Financial Valuation of Operational Flexibilities in the Aluminum Industry using Real Option Theory

Carlos de Lamare Bastian-Pinto
Department of Management - Ibmec Business School
Av. Presidente Wilson 118 - Centro
Rio de Janeiro, 20030-020, RJ, Brasil
+55 21 9496-5520
carlos.bastian@ibmecrj.br

Luiz Eduardo Teixeira Brandão
IAG Business School - Pontifícia Universidade Católica - Rio de Janeiro
Rua Marques de São Vicente 225 - Rio de Janeiro, 22451-900, RJ, Brasil
+ 55 21 2138-9304
brandao@iag.puc-rio.br

Rafael Raphael
IAG Business School - Pontifícia Universidade Católica - Rio de Janeiro
Rua Marques de São Vicente 225 - Rio de Janeiro, 22451-900, RJ, Brasil
+ 55 21 2138-9304
rafaelraphael@gmail.com

Luiz de Magalhães Ozorio
Department of Management - Ibmec Business School
Av. Presidente Wilson 118 - Centro
Rio de Janeiro, 20030-020, RJ, Brasil
+55 21 9923-1747
lmozorio@ibmecrj.br

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Abstract

In the aluminum industry, which is subject to a significant volatility in its output prices, as well as in the cost associated with one of its main input costs, electricity, the possibility of reducing production costs through the flexibility of the production process can generate important value for such real options. We study the effect of the flexibility available to a typical smelter (aluminum processing plant), that buys its electricity through long term contracts or alternatively owns a co-generation unit, of stopping production and selling its electricity in the spot market, when the price of its output, aluminum, generates negative or unrewarding cash flows, or when the spot price of electricity is high enough to give a higher cash flow than from its normal production process. Nevertheless stopping, and more specifically, restarting production involves costs associated with refurbishing the smelters units thermal revetment that may hamper the value of such stoppage options. This paper values such options through Monte Carlo simulation, modeling the prices of aluminum as a geometric mean reversion, and the price of electricity as a mean reversion with positive jumps. It also incorporates the asymmetrical costs of stopping and restarting production and checks its influence in the value of the option associated.

Keywords

Real options, switch options, managerial flexibility, stochastic processes, aluminum industry,

1. Introduction

In a high volatility economic environment, such as the present, price uncertainty of commodities can exert a high level of influence on the investment decision process as well as on return of these investments. The aluminum sector in one of those industries highly affected by market uncertainties. Aluminum producing plants usually have several electrolytic reduction units , known as Smelters, which imply in huge investment expenditures characterized by irreversibility and have their operation highly dependent on volatile prices such as electric energy and aluminum itself.

Adoption of different operation strategies may help the plant hindering the risk from input and output prices volatility, and gain from reduction costs or maximization of output income. As these plants are fundamentally dependent on electricity supply, they usually have co-generation assets which can supply at least part of their energy necessity. Alternatively they rely on long term energy supply contracts either with the Grid Company or independent power companies. At specific periods when the cash flows generated by selling of aluminum is not enough to cover production costs, or alternatively when selling of the electricity available to them either through co-generation or from their long term supply contracts, would a better rewarding cash flow, then the stoppage of aluminum production and selling of available energy may be a real option available to the smelter that can significantly increase the plant value.

This valuable decision making process, generated through the managerial flexibility linked to the uncertainties from commodities prices, does not have its value captured through traditional valuation methodologies such as discounted cash flow. Instead it can be valued as a real option which appraises the correct present value of the output switch option available to this kind of plant. But in the aluminum industry, as is also the case of the steel industry, stopping and more specifically, restarting production involves significant costs associated with refurbishing the smelters units thermal revetment. These costs work against the value created by such stoppage options. Nevertheless in this paper we value such options using Monte Carlo simulation since such switch options can be characterized as bundles of European real options. We model the prices of aluminum as a geometric mean reversion stochastic process, and the price of electricity as a mean reversion with positive jumps. We also incorporate the asymmetrical costs of stopping and restarting production and verify its influence in the value of the switch option associated. We assume that the exercise of the stoppage option is decided at the start of each period (in this case a semester) and it is modeled as a bundle of European options, solved through Monte Carlo simulation. The model was applied to a hypothetical aluminum smelter in Brazil with a capacity of 500 thousand tons per year capacity. Nevertheless all data is based on real cases.

This paper is structured as follows: chapter 2 presents a theoretical background for real optins the case to be studied and chapter 3 the methodology applied and stochastic modeling of the uncertainties involved. Chapter 4 presents the case to be studied. Results are presented in chapter 5 and conclusion in chapter 6.

2. Theoretical Background

Tourinho (1979) was actually the first to use the option appraisal technique developed by Black & Sholes (1973) do model a real asset valuation problem of natural resources reserves. This author considered that oil extraction involves costs related to reserve maintenance, extraction costs increase with time and that storage costs involve new investments for the company. Oil price is modeled as a stochastic Gauss-Wiener process.

After Tourinho´s seminal work (1979) real options techniques expanded and were broadly applied in projects of oil, gas, energy, among other, being the object of studies from several authors such as Brennan & Schwartz (1985); Morck, Schwartz, & Stangeland (1989);. Paddock, Siegel, & Smith (1988); Schwartz (1997); Schwartz (1998); Siegel, Smith, & Paddock (1987); Trigeorgis (1990); Tufano (1998), among others. The energy is expected to continue attracting academic research with the use of real options, especialy due to the recent expansion of the private sector together with the development of emerging countries.

According to Vidal, Motta, Gomes & Oliveira (2011) high expectations related to income let mining companies to reevaluate their assets and look for new business opportunities. Brennan & Schwartz (1985) apply real options theory to a mine valuation, modeling the flexibility of changing output production according to the evolution of prices, together with the possibility of abandoning the project. The authors even consider the possibility of change in the risk the project during its lifetime, due to the possible exhaustion of natural reserves and random variation of prices.

Kulatilaka (1993) analyses the value of the flexibility available in the switch options of an industrial boiler that can alternatively use oil or natural gas. The author shows that the gains obtained with the reduction in costs obtained from the operation flexibility greatly surpasses the investments necessary for the acquisition of a bi-fuel boiler.

Slade (2001) values the managerial flexibility in investments in copper mining in Canada. His study focusses on flexible operations, stressing the fact that temporary shutdowns are more commonly observed that opening or closing of operations. This author develops a Mean Reversion Model (MRM) stochastic model for prices behavior, instead of the commonly used Geometric Brownian Motion (GBM) model, a premise also adopted by Bessembinder, Coughenour, Seguin & Smoller (1995) and Schwartz (1997).

This same approach of a Mean Reversion Model (MRM) is used by Bastian-Pinto, Brandão & Hahn (2009), in an analysis of the flexibility available in the production of biofuels from sugarcane in Brazil. Ethanol producing plants in this country can easily switch from outputting ethanol or sugar from the same source, and use this flexibility as market conditions change. They use a bivariate binomial discrete model for both stochastic prices (sugar and ethanol). Results show that MRM modeling appears to be more suited for such prices and that GBM modeling tends to overestimate the switch option value, when modeling commodity prices.

