Modelling Dynamic Redemption and Default Risk for LBO Evaluation: A Boundary Crossing Approach

Alexander Lahmann, Maximilian Schreiter*, Bernhard Schwetzler

HHL Leipzig Graduate School of Management, Germany

Abstract

In this paper, we develop a model that allows evaluating the financial effects of leveraged buyouts (LBOs) from the perspective of the investor. We provide explicit form solutions for all payoffs from acquisition to exit and therefore feature the determination of net present value (NPV) and internal rate of return (IRR). The model is based on a boundary crossing approach where the default of the target firm is represented as a lower piecewise linear barrier. Those default barriers either consist of debt repayment and interest expenses or are contractually-fixed by covenants like debt-to-EBITDA. Our approach features the typical LBO debt repayment schedules: fixed and cash sweep. Furthermore, the model captures all drivers of performance and leverage identified by empirical studies: firm-specific ones like profitability, cash flow growth, volatility, and liquidation value as well as external ones like credit risk spreads and pricing discounts for debt overhang.

JEL classification: C61, C63, G12, G13, G17, G32, G33, G34

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*Corresponding author; email: maximilian.schreiter@hhl.de.

1 Introduction

Leveraged buyouts (LBOs) are a specific type of corporate transactions in which the buyer, often private equity (PE) funds, acquires the target company with a small portion of equity but a large portion of debt for a limited period of time (on average three to five years, see e.g. Kaplan and Strömberg 2008). The initial debt level is reduced stepwise over the holding period either through predefined fixed repayments or depending on the generated cash flows ("cash sweep" repayment). The role of debt in these transactions is discussed highly controversial: critics claim that high leverage exposes target firms to high bankruptcy risks and allows PE investors to reap unjustifiably high tax savings (e.g. Rasmussen February 2009). Proponents point towards lower agency costs due to the discipline imposed by corporate debt (based on Jensen and Meckling 1976) and efficiency gains (see e.g. Berg and Gottschalg 2005) increasing the value of the target firm and allowing to bear a higher sustainable debt level to create tax savings.

This paper develops a model to evaluate the financial effects in LBOs. Based on a boundary crossing approach, the model allows to include default risk and captures the particular feature of dynamic, cash flow dependent debt redemption in LBOs. The model provides an explicit form solution for the value of the entire investment, allows for the determination of the internal rate of return (IRR) and features the distinct evaluation of certain value drivers like the tax shield.

Some peculiarities of debt in LBOs provide challenges when modelling its financial effects: first, the level and the portion of debt change over the holding period. In general, target firms in LBOs carry higher debt levels and pursue a different redemption policy than their industry peers (Axelson et al. 2013). Under "cash sweep" redemption a certain percentage of the realized cash flow after interest and taxes is used to repay debt, a feature considered to be exclusive to LBOs (Jenkinson and Stucke 2011). Thus, future debt levels are turned into path dependent stochastic parameters. Second, default risk is important in the evaluation of LBOs: The public opinion combines the observed higher debt levels with higher default risks for the target firms. While the empirical evidence on this question is mixed [1], there is agreement among researchers that the higher debt levels put more weight on the importance of default risk when valuing financial effects of LBO investments.

Since our model combines particular features of debt in LBOs with a boundary crossing approach introducing potential default, it has to be couched into the literature on the impact of debt policies on corporate value. There is a well established body of literature discussing the impact of different "financing policies", i.e. strategies of redeeming, taking on new debt and adapting the level of debt to changes of the economic conditions reflected by the value of the firm (e.g. Miles and Ezzell (1980), Myers (1974), Cooper and Nyborg (2010)). These financing policies drive the risk properties of future debt levels and by doing so, the risk of the tax savings attached to them. None of the established models reflects completely the debt dynamics in LBOs: on the one hand, the policy of Miles and Ezzell (1980) assumes that firms regularly adjust the level of outstanding debt to changes in the firm value by adapting a state-independent optimal leverage ratio based on market values. On the other hand, state-independent absolute debt levels, as first proposed by Myers (1974), also do not properly reflect the "cash sweep" (path dependent) redemption dynamics of corporate debt often employed in LBOs. Some models capture the debt dynamics described but do not allow for potential default: Arzac (1996) provides two potential solutions, a recursive APV and an European call option approach. He shows that the recursive APV still leads to valuation errors since the tax shield needs to be valued explicitly but the rate of discount is unknown. The option approach overcomes this difficulty but requires another simplifying assumption: the firm cannot default on its debt prior to the end of the holding period.

Other models allow for potential default but are unable to capture the dynamics of debt typically employed in LBOs. The most recent and advanced model is a barrier option approach developed by Couch et al. (2012).

[1] Tykvová and Borell (2012) do not find evidence for bankruptcy rates of PE owned firms being different to their peers. In contrast, Hotchkiss et al. (2014) find a higher bankruptcy probability. Strömberg (2007) finds roughly 6% of the PE target firms in his sample to default; however this study does not cover the effects of the financial crisis.

The model defines the event triggering default as the EBIT hitting a certain lower constant barrier. In an extension, it allows for one time refinancing over the infinite lifetime of the firm. Braun et al. (2011) also use a barrier option approach to introduce potential default in LBOs. In their model, default occurs when the firm value drops below the face value of debt which is described by an exponentially declining function. Both models include potential default but do not allow for the specific redemption policies typically employed in LBOs: First, a fixed and stepwise redemption of debt requires a stepwise adjustment of the default barrier, imposing technical problems due to the non-differentiable nature of the barrier. Second, the "cash sweep" redemption case even necessitates multiple path-dependent adjustments.

Our paper fills the gap described above: (1) the model allows for fixed, stepwise redemption and also captures a dynamic, path dependent "cash sweep" policy. (2) At the same time, it is able to reflect potential default. We use a boundary crossing approach to construct a default condition. The mechanics are equivalent to a down-and-out barrier option with rebate. Default occurs either if a cash obligation consisting of repayment plus interest (fixed redemption) or a cash flow dependent covenant, e.g. allowed interest coverage or debt-to-EBITDA ratio, is hit within the holding period. While the classic barrier option literature (e.g. Merton (1973), Cox and Rubinstein (1985), Kunitomo and Ikeda (1992), Roberts and Shortland (1997), Lo et al. (2003)) deals with boundaries that follow a certain differentiable function, they cannot be used to capture the debt dynamics of LBOs. Hence, we apply the basic idea of Wang and Pötzelberger (1997) of using piecewise linear boundaries. This approach offers the opportunity to model any kind of boundary, also discontinuous ones. Wang and Pötzelberger (2007) extended their early approach to work also for geometric Brownian motions (gBm). Our model equations are in explicit form, but complex default boundaries require numerical integration to solve them (e.g. by Monte Carlo simulation).

Beyond this main contribution, our model meets the requirements for a realistic evaluation of an LBO. Colla et al. (2012) prove that firm-specific drivers such as profitability performance (EBITDA) and cash flow volatility are important determinants for leverage. We reflect these drivers through a stochastic cash flow process following a gBm and allowing for changes in drift and standard deviation. Axelson et al. (2013) identify another feature that should be introduced in an LBO model: the external conditions of debt markets. Particularly, they identify the credit risk premium of leveraged loans as a robust predictor of leverage. We incorporate this feature by the cost of debt and a penalty term for debt overhang at exit. Finally, the performance evaluation of LBOs for PE sponsors is different to most other financial assets. PE investors steer target companies rather by IRR than by net present value (NPV) (see e.g. Kaplan and Schoar 2005). Our LBO valuation formulas allow for inversion, thus enabling us to determine the IRR of the investment.

The remainder of the paper is organized as follows. Section 2 introduces the model, with section 2.1 stating the basic assumptions, section 2.2 illustrating the specific debt structure requirements and section 2.3 deriving payoff and present value components. Section 3 presents the stochastics behind the model resulting in solution formulas for the default probability for specific cases of debt obligations (explicit form solution) and for general ones (integral solution). In Section 4, we use the stochastic attributes derived to develop solution formulas for all NPV and IRR components. Section 5 illustrates the results by providing numerical examples. Section 6 concludes the paper. An extensive appendix is provided to underpin our results.

2 The Model

2.1 Basics of the Model

Let $(\Omega,\ \mathcal{F},\ \mathbb{P})$ be a probability space and $[0, T]$ a time interval, where $T \to \infty$ is possible. We assume that the market is free of arbitrage. For each subjective probability measure $\mathbb{P}$ exists an equivalent measure $\hat{\mathbb{P}}$ called the risk-neutral probability measure. Consider a levered firm whose value in t is given by V_t^L. According to Myers (1974), the value of the levered firm can be determined by adding the present value of the tax savings from interest payments on debt, V_t^{TS}, to the value of an otherwise identical but unlevered firm V_t^U. In every arbitrary period t, the operations of the firm generate an uncertain unlevered free cash flow stream after taxes of X_t. We assume that X_t with an initial value of $X_0 > 0$ follows a geometric Brownian motion (gBm) with constant drift rate μ and constant standard deviation σ according to

$$dX_t = \mu X_0 dt + \sigma X_0 dB_t, \tag{1}$$

with $B_t = \sqrt{t} \cdot Z$ where $Z \sim N(0,1)$ and

$$X_t = X_0 \cdot e^{(\mu - \frac{\sigma^2}{2}) \cdot t + \sigma \cdot B_t}. \tag{2}$$

Others used the gBm for example for modelling the income metric $EBIT$ (see e.g. Hackbarth et al. 2007, Sundaresan and Wang 2007). In our setting it suffices to use this assumption for modelling the unlevered after-tax cash flows.

The corporate tax rate τ_c and the risk-free rate r_f are assumed to be deterministic and constant. The firm's debt is subject to the risk of a possible default. The firm pays interests and redemption on the outstanding total amount of debt, D_t. The credit risk-adjusted cost of debt is denoted by r_D. In the subsequent analysis we pursue a risk-neutral pricing approach.

2.2 The LBO specific debt structure

Developing our model, we start with the debt structure that is imposed by the PE sponsor on the target firm since several other variables are directly linked to this.

Figure 1 shows a development of the LBO firm's debt level typically employed in LBOs throughout the holding period in detail. Prior to the buyout in $t = Pre$ (Pre-LBO) the target firm has a certain total amount of debt outstanding, D^*_{Pre}, that implies a capital structure which can be regarded as optimal for the then prevailing business strategy of the firm. One rationale for the pre-LBO capital structure could be for example the trade-off theory. In $t = 0$, the deal or PE sponsor buys the target firm for a fixed price (later referred to as initial investment) and imposes a new debt structure upon the target by redeeming the pre-LBO debt issue. The newly imposed capital structure in $t = 0$ with an initial amount of debt D_0 implies in most cases an increased debt level. During the holding period the LBO induced debt is stepwise reduced by the target. At the end of the holding period T, the realized total amount of debt is D_T. Equivalent to the pre-LBO phase, there is a certain debt level for the post-LBO phase, D_T^*, reflecting an optimal capital structure (e.g. according to the trade-off theory) for the then prevailing state of the firm. While the PE sponsor might intend to arrive at D_T^* at the end of the holding period, it is uncertain whether this is achieved. Realizing a debt level $D_T > D_T^*$ results in higher tax savings over the holding period but comes with higher default risk and an increased present value of future costs of financial distress at exit. We need to reflect this fact when we derive the payoffs of the model in section 2.3.

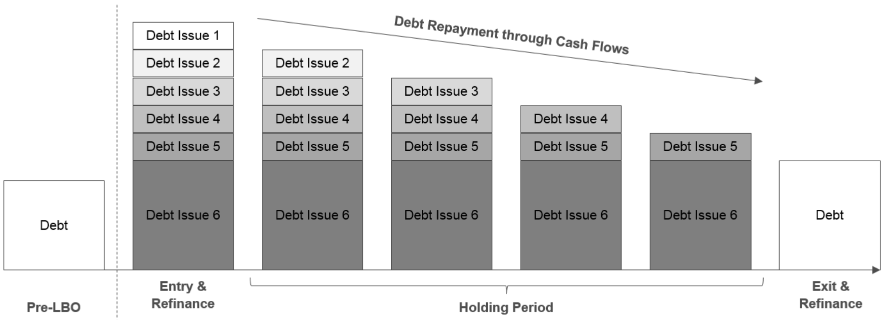

Figure 1: Capital Structure Development in an LBO

We analyze two major redemption cases popular in LBOs: fixed and cash sweep repayment. In the fixed case, there is a predetermined redemption, f_t, in each time point t during the holding period. Hence, the debt levels over the holding period can be determined by

$$
\begin{aligned}
D_{t+1}^{fixed} &= D_t - f_{t+1} \\
D_{t+2}^{fixed} &= D_t - f_{t+1} - f_{t+2} \\
&\dots \\
D_T^{fixed} &= D_t - \sum_{s=t+1}^{T} f_s .
\end{aligned}
\tag{3}
$$

In the cash sweep case, redemption is defined as a proportion a ($a \in [0,1]$) of the firm's realized unlevered after-tax cash flow X_t increased by the tax savings, $r_D \cdot \tau_c \cdot D_{t-1}$, and reduced by interest payments, $r_D \cdot D_{t-1}$.[2] The firm's future debt levels under such a regime are given as follows:

$$
\begin{aligned}
D_{t+1}^{sweep} &= D_t - a \cdot (X_{t+1} - (1-\tau_c) \cdot r_D \cdot D_t) \\
D_{t+2}^{sweep} &= D_t - a \cdot (X_{t+1} - (1-\tau_c) \cdot r_D \cdot D_t) \\
&\quad - a \cdot (X_{t+2} - (1-\tau_c) \cdot r_D \cdot D_{t+1}^{sweep}) \\
&\dots \\
D_T^{sweep} &= D_t \cdot (1 + a \cdot (1-\tau_c) \cdot r_D)^T \\
&\quad - a \cdot \sum_{s=t+1}^{T} X_s \cdot (1 + a \cdot (1-\tau_c) \cdot r_D)^{s-t}
\end{aligned}
\tag{4}
$$

In general, the total debt related cash obligations equal the sum over redemption and after-tax interest payments, here referred to as co_t per period. This definition is congruent for the fixed and the cash sweep case.

$$co_t = r_D \cdot D_{t-1} \cdot (1-\tau_c) + (D_{t-1} - D_t) \tag{5}$$

$$co_t^{fixed} = r_D \cdot D_{t-1} \cdot (1-\tau_c) + f_t \tag{6}$$

$$
\begin{aligned}
co_t^{sweep} &= r_D \cdot D_{t-1} \cdot (1-\tau_c) + (D_{t-1} - a \cdot (X_t - r_D \cdot D_{t-1} \cdot (1-\tau_c))) \\
&= D_{t-1} \cdot (1 + (1+a) \cdot r_D \cdot (1-\tau_c)) - a \cdot X_t
\end{aligned}
\tag{7}
$$

[2]For simplicity, we assume a to be a constant parameter. Note that a time dependent a_t can be easily implemented into the model.

