Dynamic Asymmetric Cournot Oligopoly under Capacity Constraints

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We consider firms with differing cost structures investing non-cooperatively in production capacity to subsequently compete in output under capacity constraints. Firms have operational flexibility to discontinue production or fully utilize their capacity in the presence of competition and stochastic demand. This paper extends the extant literature on strategic investment under uncertainty by considering competition among asymmetric firms with temporary shut-down and expansion options as well as optimal capacity investment decisions. We analyze the effect of various parameters such as initial demand, volatility and heterogeneity on firm values and concentration based on equilibrium characterizations for output choices, firm profits and values in Cournot oligopoly. We find that (ex ante) linear capacity investment cost asymmetry leads to (ex-post) heterogeneity with highly non-linear capacity distributions.

A interesting strategic feature of the model is that, while constrained firms are marginalized for high demand, unconstrained firms exert greater market power by expanding production and capture a larger/disproportional share of the growing total market value. The initial capacity decision-marking must account both for the stand-alone value of an marginal capacity unit and for its strategic effect through creating larger "strategic" convexity at large demand levels.

Key words: Asymmetric Cournot competition, real options, capacity constraints, operational flexibility

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Introduction

Assessing how industries will develop is rather challenging because firms often face *operating risk* arising from unforeseeable fluctuations in market demand or other exogenous parameters and *strategic risk* when few stakeholders, e.g., competitors, pursuing their own interests, interact and endogenously influence each others' decisions. Economists typically account for the first risk type by considering random variables or stochastic processes with parameters estimated using econometric methods. Real options analysis (see, e.g., Dixit and Pindyck 1994, Smith and Nau 1995, Trigeorgis 1996) — capitalizing on an analogy between cash-flow claims obtained in business situations and the payoff structure of financial options — enables quantifying a firm's ability to mitigate (through, e.g., production contraction) or exploit operating risk (through, e.g., production expansion). This modeling approach initially falls short of the second risk type. Industrial organization, by contrast, provides prescriptive guidance into how firms should cope with strategic risk.[1]

The Cournot model ushered theoretical research on industrial organization (see Singh and Vives 1984, Tirole 1988). In the standard Cournot model, duopolist firms simultaneously and non-cooperatively choose the quantities they will produce (their outputs) which they sell at the market-clearing price. In our paper, we greatly revise the celebrated Cournot model by considering both risk types in an integrative manner. We model a general, multistage oligopoly where, at the initial stage, firms with heterogeneous cost structures invest in production capacity (*de facto* setting their own capacity constraints) and subsequently engage repeatedly in Cournot competition with their rivals in the face of stochastic demand. In summary, we extend the standard model in several dimensions. From a static perspective, we address two shortcomings. First, while the duopoly model is an interesting benchmark, it may fail to account for certain strategic interactions compared to more general oligopoly models. Second, certain firms may enjoy and exploit firm asymmetries or competitive advantages (Porter 1980), for instance, in terms of cost structures. Incorporating stochastic demand on top yields interesting dynamics and strategic interactions. In view of demand developments, each firm may decide to contract its output or entirely shut down production, or, to the contrary, expand production until it faces capacity constraints. Under asymmetric competition, internal rivalry may evolve over time as certain firms may face capacity constraints, while other may still expand. Slack capacity gives unconstrained firms leeway in exerting greater market power: they capture a greater share of a larger market value as demand builds up.[2]

Firms face several dilemmas. Subject to strategic risk in oligopoly, each firm must account at all times for its rivals' reactions when setting its production output. Besides, firms are not necessarily on an equal footing in the product market: unconstrained firms can exert market power by expanding production, while constrained firms are marginalized. Regardless of the industry structure, each firm faces operating risk and must assess the relative advantages and disadvantages of enhancing the expected value of its expansion option by adding production capacity and incurring at the outset larger (total) cost of capacity. At the outset when firms non-cooperatively decide

[1] An analytical framework addressing strategic interactions among parties with conflicting objectives is (non-cooperative) game theory (e.g., Fudenberg and Tirole 1991). Modern industrial organization (see, e.g., Tirole 1988) borrows from these modeling techniques.

[2] An industry where the present setting fits well is the power sector. Indeed, certain utilities exert substantial market power and behave as in Cournot oligopoly (Murphy and Smeers 2005). At times of high demand and/or bad weather conditions, excess demand may rise substantially and jeopardize grid stability. Flexible Combined Cycle Gas Turbines (CCGT) are expeceted to fill in such times (IEA 2014). Given high fuels costs, such generation capacities will only operate for sufficiently above-average demand. Requiring significant investment outlays (c. €700/kW, IEA-RETD 2013) these assets are hence characterized by asymmetric profit claims similar to a call option, with zero profit at low demand and positive profits during peak hours.

on their production capacities, all these trade-offs resulting from strategic and operating risk are accounted for: choosing a high production capacity creates subsequent option value convexity and helps avoid getting marginalized and retaining a strategic stance at times of high market demand. Yet, too large capacity choices may entail a prohibitive initial cost.

To address the various trade-offs arising here we proceed backwards characterizing in turn (i) firm outputs, (ii) capacitated Cournot profits, (iii) firm values for an arbitrary industry capacity vector, and finally (iv) firms' optimal initial capacity choices. In proceeding we find several interesting economic results:

(i) The *optimal output policy* involves a continuous range of output levels, from discontinued production to full capacity utilization. At very low demand, firms decide not to sell at a price below production cost. Firm output depends linearly on the demand level at intermediate demand but is fixed (constrained) at larger demand. Rivalry among firms influences a firm's output decision as long as the latter remains unconstrained, i.e., for low to intermediate demand. Unconstrained firms factor the capacity constraints of their smaller, constrained rivals as an aggregate competitive pressure of fixed magnitude and compete with each others à la Cournot on "residual demand." Compared to the standard Cournot model, firms produce (weakly) more as long as they remain unconstrained but less once standard Cournot output exceeds the capacity limit.

(ii) The *Cournot profit* of an unconstrained firm is first nil as long as demand is insufficient for profitable operation and then convex in the demand shock as the demand level grows and firms raise the production pace (exploiting their expansion option) to leverage on increased demand. Once the capacity constraint becomes binding, profit becomes linear: the firm becomes non-strategic in the price-setting mechanism but still benefits from a higher market-clearing price. The profit function is continuous over the entire demand range but admits kinks (not continuously differentiable) at c and the various demand levels at which individual firms become capacity constrained.

(iii) For a given capacity vector, firm *values* which can be expressed in semi-closed form reflect: (a) the asset value under steady (expected) demand, (b) the upside potential for cases where demand builds up and firms gradually face capacity constraints, (c) the downside risk-mitigation value from contracting or discontinuing production when demand shrinks and constraints are no longer binding. We further investigate the effect of key parameters, such as initial demand and demand volatility, on industry value concentration.

(iv) The *optimal capacity selection* for the integrated multistage game informs us how industry asymmetry arises from investment cost differences. Besides the expected asymmetry-enhancing effect of capacity cost heterogeneity, we additionally find that demand volatility also enhances capacity asymmetry. Conversely, the higher initial demand levels will reduce industry concentration. Furthermore, we show that ex-ante investment linear cost asymmetry leads to ex-post non-linear capacity heterogeneity resembling a rank order distribution.

We make several contributions to the literature. The paper makes extensions and explores a new territory involving demand uncertainty, operational flexibility, capacity constraints, general oligopolistic competition and firm capacity distribution. We extend the literature on strategic investment under uncertainty by considering oligopolies of arbitrary size versus duopoly situations as well as firm asymmetry. Furthermore, we consider expansion and shut-down options during the production phase as well as optimal capacity investment decisions. The article is organized as follows. Section 1 discusses related research articles and positions the article within the relevant literature. Section 2 describes the basic economic setup, while Section 3 characterizes the equilibrium properties of the short-run Cournot oligopoly under capacity constraints. In Section 4 we derive firm values under a given capacity vector. Section 5 examines the optimal capacity decisions and assesses the impact of key parameters. The final section concludes.

1. Related literature

Our approach makes inroads into several research fields in operations management and economics, involving operational flexibility, capacity investment, and continuous-time stochastic games.

1.1. Operational flexibility

The literature on flexible manufacturing systems (see, e.g., Sethi and Sethi 1990) has examined production configurations allowing firms to deal with demand uncertainty by contracting or expanding production. In Kogut and Kulatilaka (1994), a firm with a global manufacturing footprint can adjust to exchange-rate fluctuations by either of two means. It can take short or long positions in correlated financial instruments, e.g., it can trade in currency options or swaps to mitigate currency variations. Alternatively, the firm can shift production between manufacturing plants exposed to different currency regimes. Modeling such situations typically involves switching options where the decision maker chooses among discrete operating modes by incurring specified non-negative switching costs (e.g., Kulatilaka and Trigeorgis 1994). This type of setting often induces a binary, "or bang-bang", strategy whereby the firm either switches or not, with no intermediary manufacturing configuration adjustment. We extend this by additionally considering strategic risk in a more general setting where the decision maker "switches" from one operating mode to another (in terms of production intensity) within a continuum of production states. This feature is somewhat more realistic as market price endogenously arises from firms' collective actions.

1.2. Capacity investment under uncertainty

Spencer and Brander (1992) consider Cournot duopoly competition and characterize in closed form the trade-off between flexibility and commitment under stochastic demand: The specific level of demand volatility determines whether commitment has more value than output flexibility. Gabszewicz and Poddar (1997) consider duopolists who simultaneously choose capacities prior to revelation of the stochastic demand level. Subsequently, they engage in one-shot Cournot quantity competition. They show that the duopolists choose capacities exceeding the Cournot certainty equivalent game. Kulatilaka and Perotti (1998) assess the possibility of a Stackelberg duopoly leader making an early commitment investment conferring greater "capability" to take advantage of future growth opportunities vis-à-vis a Stackelberg follower. Besanko and Doraszelski (2004) use a capacity accumulation model to explain firm heterogeneity in dynamic duopoly. Their model highlights product market competition mode and investment reversibility as key determinants of firm size distribution.

In Goyal and Netessine (2007), duopolist firms decide whether to adopt a production technology adapted for a single or multiple products and then in turn on capacity and output in the product-market stage. Swinney et al. (2011) analyze the investment timing decisions of established and start-up firms in a new uncertain market, with possibility for firms to invest in capacity early on or once uncertainty is resolved. In continuous time, Grenadier (2002) characterizes capacity investment trajectories for symmetric oligopolistic firms. Aguerrevere (2003) extends this symmetric setting by introducing output discretion. Novy-Marx (2007) considers firms that are asymmetric with respect to their initial capacities and can expand capacity at an increasing cost-to-scale ratio.[3] In contrast to extant literature, we consider a continuous-time model of strategic interactions with capacity constraints in a general k-firm oligopoly. Firms continuously decide on their output in view of realized demand as part of a Cournot quantity competition game. Capacity selection takes place at the outset and drives firm heterogeneity. We also investigate how the degree of ex-ante heterogeneity drives firm strategies.

