Investment under Uncertainty: a Equilibrium between Competition and Cooperation

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1 Introduction

Firms' optimal investment decisions under uncertainty has been a controversial topic for a long time due to the observed deviation from zero NPV threshold. The standard real options literature asserts that investments should be delayed until uncertainty is resolved or wait for the optimal threshold. However, the competitive real options literature argues that competition diminishes the real option values and mitigates investment delays, thus, with sufficient competition, firms' investment threshold may be pushed back to zero NPV. The most recent article, Novy-Marx (2007) shows that supply side heterogeneity can reduce the competition effect and leads to an investment threshold even later than the standard real option threshold.

This article presents an equilibrium model reconciling the contradiction in previous literature, which helps to explain firms' investment behavior – sometimes delaying the investment until the standard real option exercise threshold, sometimes later than that, sometimes don't delay at all. The analysis presented in this article shows that firms should monitor the benefit of delaying the investment to the benefit of exercising the real option. When the former is larger, firms should keep delaying the investment. When the latter is larger, firms should invest immediately. The benefit of delaying is the real option value. The benefit of exercising includes the earlier cash flows from the investment, the first mover advantage and the ability of extracting economic rents from the competitors if exercising before the competitors. Particularly, in industries with economies of scale, firms may still invest later than the zero NPV threshold to keep the real option value, but earlier than the standard real options exercise threshold in order to obtain the first mover advantage and the ability of extracting rents. Even if the competition is sufficiently fierce in these industries, it will not force firms to invest at zero NPV threshold because heterogeneity in firms will determine whichever firm invest first. Instead, sufficient competition will push firms to consider an alternative strategy – cooperation, which allows firms to benefit from the economies of scale. The equilibrium of investment threshold is formed where firms can successfully negotiate a cooperation contract.

Standard real option literature[1] shows that firms should optimally delay the investment under uncertainty until a suitable threshold for price, demand or other stochastic variable is met. Myopic firms simply apply this standard model to decide the optimal time of investment without contemplating future ramifications of their current investment decisions. However, strategic firms will deliberate the interaction of real option investments among firms when their inputs or outputs are substitutable or complementary. There may be market power, patents, proprietary expertise or location that cause these interactions. In such settings, one firm's investment decision may influence the other firm's investment decision through various factors such as the first mover advantage and the economies of scale. For example, in the petroleum industry and the real estate industry, we often observe that firms compete to become the first mover by building significant excess production capacity even when the commodity price is fairly low, and even though they realize there is a real-option value to wait. The preemptive real options literature[2] explain this as a tradeoff between the real option value to delay and the first-mover advantage. They use the intersection of real options and industrial organization theory to analyze firms' strategic preemptive investment decisions. Most of these articles develop a Bertrand, a Cournot, or a Stackelberg

[1] This includes Brennan and Schwartz (1985); Dixit and Pindyck (1994); Dixit (1995); Capozza and Sick (1991); Sick (1995); Trigeorgis (1996)

[2] This includes Fudenberg and Tirole (1985); Smit and Ankum (1993); Grenadier (1996, 2002); Mason and Weeds (2005); Garlappi (2001); Boyer et al. (2001); Murto and Keppo (2002); Lambrecht and Perraudin (2003); Murto et al. (2003); Huisman and Kort (2004); Thijssen et al. (2006); Smit and Trigeorgis (2004)

equilibrium depending on the type of competition assumed. They argue that, in extreme case, sufficient competition may deteriorate the real option value and push firms' investment threshold back to zero NPV.

Novy-Marx (2007) models an investment equilibrium in which heterogeneous firms have limited but renewable production resource, face a limited market demand and thus have to share the demand amongst themselves, produce perfect substitute outputs.

probably in the downtown area, whereas our model applies to firms have relatively unlimited land, probably in the suburb area? Firms will find it's cheaper to acquire new resources than to rebuild/redevelop the existing resource, which is true for petroleum/software/airline industry.

we are affected by other prices, but they aren't perfect substitutes. the resource is non-renewable might be the reason why the rents don't get bid to zero. Their models would work with real estate where there is an unlimited supply of land but a limited market demand.

For real estate industry, firms holding lands actually have a series of endless RO. One after another. Previous exercised RO provides cash flows working like convenience yield for later unexercised RO if firms can rebuild. If the firm wants to rebuild, it has to destroy the previous building completely. This will make the opportunity cost even larger. I guess this is the main reason that Novy-Marx's P** is higher than the standard P*. For petroleum, software and airline industry, things are different. Firms only have one RO. If they rebuild/redevelop a new project, it will only partially jeopardize the previous one. Furthermore, if single firm's output is not affecting the market price, this interaction between new/old project is minimal. Thus, I would project our optimal trigger should be lower than Novy-Marx's P** since we have smaller opportunity cost and one RO value (Novy-Marx have many RO).

Our contribution would be by allowing a choice b/t competition and cooperation, the economic rents(working like the convenience yield) extracted by the leader's will offset the benefit of delaying. Therefore, our optimal trigger, P*** should be [NPV=0, P**]. The reason is that cooperation reduces competition, but it also increases the first mover advantage since only the leader can extract the rent from the follower.

However, despite the substantial development of this literature, little attention has been paid to the effects of positive externality on firms investment decision. The network effect is also offset by the first-mover advantage that encourages early investment. The first mover advantage accrues to the first firm that builds or purchases a production facility, because it can build or purchase the facility based on its own specifications, and locational or functional preference. Moreover, once the facility is built, it can engage in a bargaining game with later movers in which it offers to lease access to its facility. The first mover has a tradeoff between the rents it can earn on a high lease rate and the opportunity to capture network benefits by having the second mover enter early. Therefore, the strategic firms not only choose the optimal investment time, but also make decisions about the optimal investment size, whether to cooperate with the competitor by sharing the facility, and how much to charge the competitor for using the facility. Decisions on these investment issues can either create or destroy significant value, which makes them important for management. Such investment opportunities share similar characteristics and can be analyzed using real option theory and cooperative game theory.

Economies of scale arises from cooperation that can yield operating synergy. The operating synergy may come in the form of lower cost structure. A single firm may not have enough production volume to make the construction or the purchase of the production facility economically viable. If it can induce others to participate, the unit costs will fall and it will face a lower cost structure including the saved fixed cost of repetitive construction or purchase of certain production facility, lower unit production costs, lower marketing costs, or lower transportation costs paid to a third party. Alternatively, the operating synergy may come in the form of higher overall revenue because the cooperative investment may generate larger market demand or improve the quality of goods.

This article studies the effect of interaction between firms' flexible investment decisions — the size (capacity choice) and timing of investment for certain industries by recognizing firms' capability of making strategic capacity choice, extracting rents from competitors, and taking the advantage of economical positive externality (network effect) depending on the level of industry concentration. In fact, this article demonstrates an equilibrium real options exercise game in which the investment cash flows are not purely exogenous (solely relying on the market demand) to the firm, but somewhat endogenous in the sense that it is affected by the firm's capacity choice and the competitor reactions. One typical application of this is the investment decisions of two adjacent gas producers. Their decisions consist of two stages. In the first stage, the natural gas price and the unproved initial reserves will determine who develops the land first and becomes the first mover. The first mover then has to decide on the optimal size and timing of construction depending on whether it plans to be cooperative or non-cooperative.[3] In the second stage, the first mover and the second mover play a sequential bargaining game to decide the optimal economic rent paid by the second mover to the first mover for use of the common facility. The second mover has to decide whether to use the first mover's facility or build its own facility, and if it decides to build its own, the optimal scale and timing of construction. By analyzing firms' behavior under a general setting of a sequential bargaining game of incomplete information in the presence of the positive externality, this article demonstrates that firms sometimes invest earlier than optimal and build excess production capacity not only for the preemptive effect or the first mover advantage, but also for being able to extract rent from the follower. Specifically, it shows that the leader can improve its enterprise value by being cooperative, i.e., building excess production capacity and leasing the excess capacity to the follower.

Furthermore, the simulation results provide several testable implications for understanding firms' bargaining behavior in these investment projects. Firstly, the relationship between firms reservation lease rate and commodity price is not monotonic. Once the leader exercises the real option, it may set the lease rate fairly high if the commodity price is below the follower's exercise threshold. But when the leader observes the price rising towards to the follower's exercise threshold, it may quickly reduce the lease rate to the lowest in order to avoid the rejection of lease. The follower tends to reject the lease contract if the commodity price and the initial reserve quantity are either relatively low or high, whereas tends to accept the lease for some medium range of price and quantity. Secondly, the relationship between firms' reservation lease rates and the network effect are also non-monotonic. The network effect is positively associated with the firms' lease rate before the peak, but becomes negatively associated after the peak.

The purpose of this article is to extend the current literature on real option exercise games by allowing size and timing decisions, as well as by incorporating the network effect into a dynamic bargaining game of incomplete information. It addresses the dynamics of optimal economic rents and capacity choice, given network effects, real options and incentives for pre-

[3]As defined in later sections of this article, the cooperative producer recognizes the economies of scale and the positive network effect and thus will construct a larger production facility for sharing, whereas the non-cooperative producer will construct a smaller facility optimal for its own reserves.

emption.

The remainder of the paper is organized as follows. Section 2 discusses the application of real options sequential bargaining games model in the relevant industries. Section 3 develops a real option exercise game model and the equilibrium of non-cooperative firms' investment decisions. Section 4 analyzes the firms' investment decisions by considering their objective functions and individual rationality constraints in a context of the sequential bargaining game. Section 5 extends the backward induction solution to this real option sequential bargaining game and explicitly analyzes the firms' prior and posterior beliefs. After ruling out non-credible threats and promises, it then develops a perfect Bayesian equilibrium for this dynamic bargaining game under incomplete information using Coasian Dynamics as discussed in Fudenberg and Tirole (1991, Ch 10). Section 6 applies the Longstaff and Schwartz (2001) least square Monte-Carlo method to simulate and optimize the real options values which leads to an efficient computational procedure to determine optimal investment time and size, when and whether firms should be cooperative, what the optimal economic rents are under the assumptions of stochastic prices and production quantities. Section 7 concludes.

2 The Application of Real Options Sequential Bargaining Games Model

In the oil and gas industry, the airline industry, the real estate industry and the software industry, investments usually require a large amount of capital to build or purchase a production facility, which may be a plant, an equipment, a jet aircraft, a R&D patent or an infrastructure depending on the nature of different industries. Investment decisions in these four industries involves a two stage game. In the first stage, firms trying to capture the first mover advantage will play a Betrand game in the case of differentiated product or a Cournot game in the case of homogeneous product. If the two firms have the same cost structure and payoff functions, there will be a simultaneous investment. If one firm has significant competitive advantage over the other, it will invest first and become the leader who gets more favorable price in the Bertrand equilibrium or larger production quantity in the Cournot equilibrium. The follower's strategy would be either to sell products at a less favorable price in Bertrand equilibrium or to produce less in Cournot equilibrium. This could have been the final equilibrium providing the price or demand is deterministic.

However, when the price or demand is stochastic, the follower has a real option to delay its investment until more favorable price or demand comes which makes both firms proceed to the second stage where the leader and the follower play a sequential bargaining game. In the second stage, the leader wants to encourage the follower to start production earlier by offering to lease part of the production facility to the follower, therefore the leader needs to determine the optimal economic rent and optimal investment scale. The follower needs to decide whether to accept the leader's offer or to wait to build its own facility. Since these investments share common characteristics and exhibit similar comparative statics, we will discuss the components of this real option bargaining game for each industry first. Then we will formally construct and analyze our real option bargaining model by solely focusing on the oil and gas industry in the rest of this article.

2.1 The airline industry

Airlines face stochastic demand for flights between city pairs. This gives them a real option to decide when to start a route between two cities, and how much capacity to put on the route.

Suppose two airlines, Air France and Lufthansa Airline have adjacent air transportation markets between central Europe and North America. Air France prefers a Paris hub whereas Lufthansa prefers a Frankfurt hub because there are two advantages of locating the hub in the airline's home countries. It allows the airport to be built on the airline's specification. It may also bring in potential future air travel demand. The property right is clearly defined because neither of them has the route authority in the other country. Since the air travel demand is uncertain, this is a real option to develop a new route. The production facility is the aircraft and the terminal facilities. The exercise price is the capital costs (mainly aircraft purchases) and the number of airplanes purchased determines the production capacity.

In the first stage, two airlines will play a real option exercise game in which the first mover (the leader) will develop the route and locate the airport in its home country (either Paris or Frankfurt), and the second mover (the follower) will choose to wait until more demand comes. In the second stage, to capture the network effect, the leader may want to encourage the follower to start selling the similar flights between Frankfurt (or Paris) and North America cities by offering a code-sharing program to the follower. The code sharing program can reduce the number of empty seats on each flight and thereby boost the revenue. The increased number of flights (the larger transportation volume) may help bring down the unit airport service fee and reduce both firms' operating costs per seat. The leader's decisions include how many planes to order, and how much to charge the follower for code sharing. The follower's decisions are to accept the leader's offer and how to bargain with the leader in terms of the code-sharing, or to delay its plane purchase until more uncertainty about the demand is revealed.

2.2 The real estate industry

Real estate developers often make decisions on whether and when to develop adjacent undeveloped properties. Suppose there are two real estate developers who own adjacent undeveloped properties that can be developed into a residential area. There is a network effect arising from shared infrastructure (roads, schools, shopping centers). The demand for houses in that area is uncertain and so is the selling price. There is a real option to develop for both companies. In the first stage real option exercise game, the leader and the follower will be determined depending on the house price and number of houses to be built. The leader becomes the main developer and the follower is the home builder. In the second stage, the leader can offer lots in its developed area, upon which the second mover can build. The leader and follower can capture the network effect if they can induce third parties to build schools, shopping centers and upgrading roads. This is more likely to happen if they cooperate and build more houses. The leader has to decide the size of the neighborhood, the construction scale of these infrastructure, how much to charge the follower for sharing the infrastructure. The follower has to decide whether to accept the leader's offer and start to develop immediately, or wait to build its own infrastructure in another neighborhood and develop in the future.

2.3 The software industry

Software companies often have to decide whether to develop multiple software packages with related functionality. Software packages can share file standards or inter-operability (plug-ins). In the first stage, the two companies will have a real option patent game to develop new software as discussed by Miltersen and Schwartz (2004). In the second stage, the leader may offer the follower a license contract which allows the follower to use the leader's patented software to develop related applications. The leader's decisions include the optimal software capability — the number of functions provided by the software as well as the optimal license fee. The follower's decision is whether to use the leader's patent by paying the license fee, or to (wait and) develop its own software later. The network effect may result from the avoidance of repetitive R&D investment, or from the increased software value due to improved compatibility and a larger customer pool.

2.4 The oil and gas Industry

In the oil and gas industry, producers often own adjacent lands from which they may produce in the future. This provides for an opportunity for joint use of infrastructure to exploit the resource. We focus on two such types of infrastructure, which typically have different ownership structure.

1. Gas processing plants remove liquids and hydrogen sulfide from the gas at the field before it can be safely shipped by pipeline. Gas plants are typically owned and operated by the first company to drill in a particular field, and they may build excess capacity and lease out that capacity to other producers in the same area.

2. Pipeline gathering systems are needed to ship the gas to central hubs, where they join the main line pipelines that distribute gas to consuming areas. These are typically owned by a company that specializes in pipelines, and it usually isn't a producer.

There are fixed costs in both of these types of infrastructure, which generates a network effect. A single gas producer may not have enough reserves to make a gas plant or pipeline connection economically viable. Also, if it can induce others to participate in the infrastructure, the unit costs will fall and it will tend to face a lower overall cost structure to produce its reserves. The first mover advantage accrues to the first company (the leader) that builds a gas processing plant to serve the field. The advantage arises because the leader can locate the plant nearest its part of the field and can customize the construction of the plant to be most efficient with the type of gas it owns. In the first stage, firms having similar size of initial reserves will invest simultaneously whereas if one firm has larger initial reserve, it will develop first and becomes the leader. In the second stage, once the plant is built, the leader can extract rents from the follower because of the fixed costs of building a competitive plant. However, the leader's efforts to extract rents are offset by its desire to have the follower agree to produce, thereby enabling the pipeline to be built or reducing the toll charges it has to pay the pipeline owner to induce it to build the pipeline. Also, there is a tradeoff between the first-mover advantage for building the gas plant and the real options incentive to delay construction until more uncertainty about volumes and prices can be resolved. The leader decides the optimal plant capacity and the leasing fee. The follower decides whether to accept the leader's offer or wait to build its own processing plant.

3 The real option model

3.1 Assumptions of the Model

There are two gas explorers, A and B, who have adjacent properties for gas exploration and production. There are two kinds of uncertainty which are going to affect firms' optimal investment scale and timing.

3.1.1 Production Uncertainty

The first is the technical uncertainty of the estimated quantity of reserves on the property. Let $Q_i(t)$ be producer i's expected remaining reserves conditional on information gathered to time t and production up to time t.

$$dQ_i = \mu_i(Q_i)dt + \sigma_i(Q_i)dz_i, \qquad i \in \{A, B\}$$

where the correlation $\rho_Q = \text{corr}(dz_A, dz_B)$.

Production at rate q_i does two things:

1. It depletes the reservoir at rate q_i;
2. It provides information that causes revised information about total reserves. So $\sigma_i(q_i)$ is non-decreasing in q_i.

$$dQ_i = -q_i dt + \sigma_i(q_i)dz_i$$

We can model exponentially declining production volume as

$$q_i = \alpha_i Q_i$$

where α_i is the production rate. But, this doesn't usually happen, because there are two constraints on production. One is a regulatory or technical upper bound on the production rate[4] $\overline{q_i} = \overline{\alpha}_i Q_i$, for some fixed $\overline{\alpha}_i$.

The other is the capacity of the processing plant, q_i^c. Therefore, the production rate q_i must satisfy the following constraint if there is one producer and one plant only:

$$q_i = \min\{q_i^c, \overline{\alpha}_i Q_i\} \tag{1}$$

At the start of production, the capacity is binding:

$$\frac{dQ_i(t)}{dt} = -q_i^c \Longrightarrow Q_i(t) = Q_i(\tau_i) - q_i^c\ t \qquad (\tau_i \le t \le \theta_{i,\text{trans}})$$

where τ_i is player i's production starting time, and $\theta_{i,\text{trans}}$ is defined as the transition time from the capacity constraint to the technology/regulatory constraint:

$$\overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) = q_i^c \tag{2}$$

$$\Longrightarrow \overline{\alpha}_i \left[Q_i(\tau_i) - q_i^c \theta_{i,\text{trans}}\right] = q_i^c$$

$$\Longrightarrow \theta_{i,\text{trans}} = \frac{Q_i(\tau_i)}{q_i^c} - \frac{1}{\overline{\alpha}_i} \tag{3}$$

After $\theta_{i,\text{trans}}$, the reserve quantity is binding, so the actual production rate is $\overline{\alpha}_i$.

$$\frac{dQ_i(t)}{dt} = -\overline{\alpha}_i Q_i(t) \Longrightarrow Q_i(t) = Q_i(\theta_{i,\text{trans}}) e^{-\overline{\alpha}_i (t-\theta_{i,\text{trans}})} \qquad (t \ge \theta_{i,\text{trans}})$$

Thus, producer i's production function is

$$q_i(t) = \begin{cases} q_i^c & t \in [\tau_i, \theta_{i,\text{trans}}] \\ \overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) e^{-\overline{\alpha}_i (t-\theta_{i,\text{trans}})} & t \in [\theta_{i,\text{trans}}, \theta_i] \end{cases} \tag{4}$$

where θ_i is producer i's maximum production time of its property.[5]

[4]Regulators often restrict the production rate to avoid damaging the rock formation and having water floods, which could reduce the ultimate production from the field. Also, there is a natural maximum flow rate for the field depending on the porosity of the rock.

[5]The remaining reserves continue drop once the production starts. After producing for

3.1.2 Price Uncertainty

The price of gas P is a source of economic uncertainty. We model it as

$$dP = \mu(P)dt + \sigma(P)dz_P$$

where we assume the correlation between technical and economic uncertainty is zero: $\text{corr}(dz_P, dz_A) = \text{corr}(dz_P, dz_B) = 0$.

3.1.3 Construction Cost

The cost of constructing a gas plant with capacity of q_i^c has fixed and variable components:

$$K(q_i^c) = a + bq_i^c \qquad i \in \{A, B\}$$

where the producers have the same construction parameters $a, b > 0$.

