Funding real options under adverse selection and moral hazard

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Abstract

We analyze optimal financing of an entrepreneur's real option investment when investors do not observe the stochastic quality process driving the investment's value. Optimal contracts encourage entrepreneurs to postpone investment by paying them for waiting. When there is adverse selection on initial quality, high quality entrepreneurs invest inefficiently early to signal their type. In the presence of moral hazard, paying before investment reduces entrepreneur incentives to work, and in response, also in this case the optimal contract induces early investment.

Key words: Adverse Selection, Dynamic contracting, Real option, Investment timing, Signalling, Principal-Agent model, Project Finance.

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1. Introduction

We study optimal investment timing for small R & D firms that need to contract with outsiders to finance major capital outlays. An example would be a cash-poor biotech start-up firm developing a new drug, setting up an alliance with an established pharma company to cover not only the costs of ongoing R & D but in particular the future large cash outlays associated with later stage clinical tests and product commercialization.[1] The timing of these investments will depend on the evolution of the project's quality, or success probability. Information accruing as a result of ongoing research will cause managers to update their estimates of their products' quality, determining the investment's success probability. If this evolution of quality is stochastic, investments in product trials or scaling-up of production processes are real options, that should be exercised when quality is judged sufficiently high.

The problem that we focus on, is the dynamic asymmetric information on the likelihood of success of such investments. The start-up managers will arguably have better information than their financing alliance partner, both as regards the initial quality of the product under development, and on the stochastic evolution of that quality as more research is done. It is then natural that they are in a better position to decide when to make the follow-on investment. However, their decision when to exercise the real option and make that investment, will likely be more reflective of their pay-off schedules than whether product quality has reached surplus maximizing investment level. In particular, if managers enjoy the upside of such investments, e.g. sharing in the profits if the investment is successful, while being shielded from the downside as they invest their alliance partner's capital, they may be tempted to paint an overly rosy picture of the project and invest too early (known as 'window dressing', see e.g. Cornelli and Yosha, 2003).

In this paper we study how alliance partners can design contracts to give incentives to the start-up manager to make the investment at the optimal time, even though the funding partner does not observe the stochastic process followed by product quality. The manager, who is protected by limited liability, gets not only a share in the firm's profits upon investment, but also a flow payment in the period before investment. This flow, which might take the form of a research fee, is essential in delaying the investment decision: when making the decision to invest, the manager not only obtains a share in the profits, but also gives up a valuable income flow.[2]

[1] Guedj (2005) gives a description of such alliances. Lerner and Malmendier (2010) analyze contract terms in biotech alliances from an incomplete contracting perspective.

[2] Ex ante research fees and royalties for completed projects are standard components in biotech alliances

We then continue to analyze how the start-up firm can signal its initial quality. The size of both profit shares upon investment and pre-investment research fees will depend on expected project value at the time of writing the contract. This value reflects the expected time until option exercise, and hence depends on initial quality. If also initial quality is unobserved to the investing alliance partner, in a pooling equilibrium, high initial quality firms will end up cross-subsidizing low quality ones. We show that if the share of high type firms in the population is not too high, high quality firms have an incentive to separate from low types by offering a contract that induces early investment. Such a contract has lower research fees, but a higher share for the start-up entrepreneur in the project upon investment. For low types, such a distorted contract is less attractive, and in the signalling equilibrium they opt for a first-best investment contract with lower pay-outs commensurate with their lower initial quality.

We next explore how moral hazard distorts contracts and investments. Research fees and unobservability of the project quality process may drive the start-up manager to divert research effort to other ('pet') projects, rather than the contracted one.[3] We model such a diversion of effort in an extreme way as the manager's decision to stop the project's quality process to get a flow of outside benefits, b, representing utility derived from the pet project. The temptation to divert effort will be larger the lower the expected pay-off from completing the contracted project. Hence, if project quality drops too far, and the expected benefits of completing the project become small, the entrepreneur will stop putting effort into the contracted project. The value of quality at which the entrepreneur prefers to shirk is endogenous to the contract. We analyze the problem of jointly determining investment and shirking thresholds, and find that with moral hazard, the optimal contract induces investment that is too early from a social surplus perspective, even including the contributions to surplus form the pet project's benefits b.

Predictions of too early investment are consistent with empirical findings in Guedj and Scharfstein (2004), who analyze differences in clinical trial success rates among research firms in biotech alliances. They find that early stage firms (where asymmetric information is likely to be worse) on average invest faster in costly trials, and have poorer success rates, than mature firms that face less severe contractibility problems. In the context of our model, this could be explained as a result from costly signalling of initial project quality, or as a contractual response to more severe moral hazard during the research phase. Cornelli and Yosha (2003) explain the use of convertible debt in staged venture capital financing as a means to overcome

(Guedj, 2005), see also Edwards (2007) for an industry description of typical contract terms.

[3] As Lerner and Malmendier (2010) describe, such projects might include work on academic output or research for other alliance partners.

window dressing by the entrepreneur. For their mechanism, it is essential that entrepreneurs engage in such window dressing before they know the evolution of the project themselves. In contrast, in our case, the asymmetric information is more extreme: entrepreneurs can choose to misreport project quality after they observe true quality, and there is no signal to the investor apart from the entrepreneur's reports.

Our model fits into a literature of real options (see Dixit and Pindyck, 1994, for an overview) in the presence of asymmetric information. Morellec and Schürhoff (2011) and Bouvard (2012) analyze investment problems like ours in which an entrepreneur designs a contract with investors and engages in costly signalling through distorted investment timing. Grenadier and Wang (2005) and Grenadier and Malenko (2011) study agency problems from the perspective of a firm that offers a screening contract to its manager. Broer and Zwart (2013) analyse a similar problem in a regulation context. The key difference with this literature is that these authors assume that the stochastic progress of the value parameter is observable and hence investment timing can be contracted upon. Asymmetric information in these contributions affects only a static, initial parameter. In contrast, in this study the value parameter, quality, can never be observed by investors and hence investment timing cannot be directly contracted upon. Adverse selection itself is truly dynamic in our case.

Since in our analysis, the stochastic process is unobservable to outsiders, we can also study continuous time moral hazard problems, in which continuous choices on the part of the entrepreneur affect that unobservable process. This differentiates our work from Grenadier and Wang (2005) who also consider static moral hazard, which ex ante affects initial project value.

