Foreign Direct Investment with Tax Benefits under Uncertainty*

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June 2017

Abstract

We study foreign direct investment (FDI) agreements where a tax holiday is offered in exchange of a commitment to not to divest during a given time period. We develop a real options model which determines the optimal timing of a FDI project, considering that the firm benefits from a more favourable tax rate over a tax holiday period and both profit and taxation policy are uncertain. Our results show that there is a non-linear effect of the (expected) tax holiday period on the investment timing, where a longer tax holiday period may deter investment. In any case a higher tax rate benefit hastens investment.

Keywords: Foreign Direct Investment, Uncertainty, Real Options, Taxation Policy, Tax Holiday.

JEL codes: G31, H25.

*This research has been financed by the European Regional Development Fund through COMPETE 2020−*Programa Operacional Competitividade e Internacionalizao* (POCI) and by Portuguese public funds through FCT (Fundao para a Cincia) in the framework of the projects POCI-01-0145-FEDER-006683 (Artur Rodrigues) and POCI-01-0145-FEDER-006890 (Paulo J. Pereira).

1 Introduction

Governments use corporate tax incentives to enhance Foreign Direct Investment (FDI). The offer to a foreign firm of a more attractive tax rate is often enough to make an investment profitable, or the relocation of a business to another country optimal. For instance, amongst the EU countries, Ireland is well known for its agressive corporate tax policy, which attracts FDI.

Attached to a FDI-related tax benefit is, usually, a set of obligations to which the foreign firm obeys, for instance, the commitment to operate in a country over a given time period, invest in human capital, create new jobs, and establish business partnerships with local firms. Therefore, a FDI can be seen as a contract between a host country and a foreign firm through which, over a given time period, the two parties are entitled to a set of financial benefits and obligations and where, before investing, the firm holds the option to invest and, after investing, the value of the FDI project is given by the expected value of its future cash flows plus the value of a forward start option (FSO), with a starting date that coincides with the expiration date of the FDI contract, that gives the firm the right to abandon the project after the contract has expired.

In this paper, we advocate that FDI contracts comprise a FSO which has been neglected by the literature and highlight that it has value and affects the timing of the investment. We develop a real option model that determines the optimal time to invest in a FDI project considering the FSO and the uncertainty about both the profit flow and the taxation policy. Our modelling setting also considers that the foreign firm has the option to abandon the project. In line with previous findings from the real options literature, we conclude that the profit flow uncertainty delays the timing of the investment. However, when the tax benefit is relatively low, we find that the effect of the taxation policy uncertainty on the timing of the investment is ambiguous.

We also determine the fair reimbursement amount that is due to the foreign firm, or the FDI host country, if there is a violation, or a renegotiation, of the FDI agreement. This aspect is important because FDI agreements hold usually over long time periods during which projects often face very adverse economic conditions, where a size contraction or an early abandonment of the project may need to be considered. For instance, the EU countries that were bailed out as a consequence of the 2008-09 financial crisis tried to renegotiate some FDI agreements. Specifically, Ireland was pressed, by the French and German officials during the negotiations of

2010 bailout, to increase its very low corporate tax rate in return for an aid package[1], Portugal renegotiated some Public-Private Partnerships (PPP) after the 2011 bailout in order to balance the public budget deficit (Burger et al. 2009, Sarmento and Renneboog 2016) and, after the 2012 bailout, Spain stopped abruptly its very generous investment incentive policy to renewable energy (Robinson 2013).

There are also more extreme cases, where the foreign firm abandons the FDI project before the termination of the FDI agreement. Yet, these cases are usually related to political disputes between countries, or the bankruptcy or financial distress of the parent firms, and should be examined very carefully because unilateral breaches of FDI agreements can damage severely the international reputation of a firm or a country (notice that when there are legal disputes these are usually set by the International Centre for the Settlement of Investment Disputes (ICSID)).[2,3]

The effect of the taxation policy on investment decisions has been a central research topic in accounting and public finance. Most of the available theoretical results are based on modelling settings which apply to contexts where the tax rate is known and either fixed or progressive, tax exemptions and fiscal depreciations are possible, the investment cost is fixed and sunk, cash flows are either stochastic or deterministic, and the early abandonment of investment and the FSO are neglected.

More specifically, Hassett and Metcalf (1999) study the effect of taxation policy uncertainty on firm's investment behaviour relying on two modelling settings. One, where the taxation policy uncertainty is due to a shock following a geometric Brownian motion (gBm) and, another, where the tax policy change is due to a random discrete jump. For the latter modelling setting, they show that the gains from delaying the investment is negatively related to the likelihood of a tax switch, whereas for the former, their results suggest that capital formation is adversely affected by increases in the uncertainty. Sureth (2002) studies the effect of taxation on firms' investment behaviour considering that cash flows are uncertain and the investment cost is only partially irreversible. She concludes that the cash flow uncertainty and the irreversibility of the investment cost do not lead to a violation of the neutrality property of a Johansson-Samuelson

[1] https://www.ft.com/content/411e7e9a-f344-11df-a4fa-00144feab49a

[2] The ICSDI is an institution which is part of the World Bank.

[3] Allee and Peinhardt (2011) show that countries suffer a significant loss of FDI if sued by a foreign firm before an agreement is made through the ICSID, and suffer even greater losses if these lose the dispute at the ICSID.

tax system.[4]

Niemann (2004) investigate the effect of the tax policy uncertainty on firms' investment behaviour and show that, under risk aversion or risk neutrality, increases in the tax rate uncertainty have an inconclusive effect on investment, depending on the fiscal depreciation policy and the cash flow structure. Niemann (2004) derive a neutral tax system using a real option model and conclude that the framework he uses does not provide a general solution for investment decisions after tax. Alvarez and Koskela (2008) examine the effect of progressive taxation on irreversible investments under uncertainty, and shows that when the tax exemption is lower than the sunk cost, a higher tax rate delays the investment by increasing its optimal threshold, whereas if the tax exemption surpasses the sunk cost, three different regimes hold, depending on the level of uncertainty. Niemann and Sureth (2013) use a discrete-time model to investigate the effect of capital gains taxation on the timing of the investment considering the casflow uncertainty and the possibility of entry and exit under different tax rates, for both ordinary income and capital gains. Their findings suggest that the investment accelerates with the capital gains taxation, in particular if there are high liquidation proceeds or low interest rates and cashflow volatility.