3.2 The players' investment decisions

3.2.1 The isolated players' investment decisions

Suppose that neither producer initially has a gas processing facility. If the producers' properties are not adjacent, the problem for each producer would be a classic two dimensional real option problem. The real option decisions are those that would be made by a monopolist owner of the project, without any consideration of interaction with the other producer. The optimal development option for producer $i \in \{A, B\}$ has a threshold $\{(P^*(Q_i), Q_i) \,|\, Q_i \in \mathbb{R}^+\}$ where $P_i^* : \mathbb{R}^+ \to \mathbb{R}^+$ is the threshold development price if the estimated reserves are Q_i. That is, producer i develops the first time $(P_t, Q_{i,t})$ are such that $P_t \geq P_i^*(Q_{i,t})$.

The cash flow for producer i at time t is $\pi_{i,t} : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ given by

$$\pi_{i,t} = (P_t - C)q_{i,t} \tag{5}$$

where C is the variable production cost. The expected payoff from an investment made by player i at time τ_i, is:

$$W_i(P, Q_i, \tau_i) = \widehat{E}_{\tau_i} \int_{\tau_i}^{\theta_i} e^{-rt} \pi_{i,t} \; dt - K(q_i^c) \tag{6}$$

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certain period of time, the remaining reserves will drop below a critical level at which it may be optimal to shut down the production because the profit may not be able to cover the variable production cost then.

This evolves according to geometric Brownian motion. It may be the case that the threshold can be simplified to a threshold level[6] of cash flow π^*, but this is not necessarily the case, since the uncertainty and risk neutral growth rates in Q and P may not be the same, so that the (threshold) level of profit may vary over the threshold. These isolated producers are non-cooperative in the sense that they do not have to consider the strategic effect from the investments by the competitors. As P and Q are assumed uncorrelated, generally, these non-cooperative firms' value of the investment opportunity(real option values) $V\big(W(P,Q),t\big)$ must satisfy the valuation PDE:[7]

$$\begin{aligned} \frac{1}{2}\left[\sigma^2(Q)V_{QQ}(P,Q)+\sigma^2(P)V_{PP}(P,Q)\right]+ \\ V_Q(P,Q)\mu(Q)+V_P(P,Q)\left[\mu(P)-\lambda_P\beta(P)\right]+V_t=rV(P,Q) \end{aligned} \tag{7}$$

and the value-matching and smooth pasting boundary conditions:[8]

$$\begin{aligned} V(P^*,Q^*) &= W(P^*,Q^*) \\ V_P(P^*,Q^*) &= \frac{\partial W_i}{\partial P_{\tau_i}} = \widehat{E}_{\tau_i}\Bigg[\int_{\tau_i}^{\theta_{i,\text{trans}}} e^{(\hat{\mu}(P)-r)(t-\tau_i)} q_i^c dt \\ &\quad + \int_{\theta_{i,\text{trans}}}^{\theta_i} e^{(\hat{\mu}(P)-r)(t-\tau_i)-\overline{\alpha}_i(t-\theta_{i,\text{trans}})}\overline{\alpha}_i Q_i(\theta_{i,\text{trans}})dt\Bigg] - K(q_i^c) \\ V_Q(P^*,Q^*) &= \frac{\partial W_i}{\partial Q_i} = \widehat{E}_{\tau_i}\left[\int_{\theta_{i,\text{trans}}}^{\theta_i} e^{-r(t-\tau_i)}(P_t-C)e^{-\overline{\alpha}_i(t-\theta_{i,\text{trans}})}dt\right] - K(q_i^c) \end{aligned} \tag{8}$$

Notice here risk-neutral drift of P is: $\mu(P,t)-\lambda_P\beta(P)=rP-\delta(P,t)$, where $\beta(P)=\frac{\text{cov}(dP,d\tilde{f})}{\sqrt{\text{var}(dP)\text{var}(d\tilde{f})}}$, r is the risk free interest rate, δ is the convenience yield of the underlying asset. λ is the risk premium for the systematic risk factor $\tilde{f}$, and $\tilde{f}$ is some systematic risk factor such that the investment asset is expected to earn a risk premium in proportion to the covariance between asset price changes and the risk factor. Similarly, the risk-neutral drift of Q is: $\mu(Q)-\lambda_Q\beta(Q)=-q_t$, where $\beta(Q)=\frac{\text{cov}(dQ,d\tilde{f})}{\sqrt{\text{var}(dQ)\text{var}(d\tilde{f})}}=0$ because the

[6]Lambrecht and Perraudin (2003) discuss the possibility of a sufficient statistic to determine the threshold.

[7]This is an extension of the classic model of operating real options by Brennan and Schwartz (1985) to finite reserves.

[8]See appendix A for detailed derivation of the two smooth-pasting conditions.

production rate $q_t = 0$ is zero before the initial investment. Equation (7) with the boundary conditions, equation (8) can be easily solved numerically as Section 6.1 will demonstrate.

3.2.2 The adjacent players' investment decisions

The cooperative producers will follow a symmetric, subgame perfect equilibrium entry strategies in which each producer's exercise strategy maximizes value conditional upon the other's exercise strategy, as in Kreps and Scheinkman (1983); Kulatilaka and Perotti (1994); Mason and Weeds (2005); Garlappi (2001); Thijssen et al. (2002); Huisman et al. (2003); Imai and Watanabe (2005). The solutions have two different exercise models: simultaneous and sequential exercise.

3.2.3 Equilibrium under simultaneous exercise

Suppose both producers exploration reveal that their initial reserve quantity are same. Denote F as the follower, and L as the leader, $F, L \in \{A, B\}$. In this case, $P_A^*(Q_A) = P_B^*(Q_B) = P_F^*(Q_F) = P_L^*(Q_L)$, producers' have the same trigger price. Therefore once the price hits the trigger, they both want to exercise the real option and build their own plant immediately. Whoever moves faster becomes the natural leader. Given that the prices P and quantities Q_A, Q_B are continuously distributed and not correlated, this is a knife-edge condition that only occurs with probability zero if the producers do not interact.

However, when their properties are adjacent, they can interact. The leader can build a plant large enough to process both producers' gas and offer a processing lease rate to the follower. The follower can accept the offer and process its gas in the leader's plant, or build its own processing plant right away. If they are cooperative, they would exercise simultaneously and play a bargaining game at that time to determine the lease rate l and plant capacity q_L^c. We define the follower in this simultaneous exercise case as a *big follower*, denoted as F_b. For simplicity, we assume that they both commit not to renegotiate the lease later.

3.2.4 Equilibrium under sequential exercise

Suppose the leader has a larger initial reserve and therefore lower optimal trigger price $P^*(Q_L)$. In this case, $P_L^*(Q_L) < P_F^*(Q_F)$ for $L, F \in \{A, B\}, L \neq$

F. The leader will enter alone, building a gas processing plant to cover its own production only. Once its production volumes decline, it will offer excess capacity to the follower at a lease rate l to be negotiated, bearing in mind the follower's reservation cost of building its own plant.

Thus, there is a bargaining game played at and after the time the leader decides to build the plant. This game determines whether the follower starts production at the same time or delays. If the follower accepts the lease, both producers start production simultaneously and the game ends. If the follower rejects the lease, they play the same sequential bargaining game at subsequent dates, where the leader offers a lease rate and capacity, and the follower decides whether to accept the offer, build its own plant or delay further. We define the follower in this sequential exercise case as a *small follower*, denoted as F_s.

4 The sequential bargaining game under incomplete information for adjacent players

One significant difference between our paper and other option exercise game papers is that we model the expected payoff $W(P, Q, q_i^c, l, t; N)$ as a result of lease vs. build (exercise the real option of investment) bargaining game when the two producers have adjacent properties in the presence of the network effect N. This bargaining game is a dynamic game of incomplete information as the leader does not have the information about the follower's payoffs function.

Denote τ_L as the first time (P, Q_L) hits the threshold $\big(P^*(Q_L), Q_L\big)$. The follower also solves for a threshold trigger price $P^*(Q_F)$ that determines the optimal condition under which it would build its own plant and start production. Denote the first hitting time to the threshold $\big(P^*(Q_F), Q_F\big)$ by the stopping time $\tau_F \in [\tau_L, \infty)$. Hence the big follower exercises at τ_{F_b} and $\tau_{F_b} = \tau_L$ because the big follower's initial reserve is of the same size as the leader's. The small follower exercises at $\tau_{F_s} > \tau_{F_b}$ because the small follower's initial reserve is smaller than the big follower. The lease will start at $\tau_{\text{lease}} \in [\tau_L, \tau_{F_s}]$. The leader's maximum production time is θ_L. The big or small follower's maximum production time is θ_{F_b} or θ_{F_s} respectively.

The timing of the game is illustrated in Figure 1.

We now formally construct this sequential bargaining game under incom-

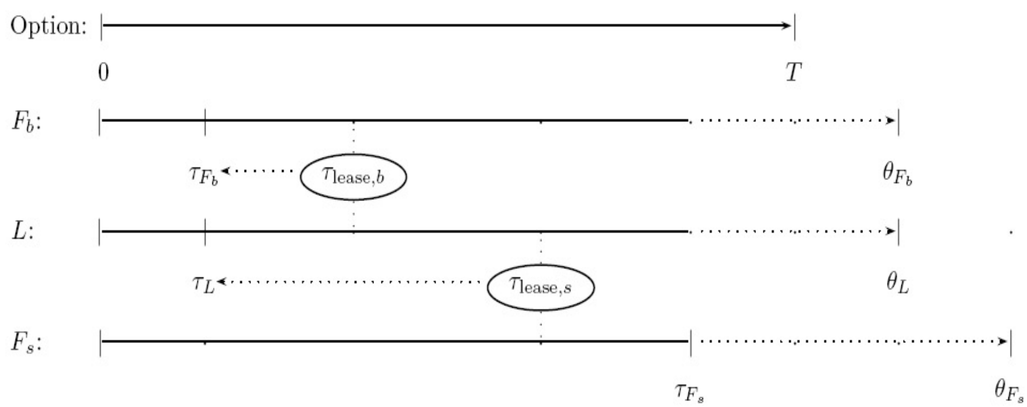


Figure 1: **The leader and the follower's timeline**

plete information. There are two players in the game, the leader and the follower. The product to be traded is the leader's (the seller) excess processing capacity. The quantity of product to be traded is the contracted fixed lease production capacity per unit of time q_{FL}. The network effect is the gain from cooperation. The transfer is the leasing fee l from the follower to the leader. The leader knows its cost of providing the excess capacity $K(\cdot)$. The follower has private information about its valuation $l_F \in \{\underline{l}_F, \bar{l}_F\}$. As shown in Section 4.3, the benefit from bargaining with the leader is smaller for the big follower than for the small follower. Hence, there are two types of buyers, the low type buyer (the big follower, F_b) who values the lease at $\underline{l}_F$ and the high type buyer (the small follower, F_s) who values the lease at $\bar{l}_F$. The leader does not know what type of buyer the follower is. Therefore, there is a conflict between efficiency (the realization of the gain from cooperation) and rent extraction in mechanism design. The leader's strategy space is to offer the lease at either $\underline{l}_F$, or $\bar{l}_F$.[9] The follower's strategy space is to either accept or reject the leader's offer. If the follower accepts, the game ends. If the follower rejects, the leader will make another offer in the next period. The decision variables are the leasing rate l, the cooperative and non-cooperative plant capacity choices q_L^Ω, or q_L^c and q_F^c, which determine

[9]We decide to analyze the mechanism bargaining on the lease rate l only, in which $q_{F_B L} = q_{F_S L} = q_{\text{FL}}$, which leads to $\int_{\tau_{F_b}}^{\theta_{F_b}} t dt > \int_{\tau_{F_s}}^{\theta_{F_s}} t dt$ because the F_b has larger initial reserve. There is another way of designing the bargaining mechanism. The leader can provide two types of contracts, $\{\underline{l}_F, q_{F_B L}\}$ and $\{\bar{l}_F, q_{F_S L}\}$, where $\underline{l}_F < \bar{l}_F$ and $q_{F_B L} > q_{F_S L}$. This is a bargaining game on both the lease rate and the lease quantity.

the construction costs $K(q_L^\Omega)$, or $K(q_L^c)$, $K(q_F^c)$ and production volumes q_L and q_F. The players' expected payoff functions will be discussed in detail in Section 4.6. The exogenous variables are the stochastic gas price P, the expected reserve quantities at the time of construction, Q_L and Q_F as assumed in Section 3.1, and the network effect N.

4.1 The adjacent players' Bellman equation

In this game, the adjacent players will maximize their own total enterprise values by optimally controlling their respective capacity choices q_L^c, q_F^c and the lease rate l, given the two stochastic variables P and Q_i that evolve over time, and the exogenous network effect N. The adjacent player i's Bellman equation can be stated as:

$$\begin{aligned} U_{i,t}(P_t, Q_{i,t}, q_{i,t}^c, l_t; N) &= V_{i,t} + \mathrm{AV}_{i,t} \\ &= \max_{\{P_{\tau_i}, Q_{\tau_i}\}} \left\{ E_{i,t}\Big[V_{i,t+1}(P_{t+1}, Q_{i,t+1}) \Big], \quad \max_{\{q_{i,t}^c, l_t\}} W_{i,t}(P_t, Q_{i,t}, q_{i,t}^c, l_t; N) \right\} \\ &\quad + \max_{\{q_{i,t}^c, l_t\}} E_{i,t}\Big[W_{i,t}(P_t, Q_{i,t}, q_{i,t}^c, l_t; N) \Big] \end{aligned} \tag{9}$$

$U_{i,t}(P_t, Q_{i,t}, q_{i,t}^c, l_t; N)$ is the total enterprise value for player i and has two components, the real option value for the investment opportunity $V_{i,t}$ and the pure asset value of the property $\mathrm{AV}_{i,t}$. The maximization of $V_{i,t}$ is a mixture of deterministic and stochastic optimal control problem for $\{P_{\tau_i}, Q_{\tau_i}, q_{i,t}^c, l_t\}$. The maximization of $\mathrm{AV}_{i,t}$ is a deterministic optimal control problem for $\{q_{i,t}^c, l_t\}$. The real option value $V_{i,t}$ still has to satisfy the stochastic PDE equation (7), but the boundary conditions are different because the optimal trigger $W^*(P_t, Q_{i,t}, q_{i,t}^c, l_t; N)$ will be determined by the equilibrium of the game. The real option will expire at time T.

The optimal trigger threshold $(P^*(Q_i), Q_i)$ which solves equation (9) for the non-cooperative producers can be achieved by different combinations of the price and expected reserves $(P^*(Q_i), Q_i)$. The players enter when (P, Q_i) first moves above the threshold $P^*(Q_i)$ so that $P_{\tau_i} \geq P^*(Q_i)$. The optimization of player i's non-cooperative enterprise value, $U_{i,\mathrm{nc}}(P, Q_i, q_i^c; N_{\tau_i})$ is done in two steps.

- Step 1: Solve for $q_i^{c*} = \mathrm{argmax}_{q_i^c} U_{i,\mathrm{nc}}(P, Q_i, q_i^c; N_{\tau_i})$. The solution is $q_i^{c*}(P, Q_i)$

- Step 2: Use $U_{i,\text{nc}}\big(P, Q_i, q_i^{c*}(P, Q_i); N_{\tau_i}\big)$ to solve for the threshold $P^*_{\tau_i}(Q_i)$

In the presence of strategic effect from the competitor, equation (9) for player $i \in \{L, F\}$ have to be solved jointly because the bargaining game and the real option to invest mutually affect each other. The equilibria of the game affect the expected payoff $W(P, Q, q_i^c, l, t; N)$, which affects the optimal exercise trigger of the option. Conversely, the exercise of option which determines the value of P^* and Q_i^*, will affect the expected payoff $W(P^*, Q^*, q_i^{c*}, l^*, t; N)$ which further affects the refinement of the players' strategy space and hence the equilibria of the game.

4.2 The network effect — gains from cooperation

The network effect N is modeled as the reduction in pipeline tolls, one component of the production cost that affects players' cash flow. Economy of scale and network effect of pipeline arise because the average cost of transporting oil or gas in a pipeline decreases as total throughput increases. As discussed in Church and Ware (1999), there are two categories of costs for pipelines which generate network effect.

1. Long-run fixed operating costs: The cost of monitoring workers is a long-run fixed cost due to the indivisibility of workers – a minimum number of monitoring workers is required. This cost is fixed as it is independent of throughput.

2. Capital investment cost

   - *Setup costs:* The expenses associated with the planning, design and installation of pipeline, and the right of way are fixed setup costs.
   - *Volumetric returns to scale:* The costs of steel are proportionate to the surface area. The capacity of the pipeline depends on its volume and the amount of horsepower required. The amount of horsepower required is determined by resistance to flow, which is decreasing in the diameter of the pipeline.

Among these two cost categories, if the total throughput increasing, the long-run fixed operating costs per unit of throughput will decrease, which generates the category 1 network effect N^1. N^1 is monotonic increasing when the total throughput increasing. Hence, producers will get N^1 only when they both produce. In addition, setup costs and volumetric return to scale will generate the category 2 network effect N^2 if the pipeline company is strategic and can anticipate the future exercise of both players. If the pipeline company observes a higher probability that players will be producing together for a certain period of time, it may build a larger pipeline to accommodate both of them. Thus, the producers will get N^2 if the producers can make a commitment to a larger throughput volume.

The pipeline company has to decide whether to build and, if it builds, what the capacity and toll rate should be. For simplicity, we will assume that, based on the information about both producer's initial reserve Q_L, Q_F and production rate q_L, q_F, the pipeline company can estimate and build a pipeline to accommodate the non-cooperative total transportation throughput, $(q_{L,\text{nc}} + q_{F,\text{nc}})$, for the leader and the follower.

$$\text{The actual non-cooperative pipeline throughput} \\ = \begin{cases} q_{L,\text{nc}}(t) & \text{when } t < \tau_F; \\ q_{L,\text{nc}}(t) + q_{F,\text{nc}}(t) & \text{when } t \geq \tau_F. \end{cases}$$

This results a higher pipeline toll rate for the leader before τ_F,[10] and a lower pipeline toll rate (category 1 network effect N^1) for both producers after τ_F as the total throughput transported increases. If the lease contract is negotiated successfully at $\tau_{\text{lease}} < \tau_F$ or even simultaneously at τ_L, the pipeline company sees the producers' commitment, and it will construct a larger pipeline to accommodate this larger cooperative total throughput, $q_{L,\text{coop}}(\tau_{\text{lease}}) + q_{F,\text{coop}}$, which will generate the category 2 network effect, N^2.

$$\text{The actual cooperative pipeline throughput} \\ = \begin{cases} q_{L,\text{coop}}(t) & \text{when } t < \tau_{\text{lease}}; \\ q_{L,\text{coop}}(t) + q_{F,\text{coop}}(t) & \text{when } t \geq \tau_{\text{lease}}. \end{cases}$$

4.3 The follower's individual rationality constraint

4.3.1 Small follower F_s's IR

The small follower can either lease the capacity from the leader at τ_{lease}, or delay further until τ_{F_s} to build its own plant. The small follower gets the

[10]We will suppress the subscript B and S for F if we are not differentiating the F_b from the F_s in the context.

network effect in both cases. The difference is that if it chooses to build its own plant, the benefit of network effect comes only after τ_{F_s} and will end at θ_L when the leader's production ends. Denote this network benefit for small follower that builds its own plant as $N^{\theta_L}_{\tau_{F_s}} = N \cdot \int_{\tau_{F_s}}^{\theta_L} q_{Lt}\, dt$. If it chooses to lease, the lease contract may allow the small follower to start production earlier than τ_{F_s} and small follower will get the network effect in the interval $[\tau_{\text{lease}}, \theta_L]$. Denote this network benefit for the small follower who leases the plant as $N^{\theta_L}_{\tau_{\text{lease}}} = N \cdot \int_{\tau_{\text{lease}}}^{\theta_L} q_{Lt}\, dt$. Clearly, $N^{\theta_L}_{\tau_{\text{lease}}} > N^{\theta_L}_{\tau_{F_s}}$ as $\tau_{\text{lease}} < \tau_{F_s}$. Therefore, for small follower, the lease contract not only saves its capital investment,[11] but also increases the total amount of network effect received. The small follower will make the comparison of $U_{F_s,\text{nc}}$ and $U_{F_s,\text{coop}}$ at the date after τ_L whenever the leader offers a lease at rate l. Thus, we have the small follower's *participation constraint*:

$$U_{F_s,\text{coop}}(P, Q_{F_s}, \bar{l}_F; N^{\theta_L}_{\tau_{\text{lease}}}) \geq U_{F_s,\text{nc}}(P, Q_{F_s}, q^{c*}_{F_s}; N^{\theta_L}_{\tau_{F_s}}) \tag{10}$$

which defines the high type buyer's valuation of lease:

$$\bar{l}_F \equiv \sup\left\{ l_F \in \mathbb{R}^+ : U_{F_s,\text{coop}} \geq U_{F_s,\text{nc}}\big|_{q^c_{F_s} = q^{c*}_{F_s}} \right\} \tag{11}$$

4.3.2 Big follower F_b's IR

Similarly, we have the big follower's *participation constraint*:

$$U_{F_b,\text{coop}}(P, Q_{F_b}, \underline{l}_F; N^{\theta_L}_{\tau_{\text{lease}} = \tau_{F_b}}) \geq U_{F_b,\text{nc}}(P, Q_{F_b}, q^{c*}_{F_b}; N^{\theta_L}_{\tau_{F_b}}) \tag{12}$$

For the big follower, $N^{\theta_L}_{\tau_{\text{lease}}} = N^{\theta_L}_{\tau_{F_b}}$, the lease does not increase its total amount of network effect received, only saves its capital cost. Hence, the low type buyer's valuation of lease:

$$\underline{l}_F \equiv \sup\left\{ l_F \in \mathbb{R}^+ : U_{F_b,\text{coop}} \geq U_{F_b,\text{nc}}\big|_{q^c_{F_b} = q^{c*}_{F_b}} \right\} \tag{13}$$

Notice the right hand sides of equation (11) and (13) are optimized over q^c_F, which means $U_{F,\text{coop}}$ has to be greater than $U_{F,\text{nc}}$ when the follower builds the optimal capacity for itself. Since $U_{F,\text{coop}}$ is decreasing in l, when equation (11) and (13) are binding, they determine a reservation lease rate $\bar{l}_F$ or $\underline{l}_F$ for the small follower or the big follower respectively.