In considering dynamic adverse selection, we tie in with a theoretical literature on dynamic mechanism design that considers optimal contracts when the agent's type evolves stochastically over time, see e.g. Baron and Besanko (1984), Besanko (1985), Board (2007), Pavan, Segal and Toikka (2014). Though most contributions are in discrete time environments, recent contributions analyze their formulation in continuous time, allowing a connection to the real option framework, (Bergemann and Strack, 2014; Arve and Zwart, 2014). We apply their insights, formulated in a principal-agent setting, in a costly signalling model in which the informed party designs the contract (Maskin and Tirole, 1992).

Moral hazard in continuous time finance environments has been studied by Sannikov (2008); DeMarzo and Sannikov (2006). In this literature, parties can contract on the stochastic process. In this paper, there is less information as the dynamic moral hazard problem is compounded by dynamic adverse selection. Instead, moral hazard interacts with that adverse selection to change optimal investment fees.[4]

2. Model

We consider an entrepreneur with an investment project that has stochastic value Q. One may view Q as the quality of the project, or a measure of the probability that the project will be successful. The costs for realizing the project are I, so that at the moment of investment, net expected value equals $V = Q - I$. Investment is irreversible. Quality Q follows geometric Brownian motion,

$$dQ = \mu Q dt + \sigma Q dz, \tag{1}$$

with μ the drift parameter and σ the volatility of the process. dz represents the standard Wiener increment. The discount rate is $r > \mu$. The project is a real option, as in McDonald and Siegel (1986), and value optimization requires finding the optimal time T at which the investment is to be made. Typically this time is formulated as the first time at which Q crosses some threshold value, $\bar{Q}$.

We introduce an agency problem into this real option framework by assuming the entrepreneur has no initial cash or other assets. The entrepreneur hence has to contract with an outside investor to obtain the funding I to make the investment. The outside investor supplies cash, and in return gets a share in the project. Outside investors are perfectly competitive, so that the entrepreneur will want to find a funding contract that maximizes value while allowing the investor to break even.

The split of funding and project execution introduces informational frictions, and potential distortions. Although investment costs I are publicly known, the evolution of project value, Q, is not observable to the investors.[5] The informational asymmetry is therefore dynamic: the project's quality (type) changes continuously, observable to the entrepreneur but not to the investors. Investors do know the parameters of the stochastic process μ, σ. In the asymmetric information setting we consider, the initial value of Q at the time of contracting, Q_0 is either high Q_H, or low Q_L, with probabilities ϕ and $1 - \phi$. From that starting point, Q evolves according to the process (1).

[4] Relatedly, Georgiadis (2014) studies a moral hazard in teams problem, where, although progress is observable, effort is induced through payments upon project completion.

[5] Q is not observable before investment, nor at investment. Q might be a success probability, or one might envisage the project to give returns only long after investment has occurred, and we assume contracts cannot be conditioned on those returns.

In section 4, we add moral hazard. In that case the agent can choose to stop working on the project and get a constant flow of benefits b instead, representing for instance leisure or benefits from working on other projects. If the agent does not work, the quality process stops, $dQ = 0$.

The entrepreneur and the investor write a funding contract. Since Q is not observable to the investor, the contract cannot depend on Q. It can only depend on the observable investment itself. To align the entrepreneur's and the investors' objectives, contracts that respect limited liability can consist of a stream of payments $p \geq 0$ to the entrepreneur before investment, and payment I in exchange for a share $1 - \alpha$ for the investor in the project at the moment of investing.

3. Signalling equilibria

3.1. Benchmark: symmetric information

To provide a benchmark, we first analyze the optimal contract if quality Q were continuously observable and contractible. Then there is no asymmetric information, and contracts can implement first-best investment. The optimal financing contract will depend on Q, and determine investment timing to maximize total value of the investment. Since the investment market is perfectly competitive, investors receive no rents, and the entrepreneur obtains all available surplus. Total expected surplus equals

$$W = \max_T E\left(e^{-rT}(Q - I)\right).$$

Here, T denotes the (stochastic) time until investment occurs. In problems that do not explicitly depend on time, optimal investment times will generally be defined as the first time that the stochastic parameter, Q, hits a particular bound $\bar{Q}$. Finding that optimal threshold is a standard real option problem (see e.g. Dixit and Pindyck, 1994). It can be solved by observing that before investment, W will satisfy a Bellman equation,

$$rW = \mu Q W_Q + \tfrac{1}{2}\sigma^2 Q^2 W_{QQ},$$

where subscripts denote derivatives. At the optimal investment quality threshold, $Q = \bar{Q}$, surplus W should be continuously differentiable,

$$W(\bar{Q}) = \bar{Q} - I, \quad W_Q(\bar{Q}) = 1.$$

Finally, in the limit $Q \to 0$, the project surplus W tends to zero.

The solution for this optimal investment threshold is

$$\bar{Q}^* = \frac{\lambda_+}{\lambda_+ - 1} I, \tag{2}$$

with λ_+ the positive root of the characteristic equation associated to the Bellman equation, $-r + \lambda\mu + \frac{1}{2}\lambda(\lambda - 1)\sigma^2$. As observed by McDonald and Siegel (1986) this expression reflects the option value of waiting to invest as an opportunity cost of investing: $\bar{Q}^*$ exceeds the break even level I. As the value of this option increases, for instance because volatility σ is higher, this margin over the break-even value increases.

Optimally, with symmetric information on Q, the entrepreneur will then negotiate a contract that specifies investment as soon as Q crosses the threshold $\bar{Q}^*$. Remuneration for the outside investors can be in terms of a fraction $1 - \alpha$ of the stocks, such that investors exactly break even,

$$(1 - \alpha)\bar{Q}^* = I.$$

The entrepreneur retains the remainder, $\alpha\bar{Q} = \bar{Q}^* - I$, which is the entire surplus at the moment of investment.

3.2. Asymmetric information on Q and paying for delay

We now abandon the assumption that quality Q is observable by investors. Since only the entrepreneur observes Q, the investment contract can no longer be contingent on that value. Investors now face the problem that for any sharing rule α, the entrepreneur will have an incentive to invest immediately, rather than wait until some value $\bar{Q}$ is reached. This follows immediately from the fact that the entrepreneur, having no cash, bears none of the costs of investment I, but does receive the benefit of the value of his share in the firm upon investment, and the assumption that $\mu < r$. Formally,

$$V = \max_T E\left(e^{-rT}\alpha Q\right)$$

is maximized by immediate investment, $T = 0$.