If the firm follows a fixed debt redemption, a default/refinancing event will be triggered if the realized cash flows, X_t, do not cover the cash obligations. Therefore, we define a default boundary, db_t^{fixed}, as follows:

$$db_t^{fixed} = co_t^{fixed} \tag{8}$$

In the cash sweep case, debt contracts usually contain also some minimum requirements, called covenants, for the firm's cash flows. A typical covenant is a certain ratio, b, of debt-to-cash flow or debt-to-EBITDA.[3] We define the default boundary, db_t^{sweep}, in such a case by

$$db_t^{sweep} = \frac{D_{t-1}}{b} \tag{9}$$

Using the equations (8) and (9), yields the following going concern and default/refinancing conditions:

$$\text{Going concern (gc)} : X_t \geq db_t, \text{ for } \forall\ 0 < t \leq T, \tag{10}$$

$$\text{Default (def)} : X_t < db_t, \text{ for } \exists\ 0 < t \leq T. \tag{11}$$

We denote the point in time where a default/refinancing happens as d. Figure 2 illustrates possible scenarios of an LBO. Hitting the default boundary triggers default or refinancing whereas the going concern condition is met as long as the cash flow stays above the default boundary.

2.3 Payoff Structure and Evaluation of an LBO

In the following we examine the evaluation of an LBO in more detail. We regard the typically considered financial decision making principles: the net present value (NPV) approach and in turn the internal rate of return (IRR). As we want to take the perspective of the deal sponsor, we evaluate the LBO purely on an equity basis.

An LBO basically generates three different payoffs that can be identified

[3] For simplicity, we assume b to be a constant parameter. Note that a time dependent b_t can be easily implemented into the model.

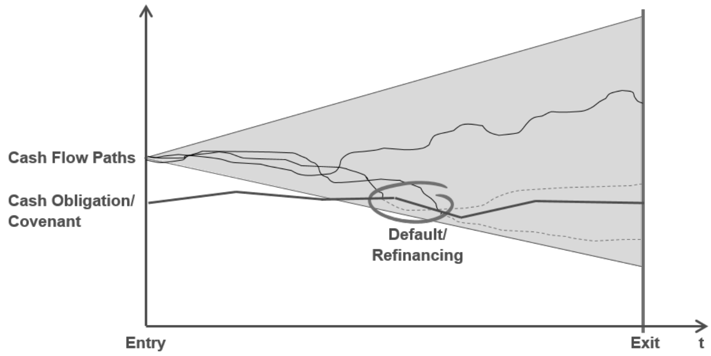

Figure 2: Potential Cash Flow Paths vs. Default Boundary

by the time of their occurrence: the initial investment to purchase the target (I_0), equity cash flows at each time point during the holding period (PO_{HP}), and the exit equity value from selling the target company (PO_{Exit}).

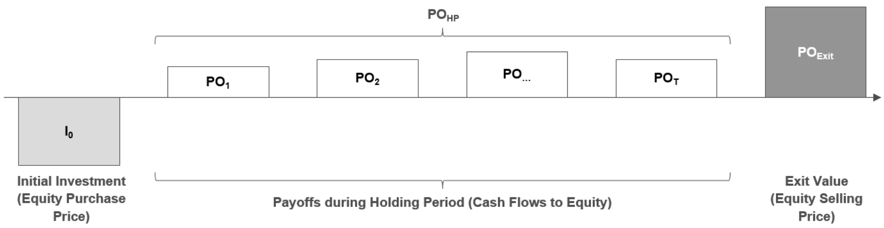

Figure 3: Payoff Structure of an LBO

The initial equity investment I_0 is equal to the enterprise deal value V_0^L minus the entry debt D^*_{Pre}. The enterprise deal value is the sum of the unlevered firm value, V_0^U, and the tax shield value, V_0^{TS}. For simplicity, we define V_0^U as a multiple, m_{Entry}, of the unlevered after-tax cash flow to firm, X_0:

$$
\begin{aligned}
I_0 &= V_0^L - D^*_{Pre} \\
&= V_0^U + V_0^{TS} - D^*_{Pre} \\
&= m_{Entry} \cdot X_0 + V_0^{TS} - D^*_{Pre}.
\end{aligned} \tag{12}
$$

The equity cash flows as payoffs over the holding period depend on whether the target company is a going concern or is in default. As long as the default boundary has not been hit, the equity payoff, PO_t, is determined as the difference between the unlevered after-tax cash flow to firm, X_t, and the cash obligations, co_t, as given in equation (6) for the fixed and in equation (7) for the cash sweep case. After default, in period d, no future unlevered after-tax cash flows are generated. The firm only realizes a liquidation payoff in period d defined as the maximum of zero and the cash flow in the default period, X_d, plus asset value, A_d, minus current debt, D_d, minus current cash obligations, co_d, minus some default or refinancing costs, c_d.

$$PO_t = \begin{cases} PO_t^{gc} = X_t - co_t, & \text{if } X_t \geq db_t \ (0 < t < d) \\ PO_t^{def,+} = (X_t + A_t - D_t - co_t - c_t)^+, & \text{if } X_t < db_t \ (t = d \leq T) \\ PO_t^0 = 0, & \text{if } t > d \end{cases} \tag{13}$$

At exit, there is an equity payoff from selling the target company. We derive the exit equity value based on the following components: the sum over the unlevered value of the firm (V_T^U) and value of the tax shield (V_T^{TS}), reduced by a penalty term for potential debt overhang (V_T^{Pen}) and the realized debt level at exit (D_T). Consistent to the entry valuation, we define $V_T^U = m_{Exit} \cdot X_T$ as a multiple of the realized unlevered after-tax cash flow at exit, and attach V_T^{TS} to the target debt level, D_T^*, which can be regarded as optimal for the target firm after exit depending on the then prevailing state of the firm (e.g. following the trade-off theory). The realized amount of debt at exit can be potentially higher than the target debt level ($D_T > D_T^*$) which translates in higher tax savings over the holding period, an increased default risk, and a higher present value of future costs of financial distress at exit. To reflect the adverse effect of increased costs of financial distress at exit, we include a penalty, $V_T^{Pen} = k \cdot (D_T - D_T^*)^+$. k denotes the penalty cost for each unit of too high debt. Note that under both redemption regimes (fixed and cash sweep) differences between D_T and D_T^* are possible, because D_T^* is path dependent[4].

[4] We assume D_T^* to be dependent on the state of the firm at exit, thus implying an active debt policy (e.g. based on the realized cash flow level). The model also captures the easier case of D_T^* being a deterministic absolute amount of debt. In this case, fixed

The PE sponsor can only plan to hit $D_T = D_T^*$ on expected values. Finally, D_T needs to be subtracted for arriving at an equity payoff. Due to the fact that the penalty payoff ($PO_{Penalty}$) has an additional condition ($D_T > D_T^*$), we separate it from the exit payoff (PO_{Exit}) to facilitate later calculations.

$$PO_{Exit} = \begin{cases} PO_{Exit}^{gc} = m_{Exit} \cdot X_T + V_T^{TS} - D_T, & \text{if } X_t \geq db_t \ (0 < t \leq T) \\ PO_{Exit}^{def} = 0, & \text{if } X_t < db_t \ (0 < t \leq T) \end{cases} \tag{14}$$

$$PO_{Penalty} = \begin{cases} PO_{Penalty,+}^{gc} = k \cdot (D_T - D_T^*), & \text{if } X_t \geq db_t \wedge D_T - D_T^* > 0 \\ PO_{Penalty,-}^{gc} = 0, & \text{if } X_t \geq db_t \wedge D_T - D_T^* \leq 0 \\ PO_{Penalty}^{def} = 0, & \text{if } X_t < db_t \ (0 < t \leq T) \end{cases} \tag{15}$$

PE funds identify worthwhile investment projects and measure their performance based on their IRR. This in turn requires calculating the NPV as discounted value of all payoffs from the investment over the holding period until exit.

$$NPV = -I_0 + PV_{HP} + PV_{Exit} - PV_{Penalty}, \tag{16}$$

where $PV_(Exit) - PV_{Penalty}$ denotes the price of the firm's equity at exit, PV_{HP} the present value of all payoffs during the holding period and I_0 the initial investment. The IRR is then a function g of the aforementioned variables by setting $NPV = 0$.

$$IRR = g(NPV = 0,\ I_0,\ PV_{HP},\ PV_{Exit}, PV_{Penalty}) \tag{17}$$

Following a risk-neutral pricing approach with continuously changing cash flows, we use $e^{-r \cdot t}$ for discounting the payoffs. The distinction between going concern and default is captured with an indicator function, $\mathbb{I}_{condition}$, that by definition is equal to one if the specified condition is satisfied and zero if it is not.

With these notations at hand, we can derive the components of the NPV:

$$I_0 = m_{Entry} \cdot X_0 + V_0^{TS} - D_0 \tag{18}$$

debt redemption should always lead to $D_T = D_T^*$.

$$
\begin{aligned}
PV_{HP} &= \sum_{t=1}^{T} e^{-r\cdot t} \cdot \mathbb{E}\left(PO_t^{gc} \cdot \mathbb{I}_{\{X_t \geq db_t, 0<t\leq d\}}\right) \\
&+ e^{-r\cdot d} \cdot \mathbb{E}\left(PO_d^{def,+} \cdot \mathbb{I}_{\{X_d < db_d, 0<d\leq T\}}\right) \\
&= \sum_{t=1}^{T} e^{-r\cdot t} \cdot \mathbb{E}\left(PO_t^{gc} \cdot \mathbb{I}_{\{X_t \geq db_t, 0<t\leq d\}}\right) \\
&+ e^{-r\cdot d} \cdot \mathbb{E}\left(PO_d^{def} \cdot \mathbb{I}_{\{D_d + co_d + c_d - A_d \leq X_d < db_d, 0<d\leq T\}}\right) && (19) \\
PV_{Exit} &= e^{-r\cdot T} \cdot \mathbb{E}\left(PO_{Exit}^{gc} \cdot \mathbb{I}_{\{X_t \geq db_t, 0<t\leq T\}}\right) && (20) \\
PV_{Penalty} &= e^{-r\cdot T} \cdot \mathbb{E}\left(PO_{Penalty,+}^{gc} \cdot \mathbb{I}_{\{X_t \geq db_t, 0<t\leq T\}}\right) \\
&= e^{-r\cdot T} \cdot \mathbb{E}\left(PO_{Penalty}^{gc} \cdot \mathbb{I}_{\{X_t \geq db_t, X_T < \frac{D_T^*}{l}, 0<t\leq T\}}\right) && (21)
\end{aligned}
$$

In the next section, we develop an approach to transform the indicator functions in explicit form solutions allowing to evaluate the financial effects of an LBO by simple numerical integration.

3 Derivation of Useful Stochastic Properties

In our model a default/refinancing is triggered by the unlevered after-tax cash flow, X_t, hitting the default barrier, db_t. For both redemption cases examined, such a structure is equivalent to a down-and-out barrier option where the default barrier is the lower boundary.

As our model captures dynamic redemption schedules, it needs to allow for stepwise changing and/or path dependent boundaries. Thus, the Black Scholes Merton framework requiring constant or exponential boundaries cannot be used to derive explicit analytic formulae. Roberts and Shortland (1997) and Lo et al. (2003) find valuable approximation approaches for any kind of boundary that can be expressed as a continuous and differentiable function throughout the examined interval. The redemption cases analyzed here need to allow for discontinuous boundaries (see figure 2). Therefore, we follow the idea of Wang and Pötzelberger (1997) to apply piecewise linear boundaries. The equations under this approach are in explicit form and can be solved by the repeated application of numerical integration (e.g. through Monte Carlo simulation).

