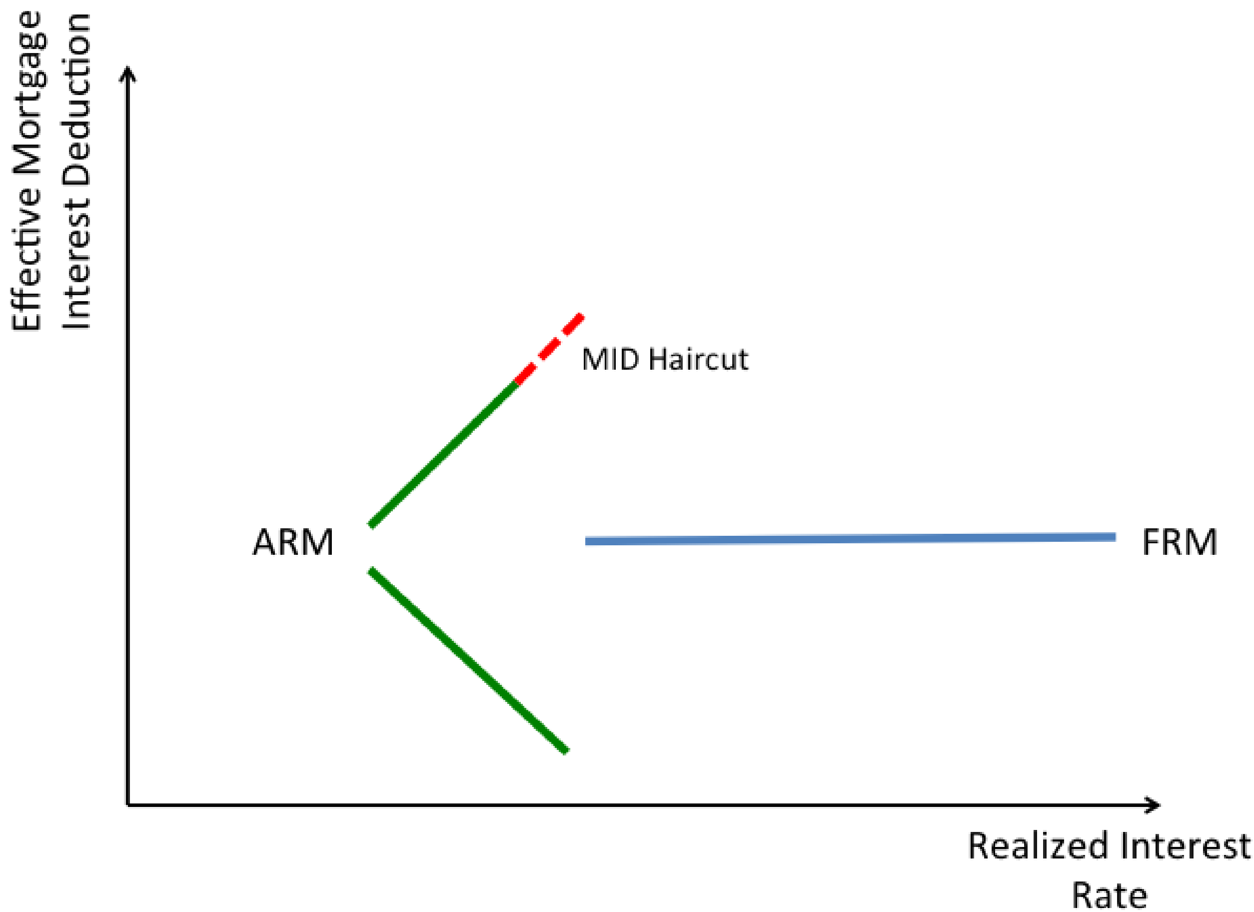

Figure 8: The MID Haircut Applied to Upside Realizations of ARM Interest Rate.

cash-flow has a higher chance to benefit from the MID. In particular, when a positive shock to interest rates takes place, the interest payment may enter the deductibility region. As a result, the mortgage owner receives a haircut on the payment (see Figure 8).

Figure 10 shows the different tax shield value of an ARM versus a FRM mortgage. We keep the mean value of the ARM equal to FRM and only allow for mean-preserving shocks[10]. This way we are able to quantity the pure value of volatility.

[10]In practice, the majority of ARM contracts have a cap on the interest rate. We abstract from this feature.