An optimal investment contract should incentivize the entrepreneur to postpone investment, even though Q is not observable to the investors. The entrepreneur should be rewarded for waiting. To achieve this, the contract needs to remunerate the entrepreneur not only at the moment of investment, but also during the waiting stage, as the next lemma indicates.[6]

Lemma 1 *A contract (p, α) that pays p to the entrepreneur before investment, and gives him a share α of the project at investment, incentivizes the entrepreneur to postpone investment until $Q = \bar{Q}$ if p and α satisfy, for any choice of threshold $\bar{Q}$*

$$p = \frac{\alpha \bar{Q} r(\lambda_+ - 1)}{\lambda_+}.$$

The value of such a contract for the entrepreneur at $t = 0$ when $Q = Q_0 < \bar{Q}$ is

$$V(Q_0) = \alpha \bar{Q} \left(\frac{\lambda_+ - 1}{\lambda_+} + \frac{1}{\lambda_+} \left(\frac{Q_0}{\bar{Q}} \right)^{\lambda_+} \right)$$

The intuition for this result is that the investor has to make it worthwhile to the entrepreneur to defer the investment: the entrepreneur has to internalize the opportunity costs of immediate investment. Since the entrepreneur is cashless, the way to do that is by giving him a stream of fees p, that is terminated at the moment of investing. When the entrepreneur invests, he gives up the present value of that stream of cash flows, p/r. The magnitude of the fee p has to be such that, at $\bar{Q}$, the benefit of investing for the entrepreneur exactly offsets the loss of the income stream p.

Notice in particular that this allows to achieve first-best investment timing: when the value of the stream of flows p/r equals α times the cost of investment I, the entrepreneur's trade-off between investing and waiting precisely coincides with that of the social planner. Substituting the value of first-best investment threshold $\bar{Q}^*$, equation (2), we indeed have $p^* = \alpha r I$.

The entrepreneur's value of the contract consists of the value of the stream of cash flows p, augmented with the option value of exchanging that stream for a share α in the project. Clearly, that value depends on the share α: the larger the entrepreneur's share in the project, the larger the cash flows needed to keep the investment threshold constant, and the larger the value upon investing. Secondly, that value depends on the current level of quality Q_0. The larger Q_0, the shorter the expected time to investment and hence the larger the option value of investing.

[6] All proofs are in the appendix.

If current quality Q_0 is known to investors, the optimal contract for the entrepreneur sets the investment threshold at first-best level $\bar{Q}^*$, and chooses α such that investors exactly break even and the entrepreneur receives the entire social surplus of the project. In that case, even without observability of the evolution of the quality process Q, first-best investment can be attained.

The situation is different if Q_0 is not observable. In that case, while it is still possible to implement optimal timing (setting $\bar{Q}$ equal to $\bar{Q}^*$), one cannot choose the share α to allocate all rents to the entrepreneur. Since the entrepreneur's value depends positively on initial quality Q_0, high quality entrepreneurs would need higher shares (and cash flows) than low quality ones.

The response to this non-observability of Q_0 is either that entrepreneurs of different initial qualities pool into the same contract, with high quality entrepreneurs cross-subsidizing the low quality ones, and investors breaking even only on average. Or, high quality entrepreneurs can try to signal their better initial position by proposing a contract that sacrifices some efficiency but allows for a separating equilibrium. We shall analyze those two potential equilibria in turn.

3.3. Pooling contract

A pooling contract (p, α) promises equal cash flows p and shares upon investment α to both types of ex ante entrepreneurs, those with high initial quality Q_H and those with low quality Q_L. As a result, by lemma 1, in a pooling contract both types will make the investment decision at the same threshold value $\bar{Q}$. An optimal pooling contract maximizes the value available to the entrepreneur, subject to the investor breaking even in expectation, averaged over both types. Let's define total surplus for a project with current quality Q, and given investment threshold $\bar{Q}$, as $W(Q, \bar{Q})$,

$$W(Q,\bar{Q}) = E\left(e^{-rT}(\bar{Q}-I) \mid Q\right) = (\bar{Q}-I)\left(\frac{Q}{\bar{Q}}\right)^{\lambda_+}.$$

Similarly, denote by $V(Q, \bar{Q})$ the expected value of the cash flows p and share $\alpha\bar{Q}$ upon investment for the entrepreneur, with p consistent with the investment threshold $\bar{Q}$ as in lemma 1,

$$V(Q,\bar{Q}) = \alpha\bar{Q}\left(\frac{\lambda_+-1}{\lambda_+} + \frac{1}{\lambda_+}\left(\frac{Q}{\bar{Q}}\right)^{\lambda_+}\right).$$

Then, with a proportion of ϕ initial high-type entrepreneurs Q_H, the break even constraint for the investors is

$$\phi\left(W(Q_H,\bar{Q}) - V(Q_H,\bar{Q})\right) + (1-\phi)\left(W(Q_L,\bar{Q}) - V(Q_L,\bar{Q})\right) = 0.$$

The contract maximizing entrepreneur value implements the first-best investment threshold, $\bar{Q} = \bar{Q}^*$, and sets the magnitude of (p, α) to leave all rents with the entrepreneurs. We then have

Proposition 1 *The optimal pooling contract has p and α consistent with first-best investment threshold $\bar{Q}^*$, or $p = \alpha r I$. Furthermore, α is chosen so as to make sure that entrepreneurs get all surplus,*

$$\phi V(Q_H, \bar{Q}^*) + (1 - \phi) V(Q_L, \bar{Q}^*) = \phi W(Q_H, \bar{Q}^*) + (1 - \phi) W(Q_L, \bar{Q}^*).$$

In this contract, high types cross-subsidize low types, $V(Q_H, \bar{Q}^*) < W(Q_H, \bar{Q}^*)$ and $V(Q_L, \bar{Q}^*) > W(Q_L, \bar{Q}^*)$.

In view of this cross-subsidy between the two entrepreneur types under pooling, the high type would be better off if it could credibly signal its higher ex ante Q_H to investors, so as to extract the full surplus associated with its higher initial quality Q_H. We next analyze how high types can send such a credible signal, but at a cost, by accepting a funding contract that distorts investment timing away from first-best. Whether the pooling configuration constitutes an equilibrium will depend on whether a separating equilibrium exists that brings more surplus to the high type entrepreneur.

