A Practical Approach to Modeling Managerial Risk Aversion in Real Option Valuation for Early Stage Investments *

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Abstract

In this work, we build on a previous real options approach that utilizes managerial cash-flow estimates to value early stage project investments, but accounting for managerial risk aversion. We introduce a market sector indicator, which is assumed to be correlated to a tradeable market index, which, through a mapping function, drives and replicates the cash-flow estimates. The mapping allows us to link the cash-flow estimates to many theoretical real options frameworks which currently can not be applied in practice. Through indifference pricing we are able to model the effect of managerial risk aversion for any given set of cash-flow estimates.

Keywords: Real Options; Managerial Information; Cash-Flow Replication; Project Valuation; Risk Aversion; Utility.

1 Introduction

Real option analysis (ROA) has been recognized as a superior method to quantify the value of real-world investment opportunities where managerial flexibility can influence their worth, as compared to standard net present value (NPV) and discounted cash-flow (DCF) analysis. ROA builds on the seminal work of Black and Scholes (1973) on financial option valuation. Shortly after, Myers (1977) correctly recognized that both financial options and project decisions are exercised after uncertainties are resolved. Early techniques therefore applied the Black-Scholes equation directly to value put and call options on tangible assets (see, for example, Brennan and Schwartz (1985)). Since this early connection, ROA has been popularized by business publications and valuation texts (Copeland and Tufano (2004), Trigeorgis (1996)), and in the last decade has transitioned from academic circles to heightened industry attention.

A number of practical and theoretical approaches for real option valuation have been proposed in the literature, yet industry's adoption of real option valuation is limited, primarily due to the inherent complexity of the models (Block (2007)). A number of leading practical approaches, some of which have been embraced by industry, lack financial rigor while many theoretical approaches are not practically implementable. The work presented in this paper is an extension of an earlier real options method developed by the authors (see Jaimungal and Lawryshyn (2010)). Our previous work assumed that future cash-flow estimates are provided by the manager in the form of

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a probability density function (PDF) at each time period. As was discussed, the PDF can simply be triangular (representing typical, optimistic and pessimistic scenarios), normal, log-normal, or any other continuous density. Second, we assumed that there exists a *market sector indicator* that uniquely determines the cash-flow for each time period and that this indicator is a Markov process. The market sector indicator can be thought of as market size or other such value. Third, we assumed that there exists a tradable asset whose returns are correlated to the market sector indicator. While this assumption may seem somewhat restrictive, it is likely that in many market sectors it is possible to identify some form of market sector indicator for which historical data exists and whose correlation to a traded asset/index could readily be determined. One of the key ingredients of our original approach is that the process for the market sector indicator determines the managerial estimated cash-flows, thus ensuring that the cash-flows from one time period to the next are consistently correlated. A second key ingredient is that an appropriate risk-neutral measure is introduced through the minimal martingale measure[1] (MMM) (Föllmer and Schweizer (1991)), thus ensuring consistency with financial theory in dealing with market and private risk, and eliminating the need for subjective estimates of the appropriate discount factor typically required in a DCF calculation.

As discussed in our original work, the advantages of the proposed approach include both practical and theoretical aspects and are listed here:

1. *The approach utilizes managerial cash-flow estimates.* In practice, managers are often expected to supply likely, optimistic and pessimistic cash-flow estimates when valuing a potential project. Our method relies on these estimates to formulate the option valuation.
2. *The approach requires little subjectivity with respect to parameter estimation.* Aside from the cash-flow estimates, the other key parameters required for the valuation will depend on the models chosen for the the market sector indicator process and the traded asset process. However, in many cases these parameters can be estimated based on historical data. Furthermore, in our approach, most of those parameters do not affect the project's valuation nor do they affect the value of the (European) option to invest.
3. *The approach provides a* ***missing link*** *between practical estimation and theoretical frameworks.* As will be discussed further, many traditional / theoretical frameworks presented in the literature bring fruitful insights regarding managerial decision making in the real options context or unique ways to value special cases of real options problems; however, these methods do not provide a mechanism to link real cash-flow estimates to the models. The approach presented here provides a mechanism to link many traditional frameworks, where the underlying process driving the cash-flows or project value is based on a Markov process, such as geometric Brownian motion (GBM) or a mean reverting process, to managerial cash-flow estimates. For example, our method can be used with entry / exit and switching frameworks such as those proposed by Sodal, Koekebakker, and Aadland (2008) and Lin (2009), to value multistage investment opportunities as proposed by Berk, Green, and Naik (2004), or to account for managerial risk aversion as proposed by Henderson (2007).
4. *The approach is theoretically consistent.* We emphasize this advantage, again, because a number of practical real options approaches fail in this respect. For example, the popular Market Asset Disclaimer (MAD) approach proposed by Copeland and Antikarov (2001) and other

