

# Economics of telecommunications: capital budgeting under competitive markets using strategic game theory and real options approach

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**Abstract** — This paper presents a novel method to evaluate the economic feasibility of Fiber-to-the-Home networks resorting to state-of-the-art techniques. The method presented in this paper is a powerful combination of game theory, advanced capital budgeting algorithms with real options and Monte Carlo simulations with complex statistical distributions to evaluate project risk.

**Index Terms** — FTTH, Game theory, Cournot, von Stackelberg, Real options, Binary trees, Monte Carlo, Beta-Pert.

## I. INTRODUCTION

Every once in while the telecommunications sector experiences certain technological waves that completely changes the way this sector works. Good examples are the appearance of the first mobile phones or the first Internet modems. This so called waves usually require a large amount of investment in new or upgraded infrastructure. The newest wave is in the next generation access networks which includes the forth mobile generation and fiber connections. Both require large amount of investments in the form of infrastructure upgrade in the first case or in the form of new deployed infrastructure in the case of fiber. Such large investments are a huge risk for operators due to the uncertainty carried with the expected profit since even macroeconomic factors can largely affect it. This paper will present a novel approach to study the feasibility of Fiber-to-the-Home (FTTH) deployment using real options as a ground basis. To found the value of the option it is necessary to obtain the volatility of the project. In order to do so a novel approach will be presented using a combination of Monte Carlo method with betaPert distributions and game theory models.

The introduction to the Monte Carlo method will be in section II.

Two models of game theory will be used to obtain the ideal number of users/market share and the ideal price (using statistical data) for a project of Fiber-to-the-Home (FTTH). This will be described in section III.

Final remarks and conclusions will be in section IV.

## II. REAL OPTIONS AND MONTE CARLO SIMULATIONS

In the traditional or classical method (the discounted cash flow, or DCF, which has as mains components the NPV and the IRR), it is assumed the project is managed passively, which is not true, because information (market information,

competitor information, macroeconomic information, etc.) carries uncertainty and varies with time, so it is common for the operator to change the strategy of the project as new information arrives. It can even abandon the project if the present value of the expected cash flows is inferior to the residual value of the project and the DCF method assumes the project will continue to exist in its lifetime, which is not always true. This is the reason why the project evaluation should be made using real options instead of the classical methods (Amram & Kulatilaka, 1999; Brealey & Myers, 1992; Copeland & Antikarov, 2003; Dixit & Pindyck, 1994, 1995; Hayes & Abernathy, 1980; Hayes & Garvin, 1982; Hertz, 1964; Hodder & Riggs, 1985; Kester, 1984; Kogut & Kulatilaka, 1994; Lander & Pinches, 1998; Magee, 1964; S.C. Myers, 1977; S. C. Myers, 1984; Ross, 1978, 1995; Trigeorgis, 1993).

Although there are several different types of real options, the following there are the ones that are extensively used by telecommunications operators when projecting a new network:

1. Abandon the project (discontinue an operation and liquidate the assets) – valued as a put option;
2. Expand the network (increase or decrease the scale of a operation in response to demand) – valued as a call option;
3. Delay the project (wait before taking an action until more is known or timing is expected to be more favourable) – valued as a call option.

These options can be put options or call options. An option is a right to buy or sell a particular good for a limited time at a specified price (*the exercise price*). A call option is the right (not the obligation) to buy.

The value of these options can be evaluated using a binomial tree. Since the value of the project can change with time according to a random walk stochastic process, more formally called geometric Brownian motion (GBM), then the options can be valued with traditional option pricing methods (Brandão, Dyer, & Hahn, 2005; Hull, Prentice Hall 2003). FIG. 1 Illustrates a binomial tree.  $q$  is the probability of an upward movement on the value of the project and  $1-q$  is the probability of a downward movement on that same movement on that same value. If the amount of money required to invest in the project is  $x$ , at the end of one period, the expected value of the project is  $q \times u \times x + (1-q) \times d \times x$ . If instead of investing on the project, the operator invested the money in a risk-free bond, the expected return would be  $x(1+r_f)$ , where  $r_f$  is the risk free

rate (if outside the Euro-zone of the European Union (EU) it is a treasury bill or bond with the expiration date closest to the project; if inside the Euro-zone of the EU it is the treasury bill or bond, of the country with best rating, according to Moody's, Fitch and Standard & Poor's rating agencies, with the expiration date closest to the project). To know how large should the probability  $q$  be in order to make neutral to either invest the money in the bond or in the project, it is necessary to set these two equations equal to one another and solve for  $q$ :

$$\begin{aligned} q \cdot u \cdot x + (1-q)d \cdot x \\ = x(1+r_f) \Leftrightarrow q = \frac{1+r_f-d}{u-d} \end{aligned} \quad (1)$$

$u$  is a number greater than 1 reflecting a proportional increase in the project value, and  $d$  is a number smaller than 1 reflecting a proportional decrease:

$$u = e^{\sigma\sqrt{T}} \quad (2)$$

$$d = \frac{1}{u} \quad (3)$$

Where  $\sigma$  is the volatility, or risk, of the project (obtained using the Monte Carlo Method) and  $T$  is the expected project duration or lifetime.