Still Bastian-Pinto, Brandão & Alves (2010) analyze the switch option available to flex fuel car owners as they can choose the cheapest fuel to fill they vehicles (ethanol or gasoline). The authors also use GBM and MRM for the prices modeling and also find the latter as better suited for these commodities yet both models prove that the flexibility has a significant value. Ozorio, Bastian-Pinto, Baidya & Brandão (2013) value the temporaty shutdown option, or even partial shutdown, in semi-integrated stell-mils, or mini-mils, in an modeling closely related to that of this present work. Yet they do not consider exercise costs since these are almost irrelevant in that sector, contrary to the case of aluminum smelters.

Relatively to the cases where real options are applied to the aluminum sector, it is common to see the modeling of value from flexible operations and incremental value of input and output switch options. Beyond the metal volatility itself this industry is highly subject to effects of uncertainties related to prices and availability of electrical energy, one of its main inputs.

Subodh, Long III, Hayden, Green & Hunt Jr. (2004) describe the implications related to aluminum supply and energy. They analyze the industry issues emphasizing the potential for energy economy in this sector through the switch of input metal to recycled metal, even in prejudice of the primary production of aluminum. These authors sustain that industries have been relocating units mostly as a function of energy costs. Primary production of the metal is being moved to areas where availability and cost of energy is favorable, whereas other locations are starting to import energy in the form of aluminum ingots.

Regarding output switch Byko (2002) emphasizes the tendency that occurred in the United States during a period of scarce energy supply and low aluminum demand, when aluminum producers shutdown their plants and started to sell the energy available to them. In this same paper the author comments on the necessity of Brazilian smelters to reduce their production due to government energy consumption reduction goals issued in face of the energy shortage the country went through at the time.

This same approach is described in Avilés (2009) in a study where smelters self-sufficient in energy are valued through real options theory. The author models a hypothetical smelter representative of the Brazilian sector, and under the effects of the volatility of energy and aluminum analyses the possible mixes of outputs of the smelter using GBM as stochastic process. Results indicate existence of a significant value for the options studied. Raphael (2010) also analyses the flexibility of changing inputs in a smelter considering cost related to the change in operation.

3. Model and Methodology

3.1. Uncertainty Modeling with Mean Reversion Models

Contrary to Geometric Brownian Motion modeling (GBM), with Mean Reversion Models (MRM) there is a tendency of the uncertainty to revert to a long term equilibrium value or mean. The logic underlying this reasoning, which applies well to most commodity prices, comes from microeconomics: when prices are below their long term mean demand will rise, while offer tends to drop due to its low remuneration, and therefore driving prices up again. The inverse will happen when prices are well above the long term equilibrium level.

In this paper we adopted a single factor geometric mean reversion model, based on that of Schwartz (1997) model 1, described by equation (0):

$$dS = \eta(\alpha - \ln[S])Sdt + \sigma Sdz \tag{0}$$

where:

- S is the stochastic variable,
- α is the log of the long term equilibrium level
- η is the mean reversion speed parameter
- σ is the volatility if the process
- dz is a standard Weiner increment, with normal distribution $dz = \varepsilon\sqrt{dt}$, $\varepsilon \sim N(0,1)$, and dt the time increment of the process.

We adapt this model to a more intuitive for our application, by using α as the log of the long term equilibrium level: $\alpha = \ln\left[\bar{S}\right]$, so equation (0) can be written as equation (0).

$$dS = \eta\left[\ln\bar{S} - \ln S\right]Sdt + \sigma Sdz \qquad (0)$$

Therefore the expected value and variance expression of the log ($x_t = \ln\left[S_t\right]$) of this process are:

$$E\left[x_t\right] = \ln\left[S_{t_0}\right]e^{-\eta(t-t_0)} + \left[\ln\left(\bar{S}\right) - \frac{\sigma^2}{2\eta}\right]\left(1 - e^{-\eta(t-t_0)}\right), \qquad \text{and:}$$

$$\operatorname{var}\left[\ln\left(S_t\right)\right] = \operatorname{var}\left[x_t\right] = \frac{\sigma^2}{2\eta}\left(1 - e^{-2\eta(t-t_o)}\right)$$

For simulation of the real process we use the following discrete time equation which is obtained through to log-normal property of the S_t process (Bastian-Pinto, 2009).

$$S_t = \exp\left\{\ln\left[S_{t-1}\right]e^{-\eta\Delta t} + \left[\ln\left(\bar{S}\right) - \frac{\sigma^2}{2\eta}\right]\left(1 - e^{-\eta\Delta t}\right) + \sigma\sqrt{\frac{1 - e^{-2\eta\Delta t}}{2\eta}}N\left(0,1\right)\right\} \qquad (0)$$

Parameter estimation based on historical series can be done using the following procedure: using the log return of the expression above, it can be written as:

$$\ln\left(S_t/S_{t-1}\right) = \underbrace{\left(1 - e^{-\eta\Delta t}\right)\left(\ln\bar{S} - \sigma^2/2\eta\right)}_{a} + \underbrace{\left(e^{-\eta\Delta t} - 1\right)}_{b-1}\ln S_{t-1} + \varepsilon_t, \qquad \text{or:}$$

$$x_t - x_{t-1} = a + (b-1)x_{t-1} + \varepsilon_t \qquad (0)$$

Running a simple linear regression on this equation:

$$\eta = -\ln(b)/\Delta t \qquad (0)$$

The volatility parameter can be estimated using the variance of errors ε of the same regression: σ_ε^2 which is given by equation: $\sigma_\varepsilon^2 = \frac{\sigma^2}{2\eta}\left(1 - e^{-2\eta\Delta t}\right)$, and using: $b^2 = e^{-2\eta\Delta t}$ we obtain:

$$\sigma = \sigma_\varepsilon\sqrt{\frac{2\ln b}{(b^2 - 1)\Delta t}} \qquad (0)$$

Also: $a = \left[\ln\bar{S} - \sigma^2/2\eta\right]\left(1 - e^{-\eta\Delta t}\right)$. With : $1 - b = 1 - e^{-\eta\Delta t}$ we have:

$$\frac{a}{(1-b)} = \left[\ln\bar{S} - \sigma^2/2\eta\right]$$

and: $$\bar{S} = \exp\left[\frac{a}{(1-b)} + \frac{\sigma^2}{2\eta}\right] \qquad (0)$$

Or:

$$\bar{S} = \exp\left[\frac{a}{(1-b)} + \frac{\sigma^2 \Delta t}{2(-\ln b)}\right]$$

Valuing projects with real options must be done using a risk neutral approach. With an MRM modeling this is converted to risk neutral through adjustment of its drift parameter (A. Dixit & R. Pindyck, 1994).