We proceed in three steps: first, we present an explicit analytic solution for the default probability of a standard (arithmetic) Brownian motion with drift versus a constant default barrier. Second, we replace the standard Brownian motion by the geometric one described in equation (2). This solution will still be in explicit analytic form. Finally, we use the results of Wang and Pötzelberger (1997) to arrive at an equation in explicit integral form for any kind of piecewise linear default barriers.

3.1 Standard Brownian Motion versus Constant Default Barrier

We start from a Brownian motion without drift, B_t, and adjust it to one with drift, $\hat{B}_t$:

$$\hat{B}_t = \alpha \cdot t + B_t \tag{22}$$

The minimum $\hat{M}_t$ of such a process under the prerequisites $\hat{M}_t \leq 0$ and $\hat{B}_t \geq \hat{M}_t$ is defined by:

$$\hat{M}_t = \min_{0 \leq t \leq T} \hat{B}_t \tag{23}$$

Hence, $\hat{M}_t$ and $\hat{B}_t$ take values in the set $\{(m, b); w \geq b, m \leq 0\}$. This allows to derive the joint density function of $\hat{M}_t$ and $\hat{B}_t$ under the real world probability measure $\mathbb{P}$ (a detailed derivation can be found in appendix 7.1):

$$f_{\hat{M}_t, \hat{B}_t}(m, b) = \frac{2 \cdot (b - 2 \cdot m)}{t \cdot \sqrt{2 \cdot \pi \cdot t}} \cdot e^{\alpha \cdot b - \frac{1}{2} \cdot \alpha^2 \cdot t - \frac{(2 \cdot m - b)^2}{2 \cdot t}} \tag{24}$$

On the basis of this density function, we are able to derive $\mathbb{P}\left\{\hat{M}_t \geq m\right\}$ which is the probability that the lower boundary, m, is not crossed during the holding period:

$$\begin{aligned}
\mathbb{P}\left\{\hat{M}_t \geq m\right\} &= \frac{1}{\sqrt{2 \cdot \pi \cdot t}} \cdot \int_m^\infty e^{-\frac{1}{2 \cdot t} \cdot (b - \alpha \cdot t)^2} db \\
&\quad - \frac{1}{\sqrt{2 \cdot \pi \cdot t}} \cdot e^{2 \cdot \alpha \cdot m} \cdot \int_m^\infty e^{-\frac{1}{2 \cdot t} \cdot (b - 2 \cdot m - \alpha \cdot t)^2} db \qquad (25) \\
&= N\left(\frac{\alpha \cdot t - m}{\sqrt{t}}\right) + e^{2 \cdot \alpha \cdot m} \cdot N\left(\frac{\alpha \cdot t + m}{\sqrt{t}}\right) \qquad (26)
\end{aligned}$$

The complementary probability is the default probability:

$$\mathbb{P}\left\{\hat{M}_t < m\right\} = \frac{1}{\sqrt{2 \cdot \pi \cdot t}} \cdot \int\limits_{-\infty}^{m} e^{-\frac{1}{2 \cdot t} \cdot (b - \alpha \cdot t)^2} db$$

$$- \frac{1}{\sqrt{2 \cdot \pi \cdot t}} \cdot e^{2 \cdot \alpha \cdot m} \cdot \int\limits_{-\infty}^{m} e^{-\frac{1}{2 \cdot t} \cdot (b - 2 \cdot m - \alpha \cdot t)^2} db \tag{27}$$

$$= N\left(\frac{m - \alpha \cdot t}{\sqrt{t}}\right) + e^{2 \cdot \alpha \cdot m} \cdot N\left(\frac{m + \alpha \cdot t}{\sqrt{t}}\right) \tag{28}$$

3.2 Geometric Brownian Motion versus Constant Default Barrier

Replacing the standard Brownian motion with drift α by our cash flow process, X_t, following a gBm yields:

$$\mathbb{P}\left\{X_0 \cdot e^{\left(r - \frac{\sigma^2}{2}\right) \cdot t + \sigma \cdot M_t} < db\right\} \tag{29}$$

$$= \mathbb{P}\left\{\frac{1}{\sigma} \cdot \left(r - \frac{\sigma^2}{2}\right) \cdot t + M_t < ln\left(\frac{db}{X_0}\right) \cdot \frac{1}{\sigma}\right\} \tag{30}$$

Transforming equation (29) into (30) reveals a structure equivalent to the one from equation (22). The term $\frac{1}{\sigma}(r - \frac{\sigma^2}{2})$ in equation (30) is equivalent to α in equation (22). Also, the lower boundary m from equations (24) to (27) has been adjusted to $ln(\frac{db}{X_0}) \cdot \frac{1}{\sigma}$ for the gBm process used in our model:

$$\mathbb{P}\left\{\alpha \cdot t + M_t = \hat{M}_t < m\right\} \tag{31}$$

$$with:$$

$$\alpha = \frac{1}{\sigma} \cdot \left(r - \frac{\sigma^2}{2}\right) \tag{32}$$

$$m = \frac{1}{\sigma} \cdot ln\left(\frac{db}{X_0}\right) \tag{33}$$

To conclude, pasting α and m from equations (32) and (33) into equations (24) and (27) yields formulas for the joint density function of $\hat{M}_t$ and $\hat{B}_t$ under the real world probability measure $\mathbb{P}$ and for the default probability, if the process follows a gBm.

3.3 Geometric Brownian Motion versus Piecewise Linear Default Barriers

In this section we generalize equations (24) and (27) for a default boundary that is a polygonal function over the holding period. We extend the approach of Wang and Pötzelberger (1997) for standard Wiener Processes without drift towards a gBm with drift.

For providing a general solution, we proceed on the assumption that the holding period ($0 \leq t \leq T$) can be divided in n-intervals ($0 = t_0 < t_1 < ... < t_n = T$) and set the lower boundary, m_t, constant on each of the intervals $[t_{j-1}, t_j]$, $j = 1, 2, ..., n$ and $m_0 < 0$. For our specific problem of LBO valuation, it is important to note that $t_0 = 0, t_1 = 1, ..., t_n = T$, and t is the parameter describing the points in time within the holding period.

The probability that the modified Wiener Process $\hat{B}_t$ does not cross m_t on the interval $[0, T]$ can be split into n conditional events that $\hat{B}_t$ does not cross m_t on the interval $[t_j, t_{j+1}]$ given that $\hat{B}(t)$ has not crossed $m(t)$ on the interval $[t_{j-1}, t_j]$. For each of these intervals, the conditional probability can be calculated by equation (25). For connecting the intervals, we restate equation (25) in a form with only one integral:

$$
\begin{aligned}
\mathbb{P}\left\{\hat{M}_t \geq m\right\} &= \frac{1}{\sqrt{2\cdot\pi\cdot t}} \cdot \int_m^\infty e^{-\frac{1}{2\cdot t}\cdot(m-\alpha\cdot t)^2} db \\
&- \frac{1}{\sqrt{2\cdot\pi\cdot t}} \cdot e^{2\cdot\alpha\cdot m} \cdot \int_m^\infty e^{-\frac{1}{2\cdot t}\cdot(m-2\cdot m-\alpha\cdot t)^2} db \\
&= \int_{m-\alpha\cdot t}^\infty \left(1 - e^{-\frac{2\cdot m\cdot(m-\alpha\cdot t-x)}{T}}\right) \cdot \frac{1}{\sqrt{2\cdot\pi\cdot t}} \cdot e^{-\frac{x^2}{2\cdot t}} dx \\
&= \int_{m-\alpha\cdot t}^\infty \left(1 - e^{-\frac{2\cdot m\cdot(m-\alpha\cdot t-x)}{T}}\right) \cdot f(x) dx \qquad (34) \\
& with: \\
& f(x) = \frac{1}{\sqrt{2\cdot\pi\cdot t}} \cdot e^{-\frac{x^2}{2\cdot t}} \qquad (35)
\end{aligned}
$$

Next, we apply and adjust Theorem 1 from Wang and Pötzelberger (1997) to derive the crossing probability for a piecewise linear boundary m_t and a Brownian

Motion with drift α. For easier expression, we define $t_j - t_{j-1} = \Delta t_j$.

$$\mathbb{P}\left\{\hat{M}_t < m_t, t \leq T\right\} = 1 - \mathbb{E}\left\{g\left(B_{t_1}, ..., B_{t_n}, m_{t_1}, ..., m_{t_n}\right)\right\}, \tag{36}$$

$$with:$$

$$g\left(x_1, ..., x_n, m_1, ..., m_n\right)$$

$$= \prod_{j=1}^{n} \mathbb{I}_{(x_j + \alpha \cdot \Delta t_j \geq m_j)} \cdot \left(1 - e^{-\frac{2 \cdot \left(m_{j-1} - \alpha \cdot \Delta t_{j-1} - x_{j-1}\right) \cdot \left(m_j - \alpha \cdot \Delta t_j - x_j\right)}{\Delta t_j}}\right) \tag{37}$$

By applying equation (34) on all time steps, we can transform equation (36) into an integral function of the form:

$$\mathbb{P}\left\{\hat{M}_t < m_t, t \leq T\right\}$$

$$= 1 - \int_{m-\alpha \cdot t}^{\infty} \left[\prod_{j=1}^{n} \left(1 - e^{-\frac{2 \cdot \left(m_{j-1} - \alpha \cdot \Delta t_{j-1} - x_{j-1}\right) \cdot \left(m_j - \alpha \cdot \Delta t_j - x_j\right)}{\Delta t_j}}\right) \cdot \prod_{j=1}^{n} \frac{1}{\sqrt{2 \cdot \pi \cdot \Delta t_j}} \cdot e^{-\frac{\left(x_j - x_{j-1}\right)^2}{2 \cdot \Delta t_j}}\right] dx$$

$$= 1 - \int_{m-\alpha \cdot t}^{\infty} \left[h(m, x) \cdot k(x)\right] dx \tag{38}$$

$$with:$$

$$h(m, x) = \prod_{j=1}^{n} \left(1 - e^{-\frac{2 \cdot \left(m_{j-1} - \alpha \cdot \Delta t_{j-1} - x_{j-1}\right) \cdot \left(m_j - \alpha \cdot \Delta t_j - x_j\right)}{\Delta t_j}}\right) \tag{39}$$

$$k(x) = \prod_{j=1}^{n} \frac{1}{\sqrt{2 \cdot \pi \cdot \Delta t_j}} \cdot e^{-\frac{\left(x_j - x_{j-1}\right)^2}{2 \cdot \Delta t_j}} \tag{40}$$

Plugging in the equations for α and m from (32) and (33), allows to arrive at an explicit formula for the default probability reflecting a gBM versus piecewise linear default barriers, thus, reflecting the dynamics of the redemption policies of LBO investments.

For $n = 1$, equation (36) can be solved analytically, otherwise numerical integration is required, e.g. via Monte Carlo simulation in MATLAB or MATHE-

MATICA. We provide an application example in chapter 5.

4 Explicit Form Solution

With equation (36) from the previous section we can solve the expected values of the NPV components (equations (18) to (21)) for both redemption cases analyzed (cash sweep or fixed). For cash sweep redemption, our model yields stochastic default boundaries. The structure of the nested integrals in our model allows to find solutions for this problem.

In general, we use the common relationship for continuous random variables,

$$\mathbb{E}(X) = \int_{-\infty}^{\infty} X \cdot f(x)dx \tag{41}$$

where $f(x)$ is the density function of the random variable X. The indicator functions in PV_{HP}, PV_{Exit} and $PV_{Penalty}$ change the regions of the integrals. In the cash sweep case, the stochastic default boundary complicates the numerical integration and requires a further adjustment of the integral regions as presented in this section.

We illustrate the necessary transformations of equations (18) to (21) for the example of PV_{Exit}. We start with equation (20) and transform it by using equation (41) to

$$PV_{Exit} = e^{-r \cdot T} \cdot \mathbb{E}\left(PO^{gc}_{Exit} \cdot \mathbb{I}_{\{X_t \geq db_t, 0<t\leq T\}}\right) \tag{42}$$

$$= e^{-r \cdot T} \cdot \int\limits_{-\infty}^{\infty} PO^{gc}_{Exit} \cdot \mathbb{I}_{\{X_t \geq db_t, 0<t\leq T\}} \cdot h(db, x) \cdot k(x)dx \tag{43}$$

In preparation for the adjustment of the integral regions, we solve the indicator function for the random variable x:

$$\begin{aligned} \mathbb{I}_{\{X_t \geq db_t, 0<t\leq T\}} &= \mathbb{I}_{\{X_0 \cdot e^{\alpha \cdot \sigma \cdot t + \sigma \cdot x} \geq db_t, 0<t\leq T\}} \\ &= \mathbb{I}_{\left\{x \geq \frac{1}{\sigma} \cdot \ln\left(\frac{db_t}{X_0}\right) - \alpha \cdot t, 0<t\leq T\right\}} \end{aligned} \tag{44}$$

To facilitate our notation, we define an adjusted default boundary, $\overline{db_t}$:

$$\overline{db_t} = \frac{1}{\sigma} \cdot \ln\left(\frac{db_t}{X_0}\right) - \alpha \cdot t \tag{45}$$

Finally, we cancel out the indicator function from equation (43) by adjusting the lower bound of the integrals according to equation (44):

$$PV_{Exit} = e^{-r\cdot T} \cdot \int_{\overline{db_t}}^{\infty} PO_{Exit}^{gc} \cdot h(db, x) \cdot k(x)dx. \tag{46}$$

Equation (46) is valid for both types of debt redemption. In the cash sweep case $\overline{db_t}$ is stochastic which might inflate the numerical integration. To facilitate the calculation, we take a closer look at $\overline{db_t}$. The expression contains a natural logarithm which is not defined for values smaller or equal to zero. We note that

$$\frac{db_t^{sweep}}{X_0} = \frac{D_{t-1}}{b \cdot X_0} > 0. \tag{47}$$

By developing this non-negative condition for the first periods, we derive a general rule for our random variables x_t. As shown in appendix 7.2 the upper boundaries to the integrals with a lag of one time period are

$$\begin{aligned}\overline{ub_{t-1}} =& \frac{1}{\sigma} \cdot \ln \left(\frac{D_0 \cdot (1 + a \cdot r_D \cdot (1 - \tau_c))^{t-1}}{a \cdot X_0 \cdot e^{\sum_{s=1}^{t-2} (\mu - \frac{\sigma^2}{2}) \cdot s}} \right. \\ & \left. - \sum_{s=1}^{t-2} e^{\sigma \cdot x_s} \cdot (1 + a \cdot r_D \cdot (1 - \tau_c))^s \right) - \alpha_{t-1} \cdot ((t-1) - (t-2)). \end{aligned} \tag{48}$$

To conclude, for cash sweep debt repayment we can adjust the upper boundaries of the integral regions from $+\infty$ to $\overline{ub_{t-1}}$.

