Investment in Electricity Markets: Equilibrium Price and Supply Function

Ryuta Takashima[1*], Yuta Naito[2] , Hiroshi Kimura[3],

Haruki Madarame[3]

[1]*Department of Nuclear Engineering and Management, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan*

[2]*Department of Quantum Engineering and Systems Science, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan*

[3]*Nuclear Professional School, The University of Tokyo, 2-22 Shirane, Shirakata, Tokai-mura, Naka-Gun, Ibaraki-ken 319-1188, Japan*

Abstract

In most real options models for evaluating power plants, it is assumed that the price fluctuates stochastically such as a geometric Brownian motion. In actual electricity market, the prices are determined by supply and demand, and the supply and demand balance may cause the price spike. Especially, the supply characteristics, which depend on the type and number of power plants in electricity market, are important factors for the price determination. In this study, we develop the model allowing for supply curve of exponential function and equilibrium quantity which is assumed to follow a geometric mean-reverting process. Thus the electricity price is determined by the supply function and the equilibrium quantity. In these setting, we examine a construction investment for power plants under uncertainty. The optimal investment timing and investment value for power plants are obtained. We show the dependence of the supply characteristics as well as uncertainty on the investment timing. Furthermore, it turns out that the opportunity of investment decreases when the supply curve is steep.

Keywords: Real options; Supply characteristics; Investment; Power plants

JEL classification: D81; Q41

*Corresponding author
Address: 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8656, Japan
Phone: +81-3-5841-2960; E-mail: takashima@n.t.u-tokyo.ac.jp

1 Introduction

Recently electricity markets in various countries, such as PJM in the United State, Nord Pool in North Europe, UKPX in the United Kingdom, Powernext in France, and JEPX in Japan, have been deregulated. The influence of these electricity deregulation on electric power industry causes difficulty in obtaining a return on investment and operation because electricity prices are determined by supply and demand rather than the management cost.

In this context, the new construction and replacement of power plants are expected due to not only the aging plants but also the exhaustion of fossil fuels, the recent sharp gains in crude oil, and global warming (IEA, 2003). Electric power supplier must make decisions about investment and operation of power plants in competitive electricity markets. Thus, the evaluation and strategy of power plants under uncertainty will be required because of electricity price uncertainty and the influence of competitor.

The real options approach has recently attracted growing attention for the evaluation method of investments projects and investment theory under uncertainty (Dixit and Pindyck, 1994; Trigeorgis, 1996). Many research groups have studied the optimal investment and operation problems of power plants using this approach. Pindyck (1993) analyzes the decision to continue or postpone the building of nuclear power plants under construction cost uncertainty [1]. Tseng and Barz (2002) evaluate the operation of a power plant with unit commitment constraints by using forward-moving Monte Carlo simulation and backward-moving dynamic programming. Deng and Oren (2003) consider the valuation of power plant incorporating operational constraints and start-up costs. Thompson, Davison and Rasmussen (2004) solve for the optimal operation strategies and value of thermal and hydro power plants taking into account the operational characteristics of real power plants. Gollier et al. (2005) make a comparison between the investment projects of a sequence of small nuclear power plants and a large nuclear power plant. Näsäkkälä and Fleten (2005) analyze the decision to upgrade a base load plant to a peak load plant under uncertainty of the spark spread which is a sum of short-term deviations and equilibrium price.

In these studies, the uncertainty of electricity prices is described as stochastic processes, and particularly, Thompson, Davison and Rasmussen (2004) use the price model allowing for periodicity, mean-reverting, and spike jump. However, the optimal management of power plants has not been analyzed from a viewpoint of both price dynamics and the characteristics of electricity supply and demand. As discussed in Lucia and Schwartz (2002) and Geman and Roncoroni (2006), the characteristics of supply is required for the model of electricity price. Additionally, from actual data it was found that the electricity supply function has the properties that the

[1] Pindyck (1993) considers the expected cost to completion rather than construction cost.

price increases exponentially for supplies as discussed in more detail below. It seems that this supply characteristics of electricity causes price spikes. Therefore, it is important to investigate the effect of supply characteristics on the investment rule for power plants.

In this paper, we propose a model for analyzing the investment problems of power plants under uncertainty of electricity prices which are determined by the supply curve expressed as a exponential function and equilibrium quantity assuming to follow the geometric mean-reverting process. In this setting, optimal investment rule and the power plant value are shown. Especially, we demonstrate the effect of not only uncertainty but also the supply characteristics on the investment threshold and value.