3.4. Separating equilibrium

A high type (Q_H) firm cannot extract the full value of its project from investors without low types, Q_L, posing as high types and causing investors to lose money on average. We here demonstrate that the high-type entrepreneur can separate from the low type by writing a contract that induces earlier investment than the first-best. For the low type, it will be optimal to concede his initial type and choose a contract consistent with the first-best investment threshold. The reason is, that for the high type, giving up some efficiency in terms of a lower investment threshold $\bar{Q} < \bar{Q}^*$ is less costly than for the low Q_L type. Specifically, the indifference curves in the α, $\bar{Q}$ plane for the different types satisfy a single-crossing condition, as the next lemma shows.

Lemma 2 *For all $\bar{Q} > Q_H$, the indifference curves of constant V as a function of α and $\bar{Q}$ are downward sloping, $\frac{d\alpha}{d\bar{Q}} < 0$. Moreover, indifference curves for the low type Q_L are steeper than those for the high type.*

To find the separating contracts, consider a hypothetical separating equilibrium consisting of contracts (t_H, p_H, α_H) and (t_L, p_L, α_L) for the two types. Here we include a possible non-negative[7] upfront payment $t_{H,L}$ at time 0 for both types. We will soon see that in fact both these payments can be set to zero without loss of generality.

For the contracts to be incentive compatible, we need the conditions

$$t_H + V(Q_H, p_H, \alpha_H) \geq t_L + V(Q_H, p_L, \alpha_L), \tag{3}$$

$$t_L + V(Q_L, p_L, \alpha_L) \geq t_H + V(Q_L, p_H, \alpha_H), \tag{4}$$

so that high types do not want to mimick low types, and similarly low types do not want to pose as high types.

In a separating equilibrium, we expect that lower value Q_L types would want to pose as higher value Q_H types. We will proceed on the assumption that the low types' incentive compatibility condition, (4) is binding with equality in equilibrium. Afterwards we check that the high types' incentive compatibility (3) is verified as well.

We know from lemma 1 that

$$V(Q_H, p_H, \alpha_H) - V(Q_L, p_H, \alpha_H) = \frac{\alpha_H \bar{Q}_H}{\lambda} \left(\left(\frac{Q_H}{\bar{Q}_H} \right)^{\lambda_+} - \left(\frac{Q_L}{\bar{Q}_H} \right)^{\lambda_+} \right). \tag{5}$$

We use this expression, combined with the low type's incentive compatibility condition, (4), to show that ex ante payment t_H needs to be zero. First observe that the left-hand side of (4) equals the low type's equilibrium rents. In the optimal separating equilibrium, high types maximize their rents, $t_H + V(Q_H, p_H, \alpha_H)$, subject to incentive compatibility. Writing the right-hand side of the incentive compatibility condition (4) as $t_H + V(Q_H, p_H, \alpha_H) - (V(Q_H, p_H, \alpha_H) - V(Q_L, p_H, \alpha_H))$, from (5) it is clear that to achieve that goal, for any given low type surplus and $\bar{Q}_H$, we have to choose maximum possible share, α_H, for the high entrepreneur in the project upon investment. However, total rents for the high type are bounded by total available surplus, and higher α means more rents are given ex post. In order to maximize α_H, we therefore have to minimize the part of the rents given ex ante, t_H. Hence, $t_H = 0$.

As long as (3) is slack, we can maximize total surplus for the low types without worrying about the constraints, choosing $\bar{Q}_L$ at its first-best value $\bar{Q}^*$. It is then sufficient to choose any pair p_L, α_L consistent with that threshold, and use t_L to make sure that all rents accrue to the entrepreneur. In particular, we may choose $t_L = 0$ and pick α_L to make sure that $V(Q_L, p_L, \alpha_L)$

[7] In view of the fact that the entrepreneur has no capital and is protected by limited liability.

equals total rents.

Finally, we consider how to choose the high types' investment threshold $\bar{Q}_H$ to maximize high type surplus subject to incentive compatibility (4). We have total surplus for initial value Q and threshold $\bar{Q}$,

$$W(Q,\bar{Q}) = E\left(e^{-rT}(\bar{Q}-I)\right) = \left(\frac{Q}{\bar{Q}}\right)^{\lambda_+}(\bar{Q}-I).$$

From binding IC, (5) and surplus extraction for both types, we have

$$W(Q_L,\bar{Q}_L) = W(Q_H,\bar{Q}_H) - \frac{\alpha_H \bar{Q}_H}{\lambda_+}\left(\left(\frac{Q_H}{\bar{Q}_H}\right)^{\lambda_+} - \left(\frac{Q_L}{\bar{Q}_H}\right)^{\lambda_+}\right), \tag{6}$$

Secondly, making sure that the entrepreneur's surplus equals social surplus requires that

$$V(Q_H,p_H,\alpha_H) = \alpha_H \bar{Q}_H\left(\frac{\lambda_+-1}{\lambda_+} + \frac{1}{\lambda_+}\left(\frac{Q_H}{\bar{Q}_H}\right)^{\lambda_+}\right) = W(Q_H,\bar{Q}_H), \tag{7}$$

from lemma 1.

Using these equations, we arrive at the following characterization of the optimal separating contract threshold $\bar{Q}_H$.

Proposition 2 *In the separating contract, low types Q_L are induced to invest at the first-best threshold, $\bar{Q}_L = \bar{Q}^*$. For high types the optimal investment threshold $\bar{Q}_H$ is given by the lowest solution to*

$$W(Q_H,\bar{Q}_H) = \frac{Q_H^{\lambda_+} + (\lambda_+-1)\bar{Q}_H^{\lambda_+}}{Q_L^{\lambda_+} + (\lambda_+-1)\bar{Q}_H^{\lambda_+}} W(Q_L,\bar{Q}_L). \tag{8}$$

Under this contract, high types invest earlier than in the first-best, $\bar{Q}_H < \bar{Q}^*$.

The intuition for the distortion is that earlier investment is achieved by lowering the flow payment, in favour of an increase in the payment upon actual investment. For entrepreneurs with high initial quality, this change is less costly than for low types: they spend shorter expected time waiting until the quality threshold is reached, and since investment occurs earlier than for low types, the gain in pay-off at investment is less heavily discounted in expectation than for their low-quality counterparts.

In the pooling equilibrium, both types invest at first-best investment times. In comparison, in the separating case, high types signal their type through costly distortions away from first-best timing. Though total surplus evidently is lower than under pooling, separation can be an equilibrium if the gain in the share of total surplus for the high types exceeds the loss in total available surplus.

To check when the separating equilibrium is in fact the unique equilibrium, we need to explore when pooling Pareto-dominates the separating equilibrium (Maskin and Tirole, 1992). Clearly, low quality types always prefer pooling over separation, since investment timing for them is identical, and under pooling they benefit from cross-subsidization. For high types, on the other hand, there is a trade-off: when the proportion of high types, ϕ, is sufficiently high, high types' losses from cross-subsidization are minor and the higher aggregate welfare under pooling will dominate.