With:

- μ - risk adjusted discount rate
- α - process drift
- δ - dividend yield of the process, or for commodities, convenience yield
- r - risk free rate

For a risk adjusted process we have: $\mu = \alpha + \delta$ or $\alpha = \mu - \delta$. In the risk neutral form the process drift α is replaced by: $r - \delta$. As with mean reversion the drift rate is $\alpha = \eta\left(\ln\left[\bar{S}\right] - \ln\left[S\right]\right)S$, and, as opposed to GBM, the dividend yield is not constant but a function of *S*: $\delta = \mu - \alpha = \mu - \eta\left(\ln\left[\bar{S}\right] - \ln\left[S\right]\right)S$. The final risk free simulation form is seen as equation (0):

$$S_t = \exp\left\{\ln\left[S_{t-1}\right]e^{-\eta\Delta t} + \left[\ln\left(\bar{S}\right) - \frac{\sigma^2}{2\eta} - \frac{\pi}{\eta}\right]\left(1 - e^{-\eta\Delta t}\right) + \sigma\sqrt{\frac{1 - e^{-2\eta\Delta t}}{2\eta}}N(0,1)\right\} \qquad (0)$$

Where: $\pi = (\mu - r)$ is the process risk premium.

3.2.Modeling Mean Reversion with Jumps

Dias & Rocha (1999) study the behavior of oil prices and point out these tend to show discrete jumps related to atypical information or events. These authors propose a stochastic modeling based on a geometric MRM coupled with random jumps. In this work a similar model is used for energy prices in the Brazilian unregulated market, which clearly appears to have a jump component. This mixed diffusion process associates softer variations described by the MRM component, together with positive random jumps which result from atypical events and are modeled through a Poisson process.

This model can be described with equation (0).

$$dS = \eta\left[\ln\bar{S} - \ln S\right]Sdt + \sigma Sdz + dq \qquad (0)$$

Where *dq* is the Poisson process, which is assumed not correlated to the Wiener *dz* process and has the following assumptions.

$dq = 0$; with probability: $1 - \lambda dt$

$dq = \phi$; with probability: λdt

Where λ is the frequency of jumps occurrence, and ϕ is the distribution of jump size. It has to be pointed out that the modeling used in this article considers that the jumps of energy prices are uncorrelated to the market and therefore have a null risk premium. With this assumption all the risk adjustment of the energy price process is made with its mean reversion component, as described in the previous chapter.

3.3. Stochastic Modeling of Aluminum Price

In order to define and calibrate the stochastic behavior of aluminum prices, historical prices in US$/metric ton published in the London Metal Exchange (LME) from December 1982 to April 2013 (monthly basis) where used as database. This series were adjusted for US inflation using Consumer Price Index – CPI obtained in Bloomberg system, transformed in april 2013 price basis. Figure 1 shows the historical behavior of aluminum prices in April 2013 US$/Ton.

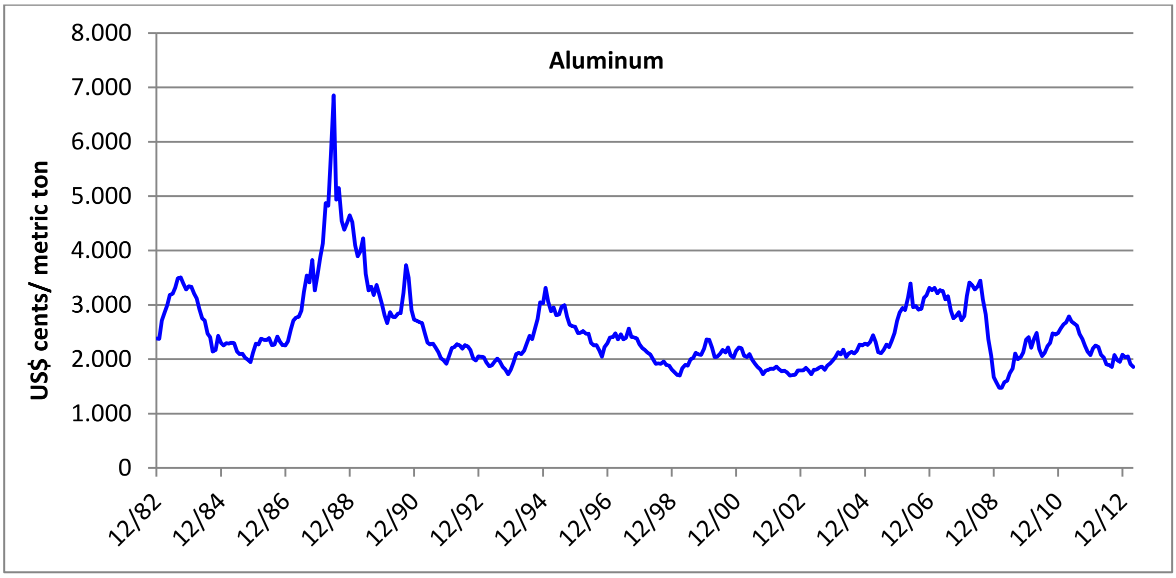

Figure 1: Monthly Aluminum Prices in 04/2013 US$
Source: London Metal Exchange

Finally using equations (0) to (0) the MRM for aluminum prices was calibrated. The regression described by equation (0) can be seen in Figure 2. Risk premium of the process was estimated by numerical procedure using the cash flow of the base case described below and discount rate adjusted and risk neutral. Parameters of the calibrated model are shown in Table 1.

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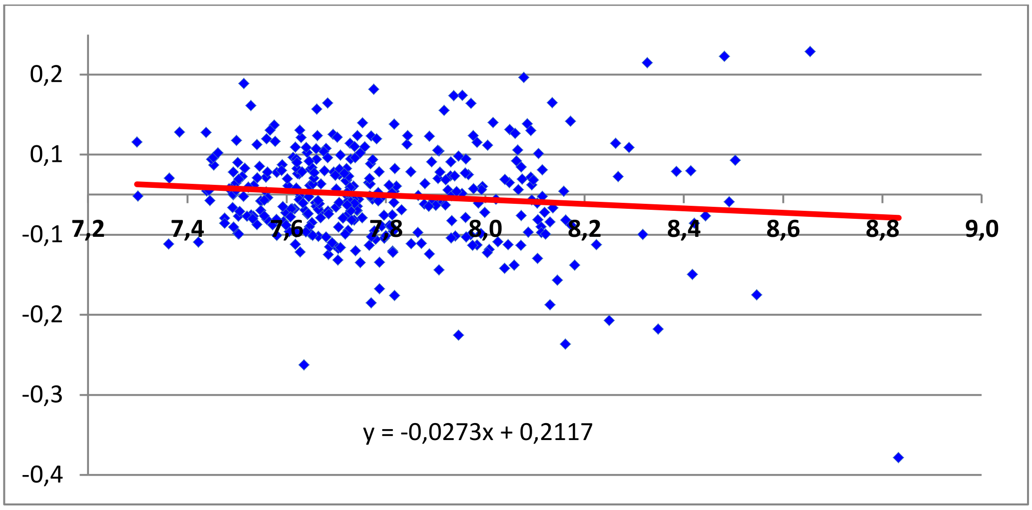