[1] As we discuss in Section **??**, the risk-neutral MMM is a particular risk-neutral measure which produces variance minimizing hedges.

approaches, such as the one proposed by Datar and Mathews (2004), require an artificial mechanism to link cash-flows from one period to the next. Because our approach links the managerial cash-flow estimates through a GBM, or some other Markov process, the correlation of cash-flows from one period to the next is captured appropriately. Recall though that the project value is not being modeled directly, rather we model the underlying sector indicator which drives the project value. Furthermore, through the risk-neutral MMM, our approach provides a theoretical mechanism to properly account for both market and private risk.

In this work, we show how our method can be used to address issues associated with managerial risk aversion (i.e. we address item 3, in the list above). In the survey by (Block 2007), it was mentioned that one of the reasons for the poor adoption rate of ROA was that the methodology leads to excessive risk taking. Here, we present a model where managers are able to asses the value of a real option by accounting for risk aversion. Due to the complexity of the mathematics, it is unlikely that this model would be used directly by practitioners, however, it does provide a framework to compare the value of a real option project from the perspective of a risk-averse manager compared to a well diversified investor, whom, in many cases, the manager has been hired to serve.

1.1 Problem Description

To frame the problem, we introduce the managerial supplied cash-flow estimates from the work of (Datar and Mathews 2004), as shown in Table 1 and depicted graphically in Figure 1[2]. The project consists of a significant investment of $325k required in two years, after which, the company will receive the estimated cash-flow stream of Table 1. Clearly, management has the option to not invest at the end of year 2, at which point the future cash-flows would be forfeited. Thus, the project should be treated as a *real* option. As will be discussed below, there have been numerous practical and theoretical approaches to value such an option. Many practical approaches lack financial rigor while most theoretical approaches cannot be applied to actual managerial estimates. For example, (Berk, Green, and Naik 2004) provide an excellent methodology to value early stage R&D projects, such as this one, however, their work assumes that the cash-flow process follows a single geometric Brownian motion (GBM) and therefore, there is no mechanism to link the managerial supplied cash-flow estimates with their proposed model. More over, there have been a number of theoretical expositions of applying indifference pricing to account for managerial utility and risk aversion in the context of ROA ((Henderson 2007),(Miao and Wang 2007),(Hugonnier and Morellec 2007)), however, again, practical implementation of these methods based on managerial supplied cash-flows is not possible given the current frameworks.

1.2 Real Options in Practice

As has been well documented ((Dixit and Pindyck 1994), (Trigeorgis 1996), (Copeland and Antikarov 2001)), the DCF approach assumes that all future cash-flows are static, and no provision for the value associated with managerial flexibility is made. To account for the value in this flexibility numerous practical real options approaches have been proposed. (Borison 2005) categorizes the five main approaches used in practice, namely, the Classical, the Subjective, the Market Asset Disclaimer

[2]We emphasize here that these are actual managerial supplied cash-flows and that often, managers are comfortable in supplying low, medium and high estimates.

Table 1: An example cash-flow for a UAV project which costs $325 to invest in at year 2.

Scenario	End of Year						
	3	4	5	6	7	8	9
Optimistic	80	116	153	177	223	268	314
Most Likely	52	62	74	77	89	104	122
Pessimistic	20	23	24	18	20	20	22

(MAD), the Revised Classical and the Integrated. As discussed by (Borison 2005), each method has its strengths and weaknesses.

