Optimal Investment in Modular Reactors in a Finite Time Decision Horizon

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Abstract

Small and Medium sized reactors (according to IAEA, 'small' are reactors with power less than 300 MWe, and 'medium' with power less than 700 MWe) are considered as an attractive option for investment in nuclear power plants. SMRs may benefit from flexibility of investment, reduced upfront expenditure, enhanced safety, and easy integration with small sized grids. Large reactors on the other hand have been an attractive option due to economy of scale. In this paper we focus on the advantage of flexibility due to modular construction of SMRs. Using real option analysis (ROA) we help a utility determine the value of sequential modular SMRs. Numerical results under different considerations, like possibility of rare events, learning, uncertain lifetimes are reported for single large unit and modular SMRs.

1 Introduction

Deregulation of the electricity market has been driven by the belief in increased cost-efficiency of competitive markets. There is a need for better valuation methods to make economic decisions for investment in power plants in these uncertain environments. The traditional net present value (NPV) approach, where investments are made if a project's expected net present value is greater than zero, is not always suitable for valuation of nuclear reactors whose costs and benefits have several sources of uncertainties. Kessides (2010) [19] raises important research questions that need to be addressed when making investment in nuclear power plants in the uncertain environment.

The real options approach for making investment decisions in projects with uncertainties became accepted in the past decade. The real options theory was pioneered by Arrow and Fischer (1974) [2], Henry(1974) [15], Brennan and

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Schwartz (1985) [6] and McDonald and Siegel (1986) [22]. Dixit and Pindyck (1991) [11] and Trigeorgis (1996) [31] comprehensively describe the real options approach for investment in projects with uncertain future cash flows. Unlike NPV, real options are able to capture the value of the option to delay, expand or abandon a project with uncertainties, when such decisions are made following an optimal policy.

Real options theory has been applied to value real assets like mines (Brennan and Schwartz (1985)), oil leases (Paddock, et. al (1988)), Patents and R&D (Schwartz (2002)). ROA for nuclear power plants was first applied by Pindyck (1992) [26], where he analyses the decision to start or continue building a nuclear power plant during the 1980's. He considers uncertain costs of reactor rather than expected cashflows for making the optimal decisions. Rothwell (2004) [28] uses ROA to compute the critical electricity price at which a new Advanced Boiling Water Reactor should be ordered in Texas.

The NEA report [24] identifies modular construction (multiple unit construction), as one of the measures to reduce the capital costs of nuclear power plants. This has raised interest in adoption of Small and Medium sized Reactors (SMR) with characteristics like simplicity of design, enhanced safety and less capital intensive investment. SMRs are especially suitable for adoption when LRs are restricted from the market due to limited capacity of electricity grids, or there is need for co-generation etc. Carelli et al. (2010) [8], Boarin and Ricotti (2011) [4] discuss the economic features of SMRs. An overview of the modularity in design and construction of nuclear power plants can be found in Upadhyay *et al.* (2011) [32].

In this paper we focus on the value of flexibility that arises from the modular construction of SMRs. Our approach is similar to Gollier *et al.* (2005) [14], where the firm needs to make a choice between a single high capacity reactor (1200 MWe) or a flexible sequence of modular SMRs (4×300 MWe). We, however, consider *finite time horizon* before which the investment decision should be made. In a competitive market the firms cannot delay an investment decision for ever and need to take one before the anticipated entry of a competitor. Also utilities need to meet the electricity demand with some minimum reliability, which restricts their decision horizon to finite time. The investment rules, such as the optimal time to start construction and the real option value of the investment can differ significantly with changing decision horizon.

We use a simulation based algorithm, called the stochastic grid method (SGM) [18], for computing the real option values of modular investment decisions. Real options can be priced with methods used for pricing American or Bermudan styled options. SGM has been used to price Bermudan options in [18] with results comparable to those obtained using least squares method (LSM) of Longstaff and Schwartz (2001) [20]. The option values are computed by generating stochastic paths for electricity prices, and thus with uncertain future cash flows. As an outcome of computing the real option price, we find the optimal electricity price at which a new module should be ordered. We extend the model to compute the option value and the critical electricity price, with uncertain lifetime of operation for the nuclear reactors and the effect of learning.

In the sections to follow we state the problem of modular investment in nuclear power plants and compare it with its counter part in the financial world. In the first half of the paper we describe the mathematical formulation behind the problem. In section 5 we describe the Stochastic Grid Method and adapt it to value sequential modular investment. In the second half of the paper we describe in detail the application of the method developed for the nuclear case. In section 6 we break down the ROA of sequential modular construction to address the following questions which a utility might face:

1. Multiple units on single site vs a Single Large Reactor.

2. Investment option value under uncertain lifetime of operation.

3. Economy of scale vs benefits of modularity of SMRs, i.e. learning and flexibility.

2 Problem Context

We consider a competitive electricity market where the price of electricity follows a stochastic process. The utility faces the choice of either constructing a single large reactor of 1200 MWe, or sequentially construct four modules of 300 MWe each. The total number of series units is denoted by n. Unit number i is characterized by discounted averaged cost per KWh equal to θ_i, its construction time is denoted by C_i and the life time of its operation by L_i. Both construction and life time are expressed in years. It is assumed that different modules are constructed in sequence, where the construction of unit $i+1$ can only begin once the construction of unit i is done. We assume a constant risk free interest rate denoted by r here.

The utility here needs to take a decision to start the construction of the modules within a finite time horizon, denoted by T_i for the *ith* module. In terms of financial options, T_i represents the expiration time for the 'option to start the construction 'of the *ith* module. Unlike financial options, it's difficult to quantify the *expiration time* for real options, and it is usually taken as the expected time of arrival of a competitor in the market. In case of an electricity utility, it also represents the time before which the utility needs to set up a plant to meet the electricity demand with certain reliability [1].

2.1 Financial Options and Pricing Methods

Real option problems are similar to their financial counterparts, i.e. Bermudan options and multiple exercise Bermudan options. A Bermudan option gives the holder the right, but not obligation, to exercise the option once, on a discretely spaced set of exercise dates. A multiple exercise Bermudan option, on the other hand, can be exercised multiple times before the option expires.

Pricing of Bermudan options, especially for multi-dimensional processes is a challenging problem owing to its path-dependent settings. The traditional valuation methods like finite difference schemes are often impractical due to the curse of dimensionality. In the past decade there have been several simulation based algorithms for pricing Bermudan options. The regression based approach proposed by Carriere (1996), Tsitsiklis and Van Roy (1999), which were popularized as least squares method (LSM) by Longstaff and Schwartz (2001) [20].

[1] Reliability is measured as the probability of the number of unplanned outages in a year due to surplus demand of electricity.

Other important approaches include the stochastic mesh method of Broadie and Glasserman (2004) [7], computing the early exercise frontier by Ibanez and Zapatero (2004) [17] and the duality based method from Haugh and Kogan (2004) [16] and Rogers (2002) [27]. More recently Jain and Oosterlee (2010) [18] proposed the stochastic grid method (SGM) for pricing high dimensional Bermudan options.

The problem of pricing Bermudan options with multiple exercise opportunity has been dealt with by Meinshausen and Hambly (2002) [23], with generalizations by Bender (2008) [5], Aleksandrov and Hambly(2008) [1] and Schoenmaker (2009) [29] who use the dual representation for such pricing problems. Chiara *et al.* (2007) [9] apply the multiple exercise real options in infrastructure projects. They use a multi-least-squares Monte Carlo method for determining the option value.