Figure 2: linear regression on aluminum prices log

Table 1: MRM parameters for aluminum prices

Parameters for Mean Reversion – Aluminum Prices	
Aluminum Initial Price	1,861.02 $ US / ton
Long term mean $\bar{S}_A$	2,513.16 $ US / ton
Volatility per year - σ_A	20.39 %
Mean Reversion Speed per year - η_A	0.332
Normalized risk premium – π/η	0.0450
Risk Adjusted Long term mean - $\bar{S}_A^*$	2,304.65 $ US / ton

In Figure 3 are plotted only 200 trajectories, for ease of visualization, using the calibrated process of equation (0)

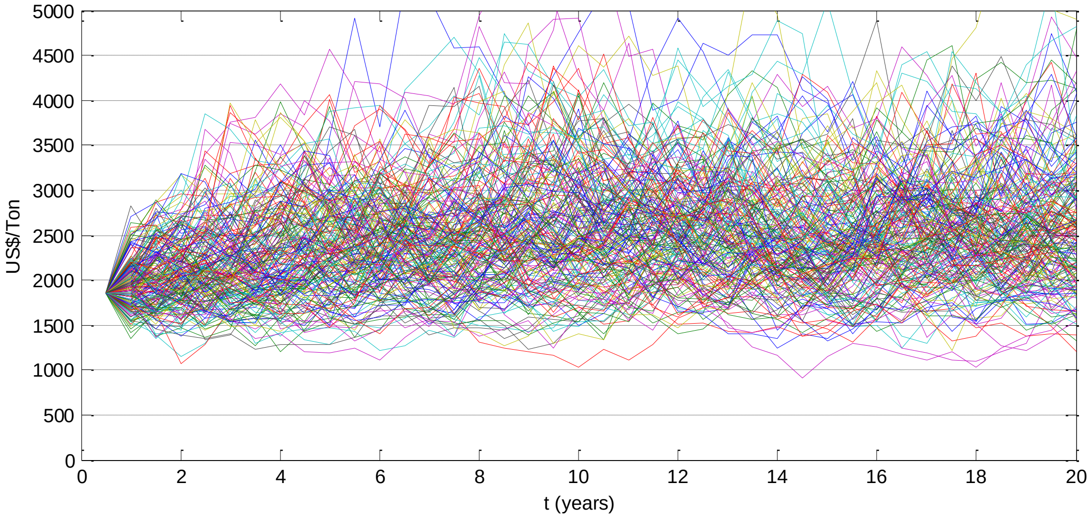

Figure 3: Aluminum prices trajectories (150) simulation

3.4. Stochastic modeling of Energy Price – MRM with Jumps

For the modeling and calibration of energy prices in the Brazilian unregulated market, the PLD (*Preço de Liquidação das Diferenças*) price was used. It stands for Settlement Price Difference, and is the price used for selling and buying energy in the short term market, which uses as base the data that the ONS (the agency that controls the dispatch of energy generation) for the optimization of the country´s integrated electrical system. And it does that considering the equilibrium between the benefits of future use and storage of water in the reservoirs or the cost of immediate generation through the dispatch of thermal plants.

The calculation of PLD prices are done through computational models that estimate a Marginal Cost of Operation (CMO) for each of the four sub-markets of the country. The CMO is obtained through an optimization method equivalent to Lagrange´s multiplier associated to a demand restriction. The trade-off in this market consists in the choice of the best time for hydro or thermal generation. This is due to the fact that an excessive use of hydro power at this moment, of which costs are minimal, might imply in a high future cost with thermal generation in times of low rainfall, periods when reservoirs are at their lowest. On the other hand if water is saved and rainfall inflows are high, a spillover of the reservoirs may be necessary representing wastage of energy and increase in operational costs. Historical values for PLD for most representative sub-market (South East-Center West: SECO) the can be observed in Figure 4.

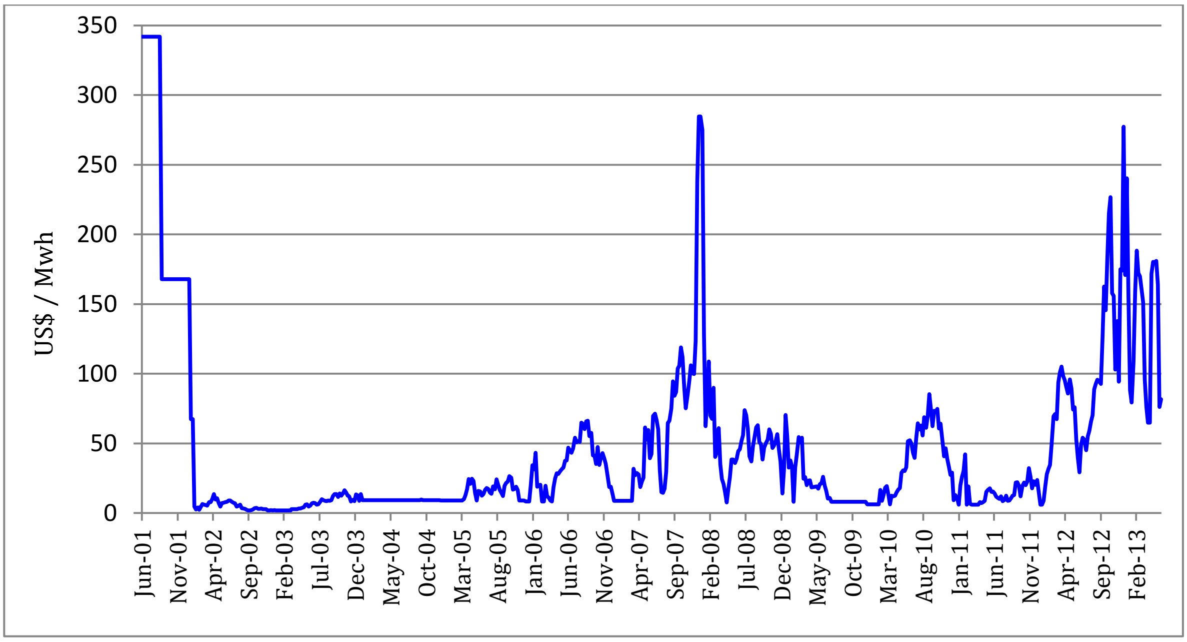

Figura 4 – Historical PDL for SE-CO sub market

Therefore the generation capacity of hydropower plants depends on rain inflows occurred up to that moment as well as forecasting of rainfall for the next periods and operative decisions taken up to that moment. So the minimum cost of energy in a time horizon takes into account different inflow scenarios which in turn generate different operational decisions (Barros, Mello, & Souza, 2011).