$S$  is the present value of expected operation cash-flows discounted at the project's cost of capital. Finally, the exercise price of the option,  $X$  is:

- For call options — the initial investment.
- For put options — the value of the project's assets if sold or shifted to a more valuable use.

The value of the option is obtained by computing all the possible scenarios. Taking FIG. 1 as an example, at the end of the project lifetime,  $T=3$ , the following scenarios are computed:

- Up, up, up, resulting in  $Su^3$  with probability  $q^3$ ;
- Up, up, down, resulting in  $Su^2d$  with probability  $q^2(1-q)$ ;
- Up, down, up, resulting in  $Su^2d$  with probability  $q^2(1-q)$ ;
- Down, up, up, resulting in  $Su^2d$  with probability  $q^2(1-q)$ ;
- Up, down, down, resulting in  $Sud^2$  with probability  $q(1-q)^2$ ;
- Down, up, down, resulting in  $Sud^2$  with probability  $q(1-q)^2$ ;
- Down, down, up, resulting in  $Sud^2$  with probability  $q(1-q)^2$ ;
- Down, down, down, resulting in  $Sd^3$  with probability  $(1-q)^3$ ;

To generalize to a binary tree of any dimension there are a total of  $[(T-1)T]+2$  scenarios, and the following outcomes:

$$S \sum_{k=0}^T u^{T-k} d^k \quad (4)$$

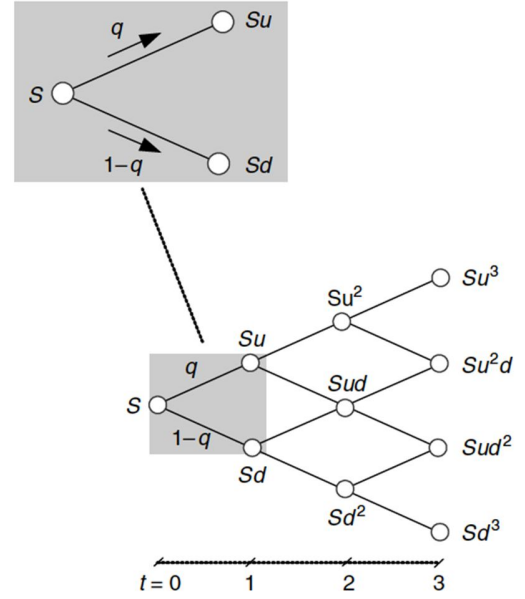


FIG. 1: BINOMIAL TREE

In a put option, the outcome of each scenario is  $\max\{X - Su^i d^j; 0\}$  (for example, in the abandonment option  $X$  is the value of the project's assets and  $Su^i d^j$  is the value of the project, so if the project value is lower than the project's assets the logical thing to do is to abandon the project; so this expression tells us by how many units are the project's assets more valuable than the project itself) and in a call option the outcome of each scenario is  $\max\{Su^i d^j - X; 0\}$  (for example, in the expansion option  $X$  is the investment required to do the expansion and  $Su^i d^j$  is the added value of the project thanks to the expansion; so this expression tells us by how many units is the added value of the project higher than the investment required)

So the value of a call option is given by:

$$f_c = e^{-r_f T} \left[ \begin{aligned} & q^T \times \max\{Su^T - X; 0\} \\ & + (1-q)^T \times \max\{Sd^T - X; 0\} \\ & + \sum_{k=1}^{T-1} (q^{T-k} (1-q)^k \times \max\{Su^{T-k} d^k - X; 0\}) \end{aligned} \right] \quad (5)$$

And the value of a put option is given by:

$$f_p = e^{-r_f T} \left[ \begin{aligned} & q^T \times \max\{X - Su^T; 0\} \\ & + (1-q)^T \times \max\{X - Sd^T; 0\} \\ & + \sum_{k=1}^{T-1} (q^{T-k} (1-q)^k \times \max\{X - Su^{T-k} d^k; 0\}) \end{aligned} \right] \quad (6)$$

To illustrate this, if the binary tree in FIG. 1 was the representation of a put option, its value would be:

$$f = e^{-r_f^3} \left[ \begin{array}{l} q^3 \times \max\{X - Su^3; 0\} \\ + Tq^2(1-q)^1 \times \max\{X - Su^2d^1; 0\} \\ + Tq^1(1-q)^2 \times \max\{X - Su^1d^2; 0\} \\ + (1-q)^3 \times \max\{X - Sd^3; 0\} \end{array} \right]$$

Since there are three options to choose from: one put (abandon the project) and two calls (delay the project and expand the project) the value of the project using real options is:

$$MarketValue = S + f \quad (7)$$

To obtain  $S$  it is assumed that the present value of the project without options is the best unbiased estimator of the market value of the project (the marketed asset disclaimer, or MAD assumption). Under this assumption, the value of the project without options serves as the underlying asset, so  $S = NPV$  (Net Present Value), i.e., the NPV without options.

The volatility, or project risk, is the standard deviation of the total returns of the project, which is obtained using the Monte Carlo Method.