[11] The annual cost of owning an asset over the its entire life is calculated as $\text{EAC}\big(K(q^c_F)\big) = \frac{K(q^c_F)r}{1-(1+r)^{-n}}$.

4.4 The leader's individual rationality constraints

At τ_L, the leader has a non-cooperative optimal capacity q_L^c which maximizes its total non-cooperative enterprise value $U_{L,\text{nc}}(P, Q_L, q_L^c; N_{\tau_F}^{\theta_L})$, where $N_{\tau_F}^{\theta_L} = N \cdot \int_{\tau_F}^{\theta_L} q_{Lt}\, dt$.

$$q_L^{c*} = \underset{q_L^c}{\operatorname{argmax}}\, U_{L,\text{nc}}(P, Q_L, q_L^c; N_{\tau_F}^{\theta_L})$$

Here a non-cooperative leader is a leader who does not consider the future possibility of leasing excess capacity to the follower. Thus $U_{L,\text{nc}}$ function does not involve a lease rate l. The network effect $N_{\tau_F}^{\theta_L}$ occurs when the follower's production starts at τ_F and ends at θ_L. This is different from the leader's cooperative enterprise value $U_{L,\text{coop}}(P, Q_L, q_L^\Omega, l; N_{\tau_\text{lease}}^{\theta_L})$ as defined in equation (9), where $N_{\tau_\text{lease}}^{\theta_L} = N \cdot \int_{\tau_\text{lease}}^{\theta_L} q_{Lt}\, dt$. This early network effect $N_{\tau_\text{lease}}^{\theta_L}$ occurs when the follower's production starts at τ_lease and ends at θ_L. We now define the leader's cooperative optimal capacity as:

$$\begin{aligned} q_L^{\Omega*} = & \underset{q_L^\Omega}{\operatorname{argmax}}\, U_{L,\text{coop}}(P, Q_L, q_L^\Omega, l; N_{\tau_\text{lease}}^{\theta_L}) \\ & \text{st.} \quad \tau_\text{lease} \le \tau_F \end{aligned} \tag{14}$$

The leader will build cooperative capacity if the following *individual rationality or participation constraint I* (IR_I) is satisfied:

$$U_{L,\text{coop}}(P, Q_L, q_L^{\Omega*}, l; N_{\tau_\text{lease}}^{\theta_L}) \ge U_{L,\text{nc}}(P, Q_L, q_L^{c*}; N_{\tau_F}^{\theta_L}) \tag{15}$$

Moreover, the leader's additional cost of building extra capacity $(q_L^\Omega - q_L^c)$ has to be compensated by the present value of all future leasing fees, plus the benefit difference between $N_{\tau_\text{lease}}^{\theta_L}$ and $N_{\tau_F}^{\theta_L}$, i.e., the leader's *participation constraint II* (IR_II):

$$\begin{aligned} & \int_{\tau_\text{lease}}^{\theta_F} e^{-rt}(q_\text{FL} \cdot l)\, dt + \left(N_{\tau_\text{lease}}^{\theta_L} - N_{\tau_F}^{\theta_L}\right) \ge b \cdot (q_L^\Omega - q_L^c) \\ \Rightarrow & \int_{\tau_\text{lease}}^{\theta_F} e^{-rt}(q_\text{FL} \cdot l)\, dt + N \cdot \int_{\tau_\text{lease}}^{\tau_F} q_{Lt}\, dt \ge b(q_L^\Omega - q_L^c) \end{aligned} \tag{16}$$

If inequalities (15) and (16) are binding, they determine the leader's cooperative capacity q_L^Ω and the lease rate l. Otherwise, they set the upper bound for q_L^Ω and lower bound for l.

If the follower is the high type F_s, the leader obtains an increase in network effect. Equation (16) then becomes:

$$\int_{\tau_{\text{lease}}}^{\theta_{F_s}} e^{-rt}(q_{\text{FL}} \cdot \bar{l}_F)\ dt + N \cdot \int_{\tau_{\text{lease}}}^{\tau_{F_s}} q_{Lt}\ dt \geq b(q_L^{\Omega} - q_L^c) \tag{17}$$

If the follower is the low type F_b, the leader obtains no increase in network effect by encouraging F_b to lease because $\tau_{F_b} = \tau_L$, and $\tau_{\text{lease}} = \tau_L \implies N^{\theta_L}_{\tau_{\text{lease}}} = N^{\theta_L}_{\tau_F}$. If F_b accepts the lease, it saves the capital cost of $K(q^c_{F_b})$. Equation (16) then becomes:

$$\int_{\tau_{\text{lease}}=\tau_L=\tau_{F_b}}^{\theta_{F_b}} e^{-rt}(q_{\text{FL}} \cdot \underline{l}_F)\ dt \geq b(q_L^{\Omega} - q_L^c) \tag{18}$$

In other words, $\bar{l}_F$ and $\underline{l}_F$ defined in equation (11) and (13) have to satisfy equation (17) and (18) respectively, in order to give the leader enough motivation to build extra capacity.

Also, it makes no sense for the leader to build cooperative capacity that cannot be used when production is at a maximum, so by (1), $q_L^{\Omega} \leq \overline{q}_{L,\tau_L} + \overline{q}_{F,\tau_L} = \overline{\alpha}_L Q_{L,\tau_L} + \overline{\alpha}_F Q_{F,\tau_L}$. If this inequality is strict, the joint production is constrained until the leader and follower have produced enough so that their combined maximum production rate is below the plant capacity. The leader's cooperative capacity has to be at least as large as its own maximum production rate, i.e., $q_L^{\Omega} \geq q_{L,\tau_L}$.

4.5 The leader's control set $\{q_L^{\Omega}, l\}$

Recall that q_L and q_F[12] are defined as the leader's and the follower's production volume respectively, q_L^c is the leader's non-cooperative capacity and $\overline{\alpha}_L$ and $\overline{\alpha}_F$ are the maximum production rates that are set by a regulator or technological constraints. From equation (4) we have the non-cooperative leader and follower's production function as:

$$q_{L,\text{nc}}(t) = \begin{cases} q_L^c & t \in [\tau_L, \theta_{L,\text{trans}}] \\ \overline{\alpha}_L Q_L(\theta_{L,\text{trans}}) e^{-\overline{\alpha}_L(t-\theta_{L,\text{trans}})} & t \in [\theta_{L,\text{trans}}, \theta_L] \end{cases} \tag{19}$$

[12]For notation simplicity, we suppress the subscripts S and B for F in this subsection as F_b and F_s's production functions share the same functional form.

and

$$q_{F,\text{nc}}(t) = \begin{cases} q_F^c & t \in [\tau_L, \theta_{F,\text{trans}}] \\ \overline{\alpha}_F Q_F(\theta_{F,\text{trans}}) e^{-\overline{\alpha}_F (t-\theta_{F,\text{trans}})} & t \in [\theta_{F,\text{trans}}, \theta_F] \end{cases} \tag{20}$$

After $\theta_{L,\text{trans}}$, the non-cooperative leader's capacity is not binding, and it can offer the follower its excess processing capacity $q_L^c - q_L$ providing the follower has not built its own plant yet.

Thus, we have the cooperative follower's production volume under leasing:

$$q_{F,\text{coop}} = \min\{q_L^c - q_L,\ \overline{\alpha}_F Q_F\}$$

Suppose that there is asymmetric information about the leader's and follower's initial reserves. The leader can only make an estimation about the follower's expected initial reserve quantity Q_F and maximum production rate and $\overline{\alpha}_F$. Based on this estimation, the leader builds a gas plant which can process the amount $q_L^\Omega \geq q_{L,\text{coop}} + q_{F,\text{coop}}$ per unit of time. The results of the bargaining game depend on the amount information of available to the leader and the follower. The cooperative leader will estimate both producers' needs and builds a gas plant with capacity $q_L^\Omega \geq q_L^c$. Therefore, the above production functions becomes:[13]

$$\begin{aligned} q_{F,\text{coop}} &= \min\{q_L^\Omega - q_{L,\text{coop}},\ \overline{\alpha}_F Q_F\} \\ 0 \leq q_{L,\text{coop}} &\leq \min\{q_L^\Omega, \overline{\alpha}_L Q_L\} \end{aligned} \tag{21}$$

The cooperative leader has an excess capacity of $q_L^\Omega - q_{L,\text{coop}}$, which will increase as the leader's production volume $q_{L,\text{coop}}$ falls over time. Assume that the cooperative follower will use all the capacity offered in the lease until reserves drop to constrain the production rate. That is, $q_{F,\text{coop}} = \min\{q_{\text{FL}}, \overline{\alpha}_F Q_F\}$. Once excess capacity reaches the contracted leasing capacity q_{FL} at τ_{lease}, the lease can start. The cooperative production function is:

$$q_{L,\text{coop}}(t) = \begin{cases} q_L^\Omega & t \in [\tau_L, \theta_{L,\text{trans}}] \\ \overline{\alpha}_L Q_L(\theta_{L,\text{trans}}) e^{-\overline{\alpha}_L (t-\theta_{L,\text{trans}})} & t \in [\theta_{L,\text{trans}}, \theta_L] \end{cases} \tag{22}$$

[13]The production volume for the leader might be set at the upper constraint in equation (1), but it is also possible that the leader will constrain production to induce the follower to enter, so it may also negotiate with the follower on the time-profile of gas plant capacity offered, as well as the lease rate.

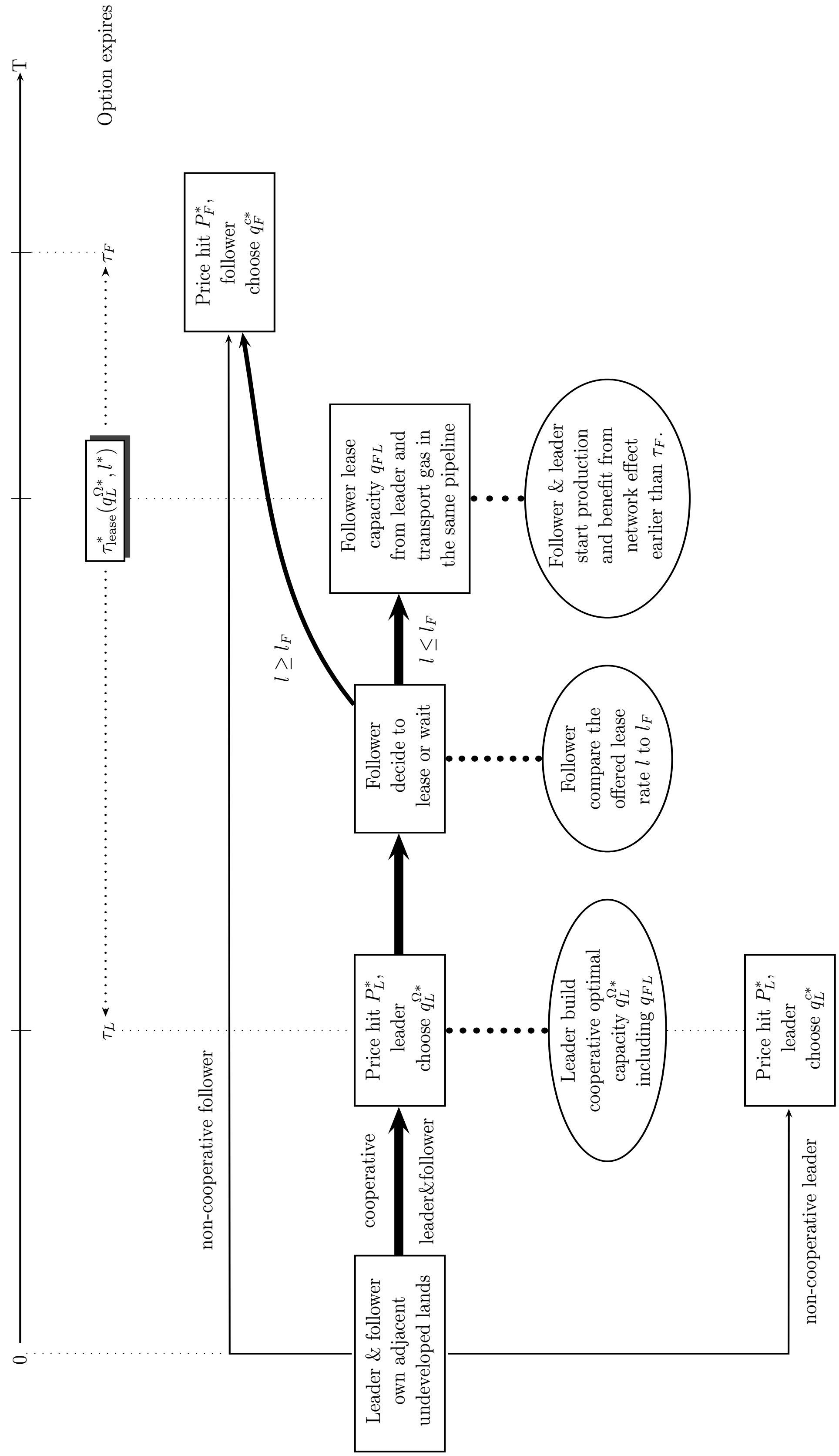
and

$$q_{F,\text{coop}}(t) = \begin{cases} q_{\text{FL}} & t \in [\tau_{\text{lease}}, \theta_{F,\text{trans}}] \\ \overline{\alpha}_F Q_F(\theta_{F,\text{trans}}) e^{-\overline{\alpha}_F (t-\theta_{F,\text{trans}})} & t \in [\theta_{F,\text{trans}}, \theta_F] \end{cases} \tag{23}$$

The cooperative leader's choices about q_L^{Ω} and l will have opposite effects on τ_{lease}. On one hand, the cooperative leader can control an early or late τ_{lease} by controlling the size of its cooperative capacity q_L^{Ω}. When q_L^{Ω} is larger, the lease can happen earlier. The earlier lease will allow the cooperative leader to benefit from the network effect earlier than τ_F. The incremental benefit of this earlier network effect is calculated as $N\big(\int_{\tau_{\text{lease}}}^{\theta_L} q_{Lt}\ dt - \int_{\tau_F}^{\theta_L} q_{Lt}\ dt\big)$ in equation (16). On the other hand, the cooperative leader wants to charge the follower the highest leasing rate up to $\overline{l}_F$ for a small follower or $\underline{l}_F$ for a big follower as defined by equation (11) and (13).[14] Thus, the lease offer is inversely related to the time the lease is accepted. The cooperative leader's objective is to find a balance among the incremental network effect benefit, the earlier leasing fee, and the extra construction costs of $q_L^{\Omega} - q_L^{c}$, bearing in mind the fact that a high lease rate will cause the follower to delay. Denote this equilibrium leader's cooperative capacity as $q_L^{\Omega *}$ which gives the leader largest total enterprise value and also ensures $\tau^*_{\text{lease}} \leq \tau_F$, as defined in equation (14).

In addition, both the leader and the follower will have to consider how much pipeline space to request and the term of the request. If the producer(s) commit(s) to a larger volume or longer-term contract, the pipeline toll rates will be even smaller, generating a category 2 network effect as discussed in Section 4.2. The leader and the follower's strategy map is shown in Figure 2.

[14] In fact, this is the standard way of extracting rents through price discrimination without losing the efficiency.



T
Option expires
0
τ_L
$\tau_{\text{lease}}^*(q_L^{\Omega *}, l^*)$
τ_F
Leader & follower own adjacent undeveloped lands
cooperative leader&follower
non-cooperative follower
non-cooperative leader
Price hit P_L^*, leader choose $q_L^{\Omega *}$
Leader build cooperative optimal capacity $q_L^{\Omega *}$ including q_{FL}
Price hit P_L^*, leader choose q_L^{c*}
Follower decide to lease or wait
Follower compare the offered lease rate l to l_F
$l \leq l_F$
$l \geq l_F$
Follower lease capacity q_{FL} from leader and transport gas in the same pipeline
Follower & leader start production and benefit from network effect earlier than τ_F.
Price hit P_F^*, follower choose q_F^{c*}

4.6 The leader's and the follower's cash flows and expected payoff

We denote the same variable production cost as C for the leader and the follower, including the pipeline tolls. The network effect N is the toll reduction that arises from transporting larger amount of oil and gas with smaller unit breakeven toll rates.

4.6.1 Non-cooperative leader and small follower

In this case, the leader and the small follower each build up a gas plant to process their own gas separately. The leader builds a plant only large enough to process its own gas. The small follower enters later and builds its own plant. The leader will not get the network effect until the small follower also start producing. The leader builds at the stopping time $\tau_L \geq 0$ and the small follower builds at $\tau_{F_s} > \tau_L$.

Stage 1: $t \in (\tau_L, \tau_{F_s})$, only the leader produces The leader has started production but the small follower is still waiting. The network effect does not exist at this stage because the pipeline can only charge the leader. The operating profit is

$$\pi^{S1}_{L,\mathrm{nc},t|F_s} = (P_t - C)q_{L,\mathrm{nc},t} \quad , t \in (\tau_L, \tau_{F_s})$$

where $q_{L,\mathrm{nc},t}$ is defined in equation (19). The risk-neutral expected payoff to the leader is

$$W^{S1}_{L,\mathrm{nc},\tau_L|F_s} = \widehat{E}_{\tau_L} \int_{\tau_L}^{\tau_{F_s}} e^{-rt} \pi^{S1}_{L,\mathrm{nc},t|F_s} \, dt$$

where the $\widehat{E}_t$ is the risk-neutral expectation conditional on information available at time t. The small follower has not built yet in this stage and therefore its cashflow is zero.