In Brazil´s case, where most of the energy matrix derives from hydropower, the total cost function is the sum of the instant and future cost. Simões & Gomes (2011) define the instant cost function (FCI) as the thermal generation costs at time *t*. This cost grows as hydro generation drops. Whereas future cost is related to expected thermal generation expenses from time *t* up to the final time of the forecasting (simulation).

Although, the historic PLD series shown in Figure 4 suggest a mean reversion behavior together with jumps, nevertheless it does not allow to calibrate the jumps behavior process in a robust approach since only three events can be observed that can be considered jumps during its 12 years length (2001. 2007/8 and 2012/3).

Newave is the computational system that calculates the optimum price policy based on instant and future costs and takes into account in its planning a time horizon of up to five years. O calibrate the jump process diffusion part of the free market energy price in Brazil, two thousand simulations of PLD from the Newave system from January 2010 to January 2014, where used. Likewise the historical behavior of the PLD, the Newave scenarios demonstrate a stochastic behavior suggesting the presence of jumps, as can be observed in Figure 5.

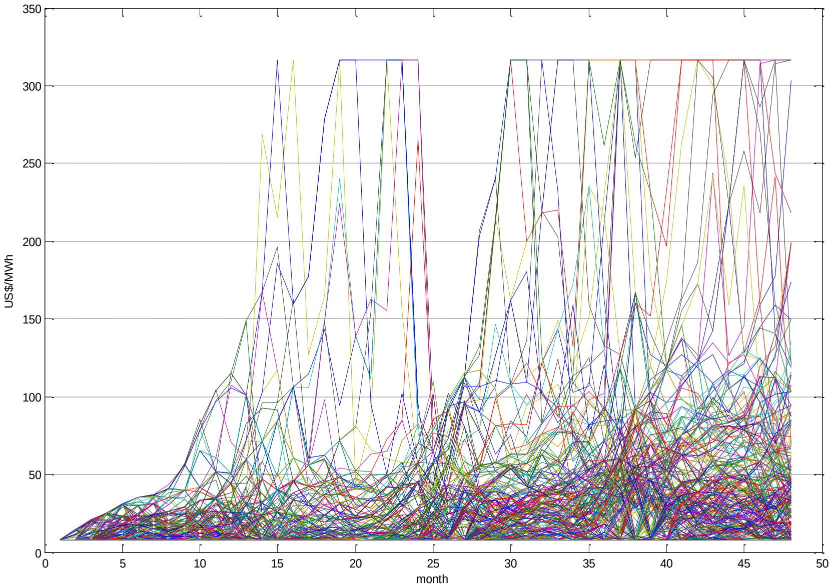

Figura 5: PLD NEWAVE scenarios (300 simulations)

It is worth mentioning that although Figure 5 is polluted by a significant number of curves (300 plots only of 2000 scenarios) generated through the PLD scenarios of Newave, its purpose is to graphically illustrate the general behavior of these prices which demonstrate a mixture of relatively smooth variations accompanied of a series of positive jumps of greater amplitude which occur at a much lower frequency. From this observation the premise of modeling of the PLD prices using a mixed MRM and jump diffusion process was adopted, using equation (11).

In order to calibrate this process some simplifying approaches are used. To estimate the frequency of jumps occurrence, a level was established above which the presence of simulated prices indicates a jump occurrence. This jump frequency was counted among the two thousand scenarios available. The premise used to define presence of a jump is a price above a level of US$125/MWh, and this level was defined through visual observation of the series plotted in Figure 5, without any sort of statistical analysis. This frequency counting gave a frequency of 5.27% of jump occurrence for each month of simulation.

But contrary to the approach used by Dias & Rocha (1999) for oil prices, where direction of the jumps is also random, observation of the series might suggest that electric energy prices long term mean appears to be closer to the bottom level of PLD scenarios and jumps occur only upwards. Besides this a model of jump intensity was adopted with a triangular frequency distribution with the following values: minimum size of jump: US$ 100/MWH, medium jump size: US$ 150/MWh, maximum jump size: US$ 200/MWh. These values are obtained by filtering all series of PLD above US$ 125/MWh, and fitting these values with a symmetrical triangular distribution, and rounding to close integer values. Dias & Rocha (1999) point out that one of the difficulties of using such a mixed process is the complexity in estimating a distribution for the size of jumps and suggest possible approaches that would surpass the objectives of this work. It was also considered that price jumps are uncorrelated to the market and therefore do not have to be compensated for risk premium, so the energy price process will only be risk adjusted in its mean reversion (MRM) component. Table 2 summarizes the premises and values used for the jump model of PLD.

Table 2: Assumptions – Jump parameters

Jump parameters for the MRM with Jumps model for PLD	
Price Level above which is considered Jump	125.00 US$/MWh
Frequency of Jumps per time period	5.27%
Size of Jumps triangular distribution - minimum	100.00 US$/MWh
Size of Jumps triangular distribution - medium	150.00 US$/MWh
Size of Jumps triangular distribution - maximum	200.00 US$/MWh

In order to calibrate the Mean Reversion component of the PLD, the historical series of PLD price was used but with a barrier at U$S 125/MWh, in order to filter for the jumps effect. Also only values after April 15 2005 were used, when the series began to behave in a more random or stochastic way, without long periods of fixed minimum values, as can be observer in Figure 4. These series where yet converted to monthly averages since the decision making to be studied in this paper is on a semester basis, and a weekly frequency such as that of the PLD series could return unrealistic parameters for this analysis. Furthermore the jumps components are also calibrated on a monthly basis.

Using the same approach as with aluminum prices applied to these series, the parameters shown in Table 3 were thus obtained. It is worth mentioning that in the simulation process of PLD values are considered restricted to a range between US$ 8.16/Mwh and US$ 316.69/MWh, since these are the minimum and maximum values accepted by the Regulating Agency (ANEEL) for the time sample studied. These prices are issued in Brazilian R$ and converted to US$ at the prevailing rate of 2,00 R$/US$ of time of the study.

Table 3: MRM parameters for energy prices - PLD

Mean Reversion Parameters for PLD	
Initial value *	109.97 US$ / MWh
Long Term Mean of Energy - $\overline{S}_E$	77.605 US$ / MWh
Standard Deviation of Energy - σ_E	127.82% aa
Mean Reversion Speed of Energy - η_E	1.007 aa
Normalized Risk Premium - PLD	0.0269
Risk Neutral Long Term Mean PLD - $\overline{S}_E^*$	72.16 US$ / MWh
Risk Neutral Long Term Mean compensated for only positive jumps - $\overline{S}_E^{**}$	57.61 US$ / MWh

* PLD value at the momento of this study (monthly average in may 25th 2013)

The Risk premium of the process for the PLD was obtained through numerical methods in order to discount the projects cash flows at the risk free rate. And it was still necessary to compensate the long term mean of the MRM process for the added jumps which are only positive in this study in order to have a process that will return the same value of the deterministic process in a simulation analysis. This was done using a numerical approach and the risk neutral and compensated long term mean $\overline{S}_E^{**}$ to be used in Monte Carlo simulations is also listed in Table 3.

