Valuing Government Guarantees in Toll Road Projects

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Luiz E. T. Brandão[1] and Eduardo C. G. Saraiva[2]

Abstract

The participation of private capital in public infrastructure investment projects has been sought by many governments who perceive this as a way to overcome budgetary constraints and foster economic growth. For some types of projects, this investment may require government participation in the form of project guarantees in order to reduce the risk of the private investor. As a consequence, the government assumes a contingent liability which may have significant future impact. For this reason, the risk analysis and valuation of these guarantees is important for both the private investor and the government. We present a real options model than can be use to assess the value of these guarantees, allows the government to analyze the cost/benefit of each level of support, and propose alternatives to limit the exposure of the government while still maintaining the benefits to the private investor. This model is then applied to the proposed BR-163 toll road that will link the Brazilian Midwest to the Amazon River. We conclude that a minimum traffic guarantee combined with a cap on the total government outlays for the project offers the best combination of risk reduction for the private investor and liability limits for the government.

Keywords: Real Options, Toll Roads, Government Guarantees, Valuation, Finance

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Introduction

The 1990's was characterized by a worldwide trend towards an increase in participation of private investment in public infrastructure projects in substitution of government investment, the main motivation being the gains in efficiency derived from the substitution of public administration for private enterprises, a better allocation of risk and budgetary constraints of governments. On the other hand, private infrastructure projects are subject to government regulation, cover services deemed essential by society, require large amounts of irreversible capital investment, have long maturity time and are usually offered monopolistically. This combination of factors assures that, once implemented, the interests of the government and that of the private investor begin to diverge, which subjects these projects to pressure from users and opportunist behavior by the government, increasing the risk to the investor. Due to this, private investors may demand that the government provide guarantees that have the effect of reducing these risks, and in doing so, turn the government into a stakeholder in the project.

Government guarantees have been used frequently in private infrastructure projects. The World Bank aided the government of Colombia to structure the El Cortino-El Vino toll road concession where traffic and construction cost guarantees were offered. For the expansion of the gas fired energy plant of Barranquilla, at a cost of $755 million dollars, the Colombian government guaranteed that the state owned Public Utility Company would honor a *take or pay* contract. (Beato, 1997, Lewis and Moody, 1998). The concession of the Santiago-Valparaíso-Viña del Mar toll road in 1998, with 130 km and a cost of $400 million dollars, offered a minimum traffic guarantee at an additional cost to the investor. (Engel, Fisher and Galetovic, 2000). The Linha Amarela expressway in Rio de Janeiro, in 1994, also includes a grant of US$ 112 million, for a total project value of US$ 174 million (Dailami and Klein, 1997).

The presence of the government as mitigator of risk may be a necessary condition since the control of many of the variables that affect important aspects of the project are under its responsibility, such as interest rates, regulation and others, because market risk is such that the project is not feasible from the perspective of the private investor. An example was the bid of the Costanera Norte toll road in Chile in 1998, a urban highway of 30 km connecting the city of Santiago to the airport, in which the government initially refused to offer guarantees deemed necessary by the private investors. Consequently, no bids were forwarded. Only after government supports were included was the road was successfully bid.

On the other hand, by offering guarantees for infrastructure projects, the government becomes responsible for all future liabilities that these supports may cause, and which, because they are determined subjectively in most cases, are not adequately valued or even accounted for in government budgets. This can become very onerous to the government if the risks involved are not adequately analyzed and quantified. The foreign exchange guarantees provided by the Spanish Government in the 1970's and the failure of the Mexican toll road concessions after the 1994 Mexican crisis eventually cost \$2.5 billion and \$8.9 billion respectively to these governments. Thus, the importance of the valuation of government supports is that it allows the government to define a level of guarantee that is high enough for the project to be economically feasible, but low enough not to burden the government and society in excess, and also to determine the value of budgetary and fiscal impacts of future contingent liabilities.

Government supports have option like characteristics, and determining the optimal level of these guarantees requires the use of option pricing methods, which cannot be achieved through traditional project analysis methods. Brandão (2002) applied real option valuation on a model of the Via Dutra highway in Brazil that incorporates the value of options to expand and to abandon. Ng and Björnsson (2004) present arguments in favor of the use of real option approach to the analysis of a toll road concession project. Rose (1998) shows that the value of the Melbourne Central Toll project in Australia increases considerably when the value of the flexibility to increase revenues is considered. Bowe and Lee (2004) analyze the Taiwan High-Speed Rail project where the concessionaire has the option to develop real estate projects along the right of way and show that the value of these options greatly reduces the risk of the project.

On the other hand, the literature on the valuation of government supports is scarce. Charoenpornpattana et.al (2002) analyze a minimum traffic guarantee and shadow toll as a bundle of independent options, but their model uses project cash flows as the underlying asset rather than traffic. Lewis and Mody (1997) and Irwin (2003) mention a World Bank study to value traffic guarantees that were offered in the El Cortijo-El Vino toll road in Colombia using option-pricing methods.

In this paper we propose a model for the valuation of revenue or traffic guarantee in a toll road project, and an estimate of the expected value of the government outlays under these guarantees under various conditions. This paper differs from Charoenpornpattana et.al (2002) in that we model the exercise of the options directly over traffic levels rather than project cash flows in order to more accurately reflect the impact of the government guarantees, and show how multiple sources of uncertainty as well as limits to the government outlays can also be included. Through a real options analysis we determine the value of guarantees which may be offered by the government, their impact of the reduction of risk of the project and the expected value of future government payments as function of the level of the guarantees and limitations these guarantees may be subject to. This allows governments to maximize return to society by designing a bid contract which incorporates the value of these supports.

This work is organized as follows. The first section presents this introduction and a summary of the main topics. In the second section we discuss possible types of government infrastructure concessions projects and their impact on private sector risk. In the third section, we present a real option valuation model for private infrastructure projects, and in the next section we illustrate this with an application to the BR-163 toll road project. In section five we conclude.

2 – Toll Road Concession Models

Toll road concession contracts can be classified according to the degree of risk the private investor is subjected to. In a traditional concession, all market risk are transferred to the concessionaire while the government provides no supports and holds no future liabilities, and this risk is reflected in a higher risk premium for the private capital. This is the most widely used type of concession, prevalent in the Argentina, Brazil, Chile and the United States, and is based on the build, operate and transfer (BOT) model (Bousquet and Fayard (2001), Hammami, Ruhashyankiko and Yehoue (2006)). According to the World Bank (2006), more than 160 such projects totaling 37 billion dollars of concessions were granted in Latin America and the Caribbean between 1990 and 2005. In the United States, 4.000 miles of toll road concessions for the US$ 100 billion dollar Trans Texas Corridor and portions of the highway I-35 are currently under construction or being auctioned (Persad et al., 2004).

This model generally breaks down when the project risk is deemed so high or the returns so uncertain that the government is unable to attract private capital for the project. This typically happens because governments usually grant out concessions of the most profitable projects first, and after this stock is depleted is left with less attractive high risk low return projects. One solution for this problem is to grant some level of government support that reduces the risk and/or increases the returns to the private investor. In Brazil in the XIX century (Summerhil (1998, 2003)), equity guarantees were given in order to foster private investment in transportation infrastructure such as railroads with great success[1]. More recently, in 2004 the Brazilian Congress voted Law 11.079/04, which allows the government to grant supports to infrastructure projects known as Public-Private-Partnerships (PPP).

PPP's have been in use by governments worldwide to increase their global efficiency and due to the lack of investment capital due to budgetary restrictions. Under this model, for example, if the returns of the project are much lower than expected, the project may receive a government subsidy proportional to the reduction in the observed demand, so that a minimum level of return is maintained. Other options may also be present, such as the option to extend (or contract) the concession period, or to postpone payments due to the government. On the other hand, PPPs require a long term commitment of the government to a project, along with the risk of taking on future liabilities that are usually not sufficiently accounted or adequately quantified. The indiscriminate granting of government supports can become a heavy burden to society, because by offering these options the government creates future liabilities and potential responsibilities. Even though they may not bear any impact on current cash flows, government supports may pose a heavy cost for future generations, since the cost of these outlays are rarely taken into consideration or even included into the budgeting process due to the limitations of the traditional valuation methods.

The participation of the government as guarantor of last resort gives it an important role in the implementation of projects that may be technically sound but not economically feasible under the classical model of concession analysis. This objective may be reached by offering guarantees that limit the losses and reduce the risk of the concessionaire in order to allow the implementation and continuity of the project. Government support in PPP contracts can take on many forms, from a simple extension of the concession period to a guarantee of a specific project NPV or a minimum return on the invested capital, representing different levels of risk reduction.

When the government offers minimum revenue of traffic guarantees, it eliminates the most unfavorable states of the distribution of the returns of the project. This fact produces two distinct effects: on one hand, it increases the average return, on the other; it reduces the risk of

[1] These guarantees were also provided to industrial projects. In March 24, 1881, the Imperial Government of Brazil granted a 7% yearly equity guarantee for the construction of a sugar mill in the village of Bracuhy, Rio de Janeiro.

the project by eliminating payoffs below a certain level. Reducing the risk of the project reduces the discount rate at which the cash flows must be discounted, which increases the project value.