#### A. Monte Carlo Simulation

The Monte Carlo Method is used to manage risk analysis in investment projects. The basic concept is quite simple: the total rate of return of the project is a function of three independent variables, sales, costs and the residual value of the investment:

$$\begin{aligned} \text{Profit} &= \text{Sales} - \text{Costs} \\ &+ (\text{Residual value of investment}) \end{aligned} \quad (8)$$

Each one of these three variables is a function of at least one variable ( $x_1, x_2, \dots, x_i$ ). All of these variables are characterized by a certain degree of uncertainty. For example, the expected market share of the operator can range from  $a\%$  to  $b\%$  ( $0\% \leq a \leq b \leq 100\%$ ), and each possible value within this range has a certain probability of occurrence associated with it (so each of these variables can be described as a probability density function, or PDF). What the Monte Carlo Method does is to randomly choose values of  $x_1, x_2, \dots, x_i$  and with these randomly chosen values computes the total rate of return. Then it repeats this process several times in order to obtain the PDF of the total rate of return.

FIG. 2 (Robert & Casella, Springer 2010) illustrates the Monte Carlo method: for each variable (in a total of nine variables) a certain value of the PDF is a randomly selected,

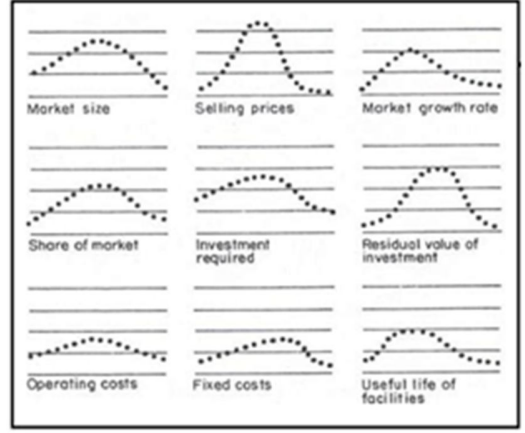


FIG. 2: MONTE CARLO SIMULATION

each of them carrying a certain degree of uncertainty; these values are then applied in the previous equations according to the process described and then this process is repeated several times until there is enough different values of the PDF to form the PDF of the profit function. Finally, the volatility of the PDF will be applied in real options models.

The residual value of the investment is given by (9) and therefor the profit is given by (10):

$$(\text{Residual value of investment}) = \frac{(L-T) \times \text{Costs}_T}{L} \quad (9)$$

$$\text{Profit} = \frac{L \times \text{Sales}_T - T \times \text{Costs}_T}{L} \quad (10)$$

Variable  $T$  is a temporary variable. In a certain moment of time  $T$ , with  $T \in [0; L]$ , the option to abandon the project can be exercised ( $T$  is the useful lifetime of the project, which is equal or less than the expected lifetime of the project,  $L$ ).

If at a certain moment of time  $T$ , the option to abandon the project is exercised (i.e., if  $T < L$ ), then the residual value of the investment is given by (9), otherwise it is zero.

Let  $g, h, i, j$  be vectors unitarily spaced with minimum value equal to one (i.e. each vector will have the form  $\{1, 2, \dots, N\}$ ) representatives of the variables *market size*, *share of market*, *selling prices* and *market growth rate* respectively, whose dimension is given by the number of iterations  $N_g, N_h, N_i, N_j$  used to obtain its PDF. Therefore:

- $g \in [1; N_g]$
- $h \in [1; N_h]$
- $i \in [1; N_i]$
- $j \in [1; N_j]$

The PDF of sales can be divided into two vectors, one for the x-axis,  $x_{\text{Sales}}$ , and one for the y-axis,  $f_{\text{Sales}}$ , both with dimension  $M_S \in [1; N_g \times N_h \times N_i \times N_j]$ . Then a value of each vector  $g, h, i, j$  is randomly selected and the mathematical operation described in (11) is executed with the resulted being stored in the vector  $M_S$  in the position  $\{(g-1) \times N_h + h - 1\} \times N_i + i -$

$1 \times N_j + j$ . Then the procedure is repeated (with a different combination of vectors and values) until all the positions of the vector  $M_S$  are fulfilled.

In the process of building the PDF of the variables, each iteration produces two outputs: one value of the x-axis of the PDF and one value of the y-axis. In (11)  $x(\text{Market size})_g$  represents the output of the iteration number  $g$  relatively to the y-axis of the PDF of the function *market size*.

$$\begin{aligned} x\text{Sales}_{M_S} &= x(\text{Market size})_g \times x(\text{Share of market})_h \\ &\times x(\text{Selling prices})_i \times \sum_{j=0}^{T-1} [1 + x(\text{Market growth rate})_j]^k \end{aligned} \quad (11)$$

To fill the vector  $f\text{Sales}$  the procedure is the same, replacing (11) with (12)

$$\begin{aligned} f\text{Sales}_{M_S} &= f(\text{Market size})_g \times f(\text{Share of market})_h \\ &\times f(\text{Selling prices})_i \times f(\text{Market growth rate})_j \end{aligned} \quad (12)$$

For each  $x$  there is an  $f$ , i.e., the value of the vector  $x\text{Sales}$  in the position  $p$  has a correspondent cross-value in the position  $p$  of the vector  $f\text{Sales}$ .