There are other real option models considering the effect of the taxation policy on FDI. Specifically, Pennings (2000) shows that increases in the investment tax credit, if financed with a rise in the tax rate, encourage investments. Agliardi (2001) examines the tax system effect on the incentive to invest or disinvest, considering the output price uncertainty and the replacement value of the firm's capital. Pennings (2005) studies the investment behaviour of a (monopolist) foreign firm considering demand uncertainty and that the firm can either export to a host country or supply the market by making an irreversible investment, benefiting from the host country's subsidies and tax benefits. Li and Rugman (2007) use a real option model to optimize the location of a FDI project and select the mode of the market entry (i.e., by building a subsidiary in a non-home region or in a home region).

Our paper also relates to the works of MacKie-Mason (1990), who investigates a specific nonlinear tax rule under uncertainty where the tax policy subsidizes assets values; Yu et al. (2007), who examines the comparative effect on the timing of FDI of the use of a tax reduction or an entry subsidy and suggests that the latter incentive is more effective and economical than

[4] The Johansson-Samuelson Theorem states that, in partial equilibrium, income taxation with a uniform tax rate has a neutral effect on investment decisions, if the fiscal depreciation allowances coincide with the economic depreciation (see Johansson (1969)).

the former; Wong (2009), who studies the effect of progressive taxation and tax exemptions on the optimal threshold to abandon the investment and shows that corporate income taxes are not neutral if the tax rate is progressive; Gries et al. (2012), who evaluates investments taking into account the cash flow uncertainty and show that, due to the optionality premium, taxes affect asymmetrically the option value to invest when projects face different risk levels; Adkins and Paxson (2013), who use a two-factor model to investigate the effect of tax depreciation on capital replacement policy and show that the capital replacement policy is affected by the tax depreciation, which reduces the optimal operating cost threshold;

Our work also intersects with those of Janeba (1995), who studies the effect of the tax rate and how the corporate income taxes are paid at the home country and the FDI host country on the FDI; Alvarez et al. (1998) who examine the effect of the timing and the nature of a corporate tax reform uncertainty on firm's behaviour; Panteghini and Schjelderup (2006) who investigate the behaviour of firms regarding FDI when countries compete to attract investment and firms can decide optimally when to invest; Zambujal-Oliveira and Duque (2011), who relies on a two-factor real option model to investigate the optimal time for a capital replacement considering the effect of both a certain taxation environment and a given depreciation policy; and Sarkar (2012), who examines whether a government should use a tax reduction or a subsidy in order to enhance investment. Barbosa et al. (2016) who study government incentives to investment considering various macroeconomic variables and the interaction of the investment host country with the local firms.

The rest of the paper is organized as follows. Section 2 presents the real option model and provides a sensitivity analysis. Section 3 studies the scenario where the taxation policy is uncertain and provides a sensitivity analsyis for its effect on the timing of the investment. Section 4 concludes.

2 The model

Let us suppose that a country and a foreign firm make an agreement regarding a FDI project, according to which the foreign firm invests I and benefits from a more favourable tax rate over a given time period (T) during which it is not allowed to divest. Thus, before investing, the foreign firm holds the option to invest whose value can be determined following standard real option-backward induction procedures.

Therefore, we start by the derivation of the firm's value function for the period where it is active and proceed then backwards in order to derive the firm's value function for the period where it is inactive.

2.1 The active firm

Let us assume that an all-equity firm is active with a FDI project that generates a pre-tax profit flow x which fluctuations over time according to the following gBm process:

$$dx(t) = \alpha x(t)dt + \sigma x(t)dw \tag{1}$$

where $\alpha < r$, σ, and dw are, respectively, the drift under the risk-neutral measure, the volatility, and the increment of a Wiener process, and r is the constant risk-free interest rate.

Let us also assume that τ_h and τ_c are, respectively, the profit tax rates which hold over and after the period during which the firm benefits from a more favourable tax rate (the tax holiday period), with $0 \leq \tau_h < \tau_c$. Therefore, the after-tax profit flow during the tax holiday period is given by $x(1-\tau_h)$, whereas the after-tax profit flow after the tax holiday period is given by $x(1-\tau_c)$.

At the time when the investment is made ($t = 0$), the firm's value is given by:

$$V(x, \tau_h, \tau_c, T) = \int_0^T x(t)(1-\tau_h)e^{-rt}dt + \int_T^{\infty} x(t)(1-\tau_c)e^{-rt}dt \tag{2}$$

whose solution is:

$$V(x, \tau_h, \tau_c, T) = \frac{x}{r-\alpha}\left(1 - \tau(\tau_h, \tau_c, T)\right) \tag{3}$$

with

$$\tau\left(\tau_h, \tau_c, T\right) = \tau_h + (\tau_c - \tau_h)e^{-(r-\alpha)T} \tag{4}$$

where $\tau\left(\tau_h, \tau_c, T\right)$ is a time-weighted average tax rate.[5]

The FDI contract gives the firm a tax benefit $(\tau_h - \tau_c)$ over the time period T during which

[5]Note that for $\tau\left(\tau_h, \tau_c, 0\right) = \tau\left(\tau_c, \tau_c, T\right) = \tau_c$, the firm does not benefit from a more favorable tax rate, whereas for $\tau\left(\tau_h, \tau_c, \infty\right) = \tau_h$ the firm benefits from a more favorable tax rate perpetually. If over a finite time period T the firm benefits from a full tax exemption, the average tax rate over a perpetual time period is $\tau\left(0, \tau_c, T\right) = \tau_c e^{-(r-\alpha)T}$.

the firm is not allowed to divest. After the tax holiday period has finished, the firm holds the option to abandon the project whose value is given by:

$$A(x,\tau_c,S)=\begin{cases} S-\dfrac{x(1-\tau_c)}{r-\alpha} & \text{if } x<x_A \\ \left(S-\dfrac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\dfrac{x}{x_A}\right)^{\beta_2} & \text{if } x\geqslant x_A \end{cases} \tag{5}$$

where S is the project's salvage value and x_A is the threshold to abandon the project, which is given by:

$$x_A=\frac{\beta_2}{\beta_2-1}\frac{S(r-\alpha)}{1-\tau_c} \tag{6}$$

with β_2 given by:

$$\beta_2=\frac{1}{2}-\frac{\alpha}{\sigma^2}-\sqrt{\left(-\frac{1}{2}+\frac{\alpha}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}} \tag{7}$$