Boyer et al. (2012) consider symmetric duopolist firms competing under demand uncertainty à la Cournot under capacity constraints and may repeatedly increase capacity by investing in lumps of fixed size. The authors investigate Markov Perfect Equilibrium (MPE) investment paths and articulate which situations favor preemption vs. tacit collusion in the investment behaviors. Hence, they focus on the optimal timing of subsequent lumpy capacity additions. Huisman and Kort (2014) study a duopoly and find that the Stackelberg leader strategically overinvests in order to deter entry. While we disregard investment timing, we consider optimal investment scale in capacity and consider closely output in the product-market stage.

[3] Chevalier-Roignant et al. (2011) summarize the literature on strategic investment under uncertainty and discuss other related articles.

1.3. Other continuous-time stochastic games

Some other literature streams have a more technical connection to our model: supergames and stochastic differential games. The literature on stochastic supergames in continuous time (Sannikov 2007, Fudenberg and Levine 2007, 2009, Sannikov and Skrzypacz 2010) considers settings in which information (e.g., on demand) arrives via diverse Levy processes and in which the players' actions may influence the parameters of the Brownian component. In this literature the question focuses on conditions under which two firms are likely to coordinate on strategies that, over the long run, yield higher value than a repeated, static Nash equilibrium.[4] Although such supergame solutions may serve as a useful benchmark, we adopt a different solution approach relying on the Markov Perfect Equilibrium (MPE) solution concept under perfect monitoring. Besides, our model is cast in a managerial context involving demand uncertainty, firm capacities and Cournot competition with more than two rivals.

In differential games (Isaacs 1965, Basar and Olsder 1999), the player values are obtained as the solution of a multidimensional Hamilton-Jacobi-Bellman (HJB) partial differential equation influenced by all actors. Such games apply in a number of business contexts, such as in industrial organization or operations management (see Dockner et al. 2000). In our model, adjusting output from one period to the other does not give rise to additional fixed costs. Consequently, the subsequent quantity-setting decisions are independent from one another and there is no need to derive and solve a k-dimensional HJB equation. Rather, we derive firm values by integrating the Cournot profits over an infinite planning horizon.

2. Model setup and solution approach

Consider a market where k firms with asymmetric cost structures choose production capacities in a non-cooperative manner and then compete à la Cournot with a flexible production technology. This basic setup resembles the model by Gabszewicz and Poddar (1997), however, we expand on their analysis by considering a general oligopoly and *repeated* Cournot competition based on continuously fluctuating demand. These demand fluctuations are observable by all market participants at all times.

We assume that sunk investment outlays and/or fixed operating costs are heterogeneous among firms. Several causes can lead to such asymmetry.[5] Function $C_i(\cdot)$ corresponding to the initial

[4] Whether a supergame solution for this setup exists is unclear. It is also questionable whether public authorities would allow such an equilibrium to sustain as it could hamper consumer well-being without encouraging innovation.

[5] For instance, in the power sector, certain utility companies may have privileged conditions with key suppliers, such as GE, Alstom, Siemens or Westinghouse/Mitsubishi or better financing terms. Firms may also incur distinct project-specific fixed operating costs since larger energy groups can spread General, Selling and Administrative (GS&A) expenses over a larger number of business units or because certain utility companies may have more efficient internal processes, e.g., maintenance management.

capital outlays and the present value of fixed operating costs as a function of firm i's capacity $\bar{q}_i$ is continuously increasing. We opt for a simple linear cost model based on two parameters: (i) the cost parameter of firm 1, C_1, and (ii) the cost differential between successive (ranked) firms, $\chi \in [0, C_1/(k-1))$. Firm i's cost function is $C_i(\bar{q}_i) = C_i \bar{q}_i$ where $C_i = C_1 - (i-1)\chi$. In the linear cost model cost curves are not intersecting.

Firm $i \in \{1, \ldots, k\}$ holds production capacity $\bar{q}_i \in \mathbb{R}_+$ throughout the game, with rivals having capacity $\bar{Q}_{-i} \in \mathbb{R}_+^{k-1}$. Vector $\bar{Q} = (\bar{q}_1, ..., \bar{q}_k)^\top$ specifies the production capacities, with total industry capacity given by $\sum_{i=1}^k \bar{q}_i$. For simplicity, we concentrate on strategy profiles for which firms are weakly ranked and indexed by *increasing capacity*:[6]

$$0 \equiv \bar{q}_0 < \bar{q}_1 \leq \cdots \leq \bar{q}_m \leq \cdots \leq \bar{q}_k < \bar{q}_{k+1} \equiv \infty. \tag{2.1}$$

Denoting firm i's share of industry capacity by $s_i \equiv \bar{q}_i / \sum_{i=1}^k \bar{q}_i$, industry concentration can be assessed by the standard Herfindahl-Hirschman Index ($H: \mathbb{R}_+^k \to [0,1]$), namely

$$H(\bar{Q}) = \sum_{i=1}^k s_i^2 \tag{2.2}$$

After capacity selection, firms compete in a repeated Cournot competition game. At each time $t \in [0, \infty)$, firm i chooses output quantity $q_i(t)$ subject to capacity constraints given by:

$$0 \leq q_i(t) \leq \bar{q}_i, \; \forall t \in \mathbb{R}_+, \; \forall i = 1, \ldots, k. \tag{2.3}$$

All firms collectively produce output $Q_t = \sum_{i=1}^k q_i(t)$. As in Kulatilaka and Perotti (1998) and Van Mieghem and Dada (1999), we assume firms face a linear (inverse) demand of the form:[7]

$$p(X_t; Q_t) = X_t - b \; Q_t, \quad b > 0. \tag{2.4}$$

Suppose the demand intercept $(X_t; t \geq 0)$ follows a geometric Brownian motion of the form:

$$\mathrm{d}X_t = \mu \; X_t \; \mathrm{d}t + \sigma X_t \; \mathrm{d}B_t, \quad X_0 = x \; (> 0), \tag{2.5}$$

[6] It is neither obvious or certain whether the (weak) ordering of investment costs lead to a (weak) ordering of production capacities in equilibrium. In this paper, we put an additional constraint to ensure that the Nash equilibrium capacity vector preserves a weak ordering (2.1) in-line with the investment cost ranking. We prove the existence and uniqueness of such a Nash equilibrium. There may exist Nash equilibria violating (2.1) but their existence will highly depend on parameter choices. Besides, the former equilibrium type would Pareto-dominate them. As common practice in economics, we focus on the Pareto-dominating Nash equilibrium in case of equilibrium multiplicity.

[7] Related contributions (see, e.g., Huisman and Kort 2014, Pawlina and Kort 2006, Grenadier 2002) typically consider an economic shock X_t affecting demand not in an *additive* but in an *multiplicative* manner with an inverse demand function of the form $p(X_t; Q_t) = X_t \mathrm{p}(Q_t)$ where function p is deterministic. While the multiplicative approach offers obvious tractability advantages, it does fail to provide a suitable benchmark to deterministic Cournot models. While multiplicative shocks are well suited for, e.g., exchange-rate fluctuations, they are less so to capture industry or market specificity in terms of customers' fluctuating willingness-to-pay or purchasing power.

where μ is the risk-neutral drift, σ (>0) is constant volatility per period, and $(B_t; t \geq 0)$ is a standard Brownian motion.[8] As standard in real options analysis, the firm can perform financial hedging to mitigate certain business risks; this allows considering a stochastic process with drift adjusted for the underlying systematic riskiness and thus discounting future risk-adjusted profit claims at the risk-free rate $(r > 0)$ under the risk-neutral probability measure.[9] If δ represents some form of convenience yield, anticipated competitive erosion, opportunity cost or below-equilibrium growth or return shortfall (see McDonald and Siegel 1986), the risk-neutral drift is given by $\mu = r - \delta$ (> 0).[10] Assuming marginal cost c common to all firms in the industry (gross) profit is

$$\pi_i(X_t; Q_t) = [p(X_t; Q_t) - c] \;\; q_i(t). \tag{2.6}$$

The term $Q_{-i}(t)$ denotes the aggregate output by all other firms except firm i at time t, with $Q_t \equiv q_i(t) + Q_{-i}(t)$.

Firm i's payoff for given capacities $(\bar{q}_i, \bar{Q}_{-i})$ and arbitrary output policies (q_i, Q_{-i}) is obtained by discounting expected risk-adjusted profits at the risk-free rate r:

$$J_i\big(x; q_i, Q_{-i}; \bar{q}_i, \bar{Q}_{-i}\big) = \mathbb{E}_x\left[\int_0^\infty e^{-rt}\pi_i(X_t; Q_t)\,\mathrm{d}t\right] - C_i(\bar{q}_i). \tag{2.7}$$

$\mathbb{E}_x[\cdot] \equiv \mathbb{E}[\cdot \mid X_0 = x]$ denotes the conditional expectation operator under the risk-neutral probability measure. Expression (2.7) differs from the standard notion of present value in that here firm i's management has flexibility in deciding what output strategy q_i to follow. Because future demand is unknown, the strategy is non-anticipative and allows for flexibility in managerial decision-making.[11]

The objective function (2.7) captures the various trade-offs and dilemmas faced by firm i. In setting its own production output $q_i(t)$, it must cope with strategic risk at each time $t \in \mathbb{R}_+$ and make predictions on its rivals' behaviors $Q_{-i}(t)$: in a holistic manner, it must assess the impact of its decision on the market-clearing price as well as its rivals' reactions. Because future demand is unknown until realized, it muss assess the initial value of each capacity unit under the premises that

[8] The process is defined on probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\mathbb{P})$ is the risk-neutral or martingale probability measure. The augmented Brownian filtration $(\mathscr{F}_t; t \geq 0)$ incorporates the historical path of the process, with $\mathscr{F}_t = \sigma(B_s, 0 \leq s \leq t)$ being the information set at time t and $\mathscr{F}_\infty = \mathcal{F}$.

[9] According to Birge (2000), option-pricing theory offers a rigorous way to incorporate risk aversion in linear profit expressions without relying on (strictly concave) utility functions.