4. Application to a Hypothetical Aluminum Smelter Case

In an aluminum smelter alumina (aluminum oxide) is reduced and transformed in metal aluminum. Among the smelter main costs is the price of electricity needed for the reduction reaction, which is generally acquired through long term contracts (or even through co-generation) so the smelter is not exposed to the high volatility of energy spot prices, as shown in Figure 4. Therefore there is an opportunity of maximizing the smelter´s cash flows when taking into account the volatilities of aluminum and energy prices.

One possibility available to the smelter is to temporarily suspend the alumina reduction process, and sell the available energy instead, since it is already contracted through long term contracts or from co-generation. It can be observed in the aluminum sector that in regions where the cost of energy is relatively high, aluminum producing plants have opted for closing smelters and or moving to regions with a higher degree of energy availability and lower costs. In the case of temporary stoppage pre-established sales contracts have been fulfilled through the acquisition of metal aluminum in the metal market, as studied by Raphael (2010).

But considering that energy price in Brazil is highly volatile and that the possibility of cash flow optimization due to the rise in these prices might be a temporary event, it is improbable that the possible stoppage of smelters be definite, since they are the result of significant capital investments, characterized by irreversibility. Therefore and considering that the uncertainty in aluminum and energy prices will most certainly re render the smelter main activity competitive again, then it is possible to envision the opportunity of economic gains through the flexibility of temporary stoppage of the smelter operation.

Depending on the level of Market price of aluminum and energy, the manager will opt for the higher return alternative, taking into account that this decision might also result in asymmetric costs for the firm. The flexibility studied in this paper considers that the smelter can temporarily shut down its aluminum production and instead sell in the spot market (highly volatile) the energy it already has pre-contracted at a fixed price.

In case the smelter is operating in the normal operational mode 1 (aluminum production) in a given moment and the cash flow estimates for the next period in mode 2 (only selling available energy at spot price) are lower compared to those projected to mode 1, the decision for the next period is to maintain the operation as is (mode 1). On the other hand if the expected cash flow from the commercialization of energy (mode 2) is higher than that of energy and compensates the costs of changing from mode 1 to mode 2, then the option will be exerted and there will be a change in operation mode. This logic applies similarly and inversely, that is, when the smelter is only selling energy (mode 2) and the cash flow of aluminum is higher and compensated the operation switching costs, then the smelter will go back to its normal operation mode.

4.1. Model Assumptions

The real operations of an aluminum smelter involve a diversity of issues whose complexity outranges the scope of this paper. Therefore only are represented essential characteristics up to a level of details necessary to illustrate a coherent approach of a typical smelter in Brazil. So simplifying assumptions are made nevertheless these are aligned with representative references of the sector. These are estimated based on market parameters and information, available from public sources.

Tables 4 and 5 represent the main assumptions adopted in the model.

Table 4 - Assumptions of operation mode 1 – Aluminum production

Assumptions	**Quantity/ Values**
Installed capacity	500,000 tons per year
Reduction units	500 units
Energy consumption	15.88 Megawatt-hour per ton of produced aluminum
Pre-contracted energy cost	34 US$/MWh
Alumina cost	14.5% of aluminum price, at the London Metal Exchange (LME), + 37 US$/t
Alumina consumption	1.92 t alumina/t produced aluminum
Aluminum price	Stochastic variable based on LME indicators.
Other production costs	640 US$/ton of produced aluminum – source: Brook Hunt & ABAL

The cost of Pre-contracted energy was considered as US$ 34 / Mwh considering average sales contracts from a distribution company. For simplification it is also considered that this is also the opportunity cost equivalent of energy from a self-sufficient smelter from co-generation.

Table 5 - Assumptions of operation mode 2 – Energy selling

Assumptions	**Quantity/ Values**
Contracted energy consumption	15.88 Megawatt-hour per ton of installed capacity
Pre-contracted energy cost	34 US$/MWh
Energy spot price	Stochastic variable based PLD price.
Other production costs	440 US$/ton of produced aluminum

Rotation cost from operation mode 1 to mode 2 – interrupting smelter aluminum production and sale of available energy in the spot market at PLD price:

- Disconnection of working force – the option of changing from mode 1 to mode 2 implies in shutting down the reduction units. This change implies in the shutdown of part of the industrial plant and, as such, the disconnection or relocation of the working force. This cost is estimated at 2 million US$, and also brings a reduction in Other production costs to 440 US/t for the installed capacity, apart from not having costs related to alumina consumption.

Rotation cost from operation mode 2 to mode 1– back to aluminum production:

- Re-hiring and capacitation of working force – again, the option of changing from mode 2 to mode 1 implies in reactivation of the reduction units, which requires re-hiring and possible capacitation of work force. Considering the necessity of training of half of the workforce for a period of one month together with an additional effort from the human resources area, this cost was estimated at 1 million dollars.
- Reactivation of the smelting furnaces – the useful life and reactivation cost of the electrolytic furnaces depend on the time length of the interruption of activity. In the case of a short stoppage period each furnace can be reactivated at a cost estimated in 10 thousand dollars, but this implies in a loss of 30% of its useful life. In the case of a longer stoppage time, a change of revetment is necessary at a cost of 100 thousand dollars per furnace. Considering the different time usage of the furnaces, it is considered that 30% of the furnaces will have their revetment reused, and the other 70% will have their revetment changed. Given the difficulty in estimating these proportions without precise data these proportions are assumed as constant for the whole study. Therefore considering a plant with 500 smelting furnaces, production restart cost (considering that all units are stopped) will imply in a cost of: 500 x (70% x US$ 100.000 + 30% x US$ 10.000) = US$ 37.500.000.

Table 6 – Model Assumptions

Assumptions	**Quantity/ Values**
Installed capacity	500,000 tons per year – metallic aluminum production;
Operation Plant Lifetime	20 years in semiannual periods;
Option exercise period	1) Semiannual decision of option exercise: temporary stoppage or normal smelter operation; 2) After one semester, management will chose between keeping or changing the operation mode;
Modeling type	Bundle of European Options and Monte Carlo Simulation
Weighted Average Cost of Capital (WACC)	1) CAPM – risk premium according to country 2) Risk free rate - T-Bonds – 10 years = 3.5% a.a. Plus country risk= 6.00% aa 3) Cost of debt - BNDES (TJLP) = 6.00% a.a. Resulting in a : WACC 9.90% a.a

4.2. Model Structure

The cash flow model of the smelter considers the cash flows of each of the operation modes and incorporates the uncertainties of both stochastic processes of energy and aluminum prices, while the temporary stoppage option exercise also considers the asymmetric costs of mode change.