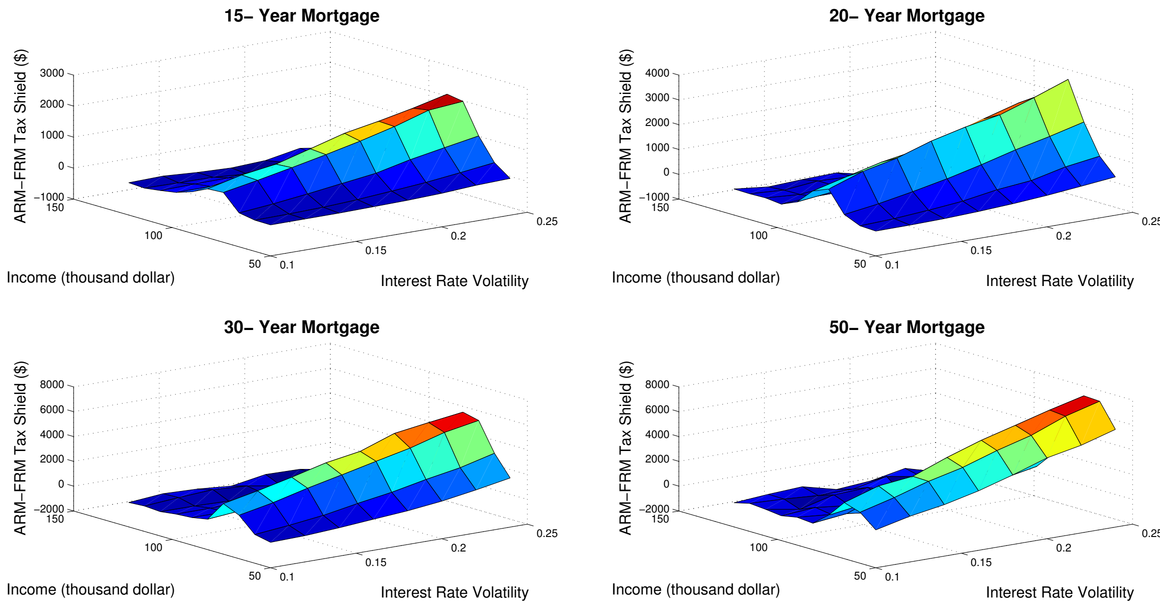

Figure 9: Difference between the Tax Shield of ARM and FRM Mortgage with Equal Average Interest Rates and a GBM Process for the Interest Rate

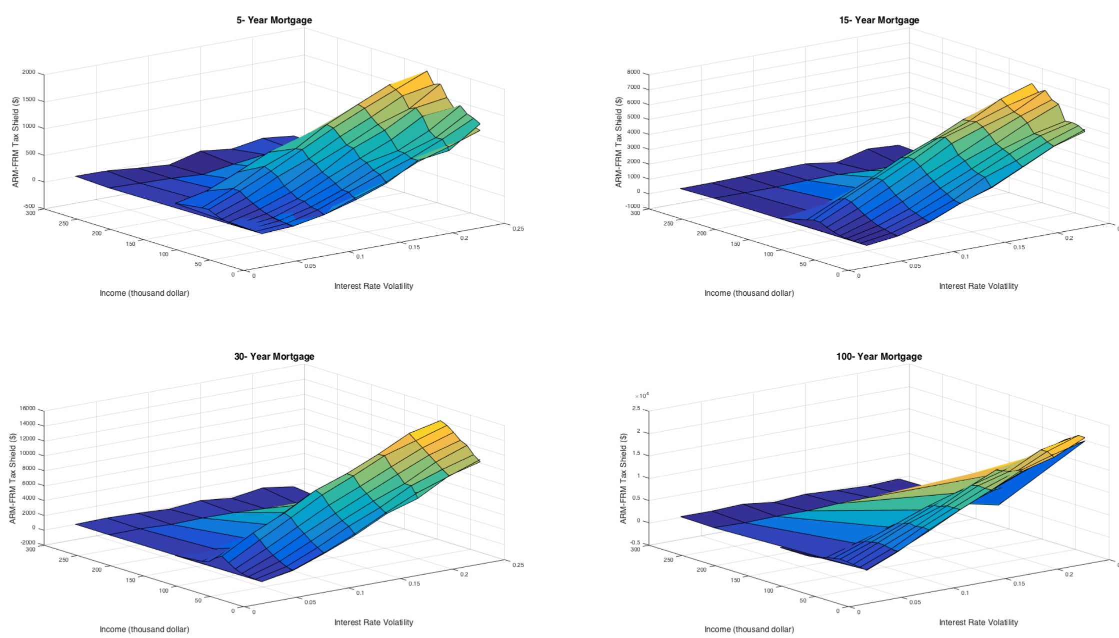

Figure 10: Difference between the Tax Shield of ARM and FRM with Equal Average Interest Rates and a CIR Process for the Interest Rate

We observe an interesting pattern in the relative attractiveness of the FRM versus the ARM. When household income and mortgage interest are high enough ARM and FRM are fully in the money and there is no advantage of the ARM over the FRM. Moreover, when household income and mortgage interest too low both the FRM and the ARM are out of money. However, with a baseline itemized expenses close to the standard deduction limit the option value of the ARM starts to increase; while, the FRM is completely out of the money. Based on these patterns we observe that the relative attractive follows an inverse U-Shape form.