Although the foreign firm cannot abandon the investment over the time period T, it can do so after it has been reached. Therefore, the FDI agreement comprises a FSO with a starting date at T that gives the right to the foreign firm to abandon the project as soon as T has been reached. The value of the FSO is given by:

$$\begin{aligned} F_A(x,\tau_c,T)=Se^{-rT}N(-d_2)-\frac{x(1-\tau_c)}{r-\alpha}e^{-(r-\alpha)T}N(-d_1) \\ +\left(S-\frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{x}{x_A}\right)^{\beta_2}N(d_3) \end{aligned} \tag{8}$$

where S is the project's salvage value, $N(.)$ represents the cumulative normal integral, and

$$d_1(x)=\frac{\ln\left(\dfrac{x}{x_A}\right)+\left(\alpha+\dfrac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \tag{9}$$

$$d_2(x)=d_1(x)-\sigma\sqrt{T} \tag{10}$$

$$d_3(x)=d_1(x)+(\beta_2-1)\,\sigma\sqrt{T} \tag{11}$$

where β_2 is given by Equation 7 and x_A is the optimal threshold to abandon the project, which is given by Equation 6.

The economic interpretation for Equation (8) is as follows: on the right-hand side, the first two terms represent the value of the option to abandon the project at time T, whereas the third term represents the value of the option to abandon the project after T has been reached if, when T is reached, it is not yet optimal to abandon.[6] Therefore, the following proposition holds:

Proposition 1. *The value of an active firm, $F(.)$, that is legally bound by a FDI agreement is given by:*[7]

$$F(x, \tau_h, \tau_c, T) = V(x, \tau_h, \tau_c, T) + F_A(x, \tau_c, T) \tag{12}$$

Corollary 1. *When $T \to 0$, there is not a tax holiday period. Therefore, the firm's value converges to the value of a firm which pays a profit tax rate τ_c and holds the option to abandon the investment:*

$$\lim_{T\to 0} F(x, \tau_h, \tau_c, T) = \frac{x(1-\tau_c)}{r-\alpha} + \left(S - \frac{x_A}{r-\alpha}(1-\tau_c)\right)\left(\frac{x}{x_A}\right)^{\beta_2} \tag{13}$$

Corollary 2. *When $T \to \infty$, the tax holiday period lasts forever. Therefore, the firm's value converges to the value of a firm which pays a profit tax rate τ_h perpetually and does not hold the option to abandon the investment.*

$$\lim_{T\to\infty} F(x, \tau_h, \tau_c, T) = \frac{x(1-\tau_h)}{r-\alpha} \tag{14}$$

2.2 The idle firm

Proposition 2. *The foreign firm, when inactive, holds the option to invest whose value is given by:*

$$O(x, \tau_h, \tau_c, T) = \Big(V(x_I, \tau_h, \tau_c, T) + F_A(x_I, \tau_c, T) - I\Big)\left(\frac{x}{x_I}\right)^{\beta_1}, \textit{ for } x < x_I \tag{15}$$

where x_I is the optimal threshold to invest, which is determined numerically by solving the following equation:

$$\begin{aligned}&(\beta_1-\beta_2)\left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{x_I}{x_A}\right)^{\beta_2} N(d_3)\\ &+(\beta_1-1)\left[V(x_I, \tau_h, \tau_c, T) - \frac{x_I(1-\tau_c)}{r-\alpha}e^{-(r-\alpha)T}N(-d_1)\right]\\ &-\beta_1\left[I - Se^{-rT}N(-d_2)\right] = 0\end{aligned} \tag{16}$$

[6] It is equivalent to the value of a European put option (*a la* Black and Scholes) which matures at time T (the time when both the tax benefit and the impossibility of abandoning the project terminate).

[7] All proofs are available in the Appendix.

where I is the investment cost, and β_1 is given by:

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{17}$$

Corollary 3. *When $T \to 0$, there is not a tax holiday period.*

$$\lim_{T\to 0} O(x, \tau_h, \tau_c, T) = \left(\frac{x_I^*(1-\tau_c)}{r-\alpha} + \left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{x_I^*}{x_A}\right)^{\beta_2} - I\right)\left(\frac{x}{x_I^*}\right)^{\beta_1}, \text{ for } x < x_I^* \tag{18}$$

where x_I^* is the optimal threshold to invest, which is a solution for the following equation:

$$(\beta_1 - \beta_2)\left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{x_I^*}{x_A}\right)^{\beta_2} + (\beta_1 - 1)\frac{x_I^*(1-\tau_c)}{r-\alpha} - \beta_1 I = 0 \tag{19}$$

Corollary 4. *When $T \to \infty$, the tax holiday period lasts forever.*

$$\lim_{T\to\infty} O(x, \tau_h, \tau_c, T) = \left(\frac{x_I^{**}(1-\tau_h)}{r-\alpha} - I\right)\left(\frac{x}{x_I^{**}}\right)^{\beta_1}, \text{ for } x < x_I^{**} \tag{20}$$

where x_I^{**} is the optimal threshold to invest:

$$x_I^{**} = \frac{\beta_1}{\beta_1 - 1}\frac{r-\alpha}{(1-\tau_h)}I \tag{21}$$

As the above findings show, the tax holiday period T and the tax benefit $(\tau_c - \tau_h)$ over the tax holiday period both play a key role in the value of the firm and the timing of the investment.

Corollary 5. *The effect of the tax holiday period T on the optimal time to invest is non-monotonic: $\frac{\partial x_I}{\partial T} \gtrless 0$.*

Corollary 6. *A lower profit tax rate (τ_h) during the tax holiday period accelerates the investment: $\frac{\partial x_I}{\partial \tau_h} > 0$.*

The non-monotonic effect of the tax holiday period on the investment threshold is illustrated in Figure 1.

[Insert Figure 1 here]

The result stated in corollary 6 is important because the non-monotonic relationship between the profit tax rate (τ_h) and the tax holiday period T suggests that, for a relatively high

τ_h, widening T does not necessarily accelerates investment. On the contrary, it may preclude investment if T is sufficiently long. This is because a higher tax holiday period reduces the tax payment (which enhances value) but it also increases the period during which the firm cannot abandon the investment (which reduces value). The result of these two opposite effects on the timing of the investment is obviously more visible as τ_h approaches τ_c. Our findings also show that, for a relatively low τ_h, the relationship between τ_h and T is monotonic.