In what concerns the calculation of the costs, the concept is the same but more simple since there is one less variable. In this case the PDF of the costs can be divided into two vectors, one for the x-axis,  $x\text{Costs}$ , and other for the y-axis,  $f\text{Costs}$ , both with size  $M_C \in [1; N_h \times N_i \times N_j]$ . A value of each vector  $h, i, j$ , is randomly select and the mathematical operation of (13) is executed and the result is stored in the vector  $M_C$  in the position  $[(h-1) \times N_i + i - 1] \times N_j + j$ . Then this operation is repeated (with a different combination of vectors and values) until all the positions of the vector  $M_C$  are fulfilled.

Besides the size of the vector  $M_C$  (and the position of the stored results) the main difference for the sales is in the calculation formula with in this case is:

$$\begin{aligned} x\text{Costs}_{M_C} &= [x(\text{Fixed costs})_h + x(\text{Operatings costs})_i] \\ &\times \sum_{j=0}^{T-1} [1 + x(\text{Market growth rate})_j]^k \end{aligned} \quad (13)$$

Once again, to fulfill the vector  $f\text{Costs}$  the procedure is the same, only replacing (13) with (14)

$$\begin{aligned} f\text{Costs}_{M_C} &= f(\text{Fixed costs})_h \\ &\times f(\text{Operating costs})_i \times f(\text{Market growth rate})_j \end{aligned} \quad (14)$$

The sales like the costs increase or decrease proportionally with the market growth rate as described in (12) e (14).

The PDF of the variable *profit* described in (10) can also be divided into two vector with size  $M \in [1; M_S \times M_C]$ , one for the x-axis e one for the y-axis. Following the same logic:

$$x\text{Profit}_M = \frac{L \times x(\text{Sales})_i - T \times x(\text{Costs})_j}{L} \quad (15)$$

Where  $x(\text{Sales})_i$ , with  $i \in [1; M_S]$  and  $x(\text{Costs})_j$ , with  $j \in [1; M_C]$  representing respectively the value of the vector  $x\text{Sales}$  in the position  $i$  and the value of the vector  $x\text{Costs}$  in the position  $j$ . The values of  $i$  and  $j$  are randomly selected and the result of this equation is stored in the position  $(i-1) \times M_C + j$  of the vector  $M$ .

Using the same reasoning:

$$f\text{Profit}_M = f(\text{Sales})_i \times f(\text{Costs})_j \quad (16)$$

The mean of the PDF is given by:

$$\mu_{\text{Profit}} = \frac{\sum_{k=1}^M [x\text{Profit}(k) \times f\text{Profit}(k)]}{\sum_{k=1}^M f\text{Profit}(k)} \quad (17)$$

And the standard deviation of a PDF is given by:

$$s = \sqrt{\int_a^b (x - \mu_{\text{Profit}})^2 f(x) dx} \quad (18)$$

The shape of the equation  $f(x)$  in the function *profit* is unknown. But since several vectors where used it is possible to obtain the standard deviation in another form:

$$s_{\text{Profit}} = \sqrt{\frac{\mu_{\text{Profit}}}{f\text{Profit} \times x\text{Profit}'} \times f\text{Profit} \times (x\text{Profit} - f\text{Profit} \times x\text{Profit}')^2} \quad (19)$$

Where  $x\text{Profit}'$  represents the the transpose of vector  $x\text{Profit}$ . But (19) gives the absolute value of the standard deviation. In percentage the standard deviation is obtained in the following way:

$$s_{\%} = \frac{\sum_{k=1}^M \left[ \frac{\max \left\{ x\text{Profit}(k) - \min \left\{ \begin{array}{l} x\text{Profit}(\min [x\text{Profit} - \mu_{\text{Profit}}])_h \\ x\text{Profit}(\min [x\text{Profit} - s_{\text{Profit}}])_j \end{array} \right\}, 0 \right\}}{x\text{Profit}(k) - \min \left\{ \begin{array}{l} x\text{Profit}(\min [x\text{Profit} - \mu_{\text{Profit}}])_h \\ x\text{Profit}(\min [x\text{Profit} - s_{\text{Profit}}])_j \end{array} \right\}} \right]}{\sum_{k=1}^M [x\text{Profit}(k) \times f\text{Profit}(k)]} \times \frac{\max \left\{ \max \left\{ \begin{array}{l} x\text{Profit}(\min [x\text{Profit} - \mu_{\text{Profit}}])_h \\ x\text{Profit}(\min [x\text{Profit} - s_{\text{Profit}}])_j \end{array} \right\} - x\text{Profit}(k), 0 \right\}}{x\text{Profit}(k) - \min \left\{ \begin{array}{l} x\text{Profit}(\min [x\text{Profit} - \mu_{\text{Profit}}])_h \\ x\text{Profit}(\min [x\text{Profit} - s_{\text{Profit}}])_j \end{array} \right\}} \right]} \quad (20)$$

This equation can seem complicated but it can be easily explained if it is decomposed. Basically what is done in order to obtain the standard deviation in percentage is:

