The Valuation for Nonrenewable Resources with Endogenous Costs: A Mature Oil Well.

Sergio Cabrales[1], Rafael Bautista

Abstract:

The economic evaluation of non-renewable resource projects is problematic given the high degree of uncertainties associated to prices, volume of the resource and production costs. We develop a model using real option for valuing nonrenewable resources given the endogenous production costs. We found circular relation (endogenous) between the international oil price and the production cost (lifting cost, geology cost, drilling cost, and transportation) in the oil industry. There are some hypothesis which might explain the endogeneous cost such as the "hold up" problem, changes of the input price for service companies and sharing the same portion with services companies. We assume a stochastic process for spot oil prices and there are no ways to improve the reserves as exploration or tertiary recovery (Enhanced Oil Recovery). The price of a mature oil well might be affected because the price volatility is absorbed by the costs affecting its final value.

1. Introduction

The valuation of an asset of the oil industry is related to the value of its oil and reserves. The task to value oil and/or gas reserves requires more information than just the actual reserve volume and spot price (Grundy & Heaney, 2011). This paper presents a valuation model using real option methodology for a mature oil well during the initial and/or secondary recovery given the endogenous production costs (lifting cost, geology cost, drilling cost, and transportation).

In the oil industry for several decades, the most common method to valuation asset has been the method of Discounted Cash Flow (DCF). During the last forty years, the number of articles using real options method have be increased because the real option method break some limitations forced by the DCF. In the DCF approach, the expected future cash flows to stockholder are determined, discounted to the present, and summed to yield the lease value (Paddock, Siegel, & Smith, 1988). If the stockholders want to have a probabilistic model to determine the free cash flows for each period, there is required a specify probability distribution for the oil and gas price, production costs, and production volumes.

The economic evaluation of non-renewable resource projects is challenging given the high degree of uncertainties associated to prices, volume of the resource and production costs. Brenna y Schwartz (1985) showed that the techniques of continuous time arbitrage and stochastic control theory might be used not only to value projects, but also to determine the optimal policy of exploration, management and give them up (Brennan & Schwartz, 1985).

The optional pricing approach has its roots in the option valuation method proposed by Black-Scholes-Merton. Although, the formula was developed to derive the discounts that should be applied to a

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[1] Corresponding author. Faculty of Management, University of Los Andes, Colombia
Email address: s-cabral@uniandes.edu.co

corporate bond incorporate the possibility of default (Black and Scholes, 1973; Merton, 1973). The applications of the real options methods came back to Myers (1977) and Ross (1978) and popularized by Myers (1984) and Kester (1984). On the one hand, probability DCF use subjective probabilities to value the project and to get the optimal strategies. On the other hand, the real option method maximize the market value of a portfolio which replicas the cash flow of the project in all possible nature's states. The project value is given by the current market price of the portfolio's replication and the principle of this approach is the "no arbitrage" or principle of the "law of one price." (Smith & McCardle, 1998).

The first work that opened the application of real option pricing methodology proposed by Black-Scholes-Merton to the oil industry was Tourinho (1979). Tourinho (1979) evaluated oil reserves using option pricing techniques including holding and storage costs. During the eighties, Brennan and Schwartz (1985) apply real option techniques to non-renewable resources. Kester (1984) recognized the flexibility in the valuation of real options to value oil assets. McDonald and Siegel (1985) studied the pricing of the irreversible project when the company has the option to shut down the production. Mason and Merton (1985) incorporated operative real options as to abandon an oil well in the large-scale oil project. And Paddock, Siegel and Smith (1988) propose a model for value an offshore petroleum lease showing the necessity of combine option pricing techniques with a model of equilibrium.

During the nineties, the applications of real options techniques to the oil industry was consolidated. Bjerksund and Ekern (1990) valued an oil field considering the options of the delay the production and abandonment. Stensland and Tjostheim (1991) using empirical dynamic programing, examined the effects of the increasing flow of the information through time reducing the uncertainties in the exploitation of natural resource projects. Bjerksund (1991) considered the cost of excluding to develop an oil and gas asset. Pickles and Smith (1993) aimed to explain how option valuation might be applied to the oil industry. Dixit and Pindyck (1994) presented in their book a methodology of pricing offshore oil reserves. And Schwartz (1997) compared the effects of three different stochastic behaviors of commodity prices analyzing the implications for the term structure of futures prices and volatilities.

In the mid 1990's, several textbooks about real option analyzed investment models for the oil and natural resources industry such as Dixit and Pindyck (1994), Trigeorgis (1996), Luenberger (1998), and Amram & Kulatilaka (1999). And by the end of the nineties, Smith and McCardle (1998) developed a model for an oil project which the production rates and oil prices behave stochastically over time. Additional, the authors incorporate the decision of shut down or growing up the production by drilling additional wells (Smith & McCardle, 1998). Laughton (1998) found that the oil prospect value increases with both oil price and reserve size uncertainties. Additional, he found that greater oil price uncertainty actions as exploration or abandonment occur later (Laughton, 1998). Cortazar and Schwartz (1997) applied Monte Carlo simulation to evaluate using real option model of an undeveloped oil field. Pindyck (1999) analyze the implications in real options methods of the long-run behavior of oil, coal, and natural gas prices.

Finally, Saito et al (2001) evaluate the real option approach and discount cash flow (DCF) method finding that the value of the oil reserves can be higher under real option approach than the value evaluated under DCF method. Kenyon and Tompaidis (2001) studied the leasing contracts such as offshore drilling rigs, marine seismic services and corporate real estate quantifying the effect of the idle time. McCormack and Sick (2001) discussed the valuation of Proven Undeveloped Reserves (PUBs) which is a rich source of option value for oil companies. And Cortazal, Schwartz and Casassus (2001) develop a real option model for valuing natural resources considering the price and geological technical uncertainty.