2.2 Problem Formulation for multiple exercise Bermudan option

Consider an economy in discrete time defined up to a finite time horizon T_n. The market is defined by the filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$. Let X_t, with $t = t_0, t_1, \ldots, t_m = T_n$, be an $\mathbb{R}^d$ valued discrete time Markov chain describing the state of the economy, the price of the underlying assets and any other variables that affect the dynamics of the underlying. Here $\mathbb{P}$ is the risk neutral probability measure. The holder of the multiple exercise Bermudan option has n exercise opportunities, that can be exercised at $t_0, t_1, \ldots, t_m$. Let $h_i(X_t)$ represent the payoff from the ith exercise of the option at time t and underlying state X_t. The time horizon for the i th exercise opportunity is given by T_i.

We define a policy π as a set of stopping times $\tau_n, \ldots, \tau_1$ with $\tau_n < \ldots < \tau_1$, which takes values in $t_0, \ldots, t_m = T_n$. τ_i determines the time where the ith remaining exercise opportunity is used. The option value when there are n early exercise opportunities remaining is then found by solving an optimization problem, i.e. to find the optimal exercise policy, π, for which the expected payoff is maximized. This can be written as:

$$V_n(t_0, X_{t_0} = x) = \sup_{\pi} \mathbb{E}\left[\sum_{k=0}^{n} h_k(X_{\tau_k}) | X_{t_0} = x\right]. \tag{1}$$

2.3 The real option formulation

The problem of modular construction can be formulated as a multiple exercise Bermudan option. In this case we consider the stochastic process X_t to be the process which models the electricity price. The payoff, $h_i(X_t = x)$, for the real option problem is the expected net cashflows per unit power of electricity sold through the lifetime of module i, when it gets operational at time t and state $X_t = x$.

Figure 1 illustrates the profit from the sale of electricity for one realized electricity price path. The cost of operation θ in the illustration is 3.5 cents/kWh and the area between the electricity path and θ gives the profit from the sale of electricity. We are interested in the expected profit, i.e. the mean profit from all possible electricity paths in the future. This expected profit (or net cash flow) is the payoff, $h_i(X_t)$, for the real option.

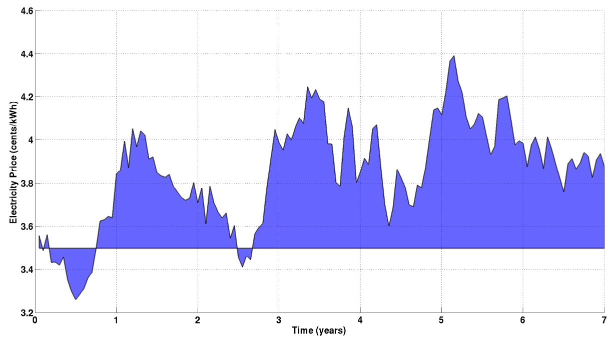


Figure 1: The area between the electricity path (starting at 3.5 cents/kWh) and cost of operation = 3.5 cents/kWh, gives cash flow for the reactor.

The revenue R_i, for the ith module, for every unit power of electricity sold through its life time L_i, starting construction at time t, when the electricity price is $X_t = x$, can be written as

$$R_i(X_t = x) = \mathbb{E}\left[\int_{t+C_i}^{t+C_i+L_i} e^{-ru} X_u du | X_t = x\right]. \tag{2}$$

R_i is the discounted expected gross revenue over all possible electricity price paths. The revenue starts flowing in once the construction is over, and therefore the limits for the integral start from $t + C_i$. and lasts as long as the plant is operational, i.e. until $t+C_i+L_i$. Similarly, the cost of operating the ith module through its lifetime for every unit power of electricity generated is:

$$K_i = \int_{t+C_i}^{t+C_i+L_i} e^{-ru}\theta_i du. \tag{3}$$

Here θ_i, the cost of operating the reactor per kWh is assumed to be constant. Therefore the net discounted cash flow, for module i is given by:

$$h_i(X_t = x) = R_i(X_t = x) - K_i. \tag{4}$$

Equations 2 to 4 gives the expected profit from the sale of electricity through the life of the nuclear reactor. Figure 1 illustrates the profit from the sale of electricity for one realized electricity price. The cost of operation θ in the illustration is 3.5 cents/kWh and the area between the electricity path and θ gives the profit from the sale of electricity. Equation 4 is the mean profit from all possible electricity paths in the future.

The expiration time T_n is the time before which the last module should be ordered. The optimal exercise policy $\pi = \{\tau_n, \dots, \tau_1\}$, is then defined by the determination of the optimal times for starting the construction of different modules, with τ_i, as the optimal time for starting the construction of module i, so that the net cash flow from the different modules is maximized. The option value for starting the first module, when there is the possibility of constructing a total of n modules, is given by equation (1).

3 Dynamic Programming Formulation

The optimal time to order[2] a new reactor under uncertain electricity price is solved using the technique of dynamic programming, where an optimal solution is found recursively moving backwards in time. Here we re-frame the problem stated above as a dynamic programming problem.

In order to construct all the modules at the optimal time, using Bellman's principle of optimality, we need to take optimal decisions starting from the last reactor. Also the optimal decision time for each of the reactors is computed starting from their respective expiration times and moving backwards in time to the initial state. The expiration time for ordering the ith module is given by

$$T_i = T_n - \sum_{k=i}^{n-1} C_k. \tag{5}$$

This constraint comes from the restriction that a new reactor can be ordered once all the prior ordered reactors have been constructed. Here T_n is the expiration time for the option to start the construction of the last module and C_i is the construction time in years for the ith module.

At the expiration time for the last module, as the firm does not have the option to delay the investment, the decision to start the construction is taken at those electricity prices for which the expected NPV of the last module is greater than zero. Therefore, the option value of the last module at the expiration time is given by

$$V_n(t_m = T_n, X_{t_m}) = \max(0, h_n(X_{t_m}). \tag{6}$$

At time t_k, $k = m-1, \cdots, 0$, the option value for the last of the series of reactors is the maximum between immediate pay-off h_n and its continuation value Q_n. The continuation value is the expected future cash flow if the decision to construct the reactor is delayed until the next time step. The reactor is constructed if at the given electricity price the net present value is greater than the expected cash flows if the reactor is constructed sometime in the future. This can be written as:

$$V_n(t_k, X_{t_k}) = \max\left(h_n(X_{t_k}), Q_n(t_k, X_{t_k})\right). \tag{7}$$

Given the present state X_{t_k}, the continuation value, or the discounted cash flows if the decision to start the construction is delayed for the last reactor is,

$$Q_n(t_k, X_{t_k}) = e^{-r(t_{k+1}-t_k)}\mathbb{E}\left[V_n(t_{k+1}, X_{t_{k+1}})|X_{t_k}\right]. \tag{8}$$

Once the option value at each time step for the last module is known, we move on to modules $n-1, \ldots, 1$. At the expiration time for the i-th module, the decision to start its construction is taken when the combined NPV of the present reactor and the expected future cash flow from the optimally constructed modules $i+1, \ldots, n$ is greater than zero. Therefore, the option value for the ith module at its expiration time T_i is given by:

[2]The optimal time to order in mathematics is called optimal stopping time. In the case of sequential modular construction optimal stopping time would refer to the time when the option to start the construction of a module be exercised. When you start the construction you *stop* the option to delay the construction to some future time.

$$V_i(T_i, X_{T_i}) = \max\left(0, h_i(X_{T_i}) + Q_{i+1}(T_i, X_{T_i})\right), \tag{9}$$

where h_i gives the direct future cash flow from the ith module and $Q_{i+1}(T_i, X_{T_i})$ gives the expected cash flow from the optimal construction of modules $i+1, \ldots, n$, given the information X_{T_i}. The option value for the module at time steps $t_k, \cdots, 0$, where $t_k < T_i$ is given by

$$V_i(t_k, X_{t_k}) = \max(h_i(X_{t_k}) + Q_{i+1}(t_k, X_{t_k}), Q_i(t_k, X_{t_k})), \tag{10}$$

i.e. the decision to start the construction of module i is taken if the cash flow from its immediate construction (given by $h_i(X_{t_k})$) and expected cash flow from the modules $i+1, \ldots, n$ constructed optimally in the future (given by $Q_{i+1}(t_k, X_{t_k})$) , is greater than the expected cash flows from the modules $i, \ldots, n$ if the decision to start its construction is delayed to the next time step (given by $Q_i(t_k, X_{t_k})$). The expected cash flow if the decision to start the construction of module $i, \ldots, n$ is delayed to the next time step is given by:

$$Q_i(t_k, X_{t_k}) = e^{-r(t_{k+1}-t_k)}\mathbb{E}\left[V_i(t_{k+1}, X_{t_{k+1}})|X_{t_k}\right], \tag{11}$$

The option value $V_i(t_k, X_{t_k})$ at time t_k for constructing the module i not only carries the information about the cash flows from module i, but also about the cashflows from the optimal construction of modules $i+1, \ldots, n$ in the future.