Another model that can also be adopted is the Develop, Build, Finance and Operate (DBOF) model used in Great Britain and Portugal, where the government pays out a contractually established yearly revenue stream directly to the concessionaire, and which may or may not involve the collection of tolls from the users. Since in this model the government bears the totality of the market risk, there is no risk to the private investor and a competitive auction for the award of the concession is likely to produce the lowest revenue stream to be paid out, reflecting a significantly lower risk premium for the private capital. On the other hand, in case of an economic downturn the government is obligated to cash outlays at a time where its budget is under greater pressure.

3 – Risk Analysis and Modeling

Toll road projects offer many distinct sources of risk to the investor (Fishbein and Babbar (1996)). Many of these are private, diversifiable risks, which we assume are of less concern to an adequately diversified investor, such as construction risk, or risks which can be hedged away, even if at a cost, such exchange rate risk. On the other hand, the uncertainty over the future levels of demand for traffic on the completed road is of great consequence and constitutes an undiversifiable market risk.

We assume there is a contractual guarantee where the government is obligated to make certain payments to the concessionaire whenever the traffic level (*AADT* – Average Anual Daily Traffic) falls below a pre-established floor during a period of time. If we assume that the toll rate is constant throughout the concession period, then the traffic guarantee is equivalent to a revenue guarantee.

Let R_t be the observed revenue of the project ($R_t = AADT_t$ x Toll Rate) in year t and P_t the minimum revenue guaranteed by the government in that year. Since we assume that the toll rate is constant, the stochastic processes of traffic and revenues will have the same parameters, so we are indifferent whether one or another is used as the underlying asset. In this case, considering the guarantee received, the effective revenue for the concessionaire in year t will be:

$R(t) = \max(R_t, P_t)$

Similarly, the value $G(t)$ of the government guarantee in that year will be:

$$G(t) = \max(0, P_t - R_t) \quad (1)$$

Given the uncertainty about the future level of traffic and revenues, in order to model this variable we consider that the traffic and the revenue vary stochastically in time, following a Geometric Brownian Motion (GBM), as is usual in the literature. This model implies that the revenue can never be negative and that its volatility is constant in time and can be represented as:

$$dR = \alpha R dt + \sigma_R R dz \quad (2)$$

where dR is the incremental change in revenue during a short period of time dt,

α is the revenue growth rate in a short interval of time dt,

σ_R is the volatility of the revenue

$dz = \varepsilon\sqrt{dt}$, where $\varepsilon \sim N(0,1)$ is the standard Wiener process.

It can be shown through an Ito process that this GBM can be represented by the stochastic evolution of the returns, as shown in Equation (3), which can be discretely modeled in yearly periods as a function of the value in the previous period, as shown in Equation (4).

$$d\ln R = \left(\alpha - \frac{\sigma_R^2}{2}\right)dt + \sigma_R dz \quad (3)$$

$$R_{t+1} = R_t e^{(\alpha_t - \frac{\sigma_R^2}{2})\Delta t + \sigma_R \varepsilon \sqrt{\Delta t}} \quad (4)$$

This process can be completely specified considering only its initial value R_0, a yearly growth rate and the volatility of the process, which we assume to be constant during the concession period, where Equation (2) represents the “true” process of the evolution of the project revenues. To value the guarantees, on the other hand, we must use a risk neutral process where we subtract the risk premium from the expected return rate of the underlying asset, substituting its “true” return by the risk free rate of return.

Given that neither the revenues nor the traffic are market assets, we cannot determine the appropriate market risk premium for this source of uncertainty directly from market data. Some authors, such as Irwin (2003) and Dixit and Pindyck (1994) suggest an exogenous solution where an arbitrary value for the risk premium is adopted. We show that the parameters for the risk premium of the revenues can be estimated from the stochastic process of the value of the project.

Let us assume that the revenue process is defined by Equation (2). Given that the revenues represent the only source of project uncertainty, we can define the evolution of the value of the project $V = f(R)$ subject to the same standard Wiener process *dz* where:

$$dV = \mu V dt + \sigma_P V dz \qquad (5)$$

where σ_P is the project volatility

By means of an Itô process, we can define:

$$dV = \underbrace{\left[\frac{\partial V}{\partial R} \alpha R + \frac{\partial V}{\partial t} + \frac{1}{2} \frac{\partial^2 V}{\partial V^2} \sigma_R^2 R^2 \right]}_{\mu V} dt + \underbrace{\frac{\partial V}{\partial R} \sigma_R R}_{\sigma_P V} dz \qquad (6)$$

From CAPM we have $\mu = r + \beta_P (E[R_m] - r)$, where μ and β_P are respectively the risk adjusted discount rate and the Beta of the project. The risk premium of $V(R)$ is then given by $\mu - r = \beta_P (E[R_m] - r)$. As we will see in this section, the risk premium of the project can also be expressed as $\lambda \sigma_P$, therefore we have:

$$\mu - r = \lambda \sigma_P \qquad (7)$$

Substituting Equation (6) into (7) we remain with:

$$\left[\frac{\partial V}{\partial R} \alpha R + \frac{\partial V}{\partial t} + \frac{1}{2} \frac{\partial^2 V}{\partial V^2} \sigma_R^2 R^2 \right] \frac{1}{V} - r = \lambda \left[\frac{\partial V}{\partial R} \sigma_R R \right] \frac{1}{V} \qquad \text{and}$$

$$\frac{\partial V}{\partial R} R \left(\alpha - \lambda \sigma_R \right) + \frac{\partial V}{\partial t} + \frac{1}{2} \frac{\partial^2 V}{\partial V^2} \sigma_R^2 R^2 - rV = 0 \qquad (8)$$

Equation (8) is the differential equation that the value of a project subject to revenue risk must conform to. With this equation we can then determine the value of options on revenues or project value, as long as we use a risk neutral process for the project revenues, with a drift rate of $\alpha - \lambda \sigma_R$ instead of α. Under the assumption that the value of the project without options is the best unbiased estimate of its market value, from CAPM we can determine the risk premium of the project cash flows. If μ is the expected rate of return of the project and β_P is its Beta, then

$\mu = r + \beta_P \left(E\left[R_m \right] - r \right)$ and the project risk premium will be $\mu - r = \beta_P \left(E\left[R_m \right] - r \right)$. Similarly, the risk premium of the revenues is given by

$$\alpha - r = \beta_R \left(E\left[R_m \right] - r \right) \tag{9}$$

We define the market price of risk λ_R as $\lambda_R = \dfrac{\alpha - r}{\sigma_R}$ (10)

Substituting (10) and the value of $\beta_R = \dfrac{\sigma_{m,R}}{\sigma_m^2}$ into Equation (9), multiplying both sides by $\left(\dfrac{\sigma_R}{\sigma_R} \right)$ and re-arranging, we obtain $\lambda_R \sigma_R = \underbrace{\left(\dfrac{\sigma_{m,R}}{\sigma_m \sigma_R} \right)}_{\rho_R} \left(\dfrac{E\left[R_m \right] - r}{\sigma_m} \right) \sigma_R$, where ρ_R represents the correlation between the change in revenues and the market returns.

Finally, we remain with

$$\lambda_R = \rho_R \left(\frac{E\left[R_m \right] - r}{\sigma_m} \right) \tag{11}$$

In a similar way, the market price of risk λ_P of the project will be

$$\lambda_P = \rho_P \left[\frac{E\left[R_m \right] - r}{\sigma_m} \right] \tag{12}$$

where ρ_P represents the correlation between the project returns and the market.

Given that we assume that the only source of uncertainty of the project are its revenues, the correlation ρ_R between the changes in revenues and the market returns will be identical to the correlation ρ_P between the project returns and the market, which implies that (11) = (12), and $\lambda_R = \lambda_P = \lambda$. From (9) and (10) we can then obtain $\lambda \sigma_R = \beta_R \left(E\left[R_m \right] - r \right)$, which defines the risk premium of the revenues. In a similar fashion we can also obtain

$$\lambda \sigma_P = \beta_P \left(E\left[R_m \right] - r \right) \tag{13}$$

Since the value of β_R is unknown, we multiply both sides of equation (13) by σ_R / σ_P and remain with Equation (14), which is the expression for the risk premium of revenues as a function of the risk premium and volatility of the project and the volatility of the revenues, all of which are known constants.

$$\lambda\sigma_R = \beta_P\left(E\left[R_m\right] - r\right)\frac{\sigma_R}{\sigma_P} \qquad (14)$$

The risk neutral process of revenues is then:

$$dR = (\alpha - \lambda\sigma_R)Rdt + \sigma_R Rdz \qquad (15)$$

where $\lambda\sigma_R$ is the risk premium of revenues previously determined in (14). We refer the reader to Hull (2006) for a more extensive analysis of this property.

The uncertainty over future levels of traffic and revenues is one of the key parameters of the model. For existing roadways, the volatility of the revenues can be observed from historical series of traffic levels. For new roadways, this volatility can be estimated if we assume that traffic levels and regional GDP are correlated. The project volatility can be determined from a Monte Carlo simulation of the stochastic cash flows of the project. Due to the leverage effect of project fixed costs, project volatility tends to be greater than traffic/revenue volatility, which reduces the risk premium of revenues.