Stage 2: $t \in (\tau_{F_s}, \theta_L)$, the leader and the small follower both produce The small follower enters at τ_{F_s}, but can only ship gas in the residual space on the pipeline, which was built to accommodate non-cooperative total throughput. The leader and the small follower will get the network effect in this stage, and their cash flows will be:

$$\pi^{S2}_{L,\text{nc},t|F_s} = (P_t - C + N)q_{L,\text{nc},t} \quad , t \in (\tau_{F_s}, \theta_L)$$
$$\pi^{S2}_{F_s,\text{nc},t} = (P_t - C + N)q_{F_s,\text{nc},t} \quad , t \in (\tau_{F_s}, \theta_L)$$

where $q_{F_s,\text{nc},t}$ is defined in equation (20) if substituting F with F_s. The expected payoff to the leader and the small follower are:

$$W^{S2}_{L,\text{nc},\tau_{F_s}|F_s} = \widehat{E}_{\tau_{F_s}} \int_{\tau_{F_s}}^{\theta_L} e^{-rt} \pi^{S2}_{L,\text{nc},t|F_s} dt$$
$$\text{and} \quad W^{S2}_{F_s,\text{nc},\tau_{F_s}} = \widehat{E}_{\tau_{F_s}} \int_{\tau_{F_s}}^{\theta_L} e^{-rt} \pi^{S2}_{F_s,\text{nc},t} dt$$

Stage 3: $t \in (\theta_L, \theta_{F_s})$, the leader's production ends and only the small follower remains in production The leader's production ends at θ_L the small follower's production ends at θ_{F_s}. We assume that the leader and follower take the same amount of time to deplete their fields. Thus $\theta_L - \tau_L = \theta_{F_s} - \tau_{F_s}$. As the leader's production starts earlier, we have $\theta_L < \theta_{F_s}$. The follower's cash flow and expected payoff are:

$$\pi^{S3}_{F_s,\text{nc},t} = (P_t - C)q_{F_s,\text{nc},t} \quad , t \in (\theta_L, \theta_{F_s})$$
$$W^{S3}_{F_s,\text{nc},\theta_L} = \widehat{E}_{\theta_L} \int_{\theta_L}^{\theta_{F_s}} e^{-rt} \pi^{S3}_{F_s,\text{nc},t} dt$$

To sum up, the non-cooperative leader and small follower's total expected payoff from all three stages are:

$$W_{L,\text{nc}|F_s} = \widehat{E}_0 \Bigg(\widehat{E}_{\tau_L} \int_{\tau_L}^{\tau_{F_s}} e^{-rt} \pi^{S1}_{L,\text{nc},t|F_s} \, dt + e^{-r(\tau_{F_s} - \tau_L)} \widehat{E}_{\tau_{F_s}} \int_{\tau_{F_s}}^{\theta_L} e^{-rt} \pi^{S2}_{L,\text{nc},t|F_s} dt - K(q^c_L) \Bigg)$$

and

$$W_{F_s,\text{nc}} = \widehat{E}_0 \Bigg(\widehat{E}_{\tau_{F_s}} \int_{\tau_{F_s}}^{\theta_L} e^{-rt} \pi^{S2}_{F_s,\text{nc},t} dt + e^{-r(\theta_L - \tau_{F_s})} \widehat{E}_{\theta_L} \int_{\theta_L}^{\theta_{F_s}} e^{-rt} \pi^{S3}_{F_s,\text{nc},t} dt - K(q^c_{F_s}) \Bigg)$$

4.6.2 Non-cooperative leader and big follower

In this case, the leader and the big follower exercise their real option to invest simultaneously at $\tau_L = \tau_{F_b}$. They each build up a gas plant to process their own gas separately. They will get the network effect during the whole production life, and their cash flows will be:

$$\pi_{L,\text{nc},t|F_b} = (P_t - C + N)q_{L,\text{nc},t} \quad , t \in (\tau_L, \theta_L)$$
$$\pi_{F_b,\text{nc},t} = (P_t - C + N)q_{F_b,\text{nc},t} \quad , t \in (\tau_{F_b}, \theta_L)$$

where $q_{F_b,\text{nc},t}$ is defined in equation (20) if substituting F with F_b. The expected payoff to the leader and the big follower are:

$$W_{L,\text{nc},t|F_b} = \widehat{E}_0\left(\widehat{E}_{\tau_L}\int_{\tau_L}^{\theta_L} e^{-rt}\pi_{L,\text{nc},t|F_b}dt - K(q_L^c)\right) \tag{24}$$

$$\text{and} \quad W_{F_b,\text{nc},t} = \widehat{E}_0\left(\widehat{E}_{\tau_{F_b}}\int_{\tau_{F_b}}^{\theta_L} e^{-rt}\pi_{F_b,\text{nc},t}dt - K(q_{F_b}^c)\right) \tag{25}$$

4.6.3 Cooperative leader and small follower

Stage 1: $t \in (\tau_L, \tau_{\text{lease}})$, only the leader produces As discussed in Section 4.5, the leader may want to build a bigger gas plant of cooperative capacity q_L^Ω with construction costs $K(q_L^\Omega)$. It then offers to lease the excess processing capacity to the small follower at a processing rate l. The leader's cash flow and risk-neutral expected payoff:

$$\pi^{S1}_{L,\text{coop},t|F_s} = (P_t - C)q_{L,\text{coop},t} \quad , t \in (\tau_L, \tau_{\text{lease}})$$
$$W^{S1}_{L,\text{coop},\tau_L|F_s} = \widehat{E}_{\tau_L}\int_{\tau_L}^{\tau_{\text{lease}}} e^{-rt}\pi^{S1}_{L,\text{coop},t|F_s}dt$$

where $q_{L,\text{coop},t}$ is defined in equation (22). The lease has not started and the small follower is waiting in this stage.

Stage 2: $t \in (\tau_{\text{lease}}, \theta_L)$, the lease starts, the leader and the small follower both produce In this stage, the small follower agrees to lease the plant capacity from the leader. They both produce and receive the network effect. The cash flows to the leader and the small follower are:

$$\pi^{S2}_{L,\text{coop},t|F_s} = (P_t - C + N)q_{L,\text{coop},t} + q_{\text{FL}}l \quad , t \in (\tau_{\text{lease}}, \theta_L)$$
$$\pi^{S2}_{F_s,\text{coop},t} = (P_t - C + N)q_{F_s,\text{coop},t} - q_{\text{FL}}l \quad , t \in (\tau_{\text{lease}}, \theta_L)$$

where $q_{F_s,\text{coop},t}$ is defined in equation (23) if substituting F with F_s. Their expected payoffs are:

$$W^{S2}_{L,\text{coop},\tau_{\text{lease}}|F_s} = \widehat{E}_{\tau_{\text{lease}}} \int_{\tau_{\text{lease}}}^{\theta_L} e^{-rt} \pi^{S2}_{L,\text{coop},t|F_s} dt$$
$$\text{and} \quad W^{S2}_{F_s,\text{coop},\tau_{\text{lease}}} = \widehat{E}_{\tau_{\text{lease}}} \int_{\tau_{\text{lease}}}^{\theta_L} e^{-rt} \pi^{S2}_{F_s,\text{coop},t} dt$$

Stage 3: $t \in (\theta_L, \theta_{F_s})$, the leader's production ends and only the small follower produces Similarly, the leader's production ends at θ_L, and the small follower continues until θ_{F_s}. The do not receive the network effect. The leader still receives the leasing fee. The leader's cash flow and expected payoff are:

$$\pi^{S3}_{L,\text{coop},t|F_s} = q_{\text{FL}} l$$
$$W^{S3}_{L,\text{coop},\theta_L|F_s} = \int_{\theta_L}^{\theta_{F_s}} e^{-rt} \pi^{S3}_{L,\text{coop},t|F_s} dt = \int_{\theta_L}^{\theta_{F_s}} e^{-rt} q_{\text{FL}} l dt$$

The small follower's cash flow and expected payoff are:

$$\pi^{S3}_{F_s,\text{coop},t} = (P_t - C) q_{F_s,\text{coop},t} - q_{\text{FL}} l \quad , t \in (\theta_L, \theta_{F_s})$$
$$W^{S3}_{F_s,\text{coop},\theta_L} = \widehat{E}_{\theta_L} \int_{\theta_L}^{\theta_{F_s}} e^{-rt} \pi^{S3}_{F_s,\text{coop},t} dt$$

To sum up, the cooperative leader and small follower's total expected payoff from all three stages are:

$$W_{L,\text{coop}|F_s} = \widehat{E}_0 \Bigg(\widehat{E}_{\tau_L} \int_{\tau_L}^{\tau_{\text{lease}}} e^{-rt} \pi^{S1}_{L,\text{coop},t} \, dt$$
$$+ e^{-r(\tau_{\text{lease}} - \tau_L)} \widehat{E}_{\tau_{\text{lease}}} \int_{\tau_{\text{lease}}}^{\theta_L} e^{-rt} \pi^{S2}_{L,\text{coop},t} dt + e^{-r(\theta_L - \tau_L)} \int_{\theta_L}^{\theta_{F_s}} e^{-rt} q_{\text{FL}} l dt - K(q^{\Omega}_L) \Bigg)$$

and

$$W_{F_s,\text{coop}} = \widehat{E}_0 \Bigg(e^{-r(\tau_{\text{lease}} - \tau_L)} \widehat{E}_{\tau_{\text{lease}}} \int_{\tau_{\text{lease}}}^{\theta_L} e^{-rt} \pi^{S2}_{F_s,\text{coop},t} dt +$$
$$e^{-r(\theta_L - \tau_L)} \widehat{E}_{\theta_L} \int_{\theta_L}^{\theta_{F_s}} e^{-rt} \pi^{S3}_{F_s,\text{coop},t} dt \Bigg)$$

4.6.4 Cooperative leader and big follower

The big follower's IR constraint ensures $\tau_{\text{lease}} \leq \tau_L$. So stage (τ_L, θ_L) converges to stage $(\tau_{\text{lease}}, \theta_L)$ in equilibrium. In this stage, the big follower agrees to lease the plant capacity from the leader. They both produce and receive the network effect. The cash flows to the leader and the big follower are:

$$\begin{aligned} \pi_{L,\text{coop},t|F_b} &= (P_t - C + N) q_{L,\text{coop},t} + q_{\text{FL}} l \quad , t \in (\tau_{\text{lease}}, \theta_L) \\ \pi_{F_b,\text{coop},t} &= (P_t - C + N) q_{F_b,\text{coop},t} - q_{\text{FL}} l \quad , t \in (\tau_{\text{lease}}, \theta_L) \end{aligned}$$

where $q_{F_b,\text{coop},t}$ is defined in equation (23) if substituting F with F_b. Their expected payoffs are:

$$\begin{aligned} W_{L,\text{coop},\tau_{\text{lease}}|F_b} &= \widehat{E}_{\tau_{\text{lease}}} \int_{\tau_{\text{lease}}}^{\theta_L} e^{-rt} \pi_{L,\text{coop},t|F_b} dt \\ \text{and} \quad W_{F_b,\text{coop},\tau_{\text{lease}}} &= \widehat{E}_{\tau_{\text{lease}}} \int_{\tau_{\text{lease}}}^{\theta_L} e^{-rt} \pi_{F_b,\text{coop},t} dt \end{aligned}$$

5 The perfect Bayesian equilibrium

We will extend the backward induction solution for a real option to this game theory setting as in Grenadier (1996); Garlappi (2001); Murto et al. (2003); Imai and Watanabe (2005). This provides a simple computation of a subgame-perfect Nash equilibrium. After explicitly analyzing the player's beliefs, i.e., ruling out incredible threats and promises, we develop a perfect Bayesian equilibrium for this dynamic bargaining game under incomplete information using Coasian Dynamics as discussed in Fudenberg and Tirole (1991, Ch 10). We assume the leader is chosen exogenously, because one of the two companies has a comparative advantage for entering early (e.g. has a larger reserve[15] or a reserve that has lower drilling costs), and that it naturally moves first.

The enterprise value of the leader (plant lessor or "seller") is common knowledge. The incomplete information aspect of the sequential bargaining is limited to the uncertainty the leader faces about the reservation lease rate of the follower (buyer). As defined in equation (11) and (13), the high type buyer F_s has a reservation lease rate of $\bar{l}_F$ and the low type buyer F_b has

[15] In Section 6.1, we shall see that larger reserve quantity will subsidize the trigger price, which gives a smaller trigger value $P_i^*(Q_i)$ and $i \in \{A, B\}$.

a reservation lease rate of $\underline{l}_F$. If the high type buyer tells the truth, its total enterprise value is $U_{F_s}(P, Q_{F_s}, \bar{l}_F; N^{\theta_L}_{\tau_{\text{lease}}})$. If the high type buyer lies successfully, its total enterprise value is $U_{F_s}(P, Q_{F_s}, \underline{l}_F; N^{\theta_L}_{\tau_{\text{lease}}})$. Since $\bar{l}_F > \underline{l}_F$ and U_{F_s} decreases on l we have

$$U_{F_s}(P, Q_{F_s}, \bar{l}_F; N^{\theta_L}_{\tau_{\text{lease}}}) < U_{F_s}(P, Q_{F_s}, \underline{l}_F; N^{\theta_L}_{\tau_{\text{lease}}}) \tag{26}$$

Thus, the high type buyer F_s is motivated to pretend to be the low type buyer F_b. In addition, notice that the follower's valuation is correlated with the leader's cost. A larger plant will allow the lease to start earlier, because of the extra capacity as noted in the discussion after equation (21). This makes a more valuable network effect, which increases the follower reservation lease rates $\bar{l}_F$ and $\underline{l}_F$. But a larger plant also incurs larger construction costs. The leader's objective is to extract maximum rents through price discrimination without losing efficiency. The leader wants the follower to accept the lease offer so that the network effect is larger.

We now consider the equilibrium of this game in a two period case. Let $t \in \{t, t+1\}$. The ex ante unconditional probability that the follower is high type (F_s) is $\overline{p}$, and $\underline{p} = 1 - \overline{p}$ is the probability that the follower is low type (F_b).

The leader offers lease rates l_t and l_{t+1} at time t and time $t+1$, respectively. Let $\overline{\eta}(l_t)$ denote the leader's posterior probability belief that the follower is high type (F_s) conditional on the rejection of offer l_t in period t, and define $\underline{\eta}(l_t) \equiv 1 - \overline{\eta}(l_t)$. The extensive form representation of this sequential bargaining game is shown in Figure 14 in Appendix B.

Definition 1. *Define the leader's critical probability as* $\chi \equiv \frac{U_L(\underline{l}_F)}{U_L(\bar{l}_F)}$.

In the last period $t+1$, the leader with probability belief $\overline{\eta}(l_t)$ makes a "take it or leave it" offer l_{t+1}. The follower will accept if and only if this l_{t+1} is not greater than its reservation lease rate.

Theorem 1. *The followers' optimal strategies at date $t+1$ is given by:*

$$\text{If } l_{t+1} = \begin{cases} \underline{l}_F, & \textit{then } F_s, F_b \textit{ both accept} \\ \bar{l}_F, & \textit{then } F_s \textit{ accepts}, F_b \textit{ rejects} \\ \text{Random}[\underline{l}_F, \bar{l}_F], & \textit{then } F_s \textit{ accepts}, F_b \textit{ rejects} \end{cases} \tag{27}$$

If the leader offers $l_{t+1} = \underline{l}_F$, both type followers will accept, the leader obtains the enterprise value of $U_{L,\text{coop}}(P, Q_L, q_L^{\Omega}, \underline{l}_F; N^{\theta_L}_{\tau_{\text{lease}}})$, simplified as $U_L(\underline{l}_F)$.

If the leader offers $l_{t+1} = \bar{l}_F$, only the high type follower accepts, so the leader has second period enterprise value of $\overline{\eta} \cdot U_{L,\text{coop}}(P, Q_L, q_L^{\Omega}, \bar{l}_F; N_{\tau_{\text{lease}}}^{\theta_L})$, simplified as $\overline{\eta} \cdot U_L(\bar{l}_F)$.[16]

Theorem 2. *The leader's optimal strategy at date $t+1$ is given by:*

$$l_{t+1} = \begin{cases} \underline{l}_F, & \text{if } \overline{\eta} < \chi \\ \bar{l}_F, & \text{if } \overline{\eta} > \chi \\ \text{Random}[\underline{l}_F, \bar{l}_F], & \text{if } \overline{\eta} = \chi \end{cases} \tag{28}$$

At time t, if the leader offers a lease rate at $l_t = \underline{l}_F$, both type followers will accept. If the leader offers a lease rate at $l_t > \underline{l}_F$, the followers' decisions are more complex.

Definition 2. *Let $y(l_t)$ be the probability that a high type follower F_s accepts l_t. According to the Bayes rule, the leader's posterior probability belief that the follower is high type conditional on the rejection is given by:*

$$\overline{\eta}(l_t) = \frac{\overline{p}\big(1 - y(l_t)\big)}{\overline{p}\big(1 - y(l_t)\big) + \underline{p}}$$

If the leader offers a lease rate at $l_t > \underline{l}_F$, the high type follower F_s should not reject this l_t with probability 1, because that will make the leader's posterior probability belief $\overline{\eta}(l_t)$ greater than χ and the leader will offer a higher second period lease rate at $l_{t+1} = \bar{l}_F$, so the high type F_s would be better off accepting l_t. On the other hand, the high type follower F_s should not accept that l_t with probability 1 either, because that will make the leader's posterior probability belief $\overline{\eta}(l_t)$ less than χ and the leader will offer a lower second period lease rate at $l_{t+1} = \underline{l}_F$, so the high type F_s would be better off rejecting l_t.

Lemma 1. *In equilibrium, when $l_t > \underline{l}_F$, the high type follower has a mixed strategy of randomizing between accept and reject in order to make the leader's posterior belief satisfy $\overline{\eta}(l_t) = \chi$. The leader will offer the second period*

[16] Since all other variables are the same, we shall simplify the cooperative leader and follower's total enterprise value function as $U_L(\underline{l}_F)$, $U_L(\bar{l}_F)$ and $U_F(\underline{l}_F)$ and $U_F(\bar{l}_F)$ throughout this subsection. The non-cooperative leader and follower do not participate in this game and their total enterprise values only helps to define the reservation lease rate.

price l_{t+1} *to be any randomization between* $\underline{l}_F$ *and* $\bar{l}_F$*. Let* $y^*(l_t)$ *denote the equilibrium probability with which the high type* F_s *accepts* l_t*. Then*

$$y^*(l_t) = 1 + \frac{\chi \underline{p}}{\overline{p}(\chi - 1)} \in [0, 1] \tag{29}$$

which satisfies the equilibrium condition $\overline{\eta}(l_t) = \chi$*.*

Since the equilibrium has to be Pareto efficient, in order for the high type follower F_s to be indifferent between accepting and rejecting l_t, we need

Definition 3. *Let* $x(l_t)$ *to be the conditional probability that the high type follower receives the lowest price* $\underline{l}_F$ *at time* $t+1$ *if it rejects* l_t*. Then*

$$x(l_t) = \frac{U_{F_s}(l_t) - U_{F_s}(\bar{l}_F)}{e^{-r}\big(U_{F_s}(\underline{l}_F) - U_{F_s}(\bar{l}_F)\big)}\,. \tag{30}$$

Definition 4. *Let* $\tilde{l}_F$ *be the lease rate at which the high type follower is indifferent between accepting* l_t *and rejecting* l_t *in order to wait for* $l_{t+1} = \underline{l}_F$ *at time* $t+1$*. It is defined implicitly by*

$$U_{F_s}(\tilde{l}_F) = (1 - e^{-r})U_{F_s}(\bar{l}_F) + e^{-r}U_{F_s}(\underline{l}_F)$$

Since the follower's enterprise value function, $U_F(l)$ decreases in l, we now summarize the optimal strategy for the follower at time t.

Theorem 3. *The low type follower only accepts* $\underline{l}_F$*. The high type follower always accepts an offer* $l_t \in [\underline{l}_F, \tilde{l}_F]$*, and accepts an offer* $l_t \in [\tilde{l}_F, \bar{l}_F]$ *with probability* y^**.*

Suppose the leader's one period discount factor is e^{-r}. The next theorem provides the equilibrium strategy for the leader at time t.

Theorem 4. *If there is a preponderance of low type followers, defined as* $\overline{p} < \chi$*, then the leader is pessimistic and its optimal strategy is one of the following:*

$$l_t = \begin{cases} \underline{l}_F, & \text{if } \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} < \dfrac{1 - e^{-r}\underline{p}}{\overline{p}} \\ \tilde{l}_F, & \text{if } \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} > \dfrac{1 - e^{-r}\underline{p}}{\overline{p}} \end{cases} \tag{31}$$

If $\overline{p} > \chi$, the leader is optimistic and the leader's first period optimal strategy is given by one of the following.

$$l_t = \begin{cases} \underline{l}_F, & \text{if } \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} < \frac{1-e^{-r}\underline{p}}{\overline{p}}, \text{ and } \frac{U_L(\bar{l}_F)}{U_L(\underline{l}_F)} < \frac{1-A}{B} \\ \tilde{l}_F, & \text{if } \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} > \frac{1-e^{-r}\underline{p}}{\overline{p}}, \text{ and } BU_L(\bar{l}_F) + (A - e^{-r}\underline{p})U_L(\underline{l}_F) < \overline{p}U_L(\tilde{l}_F) \\ \bar{l}_F, & \text{if } \frac{U_L(\bar{l}_F)}{U_L(\underline{l}_F)} > \frac{1-A}{B}, \text{ and } BU_L(\bar{l}_F) + (A - e^{-r}\underline{p})U_L(\underline{l}_F) > \overline{p}U_L(\tilde{l}_F) \end{cases} \tag{32}$$

where

$$A = e^{-r}\overline{p}(1-y)x + e^{-r}x\underline{p} > 0$$
$$B = \overline{p}y + e^{-r}\overline{p}(1-y)(1-x) > 0$$

The proof of Theorems 1 to 4 are given in Appendix C.