For sequential modular construction the payoff for module i is given by $h_i(X_{t_k}) + Q_{i+1}(t_k, X_{t_k})$. The payoff not only contains h_i, the direct discounted revenue from module i, but also Q_{i+1}, the value of the new option to start or delay the construction of new modules, that opens up with the construction of module i.

4 Electricity Price Model

The uncertain parameter in our pricing model is the electricity price. Modelling electricity spot prices is difficult primarily due to factors like:

- Lack of effective storage, which implies electricity needs to be continuously generated and consumed.
- The consumption of electricity is often localized due to constraints of poor grid connectivity
- The prices shows other features like weekly and seasonal effects, that vary from place to place.

Models for electricity spot prices have been proposed by Pilipovic (1997) [25], Lucia and Schwartz (2002) [21]. Barlow (2002) [3] develops a stochastic model for electricity prices starting from a simple supply/demand model for electricity. These models are focussed on the short term fluctuations of electricity prices which helps better pricing of electricity derivatives.

As decisions for setting up power plants look at long term evolution of electricity prices, we like Gollier [14], use the basic Geometric Brownian Motion (GBM) as the electricity price process (although we easily include other price processes).

4.1 Geometric Brownian Motion

If at any time t the electricity price is given by X_t cents/kWh, then the electricity price process is given by

$$dX_t = \alpha X_t dt + \sigma X_t dW_t, \tag{12}$$

where α represents the constant growth rate of X_t, σ is the associated volatility and W_t is the standard Brownian motion. In our model we assume α and σ to be constant. A closed form solution to the above SDE can be obtained using Ito's lemma and is given by:

$$X_t = X_0 e^{\left(\left(\alpha - \frac{\sigma^2}{2}\right)t + \sigma\sqrt{t}Z\right)}, \tag{13}$$

where Z is a standard normal variable. Also it can be seen that the above process has a log-normal distribution, i.e. $\log(X_t)$ has a Gaussian distribution with mean

$$\mathbb{E}\left[\log(X_t)\right] = \log(X_0) + \left(\alpha - \frac{\sigma^2}{2}\right)t,$$

and variance

$$Var(\log(X_t)) = \sigma^2 t.$$

5 Stochastic Grid Method for multiple exercise Bermudan options

The problem of sequential modular construction stated above can be solved using the stochastic grid method [18]. We choose the stochastic grid method as:

- The stochastic grid method (SGM) can efficiently solve the multiple exercise Bermudan option problem
- SGM can be used to compute the sensitivities of the real option value.
- The method can be easily extended to higher dimensions
- The method doesn't depend on the choice of the underlying stochastic process.
- Convergence is obtained already with few paths, when compared to LSM.

Although the problem we consider here is a one dimensional problem - with the electricity price as the stochastic variable, it should be noted that a typical real option problem tends to be high dimensional with several underlying stochastic terms. A prudent choice of pricing method would be one which could be extended to high dimensions in future.

The stochastic grid method solves a general optimal stopping time problem using a hybrid of dynamic programming and Monte Carlo methods. The method first computes the *optimal stopping policy* and direct estimator for the option price, and then computes the lower bound values as the expected pay-off when the simulated paths are exercised following the policy so obtained. Optimal stopping policy divides the electricity price into two regions, a region where it is optimal to exercise the option (early exercise region) and a region where its optimal to continue holding the option. When the electricity price path enters the early exercise region for the corresponding module, the construction of the module begins. We here briefly describe the method adapted for the sequential modular problem stated above.

SGM for multiple exercise Bermudan option begins by generating N stochastic paths for the electricity prices, starting from initial state X_0. The electricity prices realized by these paths at time step t_k constitute the grid points at t_k. The electricity price paths can be generated using equation 13.

The pricing steps for SGM can be broken down into two main parts, based on the recursive dynamic programming algorithm from the previous section.

- Parametrization of the option value: The option values at each grid points are converted into a functional approximation using piecewise regression.
- Computing the continuation value: The continuation value is computed using the conditional probability density function and the functional approximation of the option value at the next time step.

Parametrization of the Option Value

In order to obtain the continuation value for grid points at t_k we need to determine the functional approximation of the option value at t_{k+1}. Once the option value at the grid points at t_{k+1} is known, the functional approximation is obtained using a *piecewise least squares regression*. Therefore the option value at given time step is divided into two regions, seperated by the critical electricity price $X^*_{t_{k+1}}$. For two segments the functional approximation is given by

$$\widehat{V}_i(t_{k+1}, X_{t_{k+1}}) = \mathbf{1}_{\{X_{t_{k+1}} < X^*_{t_{k+1}}\}} \sum_{m=0}^{M-1} a_m X^m_{t_{k+1}} + \mathbf{1}_{\{X_{t_{k+1}} \geq \mathcal{X}^*_{t_{k+1}}\}} \sum_{m=0}^{M-1} b_m X^m_{t_{k+1}}, \tag{14}$$

$\mathbf{1}_{\{X_{t_{k+1}} < \mathcal{X}^*_{t_{k+1}}\}}$, is an indicator function whose value is 1 if the argument $\{X_{t_{k+1}} < \mathcal{X}^*_{t_{k+1}}\}$, is true and is 0 otherwise. Therefore, $\mathbf{1}_{\{X_{t_{k+1}} < \mathcal{X}^*_{t_{k+1}}\}}$ and $\mathbf{1}_{\{X_{t_{k+1}} \geq \mathcal{X}^*_{t_{k+1}}\}}$, groups the grid points into two segments, separated by the early exercise point. A better convergence is obtained by using greater number of segments (see [18]).

Computing the Continuation Value

Once the functional approximation of the option value for modules i and $(i+1)$ are known for time step t_{k+1} , the continuation value for the ith module at t_k can be computed using equation (11). In order to compute the expectation, $\mathbb{E}\left[V_i(t_{k+1}, X_{t_{k+1}}) | X_{t_k}\right]$, we need the distribution function for $X_{t_{k+1}}$ given X_{t_k}. This conditional distribution function, $f(X_{t_{k+1}} | X_{t_k} = x)$, for the basic GBM process is known in the closed form. Therefore equation (11) can be written as

$$
\begin{aligned}
\widehat{Q}_i(t_k, X_{t_k}) &= \int_{y\in[0,\mathcal{X}^*]} \left(\sum_{m=0}^{M-1} a_m y^m\right) f(y|X_{t_k}=x)dy \\
&\quad + \int_{y\in(\mathcal{X}^*,\infty]} \left(\sum_{m=0}^{M-1} b_m y^m\right) f(y|X_{t_k}=x)dy. \qquad (15)
\end{aligned}
$$

In a more generic case when the conditional distribution function is unknown, it can be approximated using the *Gram Charlier Series*. For more details on computing the continuation value refer [18].

6 Numerical Experiments

In this section we consider questions a utility might like to address when deciding between a single large reactor and sequential modular units. We break down the questions into the following.