For the next term in our analysis, $PV_{Penalty}$, we perform the same transformations as for PV_{Exit} but have to note that one additional adjustment has to be considered with respect to the indicator condition of the exit period: $X_T < \frac{D_T^*}{l}$. Hence, the integral for the exit period comprises an upper boundary in addition to the lower one. The present value $PV_{Penalty}$ is determined via

$$\begin{aligned} PV_{Penalty} &= e^{-r\cdot T} \cdot PO_{Penalty}^{gc} \cdot \mathbb{I}_{\{X_t \geq db_t, X_T < \frac{D_T^*}{l}, 0 < t \leq T\}} & (49) \\ &= e^{-r\cdot T} \cdot \underbrace{\int_{\overline{db_t}}^{\infty}}_{\text{for } t<T} \int_{\overline{db_T}}^{\frac{D_T^*}{l}} PO_{Penalty}^{gc} \cdot h(db, x) \cdot dx. & (50) \end{aligned}$$

Again, we adjust the upper boundaries under cash sweep debt repayment from $+\infty$ to $\overline{ub_{t-1}}$ for all periods prior to the exit period.

For the remaining term, PV_{HP}, we perform the same transformations as before and reflect the default case within each time period. The default probability for each period is derived as the difference between the going concern probability up to the previous period and the going concern probability up to the current period. Multiplying this default probability with the default payoff, PO_d^{def}, yields the expected default payoff.

$$
\begin{aligned}
PV_{HP} = & \sum_{t=1}^{T} e^{-r \cdot t} \cdot PO_t^{gc} \cdot \mathbb{I}_{\{X_t \geq db_t, 0<t\leq d\}} \\
& + e^{-r \cdot d} \cdot PO_d^{def} \cdot \mathbb{I}_{\{D_d + co_d + c_d - A_d \leq X_d < db_d, 0<d\leq T\}} \qquad (51) \\
= & \sum_{t=1}^{T} \left[e^{-r \cdot t} \cdot \left(\int_{\overline{db_t}}^{\infty} PO_t^{gc} \cdot h(db, x) \cdot k(x) dx \right.\right. \\
& \left.\left. + \; PO_d^{def} \cdot \left(\int_{\overline{db_{t-1}}}^{\infty} h(db, x) \cdot k(x) dx - \int_{\overline{db_t}}^{\infty} h(db, x) \cdot k(x) dx \right) \right) \right] \qquad (52)
\end{aligned}
$$

As being certain, the last component of our NPV formula, I_0, does not need any adjustment. Thus, our model contains explicit valuation equations for all NPV components allowing to evaluate any kind of leveraged buyout from a buyer perspective. Particularly, our model allows to determine the IRR of any investment by choosing the discount rate $r = IRR$ that meets the condition $NPV = 0$.

For simpler redemption schedules, where the default boundary is a linear or exponential function, the equations of our model even allow for explicit analytic solutions.

5 Example

In order to demonstrate the capabilities of our model, we present an illustrative example. Our target firm is called Illu Corp. The buyer, PREQ Funds, has a projected holding period of three years and strives to increase the current unlevered after-tax cash flow to firm of USD 100 m by 5% in year one, 15% in year two, and 10% in year three. The firm's operating risk is proxied by the industry average with a standard deviation of the cash flow's relative change of 10%. Furthermore, the initial debt level is USD 650 m. Over the three years holding period, the fund is following a fixed redemption with annual down payments of USD 70 m, USD 55 m and USD 80 m, respectively. The risk-adjusted cost of debt for such a plan is 7% p.a. The corporate tax rate is at 40%. Illu Corp has assets valued at USD 300 m that are kept constant over the next three years. In case of a default, PREQ Funds expects costs of financial distress of USD 50 m. The acquisition price PREQ Funds negotiated is at USD 920 m reflecting a multiple of 8 in relation to the current unlevered after-tax cash flow and a tax shield of USD 120 m based on the corporate tax rate and a pre-deal debt of USD 300 m.

Table 1: Assumptions for the Exemplary LBO

Assumptions					
After-tax Cash Flow		**Assets**		**Debt and Tax**	
X_0	\$100 m	A_0	\$300 m	D_0	\$650 m
μ_1	5%	A_1	\$300 m	f_1	\$70 m
μ_2	15%	A_2	\$300 m	f_2	\$55 m
μ_3	10%	A_3	\$300 m	f_3	\$80 m
σ	10%			r_D	7%
				τ	40%
Multiples		**Others**			
m_{Entry}	$8x$	r_f	5%		
m_{Exit}	$8x$	c	\$50 m		
l^*	$3x$	k	\$1.50		

Conservatively, PREQ Funds projects an exit price in three years time based on the same multiple. The target debt level at exit is determined as a multiple, $l^* = 3$, of the unlevered after-tax cash flow at exit. In case the realized debt level will be higher than the target level at exit, PREQ Funds faces a cost of USD 1.5 for each dollar of debt above the target level. Table 1 provides all relevant information of the example.

The fixed redemption schedule results in debt levels of $D_1 = \$580m$, $D_2 = \$525m$, and $D_{Exit} = \$445m$. Based on this information, the cash obligations per year, co_t^{fixed}, can be determined by equation (6): $co_1^{fixed} = \$97.30m$, $co_2^{fixed} = \$79.36m$, and $co_3^{fixed} = \$102.05m$. In our example, we consider the obligations to determine the default boundary. Hence, we look at a default boundary as in figure 4.

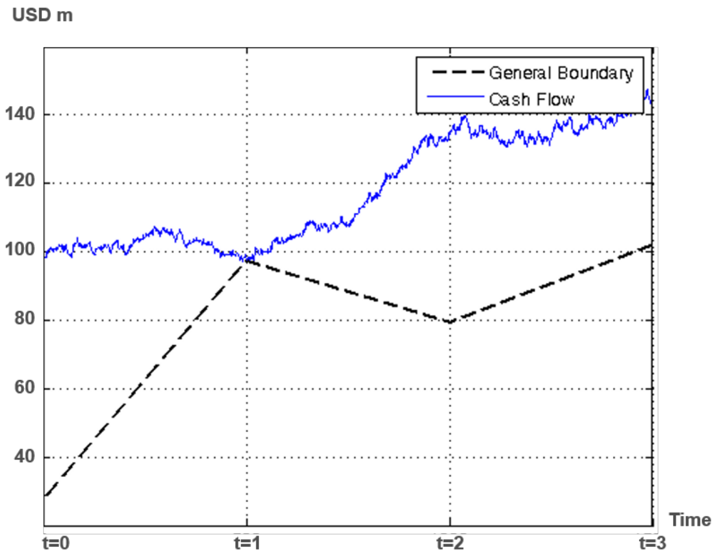

Figure 4: Cash Obligation of Exemplary LBO vs. one Potential Cash Flow Path

Having calculated the default probability and the NPV based on the equations of our model, we control the results by an extensive simulation with 200,000 cash flow paths that follow a gBm with the μ and σ parameters defined above. In order to smooth the simulation process towards a steady gBm, we use 500 time steps per year. Figure 5 illustrates the cash flow paths produced by the simulation model and their relationship to the cash obligations.

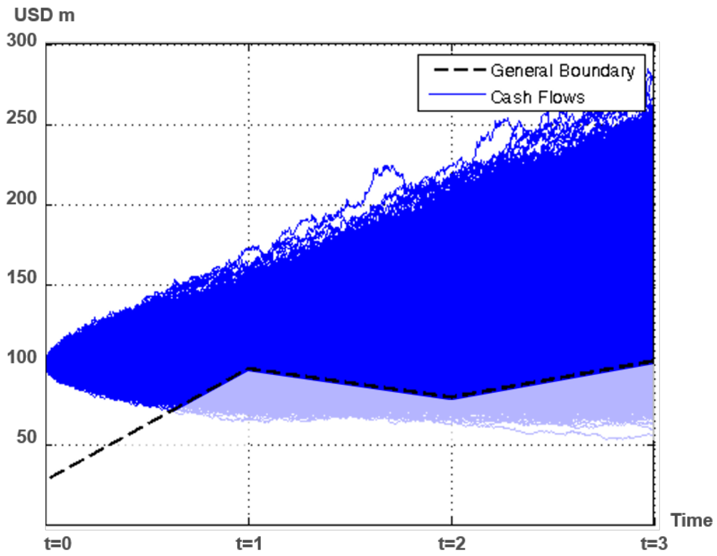

Figure 5: Cash Obligation of Exemplary LBO vs. 200,000 Cash Flow Paths

Based on equation (36), we find the cumulative default probability over the holding period to be 30.28%, while the extensive simulation results in 30.37% with a standard error of 0.10%pts. Hence, our solution lies well within a one standard error range. Figure 5 depicts the cumulative default probability over the holding period.

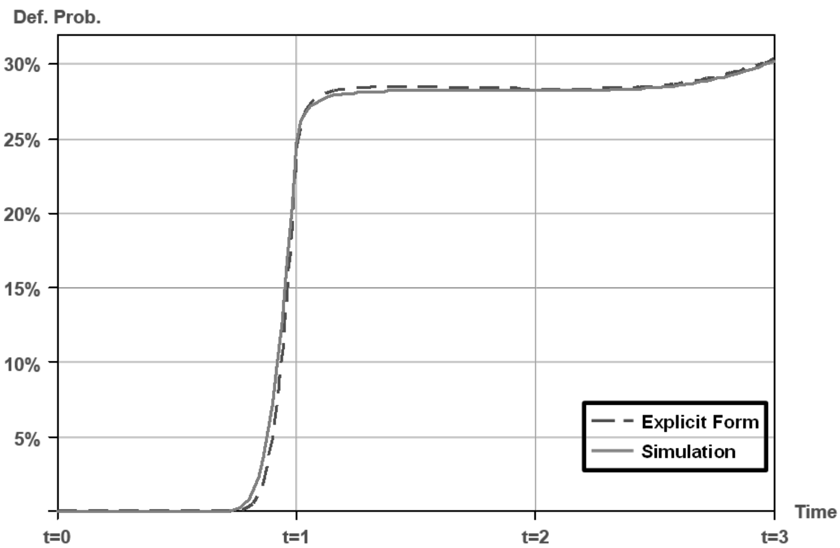

Figure 6: Cumulative Distribution Function of Default Probability - Fixed Debt Repayment

Table 2 compares the results of our model (equations (46), (49) and (52)) against the ones derived based on the simulation model. Our explicit form solution arrives at a final NPV for the equity investors of USD 286.2793 m while the extensive simulation results in USD 285.6018 m with a standard error of USD 1.1711 m. Again, the explicit form solution stays within a one standard error range.

Table 2: Results for the Exemplary LBO - Fixed Debt Repayment

Summary				
NPV	**Explicit**	**Simulation**		
Components	**Form Solution**	**Mean**	**-2 Std. Errors**	**+2 Std. Errors**
I_0	-\$270.0000 m	-\$270.0000 m	-\$270.0000 m	-\$270.0000 m
PV_1	\$8.6552 m	\$8.6476 m	\$8.6080 m	\$8.6872 m
PV_2	\$31.4998 m	\$31.4741 m	\$31.3593 m	\$31.5889 m
PV_3	\$24.3173 m	\$24.2981 m	\$24.1819 m	\$24.4143 m
PV_{Exit}	\$524.2060 m	\$523.6416 m	\$521.7312 m	\$525.5520 m
$PV_{Penalty}$	-\$32.3990 m	-\$32.4596 m	-\$32.6208 m	-\$32.2984 m
NPV	**\$286.2793 m**	**\$285.6018 m**	**\$283.2596 m**	**\$287.9440 m**

The corresponding IRRs for both calculations are determined via iteration. For the explicit form solution, the IRR is 29.9544% while the extensive simulation yields 29.9468%.

