Bubbles in Open Economies: Theory and Empirical Detection

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Abstract

Common precursors of financial crises are credit expansion and rising leverage. These fuel bubbles that result in a severe economic downturns when they burst. However, existing literature on bubbles under rationality lacks explanatory power, and this paper argues that this may be partly due to an implicit focus on closed economies. We study risk-shifting bubbles in symmetric open economies with three different investor types: fundamentalists, speculators and value investors. In open economies, credit bubbles tend to be 'displaced' abroad, have higher incidence, be larger, and last longer relative to the closed economy setting. We find that underpricing of default options embedded in loan agreements is a necessary but not a sufficient condition for risk-shifting incentives to emerge. We develop a simple empirical identification procedure of risk-shifting bubbles funded from overseas. An example of empirical identification is performed on New Zealand-Japan country pair, and we find that New Zealand's housing market has experienced a risk-shifting bubble funded from Japan in 2001-2003.

JEL Classification: D82; F30; G15; R31

Keywords: asset price bubbles; open economy; risk-shifting; housing market

1 Introduction

Credit booms and asset price bubbles have a dual nature. They may be useful in unlocking valuable investment opportunities for equity-constrained investors (Caballero and Krishnamurthy, 2006), resolving underinvestment traps created by functional dependence relations (Zavodov, 2011), and fostering economic progress (Eatwell, 2004; Eatwell and Milgate, 2011), yet they may also pose "the greatest danger to real economic activity" (Malkiel, 2010, p.14) by contributing to financial instability and potentially resulting in financial crises. The most recent economic downturn of 2008-2009 is a vivid example, as its primary cause is attributed to credit-led real estate bubbles in many parts of the world (Allen, Babus, and Carletti, 2009).

Empirical observations suggest that credit bubbles (*i.e.*, asset price bubbles that are built on leverage) are driven by a common pattern (Kaminsky and Reinhart, 1999; Mishkin, 2008; Reinhart and Rogoff, 2008a,b; Reinhart and Reinhart, 2010). Structural changes in financial markets (often induced by financial liberalisation or financial/technological innovation) induce rapid credit expansion and rising leverage that contribute to a demand expansion and the associated increase in asset prices. Frequently, these asset price booms are associated with large cross-border capital inflows and build-up of gross outstanding financial assets and liabilities position of a country (Hume and Sentance, 2009; Reinhart and Reinhart, 2009; Obstfeld and Rogoff, 2010; Ostry, Ghosh, Habermeier, Laeven, Chamon, Qureshi, and Kokenyne, 2011; Gourinchas and Obstfeld, 2012; Obstfeld, 2012; Schularick and Taylor, 2012). This run-up subsequently ends in large scale unwinding of trades, resulting in asset price collapses and a wave of defaults. In many cases, banking crises and exchange rate crises follow. The outcome of these episodes is a downturn in the real sector of the economy that lasts for several years.

However, theoretical models of credit bubbles lack explanatory power. First, they seem to underperform empirically, as local credit market conditions alone explain only a relatively small portion of large swings in asset prices (see, *e.g.*, Glaeser, Gottlieb, and Gyourko, 2010). Second, other countries are not relevant in these models. They fail to explain why asset price booms are associated with large capital inflows and build-up of gross outstanding financial assets and liabilities position of a country, and why exchange rate crises coincide with asset price collapses. Third, existing models often fail to provide a holistic framework incorporating emergence, persistence and burst phases of a bubble. Instead, they explain only specific parts of the bubble process. Fourth, models are hard to implement as they demonstrate the existence of a bubble by an explicit reference to the "fundamental value", which is difficult to observe empirically (Kohn, 2009).

This paper derives a model of credit-led asset price bubbles in an attempt to address the above issues. It is comprised of four key ingredients. First, in our model, credit-led asset price bubbles emerge because of an agency problem between borrowers and lenders introduced into the asset pricing context by Allen and Gale (2000). However, we depart from their strategy of establishing a bubble by reference to the "fundamental value". Instead, we demonstrate a bubble by considering investors' incentives. Second, we explore the relation between cross-border capital flows and asset price booms. The rationale for this is that, empirically, asset price booms and large capital inflows often go hand in hand, and, together, serve as the best leading indicator of financial crises (Kaminsky and Reinhart, 1999; Hume and Sentance, 2009; Reinhart and Reinhart, 2009; Obstfeld and Rogoff, 2010; Ostry, Ghosh, Habermeier, Laeven, Chamon, Qureshi, and Kokenyne, 2011; Gourinchas and Obstfeld, 2012; Obstfeld, 2012; Schularick and Taylor, 2012). The model accounts for this empirical observation by allowing agents to invest abroad. We thereby explicitly devise an open economy bubble model, which to our knowledge has not been done before. We show that in an open economy setting bubbles are *more likely (incidence)*, are *larger (magnitude)*, *last longer (persistence)*, and explain cross-border capital flows and build-up of gross outstanding financial assets and liabilities. Third, we account for all three phases of a bubble (emergence, persistence and burst) in a holistic framework, although we primary focus on finding the present day equilibrium pricing relations. Fourth, we consider three investor types: fundamentalists, speculators and value investors. Introduction of the speculative investor-type allows me to uncover an additional persistence mechanism of risk-shifting bubbles. The value investor-type is key to demonstrating that mispriced credit is a necessary but not a sufficient condition for risk-shifting bubbles to emerge: risk-shifting behaviour is induced only after a certain leverage amount (risk-shifting threshold) that is zero for fundamentalists (traditional agents in the existing literature) but is greater than zero for value investors. Figure 1 provides a simplified overview of the model.

The key ingredient of the model is the risk-shifting (or asset substitution) problem that arises due to inability of lender to ascertain the riskiness of borrower's

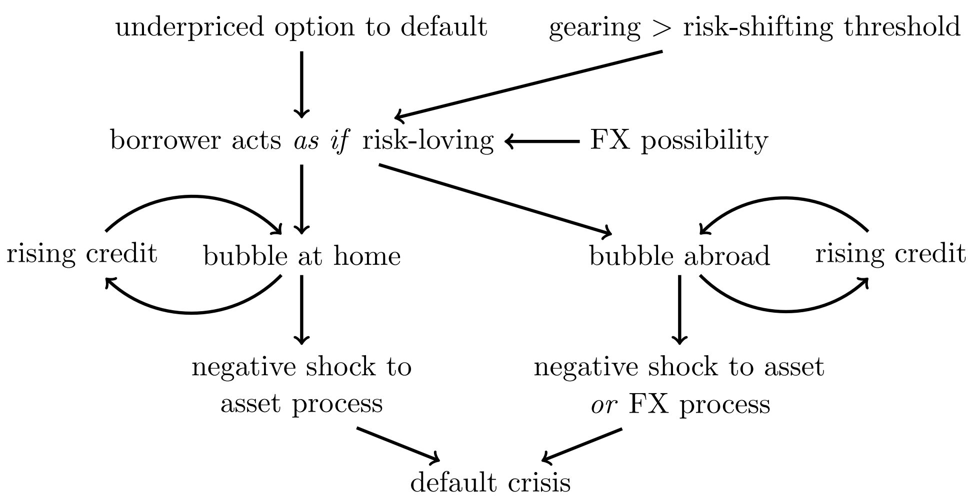


Figure 1: Simplified overview of the model

investment strategy.[1] Limited liability of debt issuers embedded in simple debt contracts (non-recourse loans) caps potential losses of borrowers in the bad state of the world.[2] Under strict priority rule, if borrower defaults in the bad state of the world, it has to "transfer" the collateral to the lender in exchange for the notional value of the outstanding debt. The expected present value of the difference between the value of the collateral and the notional value of outstanding debt upon default constitutes the value of the borrower's option to default. Although the interest rate margin charged by the lender on top of the risk-free rate of interest is supposed to compensate for this option to default, information asymmetry in borrower-lender relationship may result in inability of lender to ascertain the type and riskiness of investment strategy to be employed by borrower

[1] Galai and Masulis (1976) and Jensen and Meckling (1976) laid out the risk-shifting problem as one of the conflicts of interest between equity- and debt-holders: if debt-holders cannot control the use of funds by equity-holders and alter the terms of loan contract conditional on their actual use, then the latter may increase the value of their holdings by increasing the riskiness of the asset side of their balance sheet, thereby shifting part of risk onto debt-holders. The significance of the problem has been analysed using theoretical frameworks (Leland, 1998; Ericsson, 2000), simulation techniques (Parrino and Weisbach, 1999), managerial surveys (De Jong and Van Dijk, 2007; Graham and Harvey, 2001) and empirical evidence (Eisdorfer, 2008). Various avenues for mitigating the risk-shifting problem are available: debt covenants (Smith and Warner, 1979), debt maturity (Barnea, Haugen, and Senbet, 1980), convertible debt (Green, 1984) and managerial compensation (Brander and Poitevin, 1992; John and John, 1993). Though the question of whether these mitigation strategies can persist in equilibrium given, for example, short-termism of employees in lending institutions and external frictions emanating from mispriced government guarantees is still open. Since the seminal contribution of Stiglitz and Weiss (1981), risk-shifting also plays an important role in explaining pure credit rationing.

[2] Lenders find it optimal to write such contracts under various well-known conditions (see, *e.g.*, Townsend, 1979; Gale and Hellwig, 1985).

and thereby lead to underpricing of default option.[3] Default option underpricing effectively "removes" part of the downside risk of asset-price fluctuations, which is shifted onto the lender, thereby increasing the conditional expected return on a given geared position. Allen and Gale (2000) show that this increase in expected return induces agents to bid prices above their fundamental value and results in an asset price bubble.[4] In this paper, we note the following implication of risk-shifting behaviour induced by underpriced credit provision: *ex ante* risk-neutral borrowers act *as if* they were (unconditionally) risk-loving in bidding for risky assets. Hence, risk-shifting bubble can be identified by simply checking the sign of the effect of increased riskiness of a given risky asset on agent's reservation price for it. If it is positive, then there is a risk-shifting bubble.

We analyse the risk-shifting problem in a symmetric open economies setting, which, to our knowledge, has not been considered to date. The distribution of changes in the exchange rate represents a considerable amount of uncertainty, which *ceteris paribus* increases the variance of the core asset returns. We show that the additional gain from superior risk-shifting opportunities abroad can outweigh the additional foreign exchange transaction costs. This results in higher reservation prices for symmetric foreign risky assets compared to domestic risky assets. In equilibrium, agents invest abroad to maximise the chance of obtaining a long position in the risky asset. The analysis also allows me to show that risk-shifting bubbles in open economies have a higher incidence, are larger, and last longer relative to the closed economy setting.

We enrich the analysis by considering different investor types: fundamentalists, speculators and value investors. Fundamentalists value an asset on the basis of the discounted present value of cash flows that it is expected to generate. Speculators, in our model, do not receive any interim income in the form of dividends, and instead focus only on expected capital gains in their decision-making. The key difference between speculators' and fundamentalists' strategies

[3] Underpricing may arise for behavioural reasons. or example, Gennaioli, Shleifer, and Vishny (2010) model the neglect of certain improbable states of the world using the idea of local thinking, introduced by Gennaioli and Shleifer (2010), which is a formalisation of the notion that not all contingencies are represented in the decision maker's thought process. Lenders end up bearing risk without recognising that they are doing so. The role of the neglect of rare events in financial markets is also analysed in a historical perspective in Reinhart and Rogoff (2009).

Alternatively, it can be "induced" by mispriced explicit or implicit government guarantees for lenders (McKinnon and Pill, 1998; Krugman, 1998; Pavlov and Wachter, 2004, 2006; Schneider and Tornell, 2004; Farhi and Tirole, 2012).

[4] Recent literature attempts to generalise Allen and Gale (2000) result by showing that risk-shifting bubbles can also emerge under endogenous financial contracts (Barlevy, 2008) and endogenous loan supply (Challe and Ragot, 2007).

lies in how they opt to close an open position. Fundamentalists sell the asset they hold as soon as the bid price exceeds the perceived value of the open position (including the gain from the underpriced default option). Speculators, however, do not unwind unless the bid price sufficiently exceeds the perceived value, *i.e.*, unless a specific unwinding threshold is hit. The analysis of speculators allows me to dig deeper into the persistence mechanics of risk-shifting bubbles, and suggests that bubbles inflate not only on the basis of frequent transactions in an environment with rising leverage, but also on the basis of speculation with less frequent transactions.

Value investors are defined as agents who assess the intrinsic value of an opportunity in the same spirit as fundamentalists but require a "margin of safety" (*i.e.*, a discount as a safeguard against adverse realisations of uncertainty in the future) before opening a long position (Graham, 2006). To quantify the safety margin we draw from the insight of Yee (2008), who points out that this discount is equivalent to the value of a perpetual option to wait to invest.[5] The rationale underlying the option to wait is that an investor has an opportunity to postpone its decision to open a position in risky asset to gain additional information and obtain a sufficiently large wedge between asset's intrinsic value and price. The unleveraged risk-neutral value investor's inclination to delay investment increases in the riskiness of the investment strategy. As a result, it requires a larger margin of safety to open position in current period, and its reservation price is negatively related to volatility. However, once an underpriced option to default is introduced into decision-making, the strictly monotonic negative relationship may vanish. This is because the value of option to default increases in riskiness of the underlying investment strategy. Overall, value investors face a trade-off between margin of safety and loan mispricing considerations in determining their reservation price for a given risky asset. We show that the latter consideration is predominant for investors with leverage above a certain threshold, which we term the "risk-shifting threshold". As a result, in a setting with value investors, an underpriced default option is a necessary but not a sufficient condition for risk-shifting bubbles to emerge; sufficiency is obtained whenever leverage exceeds the risk-shifting threshold.

Although the general equilibrium analysis undertaken in this paper considers a

[5] Ability to postpone capital outlay is traditionally analysed in the context of real investment decisions that are often irreversible (Titman, 1985; McDonald and Siegel, 1986; Ingersoll and Ross, 1992; Dixit and Pindyck, 1994). Martzoukos (2001) considers the valuation of option to wait to invest in cross-border projects that are particularly relevant to the open economy context of this paper.

world comprised of two symmetric countries, it can be extended to multiple symmetric countries. This extension, however, comes with a possibility of unstable equilibria, in which an excessive number of agents invests in a particular country, thereby putting pressure on exchange rate that results in its temporal appreciation in excess of the interest rate differential implied by uncovered interest rate parity. Instability of these equilibria creates finite-lived carry trade opportunities. Availability of these opportunities, as we show, may inflate risk-shifting bubble even further.

The insights from theoretical analysis are used to provide an example of empirical identification of an open economy risk-shifting bubble in the context of New Zealand's housing market. The rationale for selecting New Zealand as the target country is two-fold: firstly, it is one of the most open economies in the world, and, second, its asset markets are sufficiently small so as to be responsive to influences from overseas. The funding country in the analysis is Japan due to its major role in New Zealand's economy both as trading partner and a major source of overseas funding during the bubble period identified. The analysis reveals a (small) negative effect of foreign exchange rate volatility between the two countries on New Zealand's house price index return in 'normal times', which is outweighed by (strong) positive effect in 2001-2003. The result is robust to controls for shifts in housing market fundamentals and the possible effects of carry trade opportunities. This suggests that a rapid increase in house prices in 2001-2003 can be due to risk-shifting incentives induced by underpriced funding from Japan through New Zealand's financial institutions. Although Eisdorfer (2008) provides a test for risk-shifting behaviour amongst financially distressed firms, the empirical analysis carried out in this paper is the first to date to consider the risk-shifting problem in the context of asset pricing (rather than corporate finance).

The rest of the paper is organised as follows. Section 2 sets up the modelling framework. In section 3, we derive the representative investor's pricing functions within a general version of the model. The specific pricing functions and their properties in the context of emergence and persistence phases for each investor type being considered are derived in section 4: fundamentalists (subsection 4.1), speculators (subsection 4.2), and value investors (subsection 4.3). Having established the properties of pricing functions, we study their implications in a two-symmetric-countries general equilibrium setting (section 5). Section 6 considers the magnifying effect of carry trade opportunities on the size of risk-shifting bubble. Section 7 provides an example of empirical identification of an open economy risk-shifting bubble in the context of New Zealand's housing market. Section 8 discusses the burst and crisis phase implications of the model and concludes.

2 The model

Consider a problem of capital allocation by a representative agent (investor) from country i. The investor is risk-neutral and has finite initial endowment W_i. The investment universe (menu of investment opportunities) is given by a safe bond in her home country i (domestic safe asset), a safe bond in country j ($j \neq i$) (foreign safe asset), a risky asset in her home country i (domestic risky asset), and a symmetric risky asset in country j ($j \neq i$) (symmetric foreign risky asset). The "symmetry" in the context of this paper means that the value dynamics of risky asset in country i in currency of country i and the value dynamics of symmetric risky asset in country j ($j \neq i$) in currency of country j are identical. We use q_{ji}^{k} to denote the quantity of the asset of type $k \in \{\mathsf{S}, \mathsf{R}\}$ from country j purchased by an investor from country i, where S is used to denote safe assets and R is reserved for risky assets. Both domestic and foreign risky assets are in fixed supply, which is normalised at 1, whereas safe assets are in variable supply.

Prices of safe assets are fixed at 1. If the investor from country i opens a long position in a domestic safe asset, it earns a fixed risk-free rate of return, $r_i > 0$, per unit time. In equilibrium, the risk-free rate of return in country i is determined by the marginal product of capital in economy i. The aggregate production function is given by $f(K_i)$, where K_i denotes the amount of capital employed for productive activity in country i. The production function satisfies the following assumptions: $f'(K_i) > 0$, $f''(K_i) < 0$, for all K_i, $f'(0) = \infty$, and $f'(\infty) = 0$.

The marginal product of capital in country i is related to that in country j ($j \neq i$) through the uncovered interest rate parity (UIP). The foreign exchange rate plays (on average) the equilibrating role. In particular, the foreign exchange rate dynamics between countries i and j in units of the domestic (numéraire) currency i, $(X_{ji,t})_{t\geqslant 0}$, follows the stochastic process of the form:

$$\mathrm{d}X_{ji,t} = (r_i - r_j) X_{ji,t}\mathrm{d}t + \sigma_j X_{ji,t}\mathrm{d}z_{j,t}^{i}, \quad X_{ji,0} \equiv X_{ji}, \text{ for all } i \neq j, \tag{1}$$

where:

$\sigma_j > 0$ denotes the instantaneous volatility of the exchange rate per unit time between countries j and i,

$\left(z_{j,t}^{i}\right)_{t \geqslant 0}$ denotes a standard Wiener process under the domestic probability measure,

$X_{ji} = 1$ to ensure initial symmetry and simplify the comparison of positions.

Two observations follow from the stochastic nature of the foreign exchange rate process. Firstly, unlike investments in the domestic safe asset, investments in the foreign safe asset are not risk-free. Secondly, if an agent decides to obtain exposure solely to foreign exchange risk, she can do so by acquiring foreign safe assets.

Apart from observing the foreign exchange rate process, the investor also considers the risky asset's dynamics, and conjectures that the risky asset in local currency (domestic risky asset), $(F_{ii,t})_{t \geqslant 0}$, follows the stochastic process of the form:

$$\mathrm{d}F_{ii,t} = \alpha F_{ii,t}\mathrm{d}t + \sigma F_{ii,t}\mathrm{d}z_t, \quad F_{ii,0} \equiv F_{ii} > 0, \tag{2}$$

where:

$0 \leqslant \alpha \leqslant r_i$ denotes the constant appreciation rate of $F_{ii,t}$ per unit time,

$\sigma > 0$ denotes the instantaneous volatility of the risky asset value in the home currency,

$(z_t)_{t \geqslant 0}$ denotes a standard Wiener process such that $\mathbb{E}\left[\mathrm{d}z_t \mathrm{d}z_{j,t}^{i}\right] = 0$.[6]

In equilibrium, the difference between the required rate of return on the risky asset, which for a risk-neutral agent is given by the risk-free rate, and the appreciation rate α, is the income (dividend) yield in country i, δ_i: $\delta_i = r_i - \alpha \geqslant 0$. Thus, although risky assets in different countries are conjectured to appreciate at the same local rate α, they may have different income yields depending on the amount allocated by local investors to safe assets (K_i) which depends *inter alia* on initial endowments, local credit conditions, and the shape of the production function, $f(K_i)$.

As it is the case with safe assets, an open position in the symmetric foreign risky asset from country j is subject to foreign exchange rate fluctuations. Thus, the value dynamics of the symmetric foreign risky asset from country j in units of the domestic currency of country i, $(F_{ji,t})_{t \geqslant 0}$ $(F_{ji,t} = X_{ji,t}F_{jj,t})$, can be obtained

[6]Note that the assumption of uncorrelated processes is made for mathematical convenience (*viz.*, to abstract from the issues associated with drift adjustments due to Siegel's Paradox), and does not materially affect our results.

by applying Itô's lemma:

$$\mathrm{d}F_{ji,t} = (r_i - r_j + \alpha) F_{ji,t}\mathrm{d}t + \sigma_{ji} F_{ji,t} \mathrm{d}z^i_{ji,t}, \quad F_{ji,0} \equiv F_{ji} > 0, \tag{3}$$

where:

$\sigma^2_{ji} = \sigma^2 + \sigma^2_j.$

Given the assumption of $X_{ji} = 1$, it is easy to see that $F_{ji} = F_{ii} = F$. Note that if the risk-free rate differential between countries i and j is zero, the appreciation rate of the symmetric foreign risky asset equals α. Then, the only difference between the dynamic properties of the domestic risky asset and a symmetric foreign risky asset (assuming the same initial conditions) is the volatility term such that $\sigma < \sigma_{ji} = \sqrt{\sigma^2 + \sigma^2_j}$ for $\sigma_j > 0$. We call this differentiating factor the *variance effect*. If this is the case, then in absence of transaction costs the comparison between the two assets is limited to the first-order effect of volatility on the agent's investment decision.