1. First a new vector equal to vector  $xProfit$ , but with the mean of vector  $xProfit$  being subtracted to all the values in all the positions of the vector (if this value is negative than it is multiplied by -1) is created:

$$NewVector_{\mu} = |xProfit - \mu Profit| \quad (21)$$

2. The index (position within a vector) correspondent to the minimum value of  $NewVector_{\mu}$  if obtained:

$$Index_{\mu} = \min\{NewVector_{\mu}\} \quad (22)$$

3. A new vector equal to vector  $xProfit$ , but with the absolute value of the standard deviation of vector  $xProfit$  being subtracted to all the values in all the positions of the vector (if this value is negative than it is multiplied by -1) is created:

$$NewVector_s = |xProfit - sProfit| \quad (23)$$

4. The index (position within a vector) correspondent to the minimum value of  $NewVector_s$  if obtained:

$$Index_s = \min\{NewVector_s\} \quad (24)$$

5. Finally the following equation is computed:

$$S_{\%} = \frac{\sum_{k=1}^M \varphi \times [xProfit(k) \times fProfit(k)]}{\sum_{k=1}^M [xProfit(k) \times fProfit(k)]} \quad (25)$$

Where  $\varphi$  is equal to one if  $xProfit(k) \in [\min\{xProfit(Index_{\mu}), xProfit(Index_s)\}; \max\{xProfit(Index_{\mu}), xProfit(Index_s)\}]$  and zero otherwise.

Since each variable carry high uncertainty, it is not possible to obtain their density probability functions based on statistical data. In these cases the operator sets the minimum, maximum and most likely (the mode) expectable value that the variable will take. For example, the operator can estimate that their most likely market share will be 40% and that it is not expectable that their market share will be higher than 50% and lower than 35%. In this example the minimum would be 35%, the maximum 50% and the mode 40%.

In these cases, where the operator does not have statistical data to build the probability density function, the distribution used must be a Beta-Pert distribution (Vose, 1996), which has the following density probability function (where  $a$  is the minimum expectable value,  $b$  is the maximum expectable value and  $m$  is the mode, i.e., the most likely value):

$$f(x) = \frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{(b-a)^{\alpha+\beta-1} \int_0^1 t^{\alpha-1}(1-t)^{\beta-1} dt} \quad (26)$$

$$a \leq x \leq b$$

$$\alpha, \beta > 0$$

$$\mu = \frac{a + 4m + b}{6} \quad (27)$$

$$\gamma = \frac{b-a}{6} \quad (28)$$

$$\alpha = \left( \frac{\mu-a}{b-a} \right) \left[ \frac{(\mu-a)(b-\mu)}{\gamma^2} - 1 \right] \quad (29)$$

$$\beta = \left( \frac{b-\mu}{\mu-a} \right) \alpha \quad (30)$$

Since  $a \leq x \leq b$ :

$$x = a + (b-a)R \quad (31)$$

Where  $R$  is a randomly chosen value between 0 and 1.

The output of the Monte Carlo Method is the PDF of the profit function and the PDF of the input variables are given as:

- $x_1$ : Market size
  - $a = 0$
  - $b = ?$
  - $m =$  current total number of broadband subscribers
- $x_2$ : Selling prices
  - $a = 0$
  - $b = ?$
  - $m = ?$
- $x_3$ : Share of market
  - $a = 0$
  - $b = 100\%$
  - $m = ?$
- $x_4$ : Market growth rate
  - $a =$  Minimum growth rate in the last 5 to 10 years
  - $b =$  Maximum growth rate in the last 5 to 10 years
  - $m =$  Average growth rate in the last 5 to 10 years
- $x_5$ : Investment required
  - $a = m \times (1 - r)$ , where  $r$  is the risk that the network is oversized.  $r$  should be determined by the operator in terms of past

experiences, but an acceptable should put  $r \in ]0; 10\%]$

- $b = m \times (1+r)$
- $m = ?$
- $x_6$ : Operating costs
  - Operating expenses (OPEX) includes employees, marketing, office administration, etc. which are defined by the operator's administrative policies in a case-by-case basis, so they cannot be defined by a specific formula.
- $x_7$ : Fixed costs
  - $a = (14)$  with market growth rate minimum
  - $b = (14)$  with market growth rate maximum
  - $m = (14)$  with market growth rate expected
- $x_8$ : Useful life
  - $a = 0$
  - $b = L$
  - $m = L$

This only gives a single point of the distribution, so it is necessary to repeat this process over and over until it is possible to plot the distribution. The total number of required iterations to form the PDF of each input is given by:

$$N = \left( \frac{9\gamma}{\frac{a+b}{2} \varepsilon} \right)^2 \quad (32)$$

Where  $\varepsilon$  is the relative error, which is set by the user of the system (the lower the chosen value, the higher the required computational capacity will be).

To create the PDF of equation (10), all the possible combinations of  $x_1, x_2, \dots, x_i$  must be used to create the output of the system.

Finally, the last process (after finding the unknown variables previously listed as '?' – all of the unknown variables can be found using game theory models<sup>1</sup>) is to calculate the volatility  $\sigma$  of the PDF of (10) and use it in (7).

### III. GAME THEORY

In the telecommunications market, operators are interdependent. The actions taken by an operator are probably going to affect the competitors, which by its turn, can respond aggressively. Game theory allows operators to obtain a broader vision of the market, the competitors and permits them to structure, analyze, and understand strategic scenarios.

In game theory, there are players (operators), situations (e.g. the current market share), possibilities (e.g. entry a market), reactions (e.g. the reaction of an incumbent to the entrance of another operator) and payoffs (the outcomes of following a strategy). Game theory is a mathematical tool that can be used by operators as a decision making auxiliary tool and to help analyze strategic decisions (Ginevičius & Krivka, 2008; Moorthy, 1985; Turocy & Stengel, 2001).