Looking at cash sweep as the second redemption case analyzed here, we assume $a = 80\%$ as the ratio of the cash flows after interests being used to repay debt during the holding period. With respect to the debt covenant, the multiple triggering the default case is $b = 7.0$ meaning that the debt level D_{t-1} should never exceed $X_t \cdot b$.

We use equation (36) to determine the default probability and perform the adjustments to the upper boundary as described in the previous section. Our model derives a default probability over the holding period of 16.93%. The highest fraction of this risk is generated in the first period (11.90% default risk in the first year) due to the ambitious covenant chosen in the example. After the first period, the incremental default risk is significantly lower than under fixed redemption. Figure 7 depicts the development of the cumulative default probability over time.

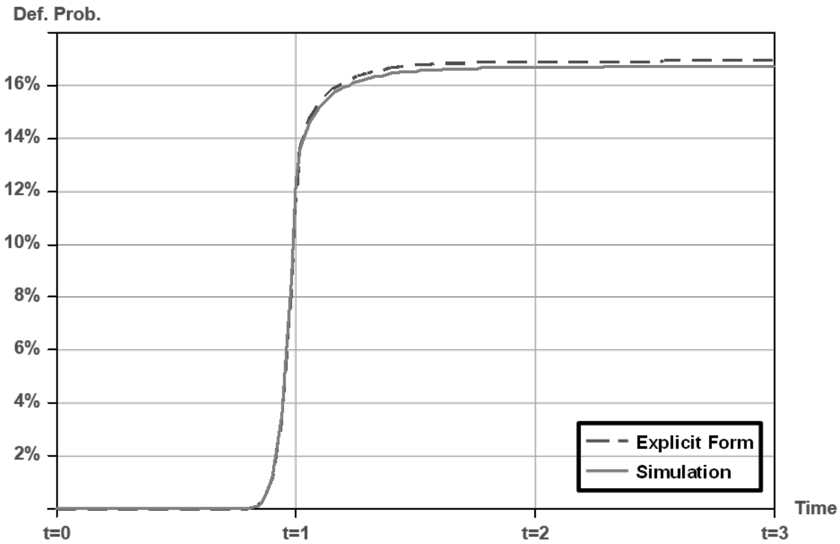

Figure 7: Cumulative Distribution Function of Default Probability - Cash Sweep Debt Repayment

Based on equations (46), (50), and (52), the NPV of this deal under cash sweep redemption is equal to USD 357.5129 m (extensive simulation: USD 358.9494), while the IRR amounts to 33.9314% (extensive simulation: 33.9038%). Table 3 illustrates the results for the different components and compares the results of the model against the extensive simulation.

Table 3: Results for the Exemplary LBO - Cash Sweep Debt Repayment

Summary				
NPV	**Explicit**	**Simulation**		
Components	**Form Solution**	**Mean**	**-2 Std. Errors**	**+2 Std. Errors**
I_0	-\$270.0000 m	-\$270.0000 m	-\$270.0000 m	-\$270.0000 m
PV_1	\$13.4192 m	\$13.3725 m	\$13.3479 m	\$13.3971 m
PV_2	\$15.2391 m	\$15.2874 m	\$15.2512 m	\$15.3236 m
PV_3	\$16.8727 m	\$16.9290 m	\$16.8856 m	\$16.9724 m
PV_{Exit}	\$620.2090 m	\$622.1391 m	\$620.4851 m	\$623.7931 m
$PV_{Penalty}$	-\$38.2271 m	-\$38.9874 m	-\$38.9874 m	-\$38.5698 m
NPV	**\$357.5129 m**	**\$358.9494 m**	**\$356.9824 m**	**\$360.9164 m**

The explicit form solutions for all NPV components lie within ±2 standard errors of the extensive simulation.

Finally, the model can also be used to optimize the debt policy of the portfolio firm. It can be applied to calculate the default probability and the IRR as a function of D_0 and a. Figure 8 and 9 depict the results.

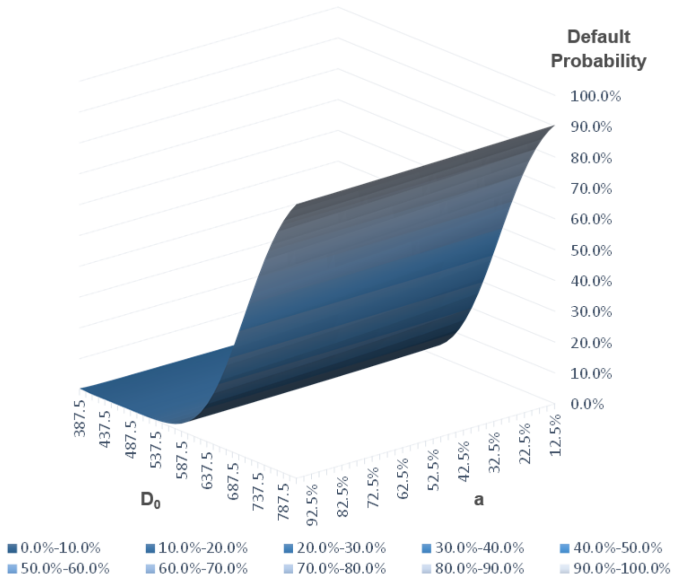

Figure 8: Default Probability for Combinations of D_0 and a

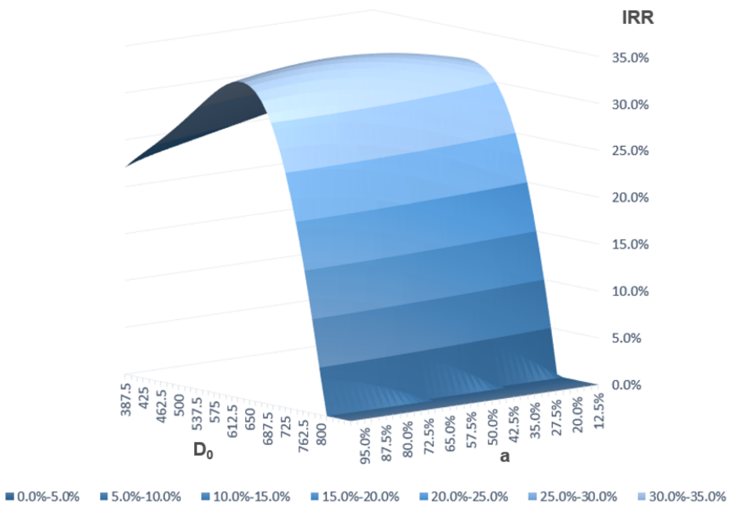

Figure 9: IRR for Combinations of D_0 and a

As expected, increasing the initial debt level, D_0, yields a higher default probability. If the buyer decides to increase the cash sweep ratio, a, the default probability slightly decreases. Figure 9 illustrates that there is an optimal leverage scenario in our example maximizing the IRR at 34.5223% with $D_0 = 625$ and $a = 70\%$. The default probability of this scenario is 9.2654%. Appendix 7.3 provides tables with default probabilities (Table 4) and IRRs (Table 5) for the different debt scenarios.

Additionally, the model also supports in optimizing the risk-return relationship for any given investor's risk appetite by combining explicit default probabilities and IRRs. Figure 10 shows the risk-return relationships for all calculated combinations of D_0 and a in our example.

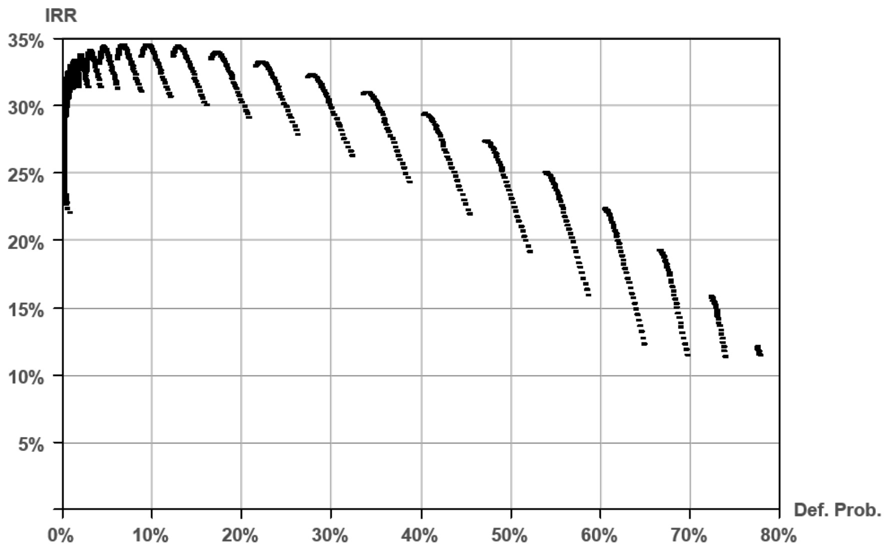

Figure 10: IRR vs. Default Probability for all Combinations of D_0 and a

Removing all dominated and non-efficient combinations of D_0 and a yields the trade-off relation shown in figure 11.

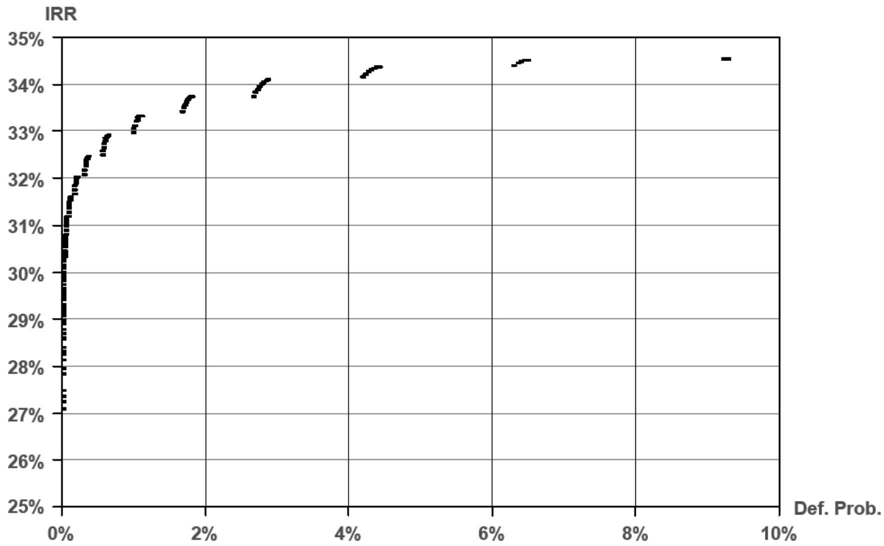

Figure 11: IRR vs. Default Probability for Dominant Combinations of D_0 and a

Table 6 in appendix 7.3 provides a table with the top 100 combinations of D_0 and a ordered by descending IRR. The table also depicts the corresponding default probabilities and efficient combinations.

The example highlights the ability of our model to evaluate and compare different structures of an LBO deal by combining return measures with default probability. Hence, the model offers support for optimizing risk-return trade-offs for different levels of investors' risk appetites.

6 Conclusion

In this paper, we derive a novel model for evaluating LBOs based on boundary crossing probabilities. It captures both types of debt redemption: the fixed predetermined one and the dynamic, path dependent one known as "cash sweep". Our model incorporates lower boundaries to the stochastic cash flow process that trigger default if they are hit. These boundaries can be either derived from cash obligations (redemption plus interest payments) or covenants (e.g. debt-to-cash flow ratio). Elaborating further on the idea of Wang and Pötzelberger (1997), the model allows to determine default probabilities by applying nested integrals that can be solved numerically. While attaching default probabilities to different redemption schedules, it also provides explicit form solutions for the valuation of an LBO. Thus, the risk-return relationship for any kind of LBO structure can be determined. Applying the model to an exemplary LBO deal shows that it works accurate and delivers insightful results.

7 Appendix

7.1 Density Function of Brownian Motion with Drift and its Minimum

In this chapter, we derive the joint density function of a Brownian motion with drift $\hat{W}(t)$ and its minimum $\hat{M}(t)$.

