# Valuation of the Options to Verticalize and Expand a Carbonatite Mine

**Haroldo Guimarães Brasil**
Ibmec Minas Gerais - Brazil
Fundação Dom Cabral - Brazil
haroldo.brasil@gmail.com

**Alexandre Aronne**
University of Verona
alexandre.aronne@gmail.com

**Alyson Rajão**
No affiliation

**Abstract**

In this paper we develop a real options model to assess managerial flexibilities embedded in a mining project, namely the options to expand and verticalize the mine. Monte Carlo simulation is used to estimate the volatility of the underlying asset and a binomial lattice is used to price the two options. Our results indicate that they represent a substantial source of value for the project owners.

## I. Introduction

This paper presents criteria to price real options in a project of exploitation and production of fertilizers. The project studied consists of the exploitation of a mine that has an estimated reserve of 780Mt of phosphate rock. The mine lasts for 38 years and commercial production is estimated to start in the fourth year from its approval. Three different alternatives were assessed in order to exploit the mine, they are:

1. Extract and sell the phosphate rock at a rate of 3.5 Mt per annum
2. Extract and sell the phosphate rock at a rate of 5.0 Mt per annum
3. Extract and treat the phosphate rock and sell its derivates, at a rate of 3.5 MT per annum

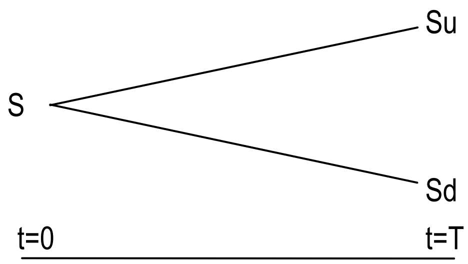
The cash flow projection of each alternative has been studied as shown below:

| | Phosphate Rock only | | Derivates – Vertical plant |
|---|---|---|---|
| **Alternatives** | **1)**<br><br>3.5 Mt p.a. | **2)**<br><br>5.0 Mt p.a. | **3)**<br><br>3.5 Mt p.a. |
| **NPV @ 12%** | 90 MUS$ | 110 MUS$ | -130 MUS$ |

It was observed that the three alternatives are not strictly mutually exclusive amongst them and they have embedded flexibilities. For example, if the project is started according to the first alternative, but is later on expanded through the payment of the expansion cost, then the project will have the same characteristics of alternative 2. Another interesting possibility consists of verticalization, which can be understood as the conversion of alternative 1 through construction of a chemical plant in order to treat the phosphate rock, which will result in a project with the same characteristics of alternative 3.

The valuation of the flexibilities above has been carried on with the use of the real options theory. The binomial model was considered the most adequate for the assessment due to its simplicity and adherence to the characteristics of the identified flexibilities, which clearly are American options to expand and to switch.

The premise of the model is that the price of the underlying asset can move after some time to one of two possible prices as shown in the figure below:




The value of an option can be found by building a portfolio that is able to replicate the same payoff from this option. This portfolio is composed of $\Delta$ units of the underlying asset and B units of a risk free asset. If S moves up with a multiplicative factor u, the payoff of the portfolio is:

$f_u = \Delta Su + Be^{rT}$,

in case S moves down with a multiplicative factor d, the payoff of the portfolio is:

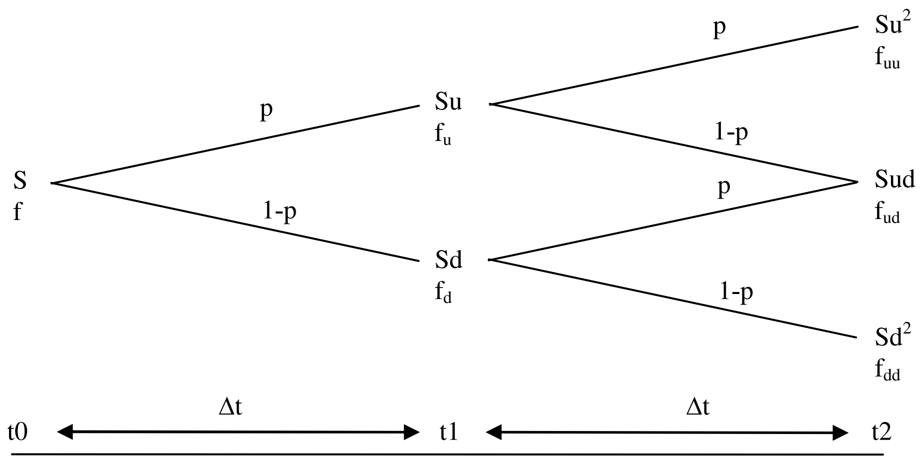
$f_d = \Delta Sd + Be^{rT}$,

where r is the risk free interest rate.