Agents face neoclassical transaction costs associated with opening positions in risky assets, $c_F(\cdot)$, and additionally in foreign assets, $c_X(\cdot)$. The latter costs are incurred when transacting currency at the time of purchase (these costs include, for example, the commission for currency exchanges), or may arise directly as a result of specific governmental intervention (for example, rationing of foreign exchange transactions that has been used in many transition economies and developing countries from time to time over the last two decades), or may simply be an outcome of indirect measures that are often approximated by the notion of investment climate.[7] Thus, agents buying $\mathsf{q}^{\mathsf{R}}_{ii}$ units of the risky asset at home have to pay $c_F\left(\mathsf{q}^{\mathsf{R}}_{ii}\right)$ only, agents buying $\mathsf{q}^{\mathsf{S}}_{ji}$ units of the foreign safe asset have to pay $c_X\left(\mathsf{q}^{\mathsf{S}}_{ji}\right)$ only, while agents buying $\mathsf{q}^{\mathsf{R}}_{ji} = \mathsf{q}^{\mathsf{R}}_{ii}$ units of the risky asset abroad have to pay both $c_F\left(\mathsf{q}^{\mathsf{R}}_{ji}\right) = c_F\left(\mathsf{q}^{\mathsf{R}}_{ii}\right)$ and $c_X\left(\mathsf{q}^{\mathsf{R}}_{ji}\right)$. Transaction cost functions satisfy the following assumptions: $c_j(0) = c'_j(0) = 0$, $c'_j(\mathsf{q}) > 0$, and $c''_j(\mathsf{q}) > 0$ for all $\mathsf{q} > 0$ and $j \in \{F, X\}$. In addition to transaction costs incurred at the time of opening a position, investors pay a constant amount (cost) of c_C per unit time for supporting an opened position in all claims other than the domestic safe asset. These are invariant in the quantity and currency of the purchased asset, and can be thought of as custodian fees.

A representative risk-neutral lender from country i has $L_i > 0$ units to lend but

[7] See, for example, the report *Doing Business 2011* by the World Bank and the International Finance Corporation for more information about investment climate and transaction costs that arise in relation to foreign investment.

cannot invest in risky asset markets directly (this includes the foreign safe asset market, as it is subject to foreign exchange rate fluctuations). The lender offers a simple (non-recourse) loan contract given by a pair (m_i, L_i), where $m_i > r_i$ denotes the interest rate charged by the lender per unit time. Under the contract, the loan does not have a maturity (akin to a consol bond) but has to be fully repaid as soon as the position is closed. There are no protective debt covenants attached to the loan contract and the borrower is guaranteed to obtain a loan if it applies for it. The borrower pledges whatever asset it acquires with the help of the loan as collateral that is seized by the lender in case of default under strict priority rule. We rule out a possibility of instantaneous default, *i.e.*, a borrower cannot rationally default at the time of opening a position. In case the borrower chooses not to take out a loan, the bank uses L_i to purchase the domestic safe asset.

It follows that the total amount available for investment purposes is $W_i + L_i$, of which K_i is invested in the real economy domestically.

The model ends either as a result of (1) selling the asset to another agent for a sufficiently high price, or (2) default by borrower and seizure of collateral by lender. Generally, default can occur in two instances: the borrower cannot service the debt (*i.e.*, the cash inflow is less than the cash outflow), or the asset value falls sufficiently low (*i.e.*, given the probability distribution of asset returns, the chance of making money on an open position is so low that the borrower does not have an incentive to service the debt). In the model, we focus on the latter default channel. We ensure that this is the only default channel by assuming that $r_i K_i \geqslant m_i L_i + c_C$ throughout the paper. In other words, proceeds from the safe asset position can be used to fund the ongoing risky investment strategy. Also, we assume that if any dividend is earned on a given asset, it is always used for consumption and is not reinvested in capital markets.

We consider three types of investors: fundamentalist, speculators and value investors. They are defined formally in the next section. For each investor type, we derive the pricing function and subject it to a risk-shifting bubble pricing test, whereby the risk-shifting bubble is defined as a situation where risk-shifting incentives outweigh any other investment considerations.

Definition 1 (Risk-shifting bubble pricing)**.** *Risk-shifting bubble pricing is present whenever the reservation price of a representative risk-neutral agent increases in the riskiness of the underlying asset, holding her conjecture about other charac-*

teristics of the asset value distribution constant:

$$\frac{\partial P}{\partial \sigma} > 0, \tag{4}$$

where:

P denotes agent's reservation price, and

σ denotes the relevant measure of risk.

This test is based on the seminal contributions of Galai and Masulis (1976) and Jensen and Meckling (1976), who note that inability of lender to adjust the terms of loan agreement to the risk profile of investment strategy employed by borrowers induces the latter to increase the riskiness of their asset base. Since they do not have to compensate the lender for an increased risk, they effectively shift risk onto her. This results in a transfer of wealth from debt- to equity-holders. Allen and Gale (2000) consider this phenomenon in the asset pricing context. They show that inability of lender to ascertain the risk of assets (or portfolios of assets) that borrower eventually decides to acquire results in an information rent for the latter, which is analogous to a transfer of wealth from lenders to investors. This additional value component induces agents to bid prices above what they perceive to be the asset's "fundamental value", which is based solely on cash flows that an asset is expected to generate in absence of any agency conflicts. Allen and Gale (2000) show the existence of bubble by comparing resulting prices with asset's fundamental values. Note, however, that risk-shifting in the sense of Allen and Gale is possible only if borrower's option to default (and hence loan contract) is underpriced. *Ceteris paribus* the magnitude of underpricing increases in risk of the investment strategy chosen. Hence, if there is any information rent component present in agent's pricing decision, then it is uncovered by checking whether her reservation price increases in underlying risk measure. Hence, risk-shifting bubble definition above is consistent with the analysis of Allen and Gale (2000). However, it is more operational than the contribution of Allen and Gale as it suggests a simpler empirical identification strategy that does not rely on the calculation of fundamental value. Empirical identification is considered in section 7.

The risk-shifting bubble definition is important for identifying a bubble in situations, where cash flows the asset generates and information rent are not the only factors that influence pricing decision. For example, value investors take into consideration the "margin of safety". Our definition of risk-shifting bubble suggests that the bubble is present only if the loan mispricing factor outweighs margin of safety considerations.

From the theoretical standpoint the assumption of *ex ante* risk-neutral investors serves particularly well for demonstrating risk-shifting bubble, as it allows for a clear-cut comparison that is not hindered by risk-aversion considerations. An agent, who is *ex ante* risk-neutral, behaves *as if* she was risk-loving whenever the bubble is present, *i.e.*, she is willing to pay an additional premium for risk revealing risk-shifting incentives.

3 Generic pricing functions

In this paper, asset pricing is governed by two sets of considerations: the value accruing to the investor when the position is open, and the direct and indirect costs of setting up a position. This determines the spirit of the analysis: we start by deriving the value of the open position, and then proceed to finding the pricing functions.

Consider first an open position in the domestic safe asset constructed through a purchase of $\mathsf{q}_{ii}^{\mathsf{S}}$ units thereof. The income stream from this asset is deterministic, and its value of equity of investor from country i upon opening a long position in domestic safe asset at time $t = 0$ is given by:

$$E_i\left(\mathsf{q}_{ii}^{\mathsf{S}}\right) = \mathbb{E}\left[\int_0^{\infty} e^{-r_i t}\left(r_i \mathsf{q}_{ii}^{\mathsf{S}} - m_i L_i\right) \mathrm{d}t\right] = \mathsf{q}_{ii}^{\mathsf{S}} - \frac{m_i L_i}{r_i}. \tag{5}$$

Given that the price of the safe asset is perfectly elastic at 1, the cost of setting this position up is $\left(L_i - \mathsf{q}_{ii}^{\mathsf{S}}\right)$. Two observations follow. Firstly, if an investment strategy consists of going long in the domestic safe asset, the rational (expected payoff-maximising) choice is not to borrow from the bank at all. Under the terms of the loan agreement (m_i, L_i), the interest rate is set above the risk-free rate, $m_i > r_i$, and hence by taking a loan out an agent would be losing money. Secondly, the quantity of the domestic safe asset is a residual of the portfolio selection problem.

Consider now an open position in $\mathsf{q}_{ji}^{\mathsf{S}}$ units of the foreign safe asset ($j \neq i$). As noted above, although income from this asset in the local currency is deterministic, it is risky in the currency of the investor as it is subject to foreign exchange rate fluctuations. In continuation region, the value of equity of investor from country i opening a long position in foreign safe asset from country j ($j \neq i$) is given by

$$E_i\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}\right) = r_j X_{ji} \mathsf{q}_{ji}^{\mathsf{S}} \mathrm{d}t - m_i L_i \mathrm{d}t - c_C \mathrm{d}t$$
$$+ e^{-r_i \mathrm{d}t} \mathbb{E}\left[E_i\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji} + \mathrm{d}X_{ji}\right)\right], \quad j \neq i, \quad X_{ji} \in \mathcal{D}_{X,ji}^{C},$$

where:

$\mathcal{D}_{X,ji}^{C}$ denotes the continuation region of equity valuation problem.

Appendix A.1 shows that, in continuation region, the value of equity, $E_i\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}\right)$, solves the following ordinary differential equation (ODE):

$$\frac{\sigma_j^2}{2} X_{ji} E_i''\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}\right) + \left(r_i - r_j\right) X_{ji} E_i'\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}\right) - r_i E_i\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}\right)$$
$$+ r_j X_{ji} \mathsf{q}_{ji}^{\mathsf{S}} - m_i L_i - c_C = 0, \quad j \neq i, \quad X_{ji} \in \mathcal{D}_{X,ji}^{C}, \quad (6)$$

where:

$E_i'\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}\right) \equiv \frac{\mathrm{d}E_i\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}\right)}{\mathrm{d}X_{ji}}$,
$E_i''\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}\right) \equiv \frac{\mathrm{d}^2 E_i\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}\right)}{\mathrm{d}X_{ji}^2}$.

The general solution to equation (6) is given by

$$E_i\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}\right) = B_{1,X,ji} X_{ji}^{\beta_{1,X,ji}} + B_{2,X,ji} X_{ji}^{\beta_{2,X,ji}}$$
$$+ \mathsf{q}_{ji}^{\mathsf{S}} X_{ji} - \frac{m_i L_i}{r_i} - \frac{c_C}{r_i}, \quad j \neq i, \quad X_{ji} \in \mathcal{D}_{X,ji}^{C}, \quad (7)$$

where:

$B_{1,X,ji}$ and $B_{2,X,ji}$ are constants to be determined as part of the solution,
$\beta_{1,X,ji}$ and $\beta_{2,X,ji}$ are non-negative and negative roots, respectively, of quadratic equation:

$$\mathcal{Q}_{X,ji} = \frac{\sigma_j^2}{2} \beta_{X,ji}\left(\beta_{X,ji} - 1\right) + \left(r_i - r_j\right) \beta_{X,ji} - r_i = 0.$$

Equation (7) decomposes the value of equity in the open position into four components: the speculative bubble component, the default option component, the core component, and the ongoing cost of supporting an open position.

To find the exact solution for the value of equity in this position, the following boundary conditions are imposed. First, the possibility of a speculative bubble in the foreign exchange rate is ruled out. Second, at the time of declaring default the equity claim is worthless. Third, since there are no protective debt covenants, default trigger, X_{ji}^{D}, is chosen optimally. Formally, these boundary conditions are:

$$\begin{aligned}
\lim_{X_{ji}\to\infty} E_i\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}\right) &= \mathsf{q}_{ji}^{\mathsf{S}} X_{ji} - \frac{m_i L_i}{r_i} - \frac{c_C}{r_i}, \quad j \neq i,\\
E_i\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}^{D}\right) &= 0, \quad j \neq i,\\
E_i'\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}^{D}\right) &= 0, \quad j \neq i,
\end{aligned}$$

The above boundary conditions determine the three constants:

$$\begin{aligned}
B_{1,X,ji} &= 0,\\
B_{2,X,ji} &= \left[\frac{m_i L_i + c_C}{r_i} - \mathsf{q}_{ji}^{\mathsf{S}} X_{ji}^{D}\right] \left(X_{ji}^{D}\right)^{-\beta_{2,X,ji}},\\
X_{ji}^{D} &= \frac{\gamma_{2,X,ji}}{\mathsf{q}_{ji}^{\mathsf{S}}} \left[\frac{m_i L_i + c_C}{r_i}\right],
\end{aligned}$$

where:

$$\gamma_{2,X,ji} = \frac{\beta_{2,X,ji}}{\beta_{2,X,ji} - 1}.$$

Instantaneous default is not possible. Hence, the value of equity in an open position in foreign safe asset at time $t = 0$:

$$\begin{aligned}
E_i\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}\right) = \mathsf{q}_{ji}^{\mathsf{S}} X_{ji} - \frac{m_i L_i}{r_i} - \frac{c_C}{r_i}&\\
+ \left[\frac{m_i L_i + c_C}{r_i\left(1 - \beta_{2,X,ji}\right)}\right] \left(\frac{X_{ji}}{X_{ji}^{D}}\right)^{\beta_{2,X,ji}}&, \quad j \neq i, \quad X_{ji} > X_{ji}^{D}, \qquad (8)
\end{aligned}$$

Given the perfect price elasticity of the safe asset's price at 1, the expected payoff of an immediately opened long position in foreign safe asset is:

$$\begin{aligned}
\Pi_i\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}\right) &= -\mathsf{q}_{ji}^{\mathsf{S}} X_{ji} - \frac{c_C}{r_i} - c_X\left(\mathsf{q}_{ji}^{\mathsf{S}}\right) + \mathsf{q}_{ji}^{\mathsf{S}} X_{ji} + \mathcal{M}_i\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}, m_i, L_i\right)\\
&= -\frac{c_C}{r_i} - c_X\left(\mathsf{q}_{ji}^{\mathsf{S}}\right) + \mathcal{M}_i\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}, m_i, L_i\right), \quad j \neq i, \quad X_{ji} > X_{ji}^{D},
\end{aligned} \qquad (9)$$

where:

$\mathcal{M}_i\left(\mathsf{q}_{ji}^{\mathsf{S}}, X_{ji}, m_i, L_i\right)$ denotes mispricing function of investor from country i, who contemplates acquisition of $\mathsf{q}_{ji}^{\mathsf{S}}$ units of safe asset from country $j \neq i$ with loan contract (m_i, L_i), and is given by:

$$\mathcal{M}_i\left(\mathsf{q}^{\mathsf{S}}_{ji}, X_{ji}, m_i, L_i\right) = \max\left[L_i - \frac{m_i L_i}{r_i} + \left[\frac{m_i L_i + c_C}{r_i\left(1-\beta_{2,X,ji}\right)}\right]\left(\frac{X_{ji}}{X^D_{ji}}\right)^{\beta_{2,X,ji}}, 0\right], \quad j \neq i, \quad X_{ji} > X^D_{ji}.$$

Two observations follow from the above formulation of expected payoff. First, investor never invests in foreign safe asset, unless the loan is underpriced. Investor does not take out an overpriced loan (hence, $\mathcal{M}_i\left(\mathsf{q}^{\mathsf{S}}_{ji}, X_{ji}, m_i, L_i\right) \geqslant 0$), as it reduces its expected payoff, and is indifferent between taking out a correctly priced loan and self-financing its investment strategy. For overpriced and correctly priced loans, the max-function in equation (9) takes the value of 0. As a result, expected payoff from opening a long position in foreign safe asset is negative for all $\mathsf{q}^{\mathsf{S}}_{ji} > 0$. Second, given the infinite price elasticity of safe asset prices, to rule out a possibility of non-zero expected profit in competitive equilibrium, an upper limit on underpricing of loan contract is imposed:[8]

$$-c_X\left(\mathsf{q}^{\mathsf{S}}_{ji}\right) - \frac{c_C}{r_i} + \mathcal{M}_i\left(\mathsf{q}^{\mathsf{S}}_{ji}, X_{ji}, m_i, L_i\right) < 0, \quad \text{for all } \mathsf{q}^{\mathsf{S}}_{ji}, \quad j \neq i, \tag{10}$$

which is satisfied either if the sum of foreign exchange transaction costs and cost of supporting an open position are sufficiently high, or if the bank imposes a sufficiently high interest rate, or both. As a result, agents are strongly biased against foreign safe asset investment *ex ante.*

The last item to be valued is the long position in a risky asset. Following the standard procedure, the value of open position in continuation region is comprised of dividend income, the cost of supporting an open position (debt service and custodian fees) and its expected appreciation:

$$\begin{aligned} E_i\left(\mathsf{q}^{\mathsf{R}}_{ji}, F_{ji}\right) &= \mathsf{q}^{\mathsf{R}}_{ji}\delta_j F_{ji}\mathrm{d}t - m_i L_i \mathrm{d}t - c_C \mathrm{d}t \\ &\quad + e^{-r_i \mathrm{d}t}\mathbb{E}\left[E_i\left(\mathsf{q}^{\mathsf{R}}_{ji}, F_{ji} + \mathrm{d}F_{ji}\right)\right], \quad \text{for all } i, j, \quad F_{ji} \in \mathcal{D}^C_{F,ji}, \end{aligned} \tag{11}$$

where:

$\mathcal{D}^C_{F,ji}$ denotes the continuation region of equity valuation problem.

Appendix A.2 shows that, in continuation region, the value of equity, $E_i\left(\mathsf{q}^{\mathsf{R}}_{ji}, F_{ji}\right)$,

[8] Despite the fact that expected payoff function decreases in $\mathsf{q}^{\mathsf{S}}_{ji}$ even if loan is underpriced, investor may be able to make a positive profit by acquiring one foreign safe bond. This assumption is required to exclude this possibility.

solves the following ODE:

$$\frac{\sigma_{ji}^2}{2} F_{ji}^2 E_i'' \left(\mathsf{q}_{ji}^{\mathsf{R}}, F_{ji}\right) + (r_i - \delta_j) F_{ji} E_i' \left(\mathsf{q}_{ji}^{\mathsf{R}}, F_{ji}\right) - r_i E_i \left(\mathsf{q}_{ji}^{\mathsf{R}}, F_{ji}\right) + \mathsf{q}_{ji}^{\mathsf{R}} \delta_j F_{ji} - m_i L_i - c_C = 0, \quad \text{for all } i, j, \quad F_{ji} \in \mathcal{D}_{F,ji}^{C}, \tag{12}$$

where:

$E_i' \left(\mathsf{q}_{ji}^{\mathsf{R}}, F_{ji}\right) \equiv \frac{\mathrm{d}E_i\left(\mathsf{q}_{ji}^{\mathsf{R}}, F_{ji}\right)}{\mathrm{d}F_{ji}}$,

$E_i'' \left(\mathsf{q}_{ji}^{\mathsf{R}}, F_{ji}\right) \equiv \frac{\mathrm{d}^2E_i\left(\mathsf{q}_{ji}^{\mathsf{R}}, F_{ji}\right)}{\mathrm{d}F_{ji}^2}$.

The general solution to equation (12) is given by

$$E_i \left(\mathsf{q}_{ji}^{\mathsf{R}}, F_{ji}\right) = B_{1,F,ji} \left(\mathsf{q}_{ji}^{\mathsf{R}}\right) F_{ji}^{\beta_{1,F,ji}} + B_{2,F,ji} \left(\mathsf{q}_{ji}^{\mathsf{R}}\right) F_{ji}^{\beta_{2,F,ji}} + \mathsf{q}_{ji}^{\mathsf{R}} F_{ji} - \frac{m_i L_i}{r_i} - \frac{c_C}{r_i}, \quad \text{for all } i, j, \quad F_{ji} \in \mathcal{D}_{F,ji}^{C}, \tag{13}$$

where:

$B_{1,F,ji} \left(\mathsf{q}_{ji}^{\mathsf{R}}\right)$ and $B_{2,F,ji} \left(\mathsf{q}_{ji}^{\mathsf{R}}\right)$ are constants to be determined as part of solution,

$\beta_{1,F,ji}$ and $\beta_{2,F,ji}$ are non-negative and negative roots, respectively, of the following quadratic equation:

$$\mathcal{Q}_{F,ji} = \frac{\sigma_{ji}^2}{2} \beta_{F,ji} \left(\beta_{F,ji} - 1\right) + (r_i - \delta_j) \beta_{F,ji} - r_i = 0. \tag{14}$$

The equity value of an open long position in risky asset is decomposed into: the speculative component, the default option component, the core component, and the cost of supporting an open position through debt service and custodian fees. The specific solution to equity valuation problem depends on investor type being considered, and is the subject of the next section.