Proposition 3 *There exists a critical fraction of high types, $\phi^* \in (0,1)$, such that pooling dominates for $\phi > \phi^*$ and the separating equilibrium is the unique equilibrium for $\phi < \phi^*$.*

With adverse selection on initial project quality, in those case when the separating equilibrium dominates, we expect to see earlier investment than would be optimal and hence lower success rates, in particular for those projects that have ex ante higher quality. Predictions of earlier investment and lower success when adverse selection is severe are in line with observations on biotech alliances in Guedj and Scharfstein (2004). In the next section we will see that not only adverse selection, but also entrepreneurial moral hazard can explain such results.

4. Moral hazard

To optimally delay the investment decision, it is necessary to have an investment contract that makes waiting attractive to the entrepreneur. We saw that we could achieve that by paying a cash flow p to the entrepreneur before the moment of investing. So far we assumed quality Q to change stochastically in a way that is exogenous to the entrepreneur. If the entrepreneur can affect the process for Q, a clear drawback of paying the entrepreneur for waiting is that this undercuts incentives to work on the project to raise Q. We now turn to analyze such moral hazard.

We adopt a simple model of moral hazard, in which the entrepreneur can either work and the quality process continues as before, or spend his time on other activities and earn private benefits b. In the latter case, the development of the funded project stops, $dQ = 0$. The investors still cannot observe Q (there is adverse selection), nor can they verify whether the entrepreneur works (there is moral hazard).

At any point in time, the entrepreneur, observing current quality level Q, will decide whether to work or to shirk, trading off the benefits of future investment to the outside benefit b of shirking. Since the stochastic process stops when the entrepreneur shirks, this trade-off will not change from that point on. Hence, once the entrepreneur decides to shirk, he will continue shirking forever, and Q will be stationary from then onwards.

Any given pay-off structure (p, α) will therefore not only determine an investment threshold $\bar{Q}$, but also an absorbing shirking boundary $\underline{Q}$, at which the entrepreneur effectively kills the diffusion. In analyzing the optimal financing contract we should therefore determine the endogenous relation between these two thresholds.

Denoting the entrepreneur's utility at current Q by $V(Q)$, for any given (p, α)we now have a value-matching and smooth-pasting condition at both boundaries:

$$V(\bar{Q}) = \alpha\bar{Q}, \tag{9}$$

$$V_Q(\bar{Q}) = \alpha, \tag{10}$$

$$V(\underline{Q}) = \frac{p+b}{r}, \tag{11}$$

$$V_Q(\underline{Q}) = 0. \tag{12}$$

The latter two conditions are new, and reflect the fact that at the shirking threshold $\underline{Q}$, the entrepreneur will receive cash flows p plus private benefits b in perpetuity. The smooth-pasting condition ensures that the entrepreneur will choose that threshold privately optimally.

Proceeding as in the previous section, we can again express the value V in terms of the thresholds $\bar{Q}$ and $\underline{Q}$ as follows

Lemma 3 *For given contract (p, α), and private benefits of shirking b, the entrepreneur will invest as soon as Q first crosses some upper threhsold $\bar{Q}$, and instead stop the process when a lower bound $\underline{Q}$ is hit. the entrepreneur's value function V satisfies*

$$V_Q(Q) = E\left(e^{-r\bar{T}} \mid Q\right) \frac{\alpha\bar{Q}}{Q}, \tag{13}$$

where $\bar{T}$ is the first passage time of $\bar{Q}$, or $\bar{T} = \infty$ if instead the absorbing lower threshold $\underline{Q}$ is hit first. An explicit expression of that conditional expectation is

$$E\left(e^{-r\bar{T}} \mid Q\right) = \frac{\underline{Q}^{\lambda_-}Q^{\lambda_+} - \underline{Q}^{\lambda_+}Q^{\lambda_-}}{\underline{Q}^{\lambda_-}\bar{Q}^{\lambda_+} - \underline{Q}^{\lambda_+}\bar{Q}^{\lambda_-}}. \tag{14}$$

The investment and shirking boundaries $\bar{Q}, \underline{Q}$ are related to the contract parameters α, p and benefits b by

$$\left(\left(\frac{\bar{Q}}{\underline{Q}}\right)^{\lambda_+} - \left(\frac{\bar{Q}}{\underline{Q}}\right)^{\lambda_-}\right) b = \tfrac{1}{2}\sigma^2 \alpha \bar{Q}(\lambda_+ - \lambda_-) \tag{15}$$

$$p = \alpha \bar{Q} r \frac{\lambda - 1}{\lambda} - b \left(\frac{\bar{Q}}{\underline{Q}}\right)^{\lambda_-} \tag{16}$$

Finally, the value function itself can be written as

$$V(Q) = \alpha \bar{Q} \left(1 - \int_{Q}^{\bar{Q}} \frac{E\left(e^{-r\bar{T}} \mid s\right)}{s} ds\right). \tag{17}$$

The expressions relating the boundaries $\bar{Q}, \underline{Q}$ to the contract parameters p, α behave intuitively as moral hazard parameter b changes. For zero b there is no moral hazard and we are in the previous regime, with $\underline{Q} = 0$ and p as in lemma 1. As private benefits b grow, keeping α and $\bar{Q}$ constant, we obtain a positive shirking boundary $\underline{Q}$. As the benefit of waiting now also includes the benefits b when the lower boundary has been hit, one has to reduce p to make sure the entrepreneur is still incentivized to invest at $\bar{Q}$.

We can of course use lemma 3 to determine $\bar{Q}$ and $\frac{\bar{Q}}{\underline{Q}}$ if we are given α and p. Vice versa, we can compute the unique combination p, α that leads to a given set of $\bar{Q}$ and $\frac{\bar{Q}}{\underline{Q}} > 1$.

Again, we first explore the first-best. We have total welfare now including both the benefits of investing, and the benefits b of inaction at the lower threshold,

$$W(\bar{Q}, \underline{Q}, Q) = E\left[\frac{b}{r} e^{-r\underline{T}} + (\bar{Q} - I) e^{-r\bar{T}} \mid Q\right], \tag{18}$$

where $\underline{T}$ represents the first time Q hits the absorbing lower boundary. It is straightforward to compute the first-best levels of $\bar{Q}, \underline{Q}$ using value matching and smooth-pasting conditions for W at those thresholds.