Theorem 5.1. *With equal expected interest rates, an ARM always weakly dominates the FRM contracts.*

Proof. With an FRM contract the MID is $\tau[r + I + Pt - \overline{tau}]^+$ and with an ARM contract the *expected* MID is given by $\tau\mathbb{E}[\tilde{r} + I + Pt - \overline{\tau}]^+$. Note that $r = \mathbb{E}[\tilde{r}]$. The $[.]^+$ operator is a convex function. Therefore, by the Jensen's inequality $\mathbb{E}[X]^+ \geqslant [\mathbb{E}(X)]^+$

Theorem 5.1 states that there is no scenario in which an ARM contracts offers a lower expected MID.

5.6. Optimal Filing Strategy

US tax code does not allow households to carry over their mortgage interest deduction to another year if their effective deductible rate is low this year. If this feature was permitted, households could form an optimal filing strategy to report zero mortgage interest for a few years and then deduct a large cumulated interest in a single year.

However, homeowners still play around with the payment of property taxes by concentrating payments of one and half years worth of payment in one year. This is achievable if the household pays the property taxes of one year within that year and also a part of previous year's taxes before Jan 15th of the current year[11]. Using this strategy they might

[11]The deadline for paying property taxes of last quarter or semi-year in many states is either Jan 1st or

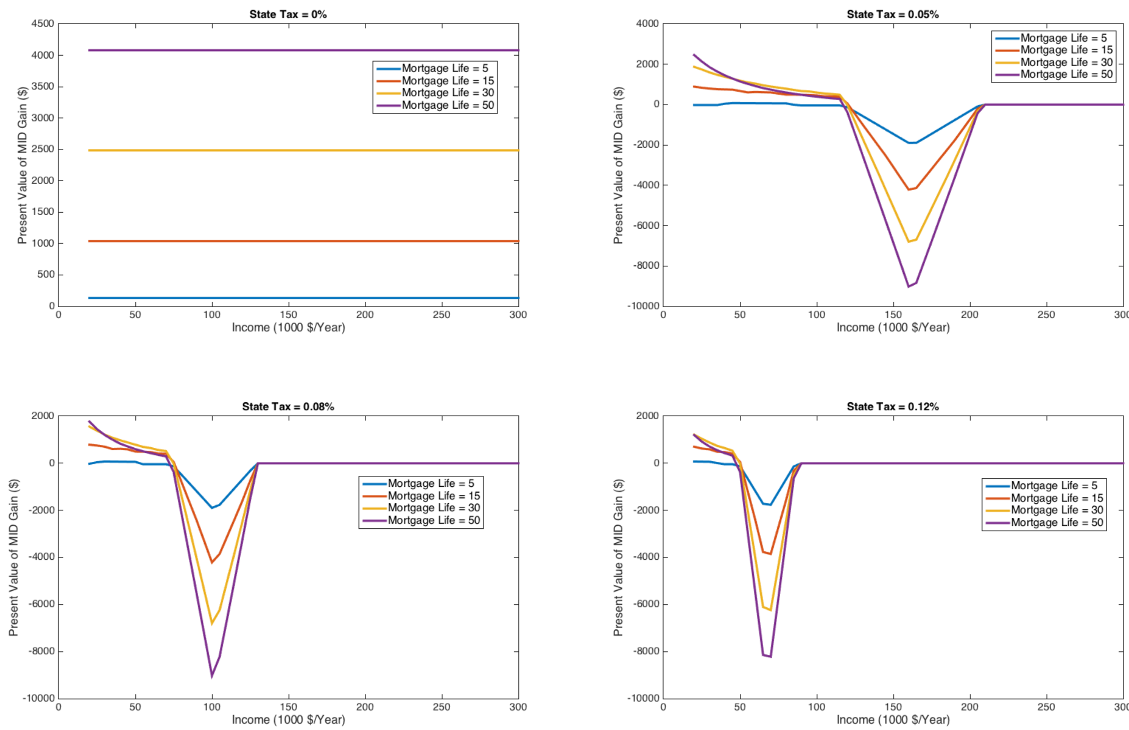

Figure 11: Benefits/Loss from Implementing a Property Tax Ramping Strategy

be able to increase their effective MID rate. Our framework can be used by households to determine the optimal timing of their property taxes.

Figure 11 shows the benefit/loss from implementing a ramping strategy for a representative house value of $300,000, property tax rate of 1.5%, and a constant marginal tax rate of 33%. In line with the theoretical results, we observe that perturbing the base of itemized expenses (through the ramping strategy) provides a small positive value to low-income households, hurts medium income houses, and finally has no impact on high-income households.