The paper is organized as follows. In the next section, we discuss the uncertainty of equilibrium quantity and supply function, and moreover, present the model for evaluating the investment of power plant. In Section 3, we illustrate results of the numerical solution and some comparative statics with regard to uncertainty and supply function.

Finally, Section 4 provides concluding remarks and future works.

2 The model

We consider the investment problems for power plants under electricity price uncertainty. The price is modeled as an stochastic processes taking into account the uncertainty of equilibrium quantity and supply curve, and thus, is determined by the equilibrium quantity corresponding to the inverse demand function and the supply function.

We investigated the shape of supply curve by using actual electricity market data. In Fig. 1, the supply curve in PJM for May 1, 2006 is shown. The supply curve shown in this figure does not have exactly an exponential shape. However, as a whole, the supply curve fits well to the exponential function. The supply function is hence assumed to be expressed by exponential function:

$$P_t = \alpha \left(\mathrm{e}^{\beta Q_t^S} - 1 \right), \tag{2.1}$$

where α and β are constants, and Q_t^S is the quantity supplied. We assume that the quantity demanded follows the geometric mean-reverting process (Dixit and Pindyck, 1994):

$$dQ_t^D = \eta(\bar{Q^D} - Q_t^D)Q_t^D dt + \sigma Q_t^D dW_t, \tag{2.2}$$

where η is the speed of reversion, $\bar{Q^D}$ is the long-run mean quantity demanded, σ is the volatility of quantity demanded, and W_t is a standard Brownian motion. When the quantity supplied and the quantity demanded are just equal to Q at the equilibrium price P, it follows from Eq. (2.1),

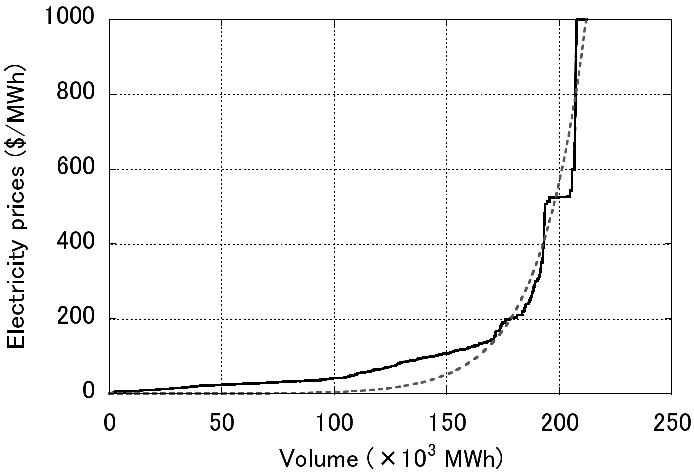

Figure 1: Supply curve in PJM for May 1, 2006 (solid line). This curve is fitted to the exponential function (dashed line).

Eq. (2.2), and Itô's lemma that the dynamics of electricity price P_t is given by,

$$\begin{aligned} dP_t =& (P_t+\alpha)\log\left(\frac{P_t+\alpha}{\alpha}\right)\left\{\frac{\eta}{\beta}\log\left(\frac{\bar{P}+\alpha}{P_t+\alpha}\right)+\frac{\sigma^2}{2}\log\left(\frac{P_t+\alpha}{\alpha}\right)\right\}dt \\ &+\sigma\left(P_t+\alpha\right)\log\left(\frac{P_t+\alpha}{\alpha}\right)dW_t \\ =& a(P_t;\alpha,\beta,\eta,\bar{P},\sigma)dt+b(P_t;\alpha,\sigma)dW_t, \qquad P_0=p, \end{aligned} \tag{2.3}$$

where

$$\bar{P}=\alpha\left(\mathrm{e}^{\beta\bar{Q}}-1\right). \tag{2.4}$$

The firm can start generating electricity by incurring the investment cost I for the construction of a power plant. The expected discounted value of the firm for power plant investment is given by the following equation:

$$\begin{aligned} J(p;\tau) &= \mathbb{E}\left[\int_\tau^{\tau+L} e^{-rt}\left(\gamma P_t - C_\gamma\right)dt - e^{-r\tau}I\right], \\ &= \mathbb{E}\left[e^{-r\tau}\left(\int_\tau^{\tau+L} e^{-r(t-\tau)}\left(\gamma P_t - C_\gamma\right)dt - I\right)\right], \\ &= \mathbb{E}\left[e^{-r\tau}h(P_t)\right], \end{aligned} \tag{2.5}$$