First, we start from a Brownian motion without drift $\tilde{W}(t)$ and adjust it in a way that a Brownian motion with drift $\hat{W}(t)$ is generated:

$$\hat{W}(t) = \alpha \cdot t + \tilde{W}(t) \tag{53}$$

Next, we define the minimum $\hat{M}(t)$ of such a process under the prerequisites $\hat{M}(t) \leq 0$ and $\hat{W}(t) \geq \hat{M}(t)$:

$$\hat{M}(t) = \min_{0 \leq t \leq T} \hat{W}(t) \tag{54}$$

According to the Girsanov Theorem, we define a new probability measure $\hat{\mathbb{P}}$ under which $\hat{W}(t)$ has zero drift:

$$\hat{Z}(t) = e^{-\alpha \cdot \tilde{W}(t) - \frac{1}{2} \cdot \alpha^2 \cdot t} = e^{-\alpha \cdot \hat{W}(t) + \frac{1}{2} \cdot \alpha^2 \cdot t} \tag{55}$$

$$\hat{P}(A) = \int_A \hat{Z}(T) d\tilde{\mathbb{P}} \tag{56}$$

For a process without drift, we know the joint density function with its minimum from the Reflection Principle (for detailed derivation see for example Shreve 2004):

$$\hat{f}_{\hat{M}(t),\hat{W}(t)}(m, w) = \frac{2 \cdot (w - 2 \cdot m)}{t \cdot \sqrt{2 \cdot \pi \cdot t}} \cdot e^{\frac{-(2 \cdot m - w)^2}{2 \cdot t}} \tag{57}$$

Knowing all this, we can finally derive the density of $\hat{M}(t)$ and $\hat{W}(t)$ under $\tilde{\mathbb{P}}$, the real-world probability:

$$\tilde{\mathbb{P}}\{\hat{M}(t) \geq m, \hat{W}(t) \geq w\} = \tilde{\mathbb{E}}\{\mathbb{I}_{\{\hat{M}(t) \geq m, \hat{W}(t) \geq w\}}\}$$

$$
\begin{aligned}
&= \hat{\mathbb{E}}\{\frac{1}{\hat{Z}(t)} \cdot \mathbb{I}_{\{\hat{M}(t)\geq m, \hat{W}(t)\geq w\}}\} \\
&= \hat{\mathbb{E}}\{e^{\alpha\cdot\hat{W}(t)-\frac{1}{2}\cdot\alpha^2\cdot t} \cdot \mathbb{I}_{\{\hat{M}(t)\geq m, \hat{W}(t)\geq w\}}\} \\
&= \int_m^\infty \int_w^\infty e^{\alpha\cdot Y-\frac{1}{2}\cdot\alpha^2\cdot T} \cdot \hat{f}_{\hat{M}(t),\hat{W}(t)}(x,y)dxdy \\
\frac{\delta^2\tilde{\mathbb{P}}\{\hat{M}(t)\geq m, \hat{W}(t)\geq w\}}{\delta m \delta w} &= e^{\alpha\cdot w-\frac{1}{2}\cdot\alpha^2\cdot t} \cdot \hat{f}_{\hat{M}(t),\hat{W}(t)}(m,w) \\
&= \frac{2\cdot(w-2\cdot m)}{t\cdot\sqrt{2\cdot\pi\cdot t}} \cdot e^{\alpha\cdot w-\frac{1}{2}\cdot\alpha^2\cdot t-\frac{(2\cdot m-w)^2}{2\cdot t}} \qquad (58)
\end{aligned}
$$

7.2 Upper Boundaries to the Integral Regions under Cash Sweep Debt Repayment

For cash sweep debt repayment we face a default barrier, $\overline{db_t}$, that is stochastic:

$$
\begin{aligned}
\overline{db_t} =& \frac{1}{\sigma} \cdot \ln\left(\frac{db_t^{sweep}}{X_0}\right) - \alpha \cdot t && (59) \\
& with: && (60) \\
& db_t^{sweep} = \frac{D_{t-1}}{b} && (61) \\
& D_{t-1} = D_{t-2} - a \cdot (X_{t-1} - (1-\tau_c) \cdot r_D \cdot D_{t-2}) && (62)
\end{aligned}
$$

Such an expression complicates numerical integrations. Hence, we look for additional limits to our integral regions in order to facilitate the calculation. By examining the term within the natural logarithm, we note that

$$
\frac{db_t^{sweep}}{X_0} = \frac{D_{t-1}}{b \cdot X_0} > 0. \qquad (63)
$$

We develop this non-negative condition for the first periods and derive a general rule for our random variables x_t:

$$
\begin{aligned}
\text{t=1: } \frac{db_1^{sweep}}{X_0} =& \frac{D_0}{b\cdot X_0} > 0 \\
& D_0 > 0 \qquad (64)
\end{aligned}
$$

$$\text{t=2: } \frac{db_2^{sweep}}{X_0} = \frac{D_0 - a \cdot (X_0 \cdot e^{(\mu - \frac{\sigma^2}{2}) \cdot 1 + \sigma \cdot x_1} - (1-\tau_c) \cdot r_D \cdot D_0)}{b \cdot X_0} > 0$$

$$\frac{1}{\sigma} \cdot \ln \left(\frac{D_0 \cdot (1 + a \cdot r_D \cdot (1-\tau_c))}{a \cdot X_0} \right) - \alpha_1 \cdot 1 > x_1 \tag{65}$$

...

$$\text{t=T: } \frac{db_T^{sweep}}{X_0} = \frac{D_{T-2} - a \cdot (X_0 \cdot e^{(\mu - \frac{\sigma^2}{2}) \cdot ((T-1)-(T-2)) + \sigma \cdot x_{T-1}} - (1-\tau_c) \cdot r_D \cdot D_{t-2})}{b \cdot X_0} > 0$$

$$\frac{\ln \left(\frac{D_0 \cdot (1 + a \cdot r_D \cdot (1-\tau_c))^{T-1}}{a \cdot X_0 \cdot e^{\sum_{t=1}^{T-2} (\mu - \frac{\sigma^2}{2}) \cdot t}} - \sum_{t=1}^{T-2} e^{\sigma \cdot x_t} \cdot (1 + a \cdot r_D \cdot (1-\tau_c))^t \right)}{\sigma} - \alpha_{T-1} \cdot ((T-1)-(T-2)) > x_{T-1} \tag{66}$$

What we find are upper boundaries to our integrals with a lag of one time period. Therefore, we define:

$$\overline{ub_{t-1}} = \frac{1}{\sigma} \cdot \ln \left(\frac{D_0 \cdot (1 + a \cdot r_D \cdot (1-\tau_c))^{t-1}}{a \cdot X_0 \cdot e^{\sum_{s=1}^{t-2} (\mu - \frac{\sigma^2}{2}) \cdot s}} - \sum_{s=1}^{t-2} e^{\sigma \cdot x_s} \cdot (1 + a \cdot r_D \cdot (1-\tau_c))^s \right) - \alpha_{t-1} \cdot ((t-1)-(t-2)) \tag{67}$$

To conclude, for cash sweep debt repayment we can adjust the upper boundaries of the integral regions from $+\infty$ to $\overline{ub_{t-1}}$.

7.3 Comparison of Leverage Scenarios

Table 4: Default Probability for Combinations of D_0 and a

	Cash sweep ratio																	
Debt level at entry	**12.5%**	**15.0%**	**17.5%**	**20.0%**	**22.5%**	**25.0%**	**27.5%**	**30.0%**	**32.5%**	**35.0%**	**37.5%**	**40.0%**	**42.5%**	**45.0%**	**47.5%**	**50.0%**	**52.5%**	**55.0%**
387.5	0.0%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
400	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
412.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
425	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
437.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
450	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
462.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
475	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
487.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
500	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
512.5	0.2%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%
525	0.3%	0.3%	0.3%	0.3%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%	0.2%
537.5	0.5%	0.5%	0.5%	0.5%	0.5%	0.4%	0.4%	0.4%	0.4%	0.4%	0.4%	0.4%	0.4%	0.3%	0.3%	0.3%	0.3%	0.3%
550	0.9%	0.9%	0.9%	0.8%	0.8%	0.8%	0.8%	0.7%	0.7%	0.7%	0.7%	0.7%	0.7%	0.6%	0.6%	0.6%	0.6%	0.6%
562.5	1.5%	1.5%	1.4%	1.4%	1.4%	1.3%	1.3%	1.3%	1.3%	1.2%	1.2%	1.2%	1.2%	1.1%	1.1%	1.1%	1.1%	1.1%
575	2.5%	2.4%	2.4%	2.3%	2.3%	2.2%	2.2%	2.1%	2.1%	2.0%	2.0%	2.0%	1.9%	1.9%	1.9%	1.9%	1.8%	1.8%
587.5	3.8%	3.8%	3.7%	3.6%	3.5%	3.5%	3.4%	3.4%	3.3%	3.3%	3.2%	3.2%	3.1%	3.1%	3.0%	3.0%	3.0%	2.9%
600	5.7%	5.6%	5.5%	5.4%	5.4%	5.3%	5.2%	5.1%	5.1%	5.0%	4.9%	4.9%	4.8%	4.7%	4.7%	4.6%	4.6%	4.5%
612.5	8.3%	8.1%	8.0%	7.9%	7.8%	7.7%	7.6%	7.5%	7.4%	7.3%	7.2%	7.2%	7.1%	7.0%	6.9%	6.9%	6.8%	6.8%
625	11.5%	11.4%	11.2%	11.1%	11.0%	10.8%	10.7%	10.6%	10.5%	10.4%	10.3%	10.2%	10.1%	10.0%	9.9%	9.8%	9.8%	9.7%
637.5	15.5%	15.3%	15.2%	15.0%	14.9%	14.7%	14.6%	14.4%	14.3%	14.2%	14.1%	14.0%	13.9%	13.7%	13.6%	13.5%	13.5%	13.4%
650	20.2%	20.0%	19.9%	19.7%	19.5%	19.4%	19.2%	19.1%	18.9%	18.8%	18.6%	18.5%	18.4%	18.3%	18.1%	18.0%	17.9%	17.8%
662.5	25.7%	25.5%	25.3%	25.1%	24.9%	24.7%	24.6%	24.4%	24.2%	24.1%	23.9%	23.8%	23.7%	23.5%	23.4%	23.3%	23.1%	23.0%
675	31.7%	31.5%	31.3%	31.1%	30.9%	30.7%	30.5%	30.3%	30.2%	30.0%	29.8%	29.7%	29.5%	29.4%	29.3%	29.1%	29.0%	28.9%
687.5	38.1%	37.9%	37.7%	37.5%	37.3%	37.1%	36.9%	36.7%	36.6%	36.4%	36.2%	36.1%	35.9%	35.8%	35.6%	35.5%	35.4%	35.2%
700	44.8%	44.6%	44.4%	44.2%	44.0%	43.8%	43.6%	43.4%	43.3%	43.1%	42.9%	42.8%	42.6%	42.5%	42.3%	42.2%	42.1%	41.9%
712.5	51.5%	51.3%	51.1%	50.9%	50.7%	50.6%	50.4%	50.2%	50.0%	49.9%	49.7%	49.6%	49.4%	49.3%	49.1%	49.0%	48.9%	48.7%
725	58.1%	57.9%	57.7%	57.6%	57.4%	57.2%	57.0%	56.9%	56.7%	56.6%	56.4%	56.3%	56.1%	56.0%	55.9%	55.7%	55.6%	55.5%
737.5	64.4%	64.2%	64.1%	63.9%	63.7%	63.6%	63.4%	63.3%	63.1%	63.0%	62.8%	62.7%	62.6%	62.4%	62.3%	62.2%	62.1%	62.0%
750	70.2%	70.1%	69.9%	69.8%	69.6%	69.5%	69.3%	69.2%	69.1%	69.0%	68.8%	68.7%	68.6%	68.5%	68.4%	68.3%	68.1%	68.0%
762.5	75.5%	75.4%	75.2%	75.1%	75.0%	74.8%	74.7%	74.6%	74.5%	74.4%	74.3%	74.2%	74.1%	74.0%	73.9%	73.8%	73.7%	73.6%
775	80.2%	80.0%	79.9%	79.8%	79.7%	79.6%	79.5%	79.4%	79.3%	79.2%	79.1%	79.0%	78.9%	78.8%	78.8%	78.7%	78.6%	78.5%
787.5	84.2%	84.1%	84.0%	83.9%	83.8%	83.7%	83.6%	83.5%	83.5%	83.4%	83.3%	83.2%	83.2%	83.1%	83.0%	82.9%	82.9%	82.8%
800	87.6%	87.5%	87.4%	87.4%	87.3%	87.2%	87.1%	87.1%	87.0%	86.9%	86.9%	86.8%	86.7%	86.7%	86.6%	86.6%	86.5%	86.4%
812.5	90.4%	90.3%	90.3%	90.2%	90.2%	90.1%	90.0%	90.0%	89.9%	89.9%	89.8%	89.8%	89.7%	89.7%	89.6%	89.6%	89.5%	89.5%