The remainder of this paper is structured as follows: Section 2 develops a real option methodology to value an oil asset under endogenous production cost; Section 3 applies the developed methodology to a mature well assuming stochastic price and endogenous production costs; and Section 4 provides the concludes.

2. The Valuation Model

The group of activities to develop an oil well might be classified into three stages: exploration, development, and extraction from the geological formation (Adelman, 1962). The exploration stage involves seismic and drilling activities to obtain information about amount of oil and/or gas reserves present, formation structure and initial develop cost of the well (REF). If the exploration results are economically and technically viable, the oil company might carry on the development stage in which the company have to build the required facilities to extract the oil (Paddock, Siegel, & Smith, 1988). Finally, the last stage is the extraction of hydrocarbon in which the company produce oil and gas depending the level of reserves.

The method of Discounting Cash Flows (DCF) to value natural resources project has some recognized restrictions as the problem of production forecast for each periods, the oil and gas price, and the inability to incorporate future operational options such as shutting down, delay or abandoned the oil production (Schwartz & Trigeorgis, 2004). According to Gaudet (1977), the rate of return on a physical asset can be disintegrated into three components: the first component is linked to the flow of products; the second component is related to the physical characteristics of assets, and the third component is connected to the rate in which the asset's market value changes over time.

The value of the marginal oil barrel for the company is given by the spot price, S, and the marginal cost of extraction (lifting cost), A. Therefore, the marginal value of an oil barrel is given by Equation 1 (Hotelling, 1931).

$$\pi = S - A \qquad (1)$$

Although, it has been recognized in the literature that the Hotelling (1931) marginal approach is suitable empirically. Some specific limitations of the valuation model include the assumption that the resource is homogeneous, the production costs are known, the rate of decline of the oil well that is constant, and non-stochastic interest rates (REF).

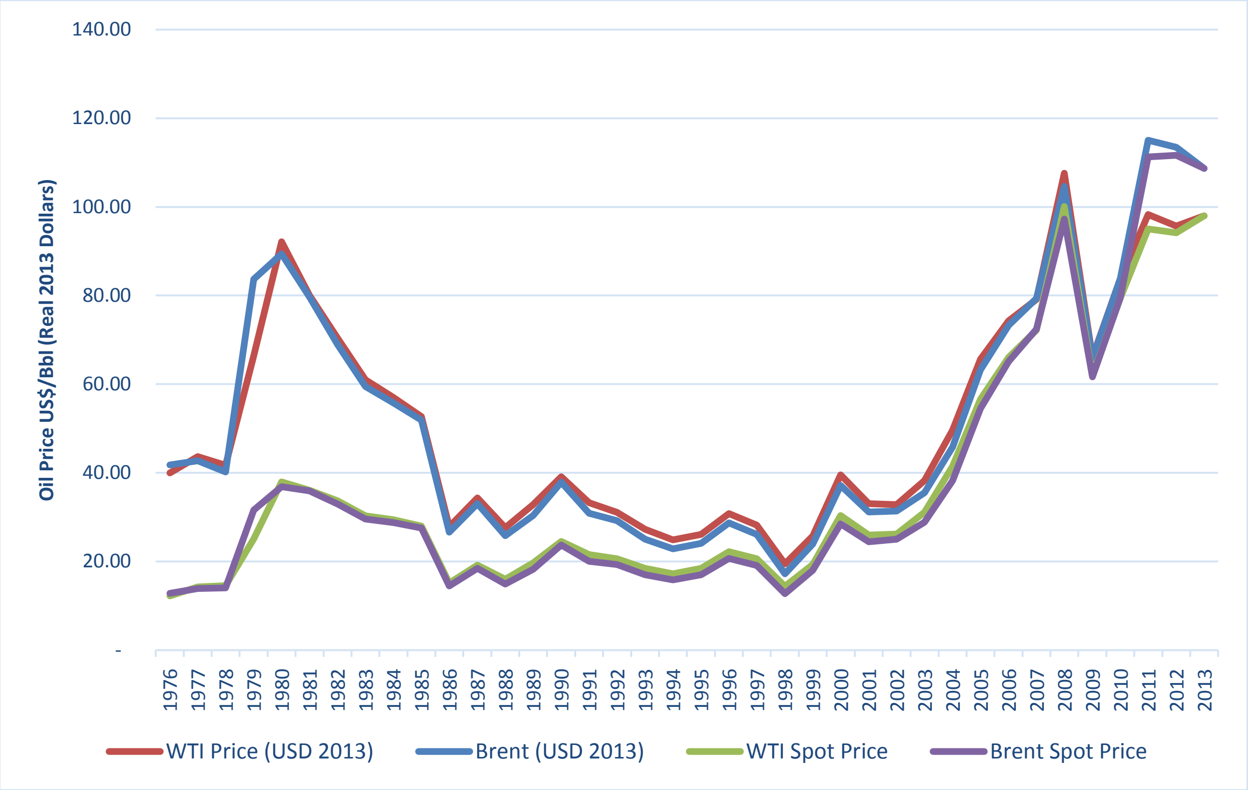


Figure 1. The oil prices (Source: BP Statistical Review World Energy 2014, and The World Bank).

