External Funding and Irreversible Investment

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Abstract

This paper endogenizes the cost of external funds and explores the implications on irreversible investments. The investment strategy incorporates equilibrium feedback that result from a bargaining process between the firm and a lender. Contrary to debt issuance, tax benefits and distress costs cannot be internalized by the firm. "Bad news" are less costly for the firm that has incentives to accelerate investment whereas creditors intend to delay it; underinvestment or overinvestment is determined by each party relative bargaining power and the size of bankruptcy costs. Default and credit market imperfections raise the cost of capital, which dampens the value of waiting. The impact of assets already in place, bankruptcy costs and leverage level are also examined.

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1 Introduction

Corporate investment decisions are often irreversible and have to be taken under a great deal of uncertainty. Building on earlier works on investment by Jorgenson (1963) and Arrow (1968), Titman (1985) and McDonald and Siegel (1986) were among the first to emphasize the benefits of delaying an irreversible investment. When the payoffs of a project are uncertain, the investor has the option to wait for new information. Therefore, a growth opportunity of a firm can be understood as a call option on a real asset and undertaking a project is optimal only when the present discounted value of future cash-flows exceeds the investment cost by the option value, the marginal benefits of investing being equal to the marginal cost of investing augmented by the option value of waiting[1].

Many companies rely on external funds to finance their projects. In January 2015, US corporations had an average market value debt ratio (market value of debt to market value of the firm) equal to 38.3 percent whereas the average book value debt ratio (market value of debt to book value of the firm) was 61.6 percent[2]. A growing number of papers has examined the interactions between financing and investment decisions. Researchers strive to understand the determinants of corporate leverage and optimal capital structure by analyzing agency conflicts (and their associated costs) between equityholders and bondholders building on the seminal work by Jensen and Meckling (1976) and Myers (1977).

This paper explores the relationship between the endogenously determined cost of debt and the timing of an irreversible investment. We use an equilibrium framework where funding is allocated through a negotiation process between the borrower (the firm) and the lender and study the implications on the value of the growth opportunity. Our setting is similar to McDonald and Siegel (1986): time is continuous and the firm has the unique ability to undertake the risky project; however, it needs to rely on external capital to finance the investment. The lender has access to capital markets at the risk free interest rate. The debt contract signed between the two parties is a consol bond. When the firm chooses to invest, it receives funding from the lender and immediately starts paying a perpetual coupon. Bankruptcy is triggered by the inability of the firm to meet its current debt obligations; liquidation takes place at no cost and the after tax proceeds of the resale of the company are shared among creditors and shareholders according to the usual priority rules. We show that after adjusting the true cost of the project to incorporate tax deductions on coupon payments and a wealth transfer from bondholders to shareholders when bankruptcy occurs, the expression of the growth opportunity is identical to the one obtained in the all equity case. The main difference is that now the timing of the investment also affects this effective cost of investing. As argued by Jensen and Meckling (1976), the presence of debt distorts the firm's ex-post choice of risk as equityholders can potentially extract value from debtholders by increasing investment risk after debt is in place. Closing the model, two types of credit markets are examined. As a benchmark, we analyze the case of a perfectly competitive lending sector. Alternatively, we assume that financing is granted after some negotiations, namely a sequential bargaining mechanism (Stackelberg type) between the two parties. When tax effects are ignored, two scenarios arise:

- When debt is repaid in full after liquidation, we show that the problem can be understood as a usual all equity irreversible investment problem with possibly a higher effective cost of investing if the lender makes a positive profit. The mark up charged over the riskfree rate rises with earnings volatility and the lender's bargaining power.

- Conversely, if creditors incur a loss at bankruptcy, we find that default risk combined with credit market imperfections increase the cost of capital, which dampens the value of waiting to invest. When default takes place, the loss is shared among parties: this makes the firm keen on hastening investment

[1]For more details, the reader can refer to Pindyck (1991) as well as the seminal book, Investment under Uncertainty, by Dixit and Pindyck (1994).

[2]These values are reported from Aswath Damodaran's website.

whereas the lender intends to delay it. Essentially, “bad news” in the spirit of Bernanke (1983) are relatively less costly for the firm, which provides incentives for earlier investment. Overinvestment (underinvestment) arises when the firm has a high (low) bargaining power. The perfect competitive lending sector corresponds to the limit case where the firm has absolute bargaining power, and thus is able to extract all the surplus in the negotiation process. Numerical simulations show that the coupon charged rises with the lender’s bargaining power, which can significantly reduce the value of the growth opportunity. As far as uncertainty effects are concerned, intuitively, a higher earnings volatility hastens liquidation inducing in equilibrium both a higher coupon and investment trigger. In the competitive case, we find that more uncertainty always increases the option value. However, under bargaining, the impact of the earnings volatility on the option value is twofold. In addition to the usual enhancing effect due to the convexity of the payoffs, more uncertainty, by raising the cost of capital, also negatively affects the value of waiting to invest. This effect is particularly strong when the irreversible investment is close to be undertaken.

The impact of taxes on the investment decision arises through two channels. First, tax reductions on coupon payments contribute to lower the effective cost of the project. Second, the higher the tax rate, the smaller the *after tax* proceeds of the resale of the company, which implies a larger wealth transfer from bondholders to equityholders when bankruptcy occurs leading in equilibrium to the higher coupon charged. The direct effect of taxes is to accelerate the undertaking of the project through the reduction of the effective cost of investing. By doing so, tax benefits shrink, so there is an induced indirect effect which tends to postpone investment. The overall impact on the growth opportunity is ambiguous. For low levels of the corporate tax rate, debt financing still negatively affects the option value. However, for high levels of the corporate tax rate, due to large tax benefits, the impact can become positive, and there may exist an optimal level of a debt that maximizes the value of the growth opportunity.

This paper builds on several strands of the literature on irreversible investment and real options and financing with risky debt. Bernanke (1983) highlights that only unfavorable outcomes actually matter for the decision to undertake or postpone an investment. He calls this effect the “bad news principle of irreversible investment”. Investment can also proceed incrementally and the firm may decide sequentially to expand the size and capacity of a factory. Pindyck (1988) addresses the issue of capacity choice under uncertainty allowing for the opportunity of not using the incremental unit when demand is low. One of the central issues of this paper, the investment-uncertainty relationship. Ingersoll and Ross (1992) study the effects of uncertain interest rates on the investment timing and obtain that uncertainty has an ambiguous impact on the option value of waiting. Similarly, Caballero (1991) demonstrates that when relaxing the hypothesis of symmetric adjustment costs, the positive relationship between investment and uncertainty may still hold. He identifies the nature of competition as the key determinant of the relationship. Under imperfect competition, the investment-uncertainty relationship can become negative when the adjustment costs are highly asymmetric and there is a strong negative relationship between marginal profitability of capital and the level of capital.

Along with debt arises the issue of capital structure and its implications on investment. The Modigliani-Miller theorem (1958) states that financing and investment decisions are completely separable in perfect capital markets. Merton (1974) and (1977) was the first to use a non-arbitrage approach to evaluate risky corporate debt. Leland (1994) focuses on the optimal capital structure by explicitly computing the value of time independent long-term protected debt and unprotected debt using contingent claim techniques. Leland (1998) analyzes the joint determination of capital structure and the level of risk and in particular the role played by tax advantages, default costs and agency costs from asset substitution[3]. Paseka (2004) endogenizes default on debt and looks at the implications on

[3]Asset substitution refers to the fact that the presence of debt may distort the firm’s ex-post choice of risk as equityholders can potentially extract value from debtholders by increasing investment risk after debt is in place. Leland

credit spreads. In two seminal articles, Jensen and Meckling (1976) and Myers (1977) argue that, when the firm has risky debt outstanding and when managers act to maximize equity value rather the total firm value, managers have incentives to under-and overinvest in future growth opportunity as well as engaging in riskier activities since they can transfer risk to bondholders and preserve upside potential. Rational bondholders anticipate conflicts and will require a higher cost of debt financing. These agency conflicts have the potential to significantly reduce the firm value and thereby temper the firm's use of debt financing. As reported by Lang, Ofek and Stultz (1996) as well as in Billet, King and Mauer (2007), there exists a negative relationship between growth opportunities and leverage for unprotected debt. Lyandres and Zhdanov (2005) reach a more balanced conclusion as they uncover two antagonistic over and under-investment effects. Finally, Westerfiled and Bradford (1993) document that 80 percent of firms prefer relying on internal sources of funds for their investments. In our paper, under-investment may be rationalized by credit market imperfections. Over-investment is also obtained in Childs, Mauer and Ott (2005) for an expansion whose underlying asset is riskier than assets-in place. reduces the wedge between the investment trigger and the cost of investing with respect to the self financing case. In Mauer and Triantis (1994), the firm dynamically manages its investment and financing decisions taking into account operating adjustment and recapitalization costs. They find that a higher production flexibility raises the debt capacity and consequently the value of tax shield.

Some papers examine investment issues within an equilibrium framework. Hugonnier, Morallec and Sudaresan (2005) study lumpy investments financed with equity within a general equilibrium model and obtain that equilibrium feedback effects also reduce the value of waiting to invest. Gomes (2001) points out that the role of cash flow on investment is probably overstated when using reduced-form investment regressions due to a combination of measurement error in q and identification problems. Sabarwal (2003) studies debt financing under limited liability in a perfect capital market. Ignoring tax effects, he finds that debt financing leads to over-investment. Mauer and Sarkar (2005) consider an expansion partially financed with debt when default is triggered at a value maximizing equity. They take into account bankruptcy costs and tax effects and assume that the amount of money lent corresponds to the fair price of the debt, i.e., the lender makes no profit. Firms maximizing equity rather than total firm value undertake the investment much earlier, which induces a significant credit spread and large agency costs. Their analysis is quite insightful but the coupon payment on debt is taken as exogenous. In contrast in this paper, we are interesting in examining the impact of equilibrium feedbacks from the credit market on the investment decision and the cost of financing. Fries, Miller, and Perraudin (1997) endogenize the output price by considering entry and exit of firms in a competitive industry and explore the implications on the price of debt and dynamic adjustments of the capital structure. They find that the value of firm can be decomposed into two components: the value of a firm with a fixed leverage and a continuum of options which are progressively exercised each time output exceeds its previous peak.

The paper is organized as follows. Section 2 describes the economic setting and provides some analytical results. In section 3, credit markets are assumed to be perfectly competitive. Conversely, in section 4, we use a sequential mechanism a la Stackelberg and alternatively a Nash bargaining to model the negotiation process between the two parties. Section 5 concludes. Proofs of all results are collected in the appendix.

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and Toft (1996) show that these costs could be reduced by relying on short term debts by providing a better convergence between shareholders and bondholders interests.

2 The Economic Setting

Time is continuous; the manager of a firm has to choose optimally the timing of an irreversible investment under uncertainty while partially relying on external capital to cover the cost of the project.

2.1 Investment Opportunity and Information Structure

Uncertainty is modeled by a probability space $(\Omega, \mathcal{F}, P^w)$ on which is defined a one dimensional (standard) Brownian motion w. A state of nature ω is an element of Ω. $\mathcal{F}$ denotes the tribe of subsets of Ω that are events over which the probability measure P is assigned. At time t, the investor's information set $\mathcal{F}_t$ is the σ-algebra generated by the observations of earnings of the project, $\{P_s; 0 \leq s \leq t)\}$ and augmented. The filtration $\mathbb{F} = \{\mathcal{F}_t,\ t \in \mathbb{R}_+\}$ is the information structure and satisfies the usual conditions (increasing, right-continuous, augmented).

Investing into the project increases the production capacity (as well as earnings) of the firm by one unit; let $\rho > 0$ denote the number of units already in place. Prior to investment, gross earnings of the firm are equal to ρP, so that once the investment completed, gross earnings are equal to $(1+\rho)P$. The project generates (observable) gross revenues P that fluctuate across time according to a geometric Brownian motion

$$dP_t = P_t \left(\alpha dt + \sigma dw_t\right),$$

where dw_t is the increment of a standard Wiener process under the probability P^w, α is the average growth rate of future revenues and σ is the earnings instantaneous volatility. The investment is irreversible with cost $I > 0$ and the risk free rate is $r > 0$. Let μ be the average return of an asset portfolio perfectly correlated with P. As argued in Dixit and Pindyck (1994), we assume that $\delta = \mu - \alpha$ is positive in order for the value of the project to be bounded. Assuming that the output of the project is tradable, under complete markets, μ is the market risk-adjusted rate of return and by the CAPM formula, we have

$$\mu = r + \rho_{Pm}\phi\sigma,$$

where ϕ is the market price of risk and ρ_{Pm} is the coefficient of correlation between P and the whole market. It follows that under the risk neutral probability Q, the dynamics of the gross revenues P are given by

$$dP_t = P_t \left((r-\delta)dt + \sigma dw_t^Q\right),$$

where dw_t^Q is the increment of a standard Wiener process under the probability Q. In the sequel, E_t^Q denotes the conditional probability at time t given the information set $\mathcal{F}_t$ under the risk neutral probability Q.

2.2 Debt Contract

Contract specifications are as follows: The lender agrees to deliver an amount $D \leq I$ when investment is undertaken and immediately after, the firm agrees to pay a perpetual fixed coupon $C > 0$ (consol bond). Equityholders have the opportunity to default and in this case bankruptcy is declared. We assume that the company files under Chapter 7[4] rather than under Chapter 11[5]. The firm's assets

[4] Under Chapter 7, the firm's assets are liquidated by a court appointed trustee; the priority of claims is respected. Recall that the Absolute Priority Rule (APR) distributes the firm's payoffs according to priority. In particular, junior claimholders receive nothing until senior claimholders are fully paid. In the United States (1) administrative expenses of the bankruptcy process are paid first, then come (2) unpaid taxes or debts to government agencies (e.g., the Pension Benefit Guarantee Corporation) (3) some wage claims (up to some ceiling), (4) secured and senior creditors, (5) junior creditors, (6) preferred shares, and, last (7) equityholders. For more details, see Tirole (2006).

[5] Alternatively, firms can file bankruptcy under Chapter 11, which allows for a workout in which a reorganization plan is designed and thus liquidation is at least temporarily avoided. Under Chapter 11, all payments to creditors are

are liquidated by a court appointed trustee. Bankruptcy costs are assumed to be proportional with factor[6] $\alpha \in [0,1]$ to the (market) value of assets sold. The priority of claims is respected; the lender receives the minimum value between the residual value of the firm once the bankruptcy costs are paid and the perpetuity $\frac{C}{r}$.

In this setting, debt is unsecured and no covenant is attached to it. It can be understood and a perpetual loan whose principal is never to be repaid. As shown in the sequel, bankruptcy is triggered as soon as the firm runs out of equity: we have in mind the case of small companies (proprietary business) that does not have the ability to issue new equity or raise additional external capital. Under the assumption that earnings follow a geometric Brownian motion, it is possible to derive the probability of bankruptcy and get some insights about the contract.

2.2.1 Probability of Bankruptcy

As shown in the sequel, it is optimal to invest as soon as earnings reach some trigger level $P = P^*$ and operate until some default threshold $P = P_D < P^*$ that is found to be proportional to the coupon charged C. Using some results on first time passage[7] of a geometric Brownian motion, the probability of bankruptcy (under the risk neutral measure) is equal to $\left(\frac{P^*}{P_D}\right)^{\left[1-\frac{2(r-\delta)}{\sigma^2}\right]^+}$, where $[x]^+ = \max(x, 0)$. Observe that this probability is non-decreasing in the ratio $\frac{P^*}{P_D}$. When the drift of the project (adjusted for volatility) is non-positive, i.e. $r - \delta - \frac{\sigma^2}{2} \leq 0$, bankruptcy occurs with certainty; the average time[8] between the investment and bankruptcy dates is finite and given by

$$E[\tau] = \frac{\log \frac{P^*}{P_D}}{\delta - r + \frac{\sigma^2}{2}}. \tag{1}$$

Earnings volatility σ has a direct effect through the travelling speed of the earning process: a higher volatility hastens bankruptcy. In addition, at the equilibrium, there is an indirect effect through the investment threshold P^* and coupon C. We shall see that both parties have incentives to delay bankruptcy; however, the lender is the one who has the greatest incentives.

We now briefly review some main results for the all-equity financed case.

2.3 All-Equity Financed

Corporate taxes are paid at a rate τ_c on earnings. At time 0, starting at P_0, the value of the equity of an all-equity financed firm $F_0(P_0)$ is given by

$$\begin{aligned} F_0(P_0) &= \sup_{\tau \geq 0} E_0^Q \left[\int_0^\tau \rho(1-\tau_c) P_s e^{-rs} ds + \left(\frac{(1-\tau_c)(1+\rho)P_\tau}{\delta} - I \right) e^{-r\tau} \right] \\ &= \frac{\rho(1-\tau_c)P_0}{\delta} + G_0(P_0), \end{aligned}$$

where $G_0(P_0) = \sup_{\tau \geq 0} E_0^Q \left[\left(\frac{(1-\tau_c)P_\tau}{\delta} - I \right) e^{-r\tau} \right]$. The value of the all-equity firm is simply equal to the value of the existing assets (incidentally the value of the equity of the firm that never expands)

suspended (automatic stay) and the firm can obtain additional financing by granting new claims seniority over existing ones. Paseka (2003) investigates this case.

[6] A study of Chapter 7 liquidations of small businesses found that the average direct costs of bankruptcy were 12% of the value of the firm's assets (see Tashjian, Lease and McConnell (1996)).

[7] See Karlin and Taylor (1981), chapter 15, from page 191 to page 202.

[8] See for instance Abel and Eberly (1998).

augmented by the option value G_0 of expanding by one unit. Clearly, assets already in place do not affect the timing of the investment. This case has been studied extensively in the literature (see for instance Dixit and Pindyck, 1994 chapter 6, p 180-185). The problem can be seen as an infinite horizon American Call option with strike price I on an underlying asset whose value is the cumulated future discounted cash flows generated by the project. The option value G_0 is given by

$$G_0(P) = \begin{cases} \left(\frac{(1-\tau_c)P_0^*}{\delta} - I\right)\left(\frac{P}{P_0^*}\right)^{\beta_1}, \text{ for } P \leq P_0^* \\ \frac{(1-\tau_c)P}{\delta} - I, \text{ for } P \geq P_0^*, \end{cases}$$

where β_1 and β_2 are respectively the positive and negative roots of the quadratic equation

$$\frac{\sigma^2}{2}x^2 + (r - \delta - \frac{\sigma^2}{2})x - r = 0,$$

and $\frac{(1-\tau_c)P_0^*}{\delta} = \frac{\beta_1}{\beta_1 - 1}I$ is the investment trigger. The implicit value of the coupon is $C_0^* = rI$. The investment decision is: invest as soon as P hits the trigger value P_0^*. Dixit and Pindyck (1994) conclude that the NPV rule is simply incorrect. There exists a wedge of size $\frac{\beta_1}{\beta_1-1} > 1$ between the value of the project that triggers investment and the cost of undertaking it.

We now investigate how relying on external funds affects the timing of the investment decision and the value of the growth opportunity. We first analyze the firm problem.

2.4 The Firm Problem

We start by determining the benefits of investing into the project.