	Cash sweep ratio																	
Debt level at entry	**57.5%**	**60.0%**	**62.5%**	**65.0%**	**67.5%**	**70.0%**	**72.5%**	**75.0%**	**77.5%**	**80.0%**	**82.5%**	**85.0%**	**87.5%**	**90.0%**	**92.5%**	**95.0%**	**97.5%**	**100.0%**
387.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%	0.4%	0.6%
400	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.2%	0.3%
412.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%
425	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%
437.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
450	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
462.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
475	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
487.5	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
500	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
512.5	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%
525	0.2%	0.2%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%
537.5	0.3%	0.3%	0.3%	0.3%	0.3%	0.3%	0.3%	0.3%	0.3%	0.3%	0.3%	0.3%	0.3%	0.2%	0.2%	0.2%	0.2%	0.2%
550	0.6%	0.6%	0.6%	0.6%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%	0.5%
562.5	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.9%	0.8%
575	1.8%	1.8%	1.7%	1.7%	1.7%	1.7%	1.6%	1.6%	1.6%	1.6%	1.6%	1.6%	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%
587.5	2.9%	2.9%	2.8%	2.8%	2.8%	2.7%	2.7%	2.7%	2.7%	2.6%	2.6%	2.6%	2.6%	2.5%	2.5%	2.5%	2.5%	2.5%
600	4.5%	4.4%	4.4%	4.4%	4.3%	4.3%	4.2%	4.2%	4.2%	4.1%	4.1%	4.1%	4.0%	4.0%	4.0%	4.0%	3.9%	3.9%
612.5	6.7%	6.6%	6.6%	6.5%	6.5%	6.4%	6.4%	6.3%	6.3%	6.2%	6.2%	6.2%	6.1%	6.1%	6.0%	6.0%	6.0%	5.9%
625	9.6%	9.5%	9.5%	9.4%	9.3%	9.3%	9.2%	9.1%	9.1%	9.0%	9.0%	8.9%	8.9%	8.8%	8.8%	8.7%	8.7%	8.6%
637.5	13.3%	13.2%	13.1%	13.0%	12.9%	12.9%	12.8%	12.7%	12.7%	12.6%	12.5%	12.5%	12.4%	12.3%	12.3%	12.2%	12.2%	12.1%
650	17.7%	17.6%	17.5%	17.4%	17.3%	17.3%	17.2%	17.1%	17.0%	16.9%	16.8%	16.8%	16.7%	16.6%	16.6%	16.5%	16.4%	16.4%
662.5	22.9%	22.8%	22.7%	22.6%	22.5%	22.4%	22.3%	22.2%	22.1%	22.0%	21.9%	21.9%	21.8%	21.7%	21.6%	21.5%	21.5%	21.4%
675	28.8%	28.6%	28.5%	28.4%	28.3%	28.2%	28.1%	28.0%	27.9%	27.8%	27.7%	27.6%	27.5%	27.4%	27.4%	27.3%	27.2%	27.1%
687.5	35.1%	35.0%	34.9%	34.7%	34.6%	34.5%	34.4%	34.3%	34.2%	34.1%	34.0%	33.9%	33.8%	33.7%	33.6%	33.5%	33.5%	33.4%
700	41.8%	41.7%	41.5%	41.4%	41.3%	41.2%	41.1%	41.0%	40.9%	40.8%	40.7%	40.6%	40.5%	40.4%	40.3%	40.2%	40.1%	40.0%
712.5	48.6%	48.5%	48.4%	48.3%	48.1%	48.0%	47.9%	47.8%	47.7%	47.6%	47.5%	47.4%	47.3%	47.2%	47.1%	47.0%	46.9%	46.9%
725	55.4%	55.2%	55.1%	55.0%	54.9%	54.8%	54.7%	54.6%	54.5%	54.4%	54.3%	54.2%	54.1%	54.0%	53.9%	53.8%	53.8%	53.7%
737.5	61.9%	61.7%	61.6%	61.5%	61.4%	61.3%	61.2%	61.1%	61.0%	60.9%	60.9%	60.8%	60.7%	60.6%	60.5%	60.4%	60.4%	60.3%
750	67.9%	67.8%	67.7%	67.6%	67.5%	67.5%	67.4%	67.3%	67.2%	67.1%	67.0%	66.9%	66.9%	66.8%	66.7%	66.6%	66.6%	66.5%
762.5	73.5%	73.4%	73.3%	73.2%	73.1%	73.1%	73.0%	72.9%	72.8%	72.8%	72.7%	72.6%	72.5%	72.5%	72.4%	72.3%	72.3%	72.2%
775	78.4%	78.4%	78.3%	78.2%	78.1%	78.1%	78.0%	77.9%	77.9%	77.8%	77.7%	77.7%	77.6%	77.5%	77.5%	77.4%	77.4%	77.3%
787.5	82.7%	82.7%	82.6%	82.5%	82.5%	82.4%	82.4%	82.3%	82.2%	82.2%	82.1%	82.1%	82.0%	82.0%	81.9%	81.9%	81.8%	81.8%
800	86.4%	86.3%	86.3%	86.2%	86.2%	86.1%	86.1%	86.0%	86.0%	85.9%	85.9%	85.8%	85.8%	85.8%	85.7%	85.7%	85.6%	85.6%
812.5	89.4%	89.4%	89.3%	89.3%	89.3%	89.2%	89.2%	89.1%	89.1%	89.1%	89.0%	89.0%	89.0%	88.9%	88.9%	88.8%	88.8%	88.8%

Table 5: IRR for Combinations of D_0 and a

Debt level at entry	Cash sweep ratio																	
	12.5%	**15.0%**	**17.5%**	**20.0%**	**22.5%**	**25.0%**	**27.5%**	**30.0%**	**32.5%**	**35.0%**	**37.5%**	**40.0%**	**42.5%**	**45.0%**	**47.5%**	**50.0%**	**52.5%**	**55.0%**
387.5	27.7%	27.5%	27.5%	27.3%	27.2%	27.1%	27.0%	26.8%	26.7%	26.5%	26.4%	26.2%	26.1%	25.9%	25.8%	25.6%	25.4%	25.3%
400	28.1%	28.0%	27.9%	27.8%	27.7%	27.6%	27.5%	27.3%	27.2%	27.1%	26.9%	26.8%	26.6%	26.5%	26.3%	26.2%	26.0%	25.8%
412.5	28.5%	28.5%	28.4%	28.3%	28.2%	28.1%	28.0%	27.9%	27.7%	27.6%	27.5%	27.3%	27.2%	27.0%	26.9%	26.7%	26.6%	26.4%
425	28.9%	28.9%	28.8%	28.8%	28.7%	28.6%	28.5%	28.4%	28.3%	28.2%	28.0%	27.9%	27.7%	27.6%	27.5%	27.3%	27.1%	27.0%
437.5	29.3%	29.3%	29.2%	29.2%	29.1%	29.1%	29.0%	28.9%	28.8%	28.7%	28.6%	28.4%	28.3%	28.2%	28.0%	27.9%	27.7%	27.6%
450	29.6%	29.6%	29.6%	29.6%	29.6%	29.5%	29.5%	29.4%	29.3%	29.2%	29.1%	29.0%	28.9%	28.7%	28.6%	28.5%	28.3%	28.2%
462.5	29.9%	30.0%	30.0%	30.0%	30.0%	30.0%	29.9%	29.9%	29.8%	29.7%	29.6%	29.5%	29.4%	29.3%	29.2%	29.1%	28.9%	28.8%
475	30.2%	30.3%	30.3%	30.4%	30.4%	30.4%	30.4%	30.3%	30.3%	30.2%	30.1%	30.1%	30.0%	29.9%	29.8%	29.6%	29.5%	29.4%
487.5	30.5%	30.6%	30.7%	30.7%	30.7%	30.8%	30.8%	30.8%	30.7%	30.7%	30.6%	30.6%	30.5%	30.4%	30.3%	30.2%	30.1%	30.0%
500	30.8%	30.9%	31.0%	31.0%	31.1%	31.1%	31.2%	31.2%	31.2%	31.2%	31.1%	31.1%	31.0%	31.0%	30.9%	30.8%	30.7%	30.6%
512.5	31.0%	31.1%	31.2%	31.3%	31.4%	31.5%	31.5%	31.6%	31.6%	31.6%	31.6%	31.6%	31.5%	31.5%	31.4%	31.3%	31.3%	31.2%
525	31.2%	31.4%	31.5%	31.6%	31.7%	31.8%	31.9%	31.9%	32.0%	32.0%	32.0%	32.0%	32.0%	32.0%	31.9%	31.9%	31.8%	31.7%
537.5	31.4%	31.6%	31.7%	31.9%	32.0%	32.1%	32.2%	32.3%	32.3%	32.4%	32.4%	32.4%	32.5%	32.4%	32.4%	32.4%	32.4%	32.3%
550	31.6%	31.8%	31.9%	32.1%	32.2%	32.4%	32.5%	32.6%	32.7%	32.7%	32.8%	32.8%	32.9%	32.9%	32.9%	32.9%	32.9%	32.8%
562.5	31.7%	31.9%	32.1%	32.3%	32.4%	32.6%	32.7%	32.8%	32.9%	33.0%	33.1%	33.2%	33.2%	33.3%	33.3%	33.3%	33.3%	33.3%
575	31.8%	32.0%	32.2%	32.4%	32.6%	32.7%	32.9%	33.0%	33.2%	33.3%	33.4%	33.5%	33.6%	33.6%	33.7%	33.7%	33.7%	33.7%
587.5	31.8%	32.1%	32.3%	32.5%	32.7%	32.8%	33.0%	33.2%	33.3%	33.5%	33.6%	33.7%	33.8%	33.9%	33.9%	34.0%	34.0%	34.1%
600	31.8%	32.0%	32.2%	32.4%	32.6%	32.8%	33.0%	33.2%	33.4%	33.5%	33.7%	33.8%	33.9%	34.0%	34.1%	34.2%	34.2%	34.3%
612.5	31.5%	31.8%	32.0%	32.2%	32.5%	32.7%	32.9%	33.1%	33.3%	33.4%	33.6%	33.7%	33.9%	34.0%	34.1%	34.2%	34.3%	34.4%
625	31.1%	31.4%	31.6%	31.9%	32.1%	32.4%	32.6%	32.8%	33.0%	33.2%	33.3%	33.5%	33.7%	33.8%	33.9%	34.1%	34.2%	34.3%
637.5	30.5%	30.8%	31.1%	31.3%	31.6%	31.8%	32.0%	32.3%	32.5%	32.7%	32.9%	33.1%	33.2%	33.4%	33.6%	33.7%	33.8%	33.9%
650	29.6%	29.9%	30.2%	30.5%	30.7%	31.0%	31.2%	31.5%	31.7%	31.9%	32.2%	32.4%	32.6%	32.7%	32.9%	33.1%	33.2%	33.3%
662.5	28.4%	28.7%	29.0%	29.3%	29.6%	29.9%	30.1%	30.4%	30.7%	30.9%	31.1%	31.4%	31.6%	31.8%	32.0%	32.1%	32.3%	32.5%
675	26.9%	27.2%	27.5%	27.8%	28.1%	28.4%	28.7%	29.0%	29.3%	29.5%	29.8%	30.0%	30.2%	30.5%	30.7%	30.9%	31.1%	31.2%
687.5	25.0%	25.3%	25.6%	26.0%	26.3%	26.6%	26.9%	27.2%	27.5%	27.8%	28.0%	28.3%	28.5%	28.8%	29.0%	29.2%	29.5%	29.6%
700	22.6%	23.0%	23.4%	23.7%	24.0%	24.4%	24.7%	25.0%	25.3%	25.6%	25.9%	26.2%	26.5%	26.7%	27.0%	27.2%	27.5%	27.7%
712.5	19.9%	20.3%	20.7%	21.0%	21.4%	21.7%	22.1%	22.4%	22.8%	23.1%	23.4%	23.7%	24.0%	24.3%	24.6%	24.8%	25.1%	25.3%
725	16.7%	17.1%	17.5%	17.9%	18.3%	18.7%	19.1%	19.4%	19.8%	20.1%	20.5%	20.8%	21.1%	21.4%	21.7%	22.0%	22.3%	22.6%
737.5	13.2%	13.6%	14.0%	14.4%	14.9%	15.3%	15.7%	16.0%	16.4%	16.8%	17.2%	17.5%	17.9%	18.2%	18.5%	18.8%	19.1%	19.4%
750	9.2%	9.6%	10.1%	10.6%	11.0%	11.4%	11.9%	12.3%	12.7%	13.1%	13.5%	13.9%	14.2%	14.6%	14.9%	15.3%	15.6%	15.9%
762.5	4.9%	5.3%	5.8%	6.3%	6.8%	7.2%	7.7%	8.1%	8.6%	9.0%	9.4%	9.8%	10.2%	10.6%	11.0%	11.4%	11.7%	12.1%
775	0.2%	0.7%	1.2%	1.8%	2.3%	2.7%	3.2%	3.7%	4.2%	4.6%	5.1%	5.5%	5.9%	6.3%	6.7%	7.1%	7.5%	7.9%
787.5	-4.7%	-4.2%	-3.6%	-3.1%	-2.5%	-2.0%	-1.5%	-1.0%	-0.5%	0.0%	0.4%	0.9%	1.3%	1.8%	2.2%	2.6%	3.1%	3.4%
800	-9.8%	-9.3%	-8.7%	-8.1%	-7.5%	-7.0%	-6.5%	-5.9%	-5.4%	-4.9%	-4.4%	-3.9%	-3.4%	-3.0%	-2.5%	-2.1%	-1.6%	-1.2%
812.5	-15.1%	-14.5%	-13.9%	-13.3%	-12.7%	-12.1%	-11.5%	-11.0%	-10.4%	-9.9%	-9.3%	-8.8%	-8.3%	-7.8%	-7.3%	-6.9%	-6.4%	-6.0%