The series of oil prices clearly show the great volatility adding a complexity to well pricing, as shown in the Figure 1. We assume that the spot price, S, is exogenously determined by a competitive market and is modeled by the following the continuous stochastic process:

$$\frac{dS}{S} = \mu dt + \sigma dz \qquad (2)$$

Where the dz is the increment to a standard Gauss-Wiener process, σ is the instantaneous standard deviation of price changes and μ is the trend in the price changes. This pricing model is based on the evaluation model of investment in renewable resources proposed by Brennan and Schwartz (1985). The function $F(S,t)$ represent the futures price on one barrel of oil at time t for delivery of at time T where $\tau = T - t$. The instantaneous change in the future oil price is given from ITO's lemma shown in the Equation 3.

$$dF = \left(-F_\tau + \frac{1}{2}F_{SS}\sigma^2 S^2\right)dt + F_S dS \qquad (3)$$

We define r as the riskless rate which is assumed known and constant during the time of analysis, and $C(S,t)$ represents marginal net convenience yield given the spot price, S. First, the riskless portfolio is composed by long a barrel of oil and short $\left(\frac{1}{F_S}\right)$ oil futures contracts. The riskless portfolio return is equal to risk-free rate, r.

$$\frac{dS}{S} + \frac{C(S)dt}{S} - \left(\frac{1}{F_S}\right)\frac{dF}{S} = rdt \qquad (4)$$

Using the Equation 2 and Equation 3 into the equation 4, the instantaneous rate of return earned by an individual who purchases one barrel and goes short $\left(\frac{1}{F_S}\right)$, and the futures contracts of oil is given by the equation 5.

$$\frac{1}{2}F_{SS}\sigma^2 S^2 + F_S(rS - C) - F_\tau = 0 \qquad (5)$$

As shown in the Equation 6, the boundary conditions for oil futures contract when the time remaining to maturity of the oil contract is zero $(\tau = 0)$. Therefore, the price of future oil contracts is exactly spot price, S.

$$F(S,0) = S \qquad (6)$$

The marginal net convenience yield $C(S,t)$ of the one barrel of oil can be written as a function of the current spot price, S, as shown the Equation 7.

$$C(S,t) = cS \qquad (7)$$

Using Equations 5, 6 and 7, the future price of oil depends of the spot price, S, and the remaining time for delivery, τ, as shown in the Equation 8 (Ross, 1978).

$$F(S,\tau) = Se^{(r-C)\tau} \qquad (8)$$

Finally, using Equation 3 which represents the instantaneous change in oil futures and the Equation 5 which represents the returns of the portfolio which replicates the cash flows of an oil well, we get an Equation 9 which represent the instantaneous changes of oil futures contracts.

$$dF = F_S[S(\mu - r) + C(S,t)]dt + F_S S\sigma dz \qquad (9)$$

The Society of Petroleum Engineer (SPE) define proved reserves as the quantities of petroleum which can be estimated with reasonable certainty to be commercially recoverable, probable reserves which are more likely than not to be recoverable, and possible reserves which are less likely to be recoverable than probable reserves.

Oil production might include three distinct phases: primary, secondary and tertiary recovery. The American Petroleum Institute (API) defines primary recovery as the oil, gas or oils and gas, recovered by natural flow or artificial lift such as pumps. Secondary recovery is defined as the oil, gas, or oil and gas, recovered by any artificial flowing or pumping obtained by the injection of liquids or gases into the reservoir (Teague, 1974). Greeb and Willhite (1998) define the tertiary recovery or Enhanced Oil Recovery (EOR) methods as miscible gases, chemical, and/or thermal energy to displace additional oil.

The model considers a single matured well during the primary or secondary recovery, where its oil is sold at spot prices, S. We define the price of the well, W, which depends of the spot oil price, S; the quantity of the oil reminder (proved reserves), Q; the actual production, q; and the calendar time, t. We write the value of the well, W, as the following function:

$$W \equiv W(S, Q, t; q) \qquad (10)$$

Applying the Ito's Lemma to the Equation 10, the instantaneous change in the value of the well is given by

$$dW = W_S dS + W_Q dQ + W_t dt + \frac{1}{2} W_{SS} S^2 \sigma^2 \qquad (11)$$

$$dW \qquad dS$$

$$\equiv W_S$$

The quantity of the oil reminder (proved reserves) are not fixed. The exploration and Enhanced Oil Recovery (EOR) might improve the volume of proved reserves. In the oil industry, independent companies certify the quantity of proved reserves, probable reserves and possible reserves. When the exploration and production company has more reservoir information, it can requested an update of its proved reserves, probable reserves and possible reserves for each fill or well.

The maximum capacity in an oil well is a priori decision that limits the maximum production rate, q, from a well. If you have the same level of reserves, Q, and under the same geological conditions, the rate of production is affected by its rate of decline, ρ. We are assuming for our model a well that is mature in the primary or secondary recovery and do not consider the maximum installed capacity. We assume that enhanced oil recovery are no applied to the mature well. There are some factors that affect the production rate such as the pressure, which decreases as it exploits a well, and geological characteristics such as Ph., Skin and KH.

It was found that the use of the decline curve for the future production of oil from deposits has been a practical methodology (REF). A common approach to evaluate the production potential of an oil well involves the use of decline production curves. Muskat (1949) proposed the first methodology to model the decline production curve. The underlying idea is that certain functions of the production rate of individual wells are represented by time-dependent functions. This methodology is believed to be especially true in the case of oil production, where production is affected by pressure and exhaustion of the resource (Chungcharoen & Fuller, 1999).

We define q as a function of the oil production rate that depend of depletion rate, ρ, and the time, t. Assuming that the geological restriction, the changes in total reserves of the well depend on the rate of production of that well and time, as shown below:

$$dQ = -q(\rho, t)dt \qquad (12)$$

In the literature, three models have been recognized to model the depletion rate of one well or fill. The declining curves of production are classified under the following three groups: Exponential, Hyperbolic and Harmonica (Hubbert, 1956). The exponential decline, also called geometric, semi-logarithmic or constant percentage, is characterized by the fact that the fall in the rate of production per unit time is proportional to the rate of production, that is:

$$q(\rho, t) = q_0 e^{-\rho t} \qquad (13)$$

One of the key factors in the oil industry is the high level of sub contraction which has great potential to hold up problems and frequent changes in prices by the supplier companies. The relation between the cost per drilled feet and the oil price as shown in the Figure 2. We found a significant positive linear relation between oil price and equipment cost, and oil price and operating cost.