1. Multiple units on single site vs a single large unit: When multiple units are constructed at the same site, the cost of the first module is usually higher than that of the following modules. Real option valuation allows us to decide the critical price at which the first unit should be ordered. This test case is similar to the test case by Gollier [14], although we consider a finite decision horizon. We also compare the option value of two twin large reactor (constructed simultaneously) with four large reactors constructed independently.

2. Uncertain life time of operation: Nuclear reactors are designed to operate for some technical lifetime. Sometimes the reactors need to be prematurely permanently shut down due to various unanticipated reasons. On the other hand the operating lifetime of reactors can be extended if they are in technically good condition and it makes economic sense to do so. The stochastic lifetime of operation needs to be included in the valuation method for making informed decisions. Specifically we analyze when modular SMRs are a better economic choice under uncertain lifetimes of operation.

3. Economy of scale vs flexibility and lLearning: This case is an extension of the test case of Gollier [14]. We consider the choice between single large reactor and sequential modular SMRs. The large reactor in this case benefits from the economy of scale, while the small reactors benefits from modularity and the learning effect.

6.1 Multiple Units vs Single Unit

We divide this test case into two parts, the first taken from Gollier [14], where he considers SMRs constructed on the same site and a single large unit. The second case involves two twin units on a single site compared with four large independent units.

Case from Gollier

The objective of this test case is to compare two investment decisions, one with single large capacity reactor of size 1200 MWe and the second a sequence of upto four modular reactors with unit capacity of 300 MWe. The construction time and electricity production costs used (see table 1) in the test case are taken from Gollier [14]. It can be seen that the cost of electricity production for the first unit is relatively expensive when compared to series units, as a large part of the fixed costs for the modular assembly, like the land rights, access by road and railway, site licensing cost, connection to the electricity grid are born by the first unit. We compute the optimal time to start the construction of each module when the decision horizon for the first module is between 5 to 15 years. Based on equation (5), we take the corresponding decision horizon for the construction of the last module between 12 to 22 years. For the single large reactor we take the final decision horizon between that of the first and the last module, i.e. between 8.5 to 18.5 years. One of the advantages of modular construction is that the increasing demand can be met gradually, which allows the spreading of the decision horizon to longer times, without encountering big gaps in demand and supply. On the other hand, in the case of a single large reactor, as meeting the increasing demand is not gradual, the decision horizon cannot be extended much when maintaining service reliability.

	Construction Time (months)	Discounted Average cost (cents/KWh)
Large Reactor	60	2.9
Modular Case		
Module 1	36	3.8
Module 2 to 4	24	2.5

Table 1: Construction time and Discounted Average cost used for the large reactor and the modular case

We assume the risk free interest rate to be 8% which is the mean value for the OECD countries. The predicted growth rate in the GBM model for the electricity prices, α, is taken as zero. We check the real option values for different scenarios, σ =10%, 20% and 30%, corresponding to low, medium and high uncertainties in future electricity prices.

Figures 2, 3, 4 compares the option value of the single large reactor with the option value of the first modular unit. The option value of the first module is less than that of the large reactor in the three given scenarios of electricity price development. However, the option value of the modular unit approaches that of the large reactor with increasing volatilities. It is clear that the large reactor would be more profitable to be ordered when the decision horizon is finite, unlike the case of infinite time horizon (see [14]).

Figure 5 compares the critical threshold price for starting construction of single large reactor and the first module. For smaller decision horizons not much information is revealed by waiting, which results in a lower threshold price. When firms can wait for infinite time to make the optimal decision, the exercise policy (or the critical electricity price) obtained is constant with time. However, when there is a finite decision horizon, the critical threshold price becomes time dependent. Figure 6 shows the time evolution of the critical

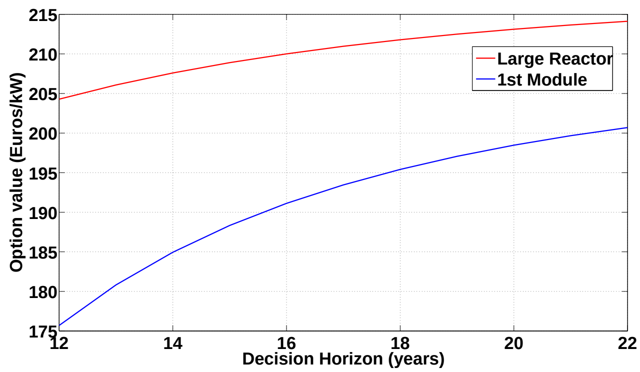


Figure 2: Option value for the large reactor and the first module when the volatility in electricity price is 10% and the price of electricity is 3 cents/kWh

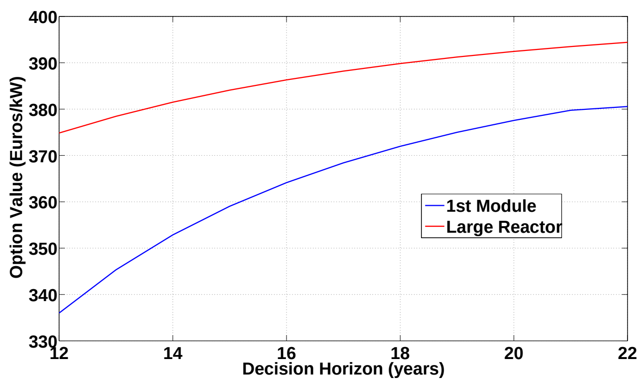


Figure 3: Option value for the large reactor and the first module when the volatility in electricity price is 20% and the price of electricity is 3 cents/kWh

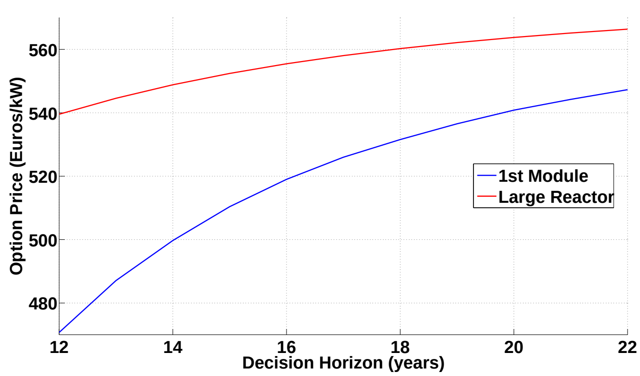
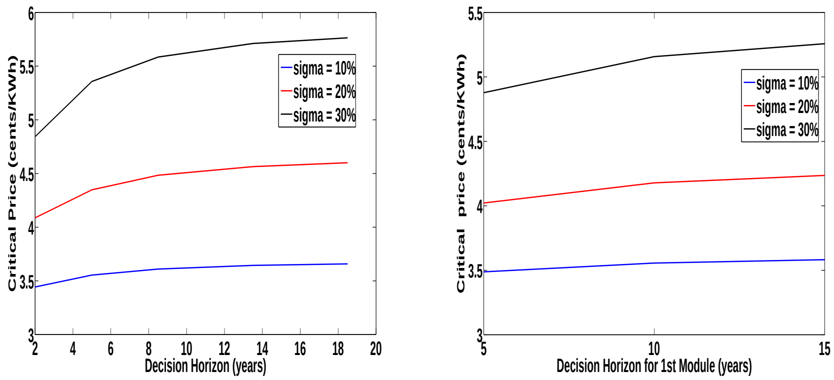


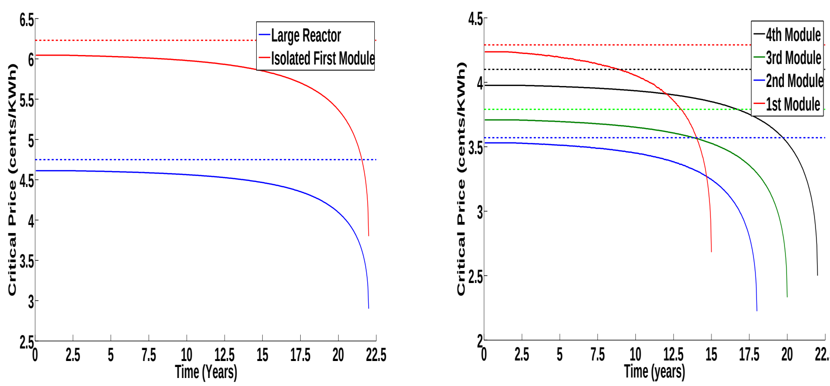
Figure 4: Option value for the large reactor and the first module when the volatility in electricity price is 30% and the price of electricity is 3 cents/kWh

electricity price at which a new reactor should be ordered for single isolated units and modular units. We see that the critical threshold price to order the first module is substantially higher when it is ordered in isolation. With the modular setting, the first module can be ordered at a much competitive electricity price due to the added value of flexibility in investment and the anticipated benefits from subsequent modules, which are more profitable. As the time approaches the final expiration time the critical price drops almost to the value obtained by NPV analysis. The reason for this drop is that, as we approach the final decision time, there is not enough time left to optimize the time to order a new reactor, and thus the value of flexibility is reduced.