2.4.1 Reward Value of Investing

After investing, shareholders operate the business and optimally choose the time to default as in Leland (1994). Let us normalize the date at which investment is completed to be 0 and denote $\tau_D = \inf\{t \geq 0,\ P_t = P_D\}$ the default stopping time for some optimal default threshold P_D to be determined. Equityholders choose τ_D so as to maximize

$$V^F(P_0) = \sup_{\tau_D \geq 0} E_0^Q\left[\int_0^{\tau_D} (1-\tau_c)((1+\rho)P_s - C)e^{-rs}ds\right],$$

and for all $0 \leq t \leq \tau_D$, $V^F(P_t) \geq 0$ or equivalently, for all $P \geq P_D$, $V^F(P) \geq 0$ and for all $P \leq P_D$, $V^F(P) = 0$. As shown in Leland (1994), optimality is achieved by imposing at $P = P_D$ a value matching, namely $V^F(P_D) = 0$ and a smooth pasting condition, $(V^F)'(P_D) = 0$; the solution is given by

$$V^F(P) = \frac{(1-\tau_c)(1+\rho)P}{\delta} - \frac{C}{r} + \frac{\tau_c C}{r}\left(1 - \left(\frac{P}{P_D}\right)^{\beta_2}\right) + \left(\frac{C}{r} - \frac{(1-\tau_c)(1+\rho)P_D}{\delta}\right)\left(\frac{P}{P_D}\right)^{\beta_2}, \quad (2)$$

where the optimal default threshold[9] is such that $\frac{(1+\rho)P_D}{\delta} = \frac{\beta_2}{\beta_2-1}\frac{C}{r}$ or equivalently $(1+\rho)P_D = \frac{\beta_1-1}{\beta_1}C$.

The first term in equation (2) $\frac{(1-\tau_c)(1+\rho)P}{\delta} - \frac{C}{r}$ is the difference between the intrinsic value of the project and the value of a perpetuity with coupon C. Next, tax benefits can be seen as a security

[9]Observe that $V^F(P_0) = (1-\tau_c)\left(\frac{(1+\rho)P_0}{\delta} - \frac{C}{r}\right) + \sup_{\tau_D \geq 0}(1-\tau_c)E_0^Q\left[(\frac{C}{r} - \frac{(1+\rho)P_{\tau_D}}{\delta})e^{-r\tau_D}\right]$. Equityholders have a put option that they exercise at the critical threshold P_D such that $\frac{(1+\rho)P_D}{\delta} = \frac{\beta_2}{\beta_2-1}\frac{C}{r}$.

that pays a coupon $\tau_c C$ as long as the firm is solvent and pays nothing in bankruptcy. The value of this security is $TB(P) = \frac{\tau_c C}{r}\left(1 - \left(\frac{P}{P_D}\right)^{\beta_2}\right)$. The last term represents the present value of the loss incurred by the bondholders and that is transferred to shareholders when bankruptcy is declared. Let $TF(P) = \left(\frac{C}{r} - \frac{(1-\tau_c)(1+\rho)P_D}{\delta}\right)\left(\frac{P}{P_D}\right)^{\beta_2}$ denote the present value of this wealth transfer. Clearly, to the extend that this expropriation is anticipated, in equilibrium, the ex-post wealth transfer will be offset by an ex ante higher price of debt. Finally, note that for a given coupon C, a higher corporate rate τ_c lowers the after tax intrinsic value of the project $\frac{(1-\tau_c)P}{\delta}$ but rises both tax benefits $TB(P)$ and the transfer value $TF(P)$.

2.4.2 Maximizing Equity Value

At time 0, starting at P_0, the value of equity $F(P_0)$ is given by

$$F(P_0) = \frac{(1-\tau_c)\rho P_0}{\delta} + G(P_0),$$

where as before $G(P_0) = \sup_{\tau \geq 0} E_0^Q\left[\left(V^F(P_\tau) - \frac{(1-\tau_c)\rho P}{\delta} - I + D\right)e^{-r\tau}\right]$ denotes the option value of waiting. Dropping the time index 0, we find that

$$F(P) = \frac{(1-\tau_c)\rho P}{\delta} + AP^{\beta_1} + BP^{\beta_2},\ P \leq P^*,$$

where P^* is the optimal investment threshold. Since $F(0) = 0$, this implies that $B = 0$ and A is a positive constant to be determined. The value matching and smooth pasting conditions respectively are

$$A(P^*)^{\beta_1} = \frac{(1-\tau_c)P^*}{\delta} - \frac{C}{r} + TB(P^*) + TF(P^*) - I + D \tag{3}$$

$$\beta_1 A(P^*)^{\beta_1 - 1} = \frac{(1-\tau_c)}{\delta} + TB'(P^*) + TF'(P^*). \tag{4}$$

For $P \leq P^*$, the value of the equity is given by

$$F(P) = \frac{(1-\tau_c)\rho P}{\delta} + \left(\frac{(1-\tau_c)P^*}{\delta} - I_E(P^*)\right)\left(\frac{P}{P^*}\right)^{\beta_1}, \tag{5}$$

with $I_E(P^*) = I + \frac{C}{r} - D - TB(P^*) - TF(P^*)$. Eliminating constant A between relationships (3) and (4) leads to the following relationship between the investment trigger P^* and the coupon C

$$\begin{aligned}\frac{(1-\tau_c)P^*}{\delta} = & \underbrace{\frac{\beta_1}{\beta_1 - 1}\left(I + \frac{C}{r} - D\right)}_{\text{cost of the project effect when no default}} \underbrace{- \frac{\beta_1 - \beta_2}{\beta_1 - 1}\left(\frac{C}{r} - \frac{(1-\tau_c)(1+\rho)P_D}{\delta}\right)\left(\frac{P^*}{P_D}\right)^{\beta_2}}_{\text{bankruptcy effect}} \\ & \underbrace{+ \frac{\tau_c C}{r}\left(-\frac{\beta_1}{\beta_1 - 1}\left(1 - \left(\frac{P^*}{P_D}\right)^{\beta_2}\right) - \frac{\beta_2}{\beta_1 - 1}\left(\frac{P^*}{P_D}\right)^{\beta_2}\right)}_{\text{tax shield effect}}.\end{aligned} \tag{6}$$

Recall that $(1+\rho)P_D = \frac{\beta_1 - 1}{\beta_1}C$ and let denote S the set of couples $(P^*, P_D) \in \mathbb{R}_+^2$ that satisfies relationship (6) with $P^* \geq P_D$.

We now interpret relationships (5) and (6) while taking the coupon C as given.

Equity Value. Relationship (5) indicates that the problem can be understood as a standard irreversible investment all equity financed with equity with effective cost $I_E(P^*)$. However, the main difference is that the investment timing through the choice of P^* has a direct impact on the reduction of the cost of investing through the tax benefits $TB(P^*)$ and the transfer of value $TF(P^*)$.

Investment Trigger. Three effects govern the optimal timing of the investment. First, should no default occur, a high cost of investing $I + \frac{C}{r} - D$ delays the investment decision. Second, the effect of bankruptcy is globally captured by the wealth transfer from bondholders to shareholders; it always accelerates the investment decision and can be expressed as

$$\frac{1}{\beta_1 - 1}(\varepsilon_{TF} - \beta_1)TF(P^*),$$

where $\varepsilon_{TF} < 0$ is the elasticity of the transfer with respect to earnings P. The term $-\frac{\beta_1 TF(P^*)}{\beta_1 - 1}$ encapsulates the direct reduction of the cost of capital due to the transfer; the term $\frac{\varepsilon_{TF} TF(P^*)}{\beta_1 - 1}$ is also negative as delaying investment decreases the present value of the wealth transfer. Third, tax deductions also affect the investment timing; the tax shield effect can be rewritten

$$\frac{1}{\beta_1 - 1}(\varepsilon_{TB} - \beta_1)TB(P^*),$$

where $\varepsilon_{TB} > 0$ is the elasticity of tax benefits with respect to earnings P. The term $-\frac{\beta_1 TB(P^*)}{\beta_1 - 1}$ reflects the direct reduction of the cost of capital due to the tax shield; the term $\frac{\varepsilon_{TB} TB(P^*)}{\beta_1 - 1} > 0$ represents the increase in tax benefits due to delaying investment. The overall sign of the tax deduction effect is ambiguous. Finally, observe that relationship (6) can be rewritten

$$\frac{(1-\tau_c)P^*}{\delta} = \frac{\beta_1 I_E(P^*)}{\beta_1 - 1} - \frac{(1-\tau_c)(1+\rho)}{\beta_1 - 1}\frac{P_D}{\delta}\left(\frac{P^*}{P_D}\right)^{\beta_2},$$

so we can conclude that the decision of investing is always made at an earlier stage with respect to some all-equity financed project with (true) cost $I_E(P^*)$. Bernanke's "bad news principle of irreversible investment" only applies partially in a context with debt since the impact of "bad news" are now shared between entrepreneurs and lenders.

2.5 The Lender Problem

Once investment is completed at some normalized date 0, the lender receives the coupon C until liquidation date τ_D; Since bankruptcy costs are assumed to account for α percent of the residual value of the firm $\frac{(1+\rho)P_D}{\delta}$, at bankruptcy debtholders receive $X(P_D) = \frac{(1-\tau_c)(1-\alpha)(1+\rho)P_D}{\delta}$. For $P_D \leq P_0 \leq P^*$, the reward value of lending V^L is given by

$$V^L(P_0) = E_0^Q\left[\int_0^{\tau_D}(1-\tau_c)Ce^{-rs}ds + e^{-r\tau_D}X(P_D)\right].$$

Given preliminary result 1. derived in appendix 1, we have $E_0^Q\left[e^{-r\tau_D}X(P_D)\right] = \left(\frac{P}{P_D}\right)^{\beta_2} X(P_D)$ and

$$E_0^Q\left[\int_0^{\tau_D} Ce^{-rs}ds\right] = M_1\left(\frac{P_0}{P_D}\right)^{\beta_1} + M_2\left(\frac{P_0}{P_D}\right)^{\beta_2} + \frac{C}{r},\ P_0 \geq P_D,$$

where M_1 and M_2 are constants to be determined. To rule out bubbles, we must have $M_1 = 0$. The boundary condition at $P = P_D$ is $M_2 + \frac{C}{r} = 0$. Dropping the time index, once the investment is completed, for $P \geq P_D$, the reward value of lending can be rewritten

$$V^L(P) = \frac{(1-\tau_c)C}{r} - (1-\tau_c)\left(\frac{C}{r} - \frac{(1-\alpha)(1+\rho)P_D}{\delta}\right)\left(\frac{P}{P_D}\right)^{\beta_2}. \quad (7)$$

Observe that $\left(\frac{P}{P_D}\right)^{\beta_2}$ is the present value of receiving \$1 contingent on bankruptcy: $V^L(P)$ is the after tax average value of a receiving coupon C weighted by the probability that bankruptcy did not occur $1-\left(\frac{P}{P_D}\right)^{\beta_2}$ and at liquidation with weight $\left(\frac{P}{P_D}\right)^{\beta_2}$ the residual value of the firm $\frac{(1+\rho)P_D}{\delta}$ less the bankruptcy costs $\frac{\alpha(1+\rho)P_D}{\delta}$. This expression is similar to the one derived in Leland (1994). It is easy to see that $V^L(P)$ is increasing in the coupon C and non-decreasing in earnings value P, which reflects the lenders' incentives to delay investment. For convenience, let us set $BC(P) = \frac{\alpha(1+\rho)P_D}{\delta}\left(\frac{P}{P_D}\right)^{\beta_2}$ the present value of the bankruptcy costs. The value of the lender L is given by

$$L(P) = \begin{cases} \left(V^L(P^*) - D\right)\left(\frac{P}{P^*}\right)^{\beta_1}, \ P \leq P^* \\ V^L(P) - D, \ P_D \leq P^* \leq P. \end{cases}$$

Finally, observe the following decomposition:

$$V^F(P) + V^L(P) = (1-\tau_c)\left(\frac{P}{\delta} - BC(P)\right).$$

The (after tax) intrinsic value of the project $\frac{(1-\tau_c)P}{\delta}$, less the (after tax tax) bankruptcy costs must be divided between equityholders and the lender.

As a benchmark, in order to highlight the effects of an equilibrium credit market mechanism on the investment timing and the option value, we now examine the case of a firm that is free to issue some unsecured debt to finance its expansion.

2.6 Unsecured Debt Issuance

2.6.1 No Restriction on the Amount of Debt Issued

In this paragraph, we assume that the firm is free to issue any amount of debt with market value

$$D(P) = \frac{C}{r} - \left(\frac{C}{r} - \frac{(1-\alpha)(1+\rho)P_D}{\delta}\right)\left(\frac{P}{P_D}\right)^{\beta_2}, \ P \geq P_D.$$

Coupon C is to be chosen by the manager of the firm and as before, the optimal default threshold and the coupon are linked by the following relationship: $(1+\rho)P_D = \frac{\beta_1 - 1}{\beta_1}C$. Immediately after investing at P^*, the value of the equity is equal to $V^F(P^*) - (I - D(P^*))$. Thus, at time 0, starting at P_0, the option value $F(P_0)$ is given by

$$F(P_0) = \frac{(1-\tau_c)\rho P_0}{\delta} + \sup_{\tau \geq 0,\, C \geq 0} E_0^Q\left[\left(\frac{(1-\tau_c)P_\tau}{\delta} + TB(P_\tau) - (1-\tau_c)BC(P_\tau) - I\right)e^{-r\tau}\right].$$

Using the matching condition at $P = P^*$, this program is equivalent to

$$G(P_0) = \max_{P^*, C}\left(\frac{(1-\tau_c)P^*}{\delta} - I_{E,F}(P^*)\right)\left(\frac{P_0}{P^*}\right)^{\beta_1}, \ P_0 \leq P^*,$$

with $I_{E,F}(P^*) = I - TB(P^*) + (1-\tau_c)BC(P^*)$. As shown in appendix 2.1, the optimal investment trigger P_F^* and the optimal coupon C_F^* are given by

$$\begin{aligned} P_F^* &= \frac{\beta_1 I \delta}{\beta_1 - 1}\frac{1}{1 - \tau_c + \tau_c(1+\rho)\left(\frac{\tau_c}{\tau_c - \alpha\beta_2}\right)^{-\frac{1}{\beta_2}}} \\ \frac{C_F^*}{r} &= \frac{\beta_1(\beta_2 - 1)I}{\beta_2(\beta_1 - 1)}\frac{1}{\tau_c + \frac{1-\tau_c}{1+\rho}\left(\frac{\tau_c - \alpha\beta_2}{\tau_c}\right)^{-\frac{1}{\beta_2}}}, \end{aligned}$$

and the option value is

$$G(P_0) = \frac{I}{\beta_1 - 1}\left(\frac{P_0}{P_F^*}\right)^{\beta_1},\ P_0 \le P_F^*.$$

This expression is similar to the one found by Sundaresan, Wang and Yang (2014).

Properties First of all, regardless of the size of the bankruptcy costs, over-investment always prevails as we have $P_F^* \le P_0^*$. However, note that the effective cost of the project is $I_{E,F}(P_F^*) = \frac{(\beta_1-1)(1-\tau_c)-\tau_c(1+\rho)\left(\frac{\tau_c}{\tau_c-\alpha\beta_2}\right)^{-\frac{1}{\beta_2}}}{(\beta_1-1)\left(1-\tau_c+\tau_c(1+\rho)\left(\frac{\tau_c}{\tau_c-\alpha\beta_2}\right)^{-\frac{1}{\beta_2}}\right)}I < I$ and it is easy to check that $\frac{(1-\tau_c)P_F^*}{\delta} \ge \frac{\beta_1 I_{E,F}(P_F^*)}{\beta_1-1}$: when able to internalize both tax benefits and distress costs, equityholders optimally decide to postpone their investment decision with respect to an all-equity financed project with cost $I_{E,F}(P_F^*)$. This result is the opposite of the one found in section 2.4.2. when the coupon is taken as given. Second, observe that for all $P \le P_F^*$, $F(P) \ge F_0(P)$: debt financing always increases the value of the equity and the percentage increase in the option value with respect to the all-equity financed firm is given by

$$\Delta G = \frac{G - G_0}{G_0} = \left(1 + \frac{\tau_c(1+\rho)\left(\frac{\tau_c}{\tau_c-\alpha\beta_2}\right)^{-\frac{1}{\beta_2}}}{1-\tau_c}\right)^{\beta_1} - 1.$$

This ratio is increasing in τ_c from 0 up to infinity, increasing in ρ and decreasing in α. All the benefits are coming from tax deductions; in fact if $\tau_c = 0$, it is optimal not to issue debt and expansion must be all-equity financed. As illustrated by the numerical simulations, the corporate tax rate is a key parameter when expansion is financed by debt issuance; a small change in the corporate tax rate have very strong quantitative effects. Furthermore, since $\frac{\partial\beta_1}{\partial\sigma} < 0$ and $\frac{\partial\beta_2}{\partial\sigma} > 0$, it is also easy to see that this ratio shrinks as uncertainty rises.

Special Case $\alpha = 0$. Default occurs right after the investment is completed. Tax benefits are at their pinnacle and fully internalized: the option value is the *same* as its all-equity financed counterpart facing a corporate tax rate equal to $-\rho\tau_c$, i.e. a credit tax rate equal to $\rho\tau_c$. The amount of cash raised at the investment date $D(P_F^*)$ is increasing in ρ but decreasing in τ_c and equal to $\frac{\beta_1 I}{\beta_1-1} \times \frac{1}{\tau_c+\frac{1-\tau_c}{1+\rho}}$, which of course is exactly the total expected value of the equity $\frac{(1+\rho)P_F^*}{\delta}$ for an all-equity financed firm facing a credit tax rate of $\rho\tau_c$.

General Case: $\alpha > 0$. As shown in the appendix 2, we have $\frac{\partial P_F^*}{\partial\alpha} > 0$, $\frac{\partial C_F^*}{\partial\alpha} < 0$, $\frac{\partial P_F^*}{\partial\rho} > 0$, $\frac{\partial C_F^*}{\partial\rho} > 0$. For small values of ρ, we have $\frac{\partial P_F^*}{\partial\tau_c} > 0$ and $\frac{\partial C_F^*}{\partial\tau_c} > 0$ and, if ρ is large enough, P_F^* and C_F^* are hump-shaped functions of the corporate tax rate τ_c. Impacts on the option value G are as follows: for all $P \le P_F^*$, $\frac{\partial G(P)}{\partial\alpha} < 0$, $\frac{\partial G(P)}{\partial\rho} > 0$, $\frac{\partial G(P)}{\partial\tau_c} < 0$ if ρ is small, otherwise the option value G is a hump-shaped function of τ_c. Finally, as far as uncertainty effects are concerned, we establish that as in the all-equity financed case, $\frac{\partial P_F^*}{\partial\sigma} > 0$ and $\frac{\partial G(P)}{\partial\sigma} > 0$: more volatility always enhances the growth opportunity but delays the investment decision.