[10] One may want to adjust the model to incorporate both demand and cost uncertainty, i.e., to account additionally for a stochastic marginal cost $(c_t; t \geq 0)$. Suppose $(X_t; t \geq 0)$ and $(c_t; t \geq 0)$ follow correlated geometric Brownian motions and note that π_i is homogeneous of degree one in these stochastic variables. Solving this problem is feasible by reducing the problem dimensionally. This would involve a change of variable replacing the two processes with a single, new "relative" process whose law of motion reflects the probability characteristics of $(X_t; t \geq 0)$ and $(c_t; t \geq 0)$. The main analytical results would still hold except, for example, the volatility values for β_1 and β_2 in Proposition 3 would be adjusted to involve relative volatility, i.e., $\sigma^2 = \sigma_X^2 + \sigma_c^2 - 2\rho\sigma_X\sigma_c$. Since such change of variables brings limited added value at the expense of exposition clarity, we concentrate on stochastic demand only.

[11] Formally, the output decisions $(q_i(t); t \geq 0)$ are $(\mathscr{F}_t; t \geq 0)$-measurable.

management will actively respond to economic shocks alongside its rivals. If firm i selects low initial capacity $\bar{q}_i$, it must endure times of high demand at which it is constrained. Each unit of capacity has positive option value since it widens the range of demand levels at which the firm can react to favorable demand developments by expanding production. The initial value of capacity depends on key parameters such as volatility, with large volatility creating larger expansion values. Regardless of the industry structure, firm i has to wave the positive value of capacity additions against the larger investment capacity cost $C_i(\bar{q}_i)$. Asymmetry among production capacities complicates the matter since, for a given demand level, some firms may be constrained and marginalized (producing at full capacity), while unconstrained rivals may use their output discretion to exert market power on the residual market. Firms are not initially aware of their rivals' capacity decisions but recognize that their rivals will factor all these strategic effects in when selecting their own production capacity. The equilibrium solution to the game provides answers to all these issues and gives guidance into which parameters prevail under which circumstances.

We now proceed backwards to solve the game, deriving in turn the equilibrium output strategies and Cournot profits (Section 3), firm values for a set capacity vector (section 4), and finally the equilibrium capacity vector (section 5). The basic problem in section 3 is close to the classic Cournot duopoly game but its solution necessitates adjustments to account for fluctuating demand, capacity constraints and an arbitrary number of oligopolist firms. In section 4, we assess firm i's value in Markov Perfect Equilibrium $\left(q_i^C, Q_{-i}^C\right)$,

$$V_i(x;\bar{Q}) = \max_{q_i(\cdot)} \mathbb{E}_x \left[\int_0^\infty e^{-rt}\pi_i(X_t;q_i(t),Q_{-i}(t))\,\mathrm{d}t\right]. \tag{2.8}$$

for a given capacity vector $\bar{Q}$.[12] As a final step in solving the game, we are seeking in section 5 a capacity vector $\bar{Q}^C$ for which no firm has an incentive to pursue a unilateral deviation from the equilibrium industry choice. In Nash equilibrium, $\bar{Q}^C = \left(\bar{q}_i^C, \bar{Q}_{-i}^C\right)$ thus satisfies

$$\hat{V}_i(x) \equiv V_i\left(x;\bar{q}_i^C,\bar{Q}_{-i}^C\right) - C_i(\bar{q}_i^C) \geq V_i\left(x;\bar{q}_i,\bar{Q}_{-i}^C\right) - C_i(\bar{q}_i) \tag{2.9}$$

for all feasible capacity choices $\bar{q}_i \in \mathbb{R}_+$ of each firm i, such that constraint (2.1) is satisfied.

3. Asymmetric *k*-firm Cournot Oligopoly with Capacity Constraints

We now consider the problem of determining the equilibrium outputs and profits for k firms with asymmetric capacity constraints in Cournot oligopoly. We identify a Markov Nash equilibrium (MNE), i.e., a profile of output choices such that no firm has an incentive to unilaterally deviate

[12] A MPE is a strategy profile, $\left(q_i^C, Q_{-i}^C\right)$, in the class of Markov or feedback policies that yields a Markov Nash equilibrium (MNE) for every demand state x. See Fudenberg and Tirole (1991, Chapter 13) for a discussion on MPE. The perfectness of a MNE strategy profile is easily proved.

from its equilibrium output decision. Let $(q_i^C, Q_{-i}^C) \equiv (q_i^C(X_t), Q_{-i}^C(X_t))$ be MNE output decisions in demand state X_t. The equilibrium Cournot (gross) profit, $\pi_i^C(X_t)$, of firm $i=1,\ldots,k$ satisfies

$$\pi_i^C(X_t) \equiv \pi(X_t; q_i^C, Q_{-i}^C) \geq \pi_i(X_t; q_i(t), Q_{-i}^C)$$

for all output decisions $q_i(t)$ satisfying capacity constraints (2.3).[13]

As a benchmark, let's recall the (equilibrium) Cournot output $\tilde{q}\,(X_t;k)$ in a unconstrained k-firm oligopoly (see, e.g., Tirole 1988)

$$\tilde{q}(X_t;k) = \begin{cases} 0 & \text{if } X_t \in (0,c), \\ \frac{X_t - c}{b(k+1)} & \text{if } X_t \in [c,\infty). \end{cases} \tag{3.1}$$

Before proceeding, we define new parameters and variables. Parameter $\bar{x}_i > 0$ is a demand threshold given by

$$\bar{x}_i \equiv b\left[\sum_{j=0}^{i-1} \bar{q}_j + (k-i+2)\,\bar{q}_i\right] + c, \quad i = 1,\ldots,k. \tag{3.2}$$

Weak capacity ordering (2.1) ensures that $0 < \bar{x}_1 \leq \bar{x}_2 \leq \cdots \leq \bar{x}_k < \infty$ partitions the state space in a non-overlapping manner. Also consider stochastic variables $\tilde{X}_t$ and $\tilde{k}_t$ defined by

$$\tilde{X}_t \equiv X_t - b\left[\sum_{m=0}^{k} \bar{q}_m \mathbb{1}_{\{X_t \geq \bar{x}_m\}}\right] \tag{3.3}$$

and

$$\tilde{k}_t \equiv k - \sum_{m=0}^{k} \mathbb{1}_{\{X_t \geq \bar{x}_m\}} \tag{3.4}$$

respectively, where $\mathbb{1}_{\{\cdot\}}$ is the indicator function. Proposition 1 below gives the MNE for the static Cournot game with capacity constraints.

Proposition 1 *In MNE, firm i produces output $q_i^C(X_t)$ given by*

$$q_i^C(X_t) = \begin{cases} 0 & \textit{with firm } i \textit{ idle if } X_t \in (0,c) \\ \frac{\tilde{X}_t - c}{b(\tilde{k}_t+1)} & \textit{with firm } i \textit{ unconstrained if } X_t \in [c,\bar{x}_i) \\ \bar{q}_i & \textit{with firm } i \textit{ constrained if } X_t \in [\bar{x}_i,\infty). \end{cases} \tag{3.5}$$

Proof of Proposition 1 See Appendix A. □

Following the MNE output policy in Proposition 1, a firm will not produce ("stay idle") if it cannot recover the marginal production cost c, i.e., in demand region $(0,c)$; this is similar to the unconstrained Cournot-Nash equilibrium, with $q_i^C(X_t) = \tilde{q}(X_t;k)$ if $X_t \in (0,c)$. Explaining the optimal output strategy in the intermediate demand region $[c,\bar{x}_i)$ is more involved as it critically

[13]For notational simplicity, we sometimes omit the dependency on vector $\bar{Q}$.

depends on the demand partitioning resulting from thresholds $\bar{x}_m$. In intermediate demand region $[\bar{x}_{m-1}, \bar{x}_m)$, firm m is the smallest unconstrained firm: the $m-1$ smaller firms are constrained, while firm m and its larger rivals ($k-[m-1]$ firms in total) remain unconstrained. The collective actions of constrained firms results in aggregate output $\sum_{j=0}^{m-1} \bar{q}_j$ in demand region $[\bar{x}_{m-1}, \bar{x}_m)$ or $\sum_{m=0}^{k} \bar{q}_m \mathbb{1}_{\{X_t \geq \bar{x}_m\}}$ in general for any arbitrary demand level $X_t \in \mathbb{R}_+$. Constrained firms exert a competitive pressure of fixed magnitude on the addressable market of unconstrained firms. This pressure is factored in in stochastic variable $\tilde{X}_t$ defined in (3.3): $\tilde{X}_t$ corresponds to the demand for unconstrained firms when current *total* demand is X_t. A related variable is $\tilde{k}_t$ which tracks down how many firms are still unconstrained for arbitrary demand X_t, with $\tilde{k}_t = k-(m-1)$ for $X_t \in [\bar{x}_{m-1}, \bar{x}_m)$ or $\tilde{k}_t = k - \sum_{m=1}^{k} \mathbb{1}_{\{x_t \geq \bar{x_m}\}}$. In our model, the $\tilde{k}_t$ unconstrained firms vie for demand $\tilde{X}_t$ in a similar fashion as k Cournot oligopolists vie for total demand X_t in the classical k-firm Cournot oligopoly model without capacity constraints, with equality $q_i^C(X_t) = \tilde{q}(\tilde{X}_t, \tilde{k}_t)$ holding when firm i is unconstrained in demand region $(c, \bar{x}_i)$.[14] For large demand ($X_t \geq \bar{x}_i$), firm i will produce at full capacity $\bar{q}_i$ (with $q_i^C(X_t) < \tilde{q}(X_t;k)$). In other words, parameter $\bar{x}_i$ given in (3.2) should be interpreted as the indifference demand level at which firm i will become capacity constrained.

Having determined the optimal output, we next derive the resulting equilibrium profits. Proposition 2 below gives the Cournot profit earned by incumbent firm i in MNE.

Proposition 2 *In Cournot oligopoly with capacity constraints, firm i's gross profit at demand level X_t is*

$$\pi_i^C(X_t) = \begin{cases} 0 & \text{if } X_t \in (0,c) \\ \frac{\left(\tilde{X}_t - c\right)^2}{b(\tilde{k}_t+1)^2} & \text{if } X_t \in [c, \bar{x}_i) \\ \bar{q}_i \frac{\left(\tilde{X}_t - c\right)}{(\tilde{k}_t+1)} & \text{if } X_t \in [\bar{x}_i, \infty). \end{cases} .$$

Cournot profit $x \mapsto \pi_i^C(x)$ is continuous, flat on $(0,c)$, strictly convex increasing on $(c, \bar{x}_i)$ and linear increasing on $(\bar{x}_i, \infty)$. Note, however, that it is not continuously differentiable as there are kinks at each x_j with $j = 1, ..., k$.