where $\tau \in \mathcal{T}$ is the stopping time for investment, $\mathcal{T}$ is the set of admissible stopping time, L is the life time of power plant, r is the discount rate, γ is the capacity factor, and C_γ is the variable cost for capacity factor of γ. Additionally, $\mathbb{E}\left[h(P_t)\right]$ in Eq. (2.5) can be described by the following equation:

$$
\begin{aligned}
V(P_t) &= \mathbb{E}\left[h(P_t)\right] \\
&= \mathbb{E}\left[\int_t^{t+L} e^{-rs}\left(\gamma P_s - C_\gamma\right) ds - I\right], \\
&= \int_t^{t+L} e^{-r(s-t)}\gamma \mathbb{E}\left[P_s\right] ds - \frac{1-e^{-rL}}{r} C_\gamma - I.
\end{aligned}
\tag{2.6}
$$

Since it is clear from Eq. (2.3) that $\mathbb{E}\left[P_s\right]$ in Eq. (2.6) can not be solved analytically, this value is obtained by numerical procedures such as Monte Carlo method. The construction problems for power plant maximize the expected discounted value by selecting an optimal investment rule:

$$
F(p) = \sup_{\tau \in \mathcal{T}} J(p;\tau) = J(p;\tau^*), \tag{2.7}
$$

where F is the value function of investment, and τ^* is an optimal investment timing. Given the constant threshold of the investment p^*, τ has the following form:

$$
\tau = \inf\left\{t > 0; P_t \geq p*\right\}. \tag{2.8}
$$

The ordinary differential equation, which is satisfied by the power plant value in Eq. (2.7), is derived from Bellman's optimality principle [2]:

$$
\frac{1}{2} b(p;\alpha,\sigma)^2 F'' + a(p;\alpha,\beta,\eta,\bar{P},\sigma)F' - rF = 0. \tag{2.9}
$$

The investment value must satisfy the following boundary conditions:

$$
F(0) = 0, \tag{2.10}
$$

$$
F(p^*) = V(p^*), \tag{2.11}
$$

$$
F'(p^*) = V'(p^*), \tag{2.12}
$$

where condition (2.10) reflect the fact that if P_t is ever zero, the investment value will forever remain at zero, and condition (2.11) and (2.12) are the standard value-matching and smooth-pasting conditions, respectively. Since Eq. (2.9) has no analytical solution, we solve this equation with boundary conditions (2.10) to (2.12) by a numerical calculation method. Consequently, we can obtain the investment threshold p^* and the investment value $F(p)$.

3 Numerical example

In the following, we present the calculation results of the investment option value for power plants, and the dependence of optimal investment rule and value on uncertainty and the supply

[2] See, for example, Dixit and Pindyck (1994).

characteristics. We analyze the power plant investment by categorizing parameters in this model into equilibrium quantity and supply function.

The base case parameters are as follows: $\gamma = 0.8$, $I = 0.3 \times 10^6$ yen/kW, $C_{\gamma=0.8} = 3.5$ yen/kWh, $L = 40$ year, $r = 0.05$, $\eta = 0.2$, $\sigma = 0.2$, $\alpha = 4.0$, $\beta = 2.0$, and $p = 8.0$ yen/kWh [3].

3.1 Expected prices

As described above, the expected value of a power plant investment must be solved by the numerical calculations such as Monte Carlo simulation. Fig. 2 shows the sample paths of electricity prices which are assumed to follow Eq. (2.3), the geometric mean-reverting process, and the geometric Brownian motions. In these simulations for each sample path, the same random number is used. It is clear from this figure that the sample path of Eq. (2.3) is more volatile than that of the geometric mean-reverting process and the geometric Brownian motions.

The expected value of electricity price can be obtained by using these paths of Eq. (2.3). In Fig 3, the expected values of Eq. (2.3) for different initial prices are shown. As can be seen from this figure, each expected value converges to the long-run mean price over time. From these simulations, we can obtain the expected value of power plant investment Eq. (2.3) for various electricity prices, and thus can calculate the option value and the threshold price.