The generic expected payoff function of an investor in risky asset is comprised the value of open long position, costs of setting it up and any margin of safety considerations that may be involved:

$$\Pi_i^{\mathfrak{T}} \left(\mathsf{q}_{ji}^{\mathsf{R}}, F_{ji}\right) = -\mathsf{q}_{ji}^{\mathsf{R}} P_{ji} + \mathsf{q}_{ji}^{\mathsf{R}} F_{ji} - \frac{c_C}{r_i} - c_F \left(\mathsf{q}_j^{\mathsf{R}}\right) - c_X \left(\mathsf{q}_j^{\mathsf{R}}\right) \mathbb{1}_{j \neq i} - \mathcal{S}_i^{\mathfrak{T}} \left(\mathsf{q}_{ji}^{\mathsf{R}}, F_{ji}\right) + \mathcal{M}_i^{\mathfrak{T}} \left(\mathsf{q}_{ji}^{\mathsf{R}}, F_{ji}, m_i, L_i\right), \quad \text{for all } i, j, \quad F_{ji} > F_{ji}^{D},$$

where:

$\mathfrak{T}$ denotes investor type to be defined,

Table 1: Assumptions characterising various investor types

Investor type, $\mathfrak{T}$	Fundamentalists, $\mathfrak{F}$	Speculators, $\mathfrak{S}$	Value investors, $\mathfrak{V}$
δ_j	>0	$=0$	>0
$B^{\mathfrak{T}}_{1,F,ji}$	$=0$	>0	$=0$
$\mathcal{S}^{\mathfrak{T}}_i(F_{ji})$	$=0$	$=0$	>0
$\mathcal{M}^{\mathfrak{T}}_i(F_{ji}, m_i, L_i)$	$\geqslant 0$	$\geqslant 0$	$\geqslant 0$

$\mathbb{1}_{j\neq i}$ is the indicator function that takes value of of 1, if $j \neq i$, and 0, otherwise,
$\mathcal{S}^{\mathfrak{T}}_i\left(\mathsf{q}^{\mathsf{R}}_{ji}, F_{ji}\right) \geqslant 0$ denotes the value of safety margin consideration, and
$\mathcal{M}^{\mathfrak{T}}_i\left(\mathsf{q}^{\mathsf{R}}_{ji}, F_{ji}, m_i, L_i\right) \geqslant 0$ denotes the loan misprricing function, which enters the valuation problem only if the loan is underpriced.

Given the market clearing conditions $\mathsf{q}^{\mathsf{R}}_{ji} = 1$ and $\Pi^{\mathfrak{T}}_i\left(\mathsf{q}^{\mathsf{R}}_{ji}, F_{ji}\right) = 0$, the generic pricing function of investor of type $\mathfrak{T}$ is given by:

$$
\begin{aligned}
P^{\mathfrak{T}}_{ji} = F_{ji} - \frac{c_C}{r_i} - c_F - c_X \mathbb{1}_{j\neq i} \\
- \mathcal{S}^{\mathfrak{T}}_i(F_{ji}) + \mathcal{M}^{\mathfrak{T}}_i(F_{ji}, m_i, L_i), \quad \text{for all } i, j, \quad F_{ji} > F^D_{ji}, \qquad (15)
\end{aligned}
$$

where:
$c_X \equiv c'_X(1)$,
$c_F \equiv c'_F(1)$,
$\mathcal{S}^{\mathfrak{T}}_i(F_{ji}) \equiv \mathcal{S}^{\mathfrak{T}}_i(1, F_{ji})$, and
$\mathcal{M}^{\mathfrak{T}}_i(F_{ji}, m_i, L_i) \equiv \mathcal{M}^{\mathfrak{T}}_i(1, F_{ji}, m_i, L_i)$.

It follows from the generic pricing equation (15) that differences in valuations between investor types depend on how they assess margin of safety and loan mispricing components of the valuation programme. In subsequent sections, we make the generic pricing function (15) more specific to each investor type, study their partial equilibrium properties and general equilibrium implications in relation to the risk-shifting bubble formation. In particular, we consider three types of investors $\mathfrak{T} \in \{\mathfrak{F}, \mathfrak{S}, \mathfrak{V}\}$: fundamentalists, speculators, and value investors.

The assumptions that characterise each investor type are summarised in Table 1. Fundamentalist, whose pricing functions are denoted with superscript $\mathfrak{F}$, value an asset in terms of cash flows that it is expected to generate and thus disregard margin of safety consideration. To derive their pricing functions, we assume non-zero income yield, abstract away from speculative and safety margin considerations. Speculators, whose pricing functions bear the superscript $\mathfrak{S}$, are assumed not to receive any interim income from a given asset and thus value it only in terms of the potential capital gain they may receive from holding and reselling it. Safety margin considerations are irrelevant for speculators. Value investors, whose pricing functions are marked with superscript $\mathfrak{V}$, value an asset based on the cash flow generating potential thereof (similar to fundamentalists) but also require a margin of safety before opening a long position. It is worth pointing out that each investor type has a distinct specification of the loan mispricing function, but none of them takes out an overpriced loan (hence $\mathcal{M}_i^{\mathfrak{T}}(F_{ji}, m_i, L_i) \geqslant 0$).

4 Asset pricing in partial equilibrium

In this section, we derive partial equilibrium reservation price functions for investor type, and study their properties.[9] These functions are the basis of general equilibrium asset pricing in section 5.

4.1 Asset pricing by fundamentalists

Fundamentalists are the investor type that forms the basis of the analysis undertaken by Allen and Gale (2000). For this investor type, we assume that assets earn a positive income yield, speculative component and safety margin adjustments are irrelevant (see Table 1). The assumptions associated with this type of investors provide a particularly useful starting point of analysis by restricting the focus to loan mispricing considerations only. As a result, in economies comprised of fundamentalists, the resulting bubbles can be characterised as pure risk-shifting bubbles, as opposed to the ones that are partly driven by speculative motives or meddled by safety margin considerations.

Following the approach for obtaining pricing functions of the previous section, we, first, derive the value equity in the open long position in risky asset, and, second, solve for the asset price. The value of equity in the open position is summarised in the proposition below.

[9]Actions of foreign agents and the risk-free rates are taken to be exogenous.

Proposition 1. *In continuation region, the value of equity in the open long position in a risky asset in country j held by fundamentalist from country i is*

$$
\begin{aligned}
E_i^{\mathfrak{F}}\left(F_{ji}\right) = F_{ji} - \frac{c_C}{r_i} - \frac{m_i L_i}{r_i} \\
+ \left(\frac{m_i L_i + c_C}{r_i}\right)\left(1-\gamma_{2,F,ji}\right)\left(\frac{F_{ji}}{F_{ji}^D}\right)^{\beta_{2,F,ji}}, \quad \textit{for all } i,j, \quad F_{ji} > F_{ji}^D
\end{aligned}
$$

where:

$$
\begin{aligned}
\gamma_{2,F,ji} &= \frac{\beta_{2,F,ji}}{\beta_{2,F,ji}-1}, \\
\beta_{2,F,ji} &= \frac{-\left(r_i - \delta_j - \frac{\sigma_{ji}^2}{2}\right) - \sqrt{\left(r_i - \delta_j - \frac{\sigma_{ji}^2}{2}\right)^2 + 2 r_i \sigma_{ji}^2}}{\sigma_{ji}^2}, \\
F_{ji}^D &= \gamma_{2,F,ji}\left(\frac{m_i L_i + c_C}{r_i}\right).
\end{aligned}
$$

Proof. See Appendix A.3. ■

Given the above result and the assumption of $\mathcal{S}_i^{\mathfrak{F}}\left(F_{ji}\right) = 0$, the generic pricing function (15) is adjusted for fundamentalists as follows:

$$
P_{ji}^{\mathfrak{F}} = F - \frac{c_C}{r_i} - c_F - c_X \mathbb{1}_{j \neq i} + \mathcal{M}_i^{\mathfrak{F}}\left(F, m_i, L_i\right), \quad \text{for all } i,j, \quad F > F_{ji}^D, \tag{16}
$$

where:

$$
\begin{aligned}
\mathcal{M}_i^{\mathfrak{F}}\left(F, m_i, L_i\right) &= \max\left[L_i - D^{\mathfrak{F}}\left(F, m_i, L_i\right), 0\right], \quad \text{for all } i,j, \quad F > F_{ji}^D, \\
D^{\mathfrak{F}}\left(F, m_i, L_i\right) &= \frac{m_i L_i}{r_i} \\
&\quad - \left(\frac{m_i L_i + c_C}{r_i}\right)\left(1-\gamma_{2,F,ji}\right)\left(\frac{F}{F_{ji}^D}\right)^{\beta_{2,F,ji}}, \text{ for all } i,j, \ F > F_{ji}^D.
\end{aligned}
$$

The reason why loan mispricing function $\mathcal{M}_i^{\mathfrak{F}}\left(F, m_i, L_i\right)$ is written as a max-function is simply because if the loan amount obtained from bank L_i is less than the value of debt contract $D^{\mathfrak{F}}\left(F, m_i, L_i\right)$, then borrowing is value-destroying for investor and it rationally chooses not to borrow. We consider this point in greater detail below.

Whenever there is no credit available ($L_i = 0$), the difference of local risky asset and symmetric foreign risky asset pricing functions is limited to additional foreign exchange transaction costs c_X that *ceteris paribus* deter an investor from going abroad. Let us now examine what may induce an investor to opt for a long position in a symmetric foreign risky asset despite the additional costs it involves. In particular, consider the role leverage plays in the pricing functions.

Under perfect information, the competitive bank designs no-arbitrage loan contracts such that $L_i - D^{\mathfrak{F}}(F, m_i, L_i) = 0$ for all i, j, *i.e.*, there is no mispricing of the loan contract. As a result, the terms characterising the loan contract (m_i, L_i) drop out from the relevant pricing function, and the capital structure does not affect the investment decision. This result is not merely partial equilibrium but also general equilibrium as it does not alter the amount invested in productive technology.

Under imperfect information, however, mispricing may arise. If the loan contract designed by the bank is overpriced from the point of view of the investor, $L_i < D^{\mathfrak{F}}(F, m_i, L_i)$, then it is losing money relative to the scenario in which it does not take out a loan. In a competitive market setting, an investor would not take out an overpriced loan, as this would decrease its reservation price and competitiveness for the scarce risky asset *vis-à-vis* other agents. Hence, if $L_i < D^{\mathfrak{F}}(F, m_i, L_i)$, the investor does not take out a loan, the loan contract terms (m_i, L_i) drop out of the pricing function, and the capital structure does not affect the investment decision. Note that although in absence of loan underpricing investor allocates more of its own initial wealth to risky investments, absence of demand for risky loans ensures that the bank puts the amount L_i into the safe domestic asset. As a result, price invariance in capital structure is a general equilibrium result. We summarise the above discussion in the proposition below.

Proposition 2. *If the bank can design loan contracts such that for any risky asset the loan is not underpriced, then the competitive equilibrium price for all assets is invariant in the capital structure employed. Furthermore, if $c_X > 0$, fundamentalists always have a higher reservation price for the local risky asset as compared to the symmetric foreign risky asset.*

However, the lender may underprice the loan contract by undervaluing the embedded option to default. Various reasons have been put forward in the literature to explain this underpricing. They can be broadly separated into those relying on asymmetric information between lenders and borrowers, short-termism of employees of lending institutions, mispriced (explicit or implicit) government guarantees for lenders, and behavioural approaches. Asymmetric information explanations rely on borrowers having superior information about risk characteristics of investment opportunity compared to lenders either as a result of inability of lenders to ascertain these characteristics *ex ante* or inability of lenders to control the use of funds (Allen and Gale, 1999, 2000, 2007). Short-termism of employees in lending institutions (Pavlov and Wachter, 2004) and mispriced government guarantees (McKinnon and Pill, 1998; Krugman, 1998; Pavlov and Wachter, 2006; Schneider and Tornell, 2004; Farhi and Tirole, 2012) result in excessive focus of financial intermediaries on increasing the volume of lending, which is achieved through loan underpricing. Behavioural explanation rely on actors ignoring some contingencies due to local thinking (Gennaioli, Shleifer, and Vishny, 2010), and thereby not realising that they underprice loan contracts.

Given that banks' funds are capped at L_i, loan underpricing in our model comes as a result of the interest rate charge being insufficient to compensate for the risk taken by the bank extending the loan. If m_i^* is the optimal interest rate, then actual interest rate satisfies:

$$m_i < m_i^* = \frac{r_i L_i + c_C\left(1-\gamma_{2,F,ji}\right)\mathbb{E}\left[e^{-r_i \tau_{F,ji}^D}\right]}{L_i - L_i\left(1-\gamma_{2,F,ji}\right)\mathbb{E}\left[e^{-r_i \tau_{F,ji}^D}\right]}, \tag{17}$$

where:

$\tau_{F,ji}^D \equiv \inf\left\{t \geqslant 0 : F_{ji,t} \leqslant F_{ji}^D\right\}$ for all j,

$$\mathbb{E}\left[e^{-r\tau_{F,ji}^D}\right] = \left(\frac{F}{F_{ji}^D}\right)^{\beta_{2,F,ji}}.$$

Once credit is underpriced, the value of mispricing function is strictly greater than 0 and investor rationally chooses to take out a loan. This leads to an increase in asset prices beyond the level that is achieved in absence of gearing. Furthermore, provided $m_i < r_i\left(1-\mathbb{E}\left[e^{-r\tau_{F,ji}^D}\right]\right)^{-1}$, mispricing function increases in loan amount and therefore asset prices also increase in the amount of leverage employed.

We now proceed to verifying whether any of the prices associated with underpriced loan contracts can generate risk-shifting bubbles, which are defined as situations in which *ex ante* risk-neutral investors act *as if* they were risk-loving (see definition 1). More specifically, we test whether the reservation price increases in the underlying risk measure.[10]

[10] Note that given the risk-neutrality of agents there is no such increase, if there is no loan

Proposition 3. *If* $\mathcal{M}_i^{\mathfrak{F}}(F, m_i, L_i) > 0$*, then the pricing function of fundamentalists increases in the riskiness of the underlying asset, and the economy is in the risk-shifting bubble state.*

Proof. See Appendix A.4. ■

The above result has two implications. Firstly, in a setting comprised of fundamentalists, an underpriced loan is a sufficient condition for risk-shifting behaviour (and hence a risk-shifting bubble) to arise. Geared agents with underpriced loan contracts (and, more specifically, underpriced default options) behave *as if* they were risk-loving, *i.e.*, they are prepared to pay a higher premium for higher risk assets. Secondly, the proposition has a direct implication for the locational characteristics of risk-shifting bubbles in open economies. Since $\sigma_{ji} > \sigma$ for $j \neq i$, if $\sigma_j > 0$, there exists a threshold marginal foreign exchange transaction cost level $c_X^{\mathfrak{F}}$ such that below it investors have a higher reservation price for the foreign symmetric risky asset from country j as compared to the domestic risky asset from country i. This threshold foreign exchange transaction cost level is given by the default option differential of the form

$$c_X^{\mathfrak{F}} = \left(\frac{m_i L_i + c_C}{r_i}\right)(1 - \gamma_{2,F,ji})\left(\frac{F}{F_{ji}^D}\right)^{\beta_{2,F,ji}} - \left(\frac{m_i L_i + c_C}{r_i}\right)(1 - \gamma_{2,F,ii})\left(\frac{F}{F_{ii}^D}\right)^{\beta_{2,F,ii}}, \quad \text{for } j \neq i, \quad F > F_{ii}^D > F_{ji}^D.$$

An example of a situation where $c_X < c_X^*$ is depicted in Figure 2. It shows that an increase in volatility leads to non-decreasing reservation prices for both assets. Furthermore, the reservation price of the symmetric foreign risky asset is greater than the reservation price of the domestic risky asset.

4.2 Asset pricing by speculators

In this subsection, we introduce a possibility of speculative behaviour. To isolate speculative effects from fundamentalist considerations, we consider "pure" speculators, who focus solely on expected capital gains in valuing assets. To do this, we set income yield to 0 in all countries: $\delta_j = 0$ for all j. Effectively, assets are priced such that investors hope to resell them for higher prices than acquired. Safety margin considerations are ignored.

underpricing and hence mispricing function is valued at 0.

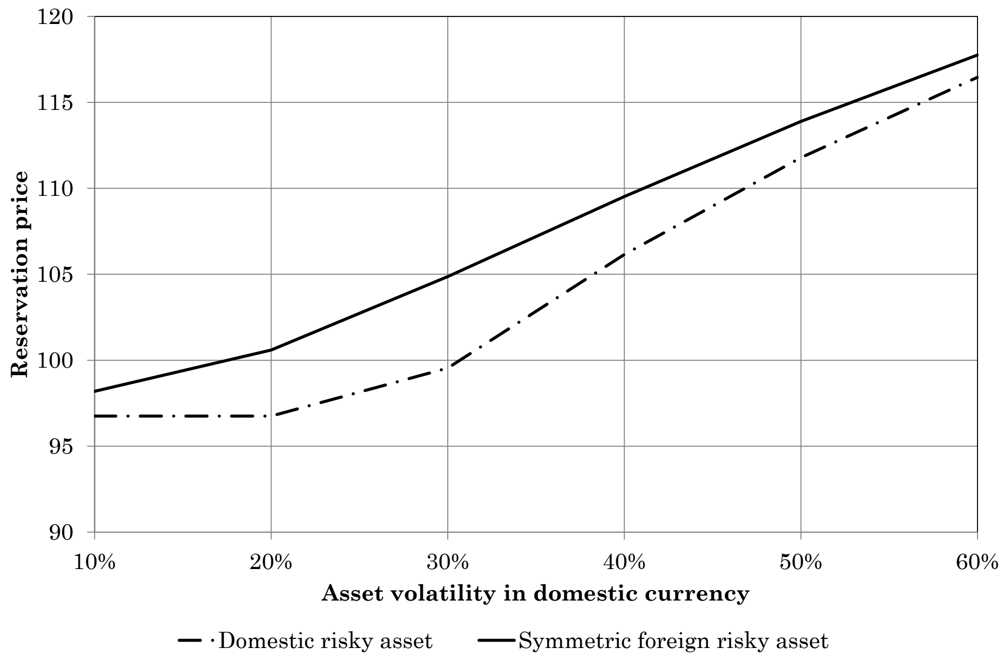


Figure 2: The effect of volatility on the pricing function of fundamentalists

Notes:
Key assumptions are $F = 100$, $X = 1$, $\alpha = 0.01$, $\sigma_j = 0.2$, $c_F = 1$, $c_X = 0.5$, $c_C = 0.09$, $L_i = 45$ and an interest rate spread of 100 bps.
The price function figure is obtained as a result of the partial equilibrium analysis with the following additional assumption: $r = 0.04$.

As previously, we start by carrying out the necessary adjustments to the value of equity in the open long position in risky asset given the newly introduced assumptions. The result is summarised in the following proposition.

Proposition 4. *In continuation region, the value of equity in the open long position in a risky asset in country j held by a speculator from country i is*

$$E_i^{\mathfrak{S}}(F_{ji}) = \frac{\sigma_{ji}^2}{2r_i}\left[\frac{m_i L_i + c_C}{r_i + \frac{\sigma_{ji}^2}{2}}\right]^{1+\frac{2r_i}{\sigma_{ji}^2}}\left[\frac{F_{ji}}{1-\left(\frac{c_C}{m_i L_i + c_C}\right)^{1+\frac{\sigma_{ji}^2}{2r_i}}}\right]^{-\frac{2r_i}{\sigma_{ji}^2}}$$
$$+\frac{F_{ji}}{1-\left(\frac{c_C}{m_i L_i + c_C}\right)^{1+\frac{\sigma_{ji}^2}{2r_i}}} - \frac{m_i L_i}{r_i} - \frac{c_C}{r_i}, \quad \textit{for all } i, j,\ F_{ji}^{D,\mathfrak{S}} < F_{ji} < F_{ji}^{C,\mathfrak{S}},$$

where:

$$
\begin{aligned}
F_{ji}^{D,\mathfrak{S}} &= \left[\frac{m_i L_i + c_C}{r_i + \frac{\sigma_{ji}^2}{2}}\right] \left(B_{1,F,ji}^{\mathfrak{S}}\right)^{-1}, \\
F_{ji}^{C,\mathfrak{S}} &= \left[\left(B_{1,F,ji}^{\mathfrak{S}} - 1\right) \left(B_{1,F,ji}^{\mathfrak{S}}\right)^{\frac{2r_i}{\sigma_{ji}^2}}\right]^{\frac{1}{-\frac{2r_i}{\sigma_{ji}^2}-1}} \left[\frac{m_i L_i + c_C}{r_i + \frac{\sigma_{ji}^2}{2}}\right], \\
B_{1,F,ji}^{\mathfrak{S}} &= \left[1 - \left(\frac{c_C}{m_i L_i + c_C}\right)^{1+\frac{\sigma_{ji}^2}{2r_i}}\right]^{-1}.
\end{aligned}
$$

Proof. See Appendix A.5. ■

Given the above result and the assumption of $\mathcal{S}_i^{\mathfrak{S}}(F_{ji}) = 0$, the generic pricing function (15) is adjusted for speculators as follows:

$$
P_{ji}^{\mathfrak{S}} = F - \frac{c_C}{r_i} - c_F - c_X \mathbb{1}_{j \neq i} + \mathcal{M}_i^{\mathfrak{S}}(F, m_i, L_i), \quad \text{for all } i, j,\ F_{ji}^{D,\mathfrak{S}} < F_{ji} < F_{ji}^{C,\mathfrak{S}}, \tag{18}
$$

where:

$$
\begin{aligned}
\mathcal{M}_i^{\mathfrak{S}}(F, m_i, L_i) &= \max\left[L_i - D^{\mathfrak{S}}(F, m_i, L_i), 0\right], \quad \text{for all } i, j,\ F_{ji}^{D,\mathfrak{S}} < F_{ji} < F_{ji}^{C,\mathfrak{S}}, \\
D^{\mathfrak{S}}(F, m_i, L_i) &= \frac{m_i L_i}{r_i} - \left(B_{1,F,ji}^{\mathfrak{S}} - 1\right) F \\
&\quad - B_{2,F,ji}^{\mathfrak{S}} F^{-\frac{2r_i}{\sigma_{ji}^2}}, \quad \text{for all } i, j, \quad F_{ji}^{D,\mathfrak{S}} < F_{ji} < F_{ji}^{C,\mathfrak{S}}, \\
B_{2,F,ji}^{\mathfrak{S}} &= \frac{\sigma_{ji}^2}{2r_i} \left[\frac{m_i L_i + c_C}{r_i + \frac{\sigma_{ji}^2}{2}}\right]^{1+\frac{2r_i}{\sigma_{ji}^2}} \left(B_{1,F,ji}^{\mathfrak{S}}\right)^{-\frac{2r_i}{\sigma_{ji}^2}}.
\end{aligned}
$$

Although mispricing function for speculators may look more complicated, same basic properties thereof can be established. It is straightforward to see that the result summarised in Proposition 2 holds for speculators as well: the speculator does not take out a loan, unless the loan contract (m_i, L_i) is underpriced, the max-function in the mispricing function $\mathcal{M}_i^{\mathfrak{S}}(F, m_i, L_i)$ takes the value of 0, and hence speculative prices turn out to be invariant in the capital structure.