Amount of Cash Raised at the Investment Date The amount of cash $D(P_F^*)$ raised at the date the investment is completed is given by

$$D(P_F^*) = \frac{\beta_2 C_F^*}{(\beta_2-1)r}\left(1 - \alpha + \frac{\alpha(1-\alpha\beta_2)}{\tau_c - \alpha\beta_2}\right).$$

One can show that $D(P_F^*)$ is an increasing (decreasing) function of the amount of assets already in place ρ (the bankruptcy cost α). Furthermore, if ρ is small (large enough), $D(P_F^*)$ is an increasing (hump-shaped) function of the corporate tax rate τ_c. The bigger the size of the firm (large ρ), the greater its ability to meet the coupon payments. Regarding the impact of the corporate tax rate τ_c, ceteris paribus, when τ_c is high, tax shield benefits are large and so are the incentives to issue a large amount of debt; note that when $\tau_c = 1$, we have $D(P_F^*) = \frac{\beta_1 I}{\beta_1 - 1} > I$. However, as shown below, a higher corporate tax rate τ_c raises the cost of the debt, which may deter the firm from issuing too much debt. We can conclude that if τ_c is low, ρ is small or/and α is large, the amount of cash the firm is able to raise may not be enough to satisfy its investment needs D; formally, we have $D(P_F^*) \leq D$ exactly when

$$\frac{\left(1 - \alpha + \frac{\alpha(1-\alpha\beta_2)}{\tau_c - \alpha\beta_2}\right) v}{\tau_c + \frac{1-\tau_c}{1+\rho}\left(\frac{\tau_c - \alpha\beta_2}{\tau_c}\right)^{-\frac{1}{\beta_2}}} \leq \frac{\beta_1 - 1}{\beta_1}. \tag{8}$$

Finally, the (average) interest rate $\overline{r}_F$ paid on the risky debt is equal to $\overline{r}_F = \frac{C_F^*}{D(P_F^*)}$ and is given by

$$\overline{r}_F = r\frac{\beta_2 - 1}{\beta_2}\frac{1}{1 - \alpha + \frac{\alpha(1-\alpha\beta_2)}{\tau_c - \alpha\beta_2}}.$$

We have $\overline{r}_F \in \left[r, \frac{\beta_2 - 1}{\beta_2} r\right]$ and clearly $\frac{\partial \overline{r}_F}{\partial \tau_c} > 0$ and we also have $\frac{\partial \overline{r}_F}{\partial \alpha} < 0$. This last result may seem paradoxical but recall that $\frac{\partial P_F^*}{\partial \alpha} > 0$ and $\frac{\partial C_F^*}{\partial \alpha} < 0$, so as α goes up, bankruptcy is delayed. We also find that $\frac{\partial \overline{r}_F}{\partial \sigma} > 0$, i.e., not surprisingly, more uncertainty raises the cost of the debt.

2.6.2 Amount of Debt Issued Must Cover Investment Needs

In this section we assume that relationship (8) holds: should no restrictions be imposed on the amount of debt optimally issued, investment needs are not covered. Therefore, the manager's must solve the following program

$$\begin{array}{ll} \max\limits_{P^*, C} & \frac{(1-\tau_c)\rho P}{\delta} + \left(V^F(P^*) - \frac{(1-\tau_c)\rho P^*}{\delta} - I + D\right)\left(\frac{P}{P^*}\right)^{\beta_1} \\ & \text{s.t. } D(P^*) = D,\ (1+\rho)P_D = \frac{\beta_1 - 1}{\beta_1} C \text{ and } P_D \leq P^*. \end{array} \tag{F}$$

The existence of a solution is established in appendix 2.2. Program F is equivalent to maximizing a simpler one dimensional program that consists in choosing the reduced variable $z = \frac{P_D}{P^*}$, the default threshold over the investment trigger ratio. On the one hand, the larger the equilibrium value z_F^*, the longer the investment is postponed, but on the other hand, once the investment completed the sooner bankruptcy is declared.

As analytical results are hard to obtain, we conclude the analysis by performing some comparative statics on the equilibrium couple (P_F^*, C_F^*) and the option value G by relying on numerical simulations.

2.6.3 Numerical Simulations

In order to access the quantitative of debt issuance, we perform some comparatives statics in the debt ratio $u = D/I$, the corporate tax rate τ_c, and the bankruptcy cost α on the after-tax investment trigger $\frac{(1-\tau_c)P_F^*}{\delta}$, the average interest rate paid on risky debt is $\overline{r}_F$ and the relative gain/loss in the value of waiting $\Delta G = \frac{G - G_0}{G_0}$. Let $r = 0.04$, $\delta = 0.04$, $I = 100$ and $\sigma = 0.2$. It follows that all-equity investment trigger is equal to $\frac{(1-\tau_c)P_0^*}{\delta} = 200$.

First, we consider the case where $\tau_c = 0.35$. For the set of parameters chosen, it turns out that the firm is free to issue any level of debt it wants. Table I summarizes the results.

	$\rho = 0$				$\rho = 1$			
α	0	0.5	0.75	1	0	0.5	0.75	1
$\frac{(1-\tau_c)P_F^*}{\delta}$	130	163.7	170.7	175.5	96.3	138.6	148.9	156.3
ΔG (%)	136.6	49.2	37.2	29.7	331.4	108.4	80.3	63.6
$\overline{r}_F$ (%)	8.00	5.79	5.54	5.40	8.00	5.79	5.54	5.40

Table I: Comparative statics for an unconstrained firm $\tau_c = 0.35$

Next, we consider the case where $\tau_c = 0$ for $u = 0.5$ and $u = 1$. For the set of parameters chosen, the firm is always constrained regarding how much debt to issue, except for the case $\alpha = 0$, which is identical to the all-equity financed case. Table II summarizes the results.

	$\rho = 0$				$\rho = 1$			
α	0	0.5	0.75	1	0	0.5	0.75	1
$u = 0.5$								
$\frac{(1-\tau_c)P_F^*}{\delta}$	200	206.1	209.5	213.2	200	202.6	204.0	205.5
ΔG (%)	0	−1.86	−2.83	−3.84	0	−1.68	−2.11	−2.54
$\overline{r}_F$ (%)	4.00	4.45	4.54	4.63	4.00	4.20	4.24	4.28
$u = 1$								
$\frac{(1-\tau_c)P_F^*}{\delta}$	200	231.3	248.5	266.5	200	211.7	218.0	224.4
ΔG (%)	0	−8.18	−11.99	−15.6	0	−6.77	−8.34	−9.88
$\overline{r}_F$ (%)	4.00	5.02	5.18	5.33	4.00	4.43	4.51	4.58

Table II: Comparative statics for a constrained firm $\tau_c = 0$

Overall, we can claim that the impact of the size of the assets already in place is to reduce (increase) the wedge between the levered firm and the all-equity financed cases when the firm's choice of debt is constrained (unconstrained), both for the timing and the cost of the investment as well as the magnitude of the option value. It also appears that the impact of parameter ρ is quite significant where the firm can freely choose its debt level ($\tau_c = 0.35$) but somewhat modest when it is forced to issue amount D ($\tau_c = 0$).

Then, we set $\rho = 0$ and graphically illustrate our findings for two values of the corporate tax rate $\tau_c = 0$ and $\tau_c = 0.1$.

Investment Trigger $\frac{(1-\tau_c)P_F^*}{\delta}$. When $\tau_c = 0$, the firm is always forced to issue amount D. Figure 1 shows that both a higher debt ratio $u = D/I$ and bankruptcy cost α always delay investment, up to 33% (for $u = 1$ and $\alpha = 1$). When $\tau_c = 0.1$, for small values of debt ratio u, the firm can freely choose its debt level and over-investment takes place (figure 2). As u rises, the higher the bankruptcy cost α, the sooner the firm becomes constrained to issue amount D. Eventually, for large enough values of the debt ratio u, under-investment prevails, but the impact is weaker than in the case $\tau_c = 0$. When $\tau_c = 0.35$, over-investment always prevails and the investment strategy is all the more aggressive as bankruptcy costs are low and assets in place large. Even for a pure development project ($\rho = 0$), the reduction in the investment trigger can be severe (down by 35%) with respect to the all-equity financed case.

Option Value G. Graphs 3 and 4 plots the relative changes in the option value as a function of the debt ratio $u = D/I$. In figure 3, for $\tau_c = 0$, we observe that the impact of the debt financing is all the more negative as the debt ratio u and the bankruptcy cost α are large, with more than 15% for $u = 1$ and $\alpha = 1$. For a higher corporate tax rate, $\tau_c = 0.1$, for low values of u, the firm can freely decide of the amount of debt to be released; in this case, issuing debt raises the option value with respect to the all-equity financed case, up to 23.5% for $\alpha = 0$ (figure 4). For large enough bankruptcy costs α, as the debt ratio u rises, covering investment needs becomes binding, and the option value decreases as parameters u and α rise, down to 6.2% (for $u = 1$ and $\alpha = 1$). For $\tau_c = 0.35$, recall that the amount of debt to be issued can always be freely decided; table I indicates that gains in the option value are always significant and potentially can be huge - up to 331% - for small (large) values of α (ρ).

Interest Rate Paid on Risky Debt $\overline{r}_F$. Figures 5 and 6 depict the effects of the debt ratio u on the (average) interest paid on risky debt. For $\tau_c = 0$, the firm is always forced to issue some debt level D; as shown in Figure 5, the interest rate charged on the debt is increasing with the debt ratio u and bankruptcy cost and the premium paid over the risk free rate $\overline{r}_F - r$ can be significant, up to 135 basis points (for $u = 1$ and $\alpha = 1$). For $\tau_c = 0.1$, figure 6 reveals that as long as the firm is not constrained, we have seen that the interest rate paid is independent of the debt ratio u, decreasing in the bankruptcy cost and notice that discrepancies are significant since the premium $\overline{r}_F - r$ can reach 400 basis points (when $\alpha = 0$). For large enough values of the debt ratio u and bankruptcy costs α, the firm is forced to issue debt amount D; the interest rate charged on debt becomes increasing both in u and α, the maximum premium paid being equal to 136 basis point for $u = 1$ and $\alpha = 1$. For $\tau_c = 0.35$, the firm always freely decides of its debt level; as in displayed in table I, the interest rate premium $\overline{r}_F - r$ ranges from 140 basis points up to 400 basis points, the latter value being charged when bankruptcy is declared right after investing ($\alpha = 0$).

In the next section, we analyze the equilibrium feedback effects between the investment trigger P^* given by relationship (6) and the coupon C under some alternative credit market structures.

3 Perfectly Competitive Credit Market

In this section, the credit market is assumed to be perfectly competitive; lenders do not make any profit and the no-profit entry condition is simply $V^L(P_C^*) - D = 0$. In equilibrium, the effective cost of investing is $I + (1-\alpha)BC(P_C^*)$, which is above the cost of the project I. However, we shall see that this does not imply that the decision to invest is always postponed with respect to the all-equity financed case. Contrary to the case where debt is issued, tax benefits and distress costs cannot be internalized by the firm.

Definition 1 *An equilibrium is a couple (P_C^*, C_C^*) that satisfies*

$$\frac{(1-\tau_c)P_C^*}{\delta} + \frac{(1-\tau_c)(\beta_1-\beta_2)}{\beta_1-1}\left(\frac{C_C^*}{r} - \frac{(1+\rho)P_D}{\delta}\right)\left(\frac{P_C^*}{P_D}\right)^{\beta_2} = \frac{\beta_1}{\beta_1-1}\left(I + \frac{C_C^*(1-\tau_c)}{r} - D\right) \quad (9)$$

$$\frac{(1-\tau_c)C_C^*}{r} - D - (1-\tau_c)\left(\frac{C_C^*}{r} - \frac{(1-\alpha)(1+\rho)P_D}{\delta}\right)\left(\frac{P_C^*}{P_D}\right)^{\beta_2} = 0, \quad (10)$$

with $\frac{(1+\rho)P_D}{\delta} = \frac{\beta_2}{\beta_2-1}\frac{C_C^*}{r}$, or equivalently $(1+\rho)P_D = \frac{\beta_1-1}{\beta_1}C_C^*$.

Proposition 1 *There exists a unique equilibrium* (P_C^*, C_C^*) *with* $P_C^* > C_C^*$ *that is given by*

$$\begin{aligned}\frac{(1-\tau_c)P_C^*}{\delta} &= -\frac{\beta_2 D x_C^*}{(1+\rho)(1-\beta_2-(1-\alpha\beta_2)(x_C^*)^{\beta_2})}\\ \frac{(1-\tau_c)C_C^*}{r} &= \frac{(1-\beta_2)D}{1-\beta_2-(1-\alpha\beta_2)(x_C^*)^{\beta_2}},\end{aligned}$$

where $x_C^* = \frac{P_C^*}{P_D} > 1$ *is the unique root the following equation*

$$M(v,\alpha)x^{\beta_2} - \frac{\beta_2(\beta_1-1)}{1+\rho}x + \beta_1 v(\beta_2-1) = 0,$$

with $M(v,\alpha) = \beta_1(v-1)(1-\alpha\beta_2) + \beta_1 - \beta_2 > 0$ *and* $v = I/D \geq 1$. *Over-investment,* $P_C^* \leq P_0^*$, *occurs exactly when bankruptcy costs are low enough, namely* $\alpha \leq 1/\beta_1$; *otherwise under-investment prevails,* $P_0^* \leq P_C^*$. *Furthermore, the equity value of the levered firm* F *is always lower than that of its all-equity financed counterpart* F_0.

Proof. See appendix 2 ■

We now examine in details the equilibrium properties. Proofs of all results are collected in appendix 3.

3.1 Properties of the Equilibrium

Unlike in the case where the firm issues debt and is free to decide of the coupon, the firm cannot internalize the benefits of tax deductions on coupon payments. Since both the borrower and the lender face the same corporate tax rate, as in the all-equity financed case, the corporate tax rate simply has a proportional impact: the after tax earning investment trigger $(1-\tau_c)P_C^*$ and after tax coupon $(1-\tau_c)C_C^*$ are independent of the tax rate.

Investment Trigger and Coupon. On the one hand, the firm bears up-front only a fraction of the total cost of the project and when bankruptcy is declared, there is loss sharing between the two parties: this induces a more daring investment decision. Bernanke's "bad news principle of irreversible investment" only applies partially in a context with debt since the impact of "bad news" are now shared between entrepreneurs and lenders. On the other hand, the lender has incentives to delay bankruptcy, in particular should the bankruptcy costs be large. The first result we derive is $P_C^* > C_C^*$ (P1), that is, even though the manager may be eager to invest, she always makes sure that the coupon can be safely met for a while after investing. Furthermore, we find that $\frac{(1-\tau_c)P_C^*}{\delta}$ always exceeds the myopic threshold[10] $\frac{\beta_1(I-D)}{\beta_1-1}$ (P6). The manager takes into account that bankruptcy will truncate earnings as well as raising the cost of financing so she requires a larger wedge between P_C^* and up-front cost of investing $I-D$. Then, from relationship (6), note that for a given coupon, the investment trigger P_C^* does not directly depends on the bankruptcy cost α; instead the bankruptcy cost only has an indirect effect through the equilibrium value of the coupon C_C^*. Property (P2) establishes that P^* defined by relationship (6) is increasing in C. A higher bankruptcy cost α induces a higher equilibrium coupon, which in turns leads to a higher equilibrium investment trigger: for all $u \leq 1$, we have $\frac{\partial P_C^*}{\partial \alpha} > 0$ and $\frac{\partial C_C^*}{\partial \alpha} > 0$ (P3). Not surprisingly, we also have $\frac{\partial C_C^*}{\partial D} > 0$, which in turn leads to $\frac{\partial P_C^*}{\partial D} > 0$ (property P4).

The next property (P5) deals with under-over investment, a phenomenon that has drawn much attention in the literature. It is generally argued that under-investment problem is more severe for firms in distress. An empirical study by Whited (1992) documents that a firm's financial position

[10] Observe that this is the investment trigger optimally chosen by an all-equity financed firm to finance a project with cost $I-D$.

affects its investment and indeed confirms that financial distress measured by a high debt to assets ratio leads to a lower level of investment. When the firm is facing an endogenous financing constraint, the investment policy can be quite sensitive to liquidity: the threat of a future cash flow shortfall may lead to early exercise of the growth option and therefore accelerating investment beyond the first best optimal level (see for instance Boyle and Guthrie (2003) and Hirth and Uhrig-Homburg (2007)). Our equilibrium model predicts that for small values of bankruptcy costs α, more specifically, $\alpha < 1/\beta_1$, over-investment takes place: $P_C^* < P_0^*$ and moreover P_C^* is increasing in ρ for all $\rho \geq 0$ and decreasing in u for all $u \leq 1$. The manager's appetite for accelerating investment is mitigating by the threat of bankruptcy and loosing the already existing activity, which leads to a more conservative investment strategy. For $\alpha = 1/\beta_1$, the accelerating force from the firm and the delaying force from the lender exactly offset each other and $P_C^* = P_0^*$ for all $u \leq 1$ and $\rho > 0$. Finally, for $\alpha > 1/\beta_1$, the high coupon charged induces under-investment: $P_C^* > P_0^*$ and moreover P_C^* is decreasing in ρ for all $\rho \geq 0$ and increasing in u for all $u \leq 1$. Since $\frac{\partial \beta_1}{\partial \sigma} < 0$, we can conclude that a higher earnings volatility contributes to the over-investment phenomenon.

Finally, property (P8) examines the effects of assets already on the timing of the investment, which are twofold. First, the larger parameter ρ, the larger revenues generated and a larger equity value to be used as collateral. Ceteris paribus, coupon payments are easier to meet, so risk for the debtholders is reduced and we obtain that $\frac{\partial C_C^*}{\partial \rho} < 0$. Second, bankruptcy will also imply the loss of the already existing activities, which could induce a more conservative investment strategy. We find that when over-investment takes place, i.e. when $\alpha \leq 1/\beta_1$, the manager's appetite for accelerating investment is mitigating by the threat of bankruptcy and loosing the already existing activity; as a consequence $\frac{\partial P_C^*}{\partial \rho} > 0$. Conversely, when $\alpha \geq 1/\beta_1$ under-investment occurs, more assets already in place accelerate the investment decision $\frac{\partial P_C^*}{\partial \rho} < 0$.

Equity Value. Contrary to the case where the project is financed by issuing some debt, proposition 1. asserts that the value of the levered firm is always below its all-financed equity counterpart. Moreover, in appendix 3.2, we prove that the higher the debt ratio u or the bankruptcy cost α, the smaller the equity value for all values of earnings P (P7). On the contrary the higher assets already in place, the higher the option value of expanding by one unit G and the (total) equity value (P8).

We conclude the analysis by performing some comparative statics on the equilibrium couple (P_C^*, C_C^*) and the value of waiting F by relying on numerical simulations.

3.1.1 Numerical Simulations

We perform some comparative statics on the after-tax investment trigger $\frac{(1-\tau_c)P_C^*}{\delta}$ and the relative gain/loss in the option value of waiting ΔG. This percentage change is independent of the earnings P and the corporate tax rate τ_c and is given by

$$\Delta G = \frac{\frac{(1-\tau_c)P_C^*}{\delta} - (1-\tau_c)BC(P_C^*) - I}{\frac{(1-\tau_c)P_0^*}{\delta} - I} \left(\frac{P_0^*}{P_C^*}\right)^{\beta_1} - 1.$$

The average interest rate paid on risky debt is $\bar{r}_C = \frac{C_C^*(1-\tau_c)}{D}$. As before, let $r = 0.04$, $\delta = 0.04$, $I = 100$ and $\sigma = 0.2$, so that $\frac{(1-\tau_c)P_0^*}{\delta} = 200$.

First we investigate the impact of the amount of assets already in place on the equilibrium when $D/I = 0.5$ for $\alpha = 0$ and $\alpha = 0.75$. Results are reported in table III. Please note that changes in the option value are expressed in tenth of percents.