Approaches more closely aligned with the Classical approach (see (Amram and Kulatilaka 1999)) rely on the assumption that the value of the real project cash-flows can be closely replicated by a *known* traded asset. Market parameters of the traded asset are assumed to be applicable to the real options model and are therefore used as a proxy in the real options model. (Borison 2005) contends that this assumption is often invalid, and furthermore, finding a traded asset that can reasonably replicate the project value may prove difficult. Arguably, DCF models suffer this same contention and yet they are currently the standard practice in industry. The Subjective approach (see (Luehrman 1997), (Luehrman 1998a) and (Luehrman 1998b)) circumvents the issue of finding appropriate parameters by allowing managers to estimate these.

(Copeland and Antikarov 2001) introduced the MAD approach in an attempt to reduce the inherent weaknesses of the Classical and Subjective approaches. The main strength of the MAD approach is its ease of implementation, requiring only a basic understanding of the binomial option valuation method, and yet it provides a mechanism to account for managerial uncertainty in estimating the project cash-flow volatility. The MAD approach is even more tractable when implemented in a decision tree setting (see (Brandao and Dyer 2005)). A number of practitioners have utilized the MAD method (see, for example, (Pendharkar 2010)), however, the theoretical basis for the model is debated. (Borison 2005) highlights two main issues with the MAD approach; one with respect to the subjectivity of inputs and the other with respect to the geometric Brownian motion (GBM) assumption placed on the project value. Further, this approach leads to consistently increasing real (call) option value as volatility increases, which contradicts the observations of (Oriani and Sobrero 2008). We discuss this last point further in Section **??** where we show that in our approach, option prices can decrease before they start increasing with volatility.

Both the Revised Classical (see (Dixit and Pindyck 1994)) and Integrated (see (Smith and Nau 1995)) approaches recognize that most projects consist of a combination of market (systematic / exogenous) and private (idiosyncratic / endogenous) risk factors. The latter provides a mechanism to value the market risk of a project through hedging with appropriate tradeable assets while private risk is valued by discounting expected values at the risk-free rate. Properly applied, the integrated approach is consistent with financial theory. As (Borison 2005) notes, the integrated approach requires more work and is more difficult to explain, but is the only approach that accounts for the fact that most corporate investments have both market and private risk. As we will show, our approach also accounts for both market and private risks and is closely aligned with the Revised Classical approach.

1.3 Theoretical Frameworks

While practitioners and "practicing" academics of real options have published a number of practical approaches for real option valuation, a significant assortment of published work has been presented that utilizes more rigorous financial mathematics to investigate specific aspects of the real option on hand. For example, as discussed above, (Berk, Green, and Naik 2004) develop a real options model of a staged R&D project. The cash flows are modeled as a GBM with partial correlation to a pricing kernel, i.e. a traded asset. Included in the model are uncertainties related to catastrophic failure, technical uncertainty and investment costs. Valuation of the project is achieved by assuming managers will invest optimally given the state of observed exogenous and endogenous parameters. While the model brings insights regarding the type of information available to managers and its impact on decision making, there is no way to use the model in a practical setting.

(Miao and Wang 2007) present four incomplete market real options models and show that standard real options approaches, which assume complete markets, can lead to contradictory results. The authors assume a constant absolute risk aversion (CARA) utility based on consumption only, however, it is noted that not capturing the wealth effect does not impact the intuition gained from the models. In Model I, where it is assumed that the real option consists of a lump-sum payoff and is not hedgeable, contrary to the standard real options approaches (see (Dixit and Pindyck 1994), for example), in the presence of *enough* risk aversion, as the idiosyncratic risk increases, the option value decreases, as does the optimal timing of investment. Model II represents the case where the lump-sum payoff is partially hedgeable. In this case the contradictory, to standard real options case, results are somewhat dampened, but continue to persist, especially for the case of increased idiosyncratic risk and increased risk aversion. In Model III the authors consider the case where upon exercise, the real option delivers an unhedgeable perpetual continuous risky cash flow. To a first order approximation, it is shown that as the idiosyncratic risk increases, both the value of the cash flow stream and the option value decrease resulting in no impact on the exercise timing. On a second order basis, the authors show that an increase in risk aversion results in a delay in exercise, when compared to the complete market case, because the impact of lowering the value of the cash flows is greater than lowering the value of the option. Model IV, similar to Model III, assumes a perpetual continuous risky cash flow stream post exercise, but in this case it is assumed to be partially hedgeable. The model provided similar results to that of Model III, i.e. incomplete hedging raises the investment threshold and delays investment, compared to the complete-markets case. This work highlights issues associated with standard real options models, and especially, the complete market assumption.