While in our previous case with only adverse selection, the pooling equilibrium turned out to lead to first-best investment timing, that relation breaks down when we include moral hazard. The reason is that now that we have two thresholds $\bar{Q}$ and $\underline{Q}$, fixing these (at first-best levels) determines both α and p according to the relations in lemma 3. Since those α, p leave a value that exceeds total welfare, given that the investor is protected by limited liability, it is not possible for an investor to finance that contract and break even.

Proposition 4 *To implement first-best levels for the investment and shirking thresholds $\bar{Q}, \underline{Q}$, one needs $\alpha = 1$ and $p = rI$. This contract does not allow the investor to break even.*

To find the optimal contract that can be financed by investors, we need to maximize the entrepreneur's value function V, subject to the constraint that investors break even. To analyze that problem, we first use lemma 3 to express V in terms of $\bar{Q}$ and $\underline{Q}$. Combining equation (17) with (15) we write

$$V = \frac{b}{\frac{1}{2}\sigma^2(\lambda_+ - \lambda_-)} \left[\frac{\lambda_+ - 1}{\lambda_+} \left(\frac{\bar{Q}}{\underline{Q}}\right)^{\lambda_+} - \frac{\lambda_- - 1}{\lambda_-} \left(\frac{\bar{Q}}{\underline{Q}}\right)^{\lambda_-} + \frac{1}{\lambda_+} \left(\frac{Q}{\underline{Q}}\right)^{\lambda_+} - \frac{1}{\lambda_-} \left(\frac{Q}{\underline{Q}}\right)^{\lambda_-} \right].$$

On the other hand we can compute total welfare for the choice of thresholds $\bar{Q}, \underline{Q}$, by substituting the expectations $E\left(e^{-r\underline{T}}\right)$ and $E\left(e^{-r\bar{T}}\right)$ into total welfare equation (18),

$$W = \frac{(\bar{Q} - I)\left(\left(\frac{Q}{\underline{Q}}\right)^{\lambda_+} - \left(\frac{Q}{\underline{Q}}\right)^{\lambda_-}\right) + \frac{b}{r}\left(\left(\frac{\bar{Q}}{\underline{Q}}\right)^{\lambda_+}\left(\frac{Q}{\underline{Q}}\right)^{\lambda_-} - \left(\frac{\bar{Q}}{\underline{Q}}\right)^{\lambda_-}\left(\frac{Q}{\underline{Q}}\right)^{\lambda_+}\right)}{\left(\frac{\bar{Q}}{\underline{Q}}\right)^{\lambda_+} - \left(\frac{\bar{Q}}{\underline{Q}}\right)^{\lambda_-}}.$$

(check: $W = \bar{Q} - I$ at $Q = \bar{Q}$, $W = \frac{b}{r}$ at $Q = \underline{Q}$).

The optimal contracting problem for given initial quality level Q now is

$$\max_{\bar{Q}, \underline{Q}} V(\bar{Q}, \underline{Q}, Q) \tag{19}$$

$$s.t. \quad V = W \tag{20}$$

Typically, the optimizing threshold will depend on the value of initial quality Q. Figure 1 shows a graph of the optimum threshold levels as a function of the initial Q, and compares these with first-best thresholds.

We have that in general, the continuation regime lies within the first-best thresholds.

Proposition 5 *The optimal contract with moral hazard has both earlier investment compared to first-best, and earlier shirking, or $\bar{Q} < \bar{Q}^*$ and $\underline{Q} > \underline{Q}^*$.*

As a result, we find that with moral hazard, it is in the investors' interest to induce earlier than first-best investment timing, to reduce the rents needed to keep the entrepreneur working. On the other side, the optimal contract also allows some inefficient shirking.

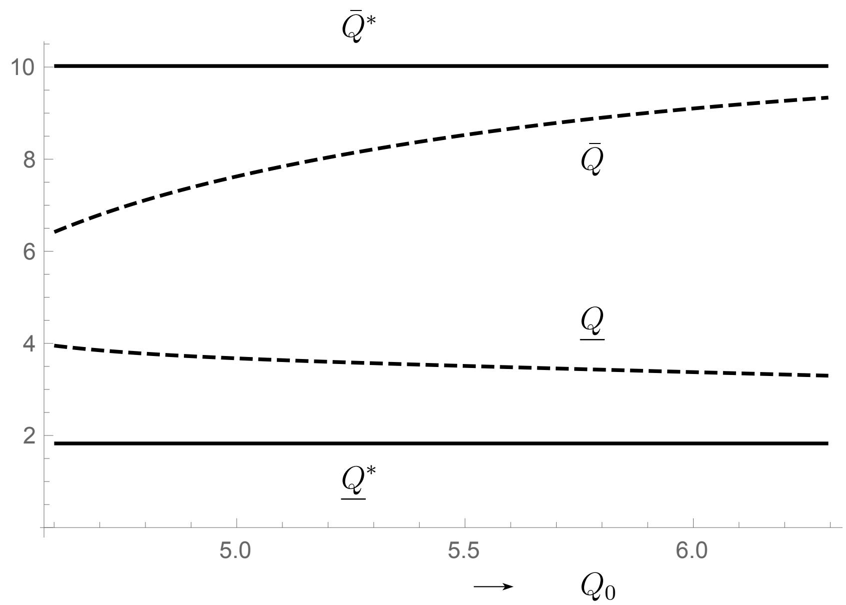

Figure 1: First-best investment and shirking thresholds (drawn lines) and optimal thresholds under information asymmetry (dashed) as a function of initial Q_0. Parameters are $\lambda_+ = 2, \lambda_- = -2, r/b = 3, I = 5$.

5. Conclusion

We analyzed financing of new firms' irreversible investments that suffer from dynamic asymmetric information. This is likely to be the case in R & D intensive industries, where entrepreneurs, often researchers themselves, try to get financing for product introduction of innovative technologies that are still being researched. Clearly, the entrepreneurs are likely to be much better informed about the evolving prospects of their projects as research progresses, but they may engage in window dressing to get financiers to put up the funds for large investments at a premature stage. After all, the entrepreneur can benefit from the upside without risking their own funds. We analyzed how contract can be designed to correct such incentives even if the investors cannot observe project quality at all, and therefore have to rely on the entrepreneurs' reports.