The conclusion is thus that there exists a unique perfect Bayesian equilibrium, and that this equilibrium exhibits Coasian dynamics — that is, $\overline{\eta}(l_t) \leq \overline{p}$ for all l_t, so the leader becomes more pessimistic over time, and $l_{t+1} \leq l_t$, so the leader's lease rate offer decreases over time.

6 Simulation of the bargaining game

6.1 Numerical Solution

This real option game problem has three stochastic variables: commodity price P and expected reserves for the two producers, Q_A, Q_B. Such a three-dimensional problem is not well-suited to numerical solution of the fundamental differential equations, so we will use the least-squares Monte Carlo method to determine the optimal policy. It has been implemented in a real options settings by Broadie and Glasserman (1997); Longstaff and Schwartz (2001); Murto et al. (2003); Gamba (2003). The essence of the technique is to replace the conditional risk-neutral one-step expectation of a binomial lattice model with a conditional expectation formed by regressing realized simulation values on observable variables (price and quantity) known at the start of the time step. With the conditional expectation, one can use the Bellman equation to determine the (approximately) optimal policy at each step. Then, given the optimal policy, the simulation can be run again (or recycled) to calculate the risk-neutral expected values arising from the policy.

The model also generates a sequential game between the two players. Sequential games often generate a large number of equilibria that have to be distinguished by a variety of refinements. However, in this setting, we can impose sequential play by the two players, except at the point where they may develop simultaneously. Even at this point, one of the players will be a natural leader, because one will have larger reserves expectation than the other. Thus, we can reduce the sequential game with simultaneous moves to one with sequential moves. Choosing the Nash Bargaining equilibrium at each point (typically a dominant strategy) will result in a unique solution with subgame-perfect strategies. This point has been established by Garlappi (2001); Murto et al. (2003); Imai and Watanabe (2005).

With the solution to the game, we propose to explore the sensitivity of the threshold boundary manifolds to the parameters faced by the players, compare the results to those of an isolated monopolist making a real options decision and assess the probability of the various game scenarios that can unfold.

The relevant variables may be categorized as:

1. Game-related variables, which players can control and optimize, including leasing rat, l, leasing quantity, q_{FL}, the leader's cooperative plant process capacities, q_L^{Ω}, and the leader and the follower's non-cooperative capacities, q_F^c and q_L^c.

2. Option related variables, which are pure exogenous and players can not control, including price P, initial reserve quantities, Q_F and Q_L, the network savings effect N, and the limiting regulatory or technical production rate $\bar{\alpha}$.

3. Other variables we are not interested, including all the parameters in the cost function (fixed, variable cost coefficients, drilling cost)[17] and maximum production life.

In order to get a clear idea about the comparative statics of these variables, we need to allow them to vary in our model, i.e., set them as a vector instead of a fixed number. Each vector will add one more dimension to our model. We have already have price P, initial reserve quantities Q_F or Q_L, network effect N and lease rate l as vectors. And the dependent variable,

[17] The drilling cost can be set as a linear function of construction cost, but here we take it simply as a fixed number.

the enterprise value is a function of those five vectors. The best thing we can do in a 3D graph is to graph the value function against any two of those vectors every time.

6.2 The comparative statics and the equilibrium reigion of the game

6.2.1 The effect of lease contract and network effect on the follower's decisions

The leasing contract is specified by quantity and lease rate, (q_{FL}, l). For the simplicity at this stage, we set the leasing quantity q_{FL} equal to the leader's excess capacity. That is, the leader will build a total capacity q_L^{Ω}, and use q_L itself. The excess capacity is leased to the follower on a "take-or-pay" basis. That is, the follower pays for $q_{\text{FL}} = q_L^{\Omega} - q_L$ whether it can use it or not.[18]

Figure 3 is the graph showing the exercise of the follower's real option.[19] There is a substantial premium associated with the right to develop early. The follower's initial reserve has very little effect on the real option value for very low commodity prices since the option never gets exercised for such low prices. When the commodity prices are higher, the probability of exercising the option is higher, the option value and sensitivity to reserves are higher. Also, as the network effect N gets larger (moving to the right and down), both the follower's real option value and exercise proceeds become larger. But as the network effect increases, the transition from the dark manifold (the real option value) to the light manifold (the exercise proceeds) falls from around commodity price of 6 to around the price of 4, especially for the larger initial reserve. This means, the exercise of real option becomes more sensitive to the larger initial reserves as network effect increases.

Figure 4 indicates that the network effect has a positive effect on the

[18] We set q_{FL} equal to the government mandated maximum production rate, $\overline{\alpha}$ multiplied the follower's initial reserve Q_F. Over time, as the reserve drops, the follower's production volume could drop below q_{FL}. In fact, the capacity q_{FL} should be an optimized variable.

[19] In our least square Monte-Carlo simulation for estimating option value, we assume the follower's real option to build its own plant has a life of 20 years with quarterly decision. To conserve the consistency and convergence of results, we choose to divide that 20 year option life into 80 time steps and 100 price paths (simulated 50 and another 50 antithetic paths). At any time step of every price path, the follower can exercise the option if it is optimal.

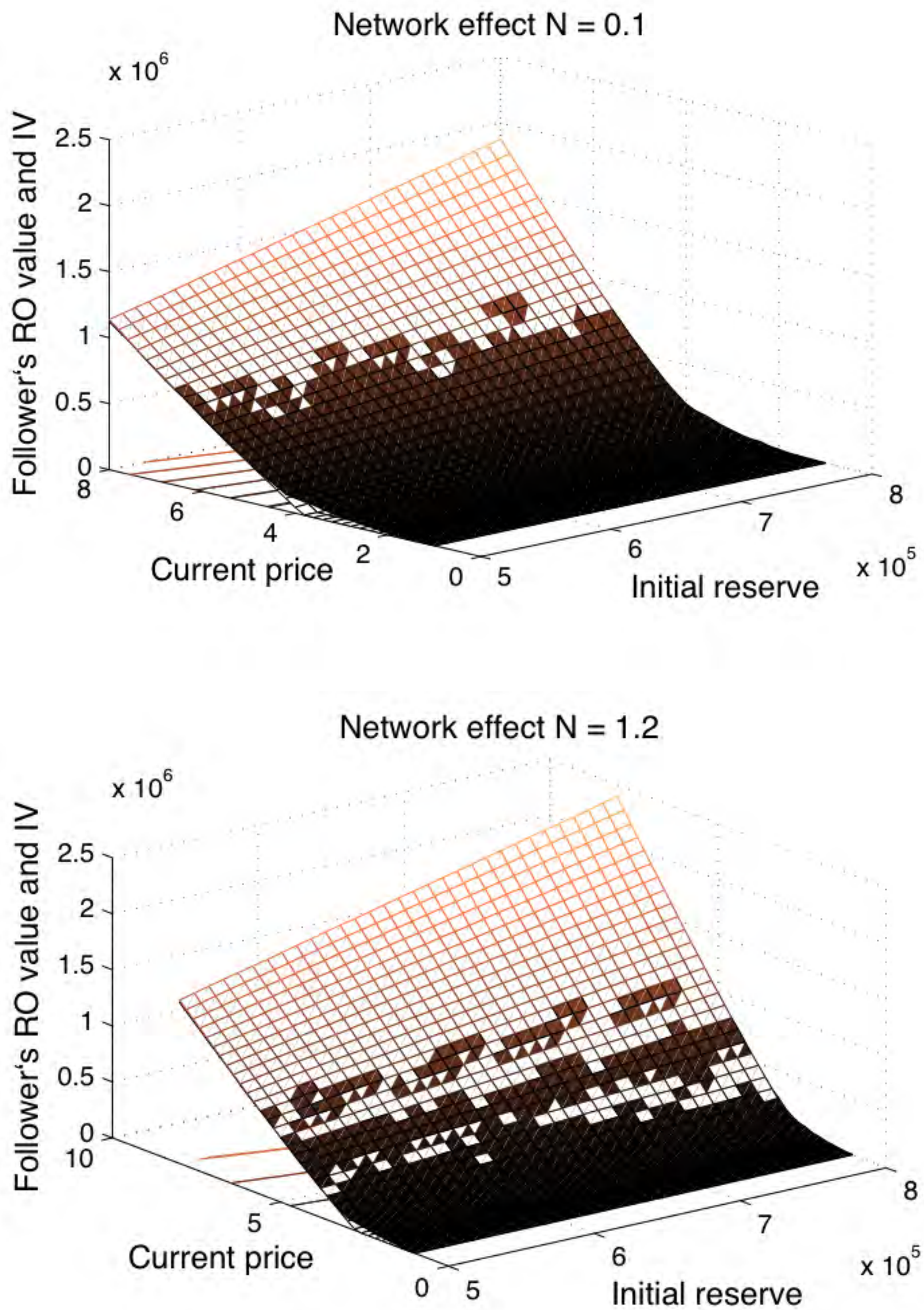


Figure 3: The exercise of the follower's real option and the smooth-pasting condition for different network effect levels. The dark manifold is the real option value and the light manifold is the exercise proceeds. The optimal exercise threshold is at the transition from dark to light. As the network effect increases the follower develops at a lower price threshold.

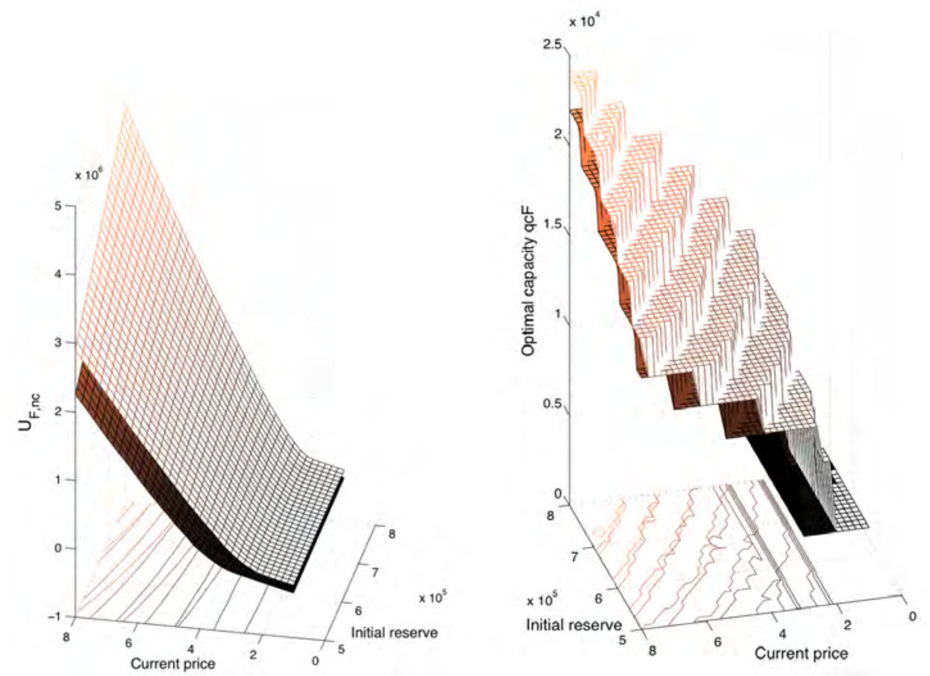


Figure 4: The left panel is the follower's maximum non-cooperative enterprise value $U^*_{F,\mathrm{nc}}$ for minimum (dark shading) and maximum (light shading) network effect levels. The right panel is the follower's optimal capacity choice q^{c*}_F for minimum and maximum network effect, with the same rule for shading. The follower has a larger enterprise value and larger reservation capacity with the larger network effect.

follower's maximum[20] non-cooperative enterprise value, $U^*_{F,\mathrm{nc}}$ and its optimal capacity choice q_F^{c*}. Also notice that the q_F^{c*} manifold is not smooth. This means that a small amount of increase in commodity price and initial reserve will not affect the follower's optimal capacity choice. Only for a large enough increase in commodity price and initial reserve, the follower should build a larger optimal capacity.

Figures 5 shows a sequence of manifolds of the follower's non-cooperative enterprise values $U_{F,\mathrm{nc}}$ and cooperative enterprise values $U_{F,\mathrm{coop}}$. The $U_{F,\mathrm{nc}}$ manifold has more curvature and stays constant whereas the $U_{F,\mathrm{coop}}$ manifold falls as l increase from the minimum of 0.1 to the maximum of 2.5. By comparing the $U_{F,\mathrm{nc}}$ manifold with the $U_{F,\mathrm{coop}}$ manifold, the follower can decide whether to accept the lease offer for various commodity price levels and initial reserve levels. In the top two sub-graphs, where the lease rate is low, the $U_{F,\mathrm{nc}}$ manifold is below the $U_{F,\mathrm{coop}}$ manifold when the commodity price is low. This indicates that for lower commodity price and smaller initial reserve, it is better for the follower to choose lease the plant from the leader. For higher commodity prices and initial reserves, the $U_{F,\mathrm{nc}}$ manifold is above the $U_{F,\mathrm{coop}}$ manifold, so the follower is better off building. In the bottom-left sub-graph ($l = 1.6$), the $U_{F,\mathrm{coop}}$ manifold moves farther below the $U_{F,\mathrm{nc}}$ manifold and they have two cross lines, which shows for extreme low and extreme high commodity price, building-own-plant is better for the follower,[21] only for some middle range of commodity price, accepting the lease is better. As we move to the highest lease rates in the bottom-right sub-graph of Figures 5, the $U_{F,\mathrm{coop}}$ manifold is completely below the $U_{F,\mathrm{nc}}$ manifold, which shows that leasing is infeasible, even for high commodity prices and high initial reserves.

If we look at the sub-graphs in Figure 5 individually, we find that $U_{F,\mathrm{coop}}$ increases linearly in P, holding other variables fixed. The $U_{F,\mathrm{nc}}$ grows non-linearly (convex upward) because it contains the follower's real option value which increases as commodity price increasing. When the commodity price is below the trigger threshold, $U_{F,\mathrm{coop}}$ grows faster. After the trigger, $U_{F,\mathrm{nc}}$ grows faster. Therefore, as commodity price get higher, $U_{F,\mathrm{nc}}$ will finally exceed $U_{F,\mathrm{coop}}$. Hence, the follower's benefit from lease decreases in increasing commodity price P because it loses the real option to delay if it leases.

[20]Recall that $U^*_{F,\mathrm{nc}}$ is achieved by the non-cooperative follower exercising its real option to invest at optimal threshold $P^*(Q_F)$ and choosing the optimal capacity q_F^{c*}

[21]Notice the lower cross line in the bottom-left sub-graph is actually below zero, which means the follower should not build or lease for extremely low commodity price.

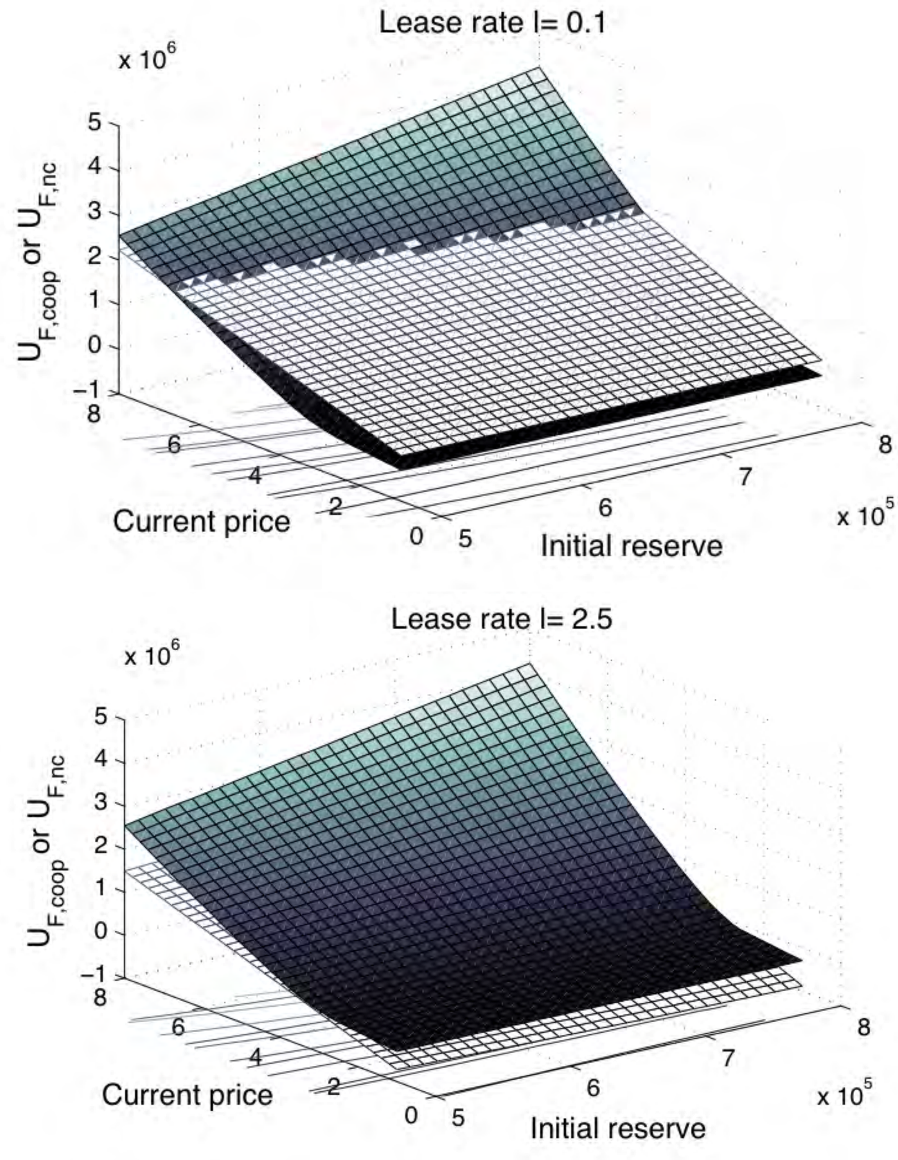


Figure 5: Manifolds of the follower's non-cooperative enterprise values $U_{F,\mathrm{nc}}$ (dark) and cooperative enterprise values $U_{F,\mathrm{coop}}$ (light) as a function of initial reserves Q_F and commodity price P for small (top) and large lease rates l (bottom). The network effect is at its mean value. When the two manifolds cross, it is optimal for the follower to switch strategies from non-cooperative (delay and build) to cooperative (lease from leader). When a low lease rate is offered, it is optimal for the follower to accept teh lease if the gas price is low or reserves are low. When a high lease rate is offered, it is not optimal for the follower to cooperate with the leader.