(a)Large Reactor (1200 MWe) (b)First Module(300 MWe)

Figure 5: The critical threshold electricity price at time t_0 at which a new reactor should be ordered for different uncertainty scenarios and decision horizon.



(a)Isolated Unit (b)Sequential Modular Construction

Figure 6: The time evolution of critical threshold electricity price for (a) Isolated units (large reactor and isolated first module), (b) Modular units. The volatility of electricity prices is taken as 20%. The dotted lines are the corresponding values reported by Gollier.

For a firm it is not only important to know the real option value for making an investment decision, but also sensitivity[3] of the value with respect to the

[3]Sensitivity analysis in financial options is done by computing the derivative with respect

parameters chosen. We here compute the *delta* values, i.e. the ratios between the change in the real option value of the reactor to the change in the underlying electricity price. High delta values imply that the investment options are sensitive to a changing electricity price. Figure 7 compares the delta values for the different modules. We see that the delta values for the final module, as expected, converge to one , i.e. when it's optimal to order a new reactor the change in option value is proportional to the change in the electricity price. However, for each prior module the delta values converge to values less than one. Therefore for the first module a unit change in the electricity price only changes the option price by a factor of 0.8. This makes modular construction investment option generally more stable, i.e. even if the electricity price drops by 1 the value of investment changes only by a factor of 0.8.

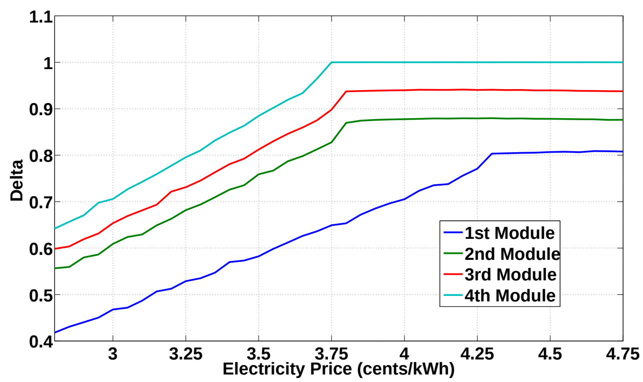


Figure 7: The delta values for the four modules when the decision horizon is 12 years and the volatility in electricity price is 20%

Table 2 gives the critical threshold prices for constructing a single large reactor and the different modules. We compare the results with those computed using the COS method [13] and Gollier [14]. Appendix A gives the details about the COS method for the modular case. Monte Carlo and deterministic method give indentical prices, although we see a difference between finite time horizon and infinite time horizon decisions.

	Final Decison Time (years)	Isolated		Modular			
		LR	Unit 1	Unit 1	Unit2	Unit3	Unit 4
SGM	12	4.56	5.98	4.05	3.44	3.65	3.93
	17	4.60	6.02	4.18	3.50	3.69	3.96
	22	4.62	6.05	4.24	3.53	3.71	3.98
COS	12	4.56	5.98	4.10	3.46	3.65	3.93
	17	4.60	6.03	4.21	3.51	3.69	3.96
	22	4.62	6.05	4.25	3.53	3.71	3.98
Gollier	∞	4.75	6.23	4.29	3.57	3.79	4.10

Table 2: Critical threshold electricity price (cents/KWh) at which new reactors should be ordered, for different decision horizon. There are twenty equally spaced exercise opportunity each year. The volatility of the electricity price is taken as 20 %.

Table 3 summarizes the two projects, for different levels of uncertainty. It

to various parameters, and these derivatives together are referred as *Greeks* is clear from the results that the option values obtained when the firms can delay its decision indefinitely are higher than that when the firm needs to make a decision in a realistic decision time horizon. However, it is clear that the possibility of more economic units in the future may make the SMRs competitive with a large reactor. Also the option values obtained are less sensitive to the electricity price when the units are constructed in a sequential manner. However, in this test case, we realize that a cheaper large unit is more competitive than modular units, when just flexibility is taken into account.

		Isolated		Modular			
	Volatility (%)	LR	Unit 1	Unit 1	Unit 2	Unit 3	Unit 4
SGM	10	206.85	64.64	191.31	425.26	444.84	464.59
	20	379.64	245.49	362.32	572.00	602.17	631.94
	30	546.68	444.76	526.13	735.83	779.01	821.76
COS	10	206.86	64.16	193.45	424.79	445.17	464.65
	20	380.03	246.33	368.39	575.62	604.25	631.97
	30	546.79	445.72	538.98	746.96	785.05	821.86
Gollier	10	217.8	-	218.1	-	-	-
	20	401.4	263.2	442.2	-	-	-

Table 3: Option value (Euro/kW) for the modular units, the large reactor and the isolated first unit for different volatilities in electricity price. The decision horizon for the modular case is taken as 17 years, for single large reactor as 13.5 years and for isolated first unit as 10 years. The option value is computed when the electricity price is 3 cents/kWh. The Gollier results are for infinite decision horizon.

Two Twin Units vs Four Single Units

We look at the case involving sharing of facilities by constructing multiple units in a single site. The parameter values are taken from a real case observed at EdF as described in the NEA report [24]. The average cost of unit reduces with increasing number of units per site. We consider the case where two pairs of units per site are constructed with a case where four individual units are constructed. All units considered here are of the same size, so the economy of scale doesn't play any role in this case.

The aim of the test case is to compare a project with two twin units on a single site with four individual units constructed at different sites. We assume that the reactors involved are of the same size (1200 MWe each), and hence the only cost difference comes from the sharing of costs if constructed on a single site. The first reactor, in both cases, is considered to be first-of-a-kind (FOAK), and we assume the cost of generating electricity for this reactor to be 3.5 cents/kWh. The costs of the other units are summarized in table 4. In order to achieve a cost benefit for the reactors constructed on the same site, the reactor units are constrained to be constructed in a phased manner. Therefore, the two twin reactors are constructed immediately one after the other, with the construction of the first unit starting when the electricity price crosses its corresponding critical price. The benefits of cheaper subsequent units comes at the loss of flexibility to order the subsequent units at more optimal electricity prices. The reasons for phased construction include considerable efficiencies and associated savings to be gained from phased construction and rolling the various craftsmen teams from one unit to the next. In addition, by construction repetition, there is the *craft labour learning effect* that reduces the time to perform a given task and correspondingly reduces labour cost and schedule. We take the decision horizon for ordering the first reactor to be 7 years. We assume that once a unit becomes operational it functions at its maximum capacity factor.

On the other hand, when the four units are constructed at separate sites, they do not have the cost benefits of sharing site specific costs, neither the productivity effects. However, when constructed individually they benefit from the flexibility to order each unit at its corresponding optimal time.