Although there are several game theoretic models, for the purpose of decision-making in market entry strategies, the von Stackelberg model fits perfectly in the telecommunications market when there is an established incumbent with market leadership and the Cournot model when two or more operators enter an empty market simultaneously.

#### A. Cournot Model

Assuming the existence of two operators,  $O_1$  and  $O_2$ .  $O_1$  has  $U_1$  users and  $O_2$  has  $U_2$  users. The price  $P$  charged to the users is a function of the total number of users  $U$  ( $U=U_1+U_2$ ), since the greater the number of users, the lower the price charged. They are related linearly and negatively as:

$$P(U) = A - U \quad (33)$$

Where  $A$  is a fixed constant.  $O_1$  has a marginal cost per user of  $c_1$  and  $O_2$  has a marginal cost per user of  $c_2$ , so the cost for  $O_1$  of serving  $U_1$  users is  $U_1c_1$  and the cost for  $O_2$  of serving  $U_2$  users is  $U_2c_2$ . Therefore, the profit functions of  $O_1$  and  $O_2$  are respectively:

$$\pi_1 = (A - U)U_1 - c_1U_1 \quad (34)$$

$$\pi_2 = (A - U)U_2 - c_2U_2 \quad (35)$$

Since  $U=U_1+U_2$ :

$$\pi_1 = (A - U)U_1 - c_1U_1 \quad (36)$$

$$= AU_1 - U_1^2 - U_1U_2 - c_1U_1$$

$$\pi_2 = (A - U)U_2 - c_2U_2 \quad (37)$$

$$= AU_2 - U_2^2 - U_1U_2 - c_2U_2$$

The reaction function is a curve that shows every optimal production level (the optimal number of subscribers) for every possible production level of the other firm. Each operator's reaction function can be found by differentiating its profit function with respect to output, giving:

<sup>1</sup> Except for the mode  $m$  of variable  $x_5$  which can be found using engineering techniques that go beyond the financial scope of this paper and therefor will be left unknown.

$$\frac{\partial \pi_1}{\partial U_1} = A - 2U_1 - U_2 - c_1 \quad (38)$$

$$\frac{\partial \pi_2}{\partial U_2} = A - 2U_2 - U_1 - c_2 \quad (39)$$

By setting these equations equal to zero (to obtain local maxima):

$$\frac{\partial \pi_1}{\partial U_1} = 0 \Leftrightarrow 4U_1 + 2U_2 = 2A - 2c_1 \quad (40)$$

$$\frac{\partial \pi_2}{\partial U_2} = 0 \Leftrightarrow U_1 + 2U_2 = A - c_2 \quad (41)$$

The Nash solution is obtained:

$$U_{N1} = \frac{A + c_2 - 2c_1}{3} \quad (42)$$

$$U_{N2} = \frac{A + c_1 - 2c_2}{3} \quad (43)$$

And the maximum price that can be afforded by the market is:

$$P(U) = A - U_{N1} - U_{N2} = \frac{A + c_1 + c_2}{3} \quad (44)$$

By following the exact same logic to apply this model to an environment with  $N$  operators, the Nash solution of operator  $i$  is:

$$U_{Ni} = \frac{A + \sum_{k=1}^N c_k - (N+1)c_i}{N+1} \quad (45)$$

And the maximum price that can be afforded by the market with  $N$  operators is:

$$P(U) = A - \sum_{k=1}^N U_{Nk} = \frac{A + \sum_{k=1}^N c_k}{N+1} \quad (46)$$

If the marginal cost is the same for all operators ( $c=c_1=c_2, \dots$ ):

$$P(U) = \frac{A + Nc}{N+1} \quad (47)$$

In this particular case, the market share would be exactly the same for each operator, which makes sense, since the cost of serving the subscribers is the same for all operators, and all operators offer the same exact product with indistinct characteristics, so there is no reason for a subscriber to prefer a certain operator against the other. What usually makes the user prefer a certain operator over the other can be the prestige of the operator, the marketing effect on the user, the uniqueness of the product's characteristics and of course, the price.

### B. von Stackelberg Model

Recalling that:

$$\begin{aligned} \pi_2 &= (A - U)U_2 - c_2U_2 \\ &= AU_2 - U_2^2 - U_1U_2 - c_2U_2 \end{aligned} \quad (48)$$

And that:

$$\frac{\partial \pi_2}{\partial U_2} = A - 2U_2 - U_1 - c_2 = 0 \quad (49)$$

$$\Leftrightarrow U_2 = \frac{A - c_2 - U_1}{2}$$

And also that:

$$\begin{aligned} \pi_1 &= (A - U)U_1 - c_1U_1 \\ &= AU_1 - U_1^2 - U_1U_2 - c_1U_1 \end{aligned} \quad (50)$$

Substituting (17) into (18):

$$\begin{aligned} \pi_1 &= AU_1 - U_1^2 - c_1U_1 \\ &\quad - U_1[(A - c_2 - U_1)/2] \end{aligned} \quad (51)$$

This differentiates with respect to  $U_1$  as:

$$\begin{aligned} \frac{\partial \pi_1}{\partial U_1} &= \frac{A}{2} - U_1 - c_1 + \frac{c_2}{2} = 0 \\ \Leftrightarrow U_1 &= \frac{A - 2c_1 - c_2}{2} \end{aligned} \quad (52)$$

Substituting (52) into (49):

$$U_2 = \frac{A + 2c_1 - 3c_2}{4} \quad (53)$$

Looking at Equations (52) and (53), it can be seen that, if the marginal cost of production for both operators is the same,  $U_2$  will be half of  $U_1$ .