(Henderson 2007) presents an incomplete market real options model based on the case of a lump-sum payoff whose value follows a GBM process that is partially correlated to a risky traded asset. Unlike (Miao and Wang 2007), Henderson assumes an exponential utility based on wealth with no consumption. Henderson presents a closed form solution to her model and establishes three distinct parameter regions. For the case where the project Sharpe ratio is less than what would be expected with the CAPM, the value of the option is shown to be convex, implying that project value and exercise timing increase with project volatility, similar to standard models, while increased risk aversion and correlation to the traded index decreases the project value and timing threshold. For a project critical Sharpe ratio that is greater than that expected by the CAPM plus half of the project volatility, the value of the option is optimized when the project value tends to infinity and exercise never occurs. For standard real options, exercise is postponed indefinitely when the project's Sharpe ratio is greater than the traded asset's. The third, and most interesting region, however occurs when the project Sharpe ratio is greater than that expected by the CAPM, but less than the critical value defined above, the value of the of the option is concave for low project value and convex as the project value increases. Thus, similar to that presented by (Miao and Wang 2007), as project volatility increases, the optjion value and exercise timing decrease. Furthermore, for the case where the project Sharpe ratio matches the CAPM, the option value is shown to be convex in project value and independent of project volatility. These results qualitatively match those reported by (Miao and Wang 2007). Another interesting aspect of the Henderson model is that in the limiting cases of perfect correlation and no risk aversion, the critical exercise regions do not approach those of non-utility based models. Henderson clearly highlights issues associated with the standard real options models. The author takes the approach that using standard models will lead to erroneous results, which is true from an entrepreneurial perspective, however,from a managerial perspective, whose ultimate duty is to create value for well diversified investors, the error is reversed – it is likely that managers will act more akin to the utility based models whereas the more appropriate actions should follow the non-utility based approach used with standard real options however assuming incomplete markets.

Based on the conclusions of the theoretical frameworks discussed above, it is clear that accounting for managerial risk aversion can lead to varied project valuations compared to standard methods. Furthermore, these frameworks provide important insights in the decision making processes of managers. However, currently, there is no method to account for managerial risk aversion given, say, cash flows as supplied in Table 1.

2 Replicating Cash-Flow Distributions

When managers think about investing in projects, they typically have in mind a cash-flow associated with three scenarios: (i) the most likely scenario (ii) the optimistic scenario and (iii) the pessimistic scenario. Datar and Mathews (2004) proposed a methodology based on such scenarios and we formulate our approach in a similar manner. An example of the three cash-flow scenarios is shown in Figure 1. The three scenarios may be derived through Monte Carlo simulations representing the technical risk inherent in the project, the corporations potential market share, the market value of the end product and so on. Regardless of how the manager comes to this cash-flow distribution, one of our main goals is to provide a consistent dynamic model which leads to a project cash-flow possessing any distribution which a manager provides. Our analysis is not limited to the triangular distribution shown in the example, although this distribution is, perhaps, the simplest form that managers employ widely.

We assume there is a traded market index I_t which the manager can invest in. This market index is, for simplicity, assumed to be a geometric Brownian motion (GBM) and satisfies the SDE

$$\frac{dI_t}{I_t} = \mu\, dt + \sigma\; dW_t, \tag{1}$$

where W_t is a standard Brownian motion under the real-world measure $\mathbb{P}$. Furthermore, we introduce an underlying observable, but not tradable, process S_t which drives the cash-flows generated from the project. This underlying process can be thought of as a *market sector indicator*. We assume