Proof of Proposition 2 See Appendix B. □

For low demand ($X_t < c$) the firm does not produce, earning zero gross profit. For intermediary demand, it is unconstrained and produces below its capacity, with profits being convex (quadratic) in demand while the demand intercept grows, the firm raises the production pace (exploiting its expansion option) to leverage on increased demand. For large demand ($X_t \geq \bar{x}_i$), firm i's capacity constraint becomes binding, although this is not necessarily the case for its larger rivals; firm i's

[14] Note the degenerate case with $q_i^C(X_t) = \tilde{q}(X_t;k)$ for intermediate demand region $(c, \bar{x}_1)$ where no firm is constrained.

profit increases in a linear fashion as the firm, faced with capacity constraints, is not in a position to expand output to fully tap on increased demand; yet, it still benefits from a higher equilibrium price.

4. Firm values in constrained Cournot oligopoly

We turn next to assessing firm value. Without adjustment costs, output decisions are independent over time, with firm value obtaining as the expected discounted sum of Cournot profits $\pi_i^C(X_t)$:

$$\begin{aligned} V_i(x;\bar{Q}) &\equiv \max_{q_i(\cdot)} \mathbb{E}_x \left[\int_0^\infty e^{-rt} \pi_i \left(X_t; q_i(t), Q_{-i}^C(t) \right) \, \mathrm{d}t \right] \\ &= \mathbb{E}_x \left[\int_0^\infty e^{-rt} \pi_i^C(X_t) \, \mathrm{d}t \right], \end{aligned}$$

where demand intercept $(X_t; t \geq 0)$ follows the GBM of equation (2.5).

4.1. Semi-analytic value expressions

The next proposition provides a value expression for firm i's flexible production capacity, $V_i(x;\bar{Q})$. The expression which seems analytical/closed-form at first actually involves two intertwined series which do not admit neat formulations; the expression is, hence, "semi-analytical."

Proposition 3 *For a given capacity vector $\bar{Q}$, firm i's value $V_i(x;\bar{Q})$, is given by:*

$$V_i(x;\bar{Q}) = \begin{cases} A_0 x^{\beta_1} & \text{if } x \in (0,c), \\ v_i(x;m) + A_m x^{\beta_1} + B_m x^{\beta_2} & \text{if } x \in [\bar{x}_{m-1}, \bar{x}_m), \ m = 1, \ldots, i, \\ \bar{v}_i(x;m) + A_m x^{\beta_1} + B_m x^{\beta_2} & \text{if } x \in [\bar{x}_{m-1}, \bar{x}_m), \ m = i+1, \ldots, k+1, \end{cases} \tag{4.1}$$

where

$$\begin{aligned} v_i(x;m) &\equiv \frac{1}{b(k-m+2)^2} \left[\frac{x^2}{2\delta - r - \sigma^2} - \frac{2\Sigma_m x}{r-\mu} + \frac{\Sigma_m^2}{r} \right], & m = 1, \ldots, i, \\ \bar{v}_i(x;m) &\equiv \frac{\bar{q}_i}{(k-m+2)} \left[\frac{x}{r-\mu} - \frac{\Sigma_m}{r} \right], & m = i+1, \ldots, k+1, \\ \beta_1, \beta_2 &\equiv -\frac{r - \delta - \sigma^2/2}{\sigma^2} \pm \sqrt{\left(\frac{r-\delta-\sigma^2/2}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} \end{aligned}$$

with $\beta_2 < 0 < 2 < \beta_1$ and $\Sigma_m \equiv c + b \sum_{j=0}^{m-1} \bar{q}_j$. Terms A_m and B_m are uniquely defined and obtained recursively with $x \mapsto V_i(x;\bar{Q})$ being continuously differentiable. Firm value as a function of demand $x \mapsto V_i(x;\bar{Q})$ is monotone increasing with $V_i(0;\bar{Q}) = 0$ and $V_i(\infty;\bar{Q}) = \infty$. Above assumes $\delta > [r + \sigma^2]/2$.

Proof of Proposition 3 See Appendix C. □

The (equilibrium) industry structure adjusts each time the capacity constraint of another firm (namely the next larger) becomes binding (demand growth scenarios) or when the capacity constraint of the next smaller firm is relaxed (demand contraction scenarios); these changes in industry structures are not accounted for in the steady-state value expressions $v_i(x;m)$ and $\bar{v}_i(x;m)$. Terms noted $A_m x^{\beta_1}$ capture the upside potential accounting for scenarios in which demand rises and firms scale up production (up to their capacity limits). $A_m x^{\beta_1}$ in effect relates to a series of options, each corresponding to an industry-structure change arising (at stochastic future time $\tau_A(x,\bar{x}_m) \equiv \inf\{t \geq 0 \mid X_t \geq \bar{x}_m\}$) when yet another previously unconstrained firm starts facing binding capacity constraints (at the new demand threshold $\bar{x}_m > x$). Since the change takes place at a future stochastic time $\tau_A(x,\bar{x}_m)$, the (future) value change from larger market power among a smaller subset of oligopolist firms is discounted to the present with use of the present value of an Arrow-Debreu derivative paying one dollar at future time $\tau_A(x,\bar{x}_m)$, $\mathbb{E}_x\left[e^{-r\tau_A(x,\bar{x}_m)}\right] = (x/\bar{x}_m)^{\beta_1}$. Terms noted $B_m x^{\beta_2}$ captures downside-risk mitigation for scenarios in which demand shrinks and the firms contract or shut-down production. These terms are similarly linked to a series of options; each time a demand threshold $\bar{x}_m$ $(< x)$ is first "hit" from above (at future stochastic time $\tau_B(x,\bar{x}_{m-1}) \equiv \inf\{t \geq 0 \mid X_t \leq \bar{x}_{m-1}\}$), the capacity constraint of yet another firm is relaxed, with rivals subsequently facing fiercer competition with more firms exerting market power over the residual market. The change in value is again discounted to the present time by use of the present value of the appropriate Arrow-Debreu derivative, $\mathbb{E}_x\left[e^{-r\tau_B(x,\bar{x}_m)}\right] = (x/\bar{x}_m)^{\beta_2}$.

For low current demand, $x \in (0,c)$, the firm is initially idle but operational flexibility is not worthless; it indeed reflects positive upside value $A_0 x^{\beta_1}$ since demand may grow sufficiently to justify production at a later time. When demand is so low that production is worthless, risk-mitigation value B_0 vanishes. In intermediate demand region $[\bar{x}_{m-1}, \bar{x}_m)$, $m-1$ firms are constrained, while $k-(m-1)$ still exert market power on the residual market and earn convex profit. Suppose firm i is ranked in capacity such that $\bar{q}_i > \bar{q}_m$. Then, in a situation where firms do not adjust their outputs in view of actual demand developments, firm i would be entitled to present value

$$v_i(x;m) = \mathbb{E}_x\left[\int_0^\infty e^{-rt}\frac{(X_t - \Sigma_m)^2}{b(k-m+2)^2}\,\mathrm{d}t\right], \quad x \in [\bar{x}_{m-1}, \bar{x}_m),\ m = 1,\ldots,i,$$

as given in Proposition 3.[15] As demand fluctuates, however, a flexible firm will adjust production in view of demand realizations and rivals' actions. In the high demand regions for $x > \bar{x}_i$, firm i is

[15]For $x \in [\bar{x}_{m-1}, \bar{x}_m)$, $m = 1,\ldots,i$, the quadratic Cournot profit expression (see Proposition 3) has terms in x^2, x and a constant, Σ_m^2. The perpetuity value of the constant amount is Σ_m^2/r. For terms that trend upwards, we employ the (continuous-time) Gordon formula: the present value of receiving a cash flow starting at y and growing in perpetuity at a rate g $(< r)$ is $y/(r-g)$. The growth rate for $(X_t; t \geq 0)$ is $r-\delta$; it is $2\delta - r - \sigma^2$ for $(X_t^2; t \geq 0)$. This confirms the $\bar{v}_i$ expression in Proposition 3.

constrained producing at full capacity $\bar{q}_i$, earning a linear profit. The present value of sustaining the initial industry structure for $x \in [\bar{x}_{m-1}, \bar{x}_m)$ forever is

$$\bar{v}_i(x;m) = \mathbb{E}_x \left[\int_0^\infty \frac{\bar{q}_i \left(X_t - \Sigma_m \right)}{k - m + 2} \, \mathrm{d}t \right].$$

Again, for a flexible firm that will not follow this committed strategy, value adjustments are needed. $A_m x^{\beta_1}$ corresponds to the upside potential resulting from flexible expansion strategies in case of increased future demand, while $B_m x^{\beta_2}$ captures the risk-mitigation strategy of contracting or discontinuing production at low demand.

4.2. Numerical illustration

Before proceeding with deriving the Nash-equilibrium, let assume a set industry capacity vector $\bar{Q}$ to illustrate various results on, e.g., firm values and demand thresholds. We adopt two approaches. First, we graph the value of a triopolist firm endowed with a fixed capacity, while the capacities of its two rivals vary (x- and y-axes). Second, we build upon the well-established Zipf's (power) law to "draw" a distribution of firm capacities.

First approach: Consider three firms A, B and C, assume $\bar{q}_A = 2.5$ and let $\bar{q}_B$ and $\bar{q}_C$ vary on $[0;5]$. By construction, constraint (2.3) on capacity ordering is relaxed. We may distinguish several industry configuration archetypes in figure 1. Firm A is largest in size in I ($q_B \leq q_C \leq q_A$ in Ia, $q_C \leq q_B \leq q_A$ in Ib); it is intermediate in II ($q_C \leq q_A \leq q_B$) and III ($q_B \leq q_A \leq q_C$) and smallest in IV ($q_A \leq q_B \leq q_C$ in IVa, $q_A \leq q_C \leq q_B$ in IVb). Besides varying rivals' capacities, Figure 1 illustrates the effect on firm A's value of increasing volatility σ (from left to right) and initial demand x (from top to bottom).

We can derive general insights:

- a steady, risk-free world is "quadratic" as (nearly) illustrated in the top left panel. Larger demand and volatility yields smoother results.
- Firm value increases with initial demand
- Firm value increases with volatility
- The contour gradient is steepest in the area of low to intermediate capacities: large production capacities at rivals flats out firm A's "strategic" convexity by limiting its ability to exert market power for large demand.
- Since larger volatility favors extremely large demand values, the latter effect arising from increased competitor capacities is exacerbated in the right panels.

In summary we can distinguish convexity arising from operating risk (driven by volatility) and convexity arising from strategic risk (driven by increased initial demand or firm A's ability to exert market power vis-a-vis its rivals). However, the two are intertwined because under large volatility the extreme demand case where capacity constraints are binding are more likely.