Jan 15.

6. Empirical Analysis (Preliminary Draft)

Our model predicts that: 1) a higher state tax should increase the incentive to borrow; 2) higher property tax rate should increase the incentive to borrow

We test some of the implications of the model using Federal Home Loan Bank Purchased Mortgage Files publicly available from Federal Housing Finance Agency (FHFA). The database contains loans from 2009-2016 (more than 45000 observations per year) and covers important loan-level characteristics of the borrower. Since loans changes from one year to another, we can not exploit panel features of the data and pool observations from all years in a single regression (after including year fixed-effects).

6.1. Econometrics Setup

The main dependent variable is the ratio of mortgage loan to income. We control for other possible factors including average local house prices, income, socio-economic and demographic characterizes of the borrower, interest rate, etc.

We consider two major specification. The first specification measures the effect of tax rates on Unpaid Balance of Mortgage Loan.

$$\text{UPB} = \beta_0 + \beta_1\tau_L + \beta_2\tau_S + \beta_3 h + \beta_4 I + \delta\xi + \alpha_1 D_1 \tag{9}$$

The second specification considers the ratio of unpaid balance to income.

$$\frac{\text{UPB}}{\text{Income}} = \beta_0 + \beta_1\tau_L + \beta_2\tau_S + \beta_3 h + \beta_4 I + \delta\xi + \alpha_1 D_1 \tag{10}$$

where UPB is the unpaid principle balance, τ_L is the local property tax, τ_S is the state income tax, h is the local house price, I is the income, ξ contains other socio-economic characteristics of the borrower and D_1 is the year fixed effect dummy.

Since we have included time-invariant the state-level average income tax rate in the regression we can not include state-level fixed effects dummies; otherwise, we will face the co-linearity problem between state fixed effects dummies and state tax rates. We will report evidence that state level characteristics (such as average income, education, etc) are not correlated with the state income tax. Thus, not including a fixed effect dummy is not likely to introduce a bias in the estimation.

6.2. Results

Table 4 shows some basic regression results of loan sizes on basic characteristics.

Model	1	2	3	4	5
State Tax Rate	16638^{***} (43.2)	4449^{***}(12.88)	4406^{***} (12.70)	4093^{***}(11.95)	-5067^{***} (-14.74)
Local Median Income		4.8^{***} (110.2)	5.02^{***} (115.23)	6.07^{***} (100.07)	5.49^{***} (96.10)
LTV		55489^{***} (19.02)	56389^{***} (19.39)	59131^{***} (20.45)	76569^{***} (28.2)
Borrower Income		0.11^{***} (50.34)		-0.62^{***} (-25.06)	$-.55^{***}$ (-24.01)
Loan to Income Ratio			8673^{***} (53.29)	51979^{***} (29.96)	47460^{***} (29.03)
Property Tax Rate					1912^{***} (76.55)
Adjusted R^2	4.0 %	28.3%	28.7 %	29.7%	37.6%

Table 4: Effect of State Tax Rate on Loan Size (t-stat in parenthesis)

A more complete version of regression results is reported in Table 5.

VARIABLES	(1) UPB	(2) UPB	(3) UPB
TaxRate	-20,732***	-22,544***	-22,544***
	(3,695)	(3,729)	(8,158)
LocMedY	3.080***	3.112***	3.112***
	(0.512)	(0.514)	(0.551)
LTV	93,749***	93,335***	93,335***
	(4,568)	(4,556)	(6,777)
Income	0.0453	0.0275	0.0275
	(0.349)	(0.351)	(0.364)
IncRat	13,601	14,588	14,588
	(19,203)	(19,288)	(21,945)
PropertyTax1	4,904*	3,847	3,847
	(2,654)	(2,560)	(7,283)
HPI	711.6***	714.0***	714.0***
	(17.94)	(17.90)	(115.3)
UnemplymentRent	-8,646***	-8,650***	-8,650***
	(997.8)	(998.2)	(2,811)
GrowthRate	-19,388***	-19,500***	-19,500***
	(4,877)	(4,893)	(5,362)
Gasoline	387,464***	391,425***	391,425***
	(95,464)	(96,043)	(78,571)
OneYMaturityRate	2.274e+06***	2.297e+06***	2.297e+06***
	(562,020)	(565,475)	(465,914)
TenYMaturityRate	154,379***	156,507***	156,507***
	(40,395)	(40,643)	(39,193)
ThirtyYMaturityRate	-194,516***	-197,064***	-197,064***
	(54,160)	(54,501)	(57,750)
Moody	-349,679***	-353,098***	-353,098***
	(91,028)	(91,554)	(82,792)
Constant	-977,299***	-933,864***	-933,864***
	(255,329)	(251,595)	(333,858)
Observations	240,418	240,418	240,418
R-squared		0.349	0.349
Number of AssignedID	104,488		

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 5: Effect of State Tax Rate on Loan Size. The first column represents a pooled-regression and the last two columns panel estimation.