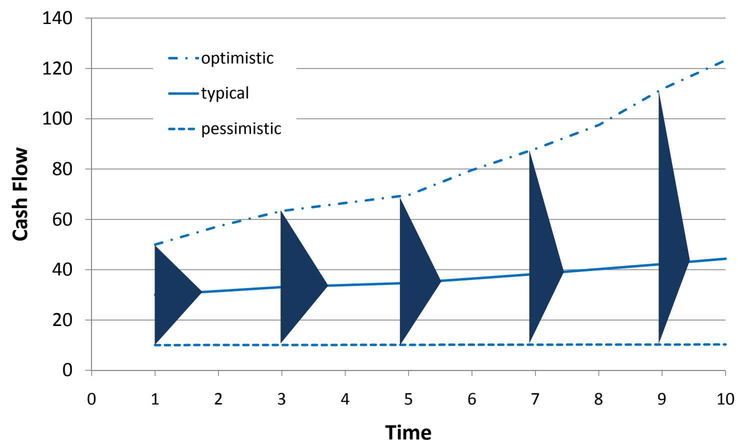

Figure 1: An example of typical, optimistic and pessimistic cash-flow streams provided by a manager.

that the market sector indicator is a standard GBM as well,

$$\frac{dS_t}{S_t} = \nu dt + \eta(\rho dW_t + \sqrt{1-\rho^2} dW_t^{\perp}), \tag{2}$$

where $dW_t^{\perp}$ is another standard Brownian motion, independent to dW_t, i.e. $dW_t dW_t^{\perp} = 0$. Here, the GBM is *not* the project value, instead, the GBM drives the cash-flows and the cash-flows V_k at times T_k ($k = 1, \ldots, n$) are each modeled as a function $\varphi_k(\cdot)$ of the underlying sector indicator, so that

$$V_k = \varphi_k(S_{T_k}). \tag{3}$$

As discussed in (Jaimungal and Lawryshyn 2010), the project value can be viewed as a strip of European contingent claims on the sector indicator with payoff functions φ_k at time T_k. When the sector indicator is high, the project value will be high, and when the sector indicator is low, the the project value will be low. Furthermore, since the cash-flows are all driven by the same underlying sector indicator, and the sector indicator is dependent on its past, there is a natural correlation between cash-flows induced by the path dependence in the sector indicator. If the sector indicator is high at the time of one cash-flow, resulting in a large cash-flow, then the probability of a large cash-flow at the next time-step is also high. This is a very desirable feature which has clear economic grounding. Contrastingly, in a number of practical approaches, the cash-flow distributions are typically assumed to be independent or correlation is introduced in a rather adhoc manner (as in the (Datar and Mathews 2004) approach).

Focusing on a single cash-flow distribution, our task is to determine φ such that at the cash-flow date T, V_T possess the manager specified distribution $F^*(v)$ – we use an asterisk to remind the reader that this distribution is provided by the manager. This requirement can be restated as, find φ such that

$$\mathbb{P}(V_T < v) = \mathbb{P}(\varphi(S_T) < v) = F^*(v), \tag{4}$$

and can be visualized as in Figure 2 for the case of a triangular managerial distribution.

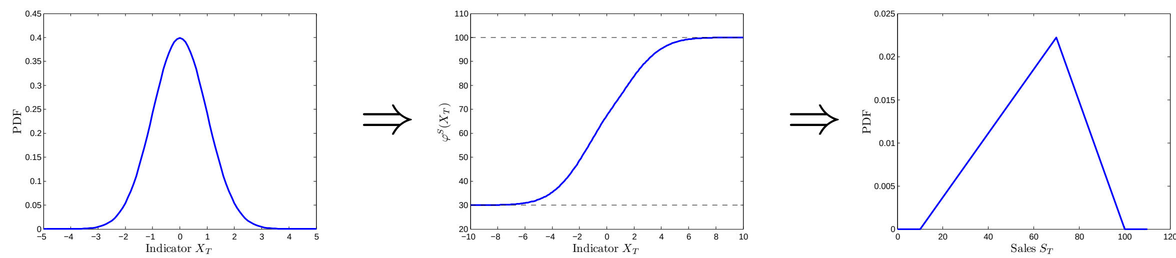

Figure 2: The underlying pdf $f_{S_T(S)}$ is mapped through the function $\varphi(S)$ to match the triangular distribution.