2.3 Numerical Analysis

In this section, we provide a sensitivity analysis regarding the effect of our model parameters on the firm's investment threshold (x_I) - see figure 2. It shows that x_I decreases with the tax holiday period (T) and increases with the tax rate that holds during the tax holiday period (τ_h), for our base-case market conditions. Thus, a higher T accelerates investments, whereas a higher τ_h deters investments. Furthermore, figures 2(a) and 2(b) show the effect of the profit drift (α) on x_I for different values of T and τ_h, respectively, from which we conclude that x_I decreases significantly with α. Therefore, a higher α accelerates investments. We also conclude that x_I is less sensitive to changes in T and τ_h as α increases. Figures 2(c) and 2(d) show the effect of the profit flow uncertainty (σ) on x_I, for different levels of T and τ_h, respectively, from which we acknowledge that x_I increases with σ. Therefore, we conclude that a higher uncertainty delays investment.

In addition, figures 2(e) and 2(f) show the effect of the project's salvage value (S) on x_I for different levels of T and τ_h, respectively. We find that x_I decreases with S, therefore, a higher degree of reversibility of the investment accelerates investment. Figures 2(g) and 2(h) show the effect of the investment cost (I) on x_I for different levels of T and τ_h, respectively, from which we learn that x_I increases linearly with I, therefore, a higher investment cost deters the investment. Figures 2(i) and 2(j) show the effect of τ_c on x_I, for different values of T and τ_h, respectively, from which we conclude that x_I increases with τ_c, therefore, a higher τ_c delays investments. These results are in line with what we would expect from a real options model.

[Insert Figure 2 here]

Figure 3 shows "iso-treshold" curves for different economic contexts where it is optimal to invest (notice that we set $x_0 = x_I$=7). It gives us a more refined analysis for the effect of our model parameters on the investment threshold, considering different tax holiday periods. Our findings confirm those of figure 2. These iso-threshold curves are helpful to design optimal taxation policies to attract FDI, considering different economic scenarios and projects' size.

[Insert Figures 3 here]

3 Taxation policy uncertainty

In the previous section, we assume that a foreign firm makes a FDI agreement with a FDI host country according to which the firm invests I in a project and is not allowed to divest over a given time period (T) during which it is has a tax benefit ($\tau_h - \tau_c$). In this section, we study the case where the FDI agreement entitles the foreign firm to a tax benefit but without specifying the termination date. Therefore, the above tax benefit can hold forever but it can also terminate suddenly in the near future.

We develop a model where, after investing, the foreign firm is entitled to a tax benefit ($\tau_h - \tau_c$) but faces the possibility that an unfavourable tax rate change (from τ_h to τ_c) may occur at an unknown future date. We assume that the arrival date of the tax rate change follows a Poisson jump process with a rate λ, where the probability of a tax rate change over an infinitesimally small time period dt is given by λdt. We also assume that, after investing, the foreign firm has the option to abandon the investment.[8] Therefore, the value of the firm after the unfavourable tax rate change is: $\frac{x(1-\tau_c)}{r-\alpha} + A(x, \tau_c, S)$, where $A(x, \tau_c, S)$ is the value of the option to abandon the investment, given by Equation (5).

3.1 The active firm

We start the analysis by the stage where the firm is active, deriving the firm's value for the stage where firm's profit is taxed at τ_h but this tax rate may change to τ_c at a random future date. Therefore, the firm's value ($V_R(.)$) must satisfy the following non-homogeneous differential equation:

$$\frac{1}{2}\sigma^2 x^2 V_R''(x) + \alpha x V_R'(x) - r V_R(x) + x(1-\tau_h) + \lambda \left(\frac{x}{r-\alpha}(1-\tau_c) + A(x, \tau_c, S) - V_R(x) \right) = 0 \quad (22)$$

[8] Notice that a rise in the tax rate can trigger the abandonment of the investment.

where $A(x, \tau_c, S)$ takes different values depending on the level of x in relation to x_A (see Equation (5)).

Notice that $V_R(x)$ is continuous and differentiable along x. Hence, we can obtain a solution for $V_R(x)$ following standard real options procedures:[9]

Proposition 3. *The value of an active firm that pays currently a profit tax rate τ_h which can increase to τ_c at a random future date is given by:*

$$V_R(x, \tau_h, \tau_c, \lambda) = \\ = \frac{x(1-\tau_h)}{r-\alpha+\lambda} + \begin{cases} b_1 x^{\eta_1} + \dfrac{\lambda}{r+\lambda} S & \textit{if } x < x_A \\ b_4 x^{\eta_2} + \dfrac{\lambda}{r-\alpha+\lambda} \dfrac{x(1-\tau_c)}{r-\alpha} + \left(S - \dfrac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\dfrac{x}{x_A}\right)^{\beta_2} & \textit{if } x \geqslant x_A \end{cases} \tag{23}$$

where x_A is the optimal threshold to abandon the investment, and

$$b_1 = \frac{S}{\eta_1 - \eta_2}\left(\frac{r-\alpha}{r-\alpha+\lambda}\frac{\beta_2}{\beta_2 - 1}(\eta_2 - 1) - \frac{r}{r+\lambda}\eta_2\right)\left(\frac{1}{x_A}\right)^{\eta_1} \tag{24}$$

$$b_4 = \frac{S}{\eta_1 - \eta_2}\left(\frac{r-\alpha}{r-\alpha+\lambda}\frac{\beta_2}{\beta_2 - 1}(\eta_1 - 1) - \frac{r}{r+\lambda}\eta_1\right)\left(\frac{1}{x_A}\right)^{\eta_2} \tag{25}$$

$$\eta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}} \tag{26}$$

$$\eta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}} \tag{27}$$

Corollary 7. *When $\lambda \to \infty$, an increase in the tax rate from τ_h to τ_c is certain. Therefore, there is not a tax holiday period and the firm's value converges to that of a firm whose profits are taxed at τ_c and holds the option to abandon the investment:*

$$\lim_{\lambda \to \infty} V_R(x, \tau_h, \tau_c, \lambda) = \frac{x(1-\tau_c)}{r-\alpha} + \left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{x}{x_A}\right)^{\beta_2} \tag{28}$$

Corollary 8. *When $\lambda \to 0$, an increase in the tax rate from τ_h to τ_c will not occur. Therefore, the tax holiday period will last forever and the firm's value converges to that of a firm with profits*

[9] In order to highlight the fact that the value of the foreign firm is a function of τ_h, τ_c and λ, henceforth, we use $V_R(x) \equiv V_R(x, \tau_h, \tau_c, \lambda)$.

taxed at τ_h, which does not holding the option to abandon the investment.

$$\lim_{\lambda \to 0} V_R(x, \tau_h, \tau_c, \lambda) = \frac{x(1-\tau_h)}{r-\alpha} \tag{29}$$

3.2 The idle firm

We analyse now the stage where the firm is inactive and has the the option to invest, $O_R(.)$. Our aim is to derive the analytical expressions for both the firm's value and the optimal investment threshold.