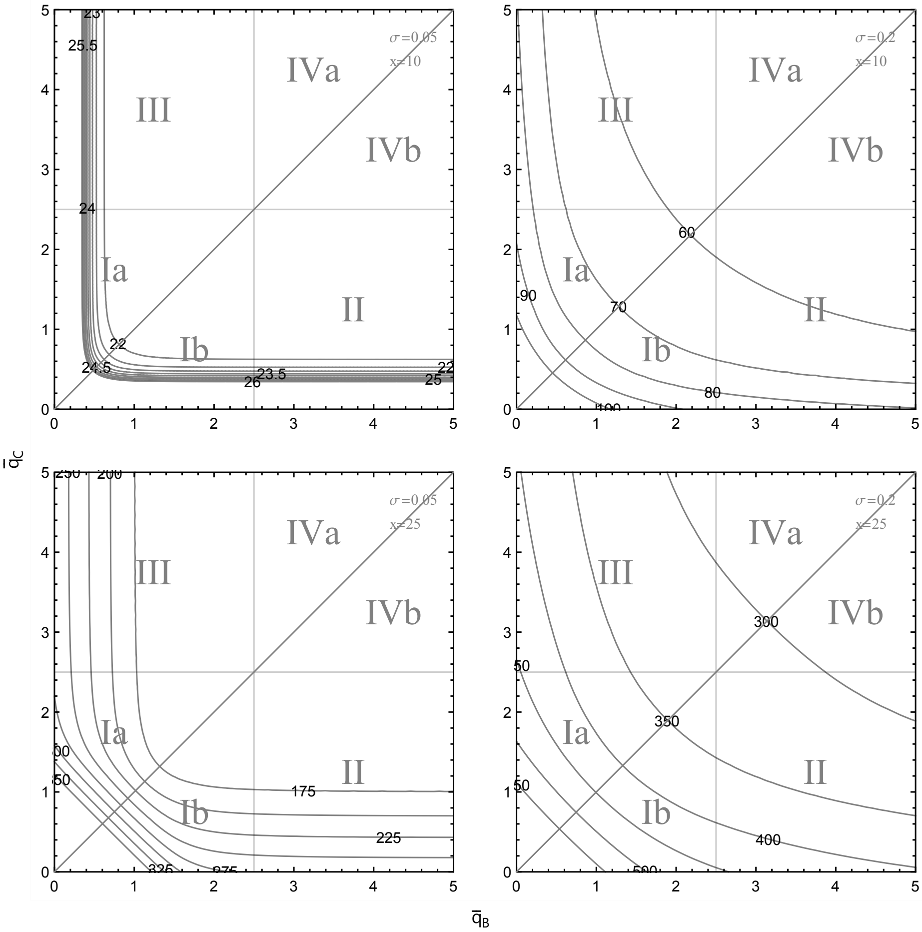

Figure 1 **Firm value V_A for different capacity configurations, initial demand and volatility levels.** $(k = 3, c = 1, \bar{q}_A = 2.5, b = 5, \mu = 0, r = \delta = 0.05)$

Second approach: We now draw results on industry values based on an empirically approved capacity distribution law. Suppose firm capacities are distributed according to Zipf's law (Ijiri et al. 1977, Axtell 2001, Fujiwara et al. 2004). Hereby, a single heterogeneity parameter, h, drives the industry structure. Firm heterogeneity ranges from homogeneity at $h = 0$ to virtual monopoly for $h \to \infty$, with firm i's capacity share given by[16]

$$s_i(h) \equiv \frac{i^{-h}}{\sum_{m=1}^{k} m^{-h}}, \quad i = 1, \cdots, k. \tag{4.2}$$

Based on this capacity distribution, we can determine firm values for different heterogeneous industry configurations as given by the heterogeneity parameter. Figure 2 depicts firm values in triopoly as well as the corresponding first-order derivatives and the critical demand thresholds $\bar{x}_1$, $\bar{x}_2$ and $\bar{x}_3$, for different combinations of the volatility parameter σ and the heterogeneity parameter

[16]Note that $\sum_{i=1}^{k} s_i(h) = 1$ for all $h \geq 0$.

h. When the 3 firms are close to being homogeneous (small h), firm values and critical threshold levels are within a small range. Firm values are strictly increasing in initial demand x; yet, whenever

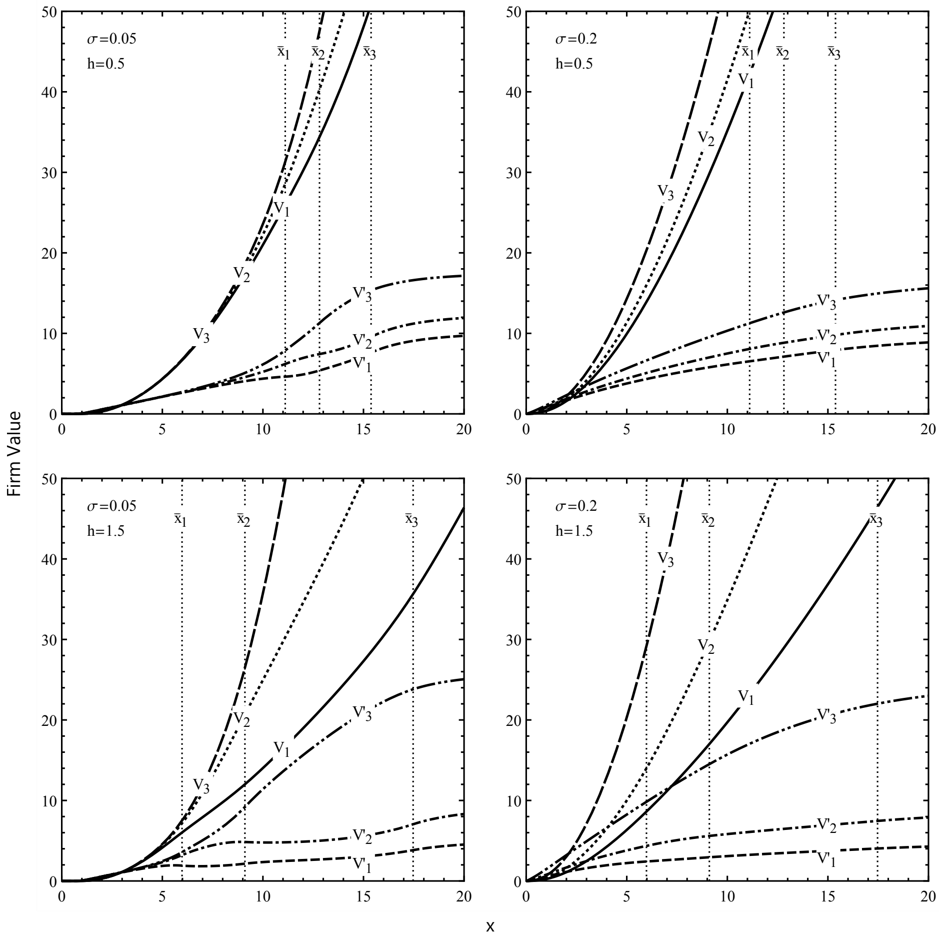

Figure 2 Firm value $V_i(x;\bar{Q})$ and first-order partial derivative $V_i'(x;\bar{Q})$ for different capacity heterogeneity levels h and demand volatility σ. $(k=3, c=1, |\bar{Q}|=2, b=5, \mu=0, r=\delta=0.05)$

a firm faces a capacity constraint, the value growth rate declines *smoothly*.[17] The firm exerts market power (convex payoff) until it faces capacity constraints and only benefits from market-clearing price upsurge with no possibility to expand production (linear profit function). Conversely, other unconstrained firms benefit from this specific firm being pushed to the fringe, with their profits increasing at a greater rate as if competition were involving fewer rivals exerting market power (see Proposition 2).

Figure 3 illustrates the industry value differential between a heterogeneous oligopoly ($h > 0$) and a symmetric one ($h = 0$). Clearly, capacity concentration enhances total industry value as a

[17]Indeed, the value of standard call options is continuously differentiable despite the payoff being kinked at the strike price. Similarly, here the firm values are continuously differentiable although the Cournot profit functions have multiple kinks.

dominant firm with larger (slack) capacity can exert greater market power once the smaller firms become capacity-constrained. This basic effect is moderated by the current demand level: For low demand levels capacity constraints are not binding, hence, there is a limited value differential. Analogously, for very high demand any industry will produce at full capacity. The effect of volatility on this value differential is ambiguous: At both the low and the high demand level the differential is largest for high volatility as this increases the probability of demand entering a domain where the dominant firm can exert market power. Conversely, in the intermediate demand area low volatility is value-enhancing since demand is less likely to leave the range where the dominant firm can exert market power.

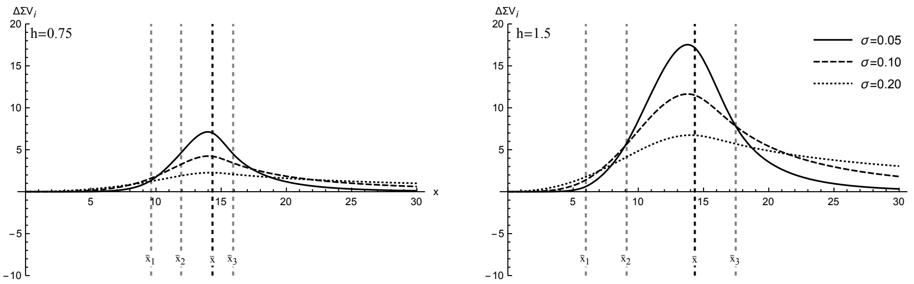

Figure 3 **Industry value differential** $\Delta \sum V_i = \sum V_i(h) - \sum V_i(0)$ **for different** σ **and** h **values** $(k = 3, c = 1, |\bar{Q}| = 2, b = 5, \mu = 0, r = \delta = 0.05)$

5. Capacity selection game

We next consider how multiple firms select their capacities in a non-cooperative manner, with the capacity vector being a set of ex-ante strategic decisions. The capacity choice directly influences firm values through, e.g., the demand partitioning (see Proposition 3), hence basic calculations, e.g., first and second-order derivatives, become intricate. In other words, it becomes cumbersome to pursue an analytic solution to the general capacity selection game. Here, we provide a numerical solution that satisfies (2.9) while preserving the monotone weak capacity ordering (2.1).