4 – Application

The Brazilian Army Corps of Engineers built the BR-163 in 1973 as a simple two lane road with wooden bridges crossing the Amazon rainforest in the South-North direction up to the Amazon River. To this day half of the extension of approximately 1,000 miles between Cuiabá, MT and Santarém, PA still remains a dirt road which is closed to traffic for several months during the rain season, and the remainder of the road is in poor condition. A significant portion of the traffic is expected to come from soybean production directed for export to world markets. Currently, one third of the Brazilian soybean crop[2] is produced in the region and travels 1,500 miles down the BR-163 and other roads to the seaports of Santos and Paranaguá. With the new road, it is expected that traffic flow will be reversed upwards towards the port of Santarém in the Amazon River, cutting down the average distance to a third.

Future traffic is difficult to estimate, since changes in commodity prices and exchange rates can affect the expected traffic. Although the road is expected to foster development in the region and increased traffic, this is far from guaranteed, so there is considerable market risk. In May 2005, the government tried to auction the road as a traditional concession but there were no bidders, and one of the alternatives currently under consideration is a PPP with some form of government supports.

Figure 1 – BR-163 highway

[2] Brazil is the world's largest soybean producer with a crop of 53,9 million metric tons in 2006.

We model the effects of a minimum traffic guarantee in order to determine the optimal level of this guarantee and its cost to the government. This guarantee provides the concessionaire the recourse to the government to receive compensatory payments whenever the observed traffic and revenue is below a pre-established level. The traffic projections data used in this paper (Appendix I) are official government estimates and are available at www.tranportes.gov.br.

We assumed that concession year 0 is calendar year 2007, that the construction and pavement of the roadway will last three years and that the first operational revenues will occur in year 2, which corresponds to calendar year 2009. There will be no toll collection in year 1, and in years 2 and 3 tolls will be collected only in the four toll plazas where construction work on the road has been completed, representing 28% of the total flow of vehicles in these two years. From the third year on the road is assumed to be completed and full toll revenues begin to be received. The basic toll rate for a standard automobile adopted in this analysis is R$ 7.60 (approximately US$ 3.50 at the current 2007 exchange rate) at each of the 13 toll plazas which are spread out at approximately 120 km (80 miles) intervals. This represents a rate of R$ 0.06 per km (US$ 0.045 per mile), which is slightly below the current average of Brazilian toll roads. The time frame is the full concession period of 25 years, so the starting year is 2007 (year 0) e the ending year is 2032 (year 25). We also assumed that the private investor cost of capital is 16% per year.

Project Model

Appendix II and III show respectively the investment and operating expenses and the static cash flow of the concession. The initial investment is R$ 966,7 million (USD $1 = R$ 2.20) distributed along the first three years, and considering a debt level of 60%, traditional DCF provides a NPV of R$ 139,8 million. These results indicate that while the project apparently is economically feasible given that its NPV is positive, the result is not sufficient for the concessionaire to undertake the project due to the difficulty of assessing the actual risks involved.

Given that there is no relevant historical traffic data for the road, the volatility of the future traffic demand was estimated assuming a correlation with the regional GDP. Based on data from IPEA[3], a government agency for economic analysis, the volatility of Brazil's Midwest GDP from 1980 to 2002 was 6.9% per year in average, and 7.0% between 1990 and 2002. We assumed a traffic volatility of 7% per year and an initial level of traffic of 106,894 Equivalent Daily Vehicles (EDV)[4] for the year 2007 for all thirteen toll plazas of the roadway. Given that this initial traffic volume is also uncertain, we assumed a triangular probability distribution around this value with a minimum of 74,826 and a maximum of 138,962 EDV, corresponding to a variation of ± 30%. (Figure 2).

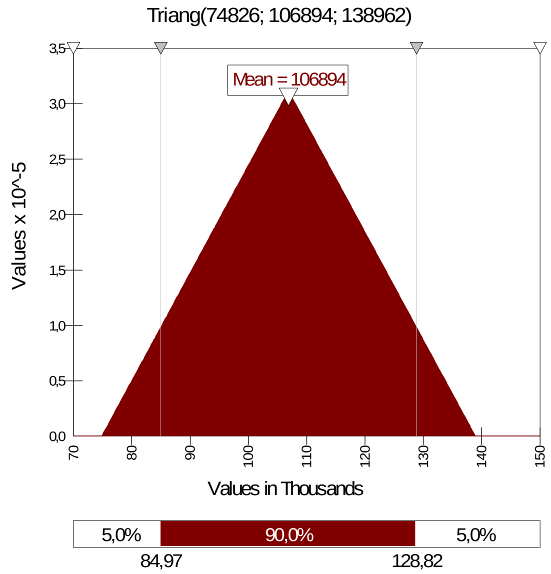

Figure 2 – Distribution of Initial Demand for Traffic

The risk analysis of the project performed through a Monte Carlo simulation considering the uncertainty over both the initial traffic level and its future evolution indicates that the project NPV, which has an expected value of R$ 139.8 million has a relatively high standard deviation of R$ 193.3 million. There is also a 24.8% probability that the project NPV will be negative, as illustrated by Figure 3.

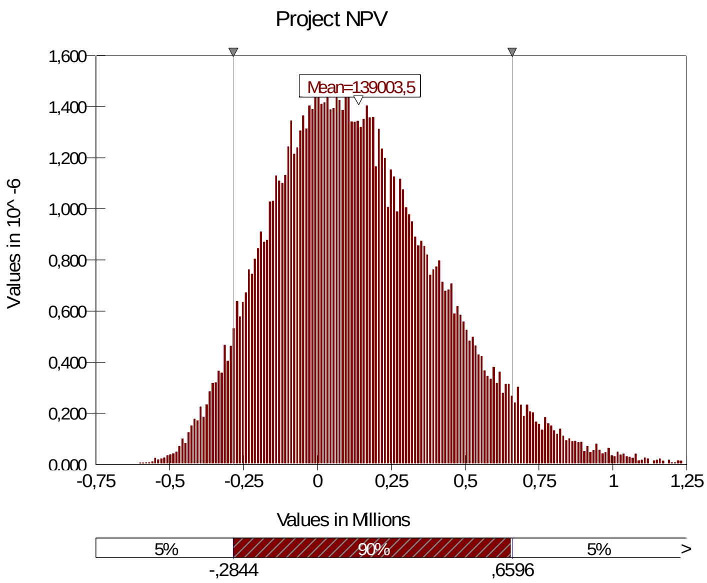

Figure 3 - Distribution of the Project NPV

[3] www.ipeadata.gov.br

[4] Equivalent to a standard two axel automobile

This analysis does not incorporate the value or the impacts on the project of any form of government supports that could be offered to make it more attractive to private investors. As shown before, in this concession model, the private investor holds all the project risk, which is considerable, and the cost to the government is zero. Therefore, the private investor will require a higher risk premium and consequently, a higher toll rate.

Valuation of Guarantees

The valuation of the government guarantees can be modeled as a series of independent European options with maturities between 1 and 25 years. While in principle, these options can be valued directly with the Black and Scholes equation, given that the traffic growth rate is non constant, we chose to use simulation methods. Given the risk neutral process of the revenues defined in (15), the value of the guarantee options can be determined by simulating different future scenarios considering the possibility of exercising the option whenever the revenue value falls below the minimum revenue. This option value is then discounted at the risk free rate. The value of the concession with the revenue guarantee can then be obtained by simply repeating this analysis for each of the 25 years of the concession and adding the present value of all these options to the static value of the project, as shown in Equation (16).

$$\text{Value of Guarantee} = \sum_{i=1}^{25} \text{Value of Option}_i \qquad (16)$$

The volatility of the project is determined by a simulation of the stochastic project cash flow adopting the criteria proposed by Brandão, Dyer and Hahn (2005b). The results indicate a volatility of 47,8%. Assuming a risk free rate of 7%, the risk premium of the project cash flows is can be determine from $\mu - r = \beta_C \left(E\left[R_m\right] - r \right) = 8\%$, and from equation (14) we obtain a value for the risk premium of the revenues (and traffic) of $\lambda = 1,32$. Given the risk neutral process of the revenues defined in (15), we determine the value of the option considering the value of exercise in each year, and the total aggregate value of all options during the concession period at each level of guarantee.

Figure 4 illustrates how the project value changes with each level of guarantee. A contract guarantees that at least 60% of the expected traffic revenue will be received by the investor, for example, increases the project value by R$ 101.9 million dollars, and this value increases as the guarantee level increases. A guarantee level of 80% has a significant impact and doubles the Net Present Value of the project, which shows that the establishment of a revenue floor is an effective way to reduce the risk of projects such as these.