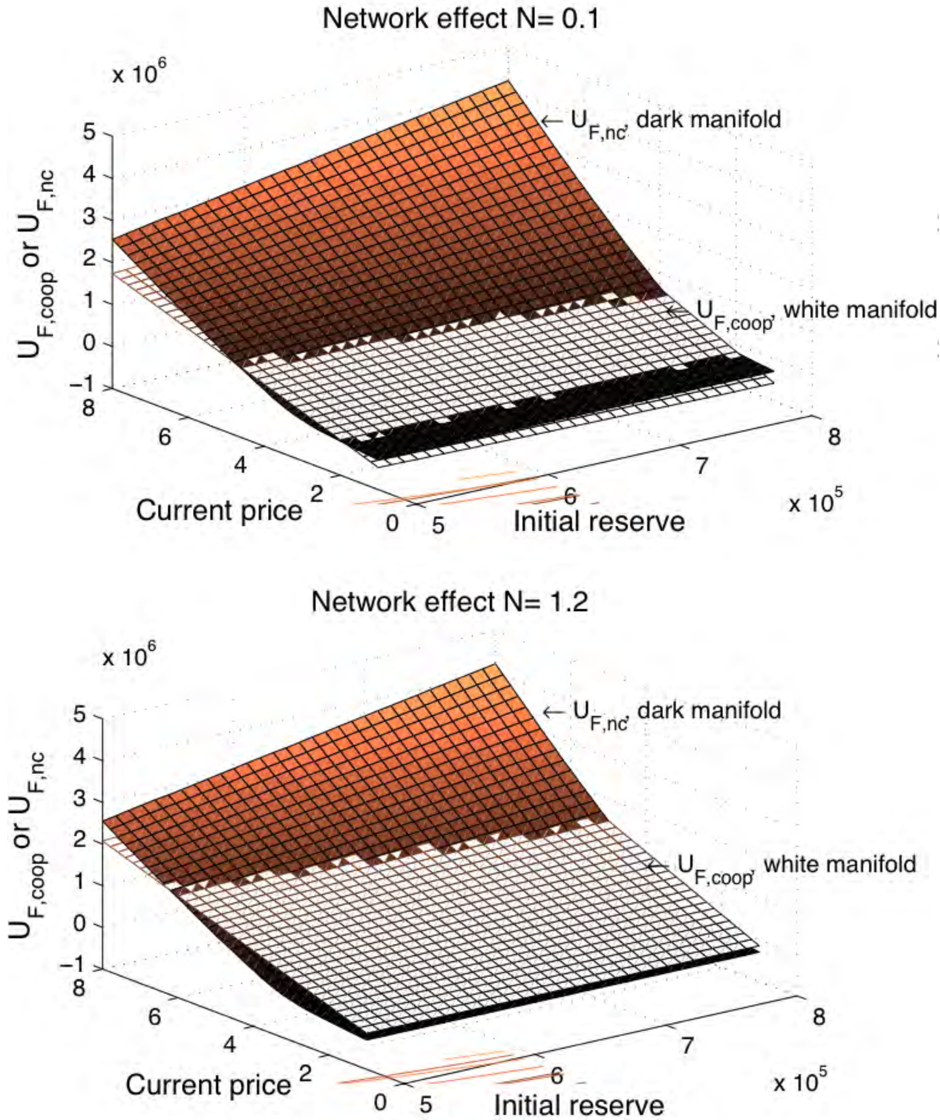


Figure 6: Manifolds of the follower's non-cooperative enterprise values $U_{F,\text{nc}}$ (dark) and cooperative enterprise values $U_{F,\text{coop}}$ (light) as a function of initial reserves Q_F and the commodity price P for small (top and large (bottom) network effect levels. The lease rate is at its mean value. When the manifolds cross, it is optimal for the follower to switch between cooperative and non-cooperative strategies. When the network effect is small, the non-cooperative strategy is optimal for moderate to high commodity prices. When the network effect is large, the cooperative strategy is optimal unless the commodity price is extremely high.

Figure 6 is similar to Figure 5 except that now the network effect level (not the lease rate) increases as we move to the right and down. The region where $U_{F,\text{coop}}$ manifold is above $U_{F,\text{nc}}$ manifold become larger as the network effect level increases. This means there is a larger probability for the follower to accept the lease if the network effect level is higher. Also, the intersection of $U_{F,\text{coop}}$ manifold and $U_{F,\text{nc}}$ manifold shifts up as the network effect level increases. This shows that higher network effect level increases the follower's benefit from lease, and thus $U_{F,\text{nc}}$ needs a higher commodity price level to exceed the $U_{F,\text{coop}}$. In addition, the intersection curve of $U_{F,\text{coop}}$ manifold and $U_{F,\text{nc}}$ manifold incline to the bottom as the initial reserve increases. This means larger initial reserve will make $U_{F,\text{nc}}$ exceed $U_{F,\text{coop}}$ at lower commodity price level. This is because, in the case of follower building-own-plant, the construction cost is relatively fixed as a function of the initial reserves. But the total leasing fee charged by the leader is proportional to the size of initial reserves Q_F. Hence, larger initial reserve makes the follower pay a larger total leasing fee while leaving the construction cost relatively constant, which reduces the lease benefit.

Figure 7 shows the reservation lease rates the follower is willing to accept corresponding to different initial reserves and current gas prices. Firstly, we analyze how the follower's reservation lease rate changes as the commodity price and initial reserve change. Since the two manifolds have similar shape, we can focus on one of them. As commodity price increases, the follower's reservation lease rate first increases then decreases. To understand this, recall that the follower's reservation lease rate is define as $\bar{l}_F \equiv \sup\{l_F \in \mathbb{R}^+ : U_{F,\text{coop}} \geq U_{F,\text{nc}}\}$. In other words, it is equivalent to the distance between $U_{F,\text{coop}}$ and $U_{F,\text{nc}}$. As we observe in Figure 5 and Figure 6, initially $U_{F,\text{coop}} > U_{F,\text{nc}}$, as commodity price increases, both $U_{F,\text{coop}}$ and $U_{F,\text{nc}}$ increase, and $U_{F,\text{coop}}$ increases faster than $U_{F,\text{nc}}$. The distance gets larger. However, above the follower's trigger threshold, $U_{F,\text{nc}}$ increases faster than $U_{F,\text{coop}}$, and the distance becomes smaller, eventually, $U_{F,\text{nc}}$ will catch up (intersect) with and then exceed $U_{F,\text{coop}}$. That is why the follower's reservation lease rate first increase then decrease. Furthermore, one can infer that the follower's peak reservation lease rate is achieved when the distance between $U_{F,\text{coop}}$ and $U_{F,\text{nc}}$ is largest, i.e., the neighbor area below the trigger threshold.

Secondly, the contours of the manifolds shows that the reservation lease rate manifold is not very sensitive to initial reserve for low commodity price, and it becomes more so when the commodity price is high. In fact, for high commodity price, the reservation lease rate decreases as the initial reserve

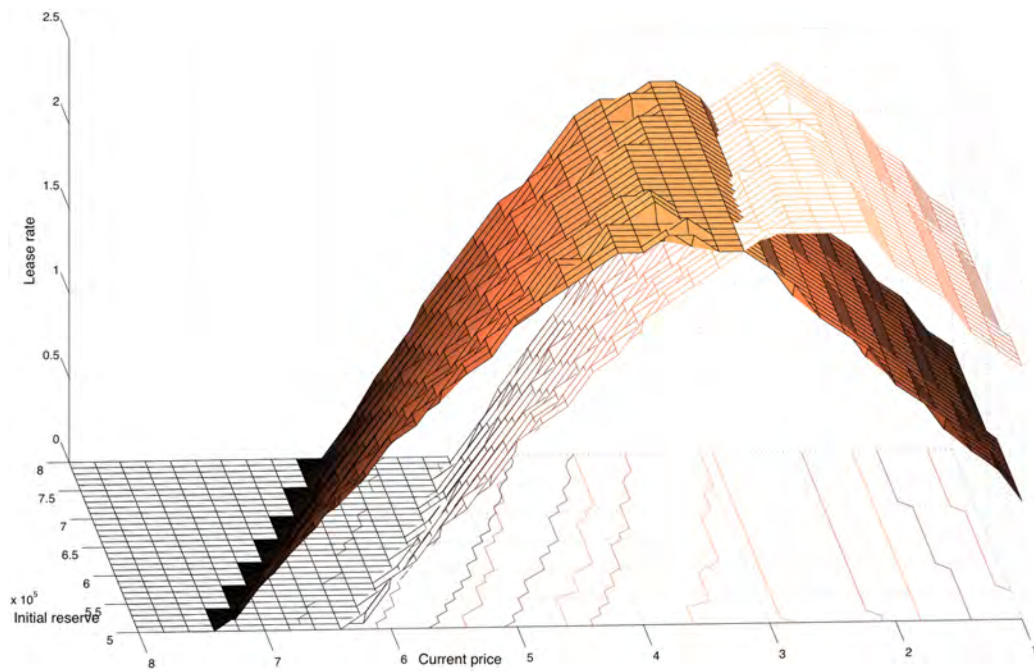


Figure 7: The follower's reservation lease rate for small (dark shading) and large (light shading) network effects as a function of initial reserves and commodity prices. The reservation lease rates are concave in the commodity price because at low commodity prices, the follower doesn't want to extract the resource at all, while at high commodity prices, the follower can afford to build its own plant, using its own timing and scale decisions, ignoring the network benefits of cooperating. When the network effect is small, the follower is more inclined to lease at high commodity prices than when the network effect is large. It reverses to being less inclined to lease when the commodity price is low, suggesting that it would prefer to delay development until the commodity price rises, which means it must opt out of any cooperative leasing arrangement.

increases. This is because larger initial reserve helps $U_{F,\text{nc}}$ exceeds $U_{F,\text{coop}}$ faster, i.e., at lower commodity price, which verifies our discuss for Figure 6.

Thirdly, by comparing the dark manifold (minimum network effect level) with the light manifold (maximum network effect level), we find that before the peak,[22] the high network effect gives the follower a larger reservation lease rate if holding the price level fixed. But after the peak, the high network effect gives the follower a smaller reservation lease rate if holding the price level fixed. This is because before real option being exercised, the network effect is favoring $U_{F,\text{coop}}$ more (making $U_{F,\text{coop}}$ increase faster), whereas after real option being exercised, the network effect is favoring $U_{F,\text{nc}}$ more (making $U_{F,\text{nc}}$ increase faster). The implication for the leader from this observation is that for extremely low commodity price, larger network effect increases the follower's willingness to pay for the lease. For high commodity price, larger network effect decreases the follower's willingness to pay for the lease, certeris paribus.

6.2.2 The effect of the lease contract and network effect on the leader's decisions

The leader has two options:

1. Build a plant with optimal non-cooperative capacity q_L^{c*} to process its own gas only for a construction cost $K(q_L^c)$. The effect of building this small plant on the optimal exercise point is mixed: it could be earlier or later than if a large plant is built.

2. Build a plant with optimal cooperative capacity $(q_L^{\Omega*})$ to process his (q_L) and the follower's gas q_{FL}. The larger plant has a construction cost $K(q_L^{\Omega}) > K(q_L^c)$. The cash flow from this decision is also larger because

   (a) leasing gives a lower toll rate (network effect)

   (b) the leasing fee is a cash inflow to the leader.

As discussed in Section 4.5, the leader wants to find a balance among the incremental network effect benefit, the earlier leasing fee, and the extra con-

[22]Notice the peak is reached below the commodity price of 3, and Figure 3 shows that the optimal option exercise region is between commodity price 4 and 6.

struction costs, $K(q_L^\Omega) - K(q_L^c)$, bearing in mind the fact that a high leasing rate will cause the follower to delay.

Figure 8 is the graph showing the exercise of the leader's real option. Similar to the follower's real option value, the leader's initial reserve has very little effect on its real option value for very low commodity prices since the option never gets exercised for such low prices. When the commodity prices are higher, the probability of exercising the option is higher, the option value and sensitivity to reserves are higher. Also, as the network effect N gets larger (moving to the right and down), both the leader's real option value and exercise proceeds become larger. However, unlike the follower's real option exercise threshold which falls from 6 to 5, the leader's optimal exercise of threshold (the transition from dark to light) does not fall significantly (stays between 6 and 5) as the network effect level increases. The leader and the follower's optimal exercise thresholds are around the same range because the leader's is also choosing the optimal capacity so that it can exercise right before the follower in order to be able to offer the lease.

Figure 9 plots the leader's maximum cooperative enterprise value and optimal capacity on one graph. In the left panel, the two value manifolds are very close to each other, which shows that the increase in network effect has very minor effect on the leader's cooperative enterprise value. The right panel shows that the leader's cooperative optimal capacity $q_L^{\Omega*}$ manifold is not smoothly increasing with the increase of commodity price and initial reserve, which is similar to the follower's optimal capacity q_F^{c*}. The right panel also shows that the leader should build larger cooperative capacity if the network effect level is higher.

Figure 10 compares the leader's optimal cooperative capacity $q_L^{\Omega*}$ with its optimal non-cooperative capacity q_F^{c*} when the network effect level is changing. From the left panel to the right panel, the difference between $q_L^{\Omega*}$ and q_F^{c*} does not increase significantly as the network effect increases. Figure 11 compares the leader's optimal cooperative capacity $q_L^{\Omega*}$ with its optimal non-cooperative capacity q_F^{c*} when the lease rate is changing. From the left panel to the right panel, the difference between $q_L^{\Omega*}$ and q_F^{c*} increases significantly as the lease rate increases. Comparison of Figure 10 and Figure 11 shows that the lease rate has larger positive effect on the leader's capacity choice than the network effect has.

Figure 12 graphs the leader's reservation lease rates against the commodity price and initial reserve. For low commodity price from 1 to 3.5, the leader's reservation lease rate is zero, meaning the leader has not exercise

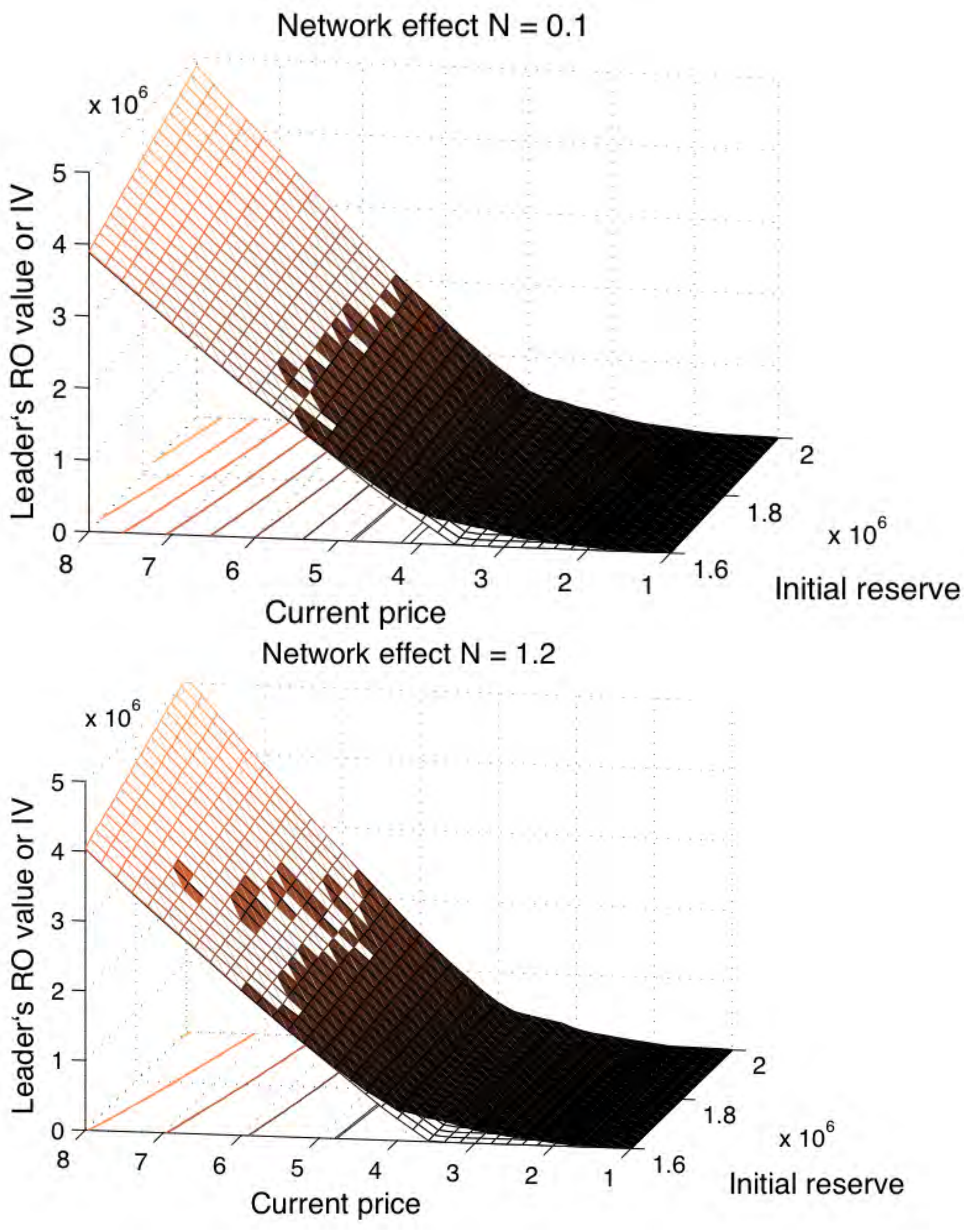


Figure 8: The leader's real option and the smooth-pasting condition for small network effects (top) and large network effects (bottom). The lease rate is at the its mean value. The dark manifold is the real option value and the light manifold is the exercise proceeds. The optimal exercise threshold is at the transition from dark to light. The leader's optimal decisions are relatively insensitive to the magnitude of the network effect. They are also insensitive to the level of initial reserves.

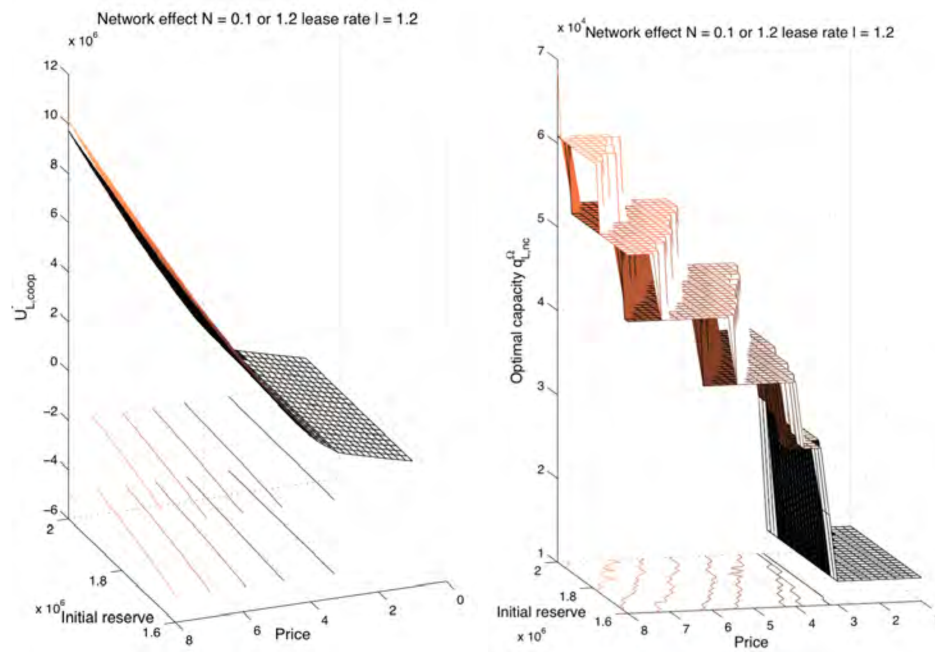


Figure 9: The left panel is the leader's maximum cooperative enterprise value $U^*_{L,\mathrm{coop}}$ for minimum (dark shading) and maximum (light shading) network effect levels. The right panel is the leader's optimal capacity choice $q_F^{\Omega*}$ for minimum and maximum network effect level, with the same shading. The lease rate is set at its mean value. The enterprise value increases strongly with commodity price, but is relatively insensitive to the network effect or initial reserves. The optimal capacity is more sensitive to these variables, with a larger capacity for higher reserves and higher network value. In addition, the optimal capacity increases with commodity price, becasue of the the increased likelihood that the follower will accept a lease when the commodity price increases.