As with fundamentalist investor type, loan underpricing changes the behaviour of speculators such that they start behaving in a risk-loving fashion but with an added speculative twist.

Proposition 5. *If* $\mathcal{M}_i^{\mathfrak{S}}(F, m_i, L_i) > 0$, *then the pricing function of speculators increases in the riskiness of the underlying asset, and the economy is in the bubble state combining risk-shifting and speculation.*

Proof. See Appendix A.6. ■

A numerical example of the above result is depicted in Figure 3. It shows that up to the point where the option to default on the geared investment in the risky asset abroad becomes underpriced ($\sigma = 0.2$), the reservation price for the domestic risky asset exceeds that of the symmetric foreign risky asset (investment at home is preferred) and is invariant in volatility of the underlying (risk-shifting incentives are absent). However, once underpricing is introduced, the price increases monotonically in volatility (risk-shifting incentives) and the reservation price of the foreign risky asset is greater than the reservation price of the domestic risky asset (investment abroad is preferred).

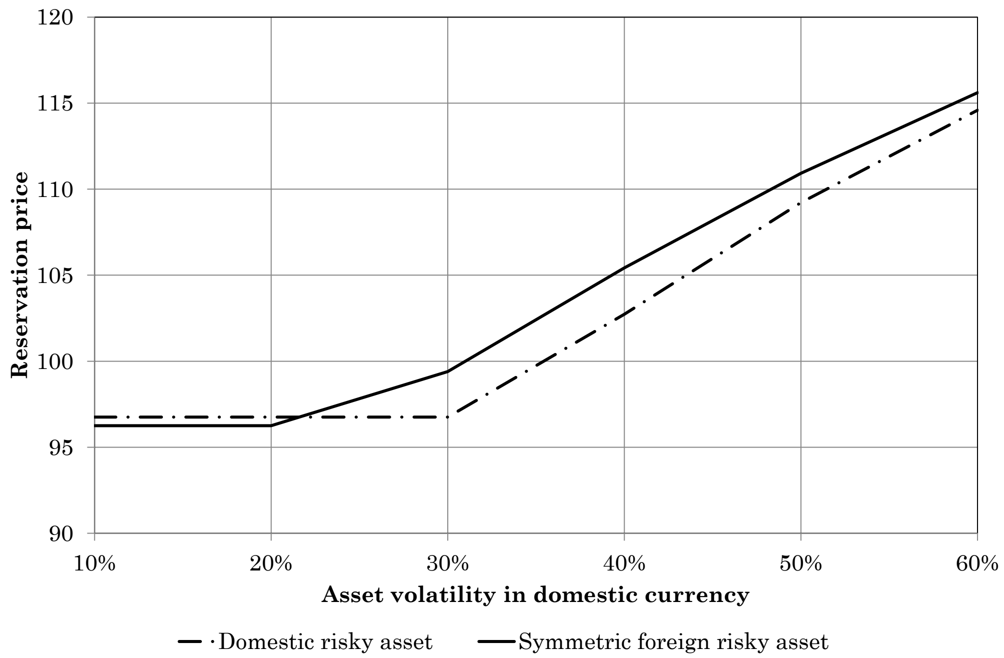


Figure 3: The effect of volatility on the pricing function of speculators

Notes:
Key assumptions are $F = 100$, $X = 1$, $\alpha = 0.01$, $\sigma_j = 0.2$, $c_F = 1$, $c_X = 0.5$, $c_C = 0.09$, $L_i = 45$, and an interest rate spread of 100 bps.
The price function figure is obtained as a result of the partial equilibrium analysis with the following additional assumption: $r = 0.04$.

A new insight relative to the previous subsection concerns the fact that once loan mispricing is introduced, speculators not only have risk-shifting but also speculative incentives, which materialise through the existence of unwinding threshold $F_{ji}^{C,\mathfrak{S}}$ (see Proposition 4). This threshold plays the role of a hysteresis mechanism in unwinding decisions, which is absent in the case of fundamentalists. Fundamentalists unwind their positions as soon as they are offered a higher price than the perceived value of the position they hold. For speculators, this alone may not be enough. Instead, they would hold on to the position until they receive a *sufficiently* higher offer than the perceived value of the asset they hold. It is interesting to study the properties of the unwinding threshold in relation to the amount of leverage employed and the volatility of the underlying asset. These are important as they influence the dynamic properties of the bubble – its persistence.

Proposition 6. *The unwinding threshold of the speculator increases in both volatility and leverage.*

Proof. See Appendix A.7. ■

The above proposition yields an important insight in relation to the persistence of risk-shifting bubbles with speculative motives working in the background. Since the expected time to unwinding increases in the level of the unwinding threshold, speculation-supported risk-shifting bubbles will exhibit less transactions in more leveraged economies, and in open economies. The latter claim is due to the fact that open economies provide opportunities for taking on extra (foreign exchange) risk in addition to the core (asset) risk. It is critical to differentiate the persistence mechanism induced by speculative behaviour set out in this subsection from the upward dynamics implied by the equilibrium price derived in subsection 4.1. Pure risk-shifting bubbles (subsection 4.1) persist in the up-market because, in the next period, new transactions take place induced by higher reservation prices of fundamentalists. Speculative risk-shifting bubbles persist in the up-market because agents hold on to their assets until prices go up sufficiently high without new transactions necessarily taking place. In this respect, speculative risk-shifting bubbles can be "quieter" relative to pure risk-shifting bubbles in terms of turnover but not necessarily so in terms of aggregate volatility. In turn, pure risk-shifting bubbles are "quieter" than (speculative) equity (but not risk-shifting) bubbles in terms of aggregate volatility.

4.3 Asset pricing by value investors

The investment strategy (and the corresponding investor type) that remains to be considered is *value investing*. The value of open position held by value investor is the same as that of fundamentalist and is provided in Proposition 1. The crucial quantitative ingredient of value investing that differentiates it value investors from fundamentalists is the "margin of safety": "a favorable difference between price on the one hand and indicated or appraised value on the other, [which] is available for absorbing the effect of miscalculations or worse than average luck" (Graham, 2006, p.517). In other words, it is a discount to the perceived intrinsic value of the asset as a safeguard against adverse realisations of uncertainty in future. None of the imminent value investors provide a rigorous way of calculating the safety margin. However, Yee (2008) points out that this discount can be thought of as the value of the option to defer investment in order to obtain the wedge between the intrinsic value and price that maximises the expected gain on the position. Yee uses a quote from Warren Buffett to illustrate this point:

> "In investments, there's no such thing as a called strike. You can stand at the plate and the pitcher can throw a ball right down the middle, and if it's General Motors at 47 and you don't know enough to decide on General Motors at 47, you let it go right on by and no one's going to call a strike. The only way you can have a strike is to swing and miss." (Lowe, 1997, p.111)

This option to wait is perpetual. It follows that the optimal entry (reservation) price for a value investor is such that the value of the option to wait is zero. To obtain the reservation price of the value investor from country i, consider the value of its option to defer investment, $G_i\left(F_{ji}\right)$. The value of option to wait to invest (or value of equity in inactive long position) in continuation region solves the following ODE:

$$\frac{\sigma_{ji}^2}{2} F_{ji}^2 G_i''\left(F_{ji}\right) + \left(r_i - \delta_j\right) F_{ji} G_i'\left(F_{ji}\right) - r_i G_i\left(F_{ji}\right) = 0, \quad \text{for all } i, j, \quad F_{ji} < F_{ji}^I. \tag{19}$$

The general solution to equation (19) is

$$G_i\left(F_{ji}\right) = A_{1,F,ji} F_{ji}^{\beta_{1,F,ji}} + A_{2,F,ji} F_{ji}^{\beta_{2,F,ji}}, \quad \text{for all } i, j, \quad F_{ji} < F_{ji}^I, \tag{20}$$

where:

$A_{1,F,ji}$ and $A_{1,F,ji}$ are constants to be determined,

$\beta_{1,F,ji}$ is the non-negative root of the quadratic equation (14).

The boundary conditions imposed to obtain the value of option to wait to invest are as follows. First, prior to opening a long position it does not generate any income, and as asset value approaches the absorbing barrier of 0, the value of long position in this risky asset vanishes. Second, at the time of opening a long position, the value of equity in this position equals the value of equity from holding this position less the costs of setting it up.[11] Third, the position is opened optimally. Formally, these boundary conditions are:

$$G_i\left(0\right) = 0, \tag{21a}$$
$$G_i\left(F_{ji}^I\right) = E_i^{\mathfrak{F}}\left(F_{ji}^I\right) - P + L_i - c_F - c_X \mathbb{1}_{j\neq i}, \tag{21b}$$
$$G_i'\left(F_{ji}^I\right) = E_i^{\mathfrak{F}\prime}\left(F_{ji}^I\right). \tag{21c}$$

Notice that for valuing the option to wait we use the value of equity in an open position held by fundamentalist since value investors are known to be relatively immune to speculative behaviour of the form described in this paper. Using the above boundary conditions, we find the investment trigger, F_{ji}^I, that satisfies the following equation:

$$F_{ji}^I\left(1-\frac{1}{\beta_{1,F,ji}}\right) + B_{2,F,ji}^{\mathfrak{F}}\left(F_{ji}^I\right)^{\beta_{2,F,ji}}\left(1-\frac{\beta_{2,F,ji}}{\beta_{1,F,ji}}\right) - \frac{m_i L_i + c_C}{r_i} + L_i$$
$$- P - c_F - c_X \mathbb{1}_{j\neq i} = 0, \quad \text{for all } i,j, \quad \mathcal{M}_i^{\mathfrak{F}}\left(F_{ji}^I, m_i, L_i\right) > 0.$$

For sufficiently low P, a value investor would extinguish the option to wait to invest immediately. Hence, we can find the reservation price of the value investor by setting $F_{ji}^I = F$ and solving for P:

$$P_{ji}^{\mathfrak{V}} = F - \frac{c_C}{r_i} - c_F - c_X \mathbb{1}_{j\neq i} - \mathcal{S}_i^{\mathfrak{V}}\left(F_{ji}\right)$$
$$+ \mathcal{M}_i^{\mathfrak{V}}\left(F_{ji}, m_i, L_i\right), \quad \text{for all } i,j, \quad F < F_{ji}^D, \tag{22}$$

[11] Alternatively, this condition can be interpreted as ensuring that when the value investor enters the market by purchasing the risky asset at price P, the value of the option to wait is 0.

where:

$$\mathcal{S}_i^{\mathfrak{V}}(F_{ji}) = \frac{F_{ji}}{\beta_{1,F,ji}}, \quad \text{for all } i,j,$$

$$\mathcal{M}_i^{\mathfrak{V}}(F_{ji}, m_i, L_i) = \begin{cases} \mathcal{M}_i^{\mathfrak{F}}(F_{ji}, m_i, L_i) - \frac{\beta_{2,F,ji}}{\beta_{1,F,ji}} B_{2,F,ji}^{\mathfrak{F}} F_{ji}^{\beta_{2,F,ji}}, & \mathcal{M}_i^{\mathfrak{F}}(F_{ji}, m_i, L_i) > 0, \\ 0, & \text{otherwise,} \end{cases}$$

$$\text{for all } i,j, \quad F_{ji} > F_{ji}^D,$$

$$B_{2,F,ji}^{\mathfrak{F}} = \left(\frac{m_i L_i + c_C}{r_i}\right) \frac{(1 - \gamma_{2,F,ji})}{\left(F_{ji}^D\right)^{\beta_{2,F,ji}}}, \quad \text{for all } i,j, \quad F_{ji} > F_{ji}^D.$$

Consider the expression for safety margin $\mathcal{S}_i^{\mathfrak{V}}(F_{ji})$. It captures the desired properties as a discount to perceived intrinsic value of asset as a safeguard against unfavourable future realisations. As uncertainty increases, absolute value of safety margin goes up. This can be verified by differentiating it with respect to volatility: $\frac{\partial \mathcal{S}_i^{\mathfrak{V}}(F_{ji})}{\partial \sigma_j} = -\frac{\mathcal{S}_i^{\mathfrak{V}}(F_{ji})}{\beta_{1,F,ji}} \frac{\partial \beta_{1,F,ji}}{\partial \sigma_j}$, and noting that $\frac{\partial \beta_{1,F,ji}}{\partial \sigma_j} < 0$.

The piecewise nature of mispricing function $\mathcal{M}_i^{\mathfrak{F}}(F_{ji}, m_i, L_i)$ can be traced to condition (21b): whenever the loan is not underpriced for a fundamentalist, it is not underpriced for the value investor, and hence drops out of the valuation problem. Thus, Proposition 2, whereby if the loan is not underpriced, asset pricing is invariant in the capital structure employed, holds for value investors as well.

Whenever loans are underpriced, investors choose to gear up. This reduces the observed discount of intrinsic asset value due to safety margin considerations. The intuition behind this result can be explained as follows. The margin of safety is a form of self-imposed insurance, whose indirect (non-pecuniary) cost is incurred by every value investor. Net of these insurance costs, investor earns zero expected profit on its position. An underpriced default option also provides an insurance because it limits investor's downside. This insurance is a form of informational rent that comes "free of charge" to the investor. There is no reason to pay for insurance that one already owns through an underpriced default option. Thus, the value investor decreases its discount to intrinsic value (and therefore the overall insurance cost), and progressively so, with higher gearing.

The risk-shifting bubble test requires the analysis of the effect of volatility increase on investor's reservation price. For value investors, there are two terms with opposing signs that are affected by an increase of position's riskiness: margin of safety and mispricing function. As a result, increase in risk is represented by the trade-off between increased safety margin and increased mispricing function.

A new insight that this subsection yields relative to subsection 4.1 concerns the sufficiency condition for generating risk-shifting incentives of agents. Previously, we established that underpricing of the embedded default option is sufficient for generating risk-shifting incentives in economies comprised of fundamentalists and speculators. The next proposition shows that if investors account for a safety margin (or a self-imposed insurance against downside risk), default option underpricing is only a necessary condition for risk-shifting incentives (and hence a risk-shifting bubble) to arise. The pricing function is no longer monotonic in the underlying measure of risk. What is also required is that the amount of leverage is sufficiently large to generate risk-shifting incentives amongst agents.

Proposition 7. *The pricing function of value investors increases in the riskiness of the underlying asset and the economy is in a risk-shifting bubble state, if loan is underpriced (*$\mathcal{M}_i^{\mathfrak{F}}(F, m_i, L_i) > 0$*) and the amount of leverage* L_i *exceeds the risk-shifting threshold,* $L_{ji}^{\mathfrak{V}}$*, given by*

$$L_{ji}^{\mathfrak{V}} = \frac{r_i F}{m_i}\left[\frac{\left(1-\beta_{2,F,ji}\right)\gamma_{2,F,ji}^{\beta_{2,F,ji}}\frac{\partial\beta_{1,F,ji}}{\partial\sigma_j}}{\beta_{1,F,ji}\left(1-\left(\beta_{1,F,ji}-\beta_{2,F,ji}\right)\ln\left(\frac{F}{F_{ji}^{D}}\right)\right)\frac{\partial\beta_{2,F,ji}}{\partial\sigma_j}-\beta_{2,F,ji}\frac{\partial\beta_{1,F,ji}}{\partial\sigma_j}}\right]^{\frac{1}{1-\beta_{2,F,ji}}} - \frac{c_C}{m_i}. \tag{23}$$

Proof. See Appendix A.8. ■

Proposition 7 demonstrates that for the risk-shifting effect to take place, the amount of leverage has to exceed a certain threshold, $L_{ji}^{\mathfrak{V}}$, which we term the *risk-shifting threshold.* If the amount of leverage afforded under the loan agreement is less than $L_{ji}^{\mathfrak{V}}$, then the pricing function is dominated by safety margin considerations and does not increase in volatility. On the other hand, above the risk-shifting threshold, *ex ante* risk-neutral investors behave *as if* they were risk-loving and pay a premium for higher risk satisfying the risk-shifting bubble condition set out in our definition above. Given the results obtained earlier, we can conclude that if there is loan underpricing and the leverage amount exceeds the risk-shifting threshold, then the premium investors are prepared to pay for risk increases in leverage. In a setting where investors have a choice between assets of varying risk, it follows that *ceteris paribus* larger reservation prices will emerge for assets with greater volatility and investors increase the riskiness in their portfolios as soon as they hit the risk-shifting threshold (if $L_i > L_{ji}^{\mathfrak{V}}$).

Existence of risk-shifting threshold $L_{ji}^{\mathfrak{V}}$ has important implications for policy in that it justifies maximum loan-to-value (LTV) ratio as a macro-prudential policy instrument. Given the state of, for example, real estate market (yields and volatility), regulator imposes a cap on LTV ratio that keeps the amount of leverage below $L_{ji}^{\mathfrak{V}}$; hence, investors do not behave in a risk-shifting fashion, which prevents a risk-shifting bubble from taking off. Hong Kong Monetary Authority has been employing this tool for nearly two decades, whilst regulators in Hungary, Norway and Sweden have committed to incorporating it into their macro-prudential toolkit (Wong, Li, and Choi, 2011).

A set of properties of the risk-shifting threshold are worth examining. Firstly, it decreases in volatility (see Figure 4). This result is important in that it shows that bubbles are not only larger in open economies (subsection 4.1) but also more likely to occur when transactions with foreign assets are available, as these may induce investors who are not risk-shifters domestically to have risk-shifting incentives when it comes to foreign risky assets (that are more volatile in the domestic currency) nevertheless. Secondly, the risk-shifting threshold decreases in custodian fees (non-debt service costs of supporting an open position). This result is clear from equation (23) and has a simple economic intuition: these costs increase the value of the option to default, making it more dominant in the valuation problem relative to the 'ungeared' safety margin considerations. Thirdly, it follows from equation (23) that foreign exchange transaction costs do not influence the risk-shifting threshold, even though they do impact the reservation price (equation (22)), and may deter investors from investing abroad.

Value investors' pricing function comparative statics with respect to the volatility of the underlying asset (Figure 4) highlights an important property concerning its piecewise nature. In previous subsections, we decompose the pricing function output into two regions: volatility levels at which the option to default is not underpriced and at which it is underpriced. Under certain assumptions regarding foreign exchange transaction costs, the reservation price of the domestic risky asset exceeds the reservation price of the symmetric foreign risky asset in the former region and the prices are not "bubbly" in the risk-shifting sense, whilst the converse held true in the latter region. In the case of value investors, three regions can be found: for low volatility levels there is no underpricing of the default option, domestic risky asset prices are greater than foreign risky asset prices

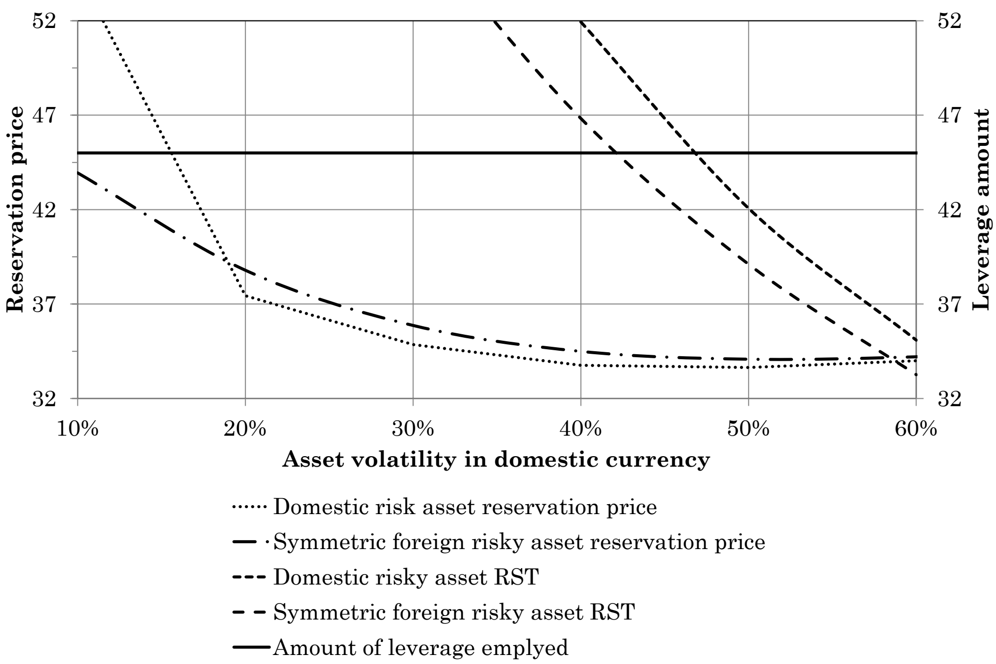


Figure 4: The effect of volatility on the risk-shifting threshold (RST) and prices

Notes:
Key assumptions are $F = 100$, $X = 1$, $\alpha = 0.01$, $\sigma_j = 0.2$, $c_F = 1$, $c_X = 0.5$, $c_C = 0.09$, and an interest rate spread of 100 bps.
The price function figure is obtained as a result of the partial equilibrium analysis with the following additional assumption: $r = 0.04$.