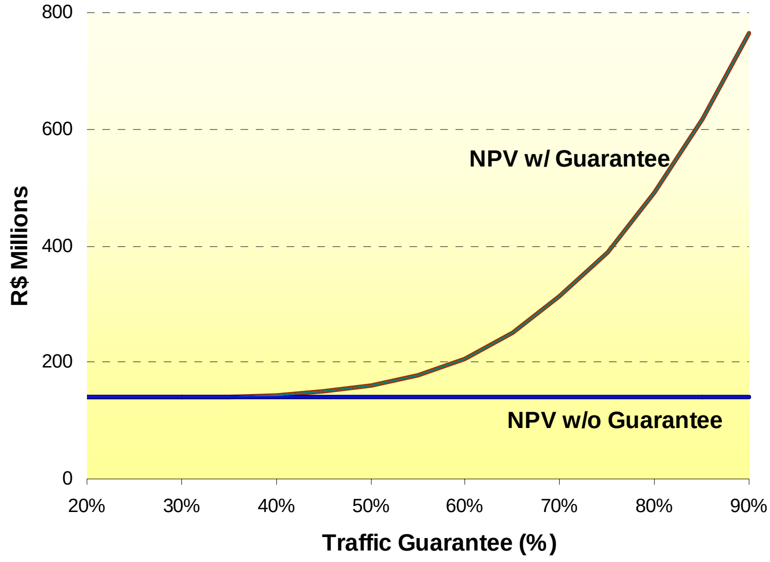

Figure 4 – Project Value at Different Levels of Guarantee

Since the revenue floor protects the investor against low traffic volume, it is only reasonable that the government appropriate revenues significantly in excess of the expected value by establishing a traffic ceiling in order to prevent excessive profits, as shown in Figure 5.

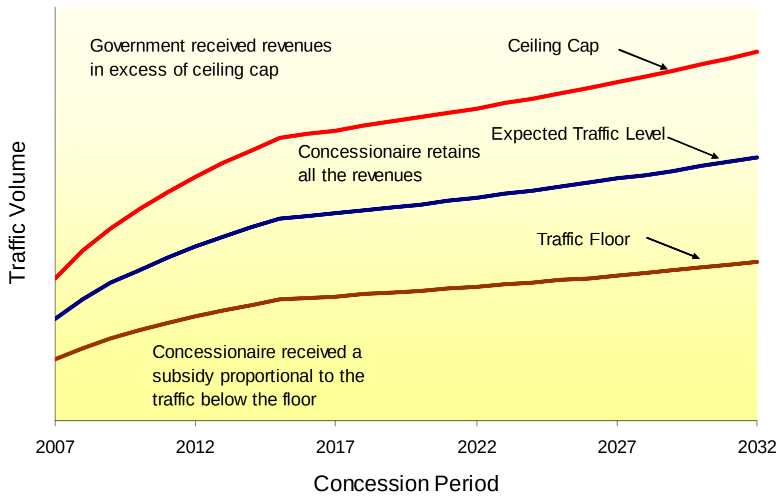

Figure 5 – Floor and Ceiling Guarantee Model

The joint modeling of a traffic floor and ceiling is a case of compound options, where distinct options can be exercised over the same underlying asset. Even though they are mutually exclusive, they exist simultaneously and must be modeled as such. This can be done by assuming that the actual traffic level will fall in any of three distinct and mutually exclusive regions: below the floor, between the floor and the ceiling or above the ceiling. For sake of simplicity, we assume that the floor and ceiling are symmetrical relative to the expected level of traffic, but other assumptions may also be adopted with ease. In this case, the revenues received by the concessionaire in each period t, assuming that the full excess amount is turned over to the government is given by:

$$R(t) = \min \{ \max (R_t, P_t), T_t \}$$

where R_t is the observed level of revenues,

P_t is the level of revenues of the traffic floor,

T_t is the level of revenues of the traffic ceiling.

Figure 6 illustrates the effect of a traffic ceiling. We can see that the net effect of this limitation is small compared to the increase in project value from the traffic floor. This is because the expected growth rates of demand for traffic beyond the first few years of the concession are relatively small.

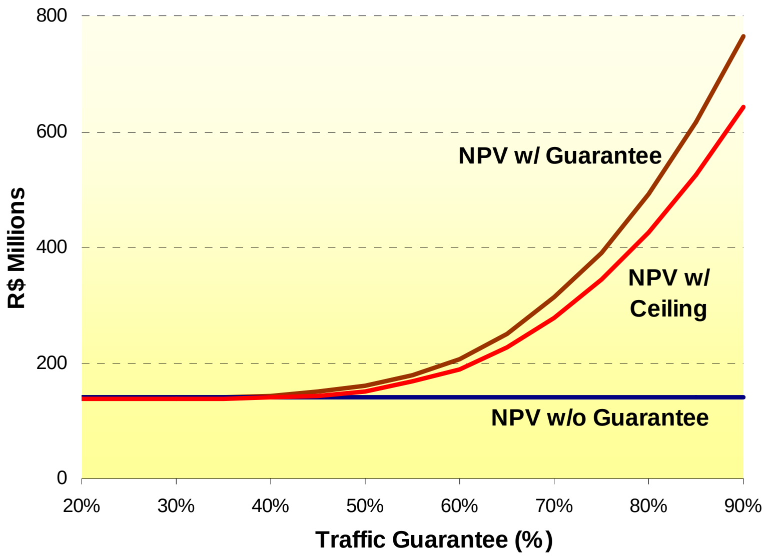

Figure 6 – Effect of a Traffic Ceiling

Effect on Risk

We can also verify the effect that revenue guarantees may have on project risk, by analyzing the changes in the probability distribution of the project NPV. The distribution for the basic concession model where there are no guarantees shown in Figure 3.

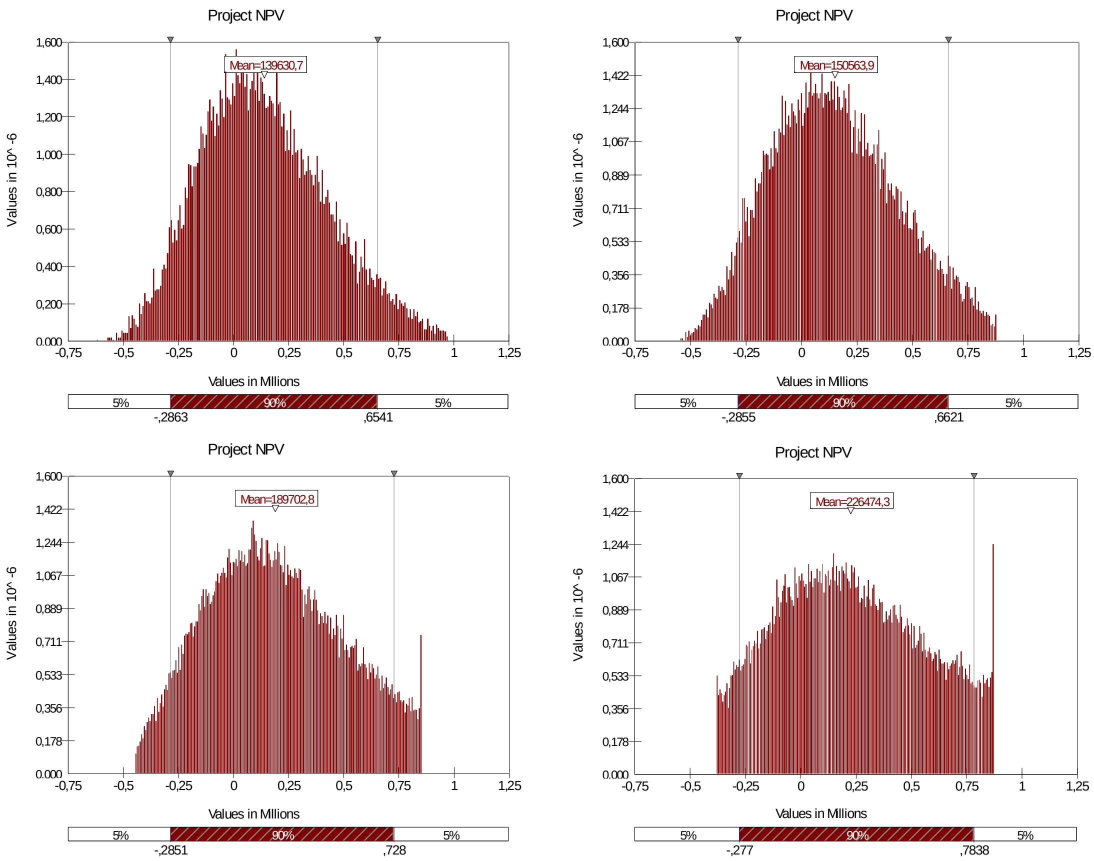

Figure 7 – NPV Distribution for Guarantee levels of 40%, 50%, 60% and 65%

The revenue floor eliminates the probability of occurrence of low NPV values, and as a consequence, increases the expected NPV, while the revenue ceiling affects the project by setting a cap on the probability of the project having very high NPVs. The two opposite project options significantly reduce the variance by increasingly eliminating both tails of the distribution. Figure 7 show the effect of revenue guarantees of 40%, 50%, 60% and 65% on the distribution of the project NPV considering both the revenue floor and ceiling.