5.1. Capacity best response

Given the semi-analytical form of firm values (4.1) for given capacity configuration, we can determine a firm's best response to competitor capacity choices. Formally, firm i's capacity best response $\bar{q}_i^*(x; \bar{Q}_{-i})$ is given by

$$\bar{q}_i^*(x; \bar{Q}_{-i}) = \underset{\bar{q}_i \in [\bar{q}_{i-1}, \bar{q}_{i+1}]}{\arg\max} \; V_i(x; \bar{q}_i; \bar{Q}_{-i}). \tag{5.1}$$

This best response function reflects the capacity ordering required by (2.3). Hence, the concrete form of the best response function is conditional on the firm index i. This is illustrated in Figure

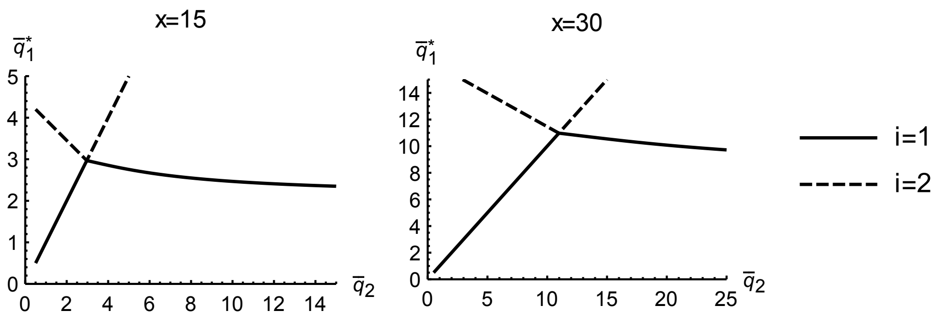

Figure 4 **Capacity best responses conditional on capacity ordering in duopoly for different** x ($C_i = 25, k = 2, c = 1, b = 5, \sigma = 0.2, \mu = 0, r = \delta = 0.05$)

4 for a duopoly. The two lines illustrate the case where firm i is the smaller firm ($i = 1$), or larger firm ($i = 2$).

Before, we noted that competitor capacities are "bad news" for a given firm reducing its value. This is reiterated here with higher capacities of the incumbent firms inducing firm i to invest with less intensity (lower $\bar{q}_i^*$)[18]—additional capacity is less valuable in the presence of large capacity stocks owned by competitors. Consequently, firm capacities in this setting are strategic substitutes. This observation is the main building block for characterizing the competitive capacity equilibrium by means of a tâtonnement process.

5.2. Competitive capacity equilibrium

Having established an individual firm's best response to competitor capacity endowments, we now want to explore the industry capacity configuration arising under strategic competition. Requiring strict capacity rankings corresponding to the ordered capacity expansions costs C_i, a tâtonnement process allows us to identify a valid capacity equilibrium. That is, we iteratively apply the firms' best response correspondences to pin-point a equilibrium capacity configuration where firm capacity choices are mutual best responses. We describe an algorithm for this purpose in Appendix D.

Figure 5 presents numerical comparative statics on the capacity equilibrium industry concentration for different values of cost differential χ, volatility σ, and initial demand x. As would be expected, cost heterogeneity χ amplifies capacity heterogeneity. However, the effect of χ is augmented by volatility as illustrated by the right panels of Figure 5.

While greater volatility enhances the output option granted by capacities for all firms, a cost-advantaged firm will expand more decisively than its competitors as the bet on future demand potentials is markedly cheaper. The converse is true for the initial demand level x—sufficiently lucrative current markets will attract also less cost-effective companies to invest at larger scale.

[18]Except for those cases where the constraint $\bar{q}_2^* \geq \bar{q}_1^*$ is binding.

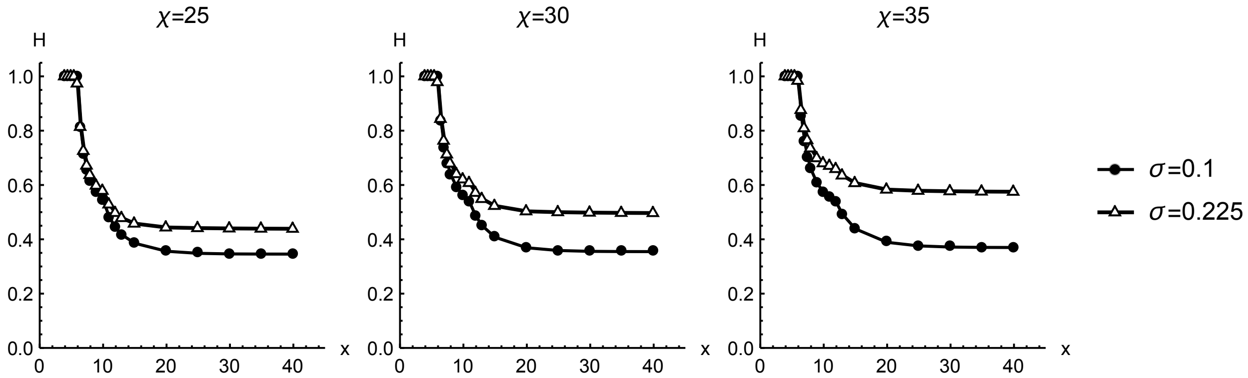

Figure 5 **Herfindahl index for different χ and σ values** $(k = 3, c = 1, b = 5, C_1 = 100, \mu = 0, r = \delta = 0.05)$

This is because the embedded call options of additional capacities will be closer to being "in the money." The demand effect is not "smooth" but rather exhibits changes in curvature whenever the underlying industry structure changes—monopoly, duopoly and finally triopoly. This illustrates that our model implicitly reflects blockaded entry in settings where the market demand is insufficient to support a larger number of active firms.

Finally, we can make observations on the arising industry structure. Figure 6 provides log-rank-log-capacity-share plots for capacity equilibria with a large number of firms as obtained by our model. These plots illustrate that simple linear capacity cost functions gives rise to highly non-linear oligopoly capacity configurations. Consequently, our analysis provides an analytic approach based on strategic interaction and investment under uncertainty to characterize probable market structures.

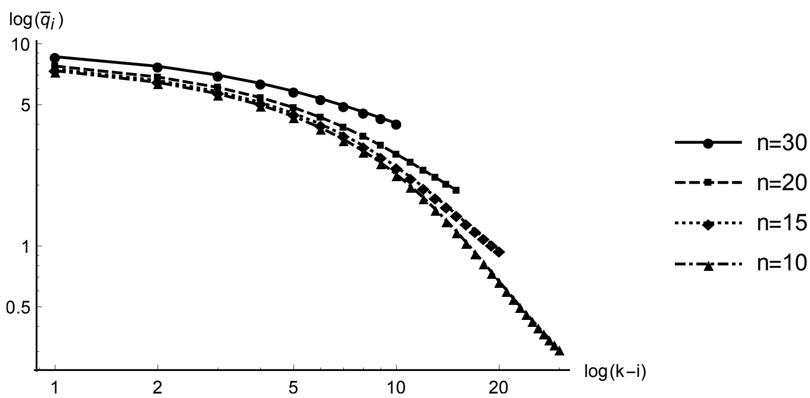

Figure 6 **Capacity rank-share correspondence for different oligopolies** $(c = 1, b = 5, C_1 = 100, \chi = 1, \sigma = 0.225, \mu = 0, r = \delta = 0.05)$

6. Conclusion

We obtain a general closed-form expression for the value of flexible assets accounting for demand uncertainty, competitive rivalry and capacity constraints. The value of outputs adjustments to the demand shock depends on the number of rivals pushed to the competitive fringe, i.e., on whether or not other firms are capacity constrained. This value reflects a steady-state component as well as risk-mitigation and upside potential values for the cases in which firms contract or expand production up to the capacity constraints. Furthermore, we characterize a numerical solution approach for the general game where firms select capacities. In summary, our analysis reinforces the link between the literature on investment under uncertainty and the theory of industrial organization.

Our modeling approach allows to address in an integrative manner various intertwined issues of strategic relevance. The analytic results in this study simultaneously account for demand uncertainty, capacity constraints and strategic interaction. Capacity has value from two perspectives as it creates growth potential with respect to the two risk sources: operating and strategic risks. First, slack capacity creates option value with the unconstrained firm profiting from a demand upsurge in a convex manner (i.e., through price increase and quantity expansion). Slack capacity also helps unconstrained firms to exert market power on the residual market, while constrained firms benefits from price increases in a linear manner. Firms with larger (slack) capacity face a more convex payoff (in the demand shock) reflecting an option value to short-run output adjustment.

Numeric analysis highlights which factors matter for the first-stage capacity choices and illustrates how ex-ante heterogeneity (in terms of cost structures) may translate into ex-post capacity asymmetry. Under capacity constraint, demand volatility favors heterogeneity. Indeed under larger volatility, firms with smaller capacity are more likely to face capacity constraints, creating a larger incentive for firms with a preferential cost position to invest heavily in capacity since (i) the stand-alone expansion option appreciates and (ii) the latter firms are more likely to exert market power, while smaller rivals drop out. Because advantaged firms adopts this stance while deciding on their production capacity, less cost-efficient firms backs off.

Natural and interesting extensions to our models include more diverse cost functions (including fixed entry costs), the opportunity of repeated and/or delayed investment as well as disinvestment. The managerial insights from our analysis can be leveraged to examine situations faced in many capital-intensive industries such as the electricity, telecommunications, chemicals or natural resources sectors. Porter (1980) highlighted the strategic importance of investment decisions for understanding the inner mechanics of these industries. These insights relate also to other general questions of strategic management, such as market entry or R&D investment.

Appendix A: Capacity-constrained Cournot Output

In MNE, the quantity-setting problem is

$$\max_{q_i} \quad \pi_i\left(x, q_i, Q_{-i}^C\right) = \left(x - b\left(q_i + Q_{-i}^C\right)\right) q_i - c q_i, \tag{A.1}$$

subject to the non-negativity constraint, $q_i \geq 0$, and the production capacity constraint, $\bar{q}_i - q_i \geq 0$.

The objective function in (A.1) is concave. The feasible region is convex. The Karush-Kuhn-Tucker (KKT) conditions are thus both necessary and sufficient. Denote the KKT multipliers by λ_1 and λ_2. In MNE, the Lagrangian is given by

$$\mathcal{L}\left(q_i, \lambda_1, \lambda_2\right) = \left(x - b\left(q_i + Q_{-i}^C\right)\right) q_i - c q_i + \lambda_1 q_i + \lambda_2\left(\bar{q}_i - q_i\right).$$

Solving the optimization problem yields individual quantity as a function of industry output:

$$q_i^C(x) = \begin{cases} 0 & \text{if } x \leq bQ_{-i}^C + c, \\ \frac{x - bQ_{-i}^C - c}{2b} & \text{if } bQ_{-i}^C + c < x < 2b\bar{q}_i + bQ_{-i}^C + c, \\ \bar{q}_i & \text{if } x \geq 2b\bar{q}_i + bQ_{-i}^C + c. \end{cases} \tag{A.2}$$

The results in Proposition are obtained by substitution and change of variables.