(investment at home), and there is no bubble in place; for medium volatility levels there is underpricing of the default option, reservation prices for the symmetric foreign risky asset exceed those of the domestic risky asset (investment abroad), but there is no risk-shifting bubble (although there is overpricing of risky assets due to default option underpricing, it does not change the incentives of investors as they do not behave *as if* they were risk-loving); for high volatility levels there is underpricing of the default option and the gearing is above the risk-shifting threshold, foreign risky asset prices are higher than domestic risky asset prices (investment abroad), and there is a risk-shifting bubble (*i.e.*, investors behave *as if* they were risk-loving and build up excessive risk).

There is another consideration that may impact the risk-shifting threshold but cannot be studied within our model: the investment horizon of value investors. We have derived the safety margin from the perpetual option to wait to invest, yet some investors may consider an infinite investment horizon as too large relative to their portfolio management careers. As a result, the option maturity may be finite, and the problem would take the form of a finite-lived American call option. There is no analytic solution to this problem but it is known that the exercise trigger of a finite-lived American call option is continuous and non-decreasing in time-to-maturity (van Moerbeke, 1979; Kim, 1990; Jacka, 1991). Therefore, the safety margin is not increasing as time-to-maturity goes down, and so is the risk-shifting threshold. Furthermore, as the time-to-maturity tends to zero, the value investor's pricing function approaches that of fundamentalist, and the risk-shifting threshold approaches zero.

It is worth pointing out that in a competitive general equilibrium, the risk-shifting threshold does not tend to zero, as it happens with option to wait models when investments by winning agents eliminates opportunities for others (Lambrecht and Perraudin, 2003). This is not the case for value investors, as the margin of safety does not play a role of additional flexibility that they are prepared to give up to win in competition, but instead is a necessary non-pecuniary insurance cost. As a result, net of this non-pecuniary cost the standard zero expected profit condition holds.

5 Asset pricing in general equilibrium

Having established the partial equilibrium properties of pricing functions for each investor type, we proceed to studying their implications in a two-symmetric-countries general equilibrium model. Consider agents of a given type $\mathfrak{T} \in \{\mathfrak{F}, \mathfrak{S}, \mathfrak{V}\}$ from two symmetric countries i and j $(i \neq j)$ that have the same initial endowment $W_i = W_j = W$ and extract information rent from underpriced loan contracts $(\mathcal{M}_i^{\mathfrak{T}}(F_{ji}, m_i, L_i) > 0$ for all $i, j)$ with leverage amounts above the risk-shifting threshold $(L_i > L_{i,j}^{\mathfrak{T}}$ for all $i, j)$.[12] If marginal foreign exchange transaction costs are below $c_X^{\mathfrak{T}}$,[13] then for each agent the reservation price for the foreign symmetric risky asset is above that for the domestic risky asset. The competitive equilibrium ensures that each agent prices assets at its respective reservation price. Since there are two categories of agents (agents from country i and country j), strategic interaction arises and the pure Nash strategy for each of them seeking to obtain a long position in a risky asset is to go abroad

[12] Note that the risk-shifting threshold for fundamentalists and speculators is at 0.

[13] Although we do not formally derive the critical level of marginal foreign exchange transaction cost $c_X^{\mathfrak{T}}$ below which investors have higher reservation rate for foreign risky assets in presence of loan underpricing for speculators and value investors, it can be obtained in the same way as in section 4.1 for fundamentalists.

(as at home they have no chance of outbidding agents coming from abroad). In equilibrium, everyone allocates the entire loan amount to the foreign risky asset to maximise the chance of obtaining this asset. The general equilibrium result follows.

Proposition 8. *For a given investor type $\mathfrak{T} \in \{\mathfrak{F}, \mathfrak{S}, \mathfrak{V}\}$, a competitive two-symmetric-countries general equilibrium is characterised by a risk-shifting bubble in each country generated by foreign investors, if the following conditions hold:*

$$L_i = L_j = L > L_{ji}^{\mathfrak{T}} \geqslant 0, \quad \mathcal{M}_i^{\mathfrak{T}}\left(F_{ji}, L, i\right) > 0,$$
$$c_X < c_X^{\mathfrak{T}}, \quad \textit{and} \quad \sigma_j > 0.$$

The resulting equilibrium $\left(P^{\mathfrak{T}}, r\right)$ is

$$P^{\mathfrak{T}} = P_i^{\mathfrak{T}} = P_j^{\mathfrak{T}}, \quad \textit{and}$$
$$r = r_i = r_j = f'\left(W + L - P^{\mathfrak{T}}\right).$$

The pricing functions for the relevant investor type are substituted in from the previous section. Each pair of conditions in the proposition above have their own significance. The first pair relates to the loan markets in the two symmetric countries, and is sufficient for the risk-shifting bubble to emerge. The second pair relates to the foreign exchange market and determines whether the bubble emerges at home or abroad.

The interesting question that has not been analysed to date is the effect of *quantitative easing* in the context of risk-shifting bubbles. We select this policy tool as the subject of the following numerical example. Consider two symmetric countries with initial wealth level $W = 200$ and zero leverage. The production function takes the form $f(K) = AK^{\gamma}$, where $A = 5$, and $\gamma = 0.3$. The risky asset is characterised by $\alpha = 0.01$, and $\sigma = 0.3$. The exchange rate process is driftless in a symmetric countries equilibrium with volatility given by $\sigma_j = 0.2$. Marginal cost of transacting a unit of the risky asset is $c_F = 1$, whereas marginal cost of transacting a unit of the foreign asset is $c_X = 0.5$. Custodian fees are $c_C = 0.09$. The spread on the loan contract is constant and is given by 100 bps. For the purposes of this exercise, we use the following terminology: risky assets are also called financial assets or financial sector of the economy, whereas safe assets are called productive technology or real sector of the economy. To emulate quantitative easing, we gradually introduce additional funds into the economy, which play the role of leverage L if used to purchase financial assets, or go to the real sector otherwise.

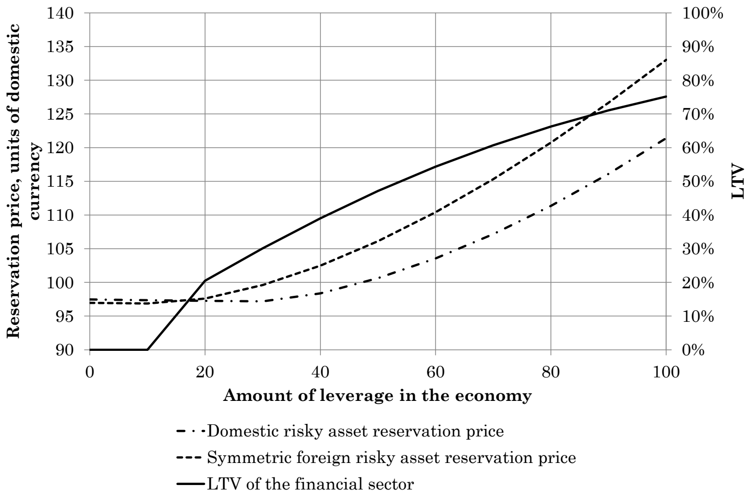
For fundamentalists, the results of the exercise are summarised in Figure 5. As can be seen in Figure 5a, an increase in leverage from 0 to 10 does not create a risk-shifting bubble, and prices for domestic risky assets are higher than the prices for foreign risky assets. Investors opt for home assets (Figure 5c) whose acquisition is funded by equity rather than debt. This happens because at $L = 10$, risky loans are overpriced, and equity is the preferred method of finance. Figure 5b shows that the introduction of additional funds into the economy results in a decreasing risk-free rate, and increased lending to the real sector (*i.e.*, an increase in the amount spent on productive technology as opposed to risky financial assets).

Increasing leverage to 20, however, results in some of the funds being channelled into the financial sector, whose loan-to-value (LTV) ratio starts to go up. However, there are still additional funds channelled into the real sector which result in a decreasing risk-free rate (Figure 5b). In the financial asset market, funds are used to purchase foreign risky assets (Figures 5a and 5c), and the risk-shifting bubble starts to emerge (loans are underpriced given the value of the option to default on foreign risky asset transactions).

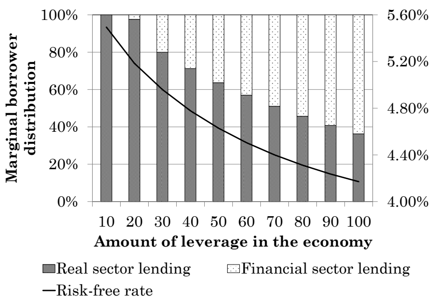
Gradually, as additional rounds of quantitative easing are carried out, more and more of the marginal funds are spent to transact financial assets whose weight in a representative agent's portfolio starts to increase (Figure 5c). This inflates the risk-shifting bubble abroad rather than financing real productive activity at home. The crowding out of real sector borrowers is demonstrated in Figure 5b, and suggests that every additional round of quantitative easing is less effective in stimulating real activity directly. On the other hand, it proves successful in inflating financial asset prices and increasing the gearing ratio of the financial sector (Figure 5a), making it more systemically important and potentially "too big to fail".

The general equilibrium results in an economy comprised of speculators are qualitatively akin to what we obtain in the setting comprised of fundamentalists. The numerical output is provided in Figure 6.

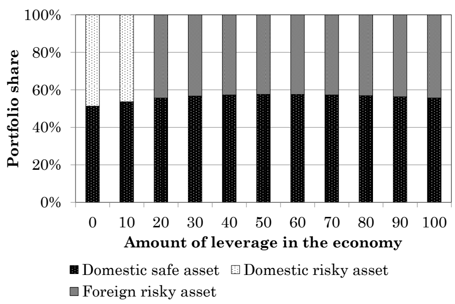
The same holds for value investors except for the additional requirement for a risk-shifting bubble in the form of $L_i > L_{ji}^{\mathfrak{F}} > 0$. The numerical output of the general equilibrium exercise is provided in Figure 7. Notice that, as one would expect, value investors allocate a larger share of their portfolio to safe asset



(a) Asset prices and loan-to-value ratio of the financial sector



(b) Marginal borrower distribution and risk-free rate

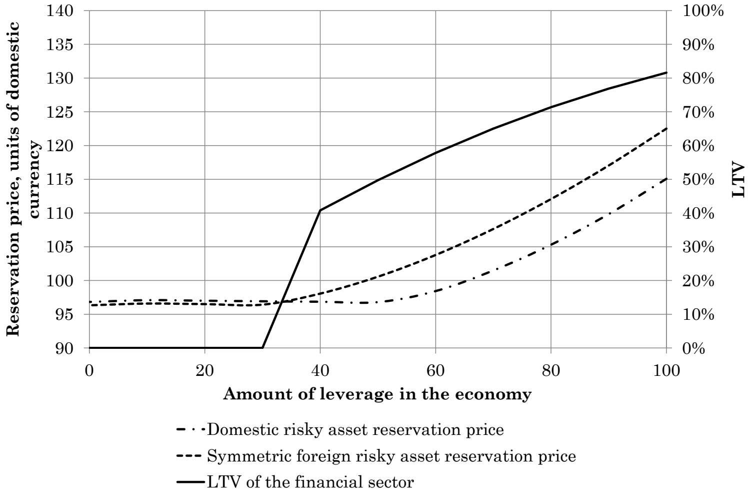


(c) Portfolio composition

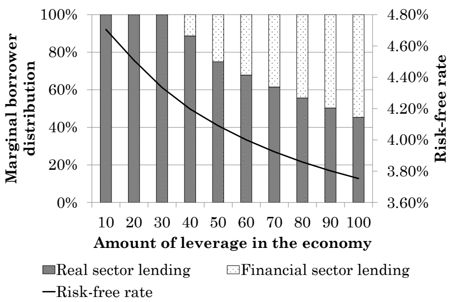
Figure 5: General equilibrium effects of quantitative easing (fundamentalists)

Notes:

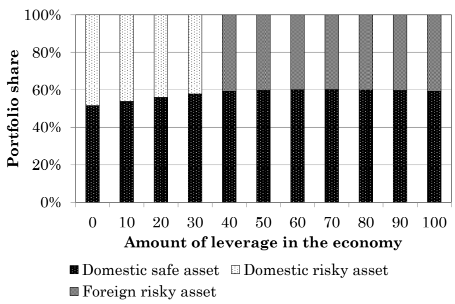
Key assumptions are $F = 100$, $X = 1$, $\alpha = 0.01$, $\sigma_j = 0.2$, $c_F = 1$, $c_X = 0.5$, $c_C = 0.09$, interest rate spread 100 bps, $f(K) = AK^{\gamma}$, $A = 5$, $\gamma = 0.3$, $W = 200$, $\sigma = 0.3$.



(a) Asset prices and loan-to-value ratio of the financial sector



(b) Marginal borrower distribution and risk-free rate



(c) Portfolio composition

Figure 6: General equilibrium effects of quantitative easing (speculators)

Notes:

Key assumptions are $F = 100$, $X = 1$, $\delta = 0$, $\sigma_j = 0.2$, $c_F = 1$, $c_X = 0.5$, $c_C = 0.09$, interest rate spread 100 bps, $f(K) = AK^\gamma$, $A = 5$, $\gamma = 0.3$, $W = 200$, $\sigma = 0.3$.

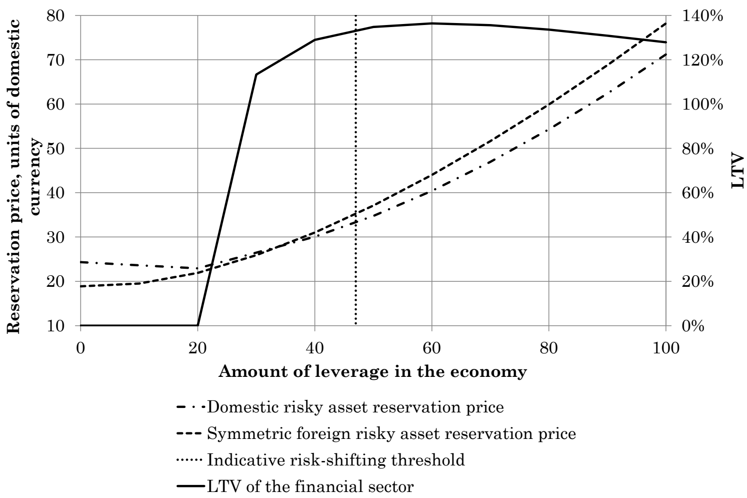
investments compared to fundamentalists (Figure 7c).

6 The possibility of global imbalances

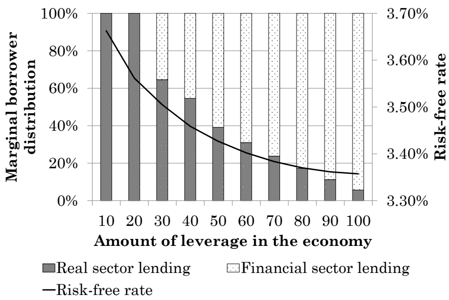
The two symmetric country setting is useful in that it enables demonstration of risk-shifting bubbles as a general equilibrium phenomenon. There is no ambiguity as to the optimal capital allocation decision of investors. This is not so in a world with more than two countries. In particular, coordination problem may arise, whereby there is overinvestment in some countries and underinvestment in others. This is an unstable outcome, because, ultimately, markets have to clear in all countries and capital allocation imbalances eliminated. Hence, if agents can adjust their locational decisions instantly, the system will generate the result that is akin to the one we obtained in a two country world: provided economies are symmetric and sufficiently geared, loans are underpriced, and foreign exchange transaction costs are sufficiently small, risk-shifting bubbles funded from abroad arise in each country. If, however, instant adjustment of locational decisions is not possible, then temporal capital account imbalances may arise. These temporal imbalances result in foreign exchange rate dynamics that deviates from the uncovered interest rate parity (UIP), which holds in stable equilibrium, and give rise to temporal carry trade opportunities.[14]

In this section, we do not model the emergence mechanics of these global imbalances and the associated carry trade opportunities explicitly. Instead, we seek to understand how the availability of carry trade opportunities affects investors' reservation prices for risky assets. We limit our analysis to fundamentalist investor type. The following reduced form formulation of global imbalances is employed. Suppose, for a given country pair, imbalances generate an appreciation rate of ψ_{ji} in excess of the stable equilibrium interest rate differential. Since imbalances are not stable, there is a probability $\mu_{ji}\mathrm{d}t$ that the abnormal appreciation rate will vanish over the next $\mathrm{d}t$ and stable equilibrium will be achieved. Given the temporary nature of imbalances, they do not affect equilibrium asset

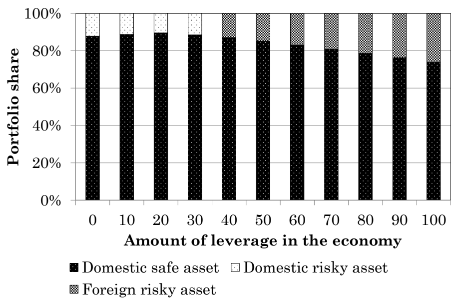
[14] Although deviations from UIP have been widely documented at least since contributions of Hansen and Hodrick (1980) and Fama (1984), there is little consensus as to the source thereof. Papers that seek to provide an explanation of deviations from UIP can be broadly divided into risk-based and and non-risk-based analyses. Recent examples for risk-based explanations are Backus, Foresi, and Telmer (2001); Lustig and Verdelhan (2007); Lustig, Roussanov, and Verdelhan (2008); Brennan and Xia (2006); Farhi and Gabaix (2008); Brunnermeier, Nagel, and Pedersen (2009). Recent non-risk-based explanations are, for instance, Lyons (2001); Burnside, Eichenbaum, and Rebelo (2007, 2009); Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011); Plantin and Shin (2011).



(a) Asset prices and loan-to-value ratio of the financial sector



(b) Marginal borrower distribution and risk-free rate



(c) Portfolio composition

Figure 7: General equilibrium effects of quantitative easing (value investors)

Notes:

Key assumptions are $F = 100$, $X = 1$, $\alpha = 0.01$, $\sigma_j = 0.2$, $c_F = 1$, $c_X = 0.5$, $c_C = 0.09$, interest rate spread 100 bps, $f(K) = AK^{\gamma}$, $A = 5$, $\gamma = 0.3$, $W = 200$, $\sigma = 0.3$.

Risk-shifting threshold is plotted for indicative purposes only: its actual value depends on the leverage employed.

yields.

Let process $(\tilde{X}_{ji,t})_{t\geqslant 0}$ capture the abnormal appreciation of foreign exchange rate (in excess of the interest rate differential between countries i and $j \neq i$) such that:

$$\mathrm{d}\tilde{X}_{ji,t} = (r_i - r_j)\,\tilde{X}_{ji,t}\mathrm{d}t + \psi_{ji}\tilde{X}_{ji,t}\mathrm{d}t + \sigma_j\tilde{X}_{ji,t}\mathrm{d}z^i_{j,t}, \quad \tilde{X}_{ji,0} \equiv \tilde{X}_{ji},\ j \neq i. \tag{24}$$

Then, the modified value dynamics of the symmetric foreign risky asset from country j in units of domestic currency of country i $(\tilde{F}_{ji,t})_{t\geqslant 0}$ $(\tilde{F}_{ji,t} = \tilde{X}_{ji,t}F_{jj,t})$ is given by:

$$\mathrm{d}\tilde{F}_{ji,t} = (r_i - r_j + \alpha)\,\tilde{F}_{ji,t}\mathrm{d}t + \psi_{ji}\tilde{F}_{ji,t}\mathrm{d}t + \sigma_{ji}\tilde{F}_{ji,t}\mathrm{d}z^i_{ji,t}, \quad \tilde{F}_{ji,0} \equiv \tilde{F}_{ji},\ j \neq i. \tag{25}$$

The value of the above-described carry trade opportunity to fundamentalist from country i, $\mathcal{C}^{\mathfrak{F}}_i(F_{ji}, \psi_{ji}, \mu_{ji})$, is given by the difference in values of equity in open position in asset that follows the process $(\tilde{F}_{ji,t})_{t\geqslant 0}$ and open position in asset that follows the benchmark process (*i.e.*, in absence of global imbalances) $(F_{ji,t})_{t\geqslant 0}$, both of which may vanish with probability $\mu_{ji}\mathrm{d}t$ over the next $\mathrm{d}t$:

$$\mathcal{C}^{\mathfrak{F}}_i(F_{ji}, \psi_{ji}, \mu_{ji}) = E^{\mathfrak{F}}_i(\tilde{F}_{ji}, \mu_{ji}) - E^{\mathfrak{F}}_i(F_{ji}, \mu_{ji}), \quad j \neq i. \tag{26}$$

The value of equity holding in an open position in the underlying asset, whose value may suddenly drop to 0, is given by the probability-weighted sum of two scenarios. Under the first scenario, asset survives over the next $\mathrm{d}t$, whereas, under the second scenario, its value vanishes with probability $\mu_{ji}\mathrm{d}t$. If the position is geared and the value of asset drops to 0, then investor defaults. Upon default, the value of outstanding debt $\frac{m_iL_i}{r_i}$ and present value of custodian fees $\frac{c_C}{r_i}$ are written off.