ρ	$\alpha = 0$					$\alpha = 0.75$				
	0	0.5	1	2	10	0	0.5	1	2	10
$\frac{(1-\tau_c)P_C^*}{\delta}$	196.3	197.7	198.2	198.9	199.7	202	201.2	200.9	200.6	200.1
ΔG ($^o/_{oo}$)	-0.35	-0.14	-0.07	-0.03	-0.00	-29.7	-18.1	-13.0	-8.37	-2.17
$\overline{r}_C$ ($^o/_o$)	4.29	4.18	4.13	4.09	4.02	4.56	4.34	4.25	4.16	4.04

Table III: Comparative statics for parameter ρ

Table III indicates that the quantitative impact of assets already in place remains quite modest - in particular in comparison with the case when debt is issued (see table II), with somewhat a more significant reduction of the equilibrium average interest rate.

Next, we consider a pure option value; we set $\rho = 0$ and investigate the impact of the debt ratio $u = D/I$ and bankruptcy costs α.

Investment Trigger $\frac{(1-\tau_c)P_C^*}{\delta}$. In Figure 7 we plot the optimal investment trigger $\frac{(1-\tau_c)P_C^*}{\delta}$ as a function of the debt ratio $u = D/I$, for several values of the bankruptcy cost α. For small debt ratio values, the investment trigger is very close to its all-equity financed value, above or below, depending on the sign of $\alpha - 1/\beta_1$ and when $u = 1$, the maximum reduction is 10% for $\alpha = 0$ whereas the maximum increase is 12.5% for $\alpha = 1$.

Interest Rate Paid on Risky Debt $\overline{r}_C$. Figure 8 depicts the effect of the debt ratio u on the (average) interest paid on risky debt. The graphs are similar to those obtained when debt is issued and $\tau_c = 0$, the larger u and α, the more costly the debt. The interest paid is found to always exceed the one charged when debt is issued, even if there are no bankruptcy costs and in particular when the debt ratio is high; when the investment is exclusively financed with debt ($u = 1$) and $\alpha = 1$, the yield spread $\overline{r}_C - r$ reaches 200 basis points.

Option Value G. Figure 9 represents changes in the option value as the debt ratio u varies. As in the case when debt is issued and $\tau_c = 0$, relying on external funds reduces the option value, but here even in absence of bankruptcy costs and the drop in option value appears to be more severe, up to -20.9% if $u = 1$ and $\alpha = 1$ instead of -15.6% when debt is issued. Distress costs cannot be internalized.

3.1.2 Summary of the Equilibrium Properties

	Investment Threshold	Coupon	Option Value
Debt Level D	$\frac{\partial P_C^*}{\partial D} > 0$	$\frac{\partial C_C^*}{\partial D} > 0$	$\frac{\partial F}{\partial D} < 0$
Bankruptcy Cost α	$\frac{\partial P_C^*}{\partial \alpha} > 0$	$\frac{\partial C_C^*}{\partial \alpha} > 0$	$\frac{\partial F}{\partial \alpha} < 0$
Corporate Tax Rate τ_c	$\frac{\partial P_C^*}{\partial \tau_c} > 0$	$\frac{\partial C_C^*}{\partial \tau_c} > 0$	
Assets Already in Place ρ	$\frac{\partial P_C^*}{\partial \rho} \geq 0$ (≤ 0) if $\alpha \leq 1/\beta_1$ $(\geq 1/\beta_1)$	$\frac{\partial C_C^*}{\partial \rho} < 0$	
Over/Under Investment	$P_C^* \leq P_0^*$ $(\geq P_0^*)$ if $\alpha \leq 1/\beta_1$ $(\geq 1/\beta_1)$		

Table IV: Summary of properties for perfectly competitive credit markets

In the next section, we depart from the perfect competitive lending sector and examine the case of a small firm that has no access to debt markets and needs to bargain with some credit institution to get funding.

4 Bargaining Over Funding

In this section, we depart from the perfectly competitive credit market hypothesis and explore the impact of several bargaining processes to get funding on the timing of the investment and the option value. To keep things simple, we assume that no assets are already in place, $\rho = 0$. First, we look at a sequential bargaining process (Stackelberg type equilibrium). Second, we assume that the negotiation process over funding is represented by a Nash Bargaining mechanism[11]. As shown in the sequel, a nice feature of these contracts is that the equilibrium outcomes are independent of the initial condition (earnings value) at the date when the agreement is signed by the two parties. Finally, observe that, contrary to Fan and Sundaresan (2000) and Sundaresan and Wang (2007), we do not allow for debt renegociation.

4.1 Sequential Negotiation Mechanism

The firm has a bargaining ability with weight $1-\theta$ whereas the lender has a bargaining ability with weight θ for some θ in $[0,1]$. The firm is free to choose the timing of the investment, which seems a reasonable assumption. This implies that the smooth pasting (4) condition still applies. Combining this latter condition with the value matching condition (3) leads to the following expressions at time 0 for the option value F and the lender's value L

$$\begin{aligned} F(P_0) &= \frac{\frac{(1-\beta_2)(1-\tau_c)P^*}{\delta} + \beta_2(I + \frac{C(1-\tau_c)}{r} - D)}{\beta_1-\beta_2}\left(\frac{P_0}{P^*}\right)^{\beta_1} \\ L(P_0) &= \frac{\frac{(\beta_1-1)(1-\tau_c)P^*}{\delta} - \beta_2(\frac{C(1-\tau_c)}{r} - D) - \beta_1 I}{\beta_1-\beta_2}\left(\frac{P_0}{P^*}\right)^{\beta_1}, \end{aligned}$$

and the investment threshold P^* and the coupon C belong to set S where

$$S = \left\{(P^*, C),\ \frac{(\beta_1-1)(1-\tau_c)P^*}{\delta} + (1-\tau_c)(\beta_1-\beta_2)\left(\frac{C}{r} - \frac{P_D}{\delta}\right)\left(\frac{P^*}{P_D}\right)^{\beta_2} = \beta_1\left(I + \frac{C(1-\tau_c)}{r} - D\right)\right\}.$$

Participation Constraints. Both parties must be willing to enter into the deal so for all $P_0 > 0$, we must have

$$\begin{aligned} F(P_0) &\geq 0 \text{ (FPC constraint)} \qquad (11) \\ L(P_0) &\geq 0 \text{ (LPC constraint).} \qquad (12) \end{aligned}$$

Contract. At time 0, the two parties agree on a couple (P_S^*, C_S^*) that maximizes the weighted average of the equity and the debt values

$$\max_{(P^*,C)\in S} \quad [F(P_0)]^{1-\theta}\,[L(P_0)]^{\theta} \quad \text{s.t. } (FPC) \text{ and } (LPC). \qquad (S)$$

[11] Nash bargaining between lenders and borrowers was originally introduced by Fan and Sundaresan (2000) who examine the implications of debt renegociation characterized by a temporary contractual coupon reduction as long the firm is under financial.

Proposition 2 *There exists an equilibrium* (P_S^*, C_S^*) *such that* $\frac{\partial P_S^*}{\partial \theta} > 0$, $\frac{\partial C_S^*}{\partial \theta} > 0$ *and for all* $P \leq P_S^*$

$$\frac{\partial F(P)}{\partial \theta} < 0 \text{ and } \frac{\partial L(P)}{\partial \theta} > 0,$$

so in particular, the equity value of the levered firm F *is always lower than that of its all-equity financed counterpart* F_0. *Furthermore, as* θ *approaches* 0, *we have* $\lim_{\theta \to 0} P_S^* = P_C^*$ *and* $\lim_{\theta \to 0} C_S^* = C_C^*$.

Proof. See appendix 4.1 ■

As in the constrained unsecured debt issuance case, program S can be transformed into a simpler one dimensional program that consists in choosing the reduced variable $z = \frac{P_D}{P^*}$, the default threshold over the investment trigger ratio. On the one hand, the larger the equilibrium value z_S^*, the longer the investment is postponed, but on the other hand, once the investment completed the sooner bankruptcy is declared. Since the lender is the party that is directly affected by the bankruptcy costs, we can expect the latter to have a higher impact on the equilibrium outcome than when markets are perfectly competitive. Finally, note that both P_S^* and C_S^* are found to be increasing in the choice of variable z.

4.1.1 Interpretation of the Results

As θ approaches 0, the LPC constraint becomes binding: the firm extracts all the surplus, the lender makes no profit and the equilibrium outcome coincides with the one found for the perfectly competitive credit market case. In some sense, this contract is an extension to the perfect competitive credit market studied in section 3. On the opposite, the case $\theta = 1$ corresponds to the standard Stackelberg equilibrium where the lender is the leader first charging a coupon and the firm is the follower that decides of the optimal timing of the investment given the coupon charged.

Not surprisingly, as θ rises, the lender can rip off a larger share of the benefits generated by the project by charging a higher coupon at the expense of the firm that is forced to delay investment. However, there is a trade-off between (i) increasing profit by charging a high coupon and (ii) inducing early investment to collect the coupon as soon as possible. Underinvestment or overinvestment is determined by each party relative bargaining power and the size of bankruptcy costs. For instance, notice that if $\alpha \leq 1/\beta_1$ and θ is small, over-investment may still take place. On the contrary, for $\alpha \geq 1/\beta_1$, under-investment always prevails and can be significant, since for $\theta = 1$ and $\alpha = 1/\beta_1$, we find that

$$\frac{(1-\tau_c)P_S^*}{\delta} = \left(\frac{\beta_1}{\beta_1 - 1}\right)^2 I.$$

Unfortunately, analytical results are hard to obtain. We rely on numerical simulations to analyze the properties of the contract.

4.1.2 Numerical Results

We perform some comparative statics on the after-tax investment trigger $\frac{(1-\tau_c)P_S^*}{\delta}$, the after-tax coupon $\frac{(1-\tau_c)C_S^*}{r}$ and the relative gain/loss in the value of waiting ΔF. This percentage change is still independent of the earnings P and the corporate tax rate τ_c and is given by

$$\Delta F = \frac{(1-\beta_2)\frac{(1-\tau_c)P_S^*}{\delta} + \beta_2(I + \frac{(1-\tau_c)C_S^*}{r} - D)}{(\beta_1 - \beta_2)\left(\frac{(1-\tau_c)P_0^*}{\delta} - I\right)} \left(\frac{P_0^*}{P_S^*}\right)^{\beta_1} - 1.$$

As before, let $r = 0.04$, $\delta = 0.04$, $I = 100$ and $\sigma = 0.2$, so that $\frac{(1-\tau_c)P_0^*}{\delta} = 200$. Contrary to the perfect credit market case, even if $D = 0$, there is still a positive coupon charged: the firm incurs some fixed cost plus a variable cost that is increasing in the amount of money borrowed D.

Investment Trigger $\frac{(1-\tau_c)P_S^*}{\delta}$. Figure 10 represents the optimal investment trigger $\frac{(1-\tau_c)P_S^*}{\delta}$ as a function of the debt ratio $u = D/I$, for several values of the bankruptcy cost α and the lender bargaining power θ. Clearly, for large values of θ, the impact on the investment trigger is quite significant and the investment decision is always delayed, the wedge between the investment trigger and its all-equity financed counterpart reaching nearly 250% for $\theta = 1$ and $\alpha = 1$. For small debt ratio values, the fixed cost effect is quite strong and the investment trigger appears to be decreasing in bankruptcy cost α, although the magnitude of the changes is small. For large values of parameter θ, the equilibrium outcome is mostly driven by the lender's profit. It turns out that the latter is the most sensitive to bankruptcy cost α when u is small (see appendix 4.1); at the equilibrium, when u is small, the equilibrium ratio z_S^* tends to be larger the larger parameter α is. For large values of u, the lender's profit is less sensitive to bankruptcy costs and the behavior of the investment trigger is similar to the one found when the credit market is perfect, the larger θ the higher the investment threshold. Although we do not report further results, simulations show that the ratio P_S^*/C_S^* that determines the magnitude of the bankruptcy cost is always increasing in u.

Coupon Paid on Risky Debt $\frac{(1-\tau_c)C_S^*}{r}$. Figure 11 depicts the effect of the debt ratio u on the coupon charged paid on risky debt. First of all, the coupon charged is quite sensitive with respect to parameter θ and always increasing in the debt ratio u. For small values of u, the higher the bankruptcy cost α the lower the coupon charged; conversely for large values of u, the graphs are similar to those obtained when $\theta = 0$, the larger u and α, the more costly the debt. Notice that when $\alpha = 1$ and $\theta = 0$, the coupon charged is 150 for $u = 1$; for $\alpha = 1$ and $\theta = 1$, when $u = 0$ and $u = 1$, the coupons charged are respectively 105.8 and 300, so even controlling for the fixed cost effect, the debt is more expensive.

Option Value G. Figure 12 displays the changes in the option value as the debt ratio u varies for several values of the bankruptcy costs α. Overall, the graphs are similar to those obtained when the credit market is perfect, except for large values of θ and low values of α for which the reduction in the option value shrinks as the debt ratio u raises. Nevertheless, observe that for small (large) values u, the higher the bankruptcy costs α, the lower (higher) reduction in the option value. Also note that even when $\theta = 1$, the lender cannot expropriate all the benefits of investing; at worst when $\alpha = 1$ and $u = 1$, the drop in the option value reaches 60%.

4.2 Nash Bargaining Mechanism

In this section, we let the timing of the investment to be determined by the interests of both parties through the negotiation process, allowing tax benefits and distress costs to be partially internalized by the firm. The main difference with the sequential equilibrium case is that the smooth pasting condition (4) that ensures that the manager of the firm acts to maximize equity value F no longer holds. Nevertheless, the value matching condition (3) that ensures that there is no jump in the equity value at the investment date still applies.

Contract. At time 0, the two parties agree on a couple (P_N^*, C_N^*) that maximizes the weighted average of the equity and the debt values[12]

$$\max_{P^*,C} \quad [F(P_0)]^{1-\theta}\,[L(P_0)]^{\theta} \qquad \text{s.t. } (FPC) \text{ and } (LPC), \tag{N}$$

[12] For simplicity, we assume that the disagreement point is $(0,0)$. This means that both parties have no outside option, which is the case if there is a unique lender and only one firm. Such a framework allows for a comparison with the perfectly competitive lending sector studied in the previous section.

where this time the option value F and the lender value are given by

$$\begin{aligned} F(P_0) &= \left(V^F(P^*) - I + D\right)\left(\frac{P_0}{P^*}\right)^{\beta_1} \\ L(P_0) &= \left(V^L(P^*) - D\right)\left(\frac{P_0}{P^*}\right)^{\beta_1}, \end{aligned}$$

and the expressions for $V^F(P^*)$ and $V^L(P^*)$ are given by relationships (2) and (7) respectively. We first examine the special case when there are no bankruptcy costs $\alpha = 0$.

4.2.1 Special Case $\alpha = 0$

As shown in appendix 4.2.1, the optimal solution is

$$\begin{aligned} P_N^* &= P_0^* \\ \frac{(1-\tau_c)C_N^*}{r} - (1-\tau_c)\left(\frac{C_N^*}{r} - \frac{P_D}{\delta}\right)\left(\frac{P_0^*}{P_D}\right)^{\beta_2} &= D + \frac{\theta I}{\beta_1 - 1}, \end{aligned}$$

and for all $P \leq P_0^*$, we have

$$F(P) = (1-\theta)F_0(P) \text{ and } L(P) = \theta F_0(P). \tag{13}$$

The timing of the investment is the same as in the all-equity financed case; however the effective cost of the project $I_{E,N} = I + V^L(P_N^*) - D = I + \frac{\theta I}{\beta_1 - 1}$ and, therefore once more, investment is undertaken too early from the firm's optimality point of view. In appendix 4.2.1. we establish that C_N^* is uniquely determined and not surprisingly, $\frac{\partial C_N^*}{\partial D} > 0$, $\frac{\partial C_N^*}{\partial \theta} > 0$. Notice that even if $D = 0$, a positive coupon is charged, so under Nash bargaining, the firm incurs a fixed cost that is increasing in the lender's bargaining power coefficient θ; inspection of relationship (**??**) reveals that on a total amount $D + \frac{\theta I}{\beta_1 - 1}$, which is increasing in the lender's bargaining power coefficient θ. Since there are no bankruptcy costs, before completion of the project, the firm and the lender split the value of the (net) intrinsic value of the project $\frac{(1-\tau_c)P}{\delta} - I$, which is equal to $F_0(P_0)$. Relationship (13) indicates that the optimal rule is quite simple: each party obtains a share of the pie that is proportional to its bargaining power coefficient.

4.2.2 General Case $\alpha > 0$

Details of the existence of an equilibrium are discussed in appendix 4.2.2 Two decisions have to be taken *simultaneously*; for convenience, we chose to reformulate the program using variable $k = -\beta_2 D/P^*$ that controls the timing of the investment and variable $z = P_D/P^*$ that controls for how long the firm remains in business. We find that both the firm and the lender have profit functions that are hump-shaped in variable k, so both party face a similar trade-off as far as the timing of the investment is concerned, the firm and the lender being eager to speed up investment when u is large and α is large respectively. Next, note that for the choice of a given investment trigger, the larger the coupon charged, the sooner bankruptcy occurs. The firm has incentives to delay bankruptcy as much as possible by choosing a small value of z in particular when the debt ratio u is small, whereas the lender has also some incentive to postpone it, but not too much[13] specially when bankruptcy costs α are small.

When the firm has full bargaining power, $\theta = 0$, the equilibrium outcome coincides with the case when the firm can decide of the coupon and is forced to raise an amount $D/(1-\tau_c)$ to finance

[13]Recall that the default threshold is proportional to the coupon charged.

investment. Nevertheless, as both parties are facing the same corporate tax rate τ_c, the firm cannot internalize any tax benefits and broadly speaking the problem can be understood as issuing debt when the corporate tax rate τ_c is equal to 0. Therefore, the Nash bargaining process can be seen as an extension of the unsecured debt issuance case studied in section 2.6.

When the lender has full bargaining power, $\theta = 1$, the firm participation constraint is binding for small values of the bankruptcy cost α and the debt ratio u as the lender intends to raise the coupon and hasten investment, not worrying too much about speeding up bankruptcy. For large values of u, the coupon is independent of the bankruptcy cost α and proportional to the amount lent D, which corresponds to an interest rate $\overline{r}_N = \frac{C_N^*(1-\tau_c)}{D} = \frac{\beta_1(\beta_2-1)}{(\beta_1-1)\beta_2} r > r$.

4.2.3 Numerical Results

We perform some comparative statics on the after-tax investment trigger $\frac{(1-\tau_c)P_N^*}{\delta}$, the after tax coupon $\frac{(1-\tau_c)C_N^*}{r}$ and the relative gain/loss in the value of waiting ΔF. This percentage change remains independent of the earnings P and the corporate tax rate τ_c and is given by

$$\Delta F = \frac{\frac{(1-\tau_c)P_N^*}{\delta} - \frac{C_N^*}{r} + TB(P_N^*) + TF(P_N^*) - I + D}{\frac{(1-\tau_c)P_0^*}{\delta} - I} \left(\frac{P_0^*}{P_N^*} \right)^{\beta_1} - 1.$$

As before, let $r = 0.04$, $\delta = 0.04$, $I = 100$ and $\sigma = 0.2$, so that $\frac{(1-\tau_c)P_0^*}{\delta} = 200$. Results are reported in table V.