Debt level at entry	Cash sweep ratio																	
	57.5%	**60.0%**	**62.5%**	**65.0%**	**67.5%**	**70.0%**	**72.5%**	**75.0%**	**77.5%**	**80.0%**	**82.5%**	**85.0%**	**87.5%**	**90.0%**	**92.5%**	**95.0%**	**97.5%**	**100.0%**
387.5	25.1%	25.0%	24.8%	24.6%	24.5%	24.3%	24.2%	24.0%	23.8%	23.7%	23.5%	23.4%	23.2%	23.0%	22.8%	22.6%	22.4%	22.1%
400	25.7%	25.5%	25.4%	25.2%	25.0%	24.9%	24.7%	24.5%	24.4%	24.2%	24.1%	23.9%	23.7%	23.6%	23.4%	23.2%	23.0%	22.7%
412.5	26.2%	26.1%	25.9%	25.8%	25.6%	25.4%	25.3%	25.1%	24.9%	24.8%	24.6%	24.4%	24.3%	24.1%	23.9%	23.8%	23.6%	23.4%
425	26.8%	26.7%	26.5%	26.3%	26.2%	26.0%	25.8%	25.7%	25.5%	25.3%	25.2%	25.0%	24.8%	24.7%	24.5%	24.3%	24.2%	24.0%
437.5	27.4%	27.3%	27.1%	26.9%	26.8%	26.6%	26.4%	26.3%	26.1%	25.9%	25.8%	25.6%	25.4%	25.3%	25.1%	24.9%	24.7%	24.6%
450	28.0%	27.9%	27.7%	27.5%	27.4%	27.2%	27.0%	26.9%	26.7%	26.5%	26.4%	26.2%	26.0%	25.9%	25.7%	25.5%	25.3%	25.2%
462.5	28.6%	28.5%	28.3%	28.2%	28.0%	27.8%	27.7%	27.5%	27.3%	27.2%	27.0%	26.8%	26.7%	26.5%	26.3%	26.1%	26.0%	25.8%
475	29.2%	29.1%	28.9%	28.8%	28.6%	28.5%	28.3%	28.1%	28.0%	27.8%	27.6%	27.5%	27.3%	27.1%	26.9%	26.8%	26.6%	26.4%
487.5	29.9%	29.7%	29.6%	29.4%	29.3%	29.1%	29.0%	28.8%	28.6%	28.5%	28.3%	28.1%	28.0%	27.8%	27.6%	27.4%	27.2%	27.1%
500	30.5%	30.3%	30.2%	30.1%	29.9%	29.8%	29.6%	29.5%	29.3%	29.1%	29.0%	28.8%	28.6%	28.5%	28.3%	28.1%	27.9%	27.7%
512.5	31.1%	31.0%	30.8%	30.7%	30.6%	30.4%	30.3%	30.1%	30.0%	29.8%	29.7%	29.5%	29.3%	29.1%	29.0%	28.8%	28.6%	28.4%
525	31.7%	31.6%	31.5%	31.3%	31.2%	31.1%	31.0%	30.8%	30.7%	30.5%	30.3%	30.2%	30.0%	29.9%	29.7%	29.5%	29.3%	29.1%
537.5	32.2%	32.2%	32.1%	32.0%	31.9%	31.7%	31.6%	31.5%	31.3%	31.2%	31.0%	30.9%	30.7%	30.6%	30.4%	30.2%	30.0%	29.9%
550	32.8%	32.7%	32.6%	32.6%	32.5%	32.4%	32.2%	32.1%	32.0%	31.9%	31.7%	31.6%	31.4%	31.3%	31.1%	30.9%	30.8%	30.6%
562.5	33.3%	33.2%	33.2%	33.1%	33.0%	33.0%	32.9%	32.7%	32.6%	32.5%	32.4%	32.2%	32.1%	31.9%	31.8%	31.6%	31.5%	31.3%
575	33.7%	33.7%	33.7%	33.6%	33.6%	33.5%	33.4%	33.3%	33.2%	33.1%	33.0%	32.9%	32.7%	32.6%	32.4%	32.3%	32.1%	32.0%
587.5	34.1%	34.1%	34.1%	34.0%	34.0%	33.9%	33.9%	33.8%	33.7%	33.6%	33.5%	33.4%	33.3%	33.2%	33.0%	32.9%	32.7%	32.6%
600	34.3%	34.3%	34.3%	34.3%	34.3%	34.3%	34.3%	34.2%	34.1%	34.0%	34.0%	33.9%	33.8%	33.6%	33.5%	33.4%	33.2%	33.1%
612.5	34.4%	34.5%	34.5%	34.5%	34.5%	34.5%	34.5%	34.4%	34.4%	34.3%	34.3%	34.2%	34.1%	34.0%	33.9%	33.7%	33.6%	33.5%
625	34.3%	34.4%	34.5%	34.5%	34.5%	34.5%	34.5%	34.5%	34.5%	34.4%	34.4%	34.3%	34.2%	34.1%	34.0%	33.9%	33.8%	33.7%
637.5	34.0%	34.1%	34.2%	34.2%	34.3%	34.3%	34.3%	34.3%	34.3%	34.3%	34.3%	34.2%	34.2%	34.1%	34.0%	33.9%	33.8%	33.7%
650	33.5%	33.6%	33.7%	33.7%	33.8%	33.9%	33.9%	33.9%	33.9%	33.9%	33.9%	33.9%	33.8%	33.8%	33.7%	33.6%	33.6%	33.5%
662.5	32.6%	32.7%	32.8%	32.9%	33.0%	33.1%	33.2%	33.2%	33.2%	33.3%	33.3%	33.2%	33.2%	33.2%	33.1%	33.1%	33.0%	32.9%
675	31.4%	31.5%	31.7%	31.8%	31.9%	32.0%	32.1%	32.2%	32.2%	32.2%	32.3%	32.3%	32.3%	32.3%	32.2%	32.2%	32.1%	32.1%
687.5	29.8%	30.0%	30.2%	30.3%	30.4%	30.6%	30.7%	30.8%	30.8%	30.9%	30.9%	31.0%	31.0%	31.0%	31.0%	31.0%	30.9%	30.9%
700	27.9%	28.1%	28.3%	28.4%	28.6%	28.7%	28.9%	29.0%	29.1%	29.2%	29.2%	29.3%	29.3%	29.3%	29.3%	29.3%	29.3%	29.3%
712.5	25.6%	25.8%	26.0%	26.2%	26.4%	26.5%	26.7%	26.8%	26.9%	27.0%	27.1%	27.2%	27.3%	27.3%	27.3%	27.4%	27.4%	27.3%
725	22.8%	23.1%	23.3%	23.5%	23.7%	23.9%	24.1%	24.2%	24.4%	24.5%	24.6%	24.7%	24.8%	24.9%	24.9%	25.0%	25.0%	25.0%
737.5	19.7%	20.0%	20.2%	20.5%	20.7%	20.9%	21.1%	21.3%	21.5%	21.6%	21.8%	21.9%	22.0%	22.1%	22.2%	22.2%	22.3%	22.3%
750	16.2%	16.5%	16.8%	17.1%	17.3%	17.6%	17.8%	18.0%	18.2%	18.4%	18.5%	18.7%	18.8%	18.9%	19.0%	19.1%	19.2%	19.2%
762.5	12.4%	12.7%	13.0%	13.3%	13.6%	13.9%	14.1%	14.3%	14.6%	14.8%	14.9%	15.1%	15.3%	15.4%	15.5%	15.6%	15.7%	15.8%
775	8.2%	8.6%	8.9%	9.2%	9.5%	9.8%	10.1%	10.4%	10.6%	10.8%	11.0%	11.2%	11.4%	11.6%	11.7%	11.9%	12.0%	12.1%
787.5	3.8%	4.2%	4.6%	4.9%	5.2%	5.5%	5.8%	6.1%	6.4%	6.6%	6.9%	7.1%	7.3%	7.5%	7.7%	7.8%	7.9%	8.1%
800	-0.8%	-0.4%	0.0%	0.4%	0.7%	1.0%	1.4%	1.7%	2.0%	2.2%	2.5%	2.7%	3.0%	3.2%	3.4%	3.5%	3.7%	3.8%
812.5	-5.5%	-5.1%	-4.7%	-4.3%	-3.9%	-3.6%	-3.2%	-2.9%	-2.6%	-2.3%	-2.0%	-1.7%	-1.5%	-1.3%	-1.1%	-0.9%	-0.7%	-0.5%

Table 6: Dominance Criterion for Top 100 Combinations of D_0 and a

Rank	D0	a	IRR	Defprob	Dominant?	Rank	D0	a	IRR	Defprob	Dominant?
1	625	70.00%	34.52%	9.27%	Yes	**51**	625	52.50%	34.17%	9.75%	No
2	625	72.50%	34.52%	9.20%	Yes	**52**	637.5	87.50%	34.17%	12.40%	No
3	625	67.50%	34.51%	9.33%	No	**53**	625	90.00%	34.14%	8.83%	No
4	612.5	67.50%	34.51%	6.48%	Yes	**54**	600	77.50%	34.13%	4.17%	Yes
5	612.5	65.00%	34.51%	6.53%	No	**55**	637.5	60.00%	34.12%	13.18%	No
6	625	75.00%	34.50%	9.15%	No	**56**	612.5	47.50%	34.11%	6.95%	No
7	612.5	70.00%	34.50%	6.43%	Yes	**57**	637.5	90.00%	34.09%	12.34%	No
8	612.5	62.50%	34.49%	6.58%	No	**58**	600	47.50%	34.09%	4.69%	No
9	625	65.00%	34.49%	9.39%	No	**59**	612.5	87.50%	34.08%	6.12%	No
10	612.5	72.50%	34.48%	6.38%	Yes	**60**	587.5	60.00%	34.07%	2.85%	Yes
11	625	77.50%	34.47%	9.09%	No	**61**	587.5	57.50%	34.07%	2.89%	No
12	612.5	60.00%	34.46%	6.64%	No	**62**	625	50.00%	34.06%	9.83%	No
13	625	62.50%	34.45%	9.46%	No	**63**	587.5	62.50%	34.06%	2.82%	Yes
14	612.5	75.00%	34.44%	6.33%	Yes	**64**	587.5	55.00%	34.06%	2.92%	No
15	625	80.00%	34.43%	9.03%	No	**65**	600	80.00%	34.05%	4.14%	No
16	612.5	57.50%	34.42%	6.70%	No	**66**	625	92.50%	34.04%	8.78%	No
17	625	60.00%	34.40%	9.53%	No	**67**	587.5	65.00%	34.03%	2.79%	Yes
18	612.5	77.50%	34.39%	6.29%	Yes	**68**	637.5	57.50%	34.03%	13.27%	No
19	625	82.50%	34.37%	8.98%	No	**69**	587.5	52.50%	34.03%	2.96%	No
20	612.5	55.00%	34.36%	6.76%	No	**70**	637.5	92.50%	34.01%	12.28%	No
21	600	62.50%	34.35%	4.40%	Yes	**71**	600	45.00%	34.00%	4.74%	No
22	637.5	75.00%	34.34%	12.72%	No	**72**	612.5	45.00%	34.00%	7.02%	No
23	600	65.00%	34.34%	4.36%	Yes	**73**	587.5	67.50%	34.00%	2.76%	Yes
24	637.5	72.50%	34.34%	12.79%	No	**74**	587.5	50.00%	33.99%	2.99%	No
25	600	60.00%	34.34%	4.44%	No	**75**	612.5	90.00%	33.98%	6.08%	No
26	625	57.50%	34.34%	9.60%	No	**76**	600	82.50%	33.96%	4.11%	No
27	637.5	77.50%	34.33%	12.65%	No	**77**	587.5	70.00%	33.94%	2.73%	Yes
28	612.5	80.00%	34.33%	6.24%	No	**78**	625	47.50%	33.94%	9.92%	No
29	600	67.50%	34.32%	4.32%	Yes	**79**	625	95.00%	33.93%	8.74%	No
30	637.5	70.00%	34.32%	12.87%	No	**80**	650	77.50%	33.93%	17.00%	No
31	600	57.50%	34.32%	4.49%	No	**81**	637.5	55.00%	33.93%	13.36%	No
32	637.5	80.00%	34.31%	12.59%	No	**82**	587.5	47.50%	33.93%	3.03%	No
33	625	85.00%	34.31%	8.93%	No	**83**	650	80.00%	33.93%	16.93%	No
34	600	70.00%	34.29%	4.28%	Yes	**84**	650	75.00%	33.92%	17.08%	No
35	637.5	67.50%	34.29%	12.94%	No	**85**	637.5	95.00%	33.92%	12.22%	No
36	612.5	52.50%	34.29%	6.82%	No	**86**	650	82.50%	33.91%	16.85%	No
37	600	55.00%	34.28%	4.53%	No	**87**	650	72.50%	33.90%	17.17%	No
38	637.5	82.50%	34.27%	12.52%	No	**88**	600	42.50%	33.90%	4.80%	No
39	625	55.00%	34.26%	9.68%	No	**89**	650	85.00%	33.89%	16.78%	No
40	612.5	82.50%	34.26%	6.20%	No	**90**	587.5	72.50%	33.88%	2.71%	Yes
41	600	72.50%	34.25%	4.24%	Yes	**91**	612.5	42.50%	33.87%	7.09%	No
42	637.5	65.00%	34.25%	13.02%	No	**92**	650	70.00%	33.86%	17.25%	No
43	600	52.50%	34.23%	4.58%	No	**93**	587.5	45.00%	33.86%	3.07%	No
44	625	87.50%	34.23%	8.88%	No	**94**	612.5	92.50%	33.86%	6.04%	No
45	637.5	85.00%	34.23%	12.46%	No	**95**	600	85.00%	33.86%	4.08%	No
46	612.5	50.00%	34.21%	6.88%	No	**96**	650	87.50%	33.84%	16.70%	No
47	600	75.00%	34.19%	4.21%	Yes	**97**	637.5	52.50%	33.82%	13.45%	No
48	637.5	62.50%	34.19%	13.10%	No	**98**	625	97.50%	33.82%	8.69%	No
49	612.5	85.00%	34.17%	6.16%	No	**99**	637.5	97.50%	33.81%	12.17%	No
50	600	50.00%	34.17%	4.63%	No	**100**	625	45.00%	33.81%	10.00%	No

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