The payoffs of the option ($f_u$ e $f_d$) at expiration date are known and the two equations can be solved for the two unknowns, obtaining the values of $\Delta$ and B. The current value of the portfolio (or option) is:

$f = \Delta S + B$

The assumption that the underlying asset can only have two possible values at the exercise date is very restrictive and can be avoided by using binomial trees of several periods, in which the option's life is split in smaller time steps, delivering several possibilities for the price of the underlying asset as demonstrated in the image below:




Source: Adapted from HULL (1998, pg 221)

The value of the option is defined by the four following equations which are a result from the generalisation of the one step model for several steps:

$f = e^{-r\Delta t} [pf_u + (1-p)f_d]$ **(1)**

$p = (e^{r\Delta t} - d) / (u - d)$ **(2)**

$u = e^{\sigma\sqrt{\Delta t}}$ **(3)**

$d = 1/u$ **(4)**

where r is the risk free interest rate and σ is the volatility of the underlying asset.

The variables p and (1-p) are called risk neutral probabilities and can be interpreted as the probability of an ascending (p) or descending (1-p) oscillation. However, according to Copeland and Antikarov (2001, pg 99), "the risk neutral probabilities are not the objective probabilities that we are used to think when estimating the likelihood of an event to happen. They are simply a mathematical convenience with the objective of adjusting the cash flows in a way that they can be discounted using the risk free interest rate."

The binomial model is adequate to price American options. In order to do that, it is necessary to analyse the tree recursively, going back in time from the exercise date, executing an optimality test in each node, in order to identify if the immediate exercise is optimal. The value of the American option in the final nodes is the same as the European option. However, in each of the previous nodes, the payoff of the immediate exercise is compared to the value of keeping the option alive (found by using equation (1) in that same node), and the highest one is chosen.

The implementation of a real options model starts by determining the underlying asset of the option. Differently from financial options, the underlying asset of a real option is often not traded in quoted markets, which might create difficulties in establishing its actual price. Moreover, the non existence of historical price series of the underlying asset makes it impossible to calculate its historical volatility.

In order to bypass these difficulties, the Present Value (PV) of the project is usually used as the underlying asset of the real option, since this is considered to be the best estimate of the market price of the analysed project.

The use of the PV as an underlying asset of the real option implies that the annually distributed operational cash flows are treated as dividends. When distributing dividends, its value is deducted from the price of the stock, because the amount paid becomes property of the stockholder and does not constitute part of the enterprise value any longer. Therefore, the distribution of the operational cash flow results in an equivalent decline in the project's PV.

The value of the managerial flexibility added to the net present value of the project is called expanded net present value. Mathematically:

ENPV = NPV + ROV,

where:

ENPV = Expanded net present value

NPV = Traditional net present value

ROV = Real Options Value

Some assumptions support the use of the Real Options Theory in evaluating projects: (a) the traditional present value method does not take into consideration flexibility in the underlying asset; (b) real options are valued in a no arbitrage world and (c) present value of cash flows fluctuates randomly (Copeland, Weston and Shastri; 2005).

## II. The Model

In this item, the various steps of the valuation process regarding the real options embedded in the mining project analysed are presented in greater detail. The real options primarily identified and assessed are:

a) Option to Expand alternative 1) by increasing the annual extraction rate from 3.5 Mt to 5.0 Mt. Once this option is exercised, another flexibility is created and could also be priced, namely the the option to contract the project back to its initial mode, which could work as a hedge against bearish market movements. However, our simulations indicate that its value is not significant, since the savings from the reduction of production are very small. Therefore, it is not presented in this paper.
b) Option to Expand / Verticalize alternative 1) by constructing a chemical plant that allows the use of the extracted rock (at the rate of 3.5 Mt p.a.) as a raw material for the production of fertilizers, which is the final product to be sold in the market. This alternative does not allow switching back and forth between operation modes, i.e., once the verticalization option is exercised, there is no way back to the previous situation. That seems to be a plausible assumption, due to the fact that the new fixed assets acquired through investing in the verticalization cannot be sold in secondary markets and have no use in other operations of the company.

After identification of the potentially valuable managerial flexibilities, the valuation process begins, following the steps below:

### a) NPV and PV Calculation

Based on the cash flow projections previously elaborated for each investment alternative it was possible to obtain the present value (PV) of the cash flows from operations, calculated before capital expenses. Mathematically,

$$PV = \sum_{t} \frac{(Cash\ flow\ from\ operations)_t}{(1 + WACC\%)^t}$$

where

WACC = Weighted Average Cost of Capital, assumed to be 12% per annum

The PVs of each alternative were used as the value of the underlying asset of the real option in $t_o$.