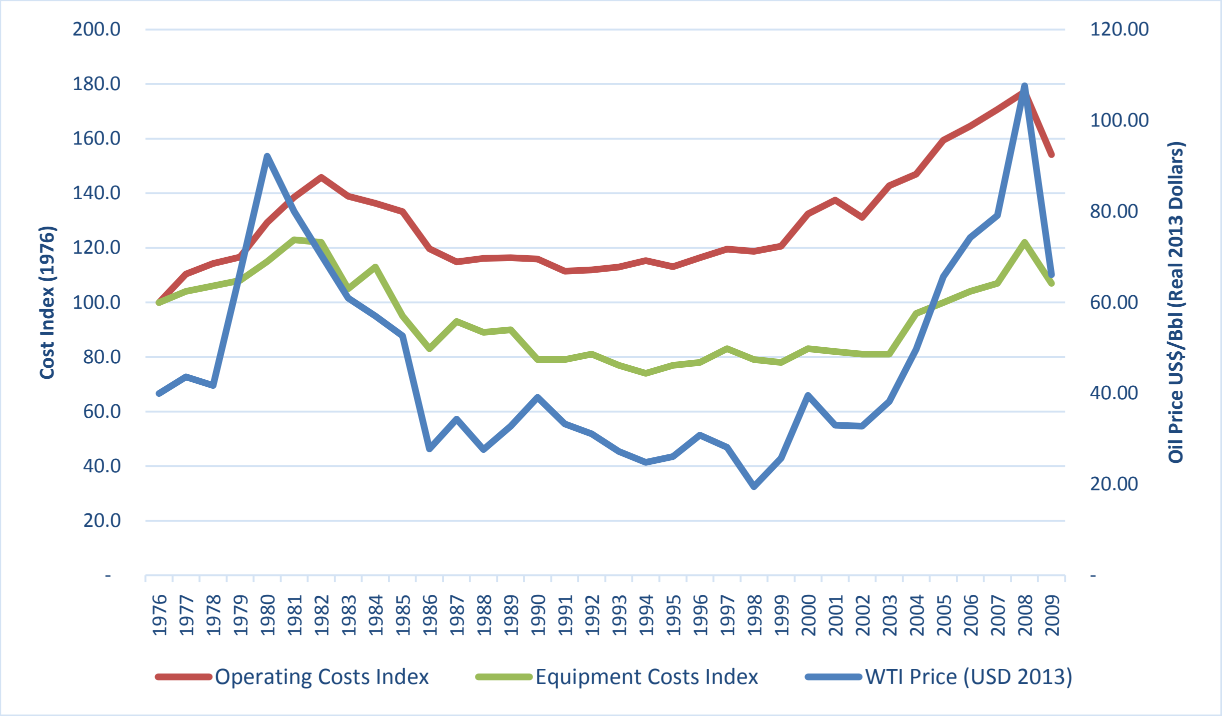


Figure 2. Indices for Oil Equipment and Annual Operating Costs and Oil Prices in Real 1976 dollars (Source: Energy Information Administration).

From the case of General Motors chassis of the relationship between suppliers and the companies have only very studied and documented cases of the company's particles. Contracts with suppliers of raw materials or services such as more customary in the oil industry are by nature incomplete. Therefore, it is possible to envisage all possible scenarios in nature and a less international commodity market.

There is deck of possible explanations of this fact of the production cost. First, this stylized fact is from the nature and relationship of production costs to the international price of oil to the ratio of energy extracted from a barrel of oil from a well with the energy required to extract that barrel. Second, another way to interpret endogeneity of costs is that outsourced firms support can see what the company's profitability is and adjust their prices because the spot price per barrel is known for all the market. The past 5 years clearly demonstrate the linear relationship between the cost of geology, transportation, exploration, drilling and production with the spot price of oil.

The transport rates of fuel depend on a high degree of international cost of steel, which is the main source of sunk costs of oil transportation projects. The steel prices are highly correlated with international oil prices, which generates a significant and positive correlation between prices of crude oil transportation and the international price of oil.

Sergio Cabrales A.
Código: 200327262
Universidad de los Andes.

When the price of oil rises, it increases the number of exploratory wells and works well; the number of companies that produce drilling rigs oil wells does not exceed five. Companies that produce holes with increases in demand can increase the price per foot of drilling, thus generating a price increase of the scan. Therefore, there is a significant positive correlation between international oil prices and the cost of exploration. The same case is presented in the cost of geology at the initial stage of exploration.