Proposition 4. *The value of an inactive firm that is attached to a FDI agreement according to which it invests I and benefits from a more favourable tax rate (τ_h), but faces the possibility that an unfavourable tax rate change (to τ_c) may occur at a random future date is:*

$$O_R(x, \tau_h, \tau_c, \lambda) = \left(V_R(x_R, \tau_h, \tau_c, \lambda) - I\right)\left(\frac{x}{x_R}\right)^{\beta_1}, \textit{ for } x < x_R \tag{30}$$

where x_R is the threshold to invest under taxation policy uncertainty and the solution of the following equation:

$$(\beta_1 - \eta_2) b_4 x_R^{\eta_2} + (\beta_1 - 1)\left(\frac{x_R(1-\tau_h)}{r-\alpha+\lambda} + \frac{\lambda}{r-\alpha+\lambda}\frac{x_R(1-\tau_c)}{r-\alpha}\right) - \beta_1 I = 0 \tag{31}$$

For $x \geqslant x_R$, the value is given by $R(x, \tau_h, \tau_c, \lambda) - I$.

Corollary 9. *When $\lambda \to \infty$, there is not a tax holiday period:*

$$\lim_{\lambda \to \infty} O_R(x, \tau_h, \tau_c, \lambda) = \left(\frac{x_R^*(1-\tau_c)}{r-\alpha} + \left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{x_R^*}{x_A}\right)^{\beta_2} - I\right)\left(\frac{x}{x_R^*}\right)^{\beta_1}, \textit{ for } x < x_R^* \tag{32}$$

where x_R^* is the optimal threshold to invest, which is a solution for the following equation:

$$(\beta_1 - \beta_2)\left(S - \frac{x_A(1-\tau_c)}{r-\alpha}\right)\left(\frac{x_R^*}{x_A}\right)^{\beta_2} + (\beta_1 - 1)\frac{x_R^*(1-\tau_c)}{r-\alpha} - \beta_1 I = 0 \tag{33}$$

Corollary 10. *When $\lambda \to 0$, the tax holiday period will last forever.*

$$\lim_{\lambda \to 0} O_R(x, \tau_h, \tau_c, \lambda) = \left(\frac{x_R^{**}(1-\tau_h)}{r-\alpha} - I\right)\left(\frac{x}{x_R^{**}}\right)^{\beta_1}, \textit{ for } x < x_R^{**} \tag{34}$$

where x_R^{**} is the optimal threshold to invest:

$$x_R^{**} = \frac{\beta_1}{\beta_1 - 1} \frac{r - \alpha}{(1 - \tau_h)} I \tag{35}$$

Corollary 11. *The threshold to invest (x_R) increases with the tax rate (τ_h) that holds during the tax holiday period: $\dfrac{\partial x_R}{\partial \tau_h} > 0$. Therefore, a higher τ_h discourages investment.*

Corollary 12. *The effect of the the taxation policy uncertainty (λ) on the timing of the investment is non-monotonic: $\dfrac{\partial x_R}{\partial \lambda} \gtreqless 0$.*

The finding expressed in corollary (12) is important because it shows that there are economic conditions in which a higher taxation policy uncertainty does not necessarily discourage investment. This is because a higher likelihood that an unfavourable change in the taxation policy will occur in the future increases the likelihood that the firm will hold the option to abandon the investment. These two opposite effects offset each other (see Figure 4), being the preponderance of one over the other dependent of λ and how close τ_h is to τ_c. When λ is very low, if it increases slightly, it means that there is a higher chance that there will be an unfavourable tax rate change in the near future, which deters the investment, but it also means there is a higher probability that the firm may will hold soon the option to abandon the investment, which enhances the investment, being the latter effect predominant over the former.

[Insert Figure 4 here]

3.3 Sensitivity analysis

In this section we provide a sensitivity analysis about the effect of our model parameters on the optimal investment threshold (x_R).

Specifically, figure 5 shows the effect on x_R of both λ and τ_h, for different values of our model parameters. We conclude that x_R increases with λ and τ_h, therefore, a higher λ or τ_h discourages investment. Additionally, figures 5(a) to 5(j) show a sensitivity analysis for the effect on x_R of other model parameters, which reveal that x_R decreases with S and increases with r, σ, I and τ_c. Hence, we conclude that a higher riskless interest rate, profit uncertainty, investment cost, or project's salvage value discourages investment. The sensitivity of x_R to changes in λ increases with τ_c and decreases with S. These findings are in line with what we would expect from a real options model.

[Insert Figures 5 here]

Figure 6 shows various iso-treshold curves that represent scenarios for which it is optimal to invest (notice that we set $x_0 = x_R = 7.0$). Therefore, points on or above each of these curves represent different (λ, τ_h) scenarios where it is optimal to invest, whereas points below each of these curves represent scenarios where it is not yet optimal to invest.

Interestingly, we find that when λ is very low, there might be economic conditions (e.g., high uncertainty and project's salvage value) where if it increases slightly, turning more likely a rise of the tax rate in the near future and deviating the firm from its optimal investment threshold, the optimal behaviour from the FDI host country (so as to keep the firm at its investment threshold) is to rise slightly the tax rate (τ_h). Furthermore, from figures 6(a) to 6(f) we can also see that, for relatively high values of λ, a small decrease in λ (*ceteris paribus*), turning less likely a rise of the tax rate in the near future makes the investment "more optimal" for the foreign firm, hence, the optimal behaviour from the FDI host country's so as to keep the firm on the iso-threshold curve is to increase τ_h. These results illustrate well the importance for FDI host countries of being perceived as countries with stable taxation policies.