As in the sequential equilibrium case, the lender's bargaining power has a strong impact on the equilibrium outcome. Worth noticing is the fact that when θ is small, the firm is better off under a Nash versus a sequential Stackelberg bargaining process, as for instance, the reduction in option being -15.6% when $\theta = 0$, $\alpha = 1$ and $u = 1$ under the former contract whereas the drop in option value is close to 20.9% for the latter. The opposite takes place since for $\theta = 0.5$, $\alpha = 1$ and $u = 1$, the option value shrinks by 51.16% under a Nash bargaining process instead of -47.30% under a sequential Stackelberg bargaining process. Results reported in table V for $\theta = 1$ confirm this intuition, in particular for small values of bankruptcy costs α and the debt ratio u for which the participation constraint of the firm is binding. The rationale for such a result is that under a Nash bargaining process, two decisions have to be taken independently, which gives some advantage to the party with a strong bargaining ability while making sure that the other party is willing to participate. Finally, observe that when both parties have the same bargaining power, the reduction in the option oscillates around 50%, weakly depending on the debt ratio and bankruptcy costs.

For small values of the debt ratio u, both the investment trigger P_N^* and the coupon C_N^* charged are found to be decreasing in bankruptcy cost α and non-increasing in θ. This result indicates that when u is small, the firm participation constraint is more likely to bind: the best trade-off for a powerful lender is to rush investment while making a concession on the coupon charged. For large values of the debt ratio u, the behavior of the couple (P_N^*, C_N^*) as a function of parameters u and α is similar to the issuance debt case.

$\theta = 0$

	u	0	0.2	0.4	0.6	0.8	1
$\alpha = 0$	$(1-\tau_c)P_F^*/\delta$	200	200	200	200	200	200
	ΔG (%)	0	0	0	0	0	0
	$(1-\tau_c)C_F^*/r$	0	20	40	60	80	100
$\alpha = 0.5$	$(1-\tau_c)P_F^*/\delta$	200	200.8	203.7	209.3	218.3	231.3
	ΔG (%)	0	-0.269	-1.551	-2.747	-5.109	-8.184
	$(1-\tau_c)C_F^*/r$	0	20.80	43.48	68.38	95.75	125.52
$\alpha = 1$	$(1-\tau_c)P_F^*/\delta$	0	201.7	207.9	220.3	240	266.5
	ΔG (%)	0	-0.550	-2.381	-5.651	-10.221	-15.597
	$(1-\tau_c)C_F^*/r$	0	21.10	44.83	71.65	101.44	133.30

$\theta = 0.5$

	u	0	0.2	0.4	0.6	0.8	1
$\alpha = 0$	$(1-\tau_c)P_F^*/\delta$	200	200	200	200	200	200
	ΔG (%)	-50	-50	-50	-50	-50	-50
	$(1-\tau_c)C_F^*/r$	53.6	77.5	103.4	131.7	163.4	199.8
$\alpha = 0.5$	$(1-\tau_c)P_F^*/\delta$	197.9	200.3	205.7	215.7	232.1	255.6
	ΔG (%)	-48.95	-48.80	-48.82	-49.02	-49.43	-50.18
	$(1-\tau_c)C_F^*/r$	51.3	76.2	104.9	138.7	178.2	222.6
$\alpha = 1$	$(1-\tau_c)P_F^*/\delta$	195.9	200.8	213.33	237.5	274.3	319.8
	ΔG (%)	-47.89	-47.44	-47.27	-47.57	-48.83	-51.16
	$(1-\tau_c)C_F^*/r$	49.0	74.8	106.7	146.0	191.5	239.8

$\theta = 1$

	u	0	0.2	0.4	0.6	0.8	1
$\alpha = 0$	$(1-\tau_c)P_F^*/\delta$	200	200	200	200	200	200
	ΔG (%)	-100	-100	-100	-100	-100	-100
	$(1-\tau_c)C_F^*/r$	116.6	146.2	180	220	271.9	378.1
$\alpha = 0.5$	$(1-\tau_c)P_F^*/\delta$	188.3	190.0	195.3	207.0	240.0	300
	ΔG (%)	-100	-100	-100	-100	-95.35	-85.21
	$(1-\tau_c)C_F^*/r$	102.2	133.5	174.1	232.0	320.0	400
$\alpha = 1$	$(1-\tau_c)P_F^*/\delta$	177.8	180	189.6	240	320.0	400
	ΔG (%)	-100	-100	-100	-86.10	-76.56	-75.00
	$(1-\tau_c)C_F^*/r$	88.9	120	165.9	240.0	320.0	400

Table V: Comparative statics for Nash equilibrium

5 Conclusion

This paper explores the impact of debt financing on the timing of an irreversible investment and the value of waiting to invest when the cost of external funds is endogenously determined. We have used a simple extension of the McDonald and Siegel (1986) setting and obtained some equilibrium feedback from the lending sector. We show that the problem can be nested in the all equity setting in which the cost of investing is altered to incorporate tax benefits and the loss incurred by the lender. This effective cost of investing is endogenous and depends on the timing of the investment. Two credit market structures have been considered: Perfect capital markets and funding allocated through a negotiation process (sequential equilibrium or Nash bargaining) between the lender and the borrower. Perfect competition turns out be a special case corresponding to lenders with no bargaining power at all. In this case, we obtain over-investment in the growth option if the lender incurs a loss when defaulting. Intuitively, from the firm's point of view, the up-front investment cost is reduced, the sooner it undertakes the project, the sooner it starts receiving cash flows. Moreover, the loss induced due to bankruptcy is shared among the parties, so "bad news" are less costly for the firm. Conversely, the lender is one who is relatively more concerned with default and intends to postpone investment. In equilibrium, both investment trigger and coupon increase with the lender bargaining power and eventually exceeds the threshold found in the self financing case. Provided that the credit market is fairly competitive (low bargaining power of lenders), consistent with the results by Jensen and Meckling (1976), over-investment can be interpreted as engaging into riskier projects since the all equity counterpart firm would have required better market conditions (higher earnings value) in order to exert its growth option. Alternatively, the model predicts that when lenders have a high bargaining power, firms that are able to self finance their projects should enter earlier in a market than firms forced to rely on external funds.

The risk of debt default induces a higher cost of capital, so independently of the credit market structure the value of growth options are reduced with respect to the self financing case. Numerical simulations reveal that this reduction is very small within the perfect competitive loan market structure but can be quite significant when the lender has a large bargaining power. This may be a reason why firms seem to mainly rely on internal sources to finance investment. Finally, as in the self financing and competitive loan market cases, a higher earnings volatility enhances the value of waiting to invest. However, under bargaining, the relationship uncertainty-option value is not monotonic but instead has an inverted U-shape. As in the all equity case, due to the convexity of the payoffs, more uncertainty increases the option value. In addition, in our setting, a greater earnings volatility raises the effective cost of investing, which reduces the value of waiting. In particular, we find that this latter effect dominates the former when the investment is close to be undertaken.

Our model is very stylized and for instance, we have ignored the effects of bankruptcy costs that can play a significant role by raising the cost of capital and making investment less attractive. Another possible extension to the model would be to assume that some renegotiation process takes place between the lender and the borrower when the coupon payment cannot be met. Clearly, when the resale value of the company exceeds that of the consol bond, both parties have incentives to renegotiate. For instance, as in Sannikov (2007), the lender could grant the borrower a credit line and tolerate losses up a limit before she declares default. This is left for future research.

6 Appendix

6.1 Appendix 1

Preliminary Result 1. If τ is a stopping time, then for all continuous function f, the function F defined as

$$F(P_0) = E_0^Q\left[\int_0^\tau f(P_s)e^{-rs}ds\right],$$

satisfies the following ODE

$$rF(P) = f(P) + (r-\delta)PF'(P) + \frac{\sigma^2}{2}P^2F''(P).$$

Proof. See Karling and Taylor (1981). ■

Then, let $P_0 \geq P_D$ and define $\tau_D = \inf\{t \geq 0,\ P_t = P_D\}$. We want to compute $E_0^Q\left[e^{-r\tau}\right]$. Let us write $1 - r\int_0^\tau e^{-rs}ds = e^{-r\tau}$. Given what precedes, $G(P_0) = E_0^Q\left[\int_0^\tau e^{-rs}ds\right]$ satisfies

$$rG(P) = 1 + (r-\delta)PG'(P) + \frac{\sigma^2}{2}P^2G''(P).$$

The general *bounded* solution to this equation is $G(P_0) = \frac{1}{r} + BP_0^{\beta_2}$. At $P_0 = P_D$, we have $E_0^Q\left[e^{-r\tau}\right] = 1$ so it must be the case that $-BP_D^{\beta_2} = 1$. It follows that

$$E_0^Q[e^{-r\tau}] = \left(\frac{P_0}{P_D}\right)^{\beta_2}. \blacksquare$$

Preliminary Result 2. $\beta_1 > (\beta_1 - \beta_2)\left(\frac{\beta_1}{\beta_1 - 1}\right)^{\beta_2}$. Set $y = \frac{\beta_1 - 1}{\beta_1} < 1$ and $a = -\beta_2 > 0$. It is equivalent to show that for all $y \in (0,1)$, $(1 + a(1-y))y^a < 1$. Define the auxiliary function

$$\begin{array}{llll} & [0,1] & \to & \mathbb{R} \\ \Psi: & y & \mapsto & a\ln y + \ln(1 + a(1-y)). \end{array}$$

Ψ is a smooth function and $\Psi'(y) = \frac{a(1+a)(1-y)}{y(1+a(1-y))}$. Thus, Ψ is strictly increasing on $[0,1]$ with $\Psi(1) = 0$, so Ψ is negative on $(0,1)$. The desired result follows. ■

Preliminary Result 3. $\frac{\partial\beta_1}{\partial\sigma} < 0$ and $\frac{\partial\beta_2}{\partial\sigma} > 0$. Recall that roots β_1 and β_2 are solutions of the following quadratic

$$\frac{\sigma^2}{2}x^2 + (r - \delta - \frac{\sigma^2}{2})x - r = 0, \tag{14}$$

and it is easy to see that $\beta_1 > 1$ and $\beta_2 < 0$. Using the Implicit Function Theorem, totally differentiating relationship (14) w.r.t. σ yields

$$\sigma x(x-1) + (\sigma^2 x + r - \delta - \frac{\sigma^2}{2})\frac{\partial x}{\partial\sigma} = 0.$$

Using relationship (14) and rearranging terms leads to $\left(\frac{\sigma^2}{2}x^2 + r\right)\frac{\partial x}{\partial\sigma} = -\sigma x^2(x-1)$. Since $\beta_1 > 1$ and $\beta_2 < 0$, the desired result follows. ■

6.1.1 Appendix 2: Debt Issuance

6.1.2 Appendix 2.1: Unconstrained Choice

For $P \leq P^*$, the equity value is given by

$$\begin{aligned} F(P) &= \frac{(1-\tau_c)\rho P}{\delta} + \left(\frac{(1-\tau_c)P^*}{\delta} + TB(P^*) - (1-\tau_c)BC(P^*) - I\right)\left(\frac{P}{P^*}\right)^{\beta_1} \\ &= \frac{(1-\tau_c)\rho P}{\delta} + \left(\frac{(1-\tau_c)P^*}{\delta} + \tau_c \frac{\beta_2 - 1}{\beta_2}\frac{(1+\rho)P_D}{\delta} + \frac{\tau_c - \alpha\beta_2}{\beta_2}\frac{(1+\rho)P_D}{\delta}\left(\frac{P^*}{P_D}\right)^{\beta_2} - I\right)\left(\frac{P}{P^*}\right)^{\beta_1}. \end{aligned}$$

Maximizing w.r.t. to C or equivalently P_D leads to $\tau_c - (\tau_c - \alpha\beta_2)\left(\frac{P^*}{P_D}\right)^{\beta_2} = 0$, so

$$P_D = \left(\frac{\tau_c}{\tau_c - \alpha\beta_2}\right)^{-\frac{1}{\beta_2}} P^*.$$

Note that indeed $P_D \leq P^*$. Plugging back into the value of the firm yields

$$F(P) = \frac{(1-\tau_c)\rho P}{\delta} + \left(\frac{1-\tau_c + \tau_c(1+\rho)\left(\frac{\tau_c}{\tau_c - \alpha\beta_2}\right)^{-\frac{1}{\beta_2}}}{\delta} P^* - I\right)\left(\frac{P}{P^*}\right)^{\beta_1}, \tag{15}$$

and the optimal investment trigger P_F^*, the optimal coupon C_F^* and the option value G are given by

$$\begin{aligned} P_F^* &= \frac{\beta_1 I\delta}{\beta_1 - 1}\frac{1}{1-\tau_c + \tau_c(1+\rho)\left(\frac{\tau_c}{\tau_c-\alpha\beta_2}\right)^{-\frac{1}{\beta_2}}} && (16) \\ \frac{C_F^*}{r} &= \frac{\beta_1(\beta_2 - 1)I}{\beta_2(\beta_1 - 1)}\frac{1}{\tau_c + \frac{1-\tau_c}{1+\rho}\left(\frac{\tau_c - \alpha\beta_2}{\tau_c}\right)^{-\frac{1}{\beta_2}}} && (17) \\ G(P) &= \frac{I}{\beta_1 - 1}\left(\frac{P}{P_F^*}\right)^{\beta_1}, \ P \leq P_F^*. && (18) \end{aligned}$$

The effective cost of the project is

$$\begin{aligned} I_{E,F}(P_F^*) &= I - \tau_c(1+\rho)\left(\frac{\tau_c}{\tau_c - \alpha\beta_2}\right)^{-\frac{1}{\beta_2}}\frac{P_F^*}{\delta} \\ &= \frac{(\beta_1 - 1)(1-\tau_c) - \tau_c(1+\rho)\left(\frac{\tau_c}{\tau_c-\alpha\beta_2}\right)^{-\frac{1}{\beta_2}}}{(\beta_1 - 1)\left(1-\tau_c + \tau_c(1+\rho)\left(\frac{\tau_c}{\tau_c-\alpha\beta_2}\right)^{-\frac{1}{\beta_2}}\right)} I < I. \end{aligned}$$

It is easy to check that $\frac{(1-\tau_c)P_F^*}{\delta} \geq \frac{\beta_1 I_{E,F}(P_F^*)}{\beta_1 - 1}$, $\frac{\partial P_F^*}{\partial \alpha} > 0$, $\frac{\partial P_F^*}{\partial \rho} < 0$, $\frac{\partial C_F^*}{\partial \alpha} < 0$ and $\frac{\partial C_F^*}{\partial \rho} > 0$. Define the auxiliary function $\Psi(\tau_c) = 1 - \tau_c + \tau_c(1+\rho)\left(\frac{\tau_c}{\tau_c - \alpha\beta_2}\right)^{-\frac{1}{\beta_2}}$. We have

$$\begin{aligned} \Psi'(\tau_c) &= -1 + (1+\rho)\left(\frac{\tau_c}{\tau_c - \alpha\beta_2}\right)^{-\frac{1}{\beta_2}}\left(1 + \frac{\alpha}{\tau_c - \alpha\beta_2}\right) \\ \Psi''(\tau_c) &= -\frac{\alpha^2(1+\rho)\beta_2}{\tau_c(\tau_c - \alpha\beta_2)^2}\left(\frac{\tau_c}{\tau_c - \alpha\beta_2}\right)^{-\frac{1}{\beta_2}} > 0. \end{aligned}$$

Ψ' is an increasing function from -1 up to $-1+(1+\rho)(1-\alpha\beta_2)^{\frac{1}{\beta_2}-1}(1+\alpha(1-\beta_2)$. If $(1-\alpha\beta_2)^{\frac{1}{\beta_2}-1}(1+\alpha(1-\beta_2) < \frac{1}{1+\rho}$, then Ψ' is always negative, so Ψ is increasing and we can conclude that $\frac{\partial P_F^*}{\partial \tau_c} > 0$. Conversely, if $(1-\alpha\beta_2)^{\frac{1}{\beta_2}-1}(1+\alpha(1-\beta_2) > \frac{1}{1+\rho}$, then Ψ' is negative on some interval $[0,\tau_c^*]$ and positive on $[\tau_c^*,1]$, with $\Psi'(\tau_c^*)=0$. We conclude that on $[0,\tau_c^*]$, $\frac{\partial P_F^*}{\partial \tau_c} > 0$ but on $[\tau_c^*,1]$, $\frac{\partial P_F^*}{\partial \tau_c} < 0$. Next, set $\Phi(\tau_c) = (1+\rho)\tau_c + (1-\tau_c)\left(\frac{\tau_c-\alpha\beta_2}{\tau_c}\right)^{-\frac{1}{\beta_2}}$. We have

$$\Phi'(\tau_c) = 1+\rho - \left(1-\frac{\alpha\beta_2}{\tau_c}\right)^{-\frac{1}{\beta_2}}\left(1+\frac{\alpha(1-\tau_c)}{\tau_c(\tau_c-\alpha\beta_2)}\right).$$

Observe that both $\tau_c \mapsto \left(1-\frac{\alpha\beta_2}{\tau_c}\right)^{-\frac{1}{\beta_2}}$ and $\tau_c \mapsto 1+\frac{\alpha(1-\tau_c)}{\tau_c(\tau_c-\alpha\beta_2)}$ are decreasing functions so Φ' is an increasing function from $-\infty$ up to $1+\rho-(1-\alpha\beta_2)^{-\frac{1}{\beta_2}}$. If $\rho < (1-\alpha\beta_2)^{-\frac{1}{\beta_2}}-1$, then Φ' is always negative, Φ is always decreasing and we can conclude that $\frac{\partial C_F^*}{\partial \tau_c} > 0$. Conversely, if $\rho > (1-\alpha\beta_2)^{-\frac{1}{\beta_2}}-1$, we can conclude that on $[0,\tau_c^*]$, $\frac{\partial C_F^*}{\partial \tau_c} > 0$ and on $[\tau_c^*,1]$, $\frac{\partial C_F^*}{\partial \tau_c} < 0$, with τ_c^* such that $\Phi'(\tau_c^*)=0$. Finally, using relationship (18), for $P \le P_F^*$, we find that $\frac{\partial G(P)}{\partial \alpha} < 0$, $\frac{\partial G(P)}{\partial \rho} > 0$ and $\frac{\partial G(P)}{\partial \tau_c} < 0$. ■

Uncertainty Effects. Set $x = -\alpha\beta_2$, it follows that $\overline{r}_F = \frac{r}{\alpha}\frac{\alpha+x}{x}\frac{\tau_c+x}{\alpha(1-\tau_c)+\tau_c+x}$. We want to show that $\overline{r}_F$ is decreasing in x, or equivalently that

$$\frac{1}{\alpha+x} - \frac{1}{x} + \frac{1}{\tau_c+x} - \frac{1}{\alpha(1-\tau_c)+\tau_c+x},$$

is negative. Gathering terms, after some simplification, the previous quantity is equal to

$$-\frac{\alpha\tau_c(x^2+2x+\alpha(1-\tau_c)+\tau_c}{x(\alpha+x)(\tau_c+x)(\alpha(1-\tau_c)+\tau_c+x)} < 0.$$

Since $\frac{\partial\beta_2}{\partial\sigma} > 0$, it follows that $\frac{\partial\overline{r}_F}{\partial\sigma} > 0$. Then, set $y=-\beta_2$; since $\frac{\beta_1\delta}{\beta_1-1} = \frac{(\beta_2-1)r}{\beta_2}$, it follows that

$$P_F^* = \frac{r}{\alpha}\frac{1+y}{y}\frac{1}{1-\tau_c+\tau_c(1+\rho)\left(1+\frac{\alpha y}{\tau_c}\right)^{-\frac{1}{y}}}.$$