The cash flow of operation mode 1 considers the production of metallic aluminum through catalytic reduction for commercialization. In this case costs considered are those of alumina consumption, energy usage and other operation costs. Cash flow of operation mode 2, on the other hand, comes exclusively from selling at PLD price, the pre-contracted (or co-generated) available energy. Once the operation mode that maximizes cash flow is determined, the algorithm values if the switch is still advantageous when considering the costs associated with the operation mode switch. Is not, the smelter continues to operate at the previous mode.

The value of the temporary stoppage of the smelter is estimated calculating the present value of semiannual cash flows of 20 years of operation, discounted at the risk free rate shown in Table 6, with the stochastic variables of the model (aluminum price and PLD price) already adjusted through their own risk premiums. It is worth pointing out that when discounting both operation modes using the risk adjusted discount rates together with the natural dynamic of the stochastic variables or using the risk free rate with the risk adjusted processes, the present values of each mode are identical.

Beyond those two operation modes, a third hybrid is also considered, in which aluminum production continues at a level of 80% of the installed capacity and 20% of the energy is available for commercialization at the PLD price. In this mode, named mode 3, there are no reduction of fixed costs, but only 80% of the alumina of mode 1 is used. There are also no costs of switching back and forth between modes 1 and 3, since the units operation is not shut down but just reduced, but they remain the same when considering mode 2 – full stoppage of the plant.

Base cases: mode 1, mode 2 and mode 3, using the assumptions set previously render respectively the following Present Values (PV) when discounted at the wacc of: 9.90% per year;

- Mode 1: smelter operation with aluminum production entirely commercialized: PV= US$ 1,010,359 (x 1000);
- Mode 2: stoppage of smelter operation, and only commercialization of available energy at PLD price: PV = US$ 1,010,359 (x 1000);
- Mode 3: smelter operation at 80% level and selling of 20% of energy available: PV = US$ 1,203,674 (x 1000).

These base case values represent the present value of 20 years of semiannual cash flows operation (40 semesters) with no residual value and will be used as comparison with the results of the Real Options Valuation.

The cash flows corresponding to the deterministic valuation are represented in Figure 6. There is represented also the analysis of the possibility of switching the operation mode between the three available, considering that at the start (time 1) since it has a higher value then mode 1, due to the extremely high present price of PLD (around US$ 180/Mwh) and relatively low of aluminum. This scenario (switch with deterministic values) has a PV calculated at: US$ 1,377,655 (x1000). It is slightly lower than the PV of mode 1, due to the high cost of conversion from mode 2 to mode 1 (or 3), making that not every time the maximum cash flow can be chosen. In a certain way the operation can find itself “stuck” to a non-optimal mode due to the high conversion mode cost. Is the conversion cost is ignored this present value would instead be PV = US$ 1,507,576 (x 1.000) and the option would chose the higher cash flow every time. It is also worth noting that apart from time=1, at no other moment mode 3 is chosen, since using the option exercise rule, this turns out to be a corner solution between modes 1 and 2.

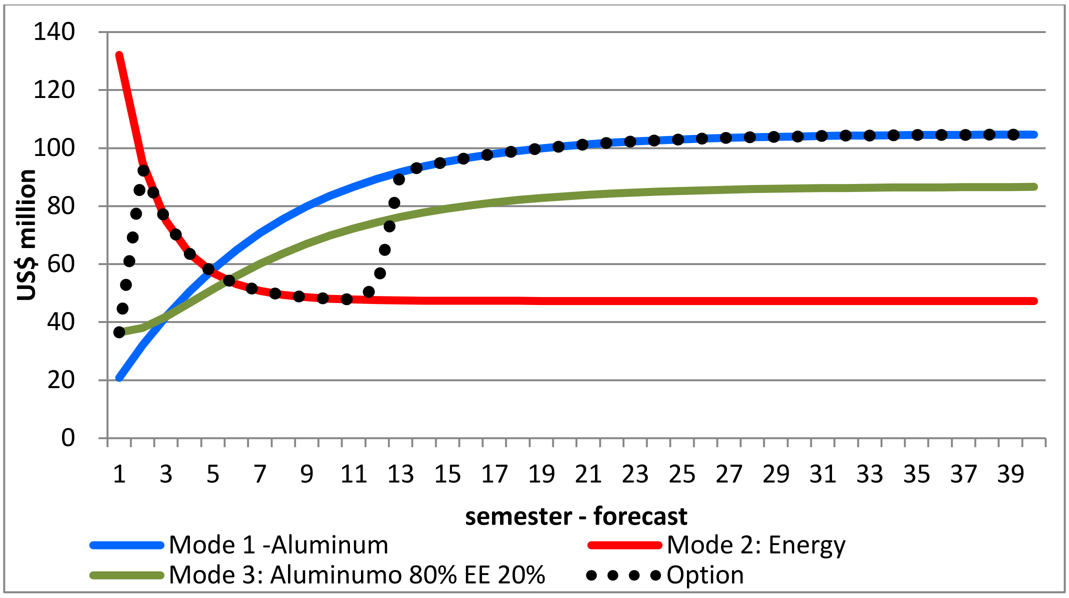

Figure 6: Deterministic Cash Flows of the different possible operation modes

5. Results

The option modeling was done using Monte Carlo Simulation with @RISK version 5.7 software. Ten thousand iterations were done for each valuation or change in parameters for sensitivity, and all were performed using the risk neutral approach. These used the risk free rate of table 7 as well as the adjustments for risk neutrality for aluminum and PLD (energy) described in the previous chapter. This latter was also compensated for the only positive jumps of the process.

As described in the previous chapter the decision to switch operation mode is taken considering the maximization of cash flows together with the cost of changing mode. This decision is based on the prices available to management at the moment of the decision taking, and these are those of the previous period, since exerting the switch based on the real prices of the next period will imply in previously knowing the future behavior of the stochastic variables, which is unrealistic. Therefore this decision is based on estimating the expected values of future prices based on the immediate past and their deterministic model behavior. Or an *ex-ante* decision. This approach differs from that adopted in other studies such as Bastian-Pinto & Brandão (2010) where the decision coincides with the realization of prices of the uncertain variables. Figure 7 illustrates one iteration showing the switch of operation mode through maximization of cash flows and considering conversion costs.

With the parameters of aluminum and PLD described in Tables 1, 2 and 3, the present value of the temporary stoppage option with selling of energy and asymmetric conversion costs is $PV_{\text{with option}}$ = US$ 2,078,351 (x 1000) giving an increase of 48.79% over the base case (mode 1).

Beyond this result the simulation points out that the temporary stoppage option (mode 2) is exerted 35.45% of the semiannual periods of the simulation, a significantly higher percentage than the 5.27% of jump frequency of PLD, which is coherent with the high volatility of this variable, but also indicates that the high conversion cost from mode 2 to mode 1, might inhibit this switch, keeping the operation "stuck" in mode 2 until the difference in cash flows overcompensates the conversion cost. This behavior can be observed in Figure 7 at least on three occasions: semester 8, 16 and 37. Also on 8.7% of periods (semesters) the case 3 is exercised. Is the historical scenario from 2001 to 2013, the smelters would had incentive to interrupt their operation on 5 of the 26 semesters.