	Construction Time (months)	Discounted Average cost (cents/KWh)
Two twin units		
Unit 1	60	3.5
Unit 2	48	1.67
Unit 3	48	1.81
Unit 4	48	1.60
Four independent units		
Unit 1	60	3.5
Unit 2 to 4	48	2.25

Table 4: Construction time and discounted average costs used for the two twin units on same site and for four units on different sites.

The following assumptions are made when determining the overnight costs.

- Unit 1 bears all of the extra first-of-a-kind (FOAK) cost.
- The cost of engineering specific to each site is assumed to be identical for each site.
- The cost of facilities specific to each site is assumed to be identical for each site.
- The standard cost (excluding the extra FOAK cost) of a unit includes the specific engineering and specific facilities for each unit

If θo is the standard cost (excluding the extra FOAK cost) of a sole unit on a site, the:

- Cost of the first unit: θ = (1+x)θo
- Cost of the following units: θo (if programme of 1 unit/site)
- Cost of the 2nd unit on a site with one pair: y θo.
- Cost of the 3rd unit on a site with two pairs: z θo.
- Cost of the 4th unit on a site with two pairs: y θo (it is assumed that the cost of the 2nd unit of a pair is independent of the rank of the pair on the site)

It is considered that a productivity effect only occurs as of the 3rd unit of a series. If n is the rank of the unit in the series, and θn is the cost which results from taking into account the sole programme effect, it follows that:

$$\theta' n = \frac{\theta n}{(1+k)^{n-2}} \quad n \geq 2$$

Where $\theta' n$ is the cost of a module if there is a productivity gain involved. Using the above formulation, for the case of EdF (x = 55%, y = 74%, z = 82

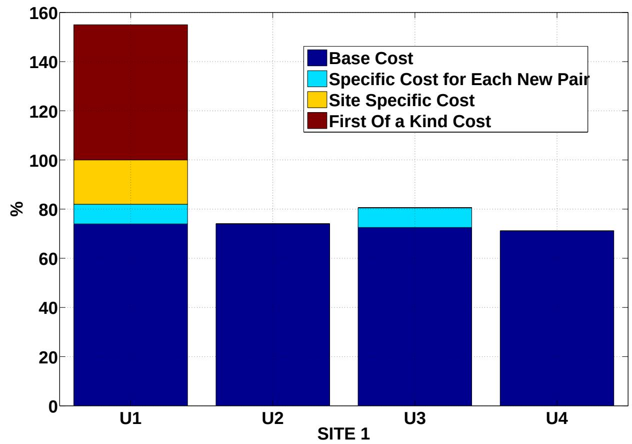


Figure 8: Relative cost of the four units when constructed as two twin units on a single site

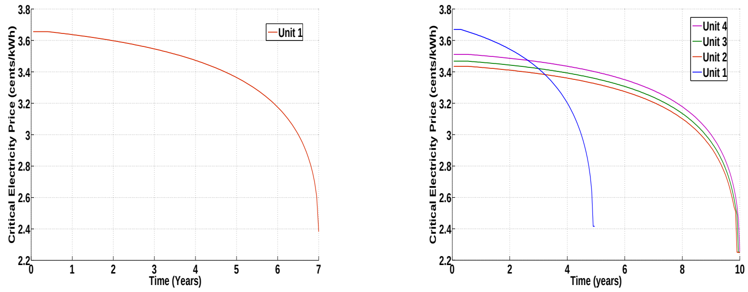
%, k = 2%) in the case of the two pairs of unit per site the relative costs are illustrated in figure 8

In the case of two twin units the cash flow due to the sales of electricity once the units become operational is modified from equation (2) to :

$$\begin{aligned} R_1(X_t = x) \quad &= \quad \mathbb{E}\left[\left(\int_{t+C_1}^{t+C_1+L_1} e^{-ru} X_u du + \int_{t+(C_1+C_2)}^{t+(C_1+C_2)+L_2} e^{-ru} X_u du + \cdots \right.\right. \\ &\left.\left. + \int_{t+(C_1+C_2+C_3+C_4)}^{t+(C_1+C_2+C_3+C_4)+L_4} e^{-ru} X_u du\right) | X_t = x\right]. \qquad (16) \end{aligned}$$

The above equation can be read as, the revenue from unit 1 when ordered at time t starts flowing in once its construction is over at $t + C_1$, and continues till the end of its lifetime, i.e. until time $t + C_1 + L + 1$. The construction of unit 2 starts at $t + C_1$, and its revenues start flowing in from $t + C_1 + C_2$, till the end of its lifetime at $t + C_1 + C_2 + L + 2$. Equation 3 can be modified accordingly. It can be seen that only the first reactor can be ordered at an optimal time, and following it the construction of the ith reactor is forced immediately after the completion of $(i - 1)$th reactor to achieve the cost reductions shown in figure 8.

For the four independent reactors the revenue is given by equation (2), however, in this case the constraint that the next reactor can be ordered only after the completion of the previous reactor is relaxed. The only constraint in this case is that the subsequent modules can be ordered only after the completion of the first reactor. This constraint is applied because the first reactor is considered to the FOAK, and significant cost reductions for the following reactors can be attained by learning from the first reactor. With the completion of the first reactor, the remaining reactors can be ordered at the optimal electricity price, with the possibility of more than one reactor being ordered simultaneously. We take the decision horizon for the first unit to be 5 years and for the remaining reactors to be 10 years.



(a)Twin Units at single site (b)4 Independent Units

Figure 9: The critical threshold electricity price at which a new reactor should be ordered for different uncertainty scenarios and decision horizon.

Figure 9 compares the critical prices at which the reactor should be ordered for the two cases. For the two twin reactors, only the first unit is allowed to be ordered at the optimal time. In the case of the four separate units, we see that the critical threshold prices for ordering the subsequent units are almost the same. Thus once the learning from the first reactor is finished, the subsequent reactors can be ordered almost simultaneously, which results in cashflows coming much quicker.

Volatility %	Critical Price cents/kWh	Real Option Value (Euro/kW)
Two twin units at single site		
10	2.95	286.79
20	3.65	337.90
30	4.51	416.86
Four independent units		
10	3.17	228.03
20	3.67	339.89
30	4.27	457.83

Table 5: Option value (Euro/kW) for the twin units, the independent units for different volatilities in electricity price. The decision horizon for the first unit (for the case of twin units) is taken as 7 years, for four independent reactors the decision horizon for the first unit is 5 years and the remaining units is 10 years. The option value is computed when the electricity price is 3 cents/kWh.

In table 5 we summarize the results for the two test cases discussed here. It can be seen that although setting up two twin reactors at a single site significantly reduces the costs of producing electricity for the units, the project loses on the value of flexibility. The four units constructed on different sites, although produces electricity at higher prices, can benefit from the possibility of being constructed at more optimal market electricity prices. It should also be noted that the construction of the units in this manner may also benefit from the point of view that in an event of natural disaster not all units would shut down (see Takashima, Yagi (2011) [33] for more details). Also it is clear that when uncertainty in electricity price increases, it appears to be advisable to go for flexibility, by constructing independent reactors, rather than increased cost efficiencies, by constructing twin units at the same site. On the other hand, when the electricity price is less uncertain not much is gained by flexibility of time to order and it appears more profitable to go for cheaper twin units.

6.2 Uncertain Life Time of Operation

Uncertain life times of operation for nuclear reactors should be taken into account when computing the value of investment in NPP. A detailed analysis would take into account not just the uncertain life time of operation but also uncertain capacity factors during the operation of the reactor. Du and Parsons (2010) [12], perform a detailed analysis on the capacity factor risk in the nuclear power plants. Rothwell(2005) [28] employs a stochastic process for varying capacity factors in his analysis. We here assume (like Gollier [14]) that the nuclear reactors operate at a mean capacity factor of 90% through their lifetime, which is true for modern reactors.

In our analysis, we assume uncertain life time of operation of nuclear power plants can be due to, premature permanent shut down or extension of operating license (or lifetime).