For $y>0$, define the auxiliary function $\Psi(y) = -\frac{1}{y}\ln\left(1+\frac{\alpha}{\tau_c}y\right)$. We have

$$\Psi'(y) = \frac{1}{y^2(1+\frac{\alpha}{\tau_c}y)}\left(\left(1+\frac{\alpha}{\tau_c}y\right)\ln\left(1+\frac{\alpha}{\tau_c}y\right)-\frac{\alpha}{\tau_c}y\right).$$

Since $\Psi'(0)=0$ and $x\mapsto(1+x)\ln(1+x)-x$ is an increasing function, we conclude that Ψ' is non-negative, so Ψ is increasing. Incidentally $\beta_2 \mapsto \left(\frac{\tau_c}{\tau_c-\alpha\beta_2}\right)^{-\frac{1}{\beta_2}}$ is a decreasing function. It follows easily that P_F^* is a decreasing function of y as the product of two decreasing functions in y. By preliminary result 3, we have $\frac{\partial\beta_2}{\partial\sigma} > 0$; given what precedes, it follows that $\frac{\partial P_F^*}{\partial\sigma} > 0$. Finally, using relationship (18), for $P \le P_F^*$, we obtain that

$$\frac{\partial G(P)}{\partial\sigma} = \frac{I}{\beta_1-1}\left(\frac{P}{P_F^*}\right)^{\beta_1}\left(-\frac{1}{\beta_1-1}+\ln\left(\frac{P}{P_F^*}\right)-\beta_1\frac{\partial\ln P_F^*}{\partial\beta_1}\right)\frac{\partial\beta_1}{\partial\sigma}.$$

First, by preliminary result 3, recall that $\frac{\partial\beta_1}{\partial\sigma} < 0$. Next

$$\beta_1\frac{\partial\ln P_F^*}{\partial\beta_1} = -\frac{1}{\beta_1-1} - \beta_1\frac{\partial\ln\left(1-\tau_c+\tau_c(1+\rho)\left(\frac{\tau_c}{\tau_c-\alpha\beta_2}\right)^{-\frac{1}{\beta_2}}\right)}{\partial\beta_2}\frac{\partial\beta_2}{\partial\beta_1}.$$

Recall that $\frac{\beta_1\delta}{\beta_1-1} = \frac{(\beta_2-1)r}{\beta_2}$, so $-\frac{\delta}{(\beta_1-1)^2}\frac{\partial\beta_1}{\partial\beta_2} = \frac{r}{\beta_2^2}$, which implies that $\frac{\partial\beta_2}{\partial\beta_1} < 0$. Then recall that $\beta_2 \mapsto \left(\frac{\tau_c}{\tau_c-\alpha\beta_2}\right)^{-\frac{1}{\beta_2}}$ is decreasing, so $\frac{\partial \ln\left(1-\tau_c+\tau_c(1+\rho)\left(\frac{\tau_c}{\tau_c-\alpha\beta_2}\right)^{-\frac{1}{\beta_2}}\right)}{\partial\beta_2} < 0$. As $P \le P_F^*$, we also have $\ln\left(\frac{P}{P_F^*}\right) \le 0$ and therefore $-\frac{1}{\beta_1-1} + \ln\left(\frac{P}{P_F^*}\right) - \beta_1\frac{\partial \ln P_F^*}{\partial\beta_1} < 0$. It follows that $\frac{\partial G(P)}{\partial\sigma} > 0$. ■

6.1.3 Appendix 2.2: Constrained Choice

In this case, the maximization program becomes

$$\max_{P^*,C} \quad \left(V^F(P^*) - \tfrac{(1-\tau_c)\rho P^*}{\delta} - I + D\right)\left(\tfrac{P}{P^*}\right)^{\beta_1}$$
$$\text{s.t. } D(P^*) = D,\ (1+\rho)P_D = \tfrac{\beta_1-1}{\beta_1}C \text{ and } P_D \le P^*.$$

Set $X = \frac{1-\tau_c}{\delta}P^*$, $Y = \frac{1-\tau_c}{\delta}P_D$ and $z = \frac{Y}{X}$. It follows that $-\frac{\beta_2 D(1-\tau_c)}{X} = (1+\rho)\left[(1-\beta_2)z - (1-\alpha\beta_2)z^{1-\beta_2}\right]$. For $z \in [0,1]$, define

$$\begin{aligned} H_{F_1}(z) &= -\beta_2(1-\tau_c) - (v-\tau_c)(1+\rho)(1-\beta_2)z + (1+\rho)N(v,\alpha,\tau_c)z^{1-\beta_2} \\ H_{F_1}(z) &= (1-\beta_2)z - (1-\alpha\beta_2)z^{1-\beta_2}, \end{aligned}$$

where $N(v,\alpha,\tau_c) = v - \tau_c - (v-1)\alpha\beta_2 > 0$.

H_{F_1} is increasing on $[0,\overline{z}]$ and decreasing on $[\overline{z},1]$ where $\overline{z} = \left(\frac{v-\tau_c}{N(v,\alpha,\tau_c)}\right)^{-\frac{1}{\beta_2}}$ and $H_{F_1}(1) = v\beta_2(v-1) < 0$. This implies that H_{F_1} is non-negative on $[0,z_1]$ with $\overline{z} < z_1 < 1$ and $H_{F_1}(z_1) = 0$. Next, H_{F_2} is increasing from 0 up to $-\beta_2(1-\alpha)$ on $[0,1]$, so it is non-negative on $[0,1]$. Define $H_F(z) = H_{F_1}(z)H_{F_2}^{\beta_1-1}(z)$; the maximization program is equivalent to

$$\max_{z\in[0,z_1]} H_F(z).$$

The objective function H_F is continuous, the maximum is attained at $z = z_F^*$ on the compact interval $[0,z_1]$. As $H_F(0) = H_F(z_1) = 0$, we have an interior solution. The first order condition implies that z_F^* must be solution of the following equation $H_F'(z) = 0$, i.e.,

$$\begin{aligned} &\beta_1(1-\alpha\beta_2)N(v,\alpha,\tau_c)z^{1-2\beta_2} - Q(v,\alpha,\tau_c)z^{1-\beta_2} - \beta_2(\beta_1-1)(1-\alpha\beta_2)\frac{1-\tau_c}{1+\rho}z^{-\beta_2} \\ &+\beta_1(1-\beta_2)(v-\tau_c)z + \beta_2(\beta_1-1)\frac{1-\tau_c}{1+\rho} = 0, \end{aligned}$$

where $Q(v,\alpha,\tau_c) = \beta_1(2-\beta_2)(v-\tau_c) - \alpha\beta_2(\beta_1(2-\beta_2)v - (\beta_1-\beta_2) - (\beta_1+\beta_2-\beta_1\beta_2)\tau_c) > 0$. Finally,

$$\frac{P_F^*}{\delta} = -\frac{\beta_2 D}{(1+\rho)H_{F_2}(z_F^*)} \text{ and } \frac{C_F^*}{r} = -\frac{Dz_F^*}{(\beta_2-1)H_{F_2}(z_F^*)}. \ ■$$

6.2 Appendix 3: Proof of Proposition 1.

6.2.1 Appendix 3.1: Existence and Uniqueness of the Equilibrium.

Set $x_C^* = \frac{P_C^*}{P_D}$. Using the fact that $\frac{(1+\rho)P_D}{\delta} = \frac{\beta_2-1}{\beta_2}\frac{C_C^*}{r}$, manipulating relationships (9) and (10), we find that x_C^* must be solution of the following equation:

$$M(v,\alpha)x^{\beta_2} - \beta_2(\beta_1-1)(1-\gamma)x + \beta_1 v(\beta_2-1) = 0, \tag{19}$$

where $M(v,\alpha)=\beta_1(v-1)(1-\alpha\beta_2)+(\beta_1-\beta_2)>0$, $v=\frac{I}{D}\geq 1$, and for convenience we have set $1-\gamma=\frac{1}{1+\rho}$, so the higher ρ, the higher γ. Next, we want to show that the function

$$\begin{array}{rlcl} & [1,\infty) & \to & \mathbb{R} \\ \Psi_C: & x & \mapsto & M(v,\alpha)x^{\beta_2}-\beta_2(\beta_1-1)(1-\gamma)x+\beta_1 v(\beta_2-1), \end{array}$$

has a unique root $x_C^*>1$. Ψ_C is a continuous differentiable convex function with

$$\begin{aligned} \Psi_C(1) &= \beta_1(v-1)(1-\alpha\beta_2)+\beta_1-\beta_2-\beta_2(\beta_1-1)(1-\gamma)+\beta_1 v(\beta_2-1) \\ &= \beta_1\beta_2(v-1)(1-\alpha)+\gamma\beta_2(\beta_1-1)\leq 0, \end{aligned}$$

and

$$\Psi_C'(x)=\beta_2 M(v,\alpha)x^{\beta_2-1}-\beta_2(\beta_1-1)(1-\gamma).$$

Note that $\Psi_C'(1)=\beta_2(\beta_1(v-1)(1-\alpha\beta_2)+1-\beta_2+\gamma(\beta_1-1))<0$, so we have $\Psi_C'\leq 0$ on $\left[1,\left(\frac{M(v,\alpha)}{(\beta_1-1)(1-\gamma)}\right)^{\frac{1}{1-\beta_2}}\right]$ and $\Psi_C'>0$ on $\left(\left(\frac{M(v,\alpha)}{(\beta_1-1)(1-\gamma)}\right)^{\frac{1}{1-\beta_2}},\infty\right)\Psi_C'$. Since $\Psi_C(1)\leq 0$ and $\lim_{\infty}\Psi_C=\infty$, it follows that Ψ_C has a unique root x_C^* on $(1,\infty)$. Notice that $\Psi_C'(x_C^*)>0$ and for all $x>1$, if $\Psi_C(x)<0$ ($\Psi_C(x)>0$), then $x<x_C^*$ ($x>x_C^*$). The optimal coupon is given by

$$\frac{(1-\tau_c)C_C^*}{r}=\frac{(1-\beta_2)D}{1-\beta_2-(1-\alpha\beta_2)\left(x_C^*\right)^{\beta_2}}, \tag{20}$$

and the optimal investment trigger $P_C^*=x_C^*C_C^*$. Finally note that x_C^* is independent of τ_c. ■

6.2.2 Appendix 3.2: Properties of the Equilibrium.

P1: $P_C^*>C_C^*$. This is equivalent to show that $x_C^*>\frac{\beta_1}{\beta_1-1}$. We actually show that $x_C^*>\frac{\beta_1}{(\beta_1-1)(1-\gamma)}$. It is enough to show that $\Psi_C\left(\frac{\beta_1}{(\beta_1-1)(1-\gamma)}\right)<0$, i.e.

$$M(v,\alpha)\left(\frac{\beta_1}{(\beta_1-1)(1-\gamma)}\right)^{\beta_2}-\beta_2(\beta_1-1)\frac{\beta_1}{\beta_1-1}+\beta_1 v(\beta_2-1)<0.$$

Note that the LHS of the inequality is decreasing in γ, so it is enough to show the result for $\gamma=0$. From preliminary result 2, recall that $\left(\frac{\beta_1}{\beta_1-1}\right)^{\beta_2}<\frac{\beta_1}{\beta_1-\beta_2}$, so it is enough to show that

$$\beta_1(v-1)(1-\alpha\beta_2)+\beta_1-\beta_2+(\beta_1-\beta_2)\left(v(\beta_2-1)-\beta_2\right)<0.$$

Observe that the RHS of the inequality is equal to $\beta_2(v-1)(1-\beta_2+\beta_1(1-\alpha))$, which indeed is non-positive. ■

P2: $\frac{\partial C_C^*}{\partial x_C^*}<0$ and $\frac{\partial P_C^*}{\partial C_C^*}>0$. Using relationships (10) and (20), we obtain that

$$\frac{\partial C_C^*}{\partial x_C^*}=\frac{1}{1-\tau_c}\frac{rD\beta_2(1-\beta_2)(1-\alpha\beta_2)(x_C^*)^{\beta_2-1}}{(1-\beta_2-(1-\alpha\beta_2)(x_C^*)^{\beta_2})^2}<0.$$

Next, observe that $\frac{\partial P_C^*}{\partial C_C^*}=\frac{\partial P_C^*}{\partial P_D}\frac{\partial P_D}{\partial C_C^*}=\frac{\partial P_C^*}{\partial P_D}\frac{\beta_2}{\beta_2-1}$. Then using relationship (9), we find that P_C^* and P_D are linked by the following relationship:

$$\frac{(1-\tau_c)}{\delta}\left((\beta_1-1)(1-\gamma)P_C^*-\frac{\beta_1-\beta_2}{\beta_2}P_D\left(\frac{P_C^*}{P_D}\right)^{\beta_2}\right)=\beta_1\left(I-D-\frac{(1-\tau_c)(\beta_2-1)P_D}{\delta\beta_2}\right). \tag{21}$$

Totally differentiating relationship (21) with respect to P_D leads to

$$\frac{1}{P_D x_C^*}\left((\beta_1-1)(1-\gamma)x_C^* - (\beta_1-\beta_2)(x_C^*)^{\beta_2}\right)\frac{\partial P_C^*}{\partial P_D} = \frac{(\beta_2-1)}{\beta_2}\left(\beta_1 - (\beta_1-\beta_2)(x_C^*)^{\beta_2}\right). \tag{22}$$

Recall that $(\beta_1-1)(1-\gamma)x_C^* > \beta_1$ so $(x_C^*)^{\beta_2} \le \left(\frac{\beta_1}{(\beta_1-1)(1-\gamma)}\right)^{\beta_2} \le \left(\frac{\beta_1}{\beta_1-1}\right)^{\beta_2} < \frac{\beta_1}{\beta_1-\beta_2}$, which implies that the RHS of relationship (22) is non-negative. Finally, observe that $\beta_1 > (\beta_1-\beta_2)(x_C^*)^{\beta_2}$, which in turn implies that the LHS of relationship (22) is non-negative. It follows that $\frac{\partial P_C^*}{\partial P_D} > 0$ and therefore $\frac{\partial P_C^*}{\partial C_C^*} > 0$. ∎

P3: Effect of Parameter α on P_C^* and C_C^*. First of all, totally differentiating relationship (19) with respect to α, we find that

$$-\beta_1\beta_2(v-1)(x_C^*)^{\beta_2} + \Psi_C'(x_C^*)\frac{\partial x_C^*}{\partial \alpha} = 0. \tag{23}$$

Since $\Psi_C'(x_C^*) > 0$, it follows that $\frac{\partial x_C^*}{\partial \alpha} < 0$. Next, in order to show that $\frac{\partial C_C^*}{\partial \alpha} > 0$, it is enough to show that $\alpha \mapsto (1-\alpha\beta_2)(x_C^*)^{\beta_2}$ is increasing, or equivalently $-x_C^* + (1-\alpha\beta_2)\frac{\partial x_C^*}{\partial \alpha} < 0$. Using relationship (23), we find that

$$\frac{\partial x_C^*}{\partial \alpha} = \frac{\beta_1(v-1)(x_C^*)^{\beta_2}}{M(v,\alpha)(x_C^*)^{\beta_2-1} - (\beta_1-1)(1-\gamma)},$$

so $-x_C^* + (1-\alpha\beta_2)\frac{\partial x_C^*}{\partial \alpha}$ has the same sign as

$$M(v,\alpha)(x_C^*)^{\beta_2} - (\beta_1-1)x_C^*(1-\gamma) - (1-\alpha\beta_2)\,\beta_1(v-1)(x_C^*)^{\beta_2},$$

which is equal to

$$(\beta_1-\beta_2)(x_C^*)^{\beta_2} - (\beta_1-1)x_C^*(1-\gamma).$$

Since $(\beta_1-1)(1-\gamma)x_C^* > \beta_1$ so $(x_C^*)^{\beta_2} < \frac{\beta_1}{\beta_1-\beta_2}$, the desired results follows. Finally, notice that P_C^* as a function of C_C^* does not directly depend on α, so $\frac{\partial P_C^*}{\partial \alpha} = \frac{\partial P_C^*}{\partial C_C^*} \times \frac{\partial C_C^*}{\partial \alpha} > 0$. ∎

P4: Effect of Parameter D on P_C^* and C_C^*. Totally differentiating relationship (19) with respect to v, we find that

$$\beta_1(\beta_2-1) + \beta_1(1-\alpha\beta_2)(x_C^*)^{\beta_2} + \Psi_C'(x_C^*)\frac{\partial x_C^*}{\partial v} = 0, \tag{24}$$

so $\Psi_C'(x_C^*)\frac{\partial x_C^*}{\partial v} = \beta_1(1-\beta_2-(1-\alpha\beta_2)(x_C^*)^{\beta_2}) > \beta_1 1 - \beta_2(1-(x_C^*)^{\beta_2}) > 0$. Since $\Psi_C'(x_C^*) > 0$, it follows that $\frac{\partial x_C^*}{\partial v} > 0$. It follows that $\frac{\partial x_C^*}{\partial D} = -\frac{v}{D}\frac{\partial x_C^*}{\partial v} < 0$. Next, differentiating relationship (20) with respect to D leads to

$$\frac{\partial C_C^*}{\partial D} = \frac{r(1-\beta_2)}{1-\beta_2-(1-\alpha\beta_2)(x_C^*)^{\beta_2}} + \frac{\partial C_C^*}{\partial x}\frac{\partial x_C^*}{\partial D}.$$

Since $\frac{\partial C_C^*}{\partial x_C^*} < 0$ and $\frac{\partial x_C^*}{\partial D} < 0$ it follows that $\frac{\partial C_C^*}{\partial D} > 0$. Then, manipulating relationships (9) and (10), we find that

$$\frac{(\beta_1-1)(1-\gamma)(1-\tau_c)P_C^*}{\delta} = \beta_1 I - \frac{(1-\tau_c)P_D}{\delta}(1-\alpha\beta_1)(x_C^*)^{\beta_2}. \tag{25}$$

Observe that $\frac{P_D}{\delta}(x_C^*)^{\beta_2}$ is increasing in D, so $\frac{\partial P_C^*}{\partial D} \ge 0$ ($\frac{\partial P_C^*}{\partial D} \le 0$) iff $\alpha \le 1/\beta_1 1$ ($\alpha \ge 1/\beta_1$). ∎

P5: Over/Under-investment. From relationship (25) it straightforwardly to see that

$$P_C^* \le P_0^* \ (P_C^* \ge P_0^*) \text{ iff } \alpha \le 1/\beta_1 \ (\alpha \ge 1/\beta_1 1),$$

and exactly when $\alpha = 1/\beta_1$, $P_C^* = P_0^*$. ■

P6: Myopic Threshold: $\frac{(1-\tau_c)P_C^*}{\delta} \geq \frac{\beta_1(I-D)}{\beta_1-1}$**.** From relationship (9) and using the fact that $\frac{(1+\rho)P_D}{\delta} = \frac{\beta_2}{\beta_2-1}\frac{C}{r}$, it is easy to see that $\frac{(1-\tau_c)P_C^*}{\delta} \geq \frac{\beta_1(I-D)}{\beta_1-1}$ iff $(\beta_1-\beta_2)(x_C^*)^{\beta_2} \leq \beta_1(1-\beta_2)$, which is satisfied as we have already established in **P4** that $(\beta_1-\beta_2)(x_C^*)^{\beta_2} \leq \beta_1$ and $\beta_1 \leq \beta_1(1-\beta_2)$. ■