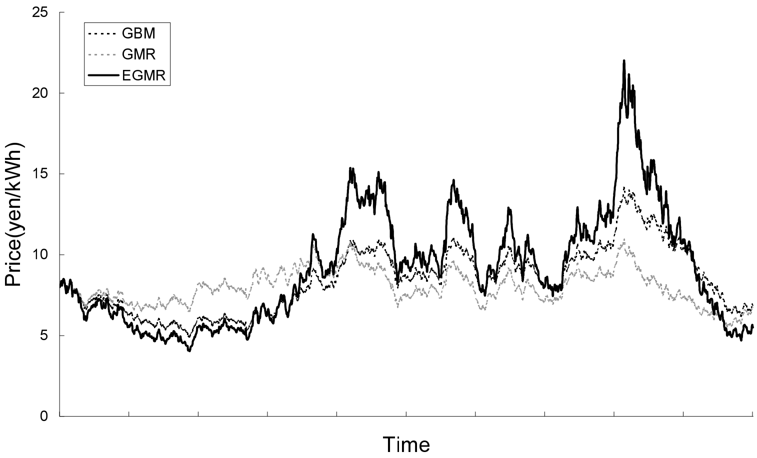

Figure 2: Simulated path of electricity prices assumed to follow Eq. (2.3) (EGMR), the geometric mean-reverting process (GMR), and the geometric Brownian motions (GBM).

[3]The profit flows of $\gamma P_t - C_\gamma$ in Eq. (2.5) include a unit conversion parameter of 8,760 hour/year.

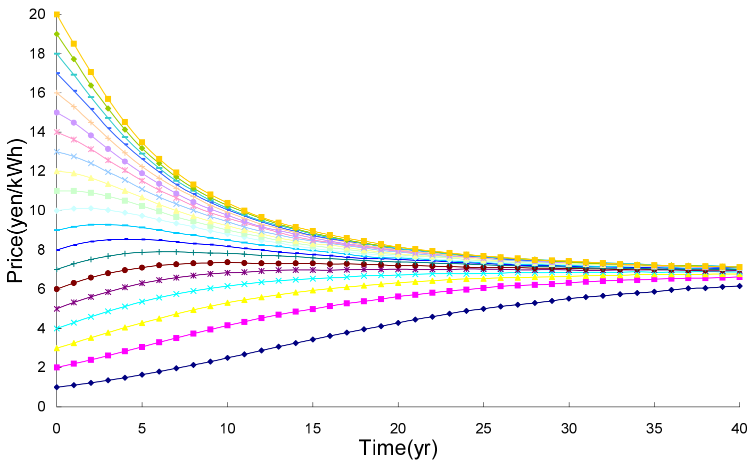

Figure 3: Expected prices obtained by Monte Carlo simulation.

3.2 Volatility

As described above, we consider the construction investment of a hypothetical power plant under electricity price uncertainty taking into account the supply function. Fig. 4 shows the investment option values and the optimal rules, for different volatilities of $\sigma = 0.1$, 0.2, and 0.3. Threshold prices for $\sigma = 0.1$, 0.2, and 0.3 are 9.4, 14.0, and 19.2, respectively. Uncertainty of the price process increases as σ increases, which is clear from $b(P_t)$ in Eq. (2.3). Unlike a standard real options model, although the expected values of power plant investment for each volatility are different functions, it is found that larger uncertainty increases the investment option value, and decreases the investment opportunity.

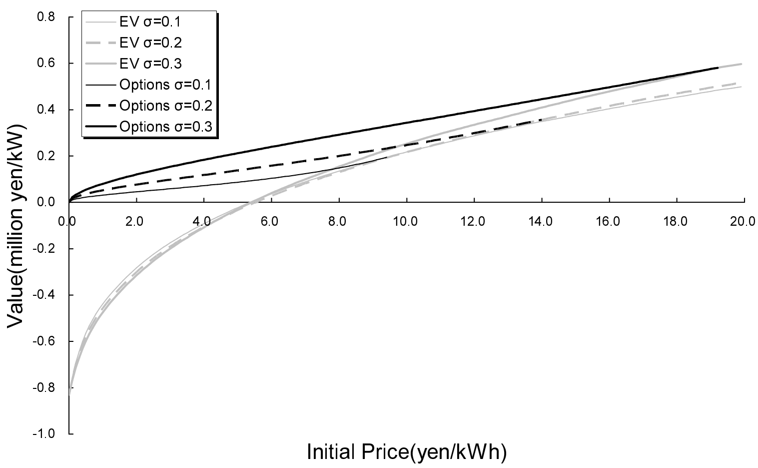

Figure 4: Investment option value of the power plant, for $\sigma = 0.1$, 0.2, and 0.3.