[Insert Figures 6 here]

4 Conclusion

We study foreign direct investment (FDI) agreements where a tax holiday is offered in exchange of a commitment to not to divest during a given time period. Our model determines the optimal timing of a FDI considering that the firm benefits from a more favourable tax rate over the tax holiday period and both profits and taxation policy are uncertain.

Our results show that when the tax holiday period is known ex-ante, there is a non-linear effect of the tax holiday period on the investment timing. Specifically, we find that when the tax benefit the firm gets over the tax holiday period is relatively small, widening the tax holiday period does not necessarily encourage investment. Furthermore, when the tax holiday period is undefined and there is some likelihood that it may end at a random future date, for relatively low tax benefits during the tax holiday period, an increase in the probability that the tax benefits will end in the near future does not necessarily delays investment.

It would be an interesting research to extend our model to cases where competition amongst FDI host countries or foreign firms is taken into account, possibly relying on Smets (1993) framework. The incorporation in our modelling setting of assets depreciation, or a more diverse set of taxation policies including, for instance, tax exemptions, tax credits or progressive taxation, would also be an interesting research.

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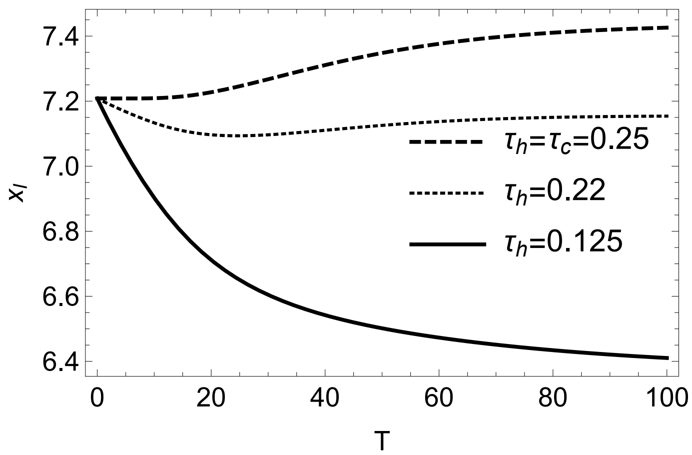
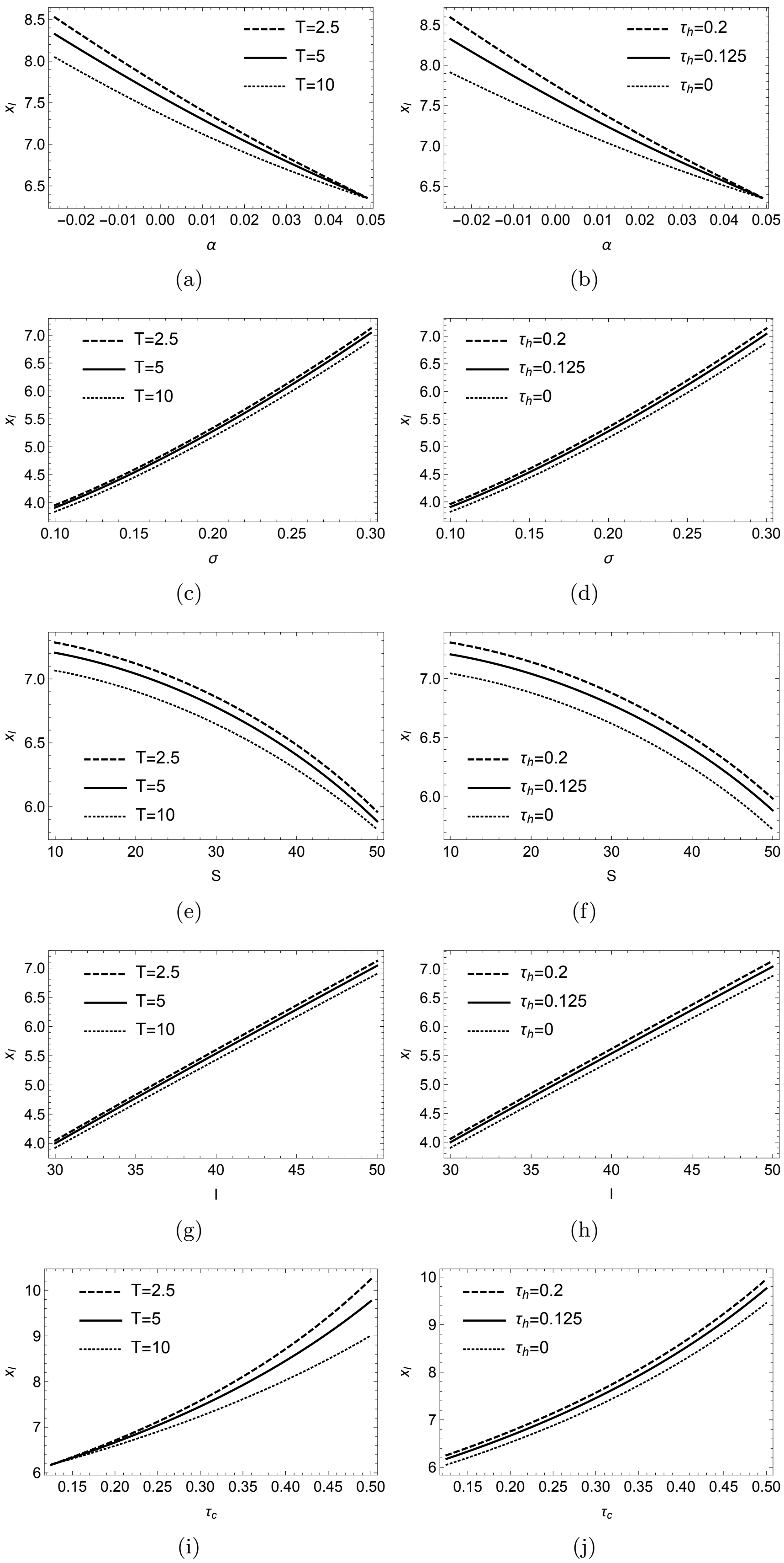