P7: Effect of Parameter α **and** D **on the Equity Value.** Recall that for $P \leq P_C^*$, the equity value is given by

$$F(P) = \frac{(1-\tau_c)\rho P}{\delta} + \left((1-\tau_c)\left(\frac{P_C^*}{\delta} - \frac{C_C^*}{r} + \left(\frac{C_C^*}{r} - \frac{(1+\rho)P_D}{\delta}\right)\left(\frac{P_C^*}{P_D}\right)^{\beta_2}\right) - I + D\right)\left(\frac{P}{P_C^*}\right)^{\beta_1}.$$

Since $\frac{\partial F(P)}{\partial P_C^*} = 0$ and using the fact that $\frac{(1+\rho)P_D}{\delta} = \frac{\beta_2-1}{\beta_2}\frac{C_C^*}{r}$, it follows that

$$\frac{\partial F(P)}{\partial \alpha} = \left(-\frac{(1-\tau_c)(\beta_2-1)(1+\rho)}{\delta\beta_2}(1-(x_C^*)^{\beta_2})\frac{\partial P_D}{\partial \alpha}\right)\left(\frac{P}{P_C^*}\right)^{\beta_1} < 0.$$

Next

$$\frac{\partial F(P)}{\partial D} = \left(-\frac{(1-\tau_c)(\beta_2-1)(1+\rho)}{\delta\beta_2}(1-(x_C^*)^{\beta_2})\frac{\partial P_D}{\partial D} + 1\right)\left(\frac{P}{P_C^*}\right)^{\beta_1}.$$

Recall that $-\frac{(1-\tau_c)}{\beta_2}\frac{(1+\rho)P_D}{\delta} = \frac{D}{1-\beta_2-(1-\alpha\beta_2)x^{\beta_2}}$ and $D\frac{\partial x_C^*}{\partial D} = -v\frac{\partial x_C^*}{\partial v}$, so

$$-\frac{(1-\tau_c)(1+\rho)}{\beta_2\delta}\frac{\partial P_D}{\partial D} = \frac{1}{1-\beta_2-(1-\alpha\beta_2)x^{\beta_2}} - \frac{\beta_2 v(1-\alpha\beta_2)(x_C^*)^{\beta_2-1}\frac{\partial x_C^*}{\partial v}}{(1-\beta_2-(1-\alpha\beta_2)x^{\beta_2})^2}.$$

Using relationship (24), we have

$$\begin{aligned}\frac{\beta_2\frac{\partial x_C^*}{\partial v}}{1-\beta_2-(1-\alpha\beta_2)x^{\beta_2}} &= \frac{\beta_1}{(\beta_1(v-1)(1-\alpha\beta_2)+\beta_1-\beta_2))(x_C^*)^{\beta_2-1}-(\beta_1-1)(1-\gamma)} \\ &= \frac{\beta_1 x_C^*}{(\beta_2-1)((\beta_1-1)(1-\gamma)x-\beta_1 v)}.\end{aligned}$$

It follows that

$$\begin{aligned}\frac{\partial F(P)}{\partial D}\left(\frac{P}{P_C^*}\right)^{-\beta_1} &= \frac{1}{1-\beta_2-(1-\alpha\beta_2)(x_C^*)^{\beta_2}}\Bigg(1-\beta_2-(1-\alpha\beta_2)(x_C^*)^{\beta_2}+(\beta_2-1)(1-(x_C^*)^{\beta_2}) \\ &\quad - \frac{\beta_2(\beta_2-1)v(1-\alpha\beta_2)(1-(x_C^*)^{\beta_2})(x_C^*)^{\beta_2-1}\frac{\partial x_C^*}{\partial v}}{1-\beta_2-(1-\alpha\beta_2)(x_C^*)^{\beta_2}}\Bigg),\end{aligned}$$

which has the same sign as

$$-(1-\alpha)\beta_2 - \frac{\beta_2(\beta_2-1)v(1-\alpha\beta_2)(1-(x_C^*)^{\beta_2})(x_C^*)^{-1}\frac{\partial x_C^*}{\partial v}}{1-\beta_2-(1-\alpha\beta_2)(x_C^*)^{\beta_2}},$$

which has the same sign as

$$-(1-\alpha)\beta_2((\beta_1-1)(1-\gamma)x_C^*-\beta_1 v) - \beta_1 v(1-\alpha\beta_2)(1-(x_C^*)^{\beta_2}).$$

Using relationship (24), the previous quantity has is equal to

$$(1-\alpha)\left(\beta_1 v - (\beta_1(v-1)(1-\alpha\beta_2)+\beta_1-\beta_2)(x_C^*)^{\beta_2}\right) - \beta_1 v(1-\alpha\beta_2)(1-(x_C^*)^{\beta_2}),$$

which is equal to

$$-\beta_1 v(1-\beta_2)\alpha + \left[\alpha\beta_1 v(1-\alpha\beta_2) + (1-\alpha)\beta_2(1-\alpha\beta_1)\right](x_C^*)^{\beta_2}.$$

If $\alpha\beta_1 v(1-\alpha\beta_2)+(1-\alpha)\beta_2(1-\alpha\beta_1)$ is negative (α small), then the previous quantity is negative. Conversely, if $\alpha\beta_1 v(1-\alpha\beta_2)+(1-\alpha)\beta_2(1-\alpha\beta_1)$ is positive (α large), since $(x_C^*)^{\beta_2} < 1$, the previous quantity is smaller than

$$-\beta_1 v(1-\beta_2)\alpha + \alpha\beta_1 v(1-\alpha\beta_2) + (1-\alpha)\beta_2(1-\alpha\beta_1),$$

which after re-arranging terms and some simplifications is equal to

$$-(1-\alpha)\left[\beta_1 v(1-\alpha\beta_2) - \beta_2(1-\alpha\beta_1\right] < 0.$$

Indeed we obtain that $\frac{\partial F(P)}{\partial D}$ is always negative. ■

P8: The Role of Assets in Place: Parameter ρ. Totally differentiating relationship (19) with respect to γ, we find that

$$\beta_2(\beta_1-1)x_C^* + \Psi_C'(x_C^*)\frac{\partial x_C^*}{\partial\gamma} = 0,$$

Since $\Psi_C'(x_C^*) > 0$, it follows that $\frac{\partial x_C^*}{\partial\gamma} > 0$ and therefore $\frac{\partial x_C^*}{\partial\rho} > 0$. Observe that relationship (20) is still valid and is independent of parameter ρ, so $\frac{\partial C_C^*}{\partial\rho} = \frac{\partial C_C^*}{\partial x}\times\frac{\partial x_C^*}{\partial\rho} < 0$. Finally, recall that

$$\begin{aligned} P_C^* &= x_C^* P_D \\ &= \frac{-\beta_2 D x_C^*(1-\gamma)}{1-\beta_2-(1-\alpha\beta_2)(x_C^*)^{\beta_2}}. \end{aligned}$$

It follows that

$$-\frac{1}{\beta_2 D}\frac{\partial P_C^*}{\partial\gamma} = \frac{-x_C^*}{1-\beta_2-(1-\alpha\beta_2)(x_C^*)^{\beta_2}} + \frac{(1-\gamma)(1-\beta_2)(1-(1-\alpha\beta_2)(x_C^*)^{\beta_2}}{\left(1-\beta_2-(1-\alpha\beta_2)(x_C^*)^{\beta_2}\right)^2}\frac{\partial x_C^*}{\partial\gamma}.$$

Recall that $\beta_2(\beta_1-1)x_C^* + \Psi_C'(x_C^*)\frac{\partial x_C^*}{\partial\gamma} = 0$; using relationship (19), we obtain that

$$\frac{\partial x_C^*}{\partial\gamma} = \frac{(\beta_1-1)(x_C^*)^2}{(1-\beta_2)\left[(\beta_1-1)(1-\gamma)x_C^* - \beta_1 v)\right]}.$$

It follows that $-\frac{1}{\beta_2 D}\frac{\partial P_C^*}{\partial\gamma}$ has the same sign as

$$-\left[(\beta_1-1)(1-\gamma)x_C^*-\beta_1 v)\right]\left[1-\beta_2-(1-\alpha\beta_2)(x_C^*)^{\beta_2}\right] + (1-\gamma)\beta_1-1)(1-(1-\alpha\beta_2)(x_C^*)^{\beta_2+1},$$

which is equal to

$$\beta_2(\beta_1-1)(1-\gamma)x_C^* + \beta_1 v(1-\beta_2) - \beta_1 v(1-\alpha\beta_2)(x_C^*)^{\beta_2}.$$

Using relationship (19), we find that the previous quantity is equal to $-\beta_2(1-\alpha\beta_1)(x_C^*)^{\beta_2}$. We conclude that $\frac{\partial P_C^*}{\partial\rho} \geq 0$ ($\frac{\partial P_C^*}{\partial\rho} \leq 0$) iff $\alpha \leq 1/\beta_1$ ($\alpha \geq 1/\beta_1$). Finally, for $P \leq P_C^*$, we examine the impact of parameter ρ on the option value of expanding by one unit, $G(P) = F(P) - \frac{(1-\tau_c)\rho P}{\delta}$. Using fact that $\frac{(1+\rho)P_D}{\delta} = \frac{\beta_2-1}{\beta_2}\frac{C_C^*}{r}$ and $\frac{\partial G(P)}{\partial P_C^*} = 0$, we obtain that

$$\frac{\partial G(P)}{\partial\rho} = \left(\frac{P}{P_C^*}\right)^{\beta_1}\frac{\partial}{\partial x_C^*}\left(\frac{(1-\tau_c)C_C^*\left[(x_C^*)^{\beta_2}-(1-\beta_2)\right]}{r(1-\beta_2)}\right)\frac{\partial x_C^*}{\partial\rho}.$$

Using relationship (20) leads to

$$\frac{\partial}{\partial x_C^*}\left(\frac{(1-\tau_c)C_C^*\left[(x_C^*)^{\beta_2}-(1-\beta_2)\right]}{r(1-\beta_2)}\right)=\frac{D\alpha\beta_2^2(1-\beta_2)(x_C^*)^{\beta_2-1}}{(1-\beta_2-(1-\alpha\beta_2)\,(x_C^*)^{\beta_2})^2}>0.$$

Since $\frac{\partial x_C^*}{\partial\rho}>0$, we conclude that $\frac{\partial G(P)}{\partial\rho}>0$. ∎

6.3 Appendix 4

6.3.1 Appendix 4.1. Sequential Equilibrium

The maximization program is

$$\begin{aligned}\max_{(P^*,C)\in S}\quad & \left[F(P_0)\right]^{1-\theta}\left[L(P_0)\right]^{\theta}.\\ & \text{s.t } (FPC) \text{ and } (LPC).\end{aligned}\tag{26}$$

For $\theta\in(0,1)$, $x\mapsto x^\theta$ and $x\mapsto x^{1-\theta}$ both have infinite derivatives at $x=0$, so at the maximum, we must have $F(P_0)>0$ and $L(P_0)>0$. Define $X=\frac{(1-\tau_c)P^*}{\delta}$ and $Y=\frac{(1-\tau_c)P_D}{\delta}$. Then for $P\le P^*$, we have

$$\begin{aligned}F(P)&=\left(\frac{1-\tau_c}{\delta}\right)^{-\beta_1}\left(X-\frac{\beta_2-1}{\beta_2}Y-\frac{Y}{\beta_2}\left(\frac{X}{Y}\right)^{\beta_2}-(v-1)D\right)\left(\frac{P}{X}\right)^{\beta_1}\\ L(P)&=\left(\frac{1-\tau_c}{\delta}\right)^{-\beta_1}\left(\frac{\beta_2-1}{\beta_2}Y+\frac{Y}{\beta_2}\left(\frac{X}{Y}\right)^{\beta_2}-\alpha Y\left(\frac{X}{Y}\right)^{\beta_2}-D\right)\left(\frac{P}{X}\right)^{\beta_1},\end{aligned}$$

and recall that $(P^*,C)\in S$, so we have

$$(\beta_1-1)X-\frac{\beta_1-\beta_2}{\beta_2}Y\left(\frac{X}{Y}\right)^{\beta_2}=\beta_1(v-1)D+\frac{\beta_1(\beta_2-1)}{\beta_2}Y.\tag{27}$$

Then, set $z=\frac{Y}{X}\le 1$. Using relationship (27), we obtain that

$$F(P)=-\frac{1}{\beta_1\beta_2}\left(\frac{1-\tau_c}{\delta}\right)^{-\beta_1}(1-z^{1-\beta_2})H_{S_2}^{\beta_1-1}(z)P^{\beta_1}\tag{28}$$

$$L(P)=-\frac{1}{\beta_1\beta_2(v-1)}\left(\frac{1-\tau_c}{\delta}\right)^{-\beta_1}H_{S_1}(z)H_{S_2}^{\beta_1-1}(z)P^{\beta_1},\tag{29}$$

with

$$\begin{aligned}H_{S_1}(z)&=\beta_2(\beta_1-1)+\beta_1 v(1-\beta_2)z-(\beta_1 v-\beta_2-\alpha\beta_1\beta_2(v-1))z^{1-\beta_2}\\ H_{S_2}(z)&=-\beta_2(\beta_1-1)+\beta_1(\beta_2-1)z+(\beta_1-\beta_2)z^{1-\beta_2}.\end{aligned}$$

Inspection of relationship (28) reveals that the (FPC) constraint is never binding even when $\theta=1$, otherwise The (LPC) constraint would also be binding.

H_{S_1} is strictly concave as $H''_{S_1}(z)=\beta_2(1-\beta_2)(\beta_1 v-\beta_2-\alpha\beta_1\beta_2(v-1))z^{\beta_2-1}<0$ with $H_{S_1}(0)=\beta_2(\beta_1-1)<0$, $H_{S_1}(1)=-\beta_1\beta_2(v-1)(1-\alpha)>0$ and $H'_{S_1}(1)=\beta_2(1-\beta_2)(1+\alpha\beta_1(v-1))<0$. It follows that H_{S_1} is hump-shaped and is non-negative on some interval $[z_1,1]$ with $0<z_1<1$ and $H_{S_1}(z_1)=0$.

H_{S_2} is strictly convex as $H''_{S_2}(z)=\beta_2(1-\beta_2)(\beta_1-\beta_2)z^{\beta_2-1}<0$ with $H_{S_2}(0)=-\beta_2(\beta_1-1)>0$ and $H_{S_2}(1)=0$. This implies that H_{S_2} has a U-shape and is non-negative on some interval $[0,z_2]$

with $0 < z_2 < 1$, $H'_{S_2}(z_2) < 0$ and $H_{S_2}(z_2) = 0$. In fact, H'_{S_2} is negative on $[0, z_2]$. Next, we want to show that $0 < z_1 < z_2 < 1$. It is enough to show that $H_{S_1}(z_2) > 0$. Using the fact that $H_{S_2}(z_2) = 0$, we can write

$$\begin{aligned} H_{S_1}(z_2) &= \beta_1(v-1)z_2\left[1-\beta_2-(1-\alpha\beta_2)z_2^{-\beta_2}\right] \\ &\geq \beta_1(v-1)z_2(1-\beta_2)(1-z_2^{-\beta_2}) > 0. \end{aligned}$$

Finally, the maximization program (26) is equivalent to

$$\max_{z\in[z_1,z_2]} H_S(z,\theta) = H^{\theta}_{S_1}(z)(1-z^{1-\beta_2})^{1-\theta}H^{\beta_1-1}_{S_2}(z).$$

The objective function is smooth and continuous so it attains its maximum z^*_S on the compact interval $[z_1, z_2]$, with $\frac{\partial H_S(z^*_S,\theta)}{\partial z} = 0$ and $\frac{\partial^2 H_S(z^*_S,\theta)}{\partial z^2} \leq 0$. It follows that

$$\begin{aligned} \frac{(1-\tau_c)P^*_S}{\delta} &= -\frac{\beta_1\beta_2(v-1)D}{H_{S_2}(z^*_S)} \\ \frac{(1-\tau_c)C^*_S}{r} &= \frac{\beta_1(1-\beta_2)(v-1)z^*_S D}{H_{S_2}(z^*_S)}. \end{aligned}$$

Notice that both P^*_S and C^*_S are increasing in z^*_S and

$$\frac{\partial^2 H_S(z^*_S,\theta)}{\partial z^2}\frac{\partial z^*_S(\theta)}{\partial\theta} + \frac{\partial^2 H_S(z^*_S,\theta)}{\partial z\partial\theta} = 0,$$

with $\frac{\partial^2 H_S(z^*_S,\theta)}{\partial z\partial\theta} = H_S(z^*_S,\theta)\left(\frac{H'_{S_1}(z^*_S)}{H_{S_1}(z^*_S)} - \frac{(\beta_2-1)(z^*_S)^{-\beta_2}}{1-(z^*_S)^{1-\beta_2}}\right) > 0$. We can conclude that $\frac{\partial z^*_S(\theta)}{\partial\theta} > 0$. Finally, since $X_S = -\frac{\beta_1\beta_2(v-1)D}{H_{S_2}(z^*_S)}$, it follows that $\frac{\partial X_S}{\partial\theta} = \frac{\beta_1\beta_2(v-1)D}{H^2_{S_2}(z^*_S)}H'_{S_2}(z^*_S)\frac{\partial z^*_S(\theta)}{\partial\theta} > 0$, so $\frac{\partial P^*_S}{\partial\theta} > 0$. Similarly, recall that $Y_S = z^*_S X_S$, so we have $\frac{\partial Y_S}{\partial\theta} > 0$, and $\frac{\partial C^*_S}{\partial\theta} > 0$. Finally, using relationship (28), we have

$$\left.\frac{\partial F(P)}{\partial z}\right|_{z=z^*_S} = -\frac{1}{\beta_1\beta_2}\left(\frac{1-\tau_c}{\delta P}\right)^{-\beta_1} H^{\beta_1-2}_{S_2}(z^*_S)\left[(\beta_2-1)(z^*_S)^{-\beta_2}H_{S_2}(z^*_S) + (\beta_1-1)H'_{S_2}(z^*_S)(1-(z^*_S)^{1-\beta_2})\right] < 0.$$

This implies that $\frac{\partial F(P)}{\partial\theta} = \left.\frac{\partial F(P)}{\partial z}\right|_{z=z^*_S} \times \frac{\partial z^*_S(\theta)}{\partial\theta} < 0$. Since $\frac{\partial H_S(z^*_S,\theta)}{\partial z} = 0$, we must have $\frac{\partial L(P)}{\partial\theta} = \left.\frac{\partial L(P)}{\partial z}\right|_{z=z^*_S} \times \frac{\partial z^*_S(\theta)}{\partial\theta} > 0$.

Limit Case $\theta = 0$. Since both $z \mapsto 1 - z^{1-\beta_2}$ and H_{S_2} are decreasing on $[z_1, z_2]$, the maximum is achieved at $z^*_S = z_1$, i.e., $H_{S_1}(z^*_S) = 0$. It is easy to verify that the solution coincides with the perfectly competitive credit market case.