TBD

6.3. Robustness Tests

To rule out the possibility of missing variable bias, we provide evidence regarding the relationship between state level tax rates and major macroeconomic variables. The graphs show the correlation of state tax and three key variables of home price index, household income, and unemployment.

TBD

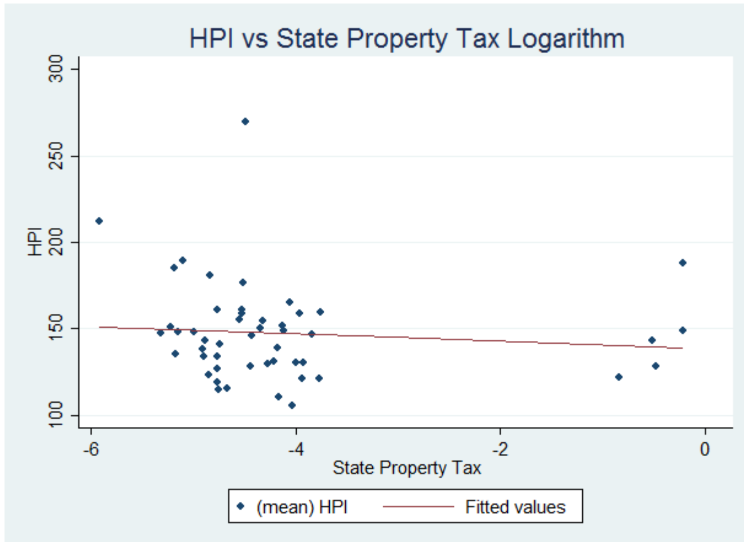

Figure 12: State Tax Rates and House Price Index (2009-2015)

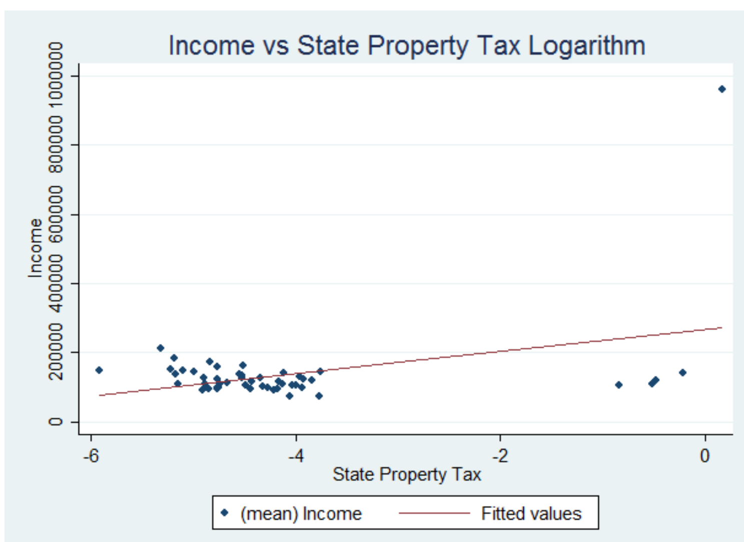

Figure 13: State Tax Rates and Income (2009-2015)

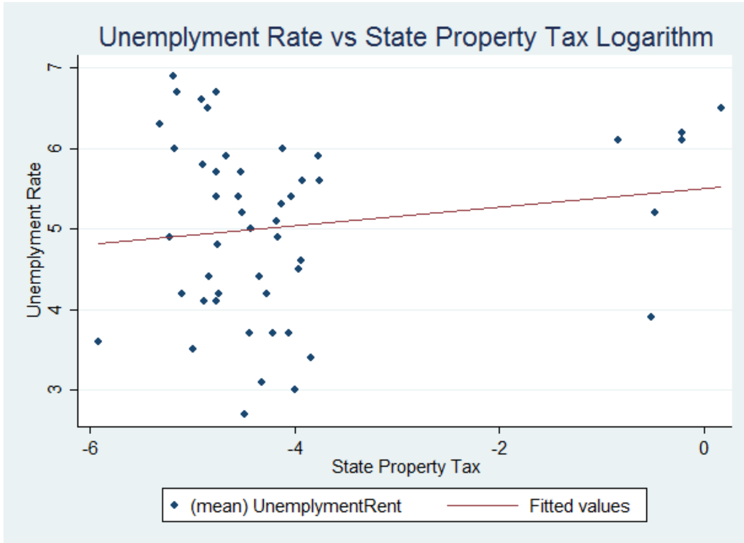

Figure 14: State Tax Rates and Unemployment (2009-2015)

7. Discussion

7.1. Effect of Expected Time to Stay

Our results show that the tax shield value of MID increases with the length of mortgage. If the household prepays the mortgage and moves to another house, the value of MID will be diluted. Recent studies show that the average tenure of a median US household is around 13 years.