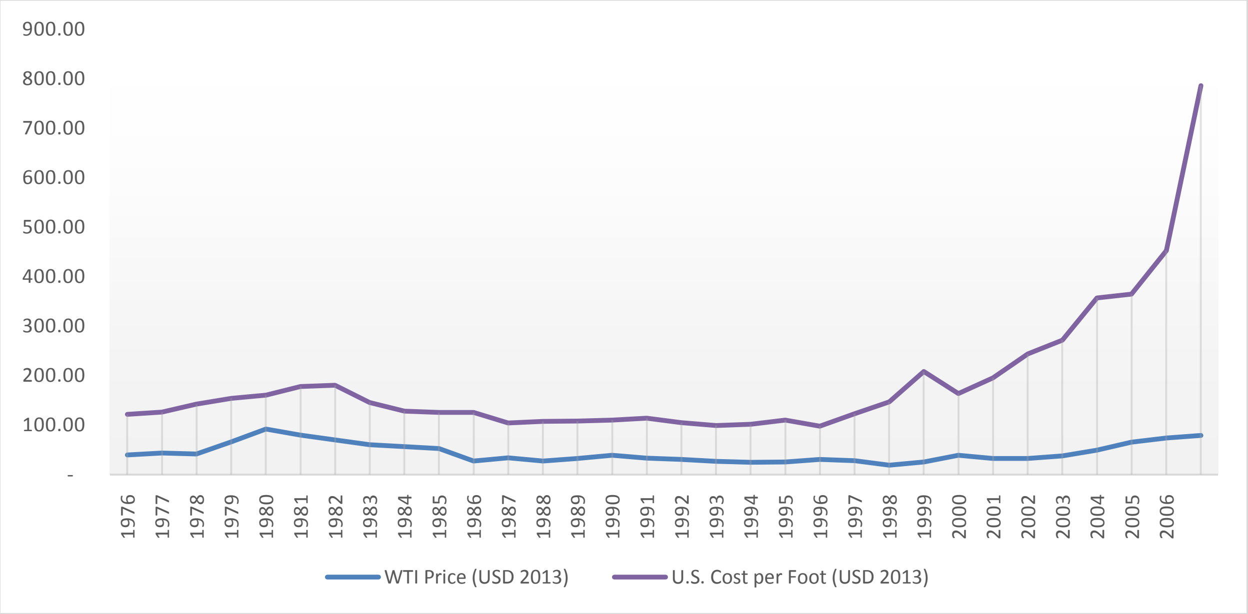
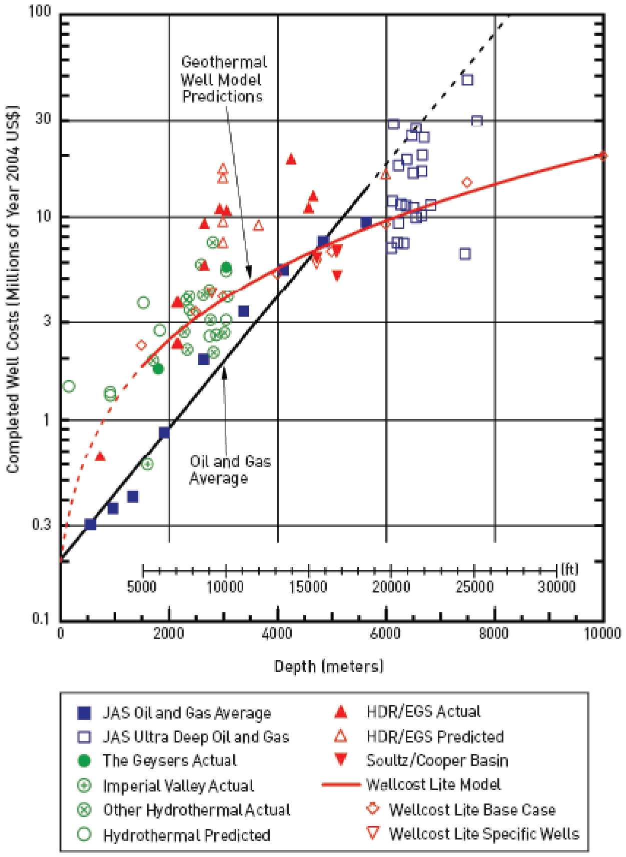


Figure 3. WTI Oil price series and drilling cost per feet series (Source: Energy Information Administration).

As shown the figure 3, the series of the WTI oil prices and the range of average annual cost of drilling per foot. It was found that there is a significant positive relationship, which indicates that drilling costs are directly affected by international oil prices.

Additionally, the reservoir characteristics (e.g. pressure) and the physical characteristics of crude oil are important factors that affect the cost of oil production. These characteristics vary considerably between different geographic locations; the cost of oil production and completed well cost also varies depending the depth, as shown in figure 4.



1. JAS = Joint Association Survey on Drilling Costs.
2. Well costs updated to US$ (yr. 2004) using index made from 3-year moving average for each depth interval listed in JAS (1976-2004) for onshore, completed US oil and gas wells. A 17% inflation rate was assumed for years pre-1976.
3. Ultra deep well data points for depths greater than 6 km are either individual wells or averages from a small number of wells listed in JAS (1994-2000).
4. "Other Hydrothermal Actual" data include some non-US wells (Source: Mansure 2004).

Figure 4. Completed geothermal and oil and gas well costs as a function of depth in year 2004 U.S. $. (Source: Energy Information Administration).

Based on the stylized fact of endogenous production costs, we define $A(S)$ as the production cost per barrel. The production cost per barrel depends on the marginal cost of the companies, β, which provide the services required for extraction and transportation of a crude to the point of sale. Additionally, these suppler companies adjust the price of their service by a premium, α, which is proportional to the spot price, S.

$$A(S) = \beta + \alpha S \qquad (14)$$

Now define div as continuous dividend (15) that is generated when it is long an oil well, which depends on the well production rate, q, the spot price, S, taxes and royalties paid for each barrel extracted, π, and operating cost per barrel, $A(S)$.

$$div = q\big(S - \pi - A(S)\big) \qquad (15)$$

Finally, we need to build a portfolio that replicates cash flows to be long an oil well. To achieve a risk-free portfolio when we are long a well, we must be short $\left(\frac{H_S}{F_S}\right)$ futures contracts of oil. The risk-free portfolio will have a return equal to risk free rate, r, per the value of the portfolio. The instantaneous rate for a portfolio, long by one matured well and goes short $\left(\frac{H_S}{F_S}\right)$ futures contracts of oil, is given by the equation 16.

$$\frac{1}{2}\sigma^2 W_{SS} + (r-h)SW_S - qW_Q + W_t - rW + q_0 e^{-\rho t}[S(1-\beta) - \pi - \alpha] - q_0 e^{-\rho t}\, W_Q = 0 \quad (16)$$