Appendix A.9 shows that whenever the probability of global imbalances correcting over the next $\mathrm{d}t$ is sufficiency large ($\mu_{ji} \geqslant r_i$), then the value of carry trade opportunity, in continuation region, is given by:

$$\mathcal{C}^{\mathfrak{F}}_i(F_{ji}, \psi_{ji}, \mu_{ji}) = \delta_j\left[\frac{\tilde{F}_{ji}}{\delta_j - \psi_{ji} + \mu_{ji}} - \frac{F_{ji}}{\delta_j + \mu_{ji}}\right], \quad j \neq i. \tag{27}$$

For $0 < \psi_{ji} < \mu_{ji} + \delta_j$, the carry trade factor is positive. As a result, if global imbalances are beneficial in the sense of the above discussion, then *ceteris paribus* the reservation price of fundamentalist from country i for symmetric foreign risky asset in country $j \neq i$ in presence of value-enhancing carry trade opportunities exceeds the reservation price in stable equilibrium.

This gives rise to the following mechanics. Risk-shifting incentives in an environment with sufficiently low foreign exchange transaction costs induce agents to look for investment opportunities abroad. In a world with more than two countries and impossibility of instant adjustment, this may lead to temporal capital account imbalances that result in temporal deviations of foreign exchange rate dynamics from UIP. Agents that observe these deviations may be attracted by an additional (but short-lived) abnormal return on risky asset and bid up prices further (beyond the stable equilibrium level). This process may be self-fulfilling for a short period of time as new entrants will keep on increasing imbalances enhancing the abnormal currency appreciation rate ψ_{ji}. However, as an increasing number of agents coordinates into a given country, not only currency appreciation rate but also the probability of "day of reckoning" $\mu_{ji}\mathrm{d}t$ arriving over the next $\mathrm{d}t$ will increase.

Two important considerations are worth mentioning. First, risk-shifting incentives may result in global imbalances that are not a stable equilibrium phenomenon and thus form a separate asset pricing factor, which we term carry trade component. Second, although underpriced credit may spark risk-shifting behaviour that in some instances may create carry trade opportunities, the latter need not arise from mispriced credit only. Their availability may lead to asset price inflation nonetheless. Thus, it is generally instructive to decompose a given bubble that is driven by overseas funding into risk-shifting and carry trade components, and to control for carry trades in empirical risk-shifting bubble identification studies.

7 Detection of risk-shifting bubble in New Zealand's housing market

Risk-shifting bubbles arise as a result of a change in investor incentives. They can be identified empirically by pinpointing these changes. In the open economy setting, if there is a risk-shifting bubble that is funded from abroad, then an exchange rate volatility between the target and funding countries should positively affect asset prices in the target country as this is a reflection of risk-shifting incentives of overseas investors. On the other hand, there should not be any positive relationship between asset price appreciation and foreign exchange rate volatility in normal times, or when the bubble is due to local risk-shifting behaviour.

To identify a risk-shifting bubble funded from abroad, the target country has to be sufficiently small and open. We select New Zealand as the target country, as it is a good example of small open economy that is susceptible to the influences of cross-border capital flows (Bordo, Hargreaves, and Kida, 2010). New Zealand is widely considered as one the most liberal and open economies in the world. Deregulatory macro-economic reforms that commenced in 1984 have integrated New Zealand's banking system into global financial markets, whilst relaxation of foreign investment guidelines allowed overseas investors to take positions in local asset markets. Indeed, liberalisation has been one of the major drivers of increased foreign ownership of capital assets: "at the turn of the century 50% of equity holdings were held offshore" (Fraser, Hoesli, and McAlevey, 2008, p.73). Foreign investment, however, is not limited to New Zealand's capital market. Given its relatively small size, an "inflow of funds can quickly lead to excess liquidity within the financial system" (Badcock, 2004, p.61). As a result, overseas investment can rapidly spill over into New Zealand's real estate market.

Indeed, this is what happened during the New Zealand's housing boom of 2000s. The country attracted large foreign capital inflows (on the relative scale to other developed countries[15]) that came from two primary sources: international business migration and offshore investment holdings (Badcock, 2004). During the period, Japan played an important role in both respects. It has been (and remains) one of New Zealand's major trading partners and sources of foreign labour (Kumarasinghe and Hoshino, 2009), whereas at the same time it has been a major source of overseas funding during the housing boom.[16] Given the Japan's role in New Zealand's economy, it was likely to exert an influence on the latter's property market during the housing boom: either directly or through funding provision.[17] The question we seek to address in this section is whether

[15]For example, Ferrero (2012) provides an instructive cross-country comparison in Figure 2 of his paper.

[16] According to Statistics New Zealand, in 2001 (the first quarter of bubble period identified in this section), New Zealand registered banks' funding denominated in Japanese Yen (JPY) amounted to 5.44bn New Zealand Dollars (NZD). This number corresponded to 7.9% of residential mortgage outstanding at the time on New Zealand registered banks' books. By 2007, JPY denominated funding reduced substantially and amounted only to NZD 1.07bn (0.7% of residential mortgages outstanding). In 2010, JPY denominated funding stood at NZD 0.78bn (0.5% of residential mortgages outstanding).

[17]An important question is whether funding from Japan went into housing market or elsewhere. There is no data on this. However, during the housing boom increase in JPY denominated liabilities of banks was accompanied by an increase in the share of residential mortgage in banks' assets. This suggests that at least part of funding from Japan may have gone into financing of housing market transactions in New Zealand.

this influence may have exhibited properties of risk-shifting behaviour amongst investors. If risk-shifting incentives in New Zealand's housing market fuelled by underpriced funding from Japan (and the corresponding risk-shifting asset price bubble) were indeed present, then, according to risk-shifting bubble theory, they would reveal themselves through a positive relationship between New Zealand's Dollar (NZD) and Japanese Yen (JPY) exchange rate volatility and house price return. The positive relationship would be immune to controls in the form of common housing market fundamentals that represent supply and demand sides of the market. Furthermore, it would vanish or reverse in a non-bubble period.

Conventional approaches to bubble identification attempt to derive the fundamental value and compare it to actual prices[18] and thus suffer from model-dependence (*i.e.*, the model for determining fundamental may be incorrect) (Gurkaynak, 2008). The strategy used in this section consists of examining whether price response to risk is consistent with the behaviour that, as theory predicts, pertains to an asset price bubble.[19] As a result, no explicit model of fundamental value determination is required, instead the results need to be controlled for fundamentals as they (rather than shifts in agents' behaviour) may be the key drivers of price changes.

Thus, the initial premise of the empirical analysis is that a given period's house price index return, $\hat{r}_t$, is explained by information contained in previous period's return, $\hat{r}_{t-1}$ and a shift in fundamentals.[20] If some of investment in housing market is due to funding from Japan (through either of the two sources described earlier) and thus originates in JPY, then the risk of cross-border operations captured by NZD-JPY exchange rate volatility, $\hat{\sigma}_t$, may provide additional explanatory power for New Zealand's housing market return determination. The impact of NZD-JPY exchange rate volatility differs depending on whether it is associated with risk-shifting incentives (and hence bubble funded from abroad) or not. To account for this potentially different impact of NZD-JPY exchange rate volatility, additional component is introduced – $\hat{\sigma}_t$ multiplied by indicator variable, d_t, that takes value of one for all quarters from 2001Q1 until 2003Q4,

[18]Fraser, Hoesli, and McAlevey (2008) is an example of such a test for the recent New Zealand housing bubble.

[19]Risk-shifting bubble theory predicts that, during the bubble period, higher prices and higher risk go hand in hand.

[20]Note that for the purposes of empirical analysis, return is calculated as percentage change in house price index.

and zero otherwise. The following regression results:

$$\hat{r}_t = a_0 + a_1\hat{r}_{t-1} + a_2\hat{\sigma}_t + a_3 d_t\hat{\sigma}_t + \sum_i b_i\Delta x_{i,t} + \epsilon_t, \tag{28}$$

where:

$\Delta x_{i,t}$ denotes a changes in fundamental parameter x_i over the previous quarter (period from $t-1$ to t).

The vector of fundamental parameters consists of the standard set of supply and demand side factors (Fraser, Hoesli, and McAlevey, 2008; Wheaton and Nechayev, 2008): floor area supply of building consents per household, disposable income and mortgage rates. To account for the possibility of asset price inflation being driven by carry trade opportunities that do not emanate from risk-shifting behaviour, we also control for interest rate differential between Japan and New Zealand, since the interest rate differential is usually the main driver of carry trades (see, *e.g.*, Brunnermeier, Nagel, and Pedersen, 2009). Given that during the period under consideration, New Zealand experienced a wave of immigration together with natural population growth, we also control for population dynamics.

7.1 Data

The data covers exactly 20 years from 1989Q4 through to 2009Q3. The dependent variable is calculated from New Zealand's housing prices. The data for these is obtained from Quotable Value New Zealand's Residential Sales Summary quarterly publications and the Reserve Bank of New Zealand (RBNZ). This quarterly house price index measures average sale prices of freehold houses and controls each quarter for the quality mix of sales. The series is deflated using the quarterly Consumer Price Index (CPI) from the RBNZ and Statistics New Zealand (SNZ).

The key explanatory variable is quarterly variance of NZD-JPY exchange rate. It is based on daily observations of NZD-JPY exchange rates that are sourced from the RBNZ, Reuters and the New Zealand Financial Markets Association (NZFMA).

Changes in supply side fundamentals are proxied by quarterly floor area supply (in m^2) of building consents. These data are obtained from SNZ, where building consents are assumed to be reflected in the price after 2 to 3 years. Therefore, quarterly housing supply is measured by the total amount of floor area supply of building consents issued 2 years ago and up to 3 years ago (*i.e.*, the sum of 4 consecutive quarters). However, a given supply of new floor space is more of a shock in 1990 than it is in 2005, given the growth in households. In order to obtain a supply measure relative to the market size, we divide the above mentioned quarterly housing supply by the number of households of the previous period. The households estimate is sourced from SNZ.

Changes in demand side fundamentals are proxied by disposable income and mortgage rates. The quarterly real disposable income (RDI) is provided by SNZ. Mortgage rates are sourced from the RBNZ. The rates used are the floating new customer rates and are weighted aggregate interest rates. All data is seasonally adjusted.

Additional controls employed are interest rate differential between two countries to account for a possibility of carry trade and population growth rate. The source of interest rate data is IMF-IFS. Population growth rates are obtained from SNZ.

7.2 Results

Results of regression analysis are reported in Table 2. Regression (1) describes the conventional relationship between house price index return and changes in fundamentals that are represented by changes in housing supply, real disposable income and access to credit through mortgage rate for new customers.[21]

Regression (2) includes the effect of NZD-JPY exchange rate volatility, whilst controlling for changes in fundamentals. Regression (3) controls (additionally) for the effect carry trades through changes in interest rate differential between the two countries, whilst regression (4) also adds population growth factor. These additional controls are required for the following reasons. As shown in previous section, availability of carry trade opportunities may exacerbate risk-shifting bubbles, but may also lead to asset price inflation in its own right. Given that NZD-JPY carry trade has been a particularly popular strategy over a large part of the period under consideration (Plantin and Shin, 2011), the analysis has to control for the effect of carry trade factor on house price index returns.

[21] Other key variables that usually appear in the literature on house price determination are construction costs and owner-occupation. Both of these did not increase during the risk-shifting bubble period, and thus are unlikely to drive house prices up. They are excluded for the purpose of not over-fitting the regression model.

Table 2: Regressions of house price index returns

	(1)	(2)	(3)	(4)
Intercept	0.0578	0.1902	0.1870	0.1856
	(0.4090)	(1.2267)	(1.1645)	(1.1502)
Lagged house price index return	0.9002	0.8441	0.8469	0.8519
	(13.4252)	(12.3584)	(11.8982)	(11.8123)
NZD-JPY exchange rate volatility		−0.0465	−0.0468	−0.04748
		(−2.4037)	(−2.3803)	(−2.3985)
Indicator * NZD-JPY exchange rate volatility		0.4147	0.4377	0.4306
		(2.1076)	(2.1095)	(2.0609)
Δ NZ-JP interest rate differential	NO	NO	YES	YES
Indicator * Δ NZ-JP interest rate differential	NO	NO	YES	YES
Δ Population growth rate	NO	NO	NO	YES
Δ Housing supply	YES	YES	YES	YES
Δ Real disposable income growth	YES	YES	YES	YES
Δ Floating new customer mortgage rate	YES	YES	YES	YES
Observations	80	80	80	80
Adjusted R^2	0.7164	0.7494	0.7435	0.7409

Notes:
Δ denotes the first difference from the previous quarter.
Indicator takes value of 1 for all quarters from 2001Q1 until 2003Q4, and 0 otherwise.
$\Delta\text{Housing supply}_t = \Delta\left(\frac{\text{Floor space supply}_t}{\text{Households}_{t-1}}\right)$.
t-statistics reported in parenthesis.
Regression diagnostics is provided in Appendix B.

Implicitly, the regression attempts to verify whether carry trades were limited to fixed-income market, or took additional advantage of increased risky asset returns as in theoretical analysis of previous section.

We separate the period under consideration into two sub-periods: "normal times" ($d_t = 0$) and the period between 2001Q1-2003Q4 ($d_t = 1$). Results indicate that in "normal times", there is a statistically significant *negative* relationship between NZD-JPY exchange rate volatility and house price index return. This is, indeed, what one would expect from risk-averse agents, who retract from market as the riskiness of asset class goes up. However, in 2001Q1-2003Q4, the effect of NZD-JPY exchange rate volatility reverses and outweighs the negative effect of "normal times". Pairwise Granger causality tests suggest that NZD-JPY exchange rate volatility impacts house price index returns and *not vice versa* (Table 3). According to theoretical analysis undertaken in this paper, a positive effect of foreign exchange rate volatility on risky asset prices

Table 3: Pairwise Granger causality test (7 lags)

Null hypothesis: x does not cause y ($x \to y$)	Observations	F-statistic	p-value
$\hat{\sigma} \to \hat{r}$	73	3.0232	0.0089
$\hat{r} \to \hat{\sigma}$		1.0148	0.4306

Note:
Lag length determination is reported in Appendix C.

(in excess of the shift in fundamentals) is an indication of risk-shifting incentives that arouse due to lax credit conditions overseas. More specifically, between 2001Q1 and 2003Q4, New Zealand experienced a risk-shifting bubble that was due to underpriced funding from Japan. It is important to note that carry trade opportunities do not add explanatory power to house price return determination. Thus, risk-shifting bubble in housing market did not have the carry trade component.

It is instructive to relate the above results to the work of Fraser, Hoesli, and McAlevey (2008), who find "dramatic overvaluation" of the New Zealand housing market in 2003-2005. Their focus is primarily on intrinsic bubbles in the sense of Froot and Obstfeld (1991) rather than risk-shifting bubbles considered in this paper. However, risk-shifting and intrinsic bubbles can be complementary. In fact, intrinsic bubbles often require an initial spark in agents' expectations that eventually becomes self-fulfilling. Risk-shifting bubbles can provide such an initial spark. What could have happened in the case of New Zealand is that initial underpricing of credit overseas induced substantial capital flows into the country and generated risk-shifting behaviour of investors. This resulted in housing price boom. Overpricing of housing assets relative to fundamentals (due to risk-shifting behaviour) created the spread required for subsequent intrinsic bubble to inflate. This spread did not have to be "dramatic" since non-linearity of intrinsic bubbles allows for inflation of any small mispricing to dramatic levels. Importantly, however, once the spread between actual and fundamental prices was in place, intrinsic bubble did not require sufficiently lax credit conditions to sustain itself. As a result, the risk-shifting bubble period of 2001-2003 identified in this section and speculative bubble period of 2003-2005 uncovered by Fraser, Hoesli, and McAlevey (2008) can be viewed as two phases of the same process.

8 Conclusion

This paper contributes to the literature on risk-shifting asset price bubbles that result from underpricing of loan contracts due to information asymmetry between borrowers and lenders. Three types of investor types are considered: fundamentalists, speculators and value investors. We show that underpricing of loan contracts (and, more specifically, of the option to default) is both necessary and sufficient condition for risk-shifting bubbles to emerge, if investors under considerations are either fundamentalists or speculators. If, however, investor type being considered is value investors, then underpriced credit is only a necessary condition for risk-shifting bubbles emergence. For this investor type, there exists a threshold leverage amount that we term "risk-shifting threshold" such that risk-shifting bubbles arise only if this leverage amount is exceeded. Once risk-shifting incentives are in place (through underpriced credit and sufficient leverage), foreign exchange transaction costs are the key determinant of bubble's locational characteristics. For sufficiently low cost of carrying out cross-border operations, bubbles tend to be "displaced" abroad. Risk-shifting bubbles that emerge abroad are usually larger than bubbles in closed economy setting, and are associated with large cross-border capital movements. If countries are symmetric, then these cross-border capital movements are not reflected in countries' (net) capital account positions but are nevertheless associated with build-up of gross outstanding financial assets and liabilities positions of these countries.

Persistence mechanics in our model comes in two forms: conventional and speculative. Conventional persistence relies on banks offering a fixed gearing ratio as a function of the price paid by investors (or any other mechanism that ensures credit expansion). Once underpricing of the default option and sufficient leverage are in the system, investors bid prices above the "fundamental value". An increase in prices induces banks to offer more credit, which in turn leads to a larger prices paid by investors. A transaction-based self-fulfilling loop results. Speculative persistence relies on the same assumption about lenders' behaviour. However, unlike fundamentalists (and value investors) who transact the asset in question at every instant and at an increasing price (as long as there is credit expansion), speculators hold on to it until the bid price is sufficiently high (*i.e.*, until it hits the unwinding threshold). Speculative risk-shifting bubbles can thus be "quieter" relative to conventional risk-shifting bubbles in terms of turnover but not necessarily in terms of volatility.

Implicit in the model are two routes from persistence to the burst of a risk-shifting bubble. Firstly and akin to Allen and Gale (2000), a sufficiently large adverse realisation of the value process may induce investors to default rationally whenever the prospects of its recovery are sufficiently low (*i.e.*, the default threshold is hit). Contrary to Allen and Gale (2000), however, if risk-shifting bubble emerged abroad, the negative shock may not only come from the asset value process but also from the foreign exchange market. This is particularly important since we show that risk-shifting bubbles are more likely to emerge abroad, and thus the dynamics of the open position is driven by a combination of "fundamental" asset and foreign exchange rate processes. If the negative shock hits the asset value process, then *ceteris paribus* busts and financial crises are symmetric in the two countries: they result in a symmetric repatriation of funds, repricing of loan agreements, and shrinkage of the economy. If, however, the negative shock affects the foreign exchange rate, then *ceteris paribus* a bust and financial crisis arises in a single country: investors repatriate funds from this country only to further deteriorate its capital account, and its economy shrinks dramatically. This is, indeed, what is often observed empirically during financial crises. Thus, the degree of symmetry of the resulting crises between two countries depends crucially on whether the shock comes from the asset or foreign exchange market. Secondly, a negative shock to either of the above variables needs not be large to induce a crisis. The persistence mechanism is key to this explanation. Note that for a self-fulfilling loop to emerge the "fundamental value" process needs not change. In particular, if the amount of leverage in the economy grows faster than the "fundamental value" process, then eventually the default threshold will gradually approach the "fundamental value" from below. In this setting, even a small downward fluctuation from the asset or foreign exchange process will result in default, loan repricing and shrinkage of the economy.[22]

The paper also contributes to empirical literature as it offers a of identifying risk-shifting bubbles. The identification does not require calculation of "fundamental value" and is therefore model independent. Instead, it makes use of the fact that in risk-shifting bubble environment prices increase in volatility. The theory of open economy risk-shifting bubbles can help identify a risk-shifting bubble in New Zealand's housing market in 2001-2003 that was fuelled by underpriced funding from Japan.