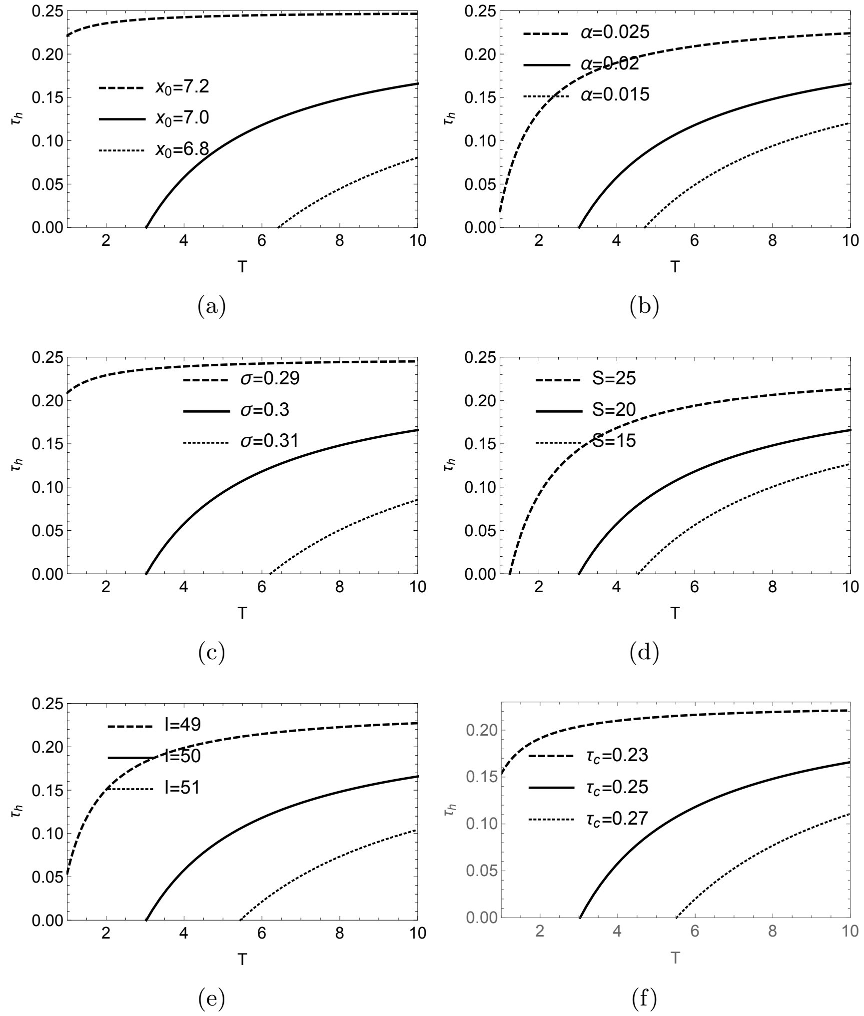
$\sigma = 0.3,\ r = 0.05,\ \alpha = 0.02,\ S = 20,\ I = 50,\ \tau_c = 0.25.$

Figure 1: this figure shows a sensitivity analysis for the effect on the optimal investment threshold (x_I) of the tax holiday period T, for different values of τ_h.



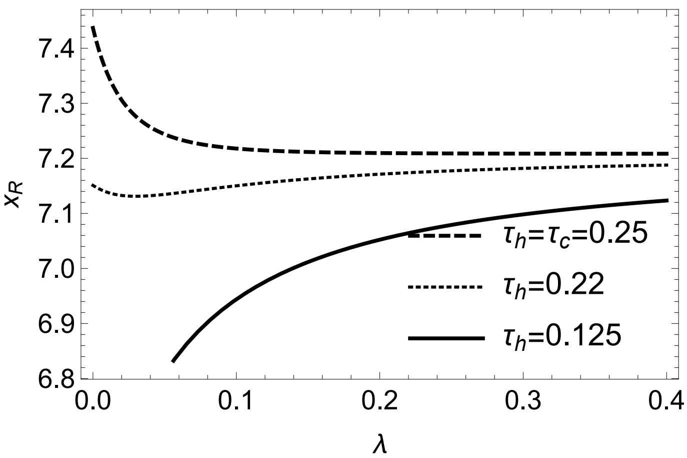
$\sigma = 0.3$, $r = 0.05$, $\alpha = 0.02$, $S = 20$, $I = 50$, $\tau_c = 0.25$, $\tau_h = 0.125$, $T = 5$.

Figure 2: this figure shows a sensitivity analysis for the effect on the optimal investment threshold (x_I) of our model parameters, for different values of T and τ_h.