Then, we have the following boundary conditions:

Well Exhaustion: When the oil reserve in the well is exhausted, the well can no longer operate. Then, the value of the well is equal to zero to zero:

$$W(S,0,t) = 0 \qquad (17)$$

Premature Well Abandonment: When the oil spot price is below the abandonment price, S^{-}, the well can no longer operate and it is preferable abandon the well. Then, the value of the well is equal to a function $g(S,Q)$ which represent the value of the abandon well with remnant reserves, Q, and spot price, S:

$$W(S,Q,t) = g(S,Q) \qquad (18)$$

End of the reservoir operation contract: when the time agreed to exploit the oil well ended, so it is a salvage value is a function of the remaining amount of crude, Q, and spot price at that time, S.

$$W(S,Q,T) = f(S,Q) \qquad (19)$$

3. Numerical example

4. Conclusion

The method of discounting cash flows to value natural resources has some recognized limitations in the literature such as the problem of production forecast for each of the periods, the future price volatility due to the high prices and the inability to model possible future operational decisions (Schwartz & Trigeorgis, 2004).

This paper proposes a pricing model which incorporates oil fields endogeneity of production costs and some geological effects that affect the rate of production. The aim was to use a more flexible framework such as real options to incorporate these two factors in the valuation of this asset class.

We argue that it is more relevant because of the use of real options methodology proposed by Black-Scholes-Merton valuation of an oil field under certain operating conditions. Taking into account that the oil industry has many uncertainties from operating as the success of onshore or offshore drilling to the market price, the use of real options methodology is highly relevant for valuing assets in the industry.

The reason to take the unit of analysis as an oil well is because it is the basic unit of a field assessment and therefore an oil production company. For the assessment of oil fields there are key factors in its assessment such as: the endogeneity of production costs, the randomness of the market price, the natural decline in production rate, the recovery phase is finding the well and geological factors that affect the well site and its pressure in both the production rate.

Annex 1.

$$dW + div - \left(\frac{W_S}{F_S}\right) dF$$

$$= \frac{1}{2} W_{SS} S^2 \sigma^2 dt + W_S dS + W_Q dQ + W_t dt + q[S - \pi - A(S)]dt$$
$$- \left(\frac{W_S}{F_S}\right) [F_S(S(\mu - r) + C)dt + F_S S \sigma dz]$$

$$= \frac{1}{2} W_{SS} S^2 \sigma^2 dt + W_S dS + W_Q dQ + W_t dt + q[S - \pi - \alpha - \beta S]dt - W_S[(S(\mu - r) + C)dt + S\sigma dz]$$

$$= \frac{1}{2} W_{SS} S^2 \sigma^2 dt + W_S dS + W_Q dQ + W_t dt + q[(1 - \beta)S - \pi - \alpha]dt$$
$$- W_S[(S(\mu - r)dt + Cdt + S\sigma dz]$$

$$dW + div - \left(W_S / F_S\right) dF$$
$$= {}^1\!/_2 W_{SS} S^2 \sigma^2 dt + W_S dS + W_Q dQ + W_t dt + q\left[S - \pi - A(S)\right]dt - \left(W_S / F_S\right)\Big[F_S\left[S(\mu - r) + C\right]dt + F_S S\sigma dz\Big]$$
$$= {}^1\!/_2 W_{SS} S^2 \sigma^2 dt + W_S dS - qW_Q dt + W_t dt + q\left[S - \pi - \alpha - \beta S\right]dt - \left(W_S\right)\Big[\left[S(\mu - r) + C\right]dt + S\sigma dz\Big]$$
$$= {}^1\!/_2 W_{SS} S^2 \sigma^2 dt + W_S dS - qW_Q dt + W_t dt + q\left[S(1 - \beta) - (\pi + \alpha)\right]dt - \left(W_S\right)\left[S\mu dt - rSdt + Cdt + S\sigma dz\right]$$
$$= {}^1\!/_2 W_{SS} S^2 \sigma^2 dt + W_S dS - qW_Q dt + W_t dt + q\left[S(1 - \beta) - \pi - \alpha\right]dt - \left(W_S\right)\left[S\left(\mu dt + \sigma dz\right) - rSdt + Cdt\right]$$
$$= {}^1\!/_2 W_{SS} S^2 \sigma^2 dt + W_S dS - qW_Q dt + W_t dt + q\left[S(1 - \beta) - \pi - \alpha\right]dt - W_S dS + rSW_S dt - CW_S dt$$
$$= {}^1\!/_2 W_{SS} S^2 \sigma^2 dt + W_t dt - qW_Q dt + q\left[S(1 - \beta) - \pi - \alpha\right]dt + rSW_S dt - CW_S dt$$
$$\Big[{}^1\!/_2 W_{SS} S^2 \sigma^2 dt + W_t - qW_Q + rSW_S - CW_S + q\left[S(1 - \beta) - \pi - \alpha\right]\Big]dt = rWdt$$
$${}^1\!/_2 W_{SS} S^2 \sigma^2 + W_t - qW_Q + rSW_S - CW_S - rW + q\left[S(1 - \beta) - \pi - \alpha\right] = 0$$
$${}^1\!/_2 \sigma^2 S^2 W_{SS} + (r - h)SW_S - qW_Q + W_t - rW + q_0 e^{-\rho t}\left[S(1 - \beta) - \pi - \alpha\right] - q_0 e^{-\rho t} W_Q = 0 \qquad (16)$$

$${}^1\!/_2 \sigma^2 S^2 W_{SS} + (r - h)SW_S + W_t - rW + q\left[S(1 - \beta) - \pi - \alpha\right] - qW = 0$$
$${}^1\!/_2 \sigma^2 S^2 W_{SS} + (r - h)SW_S - \frac{1}{2}\sigma^2 W_\tau - rW + q\left[S(1 - \beta) - \pi - \alpha\right] - qW_Q = 0$$