It is not difficult to see that if $\varphi(x)$ is assumed invertible and the cash-flow distribution $F^*(v)$ is invertible, then the solution is unique. However, invertibility is by no means necessary. Nonetheless, we restrict our analysis to this case as it leads to sound economically meaningful results. Note that the probability matching equation (4) can be also be interpreted as a quantile matching relationship where φ acts as a probability distortion function. Below we reproduce the proposition as developed by (Jaimungal and Lawryshyn 2010).

Proposition 1 The Replicating Payoff. *The payoff function $\varphi(S)$ which produces the manager specified distribution $F^*(v)$ for the cash-flow at time T, when the underlying driving uncertainty S_t is a GBM, is given by*

$$\varphi(S) = F^{*-1}\left(\Phi\left(z(S)\right)\right), \tag{5}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function and

$$z(S) = \frac{1}{\eta\sqrt{T}} \ln \frac{S}{S_0} - \frac{\left(\nu - \frac{1}{2}\eta^2\right)}{\eta}\sqrt{T}\,. \tag{6}$$

Following Jaimungal and Lawryshyn (2010), who set up the problem with GBM and geometric mean-reverting indicators, we have the following the following result.

Proposition 2 The Replicating Payoff. *The payoff function $\varphi^S(x)$ which produces the manager specified distribution $F^*(v)$ for the sales at time T, when the underlying driving uncertainty X_t is a BM, is given by*

$$\varphi^S(x) = F^{*-1}\left(\Phi\left(\frac{x - x_0}{\sqrt{T}}\right)\right), \tag{7}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function.

If the estimated cash-flow distribution has compact support, then the payoff function $\varphi(S)$ will be bounded above and below. For example, if F^* is a triangular distribution, then $\varphi(S)$ is bounded above and below by v_+ (maximum cash-flow) and v_- (minimum cash-flow), respectively, and there is a single point x_* at which the curvature changes sign. The cash-flow matching described here can be embedded into many practical real-option valuation questions ranging from an irreversible investment in a project at a fixed future, to the timing option to invest in a project, or abandon a project, and so on. As well, theoretical approaches can be layered on top of our cash-flow matching. In the following section we build on the work of (Henderson 2007) and (Miao and Wang 2007) by applying managerial utility indifference pricing to the real world example of (Datar and Mathews 2004), as depicted in Table **??**.

3 Accounting for Managerial Risk Aversion

To price the real option while accounting for managerial risk aversion, we assume the manager has exponential utility of the form

$$u(x) = -\frac{e^{-\gamma x}}{\gamma} \tag{8}$$

where $\gamma \geq 0$ represents the manager's risk aversion – the greater the value for γ, the greater the risk aversion. Given an initial wealth, X_0, the manager has two options: 1) he can invest optimally in the traded index, I_t (equation (1)) and a risk-free money market, or 2) he can invest optimally in the real options project, the traded asset and the risk-free money market. The equating the expected maximum terminal utility of the two options determines the optimal price the manager would be willing to pay for the real option project. The mathematical formulation is developed in the following subsections.

3.1 Value of the Utility for Optimal Investment in the Traded Index (Merton Model)

Here we apply standard arguments to develop an optimal investment strategy, and therefore the optimal utility, for a risk averse agent (manager) that can invest a portion of his wealth, π_t, in a risky, traded asset, I_t, and the rest in a risk-free money market (see (Merton 1971)). The wealth dynamics are given by

$$dX_t = (rX_t + \pi_t(\mu - r))\,dt + \pi_t \sigma dW_t \tag{9}$$

and we want to maximize the expected terminal utility

$$V(t,x) = \sup_{\pi_t} \mathbb{E}\left[u(X_T) \middle| X_t = x \right]. \tag{10}$$

Applying standard arguments, it can be shown that the solution to $V(t,x)$ can be achieved by solving the following PDE

$$\partial_t V - \frac{1}{2}\frac{(\mu-r)^2}{\sigma^2}\frac{(\partial_x V)^2}{\partial_{xx} V} + rx\partial_x V = 0 \tag{11}$$

with $V(T,x) = u(x)$. The solution to equation (11) is given by

$$V(t,x) = -\frac{1}{\gamma} e^{-\frac{1}{2}\left(\frac{\mu-r}{\sigma}\right)^2 - \gamma e^{r(T-t)x}}. \tag{12}$$