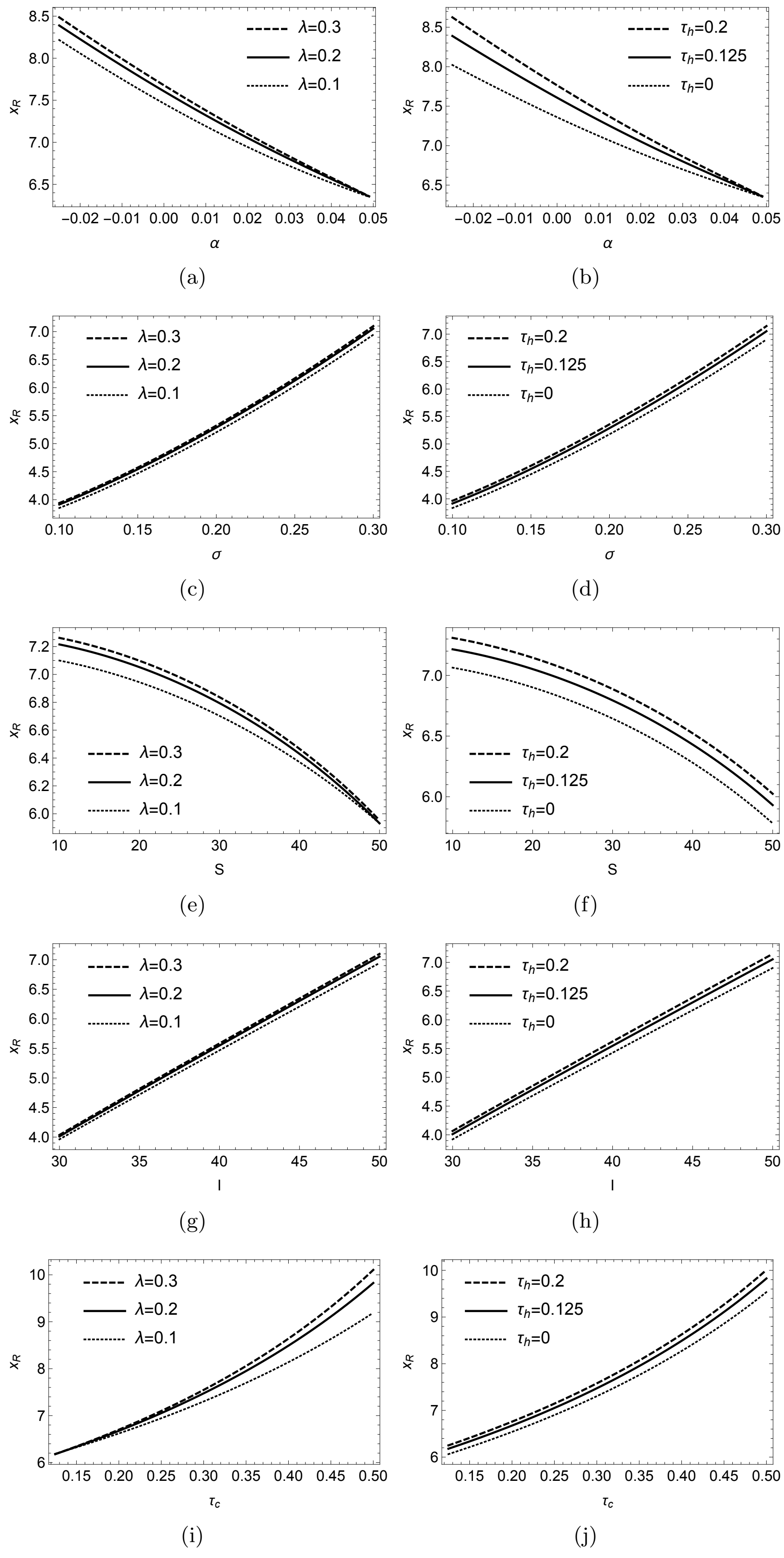
$x_I = x_0 = 7.0$, $\sigma = 0.3$, $r = 0.05$, $\alpha = 0.02$, $S = 20$, $I = 50$, $\tau_c = 0.25$.

Figure 3: this figure shows various iso-threshold curves that illustrate different economic conditions for which it is optimal to invest. Points on each of the above curves represent a pair of values (T,τ_h) that triggers the investment.



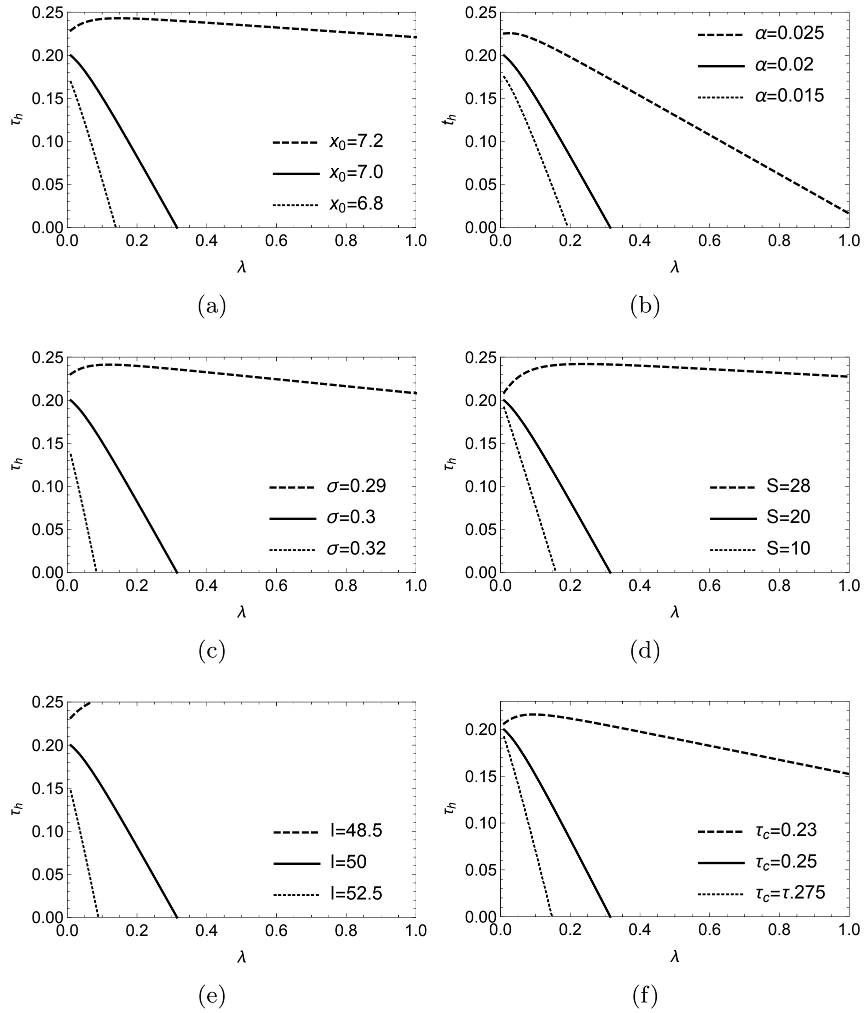
$\sigma = 0.3$, $r = 0.05$, $\alpha = 0.02$, $S = 20$, $I = 50$, $\tau_c = 0.25$, $\tau_h = 0.125$.

Figure 4: this figures shows a sensitivity analysis for the effect on the optimal investment threshold (x_R) of the taxation policy uncertainty (λ), for different tax rates that hold during the tax holiday period (τ_h).



$\sigma = 0.3$, $r = 0.05$, $\alpha = 0.02$, $S = 20$, $I = 50$, $\tau_c = 0.25$, $\tau_h = 0.125$, $\lambda = 0.2$.

Figure 5: this figures shows a sensitivity analysis for the effect of our model parameters on the optimal investment threshold (x_R), for different levels of taxation policy uncertainty (λ) and tax rate (τ_h) holding during the tax holiday period.



$x_R = x_0 = 7.0$, $\sigma = 0.3$, $r = 0.05$, $\alpha = 0.02$, $S = 20$, $I = 50$, $\tau_c = 0.25$.

Figure 6: this figures shows iso-threshold curves that represent different economic scenarios where it is optimal to invest. Each iso-thresold curve represents different sets of pairs of values (λ,τ_h), for a given set of model parameter values, which trigger the investment.