7.2. Exotic Mortgage Types

Our model implies that the chance of the MID option being in the money is higher when the level of mortgage interest is higher. Therefore, any strategy which avoids a declining interest payment schedule will increase the present value of the tax shield. Exotic mortgage types including contracts with a large ballon payments at the end, and mortgages with negative amortization are examples of mortgage contracts with larger MID benefits for the user.

7.3. Asset Pricing Implications

In an environment containing uncertainty about future tax rates (stochastic taxes), the present value of the MID will have the exposure to tax rate risks. There is an ongoing debate of tax reforms in the United States. A major proposal advocates for the total or partial elimination various tax exemptions, including the MID. Sialm (2006) shows that the effect of stochastic taxes will be more pronounced for assets with long durations.

7.4. Retirement Asset Holding

TBD

8. MID and Financial Advice Industry

We review the presentation of MID features and benefits, offered by leading real estate investment websites. A similar exercise has also been reported by Agarwal et al. (2013). The goal of the review is to find if those resources highlight the fact that MID may not apply (or only partially apply) for a large fraction of mortgage applicants. Our finding from this quick review is that the industry in general downplays the issue and presents MID as a direct benefit for every homeowner. We find only a few websites that refer to the MID as a 'misperception' or 'myth' and discuss its limitations and clarify the fact that the mortgage should exceed the itemized expense.

We also find that certain websites list the requirement to file the tax Form 1040 (to apply for itemizes deductions). However, the tone of advice is not focused on elaborating the opportunity cost of filing Form 1040; it is rather presented like a mere additional paper work.

Running a Google search for a set of phrases, including "mortgage interest tax deductibility", an analyze the content of the top 50 search results. We find that the majority of results focus on other aspects.

Advice/Comment Provided by Website	Our Critical Comment	Frequency
There is limit for MID qualified loans	The cap is too high and not relevant for the majority of mortgage owners	
MID applies to primary residence	Relevant for the majority	
Obligation to file Form 1040 and itemizes deductions	Not highlighting the limitations of the itemized approach	
Deduction limited to interest part of mortgage payment	Important to provide a realistic picture of the magnitude of saving	
Mortgage interest deducted form pre-tax income and not from taxes	Important to provide a realistic picture of the magnitude of saving	
Ownership title	Legal requirement	

Table 6: Samples of MID Advice on Websites

Based on our content review we conclude that there is a space for improving the existing practice. The information provided to public can become more precise than the status quo. First of all, the effective tax shield of mortgage interest can be calculated for different levels of income and house price. Second, we found no source that considers the MID as an option, which may be in the money and out of money, depending on the realizations of random underlying variables.

9. Conclusion and Future Research

The current paper can be extended in multiple directions. An important behavioral question would be the extent that mortgage applicants are aware of the exact capitalization of the MID. Possible misperceptions regarding the true value of the MID tax shield may cause overreactions to incentives for home ownership. Additional studies can be conducted to identify household characteristics associated with misperceptions and the welfare consequences of suboptimal housing investment decisions, driven by optimistic perceptions of the

MID value.

An important policy question is the revisiting of current MID regulations to provide a better subsidy to lower and middle-income homeowners rather than merely benefiting high-income buyers. A two-part policy can be a possible solution, in which households with an income below a threshold can claim their mortgage interest on top of their standard deductions. Another option would be to allow for tax credits of mortgage interest, which can be carried forward. Future research should elaborate the tax revenue implications of such reforms and their role in encouraging home-ownership among lower income citizens.

The nature of the MID is pro-cyclical. The subsidy is higher when the household's income is higher and is lower (or even zero) when there is a negative shock to income. A natural extension is to model the MID from an asset pricing perspective using a dynamic general equilibrium framework.

Appendix A. State-Level Taxes

Before solving an option pricing model under uncertainty we provide some stylized observations regarding the impact of state and property tax rates on the effective MID. As shown in Figure A.16 there is a great deal of heterogeneity in the income tax rates of various states. See Keightley (2014) for a detailed discussion of geographical distribution of MID in the US. Similar to state income tax rates property taxes are also heterogenous across difference states.

We change the value of state tax in a range of [0, 10%]. The lower bound corresponds to low-tax states (e.g. Texas or Alaska) and the upper bound represents high-tax states (e.g. California). The relationship between income, state tax, and tax shield effectiveness is plotted in A.15. In states with zero state tax, the full advantage of the MID only appears for annual income above $400,000; whereas, for high tax rate states (e.g. California) the income threshold for 100% tax shield effectiveness is around $200,000.