3.2 Value of the Utility for Optimal Investment in the Real Option Project

We now develop the optimal investment strategy and optimal utility value for the manager that invests in the real option project. The wealth dynamics remain similar as given in equation (9)

$$\left.\begin{array}{rcl} dX_t &=& (rX_t + \pi_t(\mu - r))\,dt + \pi_t \sigma dW_t,\ t \notin [T_0, T_1, ..., T_n] \\ X_{T_0} &=& X_{T_0^-} - K\mathbf{1}_{\mathcal{A}} \\ X_{T_j} &=& X_{T_j^-} + \varphi(S_j) e^{-rT_j}\mathbf{1}_{\mathcal{A}},\ j \in [1,2,...,n] \end{array}\right\} \tag{13}$$

where T_0 represents the exercise time when an investment of K will be required in the real option project to receive the cash-flow payments occurring at times $T_1, ..., T_n$ after T_0 and $\mathbf{1}_{\mathcal{A}}$ represents the indicator function equal to 1 if the real option is exercised.

Similar to the formulation above, the manager seeks to maximize his expected terminal utility as

$$U(t, x, s) = \sup_{\pi_t} \mathbb{E}\left[u(X_T) | \, X_t = x, S_t = s \right]. \tag{14}$$

Again, applying standard arguments, it can be shown that the solution to $U(t, x, s)$ can be achieved by solving the following PDE

$$\partial_t U + rx\partial_x U + \nu s \partial_s U + \tfrac{1}{2}\partial_{ss} U \eta^2 s^2 - \frac{1}{2}\frac{((\mu - r)\partial_x U + \rho\sigma\eta s \partial_{xx} U)^2}{\sigma^2 \partial_{xx} U} = 0, \, t \in (T_{k-1}, T_k) \tag{15}$$

for $k = 1, ..., n$, subject to the sequence boundary conditions

$$U(T_j, x, s) = U(T_j^+, x, s)e^{-\gamma\varphi(s)}, \text{ for } j = 1, ..., n-1 \tag{16a}$$

$$U(T_n, x, s) = u(x + \varphi_n(s)) \tag{16b}$$

It is easily shown that the substitution $U(t, x, s) = V(t, x)(H(t, s))^{\frac{1}{1-\rho^2}}$ in equation (15) results in the simplified PDE

$$\partial_t H + \hat{\nu} s \partial_s H + \tfrac{1}{2}\eta^2 s^2 \partial_{ss} H = 0 \tag{17}$$

with $H(T_n, s) = e^{-\gamma\varphi_n(S_{T_n})}$, and $t \in (T_{n-1}, T_n]$. We proceed in solving equation(17) in a backwards dynamic programming manner, where at each $t = T_j$, $j = \{1, 2, ..., n-1\}$, we set $H(T_j, s) = H(T_j^+, s)e^{-\gamma(1-\rho^2)\varphi_j(S_{T_j})}$. At $t = T_0$, we should invest in the real option if

$$(H(T_0^+, s))^{\frac{1}{1-\rho^2}} e^{\gamma K} \leq 1. \tag{18}$$

Defining f as the *indifference price*, i.e. the value of the real option, and setting $U(t, x - f, s) = V(t, x)$ leads to

$$f(t, s) = -\tfrac{1}{\gamma(1-\rho^2)} \ln H(t, s). \tag{19}$$

The results of the application of equation (19) are presented in the following section.

4 Results

Using the cash flow estimates presented in Table 1 and a strike price of \$325, the indifference price, $f(t, s)$, is plotted as a function of the sector indicator, S_t, and time, t, in Figure 3 for the case where $\gamma = 0.1$. At $t = T_0 = 2$ the option is exercised according to the condition given in equation (18). The indifference price for varying levels of risk aversion, γ, is plotted in Figure 4, with $S_t = S_0$. As expected, as γ approaches 0, the value of the indifference price approaches the value of the option under the minimum martingale measure (MMM) (see (Jaimungal and Lawryshyn 2010)). An interesting artifact of the valuation is that as risk aversion increases, the indifference price decreases and eventually goes to zero. In standard options and real options analysis, this would never be the case.