As the guarantee levels increase, there is an increase in the project's expected NPV and also a decrease in the dispersion of the results, which indicates a reduction in the project risk. Figure 8 illustrates the effect on project NPV and risk reduction for guarantee levels of 70%, 75%, 80% and 90%. For a guarantee level of 90% the probability of the project having a negative NPV is zero, which implies that a return above the project's hurdle rate is assured. In this sense, if the government chooses to provide such high levels of guarantees it may also require that the private investor significantly reduce its risk premium, or even eliminate it completely and earn the risk free rate of return as the project becomes essentially risk less in this case. It can be noted also that at high guarantee levels, the probability that the NPV will be at the extreme ends of the interval increase significantly.

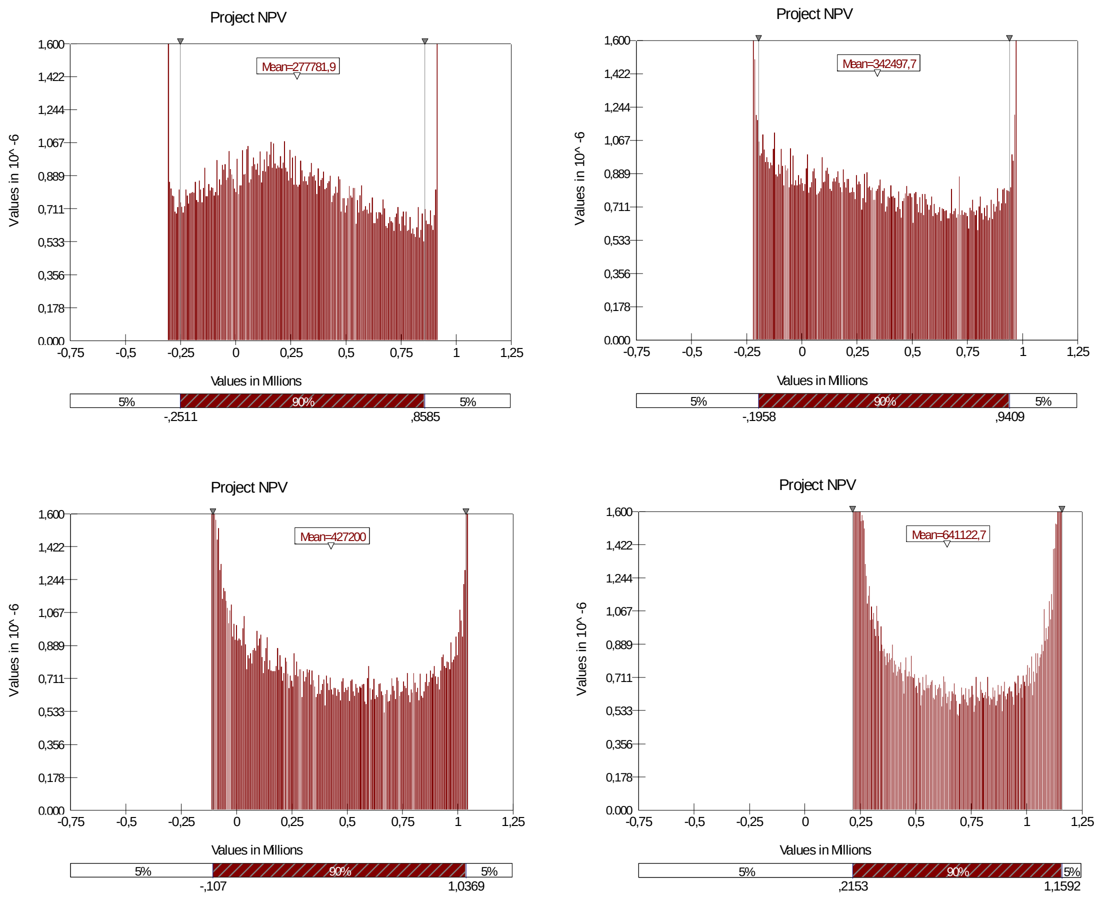

Figure 8 - NPV Distribution for Guarantee levels of 70%, 75%, 80% and 90%

Expected Value of Government Outlays

Based on non arbitrage arguments, it is clear that the expected present value of government outlays is equal to value of these guarantees to the concessionaire, in the amounts we have previously determined. On the other hand, since this is an expected value, there is a 50% probability that the actual payments be greater (or smaller) than this value, and a small probability that it will be significantly higher, which creates a budgetary risk for the government. With a Monte Carlo simulation, we can determine the probability distribution of the expected payments in order to analyze the risk the government incurs of being required to honor larger than expected outlays.

Figure 9 illustrates the probability distribution of a guarantee of 80%. We can observe that although the value of this guarantee is R$ 347,1 millions, there is a 5% probability that the actual government outlays be higher than R$ 1,216 millions.[5]

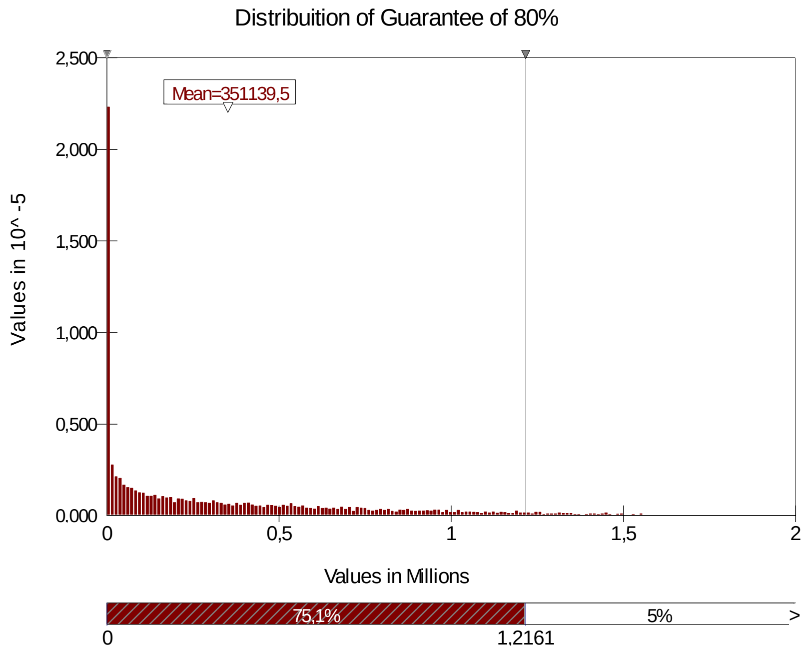

Figure 9 – Approximate Probability Distribution of a Guarantee of 80%

Figure 10 shows the cumulative probability distribution of the guarantee, where we can see that there is a 20,6% probability that the government outlays from this guarantee will be zero. The risk analysis of the guarantees shows that these contingent liabilities must be accounted for taking into consideration the risk such guarantees bring to the government budget.

[5] The probability distribution of the guarantees were determined from a risk neutral stochastic process, and not through the true process of traffic or revenues, so the values shown do not represent the actual probabilities of occurrence as these can only be determined from the true process. In this case this is not possible, since each iteration of the simulation has a distinct discount rate, so it is not possible to determine neither the present value nor the aggregate value of the options with this method. For this reason, we resorted to risk neutral valuation which provides only the risk neutral probabilities. While these are different from the true probabilities, they provide the necessary intuition for the reader to understand that the expected value of the guarantees is only an average and that there is a small probability that significantly higher probabilities may occur.

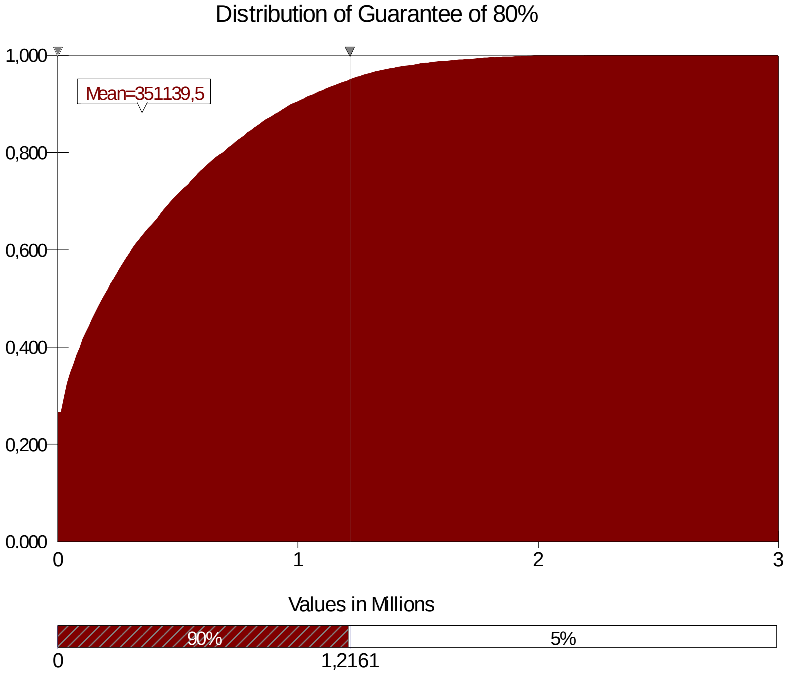

Figure 10 – Cumulative Distribution of a Guarantee of 80%

Traffic Guarantees with Caps

Government exposure can be limited with the use of guarantee caps, where the government outlays cease once a pre-established ceiling is reached. This upper limit only affect the total aggregate value of the options and do not affect the value of each option individually, except for the borderline option. The value of each option in each year is determined as shown previously, but the cumulative sum of all government outlays is limited to the cap limit, as shown in Equation (17).

$$\text{Value of Guarantee} = \min\left\{\sum_{i=1}^{25} \text{Option}_i, Cap\right\} \qquad (17)$$

Considering that the total investment cost of the project is approximately R$ 2,2 millions, for illustration purposes we established two exogenous cap limits of R$ 400 millions e R$ 600 millions, corresponding to approximately 20% and 30% of the project value, respectively. In Figure 11 we see that the impact of these caps is to reduce the value of the guarantees. Because the cap affects only the total outlays of highest value, which are the ones that have the lowest probability of occurring, its effect on the guarantee is limited and in no way cancels out its benefits. This way, it is possible that the cost of the cap relative to the guarantees be reasonably small relative to the benefits derived from the elimination of the uncertainty over the maximum government exposure in the project.