Premature Permanent Shut-down

We use the term *premature permanent shut-down* for the case when an operating reactor is permanently shut-down before completing its licensed operating life time. Historically premature permanent shut-down of a reactor has been observed for direct reasons- like accidents or serious incidents in a reactor (e.g. Three Mile Island 2- 1979, Chernobyl 4- 1986, Fukushima Daiichi 1,2,3,4 - 2011), or it could be indirect by - for example shutting down of reactors due to increased safety measures, economic reasons, changing government policies etc. (e.g. Shoreham 1989).

The arrival time of such an event (elsewhere called *rare events* or *catastrophic events*) has been modelled by a Poisson process, e.g. Clark (1997) [10] for a real options application with a single source of rare events, Schwartz (2003) [30] uses Poisson arrival times to model catastrophic events when investing in R&D. We also model the arrival time for the cause of premature permanent shut-down as a Poisson process whose arrival frequency, λ, is the expected number of such events every year.

In order to compute the frequency of premature permanent shut-down we use the data available from the IAEA report (2005) and a WNA report. In total there have been 133 reactors that have been permanently shut-down after they started operating. Out of these 11 reactors were shut down due to accidents or serious incidents, and 25 have been shut-down due to political decision or due to regulatory impediments without clear or significant economic or technical justification. The remaining 97 reactors were shut down because they completed their designated lifetime and costs associated with lifetime extension did not make economic sense for these reactors. The total cumulative life time of operation for the reactors in the world is approximately 14500 years. Therefore the number of premature permanent shut downs per reactor year is $(25 + 11)/14500$, which is 0.0025 reactors per year. Thus the rate of arrival of the cause for premature permanent shut down, is $\lambda = 0.0025$ events every year, from a statistical point of view.

Equations 2 and 3 needs to be modified to ensure that the cost and revenue are computed only as long as the plant is operational and should drop down to zero on the occurrence of a catastrophic event. For a Poisson's distribution, the probability that no premature permanent shut down occurs before time t is given by,

$$\mathbb{P}_{N_E(t)=0} = e^{-\lambda t}, \tag{17}$$

where $N_E(t)$ represents number of events until time t. Therefore, cost K_i and revenue, R_i, for a reactor are now given by

$$R_i(X_t = x) = \mathbb{E}\left[\int_{t+C_i}^{t+C_i+L_i} e^{-ru}\left(X_u(\mathbb{P}_{N_E(u)=0}) + 0(\mathbb{P}_{N_E(t)>0})\right)|X_t = x\right],$$

$$K_i = \mathbb{E}\left[\int_{t+C_i}^{t+C_i+L_i} e^{-ru}\left(\theta_i(\mathbb{P}_{N_E(u)=0}) + 0(\mathbb{P}_{N_E(t)>0})\right)du\right].$$

Using equation (17) these can be rewritten as

$$R_i(X_t = x) = \mathbb{E}\left[\int_{t+C_i}^{t+C_i+L_i} e^{-ru}(e^{-\lambda u}X_u)|X_t = x\right]. \tag{18}$$

$$K_i = \mathbb{E}\left[\int_{t+C_i}^{t+C_i+L_i} e^{-ru}(e^{-\lambda u}\theta_i)du\right]. \tag{19}$$

The inclusion of catastrophic events results in effective discount rate from r to $r + \lambda$, once the plant gets operational. Figure 10 compares the option values for different frequencies of such events, when the final decision horizon is 15 years. The value of investment option reduces with increasing probability of catastrophic event, however, the rate of decrease of option value is greater for a single large unit and for an isolated first unit when compared to the modular case.

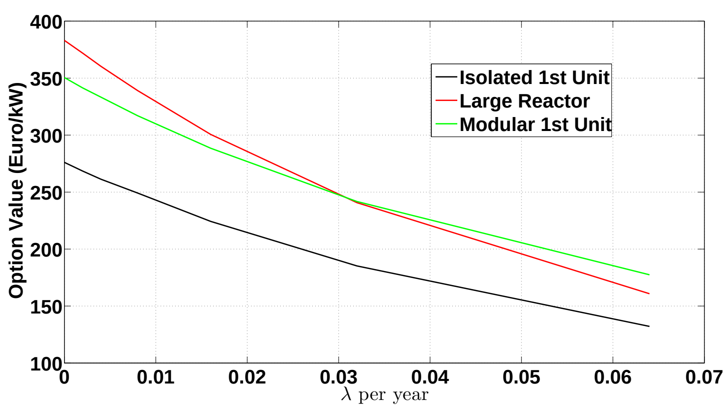


Figure 10: The option price for the large reactor, isolated first unit and modular first unit for different value of λ when the volatility of electricity price is 20% and the decision horizon is 15 years.

Life Time Extension

Most nuclear power plants originally had a nominal design lifetime of 25 to 40 years, but engineering assessments of many plants have established a longer operation time. In the USA over 60 reactors have been granted licence renewals which extend their operating lives from the original 40 to 60 years, and operators of most others are expected to apply for similar extensions. Such licence extensions at about the 30-year mark justify significant capital expenditure for replacement of worn equipment and outdated control systems. In 2010 the German government approved lifetime extension for the countrys 17 nuclear power reactors. However, after Fukushima accident in March 2011, Germany planned a complete phase-out by year 2022, reverting the previous decision. It is clear therefore that the lifetime of a nuclear power plant can be uncertain.

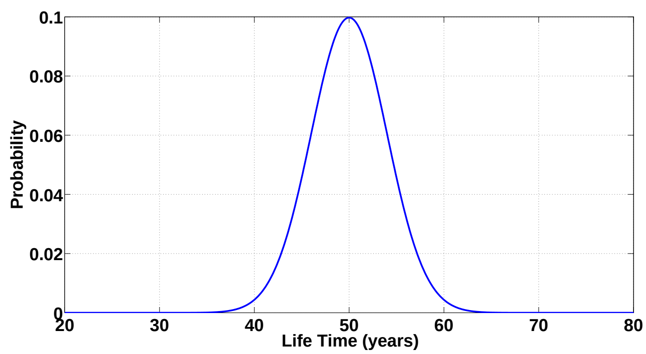


Figure 11: The distribution for the lifetime of operation of a nuclear reactor, which was originally licensed for 40 years of operation.

The Gen III and Gen IV reactors are designed for a life time of 60 years. The first discussion for extension on the lifetime of a reactor to 80 years is ongoing, however, no license as of yet has been issued (need Reference). In order to address the uncertain lifetime of operation due to the possibility of lifetime extension, we use a normal distribution with mean reactor life of $\mu_l = 50$ years and a variance of $\sigma_l = 4$, which fits well to the discussion above, as can be seen in figure 11. It should be noted that such a distribution allows for negative life time, however the probability for such lifetime is almost negligible.

In the case where electricity price follows GBM and the lifetime has a normal distribution as described above, equations (3) and (2) can be written as

$$R_i(X_t = x) = e^{-(r-\alpha)C_i} \frac{1 - e^{-((r-\alpha)\mu_l - \frac{\sigma_l^2 (r-\alpha)^2}{2})}}{r - \alpha} x, \tag{20}$$

and

$$K_i = e^{-rC_i} \frac{1 - e^{-(r\mu_l - \frac{\sigma_l^2 r^2}{2})}}{r} \theta_i. \tag{21}$$

Figure 12 compares the real option value for the first unit of two twin units in a single site with four independent units when the lifetime of the reactors is uncertain. We plot it for various standard deviations σ_l with a mean reactor

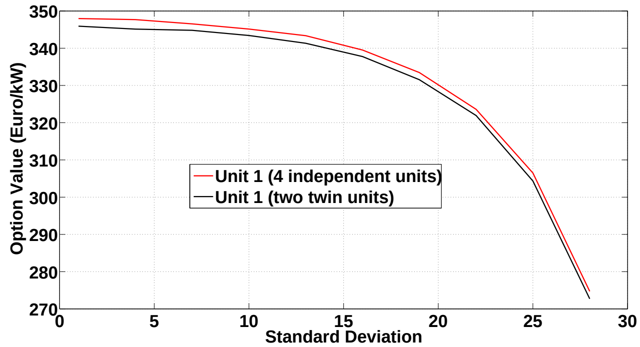


Figure 12: Option value vs uncertainty in lifetime of operation

life of 50 years. It can be seen that with increasing uncertainty in the lifetime the option value reduces, although it follows the same trend for both the cases.