Figure A.15: Effective Mortgage Interest Deductibility

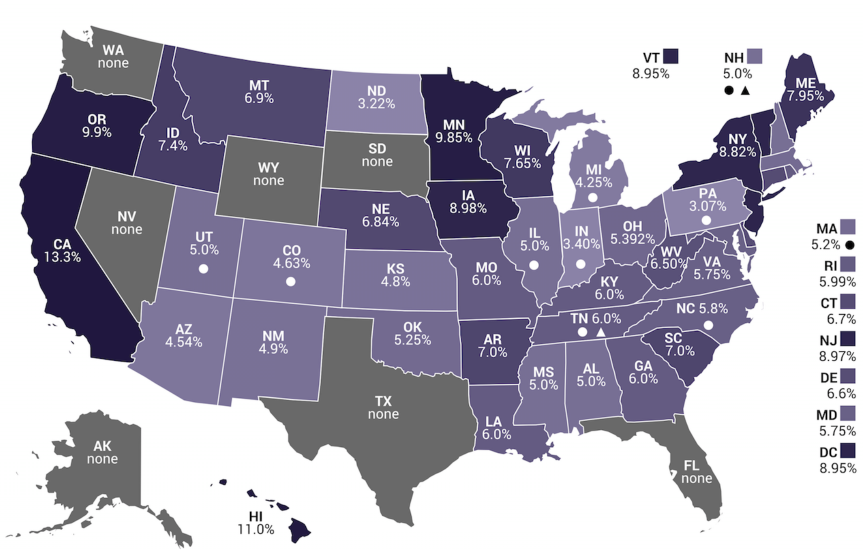

Figure A.16: State Tax Rates (for the Top Income Group). Residents of low tax states benefit from not paying a state tax; however, it is also less likely for them to benefit from mortgage tax deductibility.

An interesting question would be to compare the net benefit of living in a high or low tax state. For a baseline case of a $100,000 annual income, a 4% mortgage rate, a house price to income ratio of 2.5, LTV = 80%, and a marginal income tax rate of 33%, the household saves 2.4% of its income level due to the MID. If the household lives in a low tax state the likelihood of benefiting from MID is tiny. Whereas, for a household in a high tax state the tax shield effectiveness is close to 1.

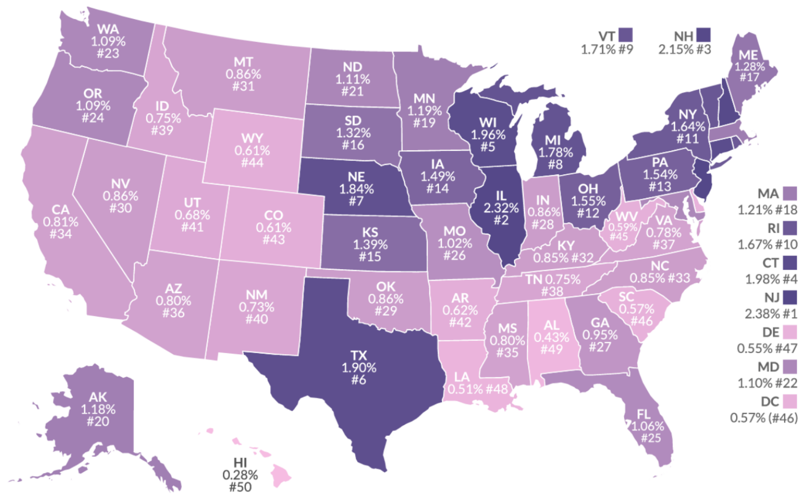

Figure A.17: State Property Tax Rates. Source of Picture: Tax Foundation. Residents of low tax states benefit from lower property taxes; however, it also makes it less likely for them to benefit from mortgage tax deductibility.

Appendix B. Option Value

$$\begin{cases} V_1'(\overline{T}-iK) = V_2'(\overline{T}-iK) \Rightarrow A_1^1\beta_1 P^{\beta_1-1} = A_1^2\beta_1 P^{\beta_1-1} + A_2^2\beta_2 P^{\beta_2-1} + \frac{\tau}{r} \\ V_1(\overline{T}-iK) = V_2(\overline{T}-iK) \Rightarrow A_1^1 P^{\beta_1} = A_1^2 P^{\beta_1} + A_2^2 P^{\beta_2} \\ V_2'(\overline{T}) = V_3'(\overline{T}) \Rightarrow A_2^3\beta_2 P^{\beta_2-1} = A_1^2\beta_1 P^{\beta_1-1} + A_2^2\beta_2 P^{\beta_2-1} + \frac{\tau}{r} \\ V_2(\overline{T}) = V_3(\overline{T}) \Rightarrow A_2^3 P^{\beta_1} = A_1^2 P^{\beta_1} + A_2^2 P^{\beta_2} \end{cases} \tag{B.1}$$