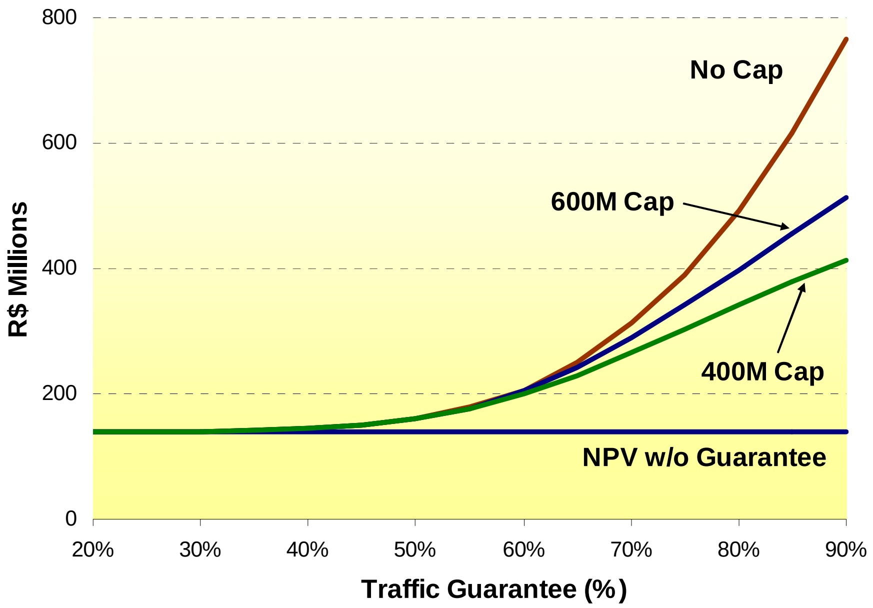

Figure 11 – Value of Guarantees with Caps

Table 1 presents the value of the project for guarantee levels ranging from 0 to 90%, and for different cap limits.

Level of Guarantee	NPV w/o Guarantee	NPV w/ Guarantee	600.000 Cap	400.000 Cap	Traffic Ceiling
0%	139,9	139,9	139,9	139,9	139,3
10%	139,9	139,9	139,9	139,9	139,0
20%	139,9	139,9	139,9	139,9	138,3
30%	139,9	140,1	140,1	140,1	137,6
35%	139,9	141,2	141,2	141,2	137,9
40%	139,9	143,5	143,5	143,5	139,5
45%	139,9	149,6	149,6	149,5	142,5
50%	139,9	159,7	159,7	159,3	150,2
55%	139,9	177,7	177,1	175,3	167,4
60%	139,9	206,5	204,4	198,9	189,2
65%	139,9	249,9	241,6	229,3	226,5
70%	139,9	312,4	290,1	265,9	278,5
75%	139,9	388,8	342,1	303,4	343,0
80%	139,9	492,0	398,6	341,7	426,0
85%	139,9	616,6	454,9	378,1	524,9
90%	139,9	764,9	512,5	414,0	640,9

Values in R$ Millions

Table 1 – Project Value as a function of Guarantees and Cap Levels

Figure 12 and 11 illustrate the approximate probability distribution of a guarantee of 80% considering caps of R$ 600 millions e R$ 400 millions, respectively. The probability that the total outlays of the government be greater than the caps is zero, although the probability that the outlays be equal to the cap are now 23,53% and 34,28% respectively. The probability of the total outlays be zero is still 20,62% as before, for both cases.

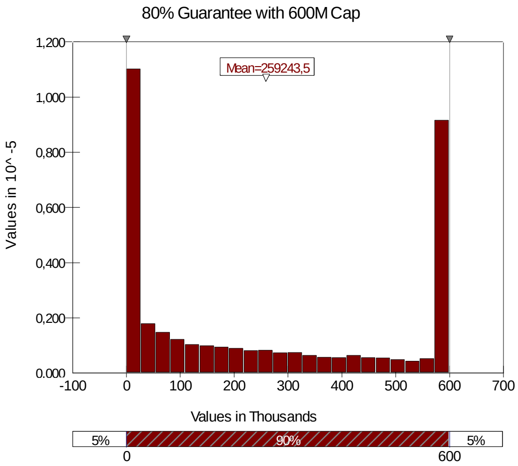

Figure 12 – Distribution of Guarantee of 80% with Cap of R$ 600 million

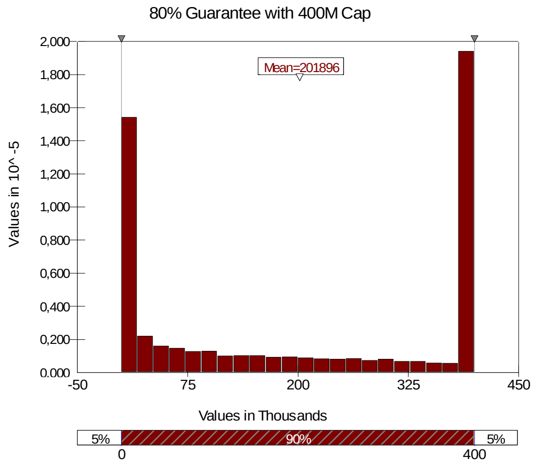

Figure 13 – Distribution of Guarantee of 80% with Cap of R$ 400 million

5 – Conclusion

We analyze the problem of private investment in public infrastructure and concluded that for some classes of projects, it may be necessary for governments to share some of the project risk by granting a level of project supports. One such type of support is a minimum traffic or revenue guarantee, which provides the concessionaire with a government subsidy if traffic falls below a pre-established level. On the other hand, determining the optimal level of these guarantees cannot be done through traditional project evaluation methods and requires the use of option pricing techniques. We show how such a model can be constructed using a real options analysis, and how different levels of support affect both the project risk and its value. We conclude that revenue guarantees can be a viable economic alternative to public infrastructure projects where the risks are such that private partner will not invest otherwise.

The approach we propose in this work can be used by governments to evaluate guarantees being offered in Public Private Partnerships and to calibrate the optimal level of guarantee required to a specific degree of risk reduction. We also analyze the impact that these supports have on government outlays, and conclude that indiscriminate granting of these guarantees can create significant future contingent liabilities for the government. We show that the use of traffic ceilings and caps on the total outlays associated with a particular level of traffic guarantee can help reduce this liability risk, and because they have an asymmetric impact on the value of the project, they may be an acceptable solution to all stakeholders involved. This would allow governments leverage their investment capabilities by redirecting scarce resources away from financing public infrastructure investment to providing a limited level of guarantees, as long as precautions are taken in selecting government project portfolio.

Although we analyze here only the case of a revenue guarantees, the model is flexible and can be easily extended to include other forms of guarantees, such as shadow tolls, exchange rate, debt and equity guarantees, and the Least Present Value of Revenues (LPVR) model suggested by Engel, Fisher and Galetovic (2000).

6 - References

BEATO, P. Road Concessions: Lessons Learned from the Experience of Four Countries. *Infrastructure and Financial Markets Division of the Sustainable Development Department,* Inter-American Development Bank, 1997.

BLACK, F.; SCHOLES, M. The Pricing of Options and Corporate Liabilities, *Journal of Political Economy,* **81** (1973), 637-59.

BOUSQUET, F.; FAYARD, A. (2001), *Road Infrastructure concession Practice in Europe*. Policy Research Working Paper Series *2675*, The World Bank.

BOWE, Michael; LEE, Ding. project evaluation in the presence of multiple embedded real options: Evidence from the Taiwan High-Speed Rail project. *Journal of Asian Economics*, 15 (2004) 71–98.

BRANDÃO, L. Uma aplicação da teoria das opções reais em tempo discreto para a valoração de uma concessão rodoviária. *Tese de Doutorado*, Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, 2002.

BRANDÃO, L; DYER, J.; HAHN, W. Response to Comments on Brandão et al. *Decision Analysis.* Vol. 2, No. 2, June 2005, pp. 103-109.