6.3 Economy of Scale vs Modularity

One of the measures identified to reduce capital costs of nuclear power by the NEA report [24] was increased plant size. The savings arising from the economy of scale when the unit size of power plants increases in the 300 MWe to 1300 MWe range have been studied by experts around the world since the early 1960s. The specific costs ($/kWe) of large nuclear power plants have been quoted within such a broad range that the derivation of scaling factors becomes difficult. In addition to savings arising from increased reactor unit size, cost reductions due to other factors such as construction of several units at the same site, effects of replication and series construction, learning effects need to be incorporated in the analysis as well. In this test case we consider two projects, one with a single large reactor which benefits from the economy of scale considerations, while the other project consists of a series of four SMRs which benefits from learning and site sharing costs. Moreover, the modular units benefit from flexibility to order the reactors at optimal times.

For many years, bigger has been better in the utility industry. Economies of scale have for some time, and in many cases, reduced the real cost of power production. The economy of scales can be expressed by the following scaling function, which relates the effect of changing unit size to the cost of the unit.

$$\frac{TC_1}{TC_0} = \left(\frac{S_1}{S_0}\right)^{\gamma}, \tag{22}$$

where TC_0, TC_1 are the total cost for construction of two reactors with size S_0, S_1, respectively, γ is the scaling factor which is usually in the range of 0.4 to 0.7. It is assumed here that the two reactors differ only in size, with *other details being equal.*

Modular SMRs benefit from learning economies which result from the replicated supply of SMR component by suppliers and from the replicated construction and operation of SMR units by the utilities and their contractors. The effect of a learning curve and the associated cost reduction for nuclear technology has been studied in detail by Zimmerman (1982). More recently, Carelli et.

al (2010) and Boarin and Ricotti (2011) discuss empirical methods for learning economies. The latter report that the cost savings for a new unit among multiple units on the same site can asymptotically reach a value of 14%.

We use the following equation for learning on site with increasing number of units.

$$P_i = P_0((1-a) + ae^{-bi}), \tag{23}$$

where a is the cost-savings factor that is asymptotically achieved with an increasing number of units and b gives the rate of on-site learning. Factor a would depend on the number of units constructed world wide, the amount of R&D efforts put in the technology, etc. Factor b depends on the contractor, the skills of the labour involved, etc. Figure 13 shows the price of subsequent units constructed at the same site for varying values of learning rate b. For increasing value of b, the subsequent reactor converges faster to the final cost efficiency gained by learning.

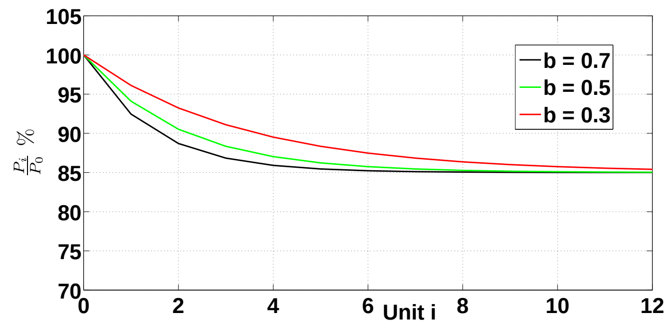


Figure 13: Cost saving factor for different local learning rates

We now consider four modules with size 300 MWe and compare this project with a single unit of size 1200 MWe. The scaling factor for economy of scale is taken to be $\gamma = 0.65$. In order to benefit from local learning, we put a constraint that the construction of a next module can begin only after a year of the start of the construction of the previous module. We take as the rate of local learning $b = 0.8$ and assume that the cost saving for large numbers of modules would approach a value of 25%. With these parameters, the construction costs of the modules can be summarized in table 6.

	Construction Time (months)	Discounted Average cost (cents/KWh)
Modular Units		
Unit 1	3	4.71
Unit 2	2	4.06
Unit 3	2	3.77
Unit 4	2	3.63
Four independent units		
Unit 1	60	2.9

Table 6: Construction time and discounted average cost used for the modular units with learning and single large unit

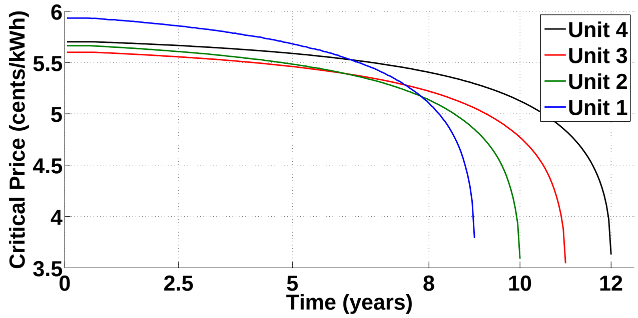


Figure 14: The optimal electricity price at which the modules should be ordered.

The option value and the optimal electricity price at which the reactor should be constructed are given in table 7 and figure 14, respectively. The gain due to the learning curve and the flexibility of construction, although improving the option value for the modular units, does not seem to be sufficient to compensate for the economies of scale factor.

%	Critical Price cents/kWh	Real Option Value (Euro/kW)
Modular Case		
Unit 1	5.93	497.27
Large Reactor		
Unit 1	4.71	811.05

Table 7: Option value (Euro/kW) for the modular case ($4 \times 300 MWe$) , and the large reactor (1200 MWe). The volatility for the electricity price is 20% and decision horizon is 12 years for the modular case and 7 years for the large reactor. The option value is computed when the electricity price is 4 cents/kWh.

7 Conclusion

In this paper we present a valuation method for finding real option value of modular construction for finite decision horizon. We use the method to analyze a few scenarios a utility might be interested in before making a choice of nuclear reactor. The conclusions drawn from the test cases can be summarized as following:

1. In a finite decision horizon, sequential modular SMRs can be ordered at more competitive electricity price, when compared to them being constructed in isolation. Also the real option value of the modules become less sensitive to change in electricity price. However, the option value of the modular SMRs in finite decision horizon would be still higher than that of a single large reactor, if they are constrained to be constructed one after the other

2. When twin units are constructed at the same site, significant cost reductions can be obtained. On the other hand in order to achieve these cost

savings, the utility loses the flexibility to order the units at optimal market conditions. When the electricity price uncertainty is low it appears that cost savings by construction twin modules at the same site would dominate, while when electricity prices are more volatile, the flexibility of choice dominates.

3. Uncertain lifetime of operations reduces the option value of both modular or single large reactor. However, with increasing frequency of premature permanent shutdown, modular construction works out more profitable than single large reactors. Uncertainty in life time extension affects the option value of a single large unit and modular units almost similarly.

4. Specific cost of SMRs can be much higher than a single large unit, because of economies of scale. Some cost reduction is achieved by the learning effect with each new module. However, it appears cost savings due to learning is not enough to make modular SMRs competitive with large units.

We would like to emphasize that the scenarios we have considered are just to demonstrate how using real option valuation in finite decision horizon results into economic decisions much different than obtained from NPV or real option valuation when the utility can wait for infinite time. A more detailed analysis should also include stochastic demand and capacity factors. For example, when the grid to which the reactor is connected can support only limited capacity, large reactors in such case might not be suitable. Also in more realistic cases, especially in case of limited transmission constraint, adding a large reactor can effect the price of electricity by changing demand-supply equilibrium.

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