Appendix B: Capacity-constrained Cournot Profits

Equation (3.5) obtains by pinning down the MNE, verifying that each firm is playing its best response, for each case in (A.2). We do so by expressing Q_{-i} as the sum of q_{-i}. To characterize firm profits, we first express total industry output as a function of x (in the following formulae $m-1$ firms are capacity constrained).

$$Q(X_t) = \begin{cases} 0 & \text{if } X_t \in (0, c), \\ \frac{(k-m+1)(X_t - \Sigma_m)}{b(k-m+2)} + \sum_{i=1}^{m-1} \bar{q}_i & \text{if } X_t \in [\bar{x}_{m-1}, \bar{x}_m), m = 1, ..., k+1. \end{cases} \tag{B.1}$$

Substituting (B.1) into (2.4) yields

$$p(X_t) = \begin{cases} X_t & \text{if } X_t \in (0, c), \\ \frac{X_t + (k-m+1)c - b\sum_{i=1}^{m-1} \bar{q}_i}{(k-m+2)} & \text{if } X_t \in [\bar{x}_m, \bar{x}_{m+1}), m = 1, ..., k. \end{cases} \tag{B.2}$$

Firm profits then obtain as

$$\pi_i = \begin{cases} 0 & \text{if } X_t \in (0, c), \\ q_i \frac{X_t - \Sigma_m}{(k-m+2)} & \text{if } X_t \in [\bar{x}_m, \bar{x}_{m+1}), m = 1, ..., k. \end{cases} \tag{B.3}$$

Appendix C: Firm Values

Let V_x and V_{xx} denoting the first and second-order derivatives of $x \mapsto V_i(x; \bar{Q})$. Applying the Feynman-Kac Theorem for function $x \mapsto V_i(x; \bar{Q})$ would yield second-order differential equation (ODE)

$$rV_i(x; \bar{Q}) = \pi_i^C(x) + \mu x \; V_x(x; \bar{Q}) + \frac{1}{2}\sigma^2 x^2 \; V_{xx}(x; \bar{Q}), \tag{C.1}$$

with $\pi_i^C(x)$ given in Proposition 2 and $V_i(0; \bar{Q}) = 0$. Note that ODE (C.1) is defined without ambiguity if $x \mapsto V_i(x; \bar{Q})$ is continuously differentiable everywhere (C^1) and twice continuously differentiable almost everywhere (piecewise C^2). These smoothness conditions are similar to the ones used for optimal stopping problems (according to the functional approach): there, one searches for a (strong) solution to a variational inequality that is C^1 and C^2 almost everywhere; hence, the value matching (continuity) and smooth-pasting conditions (continuity of the derivative). Note, however, that in optimal stopping problems as well as in the present problem, the value function happens not to be C^2 at the thresholds $\bar{x}_m$.

Functions $x \mapsto x^{\beta_1}$ and $x \mapsto x^{\beta_2}$ represent two independent solutions of the homogenous ODE

$$r f(x) = \mu x\ f'(x) + \frac{1}{2}\sigma^2 x^2\ f''(x), \tag{C.2}$$

where β_1 and β_2, given in Proposition 3, are the positive and negative roots of the quadratic function

$$\mathscr{Q}(\beta) = r - \beta\mu - \frac{1}{2}\beta(\beta-1)\sigma^2, \quad \beta \in \mathbb{R}. \tag{C.3}$$

We have $\mathscr{Q}(1) = r - \mu$ (>0) and $\mathscr{Q}(0) = r$ (>0).

Consider first a contingent claim θ with payoff structure

$$\theta(x) = \begin{cases} 0 & \text{if } x < z, \\ (x-c)^2 & \text{if } x \geq z, \end{cases}$$

and $z \geq c$. Note that θ is not C^1 if $z > c$. If one finds a solution Θ of

$$r\Theta(x) = \theta(x) + \mu x\ \Theta'(x) + \frac{1}{2}\sigma^2 x^2\ \Theta''(x) \tag{C.4}$$

that is C^1 and piecewise C^2, it corresponds to the discounted stream of contingent claims θ,

$$\Theta(x) = \mathbb{E}_x\left[\int_0^\infty e^{-rt}\theta(X_t)\ \mathrm{d}t\right].$$

Suppose $\mathscr{Q}(2) > 0$. One can verify that

$$\mathbb{E}_x\left[\int_0^\infty e^{-rt}(X_t - c)^2\ \mathrm{d}t\right] = \frac{x^2}{\mathscr{Q}(2)} - \frac{2cx}{r-\mu} + \frac{c^2}{r}, \tag{C.5}$$

is a particular solution of (C.4) for $x \geq z$. The general solution of (C.4) is

$$\Theta(x) = \begin{cases} g_1(x,z,c) & \text{if } x < z, \\ \frac{x^2}{\mathscr{Q}(2)} - \frac{2cx}{r-\mu} + \frac{c^2}{r} + g_2(x,z,c) & \text{if } x \geq z, \end{cases} \tag{C.6}$$

where $g_1(x,z,c) = x^{\beta_1} g_1(z,c)$ and $g_2(x,z,c) = x^{\beta_2} g_2(z,c)$. Θ in (C.6) is piecewise C^2. We look for a solution of (C.4) such that Θ is C^1. The smoothness conditions at z are:

$$\begin{aligned} z^{\beta_1} g_1(z,c) &= \frac{z^2}{\mathscr{Q}(2)} - \frac{2zc}{r-\mu} + \frac{c^2}{r} + z^{\beta_2} g_2(z,c) \\ \beta_1 z^{\beta_1 - 1} g_1(z,c) &= \frac{2z}{\mathscr{Q}(2)} - \frac{2c}{r-\mu} + \beta_2 z^{\beta_2 - 1} g_2(z,c). \end{aligned}$$

It easily obtains that

$$\begin{aligned} g_1(x,z,c) &= \left(\frac{x}{z}\right)^{\beta_1}\left\{\frac{2-\beta_2}{\beta_1-\beta_2}\frac{z^2}{\mathscr{Q}(2)} - \frac{1-\beta_2}{\beta_1-\beta_2}\frac{2cz}{r-\mu} - \frac{\beta_2}{\beta_1-\beta_2}\frac{c^2}{r}\right\} \\ g_2(x,z,c) &= -\left(\frac{x}{z}\right)^{\beta_2}\left\{\frac{\beta_1-2}{\beta_1-\beta_2}\frac{z^2}{\mathscr{Q}(2)} - \frac{\beta_1-1}{\beta_1-\beta_2}\frac{2cz}{r-\mu} + \frac{\beta_1}{\beta_1-\beta_2}\frac{c^2}{r}\right\}. \end{aligned}$$

We know that $\theta(x) \geq 0$ for all $x \in \mathbb{R}_+$ and $\theta(x) > 0$ for $x \geq z$. Thus by probabilistic arguments, $\Theta(x) > 0$ for all $x \in \mathbb{R}_+$. In particular, $g_1(x,z,c) > 0$ for $x < z$. Besides, $\theta(x) \leq (x-c)^2$ for all $x \in \mathbb{R}_+$ and $\theta(x) < (x-c)^2$ for $0 < x < z$. Hence,

$$\Theta(x) < \frac{x^2}{\mathscr{Q}(2)} - \frac{2cx}{r-\mu} + \frac{c^2}{r}$$

for all $x \in \mathbb{R}_+$. In particular, $g_2(x,z,c) < 0$ for $x \geq z$. Here $g_1(x,z,c)$ is the value of the adjustment made to (zero) present value to account for the optionality to receive positive profit $(X_t - c)^2$ when future demand X_t exceeds a certain demand threshold z. The term $g_2(x,z,c)$ is the (downward) value adjustment needed to account for the fact that the firm will not receive profit $(X_t - c)^2 \geq 0$ for low demand $(X_t < z)$, but instead receive zero. The term $(x/z)^{\beta_1}$ in the expression for $g_1(x,z,c)$ corresponds to the present value of a bond that pays \$1 when demand X_t exceeds demand level z $(> x)$ for the first time at (stopping) time $\tau_A(x,z) \equiv \inf\{t \geq 0 \mid X_t \geq z\}$. The second term in $\{\cdot\}$ corresponds to the positive forward value received when the firm earns positive profit for the first time at $\tau_B(x,z) \equiv \inf\{t \geq 0 \mid X_t \leq z\}$. $(x/z)^{\beta_2}$ is the present value of a bond that pays \$1 at stochastic time $\tau_B(x,z)$ with $z < x$, while the term in $\{\cdot\}$ is the forward value of losing the positive profit stream from time $\tau_B(x,z)$ onwards.

Similarly, we consider a simple contingent claim with payoff structure

$$\psi(x) = \begin{cases} 0 & \text{if } x < z, \\ x - c & \text{if } x \geq z, \end{cases}$$

with $z \geq c$. $x \mapsto x/[r-\mu] - c/r$ is a particular solution of

$$r\Psi(x) = \psi(x) + \mu x\ \Psi'(x) + \frac{1}{2}\sigma^2 x^2 \Psi''(x)$$

on $[z, \infty)$. The general solution of the second-order ODE is

$$\Psi(x) = \begin{cases} h_1(x,z,c) & \text{if } x < z, \\ \frac{x}{r-\mu} - \frac{c}{r} + h_2(x,z,c) & \text{if } x \geq z, \end{cases}$$

where $h_1(x,z,c) = x^{\beta_1} h_1(z,c)$ and $h_2(x,z,c) = x^{\beta_2} h_2(z,c)$, obtained by appropriate smoothness conditions, are given by

$$\begin{aligned} h_1(x,z,c) &= \left(\frac{x}{z}\right)^{\beta_1} \left\{ \frac{1-\beta_2}{\beta_1-\beta_2} \frac{z}{r-\mu} + \frac{\beta_2}{\beta_1-\beta_2} \frac{c}{r} \right\} \\ h_2(x,z,c) &= -\left(\frac{x}{z}\right)^{\beta_2} \left\{ \frac{\beta_1-1}{\beta_1-\beta_2} \frac{z}{r-\mu} - \frac{\beta_1}{\beta_1-\beta_2} \frac{c}{r} \right\}. \end{aligned}$$

We know that $\psi(x) \geq 0$ for all $x \in \mathbb{R}_+$ and $\psi(x) > 0$ for $x > z$. Thus, $\Psi(x) > 0$ for all $x \in \mathbb{R}_+$. In particular, $h_1(x,z,c) > 0$ for $x < z$. Besides, $\psi(x) \geq x - c$ for all $x \in \mathbb{R}_+$ and $\psi(x) > x - c$ for $x < c$. Hence, $\Psi(x) > x/[r-\mu] - c/r$ for all $x \in \mathbb{R}_+$. In particular, $h_2(x,z,c) > 0$. Here, $h_1(x,z,c)$ corresponds to the value of a call option to earn $X_t - c$ when current demand X_t is sufficiently high, i.e., when $X_t \geq z$, while $h_2(x,z,c)$ is the value of the put option to stop producing, earning zero profit, when demand is not sufficiently high, i.e., when $X_t < z$.