CHAROENPORNPATTANA S.; MINATO T., NAKAHAMA S. Government Supports as bundle of Real Options in Built-Operate-Transfer Highways projects. *Dissertação de Mestrado.* The University of Tokyo, 2002

COPELAND, T.; ANTIKAROV, V., Real Options. Texere LLC, New York, 2001.

CROCKER, K.; S. MASTEN. Regulation and Administered Contracts Revisited: Lessons from Transaction-Cost Economics for Public Utility Regulation. *Journal of Regulatory Economics* 9:5-40, 1996.

DAILAMI, M.; KLEIN M. Government support to private infrastructure projects in emerging markets. In: *Dealing with Public Risk in Private Infrastructure* (eds Irwin, Klein, M, Perry, G. E. & Thobani, M.), pp.21-42. The World Bank, Washington D.C., 1997.

DIXITt, A., and PINDYCK, R., Investment under Uncertainty, *Princeton University Press*, Princeton, NJ (1994).

ENGEL, E.; FISHER, R.; GALETOVIC, A. The Chilean Infraestructure Concessions Program: Evaluation, Lessons and Prospects for the Future. *Working Paper*, *Centro de Economia Aplicada (CEA),* Departamento de Ingenieria Industrial de Chile, 2000.

FISHBEIN, G.; BABBAR, S. Private financing of toll roads. *RMC Discussion Paper Series 117*, the World Bank, Washington D.C,. 1996.

GÓMEZ-IBÁNEZ, J.A. Mexico´s Private Toll Road Program. Kennedy School of Government Case Program. Case C15-97-1402.0, Harvard, 1997.

HAMMAMI, M.; RUHASHYANKIKO, J. F.; YEHOUE, E. (2006). *Determinants of Public-Private Partnerships in Infrastructure*. IMF Working Paper, IMF Institute.

HULL, J., *Options, Futures and Other Derivatives*, Prentice Hall, New Jersey 2003.

IRWIN, T; KLEIN, M; PERRY, G; THOBANI, M. Managing Governement Exposure do Private Infrastructure Risks. *The World Research Observer*, vol. 14, no. 2 (August 1999), pp. 229-45. IBRD, World Bank, Washington, D.C.

IRWIN, Timothy. Public Money for Private Infrastructure: Deciding When to Offer Guarantees, Output-Based Subsidies, and Other Fiscal Support. *World Bank working paper no 10*. The World Bank, Washington, D.C., July 2003.

KLEIN, B.; CRAWFORD, R.; ALCHIAN, A. Vertical Integration, Appropriable Rents, and the Competitive Contracting Process , *Journal of Law and Economics* 21: 297-326, 1978.

LEWIS, C.; MODY, A. Risk Management Systems Infrastructure Liabilities, in Timothy Irwin, Michael Klein, Guillermo E. Perry, and Mateen Thobani, eds., *Dealing with Public Risk in Private Infrastructure* (Latin American and Caribbean Studies, Washington, D.C.: World Bank, 1998).

NG F.G.; BJÖRNSSON H.C., Using Real Option and Decision Analysis to Evaluate Investments In The Architecture, Construction And Engineering Industry, *Construction Management and Economics,* (June 2004) 22, 471–482

PERSAD K., BANSAL S., MAZUMDAR D., BOMBA M., MACHEMEHL R., Trans Texas Corridor Right of Way Royalty Payment Feasibility, *Report by Center for Transportation Research*, The University of Texas at Austin, 2004.

ROSE, S. Valuation of Interacting Real Options in a Toll Road Infrastructure project. *The Quarterly Review of Economics and Finance,* Vol 38, Special Issue, 1998, pages 711-723

SUMMERHILL, W. (1998). Market Intervention in a Backward Economy: Railway Subsidy in Brazil, 1854-1913. *The Economic History Review*, v. 51, n. 3, pp. 542-568.

SUMMERHILL, W. (2003). Order Against Progress Government, Foreign Investment, and Railroads in Brazil, 1854-1913. *Stanford University Press*.

Appendix I – Traffic Projections

Expected Traffic: Equivalent Vehicles

Calendar Year	Concession Year	Toll Plaza 1	2	3	4	5	6	7	8	9	10	11	12	13	Total
2007	0	12.119	10.644	9.973	8.759	6.924	6.924	6.924	6.924	6.924	6.746	5.416	5.119	2.809	96.205
2008	1	13.964	12.230	11.734	10.535	8.558	8.558	8.558	8.558	8.558	8.323	6.821	6.514	3.768	116.680
2009	2	15.396	13.573	13.092	11.890	9.795	9.795	9.795	9.795	9.795	9.549	7.848	7.526	4.398	132.246
2010	3	16.641	14.729	14.257	13.048	10.840	10.840	10.840	10.840	10.840	10.580	8.703	8.365	4.891	145.411
2011	4	17.760	15.760	15.294	14.081	11.764	11.764	11.764	11.764	11.764	11.494	9.453	9.099	5.312	157.071
2012	5	18.792	16.709	16.244	15.016	12.600	12.600	12.600	12.600	12.600	12.317	10.126	9.758	5.683	167.644
2013	6	19.753	17.590	17.123	15.876	13.366	13.366	13.366	13.366	13.366	13.072	10.742	10.355	6.019	177.359
2014	7	20.660	18.414	17.947	16.677	14.076	14.076	14.076	14.076	14.076	13.768	11.309	10.907	6.326	186.388
2015	8	21.522	19.197	18.720	17.431	14.739	14.739	14.739	14.739	14.739	14.420	11.839	11.420	6.615	194.860
2016	9	22.001	19.484	18.979	17.626	14.916	14.916	14.916	14.916	14.916	14.581	12.002	11.563	6.779	197.593
2017	10	22.485	19.780	19.245	17.826	15.098	15.098	15.098	15.098	15.098	14.745	12.172	11.712	6.947	200.399
2018	11	22.968	20.083	19.519	18.037	15.285	15.285	15.285	15.285	15.285	14.918	12.344	11.864	7.124	203.280
2019	12	23.459	20.397	19.803	18.252	15.479	15.479	15.479	15.479	15.479	15.094	12.524	12.024	7.303	206.250
2020	13	23.953	20.720	20.091	18.475	15.680	15.680	15.680	15.680	15.680	15.278	12.712	12.186	7.490	209.302
2021	14	24.456	21.052	20.390	18.707	15.889	15.889	15.889	15.889	15.889	15.467	12.903	12.354	7.684	212.458
2022	15	24.960	21.395	20.702	18.950	16.102	16.102	16.102	16.102	16.102	15.665	13.101	12.528	7.886	215.695
2023	16	25.476	21.749	21.020	19.198	16.326	16.326	16.326	16.326	16.326	15.869	13.307	12.709	8.095	219.052
2024	17	26.002	22.111	21.349	19.456	16.558	16.558	16.558	16.558	16.558	16.078	13.523	12.896	8.308	222.513
2025	18	26.536	22.489	21.689	19.724	16.798	16.798	16.798	16.798	16.798	16.297	13.744	13.091	8.532	226.090
2026	19	27.080	22.876	22.039	20.002	17.046	17.046	17.046	17.046	17.046	16.524	13.974	13.290	8.764	229.780
2027	20	27.635	23.280	22.401	20.291	17.306	17.306	17.306	17.306	17.306	16.761	14.212	13.498	9.006	233.615
2028	21	28.206	23.694	22.777	20.592	17.574	17.574	17.574	17.574	17.574	17.006	14.460	13.714	9.256	237.577
2029	22	28.787	24.123	23.162	20.901	17.856	17.856	17.856	17.856	17.856	17.259	14.719	13.940	9.515	241.685
2030	23	29.383	24.565	23.563	21.223	18.148	18.148	18.148	18.148	18.148	17.524	14.988	14.173	9.787	245.943
2031	24	29.994	25.023	23.980	21.559	18.452	18.452	18.452	18.452	18.452	17.799	15.267	14.414	10.068	250.363
2032	25	30.619	25.496	24.405	21.904	18.767	18.767	18.767	18.767	18.767	18.086	15.558	14.667	10.362	254.932

Appendix II – Investments and Operational Expenses

(R$ 1.000,00)