[22]A consequence of such asset price and foreign exchange rate crashes can be a *negative bubble.*

A Proofs and derivations

A.1 Derivation of equation (6)

$$
\begin{aligned}
E_i\left(\mathsf{q}^{\mathsf{S}}_{ji}, X_{ji}\right) &= r_j X_{ji}\mathsf{q}^{\mathsf{S}}_{ji}\mathrm{d}t - m_i L_i \mathrm{d}t - c_C \mathrm{d}t + e^{-r_i \mathrm{d}t}\mathbb{E}\left[E_i\left(\mathsf{q}^{\mathsf{S}}_{ji}, X_{ji} + \mathrm{d}X_{ji}\right)\right] \\
&= r_j X_{ji}\mathsf{q}^{\mathsf{S}}_{ji}\mathrm{d}t - m_i L_i \mathrm{d}t - c_C \mathrm{d}t \\
&\quad + (1 - r_i \mathrm{d}t) \\
&\qquad \times \left\{E_i\left(\mathsf{q}^{\mathsf{S}}_{ji}, X_{ji}\right) + \left(\mathbb{E}\left[E_i\left(\mathsf{q}^{\mathsf{S}}_{ji}, X_{ji} + \mathrm{d}X_{ji}\right)\right] - E_i\left(\mathsf{q}^{\mathsf{S}}_{ji}, X_{ji}\right)\right)\right\} \\
&= r_j X_{ji}\mathsf{q}^{\mathsf{S}}_{ji}\mathrm{d}t - m_i L_i \mathrm{d}t - c_C \mathrm{d}t + E_i\left(\mathsf{q}^{\mathsf{S}}_{ji}, X_{ji}\right) - r_i E_i\left(\mathsf{q}^{\mathsf{S}}_{ji}, X_{ji}\right)\mathrm{d}t \\
&\quad + \mathbb{E}\left[\mathrm{d}E_i\left(\mathsf{q}^{\mathsf{S}}_{ji}, X_{ji}\right)\right], \quad j \neq i,\ X_{ji} \in \mathcal{D}^C_{X,ji}.
\end{aligned} \tag{29}
$$

By Itô's Lemma,

$$
\mathbb{E}\left[\mathrm{d}E_i\left(\mathsf{q}^{\mathsf{S}}_{ji}, X_{ji}\right)\right] = (r_i - r_j)\, X_{ji} E_i'\left(\mathsf{q}^{\mathsf{S}}_{ji}, X_{ji}\right)\mathrm{d}t + \frac{\sigma_j^2}{2} X_{ji}^2 E_i''\left(\mathsf{q}^{\mathsf{S}}_{ji}, X_{ji}\right)\mathrm{d}t, \quad j \neq i.
$$

Substituting the above into equation (29), dividing through by dt and rearranging yields equation (6). ■

A.2 Derivation of equation (12)

$$
\begin{aligned}
E_i\left(\mathsf{q}^{\mathsf{R}}_{ji}, F_{ji}\right) &= \mathsf{q}^{\mathsf{R}}_{ji}\delta_j F_{ji}\mathrm{d}t - m_i L_i \mathrm{d}t - c_C \mathrm{d}t + e^{-r_i \mathrm{d}t}\mathbb{E}\left[E_i\left(\mathsf{q}^{\mathsf{R}}_{ji}, F_{ji} + \mathrm{d}F_{ji}\right)\right] \\
&= \mathsf{q}^{\mathsf{R}}_{ji}\delta_j F_{ji}\mathrm{d}t - m_i L \mathrm{d}t - c_C \mathrm{d}t \\
&\quad + (1 - r_i \mathrm{d}t)\left\{E_i\left(\mathsf{q}^{\mathsf{R}}_{ji}, F_{ji}\right) + \mathbb{E}\left[E_i\left(\mathsf{q}^{\mathsf{R}}_{ji}, F_{ji} + \mathrm{d}F_{ji}\right) - E_i\left(\mathsf{q}^{\mathsf{R}}_{ji}, F_{ji}\right)\right]\right\} \\
&= \mathsf{q}^{\mathsf{R}}_{ji}\delta_j F_{ji}\mathrm{d}t - m_i L \mathrm{d}t - c_C \mathrm{d}t + E_i\left(\mathsf{q}^{\mathsf{R}}_{ji}, F_{ji}\right) - r_i E_i\left(\mathsf{q}^{\mathsf{R}}_{ji}, F_{ji}\right)\mathrm{d}t \\
&\quad + \mathbb{E}\left[E_i\left(\mathsf{q}^{\mathsf{R}}_{ji}, \mathrm{d}F_{ji}\right)\right], \quad \text{for all } i, j,\ F_{ji} \in \mathcal{D}^C_{F,ji}.
\end{aligned} \tag{30}
$$

By Itô's Lemma,

$$
\mathbb{E}\left[E_i\left(\mathsf{q}^{\mathsf{R}}_{ji}, \mathrm{d}F_{ji}\right)\right] = (r_i - \delta_j)\, F_{ji,t} E_i'\left(\mathsf{q}^{\mathsf{R}}_{ji}, F_{ji}\right)\mathrm{d}t + \frac{\sigma_{ji}^2}{2} F_{ji}^2 E_i''\left(\mathsf{q}^{\mathsf{R}}_{ji}, F_{ji}\right)\mathrm{d}t.
$$

Substituting the above equation into equation (30), dividing through by dt and rearranging yields equation (12). ■

A.3 Proof of Proposition 1

To solve for the value of equity in the open long position in a risky asset held by fundamentalist, the following boundary conditions are imposed. First, we rule out the speculative component of valuation. Second, at the time of default, equity-holding is worthless. Third, the default trigger F_{ji}^D is chosen optimally. Formally, these conditions are:

$$\lim_{F_{ji}\to\infty} E_i^{\mathfrak{F}}(F_{ji}) = F_{ji} - \frac{m_i L_i}{r_i} - \frac{c_C}{r_i}, \tag{31a}$$

$$E_i^{\mathfrak{F}}\left(F_{ji}^D\right) = 0, \tag{31b}$$

$$E_i^{\mathfrak{F}\prime}\left(F_{ji}^D\right) = 0. \tag{31c}$$

The above boundary conditions determine the three constants required for solution:

$$B_{1,F,ji}^{\mathfrak{F}} = 0, \tag{32a}$$

$$B_{2,F,ji}^{\mathfrak{F}} = \left(\frac{m_i L_i}{r_i} + \frac{c_C}{r_i} - F_{ji}^D\right)\left(F_{ji}^D\right)^{-\beta_{2,F,ji}}, \tag{32b}$$

$$F_{ji}^D = \gamma_{2,F,ji}\left(\frac{m_i L_i + c_C}{r_i}\right), \tag{32c}$$

where:

$$\gamma_{2,F,ji} = \frac{\beta_{2,F,ji}}{\beta_{2,F,ji} - 1}.$$

Combining the above constants with generic value of equity in open long position in risky asset given by equation (13) evaluated at q_{ji}^R completes the proof. ■

A.4 Proof of Proposition 3

If loan is underpriced and hence mispricing function enters fundamentalist's pricing decision, then differentiation of its reservation price with respect to the selected measure of risk yields:

$$\frac{\partial P_j^{\mathfrak{F}}}{\partial \sigma_{ji}} = \ln\left(\frac{F_{ji}}{F_{ji}^D}\right)\left(\frac{m_i L_i + c_C}{r_i}\right)\frac{\left(F_{ji}/F_{ji}^D\right)^{\beta_{2,F,ji}}}{(1-\beta_{2,F,ji})}\frac{\partial \beta_{2,F,ji}}{\partial \sigma_{ji}} > 0,$$

where:

$$\frac{\partial \beta_{2,F,ji}}{\partial \sigma_{ji}} = -\frac{\sigma_{ji}\beta_{2,F,ji}^2 \left(\beta_{2,F,ji} - 1\right)}{r_i + \frac{\sigma_{ji}^2 \beta_{2,F,ji}^2}{2}} > 0.$$

The risk-shifting bubble conditions are satisfied. ■

A.5 Proof of Proposition 4

In continuation region, the value of equity in open position in risky asset held by speculator ($\delta_j = 0$ for all j) solves the following ODE:

$$\frac{\sigma_{ji}^2}{2} F_{ji}^2 E_i^{\mathfrak{S}\prime\prime}\left(F_{ji}\right) + r_i F_{ji} E_i^{\mathfrak{S}\prime}\left(F_{ji}\right) - r_i E_i^{\mathfrak{S}}\left(F_{ji}\right) - m_i L_i - c_C = 0, \quad \text{for all } i, j,\ F_{ji} \in \mathcal{D}_{F,ji}^{C,\mathfrak{S}},$$

where:
$\mathcal{D}_{F,ji}^{C,\mathfrak{S}}$ denotes the continuation range of speculator.

The solution to the above equation takes the following form:

$$E_i^{\mathfrak{S}}\left(F_{ji}\right) = B_{1,F,ji}^{\mathfrak{S}} F_{ji} + B_{2,F,ji}^{\mathfrak{S}} F_{ji}^{-\frac{2r_i}{\sigma_{ji}^2}} - \frac{m_i L_i}{r_i} - \frac{c_C}{r_i}, \quad \text{for all } i, j,\ F_{ji} \in \mathcal{D}_{F,ji}^{C,\mathfrak{S}},$$

where:
$B_{1,F,ji}^{\mathfrak{S}}$ and $B_{2,F,ji}^{\mathfrak{S}}$ are constants to be determined.

The following boundary conditions are imposed. First, when speculator closes its open position by selling the risky asset, its payoff is given by the price it obtains less the present value of debt service payments it was supposed to make under the loan agreement. Second, the threshold level of value process, at which speculator closes its position, $F_{ji}^{C,\mathfrak{S}}$, is chosen optimally. Third, when speculator defaults, its equity holding is worthless. Fourth, the default threshold of speculator, $F_{ji}^{D,\mathfrak{S}}$, is chosen optimally. Formally, these boundary conditions are:

$$B_{1,F,ji}^{\mathfrak{S}} F_{ji}^{C,\mathfrak{S}} + B_{2,F,ji}^{\mathfrak{S}} \left(F_{ji}^{C,\mathfrak{S}}\right)^{-\frac{2r_i}{\sigma_{ji}^2}} - \frac{m_i L_i}{r_i} - \frac{c_C}{r_i} = F_{ji}^{C,\mathfrak{S}} - \frac{m_i L_i}{r_i}, \tag{33a}$$

$$B_{1,F,ji}^{\mathfrak{S}} - \frac{2r_i}{\sigma_{ji}^2} H_{2,F,ji}^{\mathfrak{S}} \left(F_{ji}^{C,\mathfrak{S}}\right)^{-\frac{2r_i}{\sigma_{ji}^2}-1} = 1, \tag{33b}$$

$$B_{1,F,ji}^{\mathfrak{S}} F_{ji}^{D,\mathfrak{S}} + B_{2,F,ji}^{\mathfrak{S}} \left(F_{ji}^{D,\mathfrak{S}}\right)^{-\frac{2r_i}{\sigma_{ji}^2}} - \frac{m_i L_i}{r_i} - \frac{c_C}{r_i} = 0, \tag{33c}$$

$$B_{1,F,ji}^{\mathfrak{S}} - \frac{2r_i}{\sigma_{ji}^2} B_{2,F,ji}^{\mathfrak{S}} \left(F_{ji}^{D,\mathfrak{S}}\right)^{-\frac{2r_i}{\sigma_{ji}^2}-1} = 0. \tag{33d}$$

Start by solving equation (33d) for $B^{\mathfrak{S}}_{2,F,ji}\left(F^{D,\mathfrak{S}}_{ji}\right)^{-\frac{2r_i}{\sigma^2_{ji}}}$, and substituting the result into equation (33c) to find the default trigger:

$$F^{D,\mathfrak{S}}_{ji} = \left[\frac{m_i L_i + c_C}{r_i + \frac{\sigma^2_{ji}}{2}}\right] \frac{1}{B^{\mathfrak{S}}_{1,F,ji}}. \tag{34}$$

Substitute the above equation into (33d), and solve for $B^{\mathfrak{S}}_{2,F,ji}$:

$$B^{\mathfrak{S}}_{2,F,ji} = \frac{\sigma^2_{ji}}{2r_i}\left[\frac{m_i L_i + c_C}{r_i + \frac{\sigma^2_{ji}}{2}}\right]^{1+\frac{2r_i}{\sigma^2_{ji}}} \left(B^{\mathfrak{S}}_{1,F,ji}\right)^{-\frac{2r_i}{\sigma^2_{ji}}}. \tag{35}$$

Substitute the above result into equation (33b) to find the unwinding threshold, $F^{C,\mathfrak{S}}_{ji}$:

$$F^{C,\mathfrak{S}}_{ji} = \left[\left(B^{\mathfrak{S}}_{1,F,ji} - 1\right)\left(B^{\mathfrak{S}}_{1,F,ji}\right)^{\frac{2r_i}{\sigma^2_{ji}}}\right]^{\frac{1}{-\frac{2r_i}{\sigma^2_{ji}}-1}} \left[\frac{m_i L_i + c_C}{r_i + \frac{\sigma^2_{ji}}{2}}\right]. \tag{36}$$

Substitute the above equation into (33a), and solve for $B^{\mathfrak{S}}_{1,F,ji}$:

$$B^{\mathfrak{S}}_{1,F,ji} = \frac{1}{1 - \left(\frac{c_C}{m_i L_i + c_C}\right)^{1+\frac{\sigma^2_{ji}}{2r_i}}}. \tag{37}$$

Finally, substitute the expression for $B^{\mathfrak{S}}_{1,F,ji}$ into the expression for $B^{\mathfrak{S}}_{2,F,ji}$ to obtain

$$B^{\mathfrak{S}}_{2,F,ji} = \frac{\sigma^2_{ji}}{2r_i}\left[\frac{m_i L_i + c_C}{r_i + \frac{\sigma^2_{ji}}{2}}\right]^{1+\frac{2r_i}{\sigma^2_{ji}}} \left[\frac{1}{1 - \left(\frac{c_C}{m_i L_i + c_C}\right)^{1+\frac{\sigma^2_{ji}}{2r_i}}}\right]^{-\frac{2r_i}{\sigma^2_{ji}}}. \tag{38}$$

Putting together the constants into the value of equity position equation completes the proof. ■

A.6 Proof of Proposition 5

Let $B^{\mathfrak{S}}_{1,F,ji}$ and $B^{\mathfrak{S}}_{2,F,ji}$ be the constants as defined in Appendix A.5, and let $\beta_{\mathfrak{S},ji} = -\frac{2r_i}{\sigma^2_{ji}}$. Then, by differentiating the pricing function of speculator with respect to the selected measure of risk, we obtain

$$\begin{aligned}
\frac{\partial P^{\mathfrak{S}}_{ji}}{\partial \sigma_{ji}} &= \frac{\partial}{\partial \sigma_{ji}} \left[B^{\mathfrak{S}}_{1,F,ji} F_{ji} + B^{\mathfrak{S}}_{2,F,ji} F_{ji}^{\beta_{\mathfrak{S},ji}} \right] \\
&= -\frac{\left(B^{\mathfrak{S}}_{1,F,ji}\right)^2 F_{ji}}{\beta^2_{\mathfrak{S},ji}} \ln\left(\frac{c_C}{m_i L_i + c_C}\right) \frac{\partial \beta_{\mathfrak{S},ji}}{\partial \sigma_{ji}} \\
&\quad + B^{\mathfrak{S}}_{2,F,ji} F_{ji}^{\beta_{\mathfrak{S},ji}} \left[\ln\left(\frac{F_{ji}}{F^{D,\mathfrak{S}}_{ji}}\right) - \ln\left(\frac{c_C}{m_i L_i + c_C}\right) \frac{B^{\mathfrak{S}}_{1,F,ji}}{\beta_{\mathfrak{S},ji}} \right] \frac{\partial \beta_{\mathfrak{S},ji}}{\partial \sigma_{ji}} \\
&= B^{\mathfrak{S}}_{2,F,ji} F_{ji}^{\beta_{\mathfrak{S},ji}} \ln\left(\frac{F_{ji}}{F^{D,\mathfrak{S}}_{ji}}\right) \frac{\partial \beta_{\mathfrak{S},ji}}{\partial \sigma_{ji}} \\
&\quad - \frac{B^{\mathfrak{S}}_{1,F,ji}}{\beta_{\mathfrak{S},ji}} \ln\left(\frac{c_C}{m_i L_i + c_C}\right) \left[\frac{B^{\mathfrak{S}}_{1,F,ji} F_{ji}}{\beta_{\mathfrak{S},ji}} + B^{\mathfrak{S}}_{2,F,ji} F_{ji}^{\beta_{\mathfrak{S},ji}} \right] \frac{\partial \beta_{\mathfrak{S},ji}}{\partial \sigma_{ji}} \\
&> 0.
\end{aligned}$$

The positive sign of the derivative follows from the fact that the first term on the right-hand side is clearly positive, while the positivity of the second term follows from the boundary conditions. In particular note that in continuation region $F^{D,\mathfrak{S}}_{ji} < F_{ji} < F^{C,\mathfrak{S}}_{ji}$, and

$$\begin{aligned}
\frac{B^{\mathfrak{S}}_{1,F,ji} F^{D,\mathfrak{S}}_{ji}}{\beta_{\mathfrak{S},ji}} + B^{\mathfrak{S}}_{2,F,ji} \left(F^{D,\mathfrak{S}}_{ji}\right)^{\beta_{\mathfrak{S},ji}} &= 0, \\
\frac{B^{\mathfrak{S}}_{1,F,ji} F^{C,\mathfrak{S}}_{ji}}{\beta_{\mathfrak{S},ji}} + B^{\mathfrak{S}}_{2,F,ji} \left(F^{C,\mathfrak{S}}_{ji}\right)^{\beta_{\mathfrak{S},ji}} &= \frac{F^{C,\mathfrak{S}}_{ji}}{\beta_{\mathfrak{S},ji}} < 0.
\end{aligned}$$

Hence, for all $F^{D,\mathfrak{S}}_{ji} < F_{ji} < F^{C,\mathfrak{S}}_{ji}$,

$$\frac{B^{\mathfrak{S}}_{1,F,ji} F_{ji}}{\beta_{\mathfrak{S},ji}} + B^{\mathfrak{S}}_{2,F,ji} F_{ji}^{\beta_{\mathfrak{S},ji}} < 0. \qquad \blacksquare$$

A.7 Proof of Proposition 6

Let $\beta_{\mathfrak{S},ji} = -\frac{2r_i}{\sigma_{ji}^2}$. Differentiating the unwinding threshold with respect to the measure of risk yields:

$$\begin{aligned}
\frac{\partial F_{ji}^{C,\mathfrak{S}}}{\partial \sigma_{ji}} &= F_{ji}^{C,\mathfrak{S}} \left[\frac{1}{(\beta_{\mathfrak{S},ji}-1)\left(B_{1,F,ji}^{\mathfrak{S}}-1\right)} \frac{\partial B_{1,F,ji}^{\mathfrak{S}}}{\partial \beta_{\mathfrak{S},ji}} - \frac{\ln\left(B_{1,F,ji}^{\mathfrak{S}}-1\right)}{(\beta_{\mathfrak{S},ji}-1)^2} \right] \frac{\partial \beta_{\mathfrak{S},ji}}{\partial \sigma_{ji}} \\
&\quad + F_{ji}^{C,\mathfrak{S}} \left[-\frac{\beta_{\mathfrak{S},ji}}{(\beta_{\mathfrak{S},ji}-1)\, B_{1,F,ji}^{\mathfrak{S}}} \frac{\partial B_{1,F,ji}^{\mathfrak{S}}}{\partial \beta_{\mathfrak{S},ji}} + \frac{\ln B_{1,F,ji}^{\mathfrak{S}}}{(\beta_{\mathfrak{S},ji}-1)^2} \right] \frac{\partial \beta_{\mathfrak{S},ji}}{\partial \sigma_{ji}} \\
&\quad - F_{ji}^{C,\mathfrak{S}} \frac{1}{\beta_{\mathfrak{S},ji}\,(\beta_{\mathfrak{S},ji}-1)} \frac{\partial \beta_{\mathfrak{S},ji}}{\partial \sigma_{ji}} \\
&= \frac{F_{ji}^{C,\mathfrak{S}}}{(\beta_{\mathfrak{S},ji}-1)^2} \frac{\partial \beta_{\mathfrak{S},ji}}{\partial \sigma_{ji}} \\
&\quad \times \left[\ln\left(\frac{B_{1,F,ji}^{\mathfrak{S}}}{B_{1,F,ji}^{\mathfrak{S}}-1} \right) + (\beta_{\mathfrak{S},ji}-1) \left(\frac{1}{B_{1,F,ji}^{\mathfrak{S}}-1} - \frac{\beta_{\mathfrak{S},ji}}{B_{1,F,ji}^{\mathfrak{S}}} \right) \frac{\partial B_{1,F,ji}^{\mathfrak{S}}}{\partial \beta_{\mathfrak{S},ji}} \right] \\
&\quad - F_{ji}^{C,\mathfrak{S}} \frac{1}{\beta_{\mathfrak{S},ji}\,(\beta_{\mathfrak{S},ji}-1)} \frac{\partial \beta_{\mathfrak{S},ji}}{\partial \sigma_{ji}},
\end{aligned} \tag{39}$$

where:

$$\beta_{\mathfrak{S},ji} = -\frac{2r_i}{\sigma_{ji}^2} < 0, \qquad \frac{\partial \beta_{\mathfrak{S},ji}}{\partial \sigma_{ji}} = \frac{4r_i}{\sigma_{ji}^3} > 0,$$

$$\frac{\partial B_{1,F,ji}^{\mathfrak{S}}}{\partial \beta_{\mathfrak{S},ji}} = \frac{\left(B_{1,F,ji}^{\mathfrak{S}}\right)^2}{\beta_{\mathfrak{S},ji}^2} \ln\left(\frac{c_C}{m_i L_i + c_C} \right) \left(\frac{c_C}{m_i L_i + c_C} \right)^{1-\frac{1}{\beta_{\mathfrak{S},ji}}} < 0.$$

Since the first two terms in squared brackets of equation (39) are always positive, and the third term is always negative, to prove that $\partial F_{ji}^{C,\mathfrak{S}} / \partial \sigma_{ji} > 0$, it is sufficient to show that

$$\ln\left(\frac{B_{1,F,ji}^{\mathfrak{S}}}{B_{1,F,ji}^{\mathfrak{S}}-1} \right) - \frac{\beta_{\mathfrak{S},ji}-1}{\beta_{\mathfrak{S},ji}} > 0. \tag{40}$$

Note that

$$\frac{B^{\mathfrak{S}}_{1,F,ji}}{B^{\mathfrak{S}}_{1,F,ji}-1} = \left[\frac{1}{1-\left(\frac{c_C}{m_iL_i+c_C}\right)^{1-\frac{1}{\beta_{\mathfrak{S},ji}}}}\right]\left[\frac{1-\left(\frac{c_C}{m_iL_i+c_C}\right)^{1-\frac{1}{\beta_{\mathfrak{S},ji}}}}{\left(\frac{c_C}{m_iL_i+c_C}\right)^{1-\frac{1}{\beta_{\mathfrak{S},ji}}}}\right]$$
$$= \left(\frac{c_C}{m_iL_i+c_C}\right)^{-\frac{\beta_{\mathfrak{S},ji}-1}{\beta_{\mathfrak{S},ji}}}.$$

Substituting the above result into inequality (40), rearranging and exponentiating, results in

$$\left(\frac{c_C}{m_iL_i+c_C}\right)^{-\frac{\beta_{\mathfrak{S},ji}-1}{\beta_{\mathfrak{S},ji}}} > \exp\left[\frac{\beta_{\mathfrak{S},ji}-1}{\beta_{\mathfrak{S},ji}}\right]$$
$$\exp\left[-\ln\left(\frac{c_C}{m_iL_i+c_C}\right)\left(\frac{\beta_{\mathfrak{S},ji}-1}{\beta_{\mathfrak{S},ji}}\right)\right] > \exp\left[\frac{\beta_{\mathfrak{S},ji}-1}{\beta_{\mathfrak{S},ji}}\right].$$

Taking logs and dividing through by $\frac{(\beta_{\mathfrak{S},ji}-1)}{\beta_{\mathfrak{S},ji}}$ yields

$$-\ln\left(\frac{c_C}{m_iL_i+c_C}\right) > 0. \tag{41}$$

This finishes the proof of the first part of the proposition.