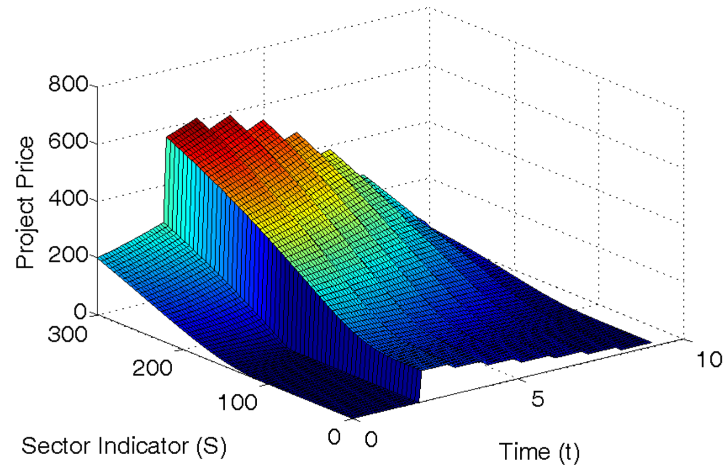

Figure 3: Indifference price as a function of the sector indicator, S_t, and time, t, for $\gamma = 0.1$, $r = 0.05$, $\rho = 0.5$, $\sigma = 0.1$.

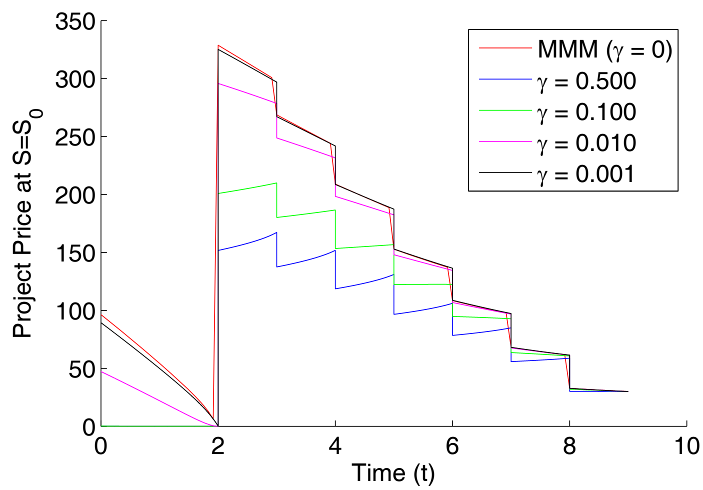

Figure 4: Indifference price as a function of time, t, with $S_t = S_0$ for varying levels of risk aversion, *gamma*. Here $r = 0.05$, $\rho = 0.5$, $\sigma = 0.1$.

5 Conclusions

This work builds on our previous proposed methodology that presents a real options approach that is practical to implement, requires limited detail regarding cash-flow estimates and minimal subjective estimation of market parameters, is consistent with financial theory, properly accounts for market and private risks, and ensures that the cash-flows are correlated appropriately among the time periods. While the model presented here may be too involved for real options practitioners, it provides a methodology to show managers how their inherent risk aversion may bias project valuation. Because managers are hired to act in the best interest of the owners / investors who are likely well diversified, we argue that managers should typically apply the minimal martingale measure to value their risky projects. Practitioners of real options valuation may, however, encounter reluctance by managers to take on risky ventures and may argue that the MMM assumption is inappropriate in real options valuation (yet this assumption is inherently made when the standard CAPM is applied). The model developed here can be used to counter this reasoning.

In the introduction we discussed a number of theoretical real options formulations that provide important insights and that have shown that standard real options modeling can lead to erroneous results. However, these models have no mechanism by which real managerial cash-flow estimates can be utilized. Here, we have provided a specific example where the method of (Jaimungal and Lawryshyn 2010) can be applied directly to model managerial risk aversion.

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