where

$$t = T - \frac{\tau}{\frac{1}{2}\sigma^2}$$

Now we need a transformation of the spot price, S, as shown below:

$$S = S_0 e^X$$

$$\frac{dS}{dX} = S_0 e^X$$

$$\frac{d}{dS} = \frac{dX}{dS}\frac{d}{dX} = \frac{1}{k}e^{-X}\frac{d}{dX}$$

$$S\frac{d}{dS} = S_0 e^X \frac{1}{k}e^{-X}\frac{d}{dX} = \frac{d}{dX}$$

$$\frac{d^2}{dS^2} = \frac{1}{S_0}e^{-X}\frac{d}{dX}\left(\frac{1}{S_0}e^{-X}\right) = -\frac{1}{S_0^{\,2}}e^{-2X}\frac{d}{dX} + \frac{1}{S_0^{\,2}}e^{-2X}\frac{d^2}{dX^2}$$

$$S^2\frac{d^2}{dS^2} = S_0^{\,2}e^{2X}\left(-\frac{1}{S_0^{\,2}}e^{-2X}\frac{d}{dX} + \frac{1}{S_0^{\,2}}e^{-2X}\frac{d^2}{dX^2}\right) = -\frac{d}{dX} + \frac{d^2}{dX^2}$$

Using the transformation to the spot price, S, we obtain:

$$\tfrac{1}{2}\sigma^2\left(-\frac{d}{dX} + \frac{d^2}{dX^2}\right) + (r-h)\frac{d}{dX} - \frac{1}{2}\sigma^2 W_\tau - rW + q\left[S_0 e^X(1-\beta) - \pi - \alpha\right] - qW_Q = 0$$

$$-\frac{d}{dX} + \frac{d^2}{dX^2} + \frac{(r-h)}{\tfrac{1}{2}\sigma^2}\frac{d}{dX} - W_\tau - \frac{r}{\tfrac{1}{2}\sigma^2}W + \frac{q}{\tfrac{1}{2}\sigma^2}\left[S_0 e^X(1-\beta) - \pi - \alpha\right] - \frac{q}{\tfrac{1}{2}\sigma^2}W_Q = 0$$

$$-\frac{d}{dX} + \frac{d^2}{dX^2} + (k_r - k_h)\frac{d}{dX} - W_\tau - \frac{r}{\tfrac{1}{2}\sigma^2}W + \frac{q}{\tfrac{1}{2}\sigma^2}\left[S_0 e^X(1-\beta) - \pi - \alpha\right] - \frac{q}{\tfrac{1}{2}\sigma^2}W_Q = 0$$

where

$$k_r = \frac{r}{\tfrac{1}{2}\sigma^2} \qquad k_h = \frac{h}{\tfrac{1}{2}\sigma^2} \qquad k_q = \frac{q}{\tfrac{1}{2}\sigma^2} = \frac{q_0 e^{-\rho\left[T - \frac{\tau}{\frac{1}{2}\sigma^2}\right]}}{\tfrac{1}{2}\sigma^2}$$

$$W_{XX} + (k_r - k_h - 1)W_X - W_\tau - k_r W + k_q\left[S_0 e^X(1-\beta) - \pi - \alpha\right] - k_q W_Q = 0$$

where

$$W = e^{AX+R\tau}u(X,Q,\tau)$$

$$W_\tau = RW + e^{AX+R\tau}u_\tau(X,Q,\tau)$$

$$W_X = AW + e^{AX+R\tau}u_X(X,Q,\tau)$$

$$W_{XX} = A\left[A + e^{AX+R\tau}u_X(X,Q,\tau)\right] + Ae^{AX+R\tau}u_X(X,Q,\tau) + e^{AX+R\tau}u_{XX}(X,Q,\tau)$$

$$W_Q = Q_0 e^{AX+R\tau}u(X,Q,\tau)$$

$$W_{XX} + (k_r - k_h - 1)W_X - W_\tau - k_r W + k_q\left[S_0 e^X(1-\beta) - \pi - \alpha\right] - k_q W_Q = 0$$

$$A\left[AW + e^{AX+R\tau}u_X(X,Q,\tau)\right] + \alpha e^{AX+R\tau}u_X(X,Q,\tau) + e^{AX+R\tau}u_{XX}(X,Q,\tau)$$

$$+(k_r - k_h - 1)AW + (k_r - k_h - 1)e^{AX+R\tau}u_X(X,Q,\tau) - RW - e^{AX+R\tau}u_\tau(X,Q,\tau)$$

$$-k_r W + k_q\left[S_0 e^X(1-\beta) - \pi - \alpha\right] - k_q Q_0 W = 0$$

$$e^{\alpha X+R\tau}u_{XX}(X,Q,\tau) + \left(2\alpha + (k_r - k_h - 1)\right)e^{\alpha X+R\tau}u_X(X,Q,\tau) + (\alpha^2 + (k_r - k_h - 1)\alpha - k_r - R - k_q Q_0)W$$

$$-e^{\alpha X+R\tau}u_\tau(X,Q,\tau) + k_q\left[S_0 e^X(1-\beta) - \pi - \alpha\right] = 0$$

$$A = \frac{1}{2}(k_h - k_r + 1) \qquad R = \frac{1}{4}(k_h - k_r + 1)^2 + k_r - k_q Q_0$$

$$\boxed{u_{XX}(X,Q,\tau) - u_\tau(X,Q,\tau) = e^{-AX-R\tau}k_q\left[S_0 e^X(1-\beta) - \pi - \alpha\right]} * heat - tranfer\ equation$$