By applying the exact same logic for a market that already has  $N$  operators, the optimal number of users of the  $(N+1)^{\text{th}}$  operator (the entrant) will be:

$$U_{N+1} = \frac{A + \sum_{k=1}^N c_k - (2N + 1)}{2(N + 1)} \quad (54)$$

And the maximum price that can be afforded by the market with  $N$  operators is:

$$P(U) = A - \sum_{k=1}^N U_{Nk} = \frac{A + \sum_{k=1}^N c_k}{N + 1} \quad (55)$$

### C. the constant A

One crucial aspect of these models is the constant  $A$ . One way of obtaining it is through the maximum theoretical price that the market can afford. TABLE I, TABLE II and TABLE III illustrates the current broadband market in ten countries. With this information, it is possible to know the following variables regarding broadband:

$$\begin{aligned} \text{Average broadband price} = & \\ & \left[ \begin{array}{l} (\text{FTTH price}) \times (\text{FTTH penetration}) \\ + (\text{DSL price}) \times (\text{DSL penetration}) \\ + (\text{Cable price}) \times (\text{Cable penetration}) \end{array} \right] \\ & \left[ \begin{array}{l} (\text{FTTH penetration}) \\ + (\text{DSL penetration}) \\ + (\text{Cable penetration}) \end{array} \right] \end{aligned} \quad (56)$$

$$\begin{aligned} \text{Weight of broadband on salary} = & \\ & \frac{\text{Average broadband price}}{\text{GDP per capita} / 12} \times 100 \end{aligned} \quad (57)$$

$$\begin{aligned} \text{Total broadband penetration} = & \\ & (\text{FTTH penetration}) \\ & + (\text{DSL penetration}) \\ & + (\text{Cable penetration}) \end{aligned} \quad (58)$$

And with these variables it is possible to build the data in TABLE IV.

The next step is to plot these values on a graph and to do a linear regression as illustrated in FIG. 3. This indicates that when the price of the fixed broadband connection tends to 3.1375% of the subscriber's salary, the penetration rate will tend to zero. Therefore:

$$\begin{aligned} P(U) = A - U \Rightarrow U = 0 \Leftrightarrow \\ A = \frac{\text{GDP per capita}}{12} \times 3.1375\% \end{aligned} \quad (59)$$

This gives  $A$  in monetary units. But also, when the price tends to zero, the number of subscribers tends to 84.3414% of the potential customers and therefore  $A$  can be given in terms of potential customers as well:

$$\begin{aligned} P(U) = A - U \Rightarrow P = 0 \Leftrightarrow \\ A = \text{Potential customers} \times 84.3414\% \end{aligned} \quad (60)$$

Now all of the unknown variables can be found:

- $x_1$ : Market size
  - $b$  = value of  $A$  in equation (60)
- $x_2$ : Selling prices
  - $b$  = value of  $A$  in equation (59)
  - $m$  = given by (46) or (55)
- $x_3$ : Share of market
  - $m$  = given by (45) or (54) divided by the total number of users

## IV. CONCLUSIONS

There is a wide range of research in any of the fields covered in this paper (real options, game theory and Monte Carlo) but this "step-by-step guide" is unique (since these precise formulas are demonstrated by the first time here) and efficient (all the information required is publicly available in the form of statistical data at OECD and United Nations websites). Also, since the error rate can be set, very accurate results can be expected. This paper demonstrated a powerful combination of "old" techniques working together to help operators evaluate the economic potential of their projected networks.

Another good characteristic is the fact that, although the method explained in this paper was applied to a particular field, it can easily be applied to any oligopoly.

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TABLE I  
FTTH BROADBAND SUBSCRIPTIONS PER 100 INHABITANTS

|               | GDP per capita (€) | FTTH price (€) | Weight of FTTH price on Salary | FTTH penetration | Operator        |
|---------------|--------------------|----------------|--------------------------------|------------------|-----------------|
| Portugal      | 16600              | 65             | 4.698795181%                   | 0.3              | Sonaecom        |
| Spain         | 21300              | 39             | 2.197183099%                   | 0.1              | Nostracom       |
| Australia     | 29800              | 84             | 3.382550336%                   | 0.1              | Internode       |
| United States | 34200              | 76             | 2.666666667%                   | 1.3              | Paxio           |
| Finland       | 25500              | 40             | 1.882352941%                   | 0.2              | Sonera          |
| France        | 24000              | 45             | 2.250000000%                   | 0.1              | Orange          |
| Iceland       | 27700              | 36             | 1.559566787%                   | 2.2              | Rejkavik Energy |
| Switzerland   | 31000              | 85             | 3.290322581%                   | 0.3              | Green.ch        |
| Denmark       | 26700              | 29             | 1.303370787%                   | 4.2              | Jay Net         |
| Netherlands   | 29300              | 52             | 2.129692833%                   | 0.8              | Tele2           |