Concession Year	Calendar Year	Initial Investment	Mainte-nance	Improve-ments	Equip-ments	Vehicles	Total	Acumulated Investment	Depreciation	Net Acum Investment
0	2007	283.534					283.534	283.534		283.534
1	2008	268.580					268.580	552.114	(11.341)	540.773
2	2009	414.634					414.634	955.407	(23.005)	932.402
3	2010						0	932.402	(41.539)	890.862
4	2011		0	1.012	1.189	490	2.691	893.554	(42.382)	851.172
5	2012		46.063	46.128	1.189	781	94.160	945.332	(42.550)	902.782
6	2013		46.063	61.837	1.638	4.167	113.706	1.016.488	(47.267)	969.221
7	2014		46.063	62.734	1.189	1.072	111.058	1.080.279	(53.499)	1.026.780
8	2015		96.438	0	1.638	4.231	102.307	1.129.087	(60.016)	1.069.072
9	2016		65.915	0	1.189	1.542	68.646	1.137.718	(66.417)	1.071.301
10	2017		65.915	0	1.189	490	67.593	1.138.894	(71.107)	1.067.787
11	2018		65.915	0	4.081	3.988	73.984	1.141.771	(75.926)	1.065.844
12	2019		65.915	0	1.189	762	67.865	1.133.710	(81.555)	1.052.155
13	2020		65.915	0	18.534	5.501	89.949	1.142.104	(87.208)	1.054.895
14	2021		15.539	0	1.189	0	16.728	1.071.623	(95.175)	976.448
15	2022		15.539	0	9.093	1.344	25.976	1.002.424	(97.420)	905.003
16	2023		25.302	0	1.470	3.895	30.667	935.670	(100.242)	835.428
17	2024		25.302	0	1.189	781	27.272	862.700	(103.963)	758.736
18	2025		25.302	0	1.638	4.992	31.932	790.669	(107.837)	682.831
19	2026		25.302	0	1.189	1.072	27.563	710.394	(112.953)	597.441
20	2027		99.179	0	1.189	0	100.369	697.810	(118.399)	579.411
21	2028		99.179	0	3.913	4.948	108.040	687.451	(139.562)	547.889
22	2029		99.179	0	1.189	490	100.858	648.748	(171.863)	476.885
23	2030		99.179	0	3.545	4.813	107.538	584.422	(216.249)	368.173
24	2031		99.179	0	1.189	762	101.130	469.303	(292.211)	177.092
25	2032		99.179	0	16.178	490	115.847	292.939	(292.939)	0
		966.748	1.291.561	171.712	75.998	46.608	2.552.627	2.258.309		

Concession Year	Calendar Year	Conser-vation	Salaries	G&A Expenses	Other Services	Insurance	Operational Costs
0	2007						
1	2008	0	15.689	5.570	1.080	8.137	30.476
2	2009	2.249	15.689	5.570	1.080	8.137	32.725
3	2010	4.498	15.689	6.234	1.080	8.077	35.578
4	2011	7.252	33.032	11.146	1.080	7.857	60.367
5	2012	9.997	33.032	11.146	1.080	7.610	62.864
6	2013	12.779	33.032	11.146	1.080	7.338	65.374
7	2014	12.779	33.032	11.146	1.080	7.045	65.082
8	2015	12.779	33.032	11.146	1.080	6.734	64.770
9	2016	12.779	33.032	11.146	1.080	6.404	64.440
10	2017	12.779	33.032	11.146	1.080	6.058	64.094
11	2018	12.779	33.032	11.146	1.080	5.696	63.732
12	2019	12.779	33.032	11.146	1.080	5.329	63.365
13	2020	12.779	33.032	11.146	1.080	4.956	62.992
14	2021	12.779	33.032	11.146	1.080	4.578	62.614
15	2022	12.779	33.032	11.146	1.080	4.194	62.230
16	2023	12.779	33.032	11.146	1.080	3.804	61.841
17	2024	12.779	33.032	11.146	1.080	3.409	61.445
18	2025	12.779	33.032	11.146	1.080	3.006	61.043
19	2026	12.779	33.032	11.146	1.080	2.598	60.634
20	2027	12.779	33.032	11.146	1.080	2.183	60.219
21	2028	12.779	33.032	11.146	1.080	1.761	59.797
22	2029	12.779	33.032	11.146	1.080	1.332	59.368
23	2030	12.779	33.032	11.146	1.080	895	58.932
24	2031	12.779	33.032	11.146	1.080	452	58.488
25	2032	12.779	33.032	11.146	1.080	8.647	66.683
		279.570	773.763	262.580	27.000	126.238	1.469.152

Appendix III – Project Cash Flows

Free Cash Flow To Equity (R$ 1.000,00)

Concession Year	0	1	2	3	4	5	6	7	8	9	10	11	12
Calendar Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Initial Investment	(283.534)	(268.580)	(414.634)	0	0	0	0	0	0	0	0	0	0
Financing	170.120	161.148	248.780										
Net Investment	(113.414)	(107.432)	(165.854)	0	0	0	0	0	0	0	0	0	0
PV of Net Investment	(358.680)												
Toll Revenues		0	149.661	403.371	435.714	465.044	491.992	517.041	540.541	548.124	555.905	563.900	572.138
Toll Tax		0	(20.997)	(56.593)	(61.131)	(65.246)	(69.027)	(72.541)	(75.838)	(76.902)	(77.994)	(79.115)	(80.271)
Net Revenues		0	128.664	346.778	374.583	399.798	422.966	444.500	464.703	471.222	477.912	484.784	491.867
Operating Costs		30.476	32.725	35.578	60.367	62.864	65.374	65.082	64.770	64.440	64.094	63.732	63.365
Interest		15.311	29.814	52.204	52.204	48.724	45.244	41.764	38.283	34.803	31.323	27.842	24.362
Depreciation		11.341	23.005	41.539	42.382	42.550	47.267	53.499	60.016	66.417	71.107	75.926	81.555
Total Costs		57.129	85.544	129.321	154.953	154.139	157.885	160.344	163.068	165.660	166.524	167.501	169.282
EBT		(57.129)	43.120	217.456	219.631	245.660	265.081	284.156	301.635	305.562	311.388	317.284	322.585
Tax		0	(14.661)	(73.935)	(74.674)	(83.524)	(90.128)	(96.613)	(102.556)	(103.891)	(105.872)	(107.876)	(109.679)
Net Earnings		(57.129)	28.459	143.521	144.956	162.136	174.954	187.543	199.079	201.671	205.516	209.407	212.906
+ Depreciation		11.341	23.005	41.539	42.382	42.550	47.267	53.499	60.016	66.417	71.107	75.926	81.555
- Amortization		0	0	0	(38.670)	(38.670)	(38.670)	(38.670)	(38.670)	(38.670)	(38.670)	(38.670)	(38.670)
- Maintenance		0	0	0	0	(68.980)	(74.782)	(81.072)	(184.011)	(136.348)	(147.817)	(160.250)	(173.729)
- Improvements		0	0	0	(490)	(781)	(4.167)	(1.072)	(4.231)	(1.542)	(490)	(3.988)	(762)
FCFE	**(358.680)**	(45.787)	51.464	185.061	148.179	96.255	104.602	120.228	32.183	91.527	89.647	82.426	81.301

Discount Rate = 16% **IRR =** **21,9%** **MIRR =** **17,3%**

PV_0 = **498.531** **Investment =** **(358.680)** **NPV_0 =** **139.850**

Free Cash Flow To Equity (R$ 1.000,00)

Concession Year	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Calendar Year	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032
Initial Investment	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Financing														
Net Investment	0	0	0	0	0	0	0	0	0	0	0	0	0	0
PV of Net Investment														
Toll Revenues	572.138	580.604	589.357	598.338	607.650	617.252	627.173	637.409	648.047	659.037	670.434	682.246	694.507	707.182
Toll Tax	(80.271)	(81.459)	(82.687)	(83.947)	(85.253)	(86.600)	(87.992)	(89.429)	(90.921)	(92.463)	(94.062)	(95.719)	(97.439)	(99.218)
Net Revenues	491.867	499.146	506.671	514.391	522.397	530.651	539.181	547.981	557.126	566.575	576.373	586.527	597.067	607.964
Operating Costs	63.365	62.992	62.614	62.230	61.841	61.445	61.043	60.634	60.219	59.797	59.368	58.932	58.488	66.683
Interest	24.362	20.882	17.401	13.921	10.441	6.961	3.480	0	0	0	0	0	0	0
Depreciation	81.555	87.208	95.175	97.420	100.242	103.963	107.837	112.953	118.399	139.562	171.863	216.249	292.211	292.939
Total Costs	169.282	171.083	175.191	173.572	172.524	172.369	172.360	173.587	178.618	199.359	231.231	275.181	350.699	359.622
EBT	322.585	328.063	331.480	340.819	349.873	358.283	366.821	374.394	378.509	367.216	345.142	311.346	246.369	248.342
Tax	(109.679)	(111.541)	(112.703)	(115.878)	(118.957)	(121.816)	(124.719)	(127.294)	(128.693)	(124.853)	(117.348)	(105.858)	(83.765)	(84.436)
Net Earnings	212.906	216.522	218.776	224.941	230.916	236.467	242.102	247.100	249.816	242.362	227.794	205.488	162.603	163.906
+ Depreciation	81.555	87.208	95.175	97.420	100.242	103.963	107.837	112.953	118.399	139.562	171.863	216.249	292.211	292.939
- Amortization	(38.670)	(38.670)	(38.670)	(38.670)	(38.670)	(38.670)	(38.670)	0	0	0	0	0	0	0
- Maintenance	(173.729)	(188.342)	(48.135)	(52.183)	(92.117)	(99.865)	(108.265)	(117.371)	(498.776)	(540.730)	(586.212)	(635.519)	(688.974)	(746.925)
- Improvements	(762)	(5.501)	0	(1.344)	(3.895)	(781)	(4.992)	(1.072)	0	(4.948)	(490)	(4.813)	(762)	(490)
FCFE	81.301	71.218	227.147	230.163	196.476	201.114	198.012	241.609	(130.562)	(163.753)	(187.045)	(218.594)	(234.921)	(290.570)