The second part of the proposition is proven by differentiating the unwinding threshold with respect to L_i:

$$\frac{\partial F^{C,\mathfrak{S}}_{ji}}{\partial L_i} = F^{C,\mathfrak{S}}_{ji}\frac{m_i}{m_iL_i+c_C}$$
$$+\left(\frac{1+\frac{2r_i}{\sigma^2_{ji}}\frac{\left(B^{\mathfrak{S}}_{1,F,ji}-1\right)}{B^{\mathfrak{S}}_{1,F,ji}}}{1+\frac{2r_i}{\sigma^2_{ji}}\left(B^{\mathfrak{S}}_{1,F,ji}-1\right)}\right)\frac{F^{C,\mathfrak{S}}_{ji}}{\left(B^{\mathfrak{S}}_{1,F,ji}\right)^2}\left(\frac{c_C}{m_iL_i+c_C}\right)^{1+\frac{\sigma^2_{ji}}{2r_i}}\frac{m_i}{(m_iL_i+c_C)}$$
$$> 0.$$

This completes the proof of the second part of the proposition. ■

A.8 Proof of Proposition 7

Differentiate the pricing function of the value investor with respect to volatility of the underlying:

$$
\begin{aligned}
\frac{\partial P_{ji}^{\mathfrak{V}}}{\partial \sigma_{ji}} &= F_{ji} \frac{\partial \beta_{1,F,ji} / \partial \sigma_{ji}}{\beta_{1,F,ji}^2} \\
&\quad + B_{2,F,ji}^{\mathfrak{V}} F_{ji}^{\beta_{2,F,ji}} \left[\left(1 - \frac{\beta_{2,F,ji}}{\beta_{1,F,ji}}\right) \ln\left(\frac{F_{ji}}{F_{ji}^D}\right) - \frac{1}{\beta_{1,F,ji}} \right] \frac{\partial \beta_{2,F,ji}}{\partial \sigma_{ji}} \\
&\quad + B_{2,F,ji}^{\mathfrak{V}} F_{ji}^{\beta_{2,F,ji}} \frac{\beta_{2,F,ji}}{\beta_{1,F,ji}^2} \frac{\partial \beta_{1,F,ji}}{\partial \sigma_{ji}}.
\end{aligned}
$$

Solving $\frac{\partial P_{ji}^{\mathfrak{V}}}{\partial \sigma_{ji}} > 0$ for L_i yields the risk-shifting threshold $L_{ji}^{\mathfrak{V}}$. ■

A.9 Derivation of equation (27)

Consider an open position in asset with abnormal appreciation rate. In continuation region, the value of equity in this open position is:

$$
\begin{aligned}
E_i^{\mathfrak{F}}\left(\tilde{F}_{ji}, \mu_{ji}\right) &= (1 - \mu_{ji} \mathrm{d}t) \\
&\quad \times \left\{ \delta_j \tilde{F}_{ji} \mathrm{d}t - m_i L_i \mathrm{d}t - c_C \mathrm{d}t + \mathbb{E}\left[e^{-r\mathrm{d}t} E_i^{\mathfrak{F}}\left(\tilde{F}_{ji} + \mathrm{d}\tilde{F}_{ji}, \mu_{ji}\right) \right] \right\} \\
&\quad + \mu_{ji} \mathrm{d}t \left\{ \frac{m_i L_i}{r_i} + \frac{c_C}{r_i} \right\}, \quad i \neq j,
\end{aligned} \tag{42}
$$

where:

the first term on the right-hand side pertains to the value of an open position with abnormal foreign exchange rate appreciation provided it does not vanish over the next $\mathrm{d}t$,

the second term is the present value of debt service and non-debt service costs of supporting an open position that is discharged if abnormal foreign exchange rate appreciation vanishes and the position collapses over the next $\mathrm{d}t$.

Using the standard techniques, it can be shown that the value of the equity in the open position $E_i^{\mathfrak{F}}\left(\tilde{F}_{ji}, \mu_{ji}\right)$, in continuation region, solves the following

ODE:

$$\delta_j \tilde{F}_{ji} - (m_i L_i + c_C)\left(1 - \frac{\mu_{ji}}{r_i}\right) - (r_i + \mu_{ji})\, E_i^{\mathfrak{F}}\left(\tilde{F}_{ji}, \mu_{ji}\right)$$
$$+ (r_i - \delta_j + \psi_{ji})\, \tilde{F}_{ji} E_i^{\mathfrak{F}\prime}\left(\tilde{F}_{ji}, \mu_{ji}\right) + \frac{\sigma_{ji}^2}{2}\tilde{F}_{ji}^2 E_i^{\mathfrak{F}\prime\prime}\left(\tilde{F}_{ji}, \mu_{ji}\right) = 0. \qquad (43)$$

The general solution to equation (43) is given by:

$$E_i^{\mathfrak{F}}\left(\tilde{F}_{ji}, \mu_{ji}\right) = \frac{\delta_j \tilde{F}_{ji}}{\delta_j - \psi_{ji} + \mu_{ji}} - \left(\frac{r_i - \mu_{ji}}{r_i + \mu_{ji}}\right)\frac{(i_i L_i + c_C)}{r_i}$$
$$+ B_{1,\tilde{F},ji}^{\mathfrak{F},\mu,ji}\tilde{F}_{ji}^{\beta_{1,\tilde{F},ji}} + B_{2,\tilde{F},ji}^{\mathfrak{F},\mu,ji}\tilde{F}_{ji}^{\beta_{2,\tilde{F},ji}}, \qquad (44)$$

where:

$B_{1,\tilde{F},ji}^{\mathfrak{F},\mu,ji}$ and $B_{2,\tilde{F},ji}^{\mathfrak{F},\mu,ji}$ are constants to be determined as part of the solution,
$\beta_{1,\tilde{F},ji}$ and $\beta_{2,\tilde{F},ji}$ are positive and negative roots, respectively, of quadratic equation:

$$\tilde{\mathcal{Q}}_{ji} = \frac{\sigma_{ji}^2}{2}\beta_{\tilde{F},ji}\left(\beta_{\tilde{F},ji} - 1\right) + (r_i - \delta_j + \psi_{ji})\,\beta_{\tilde{F},ji} - (r_i + \mu_{ji}) = 0. \qquad (45)$$

The boundary conditions are as follows. There is no speculative bubble component. Other conditions concern the default option value. Note that default can take place either as a result of asset value process falling sufficiently low that equity claim is worthless, or if abnormal foreign exchange rate appreciation vanishes. Investor identifies its preference towards one or another default route depending on the value of μ_{ji}. Consider equation (44): if $\mu_{ji} \geqslant r_i$, then it is optimal for investor to default in the latter fashion. If, however, $\mu_{ji} < r_i$, then default happens in the standard fashion such that the default trigger is selected optimally. Formally, the boundary conditions are are:

$$\lim_{\tilde{F}_{ji}\to\infty} E_i^{\mathfrak{F}}\left(\tilde{F}_{ji}, \mu_{ji}\right) = \frac{\delta_j \tilde{F}_{ji}}{\delta_j - \psi_{ji} + \mu_{ji}} - \left(\frac{r_i - \mu_{ji}}{r_i + \mu_{ji}}\right)\frac{(m_i L_i + c_C)}{r_i}, \qquad (46a)$$
$$E_i^{\mathfrak{F}}\left(\tilde{F}_{ji}^D, \mu_{ji}\right) = 0, \quad \mu_{ji} < r_i, \qquad (46b)$$
$$E_i^{\mathfrak{F}\prime}\left(\tilde{F}_{ji}^D, \mu_{ji}\right) = 0, \quad \mu_{ji} < r_i. \qquad (46c)$$

The above system yields the following constants:

$$B_{1,\tilde{F},ji}^{\mathfrak{F},\mu,ji} = 0, \tag{47a}$$

$$B_{2,\tilde{F},ji}^{\mathfrak{F},\mu,ji} = \mathbb{1}_{\mu_{ji}<r_i} \left[\left(\frac{r_i - \mu_{ji}}{r_i + \mu_{ji}} \right) \frac{(m_i L_i + c_C)}{r_i} - \frac{\delta_j \tilde{F}_{ji}^D}{\delta_j - \psi_{ji} + \mu_{ji}} \right] \left(\tilde{F}_{ji}^D \right)^{-\beta_{2,\tilde{F},ji}}, \tag{47b}$$

$$\tilde{F}_{ji}^D = \gamma_{2,\tilde{F},ji} \left(\frac{r_i - \mu_{ji}}{r_i + \mu_{ji}} \right) \frac{(m_i L_i + c_C)}{r_i} \frac{\delta_j - \psi_{ji} + \mu_{ji}}{\delta_j}, \quad \mu_{ji} < r_i, \tag{47c}$$

where:

$$\mathbb{1}_{\mu_{ji}<r_i} = \begin{cases} 1, & \mu_{ji} < r_i, \\ 0, & \mu_{ji} \geqslant r_i, \end{cases}$$
$$\gamma_{2,\tilde{F},ji} = \frac{\beta_{2,\tilde{F},ji}}{\beta_{2,\tilde{F},ji} - 1}.$$

Hereinafter we assume that $\mu_{ji} \geqslant r_i$. Then, the value of the open position is:

$$E_i^{\mathfrak{F}} \left(\tilde{F}_{ji}, \mu_{ji} \right) = \frac{\delta_j \tilde{F}_{ji}}{\delta_j - \psi_{ji} + \mu_{ji}} - \left(\frac{r_i - \mu_{ji}}{r_i + \mu_{ji}} \right) \frac{(m_i L_i + c_C)}{r_i}, \quad \mu_{ji} \geqslant r_i. \tag{48}$$

Similarly, the value of open position in an asset, whose dynamics follows the benchmark process is

$$E_i^{\mathfrak{F}} \left(F_{ji}, \mu_{ji} \right) = \frac{\delta_j F_{ji}}{\delta_j + \mu_{ji}} - \left(\frac{r_i - \mu_{ji}}{r_i + \mu_{ji}} \right) \frac{(m_i L_i + c_C)}{r_i}, \quad \mu_{ji} \geqslant r_i. \tag{49}$$

The difference between the two valuations is the value-enhancing component due to carry trade factor and is added to the value of open position in symmetric foreign risky asset in country j held by investor from country i. The pricing factor follows. ■

B Regression diagnostics

The purpose of this section is to provide the diagnostics for the principal assumptions of linear regression models to justify the use of OLS regression in the context of this paper. Consider the ordinary least squares (OLS) regression (2) reported in Table 2.

To test for linearity of the relationship between dependent and independent variables, we plot the residuals of the regression versus the predicted values (Figure 8a) and the observed versus the predicted values (Figure 8c). As required for the assumption of linearity, the points are symmetrically distributed around the horizontal line in figure 8a and the diagonal line in figure 8c. There is no indication that the model makes systematic errors.

To diagnose the independence of the errors (no serial correlation) we look at the autocorrelation of the residuals (Table 4).[23] The two columns reported in table 4 are the Ljung-Box Q-statistics and their p-values. The Q-statistic at the given lag is a test statistic for the null hypothesis that there is no autocorrelation up to the order of the lag. The null hypothesis is not rejected at all lags at the 5% significance level.

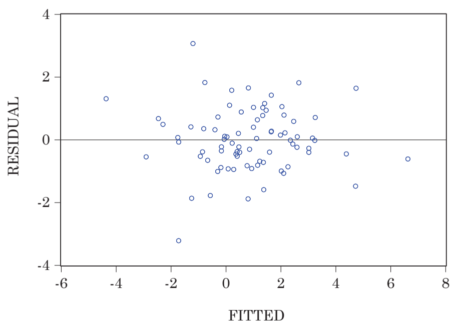
Table 4: Ljung-Box Q-statistics

Lag	Q-statistic	p-value
1	0.0046	0.946
2	0.2698	0.874
3	2.4583	0.483
4	9.0635	0.060

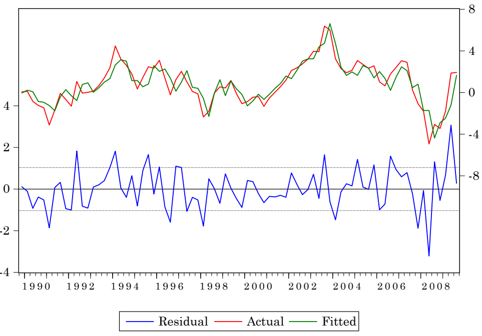
To test for homoscedasticity (constant variance) of the errors we look at plots of residuals versus time (Figure 8b) and residuals versus predicted values (Figure 8a). There is no significant evidence of residuals getting more spread-out as a function of time or as a function of the predicted value. Furthermore, we apply the Breusch-Pagan/Cook-Weisberg test for heteroscedasticity that yields $\chi^2 = 3.18$ (p-value of 0.7854); hence, the null hypothesis of a constant variance cannot be rejected at the 5% significance level.

To diagnose for normality of the error distribution we look at normal probabil-

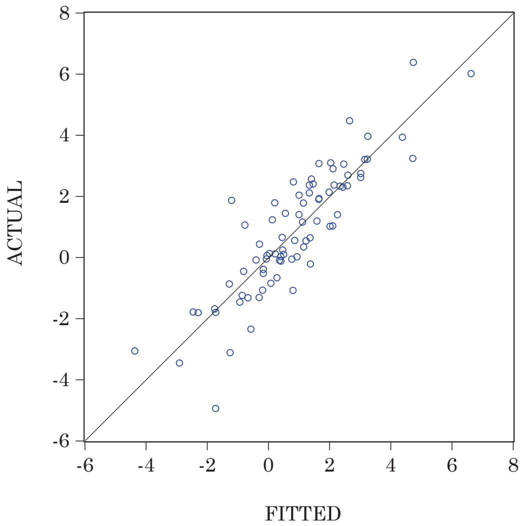
[23]An alternative way to look at the autocorrelation of the residuals is the Durbin-Watson statistic that for regression under consideration is 2.0137.



(a) Plot of residuals versus predicted values



(b) Plot of residuals versus time



(c) Plot of observed versus predicted values

Figure 8: Diagnostics plots

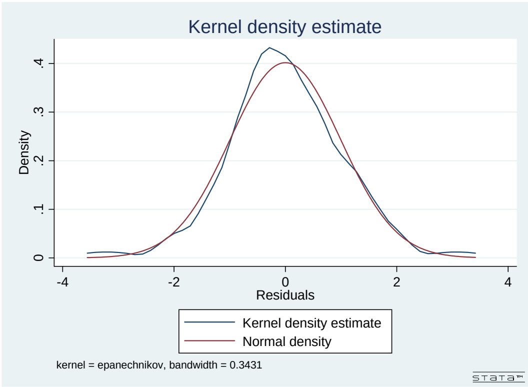
Table 5: Variance Inflation Factors

Variable	Coefficient Variance	Uncentered VIF	Centered VIF
Intercept	0.024049	1.803365	NA
Lagged house price index return	0.004666	1.710362	1.469536
NZD-JPY exchange rate volatility	0.000374	1.570894	1.127750
Indicator * NZD-JPY exchange rate volatility	0.038717	1.368837	1.193241
Δ Housing supply	0.977629	1.081168	1.065334
Δ Real disposable income growth	0.004267	1.022048	1.021977
Δ Floating new customer mortgage rate	0.034511	1.427774	1.396886

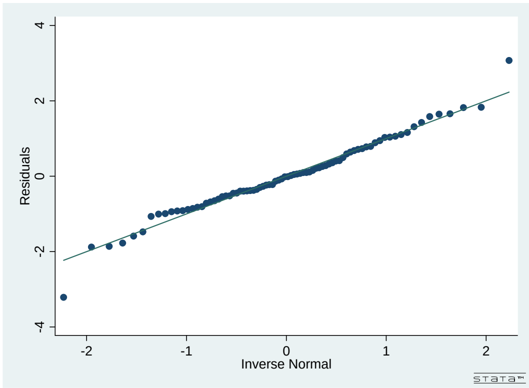
ity plots of the residuals (Figures 9a, 9b and 9c). The points on the plot of quantiles of the error distribution against quantiles of the normal distribution (Figure 9b) fall close to the diagonal line, *i.e.*, the error distribution is close to normal. Figures 9a and 9c provide additional support for normality of the error distribution.

We also carry out the Ramsey regression equation specification-error test (RESET) for omitted variables (F-statistic is 0.99, and p-value of 0.4049) and provide variance inflation factors for the independent variables as a method of measuring the level of collinearity between the regressors in Table 5. The null hypothesis of no omitted variables cannot be rejected at the 5% significance level and the variance inflation factors of the independent variables indicate no sign of collinearity issues.

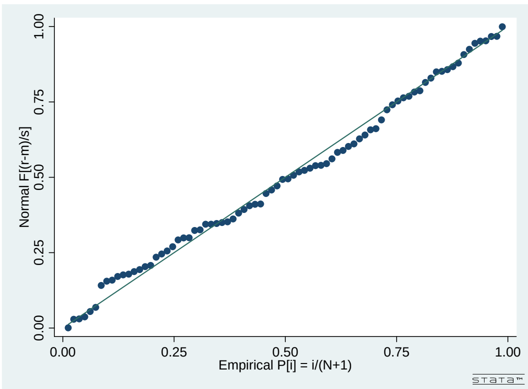
We now look for influential data as the product of outlierness (*i.e.*, an observation with an extremely large absolute residual) and leverage (*i.e.*, an observation with an extreme value on a predictor variable). A case is "influential" if removing it from the regression would markedly change a parameter (or parameters) in the model. Outliers are analysed in Figure 10 by looking at studentised residuals. Studentised residuals follow a t-distribution, so a 5% cut off where observations are generally considered to be potentially "influential" is given by ± 1.993 for 73 ($= 80 - 6 - 1$) degrees of freedom. Three outliers with studentised residuals that exceed an absolute value of 1.993 can be identified. Leverage is analysed in Figure 11. Generally, leverage greater than $(2k + 2)/n$ may be "influential", where k is the number of predictors and n is the number of observations. Five observations exceed leverage values of $\frac{2 \cdot (6+1)+2}{80} = 0.2$. Displacing both outliers and observations with leverage higher than 0.2 from the regression results in a valid model that does not change parameters materially. Likewise, the exclusion



(a) Kernel density estimate against normal distribution



(b) Quantiles of error distribution against quantiles of normal distribution



(c) Standardized normal probability plot

Figure 9: Normal probability plots

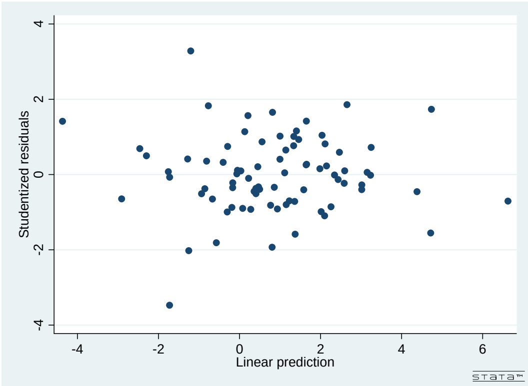


Figure 10: Plot of studentized residuals versus the linear prediction

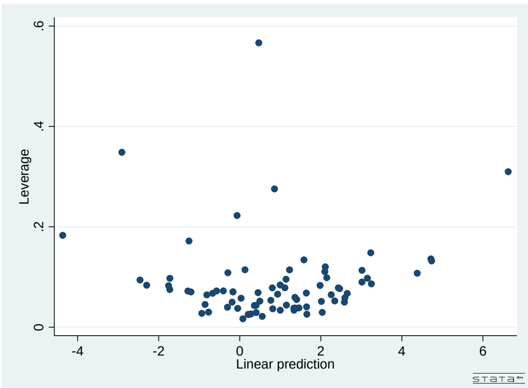


Figure 11: Plot of leverage versus the linear prediction

of *all* post-2006 observations (*i.e.*, the burst of the bubble) results in a valid model that does not change parameters materially.

C Granger causality test: Lag order selection

The lag order for the Granger causality test reported in Table 3 is determined by the largest lag order suggested by the lag length criteria listed in Table 6, because it is generally better to use more rather than fewer lags to account for the relevance of all past information. The lag length criteria in Table 6 are obtained by estimating the corresponding vector autoregression (VAR) models.

Table 6: VAR lag order selection criteria

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-388.7756	NA	177.5277	10.85488	10.91812	10.88005
1	-346.7742	80.50269	61.78279	9.799282	9.989005*	9.874811
2	-343.1128	6.814150	62.38866	9.808690	10.12489	9.934571
3	-334.9412	14.75438	55.60586	9.692810	10.13550	9.869045
4	-328.8365	10.68316	52.52148	9.634348	10.20351	9.860934
5	-319.7085	15.46697	45.64847	9.491902	10.18755	9.768841*
6	-317.0438	4.367032	47.52456	9.528995	10.35112	9.856288
7	-309.2706	12.30769*	42.98316*	9.424182*	10.37279	9.801827
8	-306.0044	4.989901	44.12275	9.444568	10.51966	9.872565

Notes:
* indicates lag order selected by the criterion.
LogL: Log likelihood value
LR: Sequential modified likelihood ratio test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

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