TABLE II  
DSL BROADBAND SUBSCRIPTIONS PER 100 INHABITANTS

|               | GDP per capita (€) | DSL price (€) | Weight of DSL price on Salary | DSL penetration | Operator        |
|---------------|--------------------|---------------|-------------------------------|-----------------|-----------------|
| Portugal      | 16600              | 30            | 2.168674699%                  | 10.4            | Sonaecom        |
| Spain         | 21300              | 35            | 1.971830986%                  | 17.1            | Nostracom       |
| Australia     | 29800              | 55            | 2.214765101%                  | 19              | Bigpond         |
| United States | 34200              | 36            | 1.263157895%                  | 10.7            | Verizon         |
| Finland       | 25500              | 40            | 1.882352941%                  | 22.2            | Sonera          |
| France        | 24000              | 40            | 2.000000000%                  | 28.7            | Orange          |
| Iceland       | 27700              | 44            | 1.906137184%                  | 30.7            | Rejkavik Energy |
| Switzerland   | 31000              | 41            | 1.587096774%                  | 25.1            | Orange          |
| Denmark       | 26700              | 47            | 2.112359551%                  | 22.4            | Yousee          |
| Netherlands   | 29300              | 35            | 1.433447099%                  | 22.1            | KPN             |

TABLE III  
CABLE BROADBAND SUBSCRIPTIONS PER 100 INHABITANTS

|               | GDP per capita (€) | Cable price (€) | Weight of cable price on Salary | Cable penetration | Operator    |
|---------------|--------------------|-----------------|---------------------------------|-------------------|-------------|
| Portugal      | 16600              | 50              | 3.6144578%                      | 7.2               | TVTEL       |
| Spain         | 21300              | 60              | 3.3802817%                      | 4                 | ONO         |
| Australia     | 29800              | 55              | 2.2147651%                      | 4.2               | Bigpond     |
| United States | 34200              | 71              | 2.4912281%                      | 14.1              | Comcast     |
| Finland       | 25500              | 55              | 2.5882353%                      | 4.2               | Welho       |
| France        | 24000              | 33              | 1.6500000%                      | 1.6               | Numericable |
| Iceland       | 27700              | 0               | 0.0000000%                      | 0                 | -           |
| Switzerland   | 31000              | 66              | 2.5548387%                      | 10                | Cablecom    |
| Denmark       | 26700              | 34              | 1.5280899%                      | 10.1              | Yousee      |
| Netherlands   | 29300              | 60              | 2.4573379%                      | 14.2              | UPC         |

TABLE IV  
BROADBAND SUBSCRIPTIONS PER 100 INHABITANTS

|               | GDP per capita (€) | Average broadband price (€) | Weight of broadband on Salary | Total broadband penetration |
|---------------|--------------------|-----------------------------|-------------------------------|-----------------------------|
| Portugal      | 16600              | 38.63128492                 | 2.792623006%                  | 17.9                        |
| Spain         | 21300              | 39.73584906                 | 2.238639383%                  | 21.2                        |
| Australia     | 29800              | 55.12446352                 | 2.219777054%                  | 23.3                        |
| United States | 34200              | 56.90038314                 | 1.996504672%                  | 26.1                        |
| Finland       | 25500              | 42.36842105                 | 1.993808050%                  | 26.6                        |
| France        | 24000              | 39.64802632                 | 1.982401316%                  | 30.4                        |
| Iceland       | 27700              | 43.46504559                 | 1.882962264%                  | 32.9                        |
| Switzerland   | 31000              | 48.43502825                 | 1.874904319%                  | 35.4                        |
| Denmark       | 26700              | 41.36239782                 | 1.858984172%                  | 36.7                        |
| Netherlands   | 29300              | 44.93530997                 | 1.840353992%                  | 37.1                        |

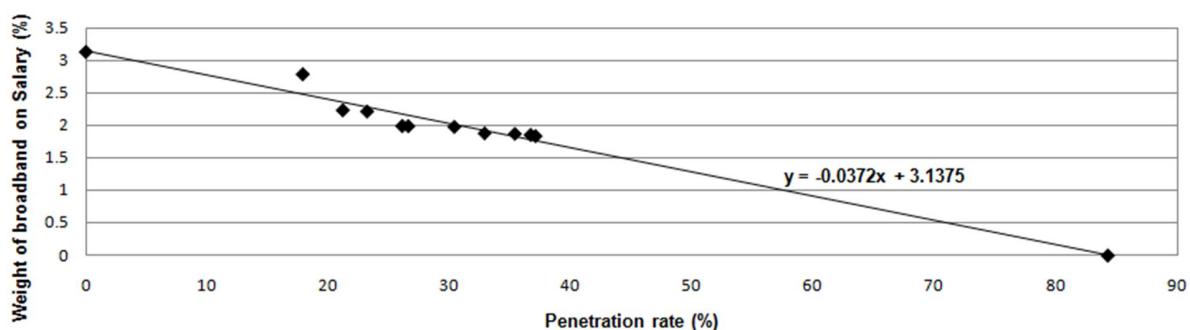


FIG. 3: LINEAR REGRESSION

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