3.3 Supply function

In the previous section, we analyzed the dependence of optimal investment rule on parameters concerning the state variable such as the volatility of electricity price, like earlier research. This section provides some calculation results on the effect of supply characteristics on the investment for power plant.

Table 1 indicates the dependence of threshold price on the slope of supply function, for $\alpha = 2.0$, $\alpha = 4.0$, and $\alpha = 6.0$. We can see that the increase in the slope β induces a decrease in investment opportunity. When the slope β is large, the expectation of the future realization of high price discourages the current decision making of investment. On the contrary, larger α causes increment of investment opportunity because the supply function shift to the upper side.

Fig. 5 presents the dependence of investment value on the slope of supply function, for $\alpha = 2.0$, $\alpha = 4.0$, and $\alpha = 6.0$. This figure shows that the investment value of power plant is increasing with respect to the slope β. From these results, it is shown that the effect of slope of supply function on optimal investment rule is similar to that of volatility. When $p < \bar{P}$,

Table 1: Threshold prices for different values of α and β.

$\alpha \setminus \beta$	0.5	1.0	1.5	2.0	2.5	3.0
2.0	9.613	13.247	13.532	13.850	16.212	16.237
4.0	9.675	12.971	13.367	13.911	14.594	15.030
6.0	9.728	12.671	13.118	13.628	13.960	14.578

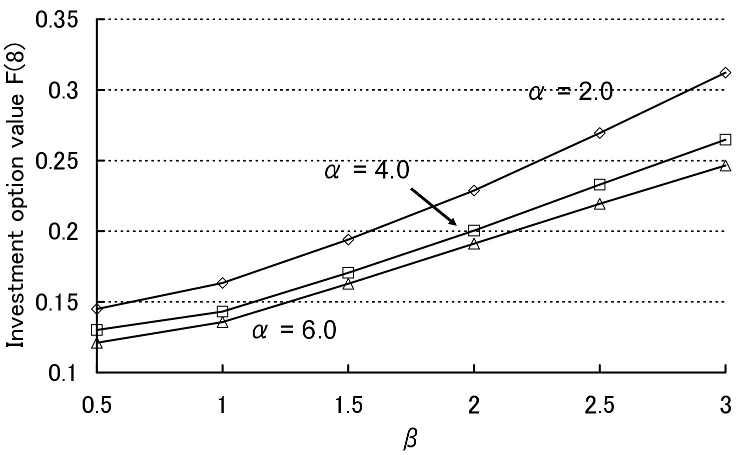

Figure 5: Investment option value as a function of the slope of supply curve, for $\alpha = 2.0$, $\alpha = 4.0$, and $\alpha = 6.0$.

the expected price ($\mathbb{E}[P_s]$ in Eq.(2.6)) with smaller α is always larger than that with larger α. Otherwise, the inverse relationship is obtained. As a result, the expected value with larger α is smaller than that with smaller α when $p < \bar{P}$. This influences the investment value at $p = 8.0 < \bar{P}$. Thus, the investment value decreases as α increases.

4 Conclusions

In this paper, we have developed the model for analyzing power plant investment under uncertainty of electricity prices which are determined by the supply function and equilibrium quantity. The dependence of optimal investment rule on the volatility of electricity price was shown. We also derived the effect of the supply function characteristics on the investment rule and value. From this analysis, it was found that the opportunity of investment decreases if the supply curve is steep. The influence of slope of supply curve on optimal investment rule is similar to that of uncertainty.

Although the supply curves used in this study are the time-independent fixed function, the supply function in actual market changes hourly due to the strategy of power producers as pointed out in Guan et al. (2001). Future work will include the extension of supply function which is dependent on time and firms' decision-making. Additionally, it will be necessary to consider not only the construction investment of power plants but also start-up and shut-down (entry and exit) of those, particularly using non-Gaussian stochastic process such as Lévy process (Deng and Wenjiang, 2005). For example, Boyarchenko and Levendorskiĭ (2000) develop a model of entry and exit problems under price uncertainty which is described as Lévy process. The model presented in this study can represent the volatile behavior of electricity prices from the characteristics of supply curve. We will analyze the start-up and shut-down problems by using the setting in this study.

Acknowledgment

The authors would like to thank Hiroshi Takamori, Motoo Tsujimura, and Makoto Goto for useful comments. This research was partially supported by Collaborative Research Grant, Value Creation Initiative (Sumitomo Corporation), Research into Artifacts, Center for Engineering, The University of Tokyo.

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