Optimal contracts reward the entrepreneur for waiting, to ensure that investments are not sunk prematurely. If there is adverse selection on initial project quality as well, good entrepreneurs may signal their type by concluding contracts that sacrifice some real option value by reducing this payment for waiting, hence inducing too early investment. If there is moral hazard, payments for waiting are risky since they undercut incentives to bring the project to fruition. In that case, again, waiting fees are reduced compared to the first-best, resulting in early investment compared to real option optimal timing.

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A. Appendix

Proof of lemma 1: We use a result from Arve and Zwart (2014) that any combination of payments that incentivizes the entrepreneur to invest at $Q = \bar{Q}$ should satisfy a condition on the entrepreneur's value's derivative,

$$V_Q(Q) = E\left[e^{-rT}\frac{\alpha\bar{Q}}{Q}\right]. \tag{21}$$

This follows from the observations that at $\bar{Q}$, V should satisfy smooth pasting, $V_Q(\bar{Q}) = \alpha$, and furthermore, $QV_Q(Q)$ satisfies a Bellman equation without source term p:

$$r\left(QV_Q\right) = \mu Q(QV_Q)_Q + \tfrac{1}{2}\sigma^2Q^2(QV_Q)_{QQ}.$$

Solving that partial differential equation for QV_Q with boundary condition defined by the smooth pasting condition leads to (21). Then, since $V(\bar{Q}) = \alpha\bar{Q}$, we have

$$V(Q) = \alpha\bar{Q} - \int_Q^{\bar{Q}} e\left[\alpha\frac{\bar{Q}}{Q'}e^{-rT}\right]dQ'.$$

The required value for p can be computed by noting that V itself satisfies the Bellman equation with source term p,

$$rV = p + \mu QV_Q + \tfrac{1}{2}\sigma^2Q^2V_{QQ},$$

and substituting the above expression for $V(Q)$ evaluated at $\bar{Q}$. Using that $E\left[e^{-rT}|Q_0\right] = \left(\frac{Q_0}{Q}\right)^{\lambda_+}$, and that $r = \mu\lambda_+ + \frac{1}{2}\sigma^2\lambda_+(\lambda_+ - 1)$ by the definition of λ_+, we find the expressions for p and V. *Q.E.D.*

Proof of proposition 1: The optimal pooling contract maximizes entrepreneur surplus. Since all surplus can be extracted, and both types' social surplus is maximized at the same threshold $\bar{Q}^*$, the optimal contract implements this first-best level. To see that high types cross-subsidize low types, first note that full rent extraction implies

$$\phi\left(V(Q_H,\bar{Q}^*) - W(Q_H,\bar{Q}^*)\right) = (1-\phi)\left(W(Q_L,\bar{Q}^*) - V(Q_L,\bar{Q}^*)\right),$$

so $V - W$ have opposite signs for high and low types. With first-best $\bar{Q}^* = \frac{\lambda_+}{\lambda_+-1}I$, we have $\frac{\partial W(Q,\bar{Q}^*)}{\partial Q} = \left(\frac{Q}{\bar{Q}^*}\right)^{\lambda_+-1}$. Also, $\frac{\partial V(Q,\bar{Q}^*)}{\partial Q} = \alpha\left(\frac{Q}{\bar{Q}^*}\right)^{\lambda_+-1}$, so $\frac{\partial(V-W)}{\partial Q}$ is negative. Hence, $V(Q_H,\bar{Q}^*) - W(Q_H,\bar{Q}^*) < 0$ and $V(Q_L,\bar{Q}^*) - W(Q_H,\bar{Q}^*) > 0$. *Q.E.D.*

Proof of proposition 2: Rewriting the requirement that the entrepreneur gets all of the surplus, (7), we find that

$$\frac{\alpha_H \bar{Q}_H^{1-\lambda_+}}{\lambda_+} = \frac{W(Q_H, \bar{Q}_H)}{Q_H^{\lambda_+} + (\lambda_+ - 1)\bar{Q}_H^{\lambda_+}}. \tag{22}$$

Combining this with binding incentive compatibility, (6), we arrive at equation (8) in the proposition. To see that this equation has solutions, note first that the right-hand side is always larger that $W(Q_L, \bar{Q}_L)$ and is decreasing in $\bar{Q}_H$, since $Q_H > Q_L$ and $\lambda_+ > 1$. The left hand side, $W(Q_H, \bar{Q}_H) = \left(\frac{Q_H}{\bar{Q}_H}\right)^{\lambda_+} (\bar{Q}_H - I)$, is quasi-concave in $\bar{Q}_H$, attains its maximum at $\bar{Q}_H = \bar{Q}^* > I$, and is zero at $\bar{Q}_H = I$. At $\bar{Q}_H = \bar{Q}^*$, we have $W(Q_H, \bar{Q}^*) = \left(\frac{Q_H}{Q_L}\right)^{\lambda_+} W(Q_L, \bar{Q}_L)$, which is larger than the right-hand side of (8). Hence, the equation has a unique solution $\bar{Q}_H$ on the interval $(I, \bar{Q}^*)$. In addition, there exists a solution for which $\bar{Q}_H > \bar{Q}^*$. But any higher solution has lower surplus since the right-hand side is decreasing.

Finally, we still need to verify that high types do not want to mimick low types, incentive compatibility condition (3). Subtracting (4) from (3), we equivalently need to show that $\alpha_H \bar{Q}_H^{1-\lambda_+} \geq \alpha_L \bar{Q}_L^{1-\lambda_+}$. To see that his holds, consider equation (22) and its analogon for low type entrepreneurs. Comparing these, and using (8) characterizing the separating equilibrium, we find that

$$\frac{\alpha_H \bar{Q}_H^{1-\lambda_+}}{\lambda_+} = \frac{\alpha_L \bar{Q}_L^{1-\lambda_+}}{\lambda_+} \frac{Q_L^{\lambda_+} + (\lambda_+ - 1)\bar{Q}_L^{\lambda_+}}{Q_L^{\lambda_+} + (\lambda_+ - 1)\bar{Q}_H^{\lambda_+}}.$$

Since $\bar{Q}_H < \bar{Q}^* = \bar{Q}_L$, we find that high-type incentive compatibility is verified. *Q.E.D.*

Proof of lemma 2: We have

$$V = \alpha \bar{Q} \left(\frac{\lambda_+ - 1}{\lambda_+} + \frac{1}{\lambda_+} \left(\frac{Q}{\bar{Q}}\right)^{\lambda_+} \right).$$

Hence,

$$\frac{\partial V}{\partial \bar{Q}} = \frac{\alpha(\lambda_+ - 1)}{\lambda_+} \left(1 - \left(\frac{Q}{\bar{Q}}\right)^{\lambda_+}\right), \quad \text{and} \quad \frac{\partial V}{\partial \alpha} = \frac{\bar{Q}(\lambda_+ - 1)}{\lambda_+} \left(1 + \frac{1}{\lambda_+ - 1} \left(\frac{Q}{\bar{Q}}\right)^{\lambda_+}\right),$$

so that on indifference curves of constant V, $\frac{d\alpha}{dQ}$ is negative. Moreover, at given $\alpha, \bar{Q}$, higher $Q < \bar{Q}$ leads to a less negative slope. *Q.E.D.*

Proof of proposition 3: We need to compare V_H in the pooling and the separating cases.