We first eliminate A_1^1. For this purpose we use the first equations

$$\begin{cases} A_1^1 P^{\beta_1-1} = A_1^2 P^{\beta_1-1} + A_2^2\frac{\beta_2}{\beta_1} P^{\beta_2-1} + \frac{\tau t}{\beta_1 r} \Rightarrow A_1^1 P^{\beta_1} = A_1^2 P^{\beta_1} + A_2^2\frac{\beta_2}{\beta_1} P^{\beta_2} + P\frac{\tau t}{\beta_1 r} \\ A_1^1 P^{\beta_1} = A_1^2 P^{\beta_1} + A_2^2 P^{\beta_2} \end{cases} \tag{B.2}$$

$$\begin{cases} A_1^2 P^{\beta_1} + A_2^2 P^{\beta_2} = A_1^2 P^{\beta_1} + A_2^2 \frac{\beta_2}{\beta_1} P^{\beta_2} + P \frac{\tau}{\beta_1 r} \Rightarrow \\ A_2^2 (1 - \frac{\beta_2}{\beta_1}) \overline{P}^{\beta_2} = \overline{P} \frac{\tau}{\beta_1 r} \Rightarrow A_2^2 = \frac{1}{\beta_1 - \beta_2} \overline{P}^{1-\beta_2} \frac{\tau}{r} \\ A_1^2 = \frac{1}{\beta_1 - \beta_2} \underline{P}^{1-\beta_1} \frac{\tau}{r} \end{cases} \tag{B.3}$$

The same approach applies to A_2^3.

$$\begin{cases} V_2'(\overline{T}) = V_3'(\overline{T}) \Rightarrow A_2^3 \beta_2 P^{\beta_2 - 1} = A_1^2 \beta_1 P^{\beta_1 - 1} + A_2^2 \beta_2 P^{\beta_2 - 1} + \frac{\tau}{r} \\ V_2(\overline{T}) = V_3(\overline{T}) \Rightarrow A_2^3 P^{\beta_1} = A_1^2 P^{\beta_1} + A_2^2 P^{\beta_2} \end{cases} \tag{B.4}$$

Multiplying the first line by P and the second line by β_2

$$\begin{cases} A_2^3 \beta_2 P^{\beta_2} = A_1^2 \beta_1 P^{\beta_1} + A_2^2 \beta_2 P^{\beta_2} + P \frac{\tau}{r} \\ A_2^3 \beta_2 P^{\beta_1} = A_1^2 \beta_2 P^{\beta_1} + A_2^2 \beta_2 P^{\beta_2} \end{cases} \tag{B.5}$$

and then subtracting the two equations:

$$\begin{cases} A_1^2 (\beta_1 - \beta_2) P^{\beta_1} + P \frac{\tau}{r} = 0, P \to \overline{P}_2 \Rightarrow \\ A_1^2 = \frac{-1}{\beta_1 - \beta_2} \overline{P}_2^{1-\beta_2} \frac{\tau}{r} \end{cases} \tag{B.6}$$

Using the solved values of A_1^2 and A_2^2 one can calculate the value of A_1^1

$$\begin{cases} A_1^1 P^{\beta_1} = A_1^2 P^{\beta_1} + A_2^2 P^{\beta_2}, P \to \overline{P}_1 \Rightarrow \\ A_1^1 = A_1^2 + A_2^2 \overline{P}_1^{\beta_2 - \beta_1} \end{cases} \tag{B.7}$$

Using the solved values of A_1^2 and A_2^2 one can calculate the value of A_3^1 too.

$$
\begin{cases}
A_2^3 P^{\beta_1} = A_1^2 P^{\beta_1} + A_2^2 P^{\beta_2}, P \to \overline{P}_2 \Rightarrow \\
A_2^3 = A_1^2 + A_2^2 \overline{P}_2^{\beta_2 - \beta_1}
\end{cases}
\tag{B.8}
$$

$$
\begin{cases}
A_1^1 = \frac{\tau}{r} \frac{\underline{P}^{1-\beta_1} - \overline{P}^{1-\beta_1}}{\beta_1 - \beta_2} \\
A_1^2 = -\frac{\tau}{r} \frac{\overline{P}^{1-\beta_1}}{\beta_1 - \beta_2} \\
A_2^2 = \frac{\tau}{r} \frac{\underline{P}^{1-\beta_2}}{\beta_1 - \beta_2} \\
A_2^3 = \frac{\tau}{r} \frac{\underline{P}^{1-\beta_2} - \overline{P}^{1-\beta_2}}{\beta_1 - \beta_2}
\end{cases}
\tag{B.9}
$$

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