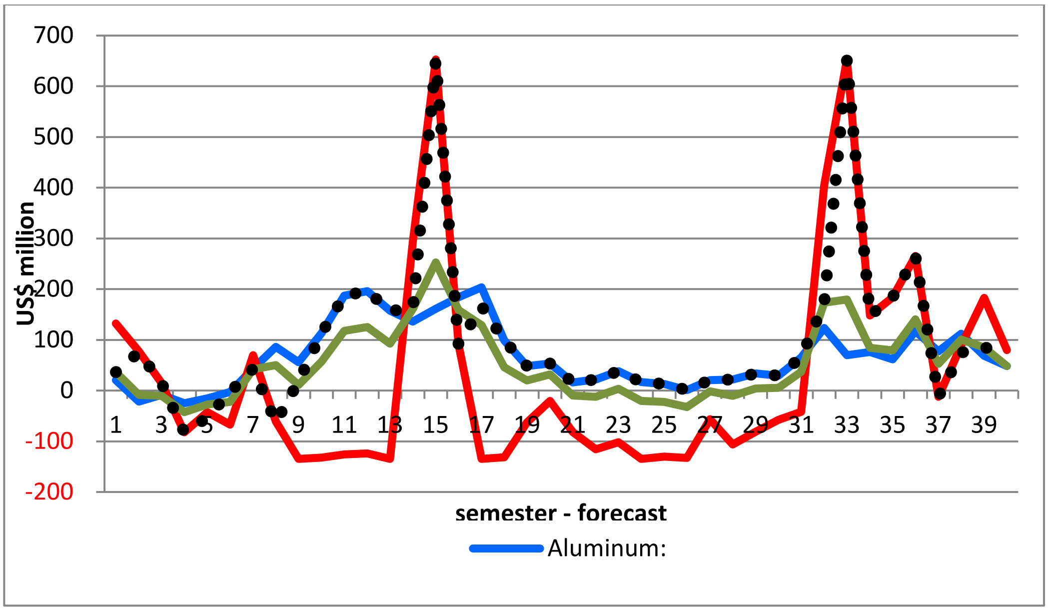

Figure 7: Simulation of Cash Flow trajectories for the different possible modes, showing non-optimal exercise in several instances.

If only modes 1 (aluminum) and 3 (hybrid) are considered (no conversion costs) and ignoring mode 2 (full stoppage of smelter) the present value of the operation with option is: PV = US$ 1,672,396 (x 1000), or an increase in 19.73% above the base case, with exercise of mode 3 in 25.4% of semesters. This shows also a significant value for the simple production reduction option.

The same simulation considering that no costs exist for full stoppage (modes 1, 2 and 3) bring a PV of: US$ 2,204,336 (x 1000), or 57.81% above the mode 1 base case, and a frequency of full stoppage (mode 2) of 32.13%. In this case in 9.7% of semesters mode 3 (hybrid) is enforced. As expected the switch cost has an effect on the stoppage option, but it is not as significant as one could expect.

This extremely high value for a temporary stoppage option, might also be overvalued due to the parameters used for the stochastic variables, especially for the PLD (energy) price, since these are released in weekly frequency but in this paper monthly averages were used to calibrate the stochastic model. Therefore a sensitivity analysis on the most important parameters is also done in order to check is these might be significantly distorting the real option value. PLD parameters on which this sensitivity is made are: long term mean $\overline{S}_E^{**}$ (risk neutral and compensated for jumps) and model volatility $\boldsymbol{\sigma_E}$ on one side, and with or without positive jumps. Results are shown in Table 7.

Table 7: Sensitivity of Assumptions of PLD price modeling

PV total (US$ x10,000)

With jumps	Volatility			
Mean*	70%	100%	127.82%	150%
40.00	1,545,122	1,603,001	1,684,957	1,727,972
57.61	1,907,202	2,028,226	**2,078,351**	2,082.264
70,00	2,297,499	2,379,611	2,390,916	2,371,424

Without jumps	Volatility			
Mean**	70%	100%	127.82%	150%
50.00	1,405,698	1,497,950	1,585,753	1,624,821
72.61	1,820,490	1,968,364	**2,058,378**	2,030,306
90.00	2,390,715	2,488,839	2,449,888	2,371,984

% of value of the option over the base case

With jumps	Volatility			
Mean*	70%	100%	127.82%	150%
40.00	10.6%	14.8%	20.6%	23.7%
57.61	36.5%	45.2%	**48.8%**	49.1%
70,00	64.5%	70.4%	71.2%	69.8%

Without jumps	Volatility			
Mean**	70%	100%	127.82%	150%
50.00	0.6%	7.2%	13.5%	16.3%
72.61	30.3%	40.9%	**47.4%**	45.3%
90.00	71.1%	78.2%	75.4%	69.8%

*: adjusted mean for risk neutrality and compensated for positive jumps
**: adjusted mean for risk neutrality
Bolt: base cases
Color variation from red to green indicates proportion of higher values

As can be observed the change of parameters of the PLD modeling does have influence over the option value, but, apart from extreme cases (very low mean and volatility) these will not significantly affect the decision as to the exercise of possible stoppage of the smelter.

6. Conclusions and Final Considerations

Results shown in the previous chapter indicate that the methodology used and based on real options theory is capable of valuing economic gains which are not appraised through traditional valuation methods.

In the hypothetical (but based on real data) smelter valued in this paper, the present study used references of an existing case in the Brazilian aluminum industry to demonstrate the existence of a temporary stoppage option in light of all elements necessary to the characterization of a real options problem: the flexibility existing in the possibility of temporary stopping the smelter operation, either totally or only partially, and selling the available energy in the spot market; uncertainties represented by the volatilities of prices of energy and aluminum; and irreversibility characterized by the significant investments made in the smelters.

The analysis indicates a significant value for the operational flexibility of stoppage (totally or partially) of the aluminum plant, which can provide competitive gains in this industry through maximization of cash flows and a as a consequence company value.

On top of that, the approach used suggests some questions that might bring contributions to future analysis of the theme of switch options. The use of a composed MRM and jump stochastic process, although already used in the literature, might indicate a possible treatment of certain variables. Likewise incorporating switch costs in the simulation of a bundle of European options might attract interest in new studies with similar assumptions.

It should also be pointed out that the methodologies and results presented in this work are subject to simplifications that require improvement that, further from limiting results and conclusions achieved, might also breed opportunities for future researches. Among these can be mentioned the timing of decision making of the stoppage or switch option: in the presented case a simplification is adopted considering that the expected value of the stochastic variables for the future period is based on the last information available which is that of the previous period, therefore considering an *ex-ante* decision of the realization of the stochastic variables. This approach can be improved considering that at each period the decision is no made on the expectation from the previous period but from the stochastic behavior itself of the variables, and that the option is no longer an European one but an American option assuming a time horizon up to the end of the modeling, at each period, transforming the problem into a bundle of American options.

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