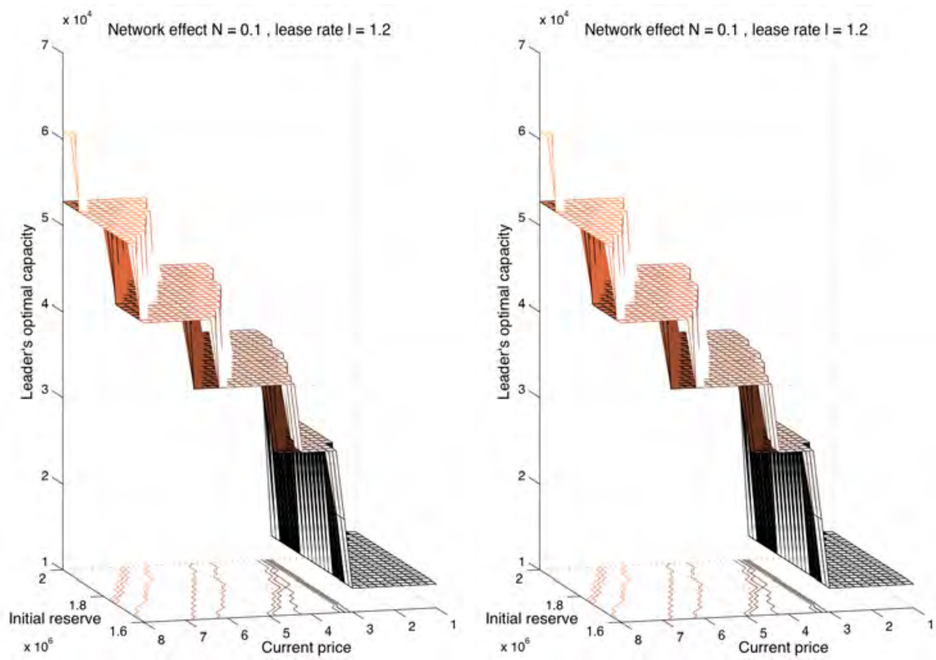


Figure 10: The leader's optimal cooperative capacity (light shading) and optimal non-cooperative capacity (dark shading). The left panel is for the minimum network effect level and the right panel is for maximum network effect level. The lease rate is set at its mean value. It builds larger capacity if the follower will cooperate by leasing part of the plant. The optimal capacity increases with the reserves estimate and with the commodity price.

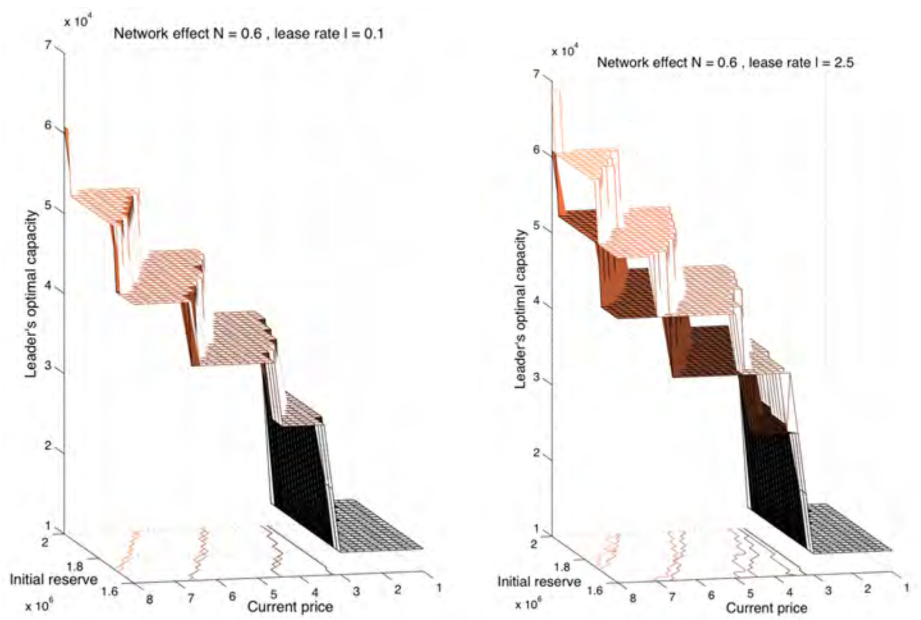


Figure 11: The leader's optimal cooperative capacity (light shading) and its optimal non-cooperative capacity (dark shading). The left panel is for the minimum lease rate and the right panel is for the maximum lease rate. The network effect level stays at its mean value. At low lease rates, the leader sets approximately the same optimal capacity at about the same level as it sets if it will not lease, because it wants to keep leasing activity to a minimum. But, at a high lease rate, it is more willing to build extra capacity to accommodate the lease demand.

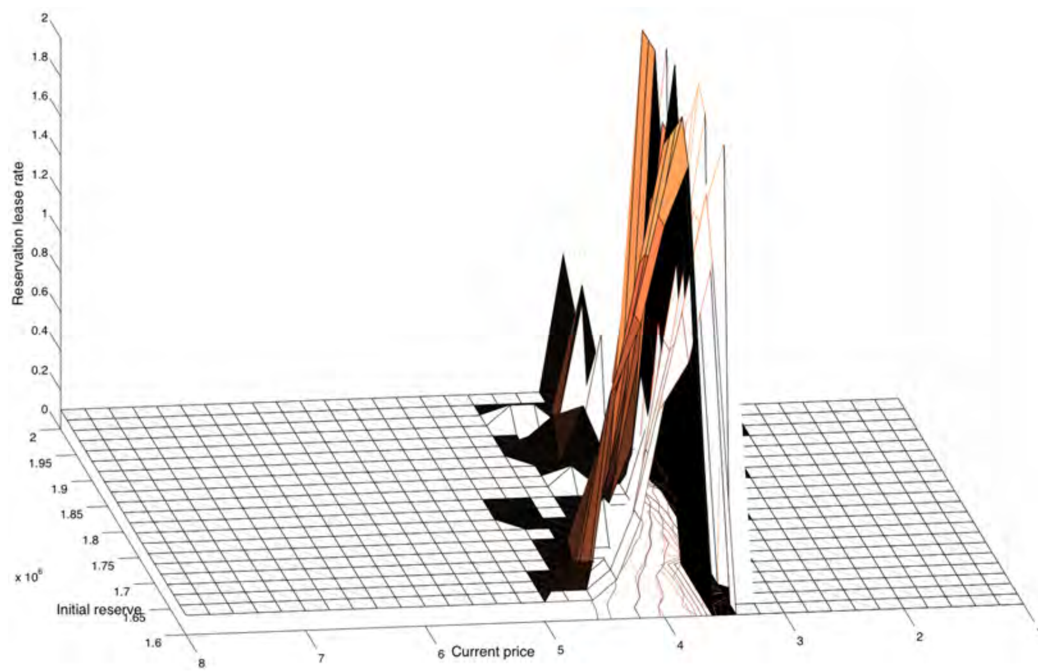


Figure 12: Manifolds of the leader's reservation lease rates for minimum (dark shading) and maximum (light shading) network effect levels. The leader will accept a lower lease rate if it has larger reserves, because it has a lower marginal cost of production with larger volume. The leader's reservation lease rate is concave in the price, just as the follower's lease rate was concave.

the real option yet and hence can not provide the lease. After commodity price of 4, the leader's reservation lease rate quickly drops to the lowest lease rate of 0.1 before the commodity price hit 5. This means that, when the commodity price gets very close to the follower's exercise threshold (between the commodity price of 5 and 6), the leader is willing to accept the lowest lease rate in order to avoid the follower's rejection of lease. The contours of the manifolds indicate that as initial reserve increases, the leader's reservation lease rate decreases. Moreover, by comparing the dark manifold (minimum network effect level) with the light manifold (maximum network effect level), we find that before the peak the high network effect gives the leader a larger reservation lease rate if holding the price level fixed. But after the peak, the high network effect gives the leader a smaller reservation lease rate if holding the price level fixed. The implication for the leader from this observation is that, once it exercises the real option, larger network effect will reduce the leader's reservation lease rate even further, i.e., the leader is willing to set a lower lease rate to capture the larger network effect, certeris paribus.

6.2.3 The possible equilibrium region for the lease rate

Figure 13 really is a combination of Figure 7 and Figure 12. It indicates the possible region of the equilibrium lease rate for bargaining. In the left panel, the leader's reservation lease rate exceeds the follower's reservation lease rate for the commodity price range of 3 to 4, which means the leader and the follower cannot cut a deal for that low commodity price. However, there is an empty region (from commodity price 4.6 to 5.6) below the lowest thick red line and above the highest dotted black line. That empty region is the gain from cooperation for which the leader and the follower will bargain. Also notice that as the commodity price gets higher than 5.7, the follower's reservation lease rate drops zero because at that commodity price level, the follower's optimal option exercise threshold[23] has already been hit, therefore the follower would rather build its own plant.

[23]As shown in Figure 3, the follower's optimal option exercise threshold is definitely below commodity price of 6 no matter how much initial reserve it has.

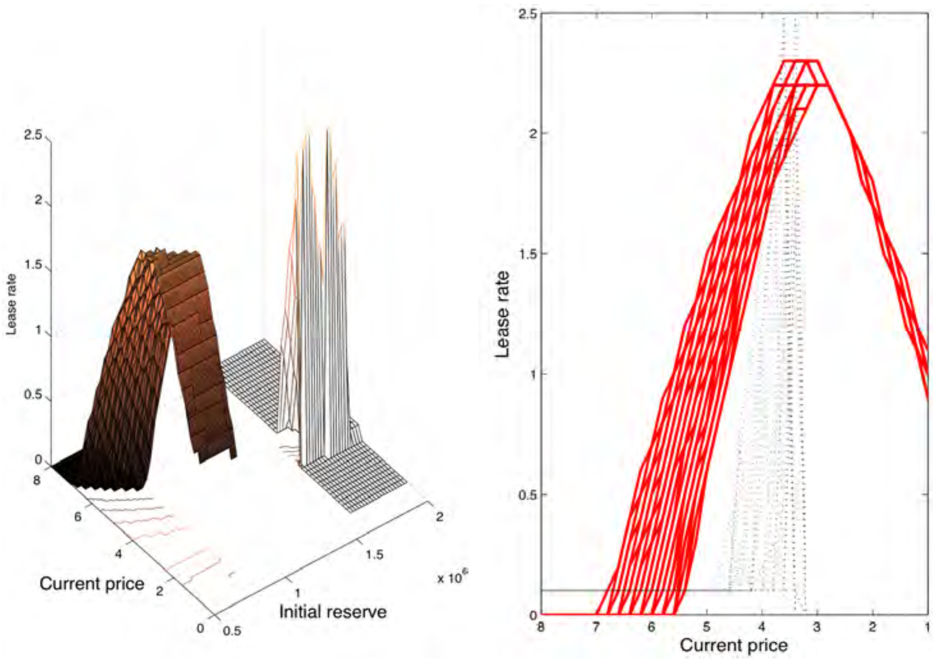


Figure 13: The leader's reservation lease rate compared with the follower's reservation lease rate. The right panel is a 2-D projection of the manifolds in the left panel. The network effect is at its mean value. In the left panel, the dark manifold corresponding to the low initial reserves is the follower's reservation lease rate, and the light manifold corresponding to the high initial reserve is the leader's reservation lease rate. The leader and follower have different reserves, so it is sensible to project these onto the price axis only, as in the right panel, where the thick red line is the follower's reservation lease rate and the dotted black line is the leader's reservation lease rate. The two parties can negotiate a cooperative lease when the red line is above the dotted black line. Otherwise, they do not cooperate.

7 Conclusion

In this paper we have developed a model to analyze the real option exercise game with two asymmetric players, the leader and the follower. In this game, two players have to decide when to explore and develop their adjacent oil or gas lands. The game is a dynamic sequential bargaining game of one-sided incomplete information. Players are bargaining over the lease rate which is going to be specified by the leasing contract.

Our simulation model illustrates the region of the equilibrium lease rate while treating other variables such as leasing quantity, and government regulated maximum production rate as fixed variables. In the three dimensional space spanned by enterprise value, current commodity price, initial reserve, the equilibrium lease rate may be located inside the 3-D space bounded by the follower's and the leader's reservation lease rate.

We observe that the follower tends to accept the lease contract if the commodity price and its initial reserves are low, and rejects the lease contract if the commodity price and its initial reserves are high. With high commodity price and initial reserves, the follower has more bargaining power, so the leader should charge a relatively low lease rate to encourage the follower's immediate start of production. If the commodity price and the leader's initial reserves are high, the leader should lower the lease rate, which coincides with the behavior of its reservation lease rate. Furthermore, the network effect positively affects the follower's reservation lease rate, which creates a larger space for cooperative bargaining.

On the other hand, when considering whether to be cooperative or noncooperative, the leader is always better off by being cooperative as long as the incremental construction cost of the excess capacity can be covered by the present value of the leasing fee and increase in total network effect.

One possible extension of our model would be to change the underlying process from GBM to a mean reverting process with/without jump. Alternatively, from the game theory perspective, one can change our one-sided incomplete information setting to two-sided incomplete information, or allow the leader and follower to provide alternating offers, or extend the two type followers assumption to continuous type followers, or extend the game from multi-period finite time horizon to infinite time horizon.

A Appendix: The derivation of the smooth-pasting conditions for non-cooperative player's investment decision

In order to derive the smooth-pasting conditions for non-cooperative player's investment decision, we need to partially differentiate the expected payoff function W_i with respect to P_{τ_i} and Q_i. This requires differentiating a definite integral with respect to a parameter that appears in the integrand and in the limits of the integral. The following formula is discovered by Gottfried Wilhelm von Leibniz. Let f be a differentiable function of two variables, let a and b be differentiable functions of a single variable, and define the function F by

$$F(t) = \int_{a(t)}^{b(t)} f(t,x)dx \qquad \forall\ t.$$

Then

$$F'(t) = f\big(t, b(t)\big)b'(t) - f\big(t, a(t)\big)a'(t) + \int_{a(t)}^{b(t)} f_t(t,x)dx$$

We now apply this Leibniz formula to differentiate W_i with respect to P_{τ_i} and Q_i separately, where τ_i is the first time the manifold (P(Q),Q) hits the threshold (P*(Q),Q). Producer i's cash flow function:

$$\pi_{i,t} = (P_t - C)q_{i,t}$$

Producer i's production function:

$$q_i(t) = \begin{cases} q_i^c & t \in [\tau_i, \theta_{i,\text{trans}}] \\ \overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) e^{-\overline{\alpha}_i (t-\theta_{i,\text{trans}})} & t \in [\theta_{i,\text{trans}}, \theta_i] \end{cases}$$

The production transition time is defined as:

$$\theta_{i,\text{trans}} = \frac{Q_i(\tau_i)}{q_i^c} - \frac{1}{\overline{\alpha}_i} \implies \frac{\partial \theta_{i,\text{trans}}}{\partial P_{\tau_i}} = 0 \quad \text{and} \quad \frac{\partial \theta_{i,\text{trans}}}{\partial Q_i} = \frac{1}{q_i^c}$$
$$\overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) = q_i^c$$

Therefore, we have

$$
\begin{aligned}
W_i \equiv \widehat{E}_{\tau_i}\bigg[& \int_{\tau_i}^{\theta_{i,\text{trans}}} e^{-r(t-\tau_i)}(P_t - C)q_i^c dt \\
& + \int_{\theta_{i,\text{trans}}}^{\theta_i} e^{-r(t-\tau_i)}(P_t - C)\overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) e^{-\overline{\alpha}_i(t-\theta_{i,\text{trans}})} dt \bigg] - K(q_i^c)
\end{aligned}
$$

Also, to simplify the notation, we denote $\hat{\mu}$ as the risk-neutral drift rate of the price P, i.e., $\hat{\mu}(P) = \mu$. Since the price is assumed to follow the GBM, we have

$$
P_t = P_{\tau_i} e^{\hat{\mu}(t-\tau_i)}
$$

Therefore, the first smooth-pasting condition is:

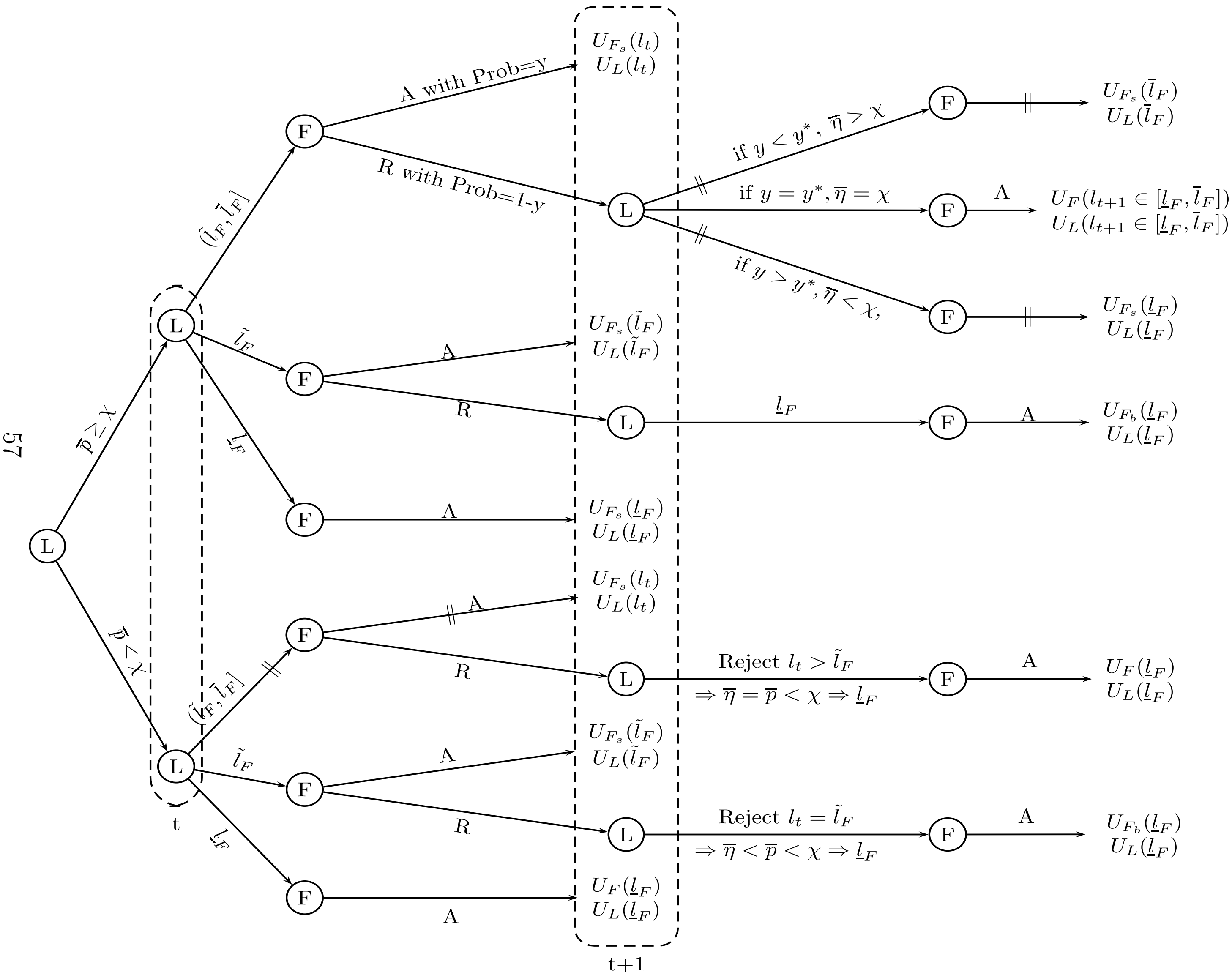
$$
\begin{aligned}
V_P(P^*, Q^*) &= \frac{\partial W_i}{\partial P_{\tau_i}} \\
&= \widehat{E}_{\tau_i}\bigg[\int_{\tau_i}^{\theta_{i,\text{trans}}} e^{-r(t-\tau_i)} e^{\hat{\mu}(t-\tau_i)} q_i^c dt \\
&\qquad + \int_{\theta_{i,\text{trans}}}^{\theta_i} e^{-r(t-\tau_i)} e^{\hat{\mu}(t-\tau_i)} \overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) e^{-\overline{\alpha}_i(t-\theta_{i,\text{trans}})} dt \bigg] - K(q_i^c) \\
&= \widehat{E}_{\tau_i}\bigg[\int_{\tau_i}^{\theta_{i,\text{trans}}} e^{(\hat{\mu}-r)(t-\tau_i)} q_i^c dt \\
&\qquad + \int_{\theta_{i,\text{trans}}}^{\theta_i} e^{(\hat{\mu}-r)(t-\tau_i) - \overline{\alpha}_i(t-\theta_{i,\text{trans}})} \overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) dt \bigg] - K(q_i^c)
\end{aligned}
$$

The second smooth-pasting condition is

$$
\begin{aligned}
V_Q(P^*, Q^*) &= \frac{\partial W_i}{\partial Q_i} \\
&= \widehat{E}_{\tau_i}\Bigg[\frac{1}{q_i^c} e^{-r(\theta_{i,\text{trans}}-\tau_i)}(P_t - C)q_i^c - 0 + 0 \\
&\quad 0 - \frac{1}{q_i^c} e^{-r(\theta_{i,\text{trans}}-\tau_i)}(P_t - C)\overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) e^{-\overline{\alpha}_i(\theta_{i,\text{trans}}-\theta_{i,\text{trans}})} \\
&\quad + \int_{\theta_{i,\text{trans}}}^{\theta_i} e^{-r(t-\tau_i)}(P_t - C)e^{-\overline{\alpha}_i(t-\theta_{i,\text{trans}})} dt\Bigg] - K(q_i^c) \\
&= \widehat{E}_{\tau_i}\Bigg[e^{-r(\theta_{i,\text{trans}}-\tau_i)}(P_t - C) - \frac{1}{q_i^c} e^{-r(\theta_{i,\text{trans}}-\tau_i)}(P_t - C)\overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) \\
&\quad + \int_{\theta_{i,\text{trans}}}^{\theta_i} e^{-r(t-\tau_i)}(P_t - C)e^{-\overline{\alpha}_i(t-\theta_{i,\text{trans}})} dt\Bigg] - K(q_i^c) \\
&= \widehat{E}_{\tau_i}\Bigg[e^{-r(\theta_{i,\text{trans}}-\tau_i)}(P_t - C) - \frac{1}{q_i^c} e^{-r(\theta_{i,\text{trans}}-\tau_i)}(P_t - C)q_i^c \\
&\quad + \int_{\theta_{i,\text{trans}}}^{\theta_i} e^{-r(t-\tau_i)}(P_t - C)e^{-\overline{\alpha}_i(t-\theta_{i,\text{trans}})} dt\Bigg] - K(q_i^c) \\
&= \widehat{E}_{\tau_i}\Bigg[\int_{\theta_{i,\text{trans}}}^{\theta_i} e^{-r(t-\tau_i)}(P_t - C)e^{-\overline{\alpha}_i(t-\theta_{i,\text{trans}})} dt\Bigg] - K(q_i^c)
\end{aligned}
$$

B Appendix:ExtensiveForm

Figure 14: **Appendix B: The extensive form of the bargaining game**



C Appendix: The derivation of the perfect Bayesian equilibrium for the leader and follower bargaining game.