First, from the explicit expressions for V and W, as well as the full surplus extraction equation

$$\phi V_H^{pool} + (1-\phi)V_L^{pool} = \phi W_H^{pool} + (1-\phi)W_L^{pool}$$

it is straightforward to show that V_H is monotonically increasing in ϕ. In the pooling case, in the limit $\phi = 1$ where all firms are high types, we have that high-type surplus is maximal at first-best surplus for the high type, $V_H^{pool} = W_H(\bar{Q}^*)$. Finally, as $\phi \to 0$, contract parameters converge to the low type's parameters α_L, p_L, and hence $V_H^{pool} \to V(Q_H, p_L, \alpha_L)$. Comparing this with the separating solution, we have that $V_H^{sep} < V_H^{pool}(\phi = 1)$, since total high-type surplus under separation is lower than first-best surplus. And from high type incentive compatibility, (3), $V_H^{sep} > V_H^{pool}(\phi = 0)$. Hence there is a critical $\phi^* \in (0,1)$ below which $V_H^{sep} > V_H^{pool}$. *Q.E.D.*

Proof of lemma 3: From the smooth-pasting conditions, we know V_Q at both boundaries. Between those boundaries, QV_Q satisfies an HJB equation without source term. Equation (13) solves that differential equation with those boundary conditions.

To compute the explicit expression for the conditional expectation, note that the general solution to its differential equation takes the form

$$E\left(e^{-r\bar{T}} \mid Q\right) = AQ^{\lambda_+} + BQ^{\lambda_-},$$

with $\lambda_\pm$ the positive and negative roots to the fundamental quadratic, as before. Using the boundary conditions,

$$E\left(e^{-r\bar{T}} \mid \bar{Q}\right) = 1, \quad E\left(e^{-r\bar{T}} \mid \underline{Q}\right) = 0,$$

we can compute the coefficients A, B to find expression (14) .

Now consider the HJB equation for V,

$$rV = p + \mu Q V_Q + \tfrac{1}{2}\sigma^2 Q^2 V_{QQ}.$$

Evaluating this at $Q = \underline{Q}$, where $V = (p+b)/r$ and $V_Q = 0$, we find

$$b = \tfrac{1}{2}\sigma^2 Q^2 V_{QQ},$$

which leads to (15) upon computing the derivative from (13) and (14). Instead evaluating it at $\bar{Q}$, where $V = \alpha\bar{Q}$ and $V_Q = \alpha$, leads to (16) upon using (15). Finally, the equation for V follows from the derivative condition (13) combined with the boundary value $V(\bar{Q}) = \alpha\bar{Q}$. *Q.E.D.*

Proof of proposition 4: Setting p, α as prescribed identifies the entrepreneur's problem of setting the thresholds with the social planner's. By investing, the entrepreneur gives up a perpetuity of $p = rI$, hence the opportunity costs of investing is I. Since $\alpha = 1$, the net pay-out at $\bar{Q}$ is identical to the social planner's. Similarly, the decisions at the lower threshold coincides. With these pay-offs, the total value for the entrepreneur equal W plus a perpetuity of $p = rI$. Hence, the entrepreneur's value exceeds total welfare by I, and outside investors funding this contract would therefore get net surplus equal to $-I$.

For completeness: optimization of total welfare W, using value-matching and smooth pasting, leads to conditions on the thresholds

$$\bar{Q} = \frac{\lambda_-}{\lambda_- - 1}\left(I + \frac{b}{r}\left(\frac{\bar{Q}}{\underline{Q}}\right)^{\lambda_+}\right)$$
$$\bar{Q} = \frac{\lambda_+}{\lambda_+ - 1}\left(I + \frac{b}{r}\left(\frac{\bar{Q}}{\underline{Q}}\right)^{\lambda_-}\right).$$

Solving these two equation gives the first-best. Solving also for the value of W at this optimum gives

$$W = \frac{b}{r(\lambda_+ - \lambda_-)}\left(-\lambda_-\left(\frac{Q}{\underline{Q}}\right)^{\lambda_+} + \lambda_+\left(\frac{Q}{\underline{Q}}\right)^{\lambda_-}\right).$$

Now compare with V, the entrepreneur's value at these thresholds.

$$V = \frac{b}{\frac{1}{2}\sigma^2(\lambda_+ - \lambda_-)}\left[\frac{\lambda_+ - 1}{\lambda_+}\left(\frac{\bar{Q}}{\underline{Q}}\right)^{\lambda_+} - \frac{\lambda_- - 1}{\lambda_-}\left(\frac{\bar{Q}}{\underline{Q}}\right)^{\lambda_-} + \frac{1}{\lambda_+}\left(\frac{Q}{\underline{Q}}\right)^{\lambda_+} - \frac{1}{\lambda_-}\left(\frac{Q}{\underline{Q}}\right)^{\lambda_-}\right].$$

Using the equations for the optimal boundaries, and $-r = \frac{1}{2}\sigma^2\lambda_+\lambda_-$, some rewriting reveals that the first two terms equal I, while the last two terms equal W. It is then straightforward to also calculate the values for α and p. *Q.E.D.*

Proof of proposition 5: Idea of proof: For fixed Q, we can look at the W-optimizing $\bar{Q}$ given $\underline{Q}$, $\bar{Q}(\underline{Q})$. And similarly we can find the optimal $\underline{Q}$ given $\bar{Q}$, $\underline{Q}(\bar{Q})$. Both lines are decreasing. The intersection of these lines gives the first-best. For given $\underline{Q}$, demanding that $W \geq V$ means that $\bar{Q} < \bar{Q}(\underline{Q})$. Similarly, for given $\bar{Q}$, the condition $W \geq V$ implies $\underline{Q} > \underline{Q}(\bar{Q})$. Hence, maximizing W s.t. $W \geq V$ means we will be to the south east of the intersection. *Q.E.D.*