$$u_{XX}(X,Q,\tau) - u_\tau(X,Q,\tau) = p(X,\tau)$$

$$u(X,Q,\tau) = \int_0^{\tau}\int_0^{\infty}\frac{p(y,n)}{2\sqrt{\pi(\tau-n)}}\left\{e^{\left[-\frac{(X-y)^2}{4(\tau-n)}\right]} - e^{\left[-\frac{(X+y)^2}{4(\tau-n)}\right]}\right\}dydn$$

where

$$p(X,\tau) = e^{-AX-R\tau}q_0 e^{-\rho\left[T - \frac{\tau}{{}^1\!/_2\sigma^2}\right]}\left[S_0 e^X(1-\beta) - T - \alpha\right]$$

$$W(X,Q,\tau) = e^{\alpha X+R\tau}u(X,Q,\tau)$$

$$W(X,Q,\tau) = \int_0^{\tau}\int_0^{\infty}\frac{q_0 S_0(1-\beta)e^{-\rho\left[T - \frac{\tau}{{}^1\!/_2\sigma^2}\right]+X}}{2\sqrt{\pi(\tau-n)}}\left\{e^{\left[-\frac{(X-y)^2}{4(\tau-n)}\right]} - e^{\left[-\frac{(X+y)^2}{4(\tau-n)}\right]}\right\}dydn$$

$$+\int_0^{\tau}\int_0^{\infty}\frac{-q_0(\pi+\alpha)e^{-\rho\left[T - \frac{\tau}{{}^1\!/_2\sigma^2}\right]}}{2\sqrt{\pi(\tau-n)}}\left\{e^{\left[-\frac{(X-y)^2}{4(\tau-n)}\right]} - e^{\left[-\frac{(X+y)^2}{4(\tau-n)}\right]}\right\}dydn$$

$$W(X,Q,\tau)=\frac{q_0S_0(1-\beta)e^{-\rho T}}{2}\int_0^{\tau}\int_0^{\infty}\frac{e^{\frac{\rho n}{\frac{1}{2}\sigma^2}-\frac{[2(\tau-n)+X]^2+X^2}{4(\tau-n)}}}{\sqrt{\pi(\tau-n)}}\left\{e^{-\frac{([2(\tau-n)+X]-y)^2}{4(\tau-n)}}-e^{-\frac{([2(\tau-n)+X]+y)^2}{4(\tau-n)}}\right\}dydn$$

$$+\int_0^{\tau}\int_0^{\infty}\frac{-q_0(\pi+\alpha)e^{-\rho\left[T-\frac{\tau}{\frac{1}{2}\sigma^2}\right]}}{2\sqrt{\pi(\tau-n)}}\left\{e^{\left[-\frac{(X-y)^2}{4(\tau-n)}\right]}-e^{\left[-\frac{(X+y)^2}{4(\tau-n)}\right]}\right\}dydn$$

where

$$S=S_0e^X \qquad x=\ln(S/S_0) \qquad t=T-\frac{\tau}{\frac{1}{2}\sigma^2}\rightarrow\tau=(T-t)\frac{1}{2}\sigma^2$$

Boundary Conditions:

$W(S,0,t)=0$

$W(0,Q,t)=0$

$W(S,Q,T)=k*$

Annex 2.

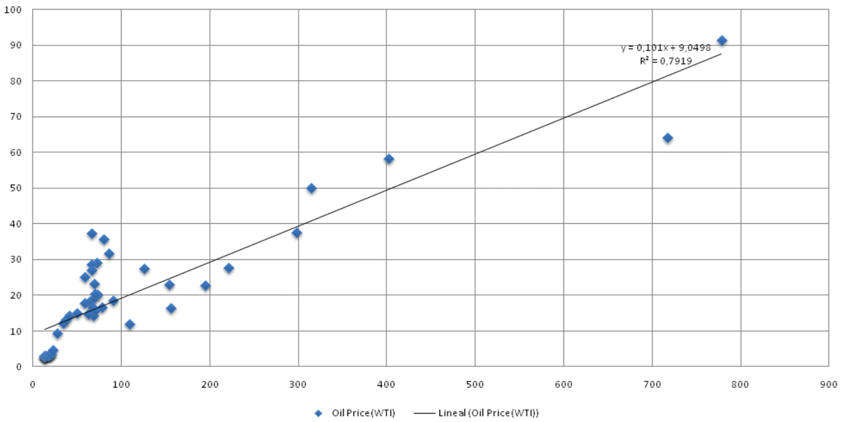


Figure 5. The regression WTI oil price versus drilling cost per feet (source: EIA)

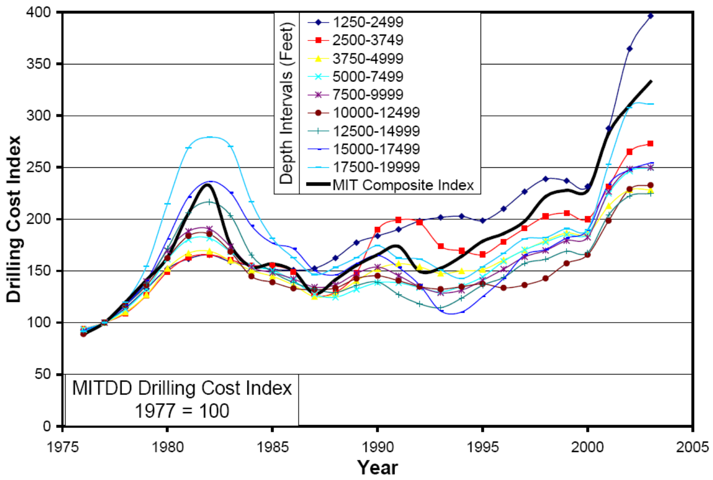


Figure 6. Drilling cost index 1977 U.S. $. Source: MIT & EAI

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