The extensive form representation of this sequential bargaining game is shown in Figure 14 of Appendix B, which will be used throughout the discussion of the perfect Bayesian equilibrium.

C.1 The leader and follower optimal strategies at time $t+1$

In period $t+1$, the leader with beliefs $\overline{\eta}(l_t)$ makes a "take it or leave it" offer l_{t+1} so as to maximize that period's profit. Because period $t+1$ is the last period, the leader's threat of offering no other contract in the future is credible, so the follower will accept if and only if his reservation is at least l_{t+1}. The follower's optimal strategy at date $t+1$ is defined as: [24]

$$\text{If } l_{t+1} = \begin{cases} \underline{l}_F, & F_s, F_b \text{ both accept} \\ \bar{l}_F, & F_s \text{ accepts}, F_b \text{ rejects} \\ \text{Random}[\underline{l}_F, \bar{l}_F], & F_s \text{ accepts}, F_b \text{ rejects} \end{cases} \tag{33}$$

The leader's offer l_{t+1} ranges from $\underline{l}_F$ to $\bar{l}_F$. If offering $l_{t+1} = \underline{l}_F$, the leader sells for sure and obtains the enterprise value of $U_{L,\text{coop}}(P, Q_L, q_L^{\Omega}, \underline{l}_F; N_{\tau_{\text{lease}}}^{\theta_L})$, simplified as $U_L(\underline{l}_F)$. If offering $l_{t+1} = \bar{l}_F$, the leader sells with probability $\overline{\eta}$ and has second period enterprise value of $\overline{\eta} \cdot U_{L,\text{coop}}(P, Q_L, q_L^{\Omega}, \bar{l}_F; N_{\tau_{\text{lease}}}^{\theta_L})$, simplified as $\overline{\eta} \cdot U_L(\bar{l}_F)$.Therefore, there exists a unique critical probability $\chi \equiv \frac{U_L(\underline{l}_F)}{U_L(\bar{l}_F)}$, and the leader's optimal strategy at date $t+1$ is defined as:

$$l_{t+1} = \begin{cases} \underline{l}_F, & \text{if } \overline{\eta} < \chi \\ \bar{l}_F, & \text{if } \overline{\eta} > \chi \\ \text{Random}[\underline{l}_F, \bar{l}_F], & \text{if } \overline{\eta} = \chi \end{cases} \tag{34}$$

[24] Each type follower is actually indifferent between accepting and rejecting a lease rate of l_{t+1} that exactly equals that type's reservation rate. However, as long as the supremum of the leader's total enterprise value is achieved in the limit of lease rate $l_{t+1} = l - |\varepsilon|$ as $\varepsilon \to 0$, we could assume, without loss of generality, the existence of an equilibrium given the leader's beliefs requires that type l accept $l_{t+1} = l$, and whether the other type accepts a lease rate equal to its reservation rate is irrelevant.

C.2 The leader and the follower's optimal strategy at time t

At time t, the leader and the follower's decisions are more complex. Ideally, the leader would want to offer the high type follower at $\bar{l}_F$ and the low type follower at $\underline{l}_F$. But we have already shown that the high type follower is motivated to lie. Therefore, the leader's task is to differentiate the high type follower from the low type follower by testing them with different lease rate. At time t, the low type follower F_b will accept if and only if $l_t = \underline{l}_F$ since it will never obtains a surplus at next period. Of course, the high type follower F_s accepts $l_t = \underline{l}_F$ too. The high type follower, however, if offered $l_t > \underline{l}_F$, has to consider how its rejection might affect the leader's posterior belief about the follower's type. High type follower F_s obtains a surplus only if the leader is sufficiently convinced that it is the low type follower, i.e., $\overline{\eta} < \chi$.

C.2.1 The consequence of the follower's rejection on the leader's posterior belief

We now discuss how the follower's rejection might affect the leader's posterior belief.

1. Choice of mixed and pure strategy

   Suppose the rejection of $l_t > \underline{l}_F$ generates "optimistic posterior beliefs": $\overline{\eta} > \chi$. From equation (28) the leader charges $l_{t+1} = \bar{l}_F$. High type F_s has no second period surplus from rejecting (continue lying) that $l_t > \underline{l}_F$. Therefore, the high type F_s is better off accepting $l_t > \underline{l}_F$. And since l_t is rejected by the low type F_b, Bayes' rule yields $\overline{\eta}(l_t) = \frac{\overline{p}\cdot 0}{\overline{p}\cdot 0 + \underline{p}} = 0$, a contradiction. Thus neither of the pure strategies, accept or reject, is optimal here. In the following subsections, we will develop a mixed strategy for the follower and the leader in the case of the rejection generating optimistic posterior, and we will also elaborate the leader and follower's pure strategy in the case of the rejection generating "pessimistic posterior beliefs".

   Let $y(l_t)$ denote the probability that the high type F_s accepts l_t. Then the high type follower consider how its probability of rejection will affect the leader's posterior according to the following formula:

$$\overline{\eta}(l_t) = \frac{\overline{p}\big(1 - y(l_t)\big)}{\overline{p}\big(1 - y(l_t)\big) + \underline{p}}$$

(a) If F_s accept with probability of 1, then $y = 1 \Rightarrow 1 - y = 0$, then $\overline{\eta}(l_t) = \frac{\overline{p}\cdot 0}{\overline{p}\cdot 0 + \underline{p}\cdot 1} = 0 < \chi$. According to equation (28), the leader with posterior $\overline{\eta} < \chi$ will offer $l_{t+1} = \underline{l}_F$. So F_s who anticipates this lower second period price l_{t+1} should not accept l_t with probability of 1. A contradictory.

(b) If F_s reject with probability of 1, then $y = 0 \Rightarrow 1 - y = 1$, then $\overline{\eta}(l_t) = \frac{\overline{p}\cdot 1}{\overline{p}\cdot 1 + \underline{p}\cdot 1} = \overline{p}$. Now since in the top branch of the extensive form of game, we have $\overline{p} > \chi$. Therefore, $\overline{\eta} = \overline{p} > \chi$. According to equation (28), the leader with posterior $\overline{\eta} > \chi$ will offer $l_{t+1} = \bar{l}_F$. So F_s who anticipates this higher second period price l_{t+1} should not reject l_t with probability of 1. A contradictory.

In equilibrium the high type F_s should not reject l_t with probability 1, because in that case we would have $\overline{\eta}(l_t) = \overline{p} > \chi$ and the leader charging $l_{t+1} = \bar{l}_F$, so the high type F_s would be better off accepting l_t. But we already saw that the high type F_s cannot accept such an l_t with probability 1 either. Hence, the high type follower needs a mixed strategy here by randomizing between accept and reject, i.e., controlling the y so that the leader's posterior is $\overline{\eta}(l_t) = \chi$.

2. Rejection deteriorates the leader's ex ante belief.
According to the Bayes rule, for any rejection of $l_t > \underline{l}_F$, the leader's posterior belief is calculated as:

$$\begin{aligned}\overline{\eta}(l_t) &= \frac{\text{Prob}(\text{type} = F_s \ \& \ \text{reject } l_t > \underline{l}_F)}{\text{Prob}(\text{reject } l_t > \underline{l}_F)} = \frac{\overline{p}\cdot \text{Prob}(l_t > \tilde{l}_F)}{\overline{p}\cdot \text{Prob}(l_t > \tilde{l}_F) + \underline{p}} \\ &= \frac{\overline{p}}{\overline{p} + \dfrac{\underline{p}}{\text{Prob}(l_t > \tilde{l}_F)}} \leq \overline{p}\end{aligned} \tag{35}$$

which means the posterior is always less than or equal to the prior conditional on the rejection of $l_t > \underline{l}_F$.

C.2.2 The follower's indifference lease rate $\tilde{l}_F$

To analyze the high type follower's behavior at t when offered price $l_t \in (\underline{l}_F, \bar{l}_F]$, we have to define a critical indifference least rate $\tilde{l}_F$. The high type follower F_s should accept l_t only if

$$\begin{aligned} U_{F_s}(l_t) - U_{F_s}(\bar{l}_F) &\geq e^{-r}\big(U_{F_s}(\underline{l}_F) - U_{F_s}(\bar{l}_F)\big) \\ \Rightarrow \qquad U_{F_s}(l_t) &\geq (1-e^{-r})U_{F_s}(\bar{l}_F) + e^{-r}U_{F_s}(\underline{l}_F) \end{aligned} \tag{36}$$

To see this, note that $U_{F_s}(l_t) - U_{F_s}(\bar{l}_F)$ is the realized gain from lying at time t and $U_{F_s,\text{coop}}(\underline{l}_F) - U_{F_s,\text{coop}}(\bar{l}_F)$ is the maximum possible gain from continuing lying at time $t+1$. Denote $\tilde{l}_F$ as the l_t which makes the above inequality equal. That is

$$U_{F_s}(\tilde{l}_F) = (1-e^{-r})U_{F_s}(\bar{l}_F) + e^{-r}U_{F_s}(\underline{l}_F)$$

Obviously, when $l_t = \tilde{l}_F$, the high type follower F_s is indifferent between accepting this l_t and getting $l_{t+1} = \underline{l}_F$ at time $t+1$ by rejecting this l_t. As the high type follower's enterprise value function, $U_{F_s}(l)$ decreases in l, we have the optimal strategy for the high type follower when facing the lease offer at $l_t > \underline{l}_F$.

- If $\underline{l}_F < l_t \leq \tilde{l}_F \Rightarrow U_{F_s}(l_t) \geq U_{F_s}(\tilde{l}_F) = (1-e^{-r})U_{F_s}(\bar{l}_F) + e^{-r}U_{F_s}(\underline{l}_F)$. Equation (36) is satisfied. High type F_s accepts this $l_t \in (\underline{l}_F, \tilde{l}_F]$.
- If $l_t > \tilde{l}_F$, rejecting l_t is optimal for the high type F_s as it is for the low type F_b, and therefore Bayes' rule yields

  $$\overline{\eta}(l_t > \tilde{l}_F) = \frac{\overline{p}\cdot 1}{\overline{p}\cdot 1 + \underline{p}\cdot 1} = \overline{p}$$

  which means the posterior beliefs coincide with the prior beliefs. In other words, the follower is safe to reject any offer $l_t > \tilde{l}_F$ at time t without improving the leader's information about the follower's type.

C.2.3 The strategy of the pessimistic leader $\overline{p} < \chi$

Equation (35) shows $\overline{\eta} \leq \overline{p}$, combined with $\overline{p} < \chi$, we have $\overline{\eta} < \chi$. This means no matter what the first period offer is, the follower's rejection always makes the leader pessimistic. Therefore the leader's second period strategy is limited to $l_{t+1} = \underline{l}_F$ whenever it observes a rejection at time t. We now compare the leader's expected total enterprise values from three different first period strategies, as illustrated in the bottom branch of Figure 14.

1. Bottom-Bottom strategy (BB): $l_t = \underline{l}_F$
   Both type followers will accept this l_t as they knows this is the most favorable price. BB therefore leads to a pooling equilibrium. The leader has an enterprise value of $U_L(\underline{l}_F)$.

2. Bottom-Middle strategy (BM): $l_t = \tilde{l}_F$
   The high type F_s would accept this l_t because it is indifferent as discussed in Section C.2.2. The low type F_b rejects this offer because leasing would give him a negative surplus, i.e., $U_{F_b,\text{coop}}(\tilde{l}_F) < U_{F_b,\text{nc}}$ according to equation (13). Thus if the leader observes a rejection, it knows the follower is low type and will set $l_{t+1} = \underline{l}_F$. BM therefore leads to a separating equilibrium. The leader's expected enterprise value from BM strategy is: $\overline{p} \cdot U_L(\tilde{l}_F) + e^{-r}\underline{p} \cdot U_L(\underline{l}_F)$.

3. Bottom-Top strategy (BT): $l_t \in (\tilde{l}_F, \overline{l}_F]$
   Again, the low type follower F_b rejects this offer because $U_{F_b,\text{coop}}(\tilde{l}_F+) < U_{F_b,\text{nc}}$. The high type follower F_s would rather reject this l_t since it knows that the consequence of rejecting the leader's offer is $\overline{\eta} = \overline{p} < \chi$ and the leader will offer a lower lease rate next period, $l_{t+1} = \underline{l}_F$. BT therefore leads to a pooling equilibrium as both type followers reject. BT strategy will give the leader a total enterprise value of $e^{-r} \cdot U_L(\underline{l}_F)$.

Clearly, BB is better than BT and BM is better than BT.[25] Either BB or BM can give the leader higher value depending on the generic values of parameters. Thus, we summarize the pessimistic leader's optimal strategy as:

$$l_t = \begin{cases} \underline{l}_F, & \text{if } \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} < \dfrac{1 - e^{-r}\underline{p}}{\overline{p}} \\ \tilde{l}_F, & \text{if } \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} > \dfrac{1 - e^{-r}\underline{p}}{\overline{p}} \end{cases} \tag{37}$$

[25] The leader's valuation function $U_L(l)$ increases at l. Hence, $\overline{p}U_L(\tilde{l}_F) > \overline{p}U_L(\underline{l}_F)$. Therefore, $\overline{p}U_L(\tilde{l}_F) + e^{-r}\underline{p}U_L(\underline{l}_F) > \overline{p}U_L(\underline{l}_F) + e^{-r}\underline{p}U_L(\underline{l}_F) > \overline{p}e^{-r}U_L(\underline{l}_F) + e^{-r}\underline{p}U_L(\underline{l}_F) = e^{-r}U_L(\underline{l}_F)$.

C.2.4 The strategy of the optimistic leader $\overline{p} > \chi$

1. Top-Bottom strategy (TB): $l_t = \underline{l}_F$
   The TB strategy is same as the BB strategy. Both type followers accept the lease and the leader's enterprise value is $U_L(\underline{l}_F)$, a pooling equilibrium.

2. Top-Middle strategy (TM): $l_t = \tilde{l}_F$
   This is also similar to BM strategy. The high type F_s accepts whereas the low type F_b rejects this offer, a separating equilibrium. The leader's expected enterprise value from TM strategy is: $\overline{p} \cdot U_L(\tilde{l}_F) + e^{-r}\underline{p} \cdot U_L(\underline{l}_F)$.

3. Top-Top strategy (TT): $l_t \in (\tilde{l}_F, \bar{l}_F]$
   The low type follower F_b rejects this offer. The high type follower F_s has a more complex decision because it has to consider the consequence of rejecting the leader's offer, i.e., whether the leader is going to charge a higher or lower l_{t+1}. In equilibrium the high type F_s cannot reject l_t with probability 1, because in that case we would have $\overline{\eta}(l_t) = \overline{p} > \chi$ and the leader charging $l_{t+1} = \bar{l}_F$, so the high type F_s would be better off accepting l_t. But we already saw that the high type F_s cannot accept such an l_t with probability 1 either. In fact, the offer of $l_t \in (\tilde{l}_F, \bar{l}_F]$ is a dilemma for the high type because if it rejects, the leader will charge an even higher $l_{t+1} = \bar{l}_F$; if it accepts, it gets the smallest expected enterprise value.

   Hence, the high type follower needs a mixed strategy here by randomizing between accept and reject. In equilibrium the high type F_s must randomize in order to make the leader's posterior belief satisfy $\overline{\eta}(l_t) = \chi$ so that the leader will offer the price l_{t+1} to be any randomization between $\underline{l}_F$ and $\bar{l}_F$. Let $y(l_t)$ denote the probability that the high type F_s accepts l_t. Then $\overline{\eta}(l_t) = \chi$ will give:

$$\overline{\eta}(l_t) = \frac{\overline{p}\big(1 - y^*(l_t)\big)}{\overline{p}\big(1 - y^*(l_t)\big) + \underline{p}} = \chi \;\Rightarrow\; y^*(l_t) = 1 + \frac{\chi\underline{p}}{\chi\overline{p} - \overline{p}}$$

   which defines a unique $y^*(l_t) = y^* \in [0, 1]$. Note that $y^*(l_t)$ is independent of l_t. Any $y < y^*$ will make the leader's posterior belief $\overline{\eta} > \chi$, which leads to $l_{t+1} = \bar{l}_F$. Any $y > y^*$ will make the leader's posterior belief $\overline{\eta} < \chi$, which leads to $l_{t+1} = \underline{l}_F$. Since the equilibrium has to be Pareto efficient, in order for the high type F_s to be indifferent between accepting and rejecting l_t, we need to define another probability $x(l_t)$ for the high type follower to realize its maximum second period gain.

$$U_{F_s}(l_t) - U_{F_s}(\bar{l}_F) = e^{-r}x(l_t)\big(U_{F_s}(\underline{l}_F) - U_{F_s}(\bar{l}_F)\big)$$

which defines a unique probability $x(l_t)$ for $l_{t+1} = \underline{l}_F$. The leader's expected enterprise value can be calculated as:

$$\overline{p}yU_L(\bar{l}_F) + e^{-r}\big[\overline{p}(1-y)(1-x)U_L(\bar{l}_F) + \overline{p}(1-y)xU_L(\underline{l}_F) + x\underline{p}U_L(\underline{l}_F)\big] \tag{38}$$

Any of those strategies, TT, TM and TB can generates the highest total enterprise value for the leader depending on the parameter values. We summarize the optimistic leader's optimal strategy and expected enterprise value in the first period as one the following:

$$l_t = \begin{cases} \underline{l}_F, & \text{which generates value } U_L(\underline{l}_F); \\ \tilde{l}_F, & \text{which generates value } \overline{p}\cdot U_L(\tilde{l}_F) + e^{-r}\underline{p}\cdot U_L(\underline{l}_F); \\ \bar{l}_F, & \text{which generates value } \overline{p}y\cdot U_L(\bar{l}_F) + e^{-r}\big(\overline{p}(1-y) + \underline{p}\big)U_L(\underline{l}_F). \end{cases}$$

where the third enterprise value is computed using the fact that, for posterior beliefs $\overline{\eta} = \chi$, $l_{t+1} = \underline{l}_F$ is an optimal price in the second period for the seller as $x(\underline{l}_F) = 1$. Note that if the third value is highest, the leader never sells to the low type F_b as $x(\bar{l}_F) = 0$.

The conclusion is thus that there exists a unique perfect Bayesian equilibrium, and that this equilibrium exhibits Coasian dynamics — that is, $\overline{\eta}(l_t) \leq \overline{p}$ for all l_t, so the leader becomes more pessimistic over time, and $l_{t+1} \leq l_t$, so the leader's offer decreases over time.

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