Special Case $\theta = 1$ and $\alpha = 1/\beta_1$. In this case, we have $\beta_1 H_{S_2}(z^*_S) = -\beta_2(\beta_1-1)^2(v-1)$ and

$$\begin{aligned} \frac{(1-\tau_c)P^*_S}{\delta} &= \left(\frac{\beta_1}{\beta_1-1}\right)^2 I \\ \frac{(1-\tau_c)C^*_S}{r} &= \frac{(\beta_2-1)z^*_S}{\beta_2}\left(\frac{\beta_1}{\beta_1-1}\right)^2 I. \blacksquare \end{aligned}$$

6.3.2 Appendix 4.2. Nash Bargaining

Appendix 4.2.1: Special Case $\alpha = 0$. The program is

$$\max_{P^*,C} \left[\frac{(1-\tau_c)P^*}{\delta} - I - (V^L(P^*) - D)\right]^{1-\theta} \left[V^L(P^*) - D\right]^{\theta} (P^*)^{-\beta_1}.$$

The maximization with respect to C yields

$$\frac{-(1-\theta)}{\frac{(1-\tau_c)P^*}{\delta} - I - (V^L(P^*) - D)} + \frac{\theta}{V^L(P^*) - D} = 0.$$

Hence we obtain that $V^L(P^*) - D = \theta\left(\frac{(1-\tau_c)P^*}{\delta} - I\right)$. Plugging back this identity into the initial objective function, the maximization with respect to P^* becomes

$$\max_{P^*} (1-\theta)^{(1-\theta)}\theta^{\theta}\left[\frac{(1-\tau_c)P^*}{\delta} - I\right](P^*)^{-\beta_1},$$

and therefore $P^*_N = P^*_0$. The optimal coupon C^*_N is implicitly defined by

$$\frac{(1-\tau_c)C^*_N}{r} - (1-\tau_c)\left(\frac{C^*_N}{r} - \frac{P_D}{\delta}\right)\left(\frac{P^*_0}{P_D}\right)^{\beta_2} = D + \frac{\theta I}{\beta_1 - 1}.$$

Recall that $\frac{P_D}{\delta} = \frac{\beta_2}{\beta_2 - 1}\frac{C^*_N}{r}$ and set $z^*_N = \frac{P_D}{P^*_0}$; z^*_N satisfies

$$(1-\beta_2)z^*_N - (z^*_N)^{1-\beta_2} = -\beta_2\frac{\beta_1 - 1 + \theta v}{\beta_1 v}.$$

For $z \in [0,1]$, $\Psi : z \mapsto (1-\beta_2)z - z^{1-\beta_2}$ is a continuous and $\Psi'(z) = (1-\beta_2)(1 - z^{-\beta_2}) > 0$. It follows that Ψ is a strictly increasing with $\Psi(0) = 0$ and $\Psi(1) = -\beta_2 > -\beta_2\frac{\beta_1-1+\theta v}{\beta_1 v}$. We conclude that the equation $\Psi(z) = 0$ has a unique root z^*_N in $(0,1)$. Furthermore observe that $\Psi'(z^*_N) > 0$ and $\Psi'(z^*_N)\frac{\partial z^*_N}{\partial \theta} = -\frac{\beta_2}{\beta_1} > 0$. Since $\frac{\partial z^*_N}{\partial \theta} = \frac{\beta_1 - 1}{\beta_1 P^*_0}\frac{\partial C^*_N}{\partial \theta}$, we must have $\frac{\partial C^*_N}{\partial \theta} > 0$. Similarly, $\Psi'(z^*_N)\frac{\partial z^*_N}{\partial v} = \frac{\beta_2(\beta_1 - 1)}{\beta_1 v^2} < 0$. Since $\frac{\partial z^*_N}{\partial v} = \frac{\beta_1 - 1}{\beta_1 P^*_0}\frac{\partial C^*_N}{\partial v}$, we must have $\frac{\partial C^*_N}{\partial v} < 0$ and therefore $\frac{\partial C^*_N}{\partial D} > 0$. Finally, note that for all $P \leq P^*_0$, $F(P) = (1-\theta)F_0(P)$ and $L(P) = \theta F_0(P)$. ■

Appendix 4.2.2: General Case $\alpha > 0$. Set $X = \frac{(1-\tau_c)P^*}{\delta}$, $Y = \frac{(1-\tau_c)P_D}{\delta}$, $k = -\frac{\beta_2 D}{X}$ and $z = \frac{Y}{X}$. Option value and lender's value can be rewritten

$$\begin{aligned} F(P) &= D\left(-\frac{(1-\tau_c)\beta_2 D}{\delta}\right)^{-\beta_1} H_{N_1}(k,z)k^{\beta_1 - 1}P^{\beta_1} \\ L(P) &= D\left(-\frac{(1-\tau_c)\beta_2 D}{\delta}\right)^{-\beta_1} H_{N_2}(k,z)k^{\beta_1 - 1}P^{\beta_1}, \end{aligned}$$

where

$$\begin{aligned} H_{N_1}(k,z) &= -\beta_2 + (\beta_2 - 1)z + z^{1-\beta_2} - (v-1)k \\ H_{N_2}(k,z) &= (1-\beta_2)z - (1-\alpha\beta_2)z^{1-\beta_2} - k. \end{aligned}$$

The FPC and LPC can be written respectively

$$\begin{aligned} -\beta_2 + (\beta_2 - 1)z + z^{1-\beta_2} - (v-1)k &\geq 0 \\ (1-\beta_2)z - (1-\alpha\beta_2)z^{1-\beta_2} - k &\geq 0. \end{aligned}$$

The maximization program is equivalent to

$$\begin{array}{ll} \max\limits_{k\geq 0, z\in[0,1]} & H_N(k,z) = H_{N_1}^{1-\theta}(k,z) H_{N_2}^{\theta}(k,z) k^{\beta_1 - 1} \\ \text{s.t.} & (FPC) \text{ and } (LPC). \end{array}$$

Special Case: $\theta = 1$. In this case, the maximization program is

$$\begin{array}{ll} \max\limits_{k\geq 0, z\in[0,1]} & H_{N_2}(k,z) k^{\beta_1 - 1} \\ \text{s.t.} & (FPC) \text{ and } (LPC). \end{array}$$

The interior solution of the maximization program is $z_N^* = (1-\alpha\beta_2)^{\frac{1}{\beta_2}}$ and $k_N^* = -\frac{\beta_2(\beta_1-1)}{\beta_1} z_N^*$, so that

$$\begin{aligned} \frac{(1-\tau_c)P_N^*}{\delta} &= \frac{\beta_1 D}{\beta_1 - 1}(1-\alpha\beta_2)^{-\frac{1}{\beta_2}} \\ \frac{(1-\tau_c)C_N^*}{r} &= \frac{\beta_1(\beta_2-1)D}{\beta_2(\beta_1-1)}. \end{aligned}$$

This is the solution provided that FPC constraint is satisfied. Set $\Psi(\alpha) = -\beta_2 + (\beta_2 - 1)z_N^* + (z_N^*)^{1-\beta_2} - (v-1)k_N^*$. Ψ is an increasing function with $\Psi(0) = \frac{\beta_2(\beta_1-1)(v-1)}{\beta_1} < 0$. Thus, the solution is interior iff v is small enough (large value of D) and if α is large enough. When the FPC constraint is binding, the option value F is always zero and the maximization program is equivalent to

$$\max_{z\in[0,1]} J(z) = \left[\beta_2 + v(1-\beta_2)z - (v-(v-1)\alpha\beta_2)z^{1-\beta_2}\right]\left[-\beta_2 + (\beta_2-1)z + z^{1-\beta_2}\right]^{\beta_1 - 1},$$

and the optimal investment trigger and coupon are given by

$$\begin{aligned} \frac{(1-\tau_c)P_N^*}{\delta} &= -\frac{\beta_2(I-D)}{-\beta_2 + (\beta_2-1)z_N^* + (z_N^*)^{1-\beta_2}} \\ \frac{(1-\tau_c)C_N^*}{r} &= -\frac{(\beta_2-1)(I-D)z_N^*}{-\beta_2 + (\beta_2-1)z_N^* + (z_N^*)^{1-\beta_2}}, \end{aligned}$$

where z_N^* is such that $J'(z_N^*) = 0$. ■

Special Case: $\theta = 0$. In this case, the maximization program is

$$\begin{array}{ll} \max\limits_{k\geq 0, z\in[0,1]} & H_{N_1}(k,z) k^{\beta_1 - 1} \\ \text{s.t.} & (FPC) \text{ and } (LPC). \end{array}$$

It is easy to see that we cannot have an interior solution and therefore the LPC constraint must be binding, so $V^L(P^*) = D$ or equivalently $(1-\tau_c)D(P^*) = D$, where $D(P)$ denotes the market value of an unsecured debt. This problem is identical to the one solved in section 2.6. after replacing D by $D/(1-\tau_c)$ or equivalently replacing v by $v(1-\tau_c)+\tau_c$ (see appendix 2.2). Observe that in this case taxes play no role as both the $\frac{(1-\tau_c)P_N^*}{\delta}$ and $\frac{(1-\tau_c)C_N^*}{r}$ are independent of the corporate tax rate τ_c. ■

General Case: $\theta \in (0,1)$. Since both $x \mapsto x^\theta$ and $x \mapsto x^{1-\theta}$ have infinite derivatives at $x = 0$, we must have $F(P_0) > 0$ and $L(P_0)$ at the maximum. In particular, this implies that $0 \leq k \leq (1-\beta_2)z - (1-\alpha\beta_2)z^{1-\beta_2} \leq -\beta_2(1-\alpha)(1-\alpha\beta_2)^{\frac{1}{\beta_2}}$ for $z \in [0,1]$. The objective function H_N is continuous so it attains its maximum at (z_N^*, k_N^*) on the compact set $\left[0, -\beta_2(1-\alpha)(1-\alpha\beta_2)^{\frac{1}{\beta_2}}\right] \times [0,1]$. Observe that if $z_N^* \in \{0,1\}$, then $k_N^* = 0$, and either $F(P) = 0$ or $L(P) = 0$, which is impossible. Similarly, if $k_N^* = 0$, $H_N(0, z_N^*) = 0$ and if $k_N^* = -\beta_2(1-\alpha)(1-\alpha\beta_2)^{\frac{1}{\beta_2}}$, we must $z_N^* = (1-\alpha\beta_2)^{\frac{1}{\beta_2}}$ and $L(P) = 0$. Thus we must have an interior solution and the first order conditions are:

$$\begin{aligned}
\frac{\beta_1 - 1}{k_N^*} - \frac{(v-1)(1-\theta)}{-\beta_2 + (\beta_2 - 1)z_N^* + (z_N^*)^{1-\beta_2} - (v-1)k_N^*} - \frac{\theta}{(1-\beta_2)z_N^* - (1-\alpha\beta_2)(z_N^*)^{1-\beta_2} - k_N^*} &= 0 \\
\frac{(1-\theta)(1-(z_N^*)^{-\beta_2})}{-\beta_2 + (\beta_2 - 1)z_N^* + (z_N^*)^{1-\beta_2} - (v-1)k_N^*} - \frac{\theta(1-(1-\alpha\beta_2)(z_N^*)^{-\beta_2})}{(1-\beta_2)z_N^* - (1-\alpha\beta_2)(z_N^*)^{1-\beta_2} - k_N^*} &= 0.
\end{aligned}$$

Thus

$$\begin{aligned}
k_N^* &= \frac{(\beta_1 - 1)(1-(z_N^*)^{-\beta_2})((1-\beta_2)z_N^* - (1-\alpha\beta_2)(z_N^*)^{1-\beta_2})}{(\beta_1 - 1)(1-(z_N^*)^{-\beta_2}) + \theta(v - (v-(v-1)\alpha\beta_2)(z_N^*)^{-\beta_2})} \\
k_N^* &= \frac{(\beta_1 - 1)(1-(1-\alpha\beta_2)(z_N^*)^{-\beta_2})(-\beta_2 + (\beta_2 - 1)z_N^* + (z_N^*)^{1-\beta_2})}{(v-1)(\beta_1 - 1)(1-(1-\alpha\beta_2)(z_N^*)^{-\beta_2}) + (1-\theta)(v-(v-(v-1)\alpha\beta_2)(z_N^*)^{-\beta_2})}.
\end{aligned}$$

Observe that for P_N^* to be non-negative, we must have

$$k^* \leq (1-\beta_2)z_N^* - (1-\alpha\beta_2)(z_N^*)^{1-\beta_2}.$$

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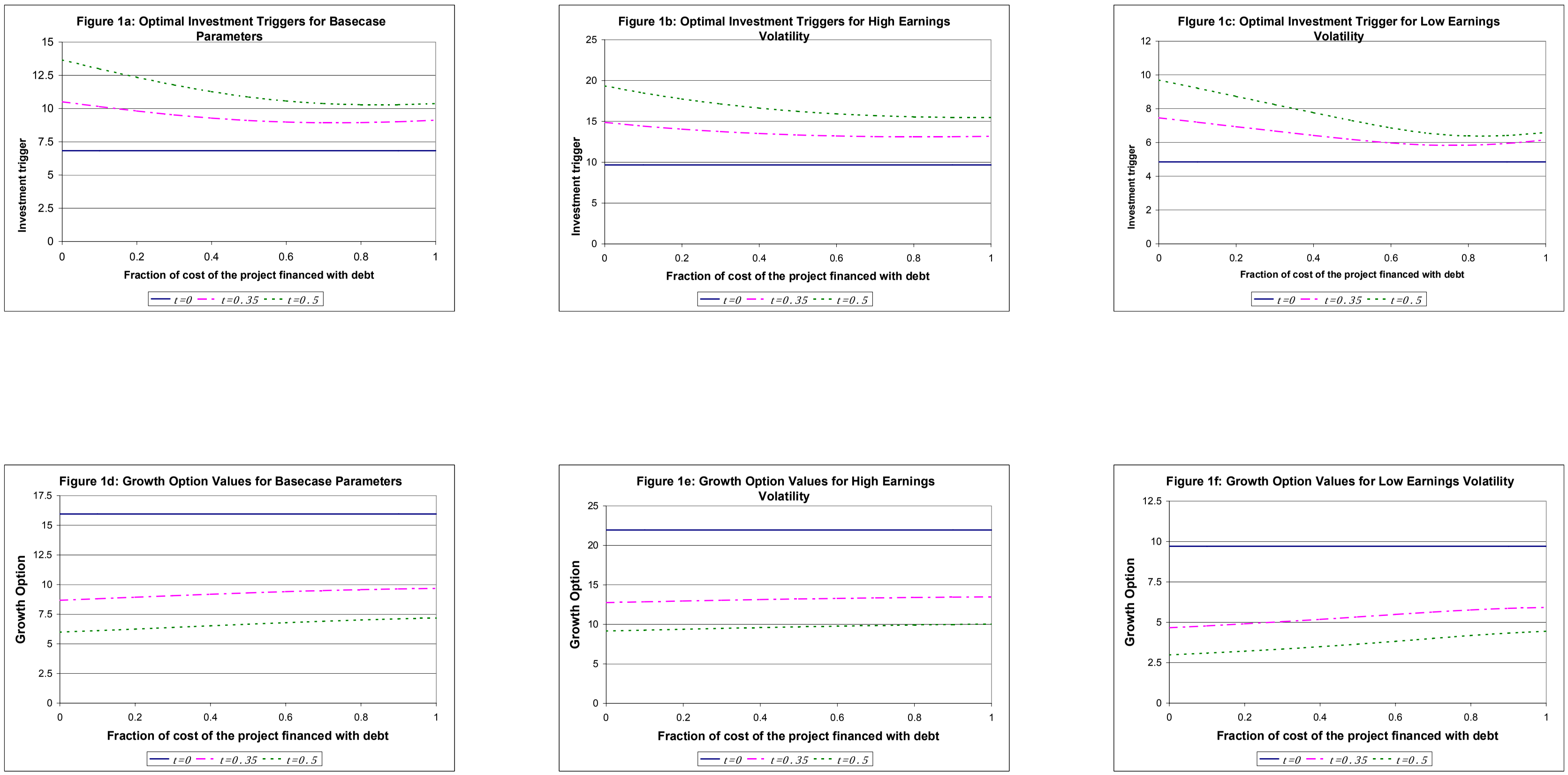

Figure 1: No loss incurred

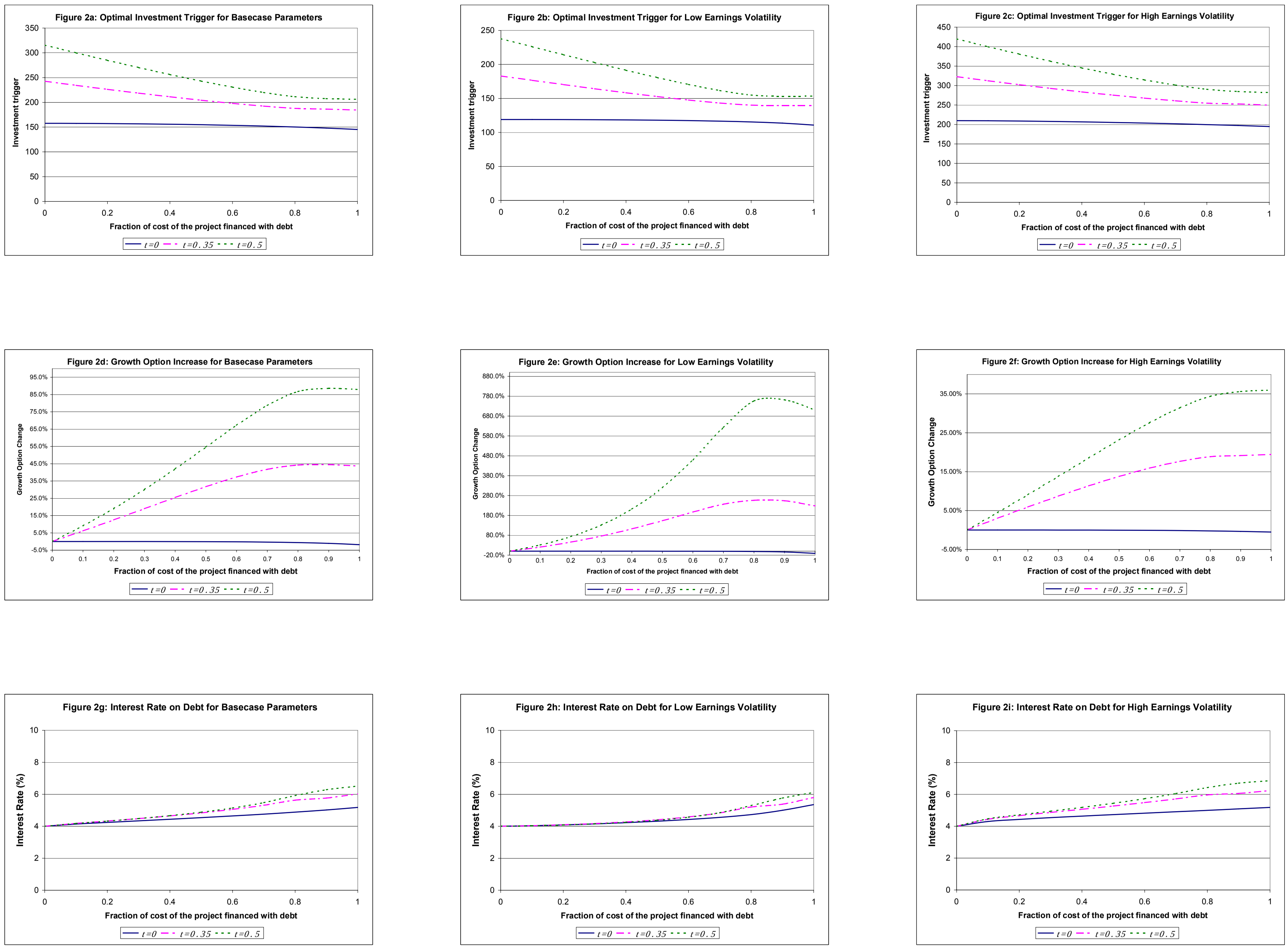

Figure 2: Loss incurred