We next consider the nonhomogenous ODE (C.1). The general solution is

$$V_i(x;\bar{Q}) = \begin{cases} A_0 x^{\beta_1} + B_0 x^{\beta_2}, & \text{if } x \in (0, x_0)\,, \\ \frac{1}{b(k-m+2)^2}\left[\frac{x^2}{\mathscr{Q}(2)} - \frac{2\Sigma_m x}{r-\mu} + \frac{\Sigma_m^2}{r}\right] + A_m x^{\beta_1} + B_m x^{\beta_2}, & \text{if } x \in [\bar{x}_{m-1}, \bar{x}_m)\,, m = 1, \ldots, i, \\ \frac{\bar{q}_i}{(k-m+2)}\left[\frac{x}{r-\mu} - \frac{\Sigma_m}{r}\right] + A_m x^{\beta_1} + B_m x^{\beta_2}, & \text{if } x \in [\bar{x}_{m-1}, \bar{x}_m)\,, m = i+1, \ldots, k+1, \end{cases} \tag{C.7}$$

where A_m and B_m, $m = 0, \ldots, k+1$ are constants determined by "enforcing" the solution of the second-order differential equation to be C^1. Note that $x \mapsto V_i(x;\bar{Q})$ given in (C.7) is piecewise C^2 in $(-\infty, \bar{x}_0)$

and $[\bar{x}_{m-1}, \bar{x}_m)$ for $m = 1, \ldots, k+1$. It remains to select values for A_m and B_m, $m = 0, \ldots, k+1$, such that $x \mapsto V_i(x; \bar{Q})$ is C^1 at $\bar{x}_m$, $m = 0, \ldots, k$.

To ensure that $V_i(0; \bar{Q}) = 0$ and avoid bubble solutions, we set $B_0 = A_{k+1} = 0$. The smoothness conditions at $\bar{x}_0 = c$ read

$$\begin{aligned}(A_0 - A_1)\,\bar{x}_0^{\beta_1} + (B_0 - B_1)\,\bar{x}_0^{\beta_2} &= \frac{1}{b(k+1)^2}\left[\frac{\bar{x}_0^2}{\mathscr{Q}(2)} - \frac{2c\bar{x}_0}{r-\mu} + \frac{c^2}{r}\right] \\ \beta_1(A_0 - A_1)\bar{x}_0^{\beta_1 - 1} + \beta_2(B_0 - B_1)\bar{x}_0^{\beta_2 - 1} &= \frac{1}{b(k+1)^2}\left[\frac{2\bar{x}_0}{\mathscr{Q}(2)} - \frac{2c}{r-\mu}\right].\end{aligned}$$

It obtains

$$A_0 = A_1 + \frac{g_1(\bar{x}_0, c)}{b(k+1)^2}\ (> A_1) \quad \text{and} \quad B_1 = B_0 + \frac{g_2(\bar{x}_0, c)}{b(k+1)^2}\ (< B_0).$$

The smoothness conditions at $\bar{x}_m$, $m = 1, \ldots, i-1$, are

$$\begin{aligned}(A_m - A_{m+1})\bar{x}_m^{\beta_1} + (B_m - B_{m+1})\bar{x}_m^{\beta_2} =& \frac{1}{b(k-m+1)^2}\left[\frac{\bar{x}_m^2}{\mathscr{Q}(2)} - \frac{2\Sigma_{m+1}x_m}{r-\mu} + \frac{\Sigma_{m+1}^2}{r}\right] \\ &- \frac{1}{b(k-m+2)^2}\left[\frac{\bar{x}_m^2}{\mathscr{Q}(2)} - \frac{2\Sigma_m\bar{x}_m}{r-\mu} + \frac{\Sigma_m^2}{r}\right] \\ \beta_1(A_m - A_{m+1})\bar{x}_m^{\beta_1-1} + \beta_2(B_m - B_{m+1})\bar{x}_m^{\beta_2-1} =& \frac{1}{b(k-m+1)^2}\left[\frac{2\bar{x}_m}{\mathscr{Q}(2)} - \frac{2\Sigma_{m+1}}{r-\mu}\right] - \frac{1}{b(k-m+2)^2}\left[\frac{2\bar{x}_m}{\mathscr{Q}(2)} - \frac{2\Sigma_m}{r-\mu}\right]\end{aligned}$$

We thus have

$$A_m = A_{m+1} + \frac{g_1(\bar{x}_m, \Sigma_{m+1})}{b(k-m+1)^2} - \frac{g_1(\bar{x}_m, \Sigma_m)}{b(k-m+2)^2}, \quad \text{and} \quad B_{m+1} = B_m + \frac{g_2(\bar{x}_m, \Sigma_{m+1})}{b(k-m+1)^2} - \frac{g_2(\bar{x}_m, \Sigma_m)}{b(k-m+2)^2},\ m = 1, \ldots, i-1.$$

The smoothness conditions at x_i are

$$\begin{aligned}(A_i - A_{i+1})\,\bar{x}_i^{\beta_1} + (B_i - B_{i+1})\,\bar{x}_i^{\beta_2} &= \frac{\bar{q}_i}{(k-i+1)}\left[\frac{\bar{x}_i}{r-\mu} - \frac{\Sigma_{i+1}}{r}\right] - \frac{1}{b(k-i+2)^2}\left[\frac{\bar{x}_i^2}{\mathscr{Q}(2)} - \frac{2\Sigma_i\bar{x}_i}{r-\mu} + \frac{\Sigma_i^2}{r}\right] \\ \beta_1\,(A_i - A_{i+1})\,\bar{x}_i^{\beta_1-1} + \beta_2\,(B_i - B_{i+1})\,\bar{x}_i^{\beta_2-1} &= \frac{\bar{q}_i}{(k-i+1)}\frac{1}{r-\mu} - \frac{1}{b(k-i+2)^2}\left[\frac{2\bar{x}_i}{\mathscr{Q}(2)} - \frac{2\Sigma_i}{r-\mu}\right].\end{aligned}$$

Similarly, we obtain

$$A_i = A_{i+1} + \bar{q}_i\frac{h_1(\bar{x}_i, \Sigma_{i+1})}{(k-i+1)} - \frac{g_1(\bar{x}_i, \Sigma_i)}{b(k-i+2)^2} \quad \text{and} \quad B_{i+1} = B_i + \bar{q}_i\frac{h_2(\bar{x}_i, \Sigma_{i+1})}{(k-i+1)} - \frac{g_2(\bar{x}_i, \Sigma_i)}{b(k-i+2)^2}.$$

Finally, the smoothness conditions at x_m, $m = i+1, \ldots, k$, read

$$\begin{aligned}(A_m - A_{m+1})\,\bar{x}_m^{\beta_1} + (B_m - B_{m+1})\,\bar{x}_m^{\beta_2} &= \frac{\bar{q}_i}{(k-m+1)}\left[\frac{\bar{x}_m}{r-\mu} - \frac{\Sigma_{m+1}}{r}\right] - \frac{\bar{q}_i}{(k-m+2)}\left[\frac{\bar{x}_m}{r-\mu} - \frac{\Sigma_m}{r}\right] \\ \beta_1\,(A_m - A_{m+1})\,\bar{x}_m^{\beta_1-1} + \beta_2\,(B_m - B_{m+1})\,\bar{x}_m^{\beta_2-1} &= \frac{\bar{q}_i}{(k-m+1)}\frac{1}{r-\mu} - \frac{\bar{q}_i}{(k-m+2)}\frac{1}{r-\mu}.\end{aligned}$$

It thus obtains

$$A_m = A_{m+1} + \bar{q}_i\left[\frac{h_1(\bar{x}_m, \Sigma_{m+1})}{k-m+1} - \frac{h_1(\bar{x}_m, \Sigma_m)}{k-m+2}\right]$$

$$B_{m+1} = B_m + \bar{q}_i\left[\frac{h_2(\bar{x}_m, \Sigma_{m+1})}{k-m+1} - \frac{h_2(\bar{x}_m, \Sigma_m)}{k-m+2}\right],\ m = i+1, \ldots, k.$$

Appendix D: Numerical algorithm for capacity choice equilibrium

The algorithm proceeds as follows in iteration steps $l \geq 0$; to each iteration corresponds a tentative MNE noted $\bar{Q}^l$. At the outset $(l=0)$ firm capacities are set to zero with $\bar{Q}^0 = (0, \cdots, 0)^\top$. In each iteration $l \geq 1$, one firm i determines the best response $\bar{q}_i^l$ to the industry capacity vector $\bar{Q}_{-i}^{l-1}$, i.e.,

$$\bar{q}_i^l = \underset{q_i^l \in [q_{i-1}^l, q_{i+1}^l]}{\arg\max} \; V_i(x; q_i^l, \bar{Q}_{-i}^{l-1}) - C_i(q_i^l)$$

such that $\bar{q}_i^l \in [\bar{q}_{i-1}^{l-1}, \bar{q}_{i+1}^{l-1}]$. The latter constraint is due to the posited capacity ordering (2.1). The algorithm calculates the best response of firm k at $l=1$, of firm $k-1$ at $l=2$,..., of firm 1 at $l=k$. At step $l=1$, firm k selects the monopoly capacity. At step $l=2$, firm $k-1$ reacts as a Stackelberg follower... At $l=k+1$, firm k brings down its monopoly capacity to account for capacity expansions by firms $k-1$ to 1. At $l=k+1$, firm $k-1$ proceeds similarly. The left panel of Figure D summarizes this procedure.

Iteration (l)	Optimizing firm i	Conjectured MNE
0	n/a	$\bar{Q}^0 = (0,0,0)$
1	3	$\bar{Q}^1 = (0,0,\bar{q}_3^1)$
2	2	$\bar{Q}^2 = (0,\bar{q}_2^2,\bar{q}_3^1)$
3	1	$\bar{Q}^3 = (\bar{q}_1^3,\bar{q}_2^2,\bar{q}_3^1)$
4	3	$\bar{Q}^4 = (\bar{q}_1^3,\bar{q}_2^2,\bar{q}_3^4)$
$\vdots$	$\vdots$	$\vdots$

By this procedure one narrows down progressively the space of MNE candidates. One obtains a fixed point once convergence is achieved at iteration step $l^\star$ given by

$$l^\star = \inf\left\{ l \in \mathbb{N} \;\middle|\; \sum_{i=1}^{k} \mid \bar{q}_i^l - \bar{q}_i^{l-1} \mid \leq \epsilon \right\}.$$

Convergence of the algorithm is ensured by existence and unicity of a Nash equilibrium preserving weak capacity ordering (2.1). Our numerical analysis uses $\epsilon = 0.1$.

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