**b) Volatility estimation using Monte Carlo simulation**

Volatility estimation was preceded by the selection of the sources of uncertainty of each investment alternative considered. In every case, the stochastic variables taken into consideration were commodity prices, according to the table below:

| | **Phosphate Rock only** | | **Derivates – Vertical plant** |
|---|---|---|---|
| **Alternative** | **1)** 3,5 Mt p.a. | **2)** 5,0 Mt p.a. | **3)** 3,5 Mt p.a. |
| **Commodity prices considered stochastic** | • Phosphate rock | • Phosphate rock | • Phosphate rock<br>• MAP<br>• DAP<br>• TSP<br>• Sulfur<br>• Ammonia |

*Obs: MAP, DAP and TSP are phosphate fertilizers, and stand for, respectively: Monoammonium phosphate, Diammonium phosphate and Trisodium phosphate*

Once the sources of uncertainty of each alternative were determined, the average growth rate of prices (drift term of a geometric Brownian motion) and the volatility of the returns of each stochastic variable were estimated, after detection and substitution of outliers[1].

The parameters obtained for each stochastic variable is presented in the table below, in annual terms.

| Variables | Growth/drift rate | Volatility (σ) |
|---|---|---|
| Phosphate rock | 14.21% | 17.05% |
| MAP | 17.11% | 28.62% |
| DAP | 14.25% | 24.70% |
| TSP | 13.39% | 21.51% |
| Sulfur | 23.44% | 52.01% |
| Ammonia | 12.26% | 42.52% |

Given that alternative 3) presents more than one source of uncertainty, a correlation matrix of the input variables has been estimated, as shown in the table below:

| Correlation | Rock | MAP | DAP | TSP | Sulfur | Ammonia |
|---|---|---|---|---|---|---|
| Rock | 1.000 | 0.898 | 0.898 | 0.677 | 0.777 | 0.696 |
| MAP | 0.898 | 1.000 | 0.863 | 0.897 | 0.916 | 0.834 |
| DAP | 0.677 | 0.863 | 1.000 | 0.847 | 0.796 | 0.632 |
| TSP | 0.644 | 0.897 | 0.847 | 1.000 | 0.931 | 0.733 |
| Sulfur | 0.777 | 0.916 | 0.796 | 0.931 | 1.000 | 0.756 |
| Ammonia | 0.696 | 0.834 | 0.632 | 0.733 | 0.756 | 1.000 |

[1] Since the outliers detected represented consecutive observations in time, it's been decided to substitute these values by linear interpolation of the last observation before the series of outliers and the first observation after this series.

The growth rate of prices, volatility of each variable and correlations between each of them were used as inputs in the Monte Carlo simulation, in order to determine the volatility of the present value of the cash flows from operations. In order to simulate the annual trajectories of a geometric Brownian motion[2], the following formula was used:

$$S_{t+1} = S_t e^{(g-0.5\sigma^2)N(0,1)}$$

where g represents the average growth rate of prices, $\sigma$ represents the annual volatility of the returns of the price series and N(0,1) represents a withdrawal from the standard normal distribution table.

The Monte Carlo simulation was modeled considering the price function as an input variable and the logarithm of the instantaneous rate of return of the PV as output variable.

After the simulation of each path, a new value for the instantaneous rate of return is calculated following the formula below:

$\ln\left(\frac{PV_1}{PV_0}\right)$ = instantaneous rate of return

The standard deviation of the instantaneous rates of return was used as a proxy for the volatility of the present value of the cash flows from operations.

**c) Binomial lattice of the underlying asset**

All the parameters needed to construct the binomial lattice have already been estimated/calculated, except for the *dividend yield*. It´s been assumed that the cash flows generated for each alternative are equal to the value of the dividends. In other words, the *dividend yield* at time t is calculated by dividing the cash flow from operations (at time t) by the project NPV (at time t):

$$Dividend\ yield_t = \frac{(Cash\ Flow\ from\ Operations)_t}{NPV_t}$$

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[2] Following Dixit & Pindyck discussion about cooper prices behavior (DIXIT & PINDYCK, 1994, p.264).

The binomial lattice of the possible prices of the underlying asset has been constructed taking into consideration the decrease that the distribution of the cash flows generates in the PV obtained at each node. In order to perform this adjustment, it´s sufficient to multiply the value at each node at time t by $(1 - Dividend\ yield_t)$.

## III. Valuation of the embedded real options

The valuation of both options is detailed below.

### a) Option to expand from 3.5 to 5.0 Mty

This option has been modeled as an American call option, in which the exercise prices changes at each time period ($X_t$) and are equal to the incremental investments needed to expand the project at a given point in time. The exercise prices ($X_t$) at time t have been determined by the difference between the PV of the CAPEX expected to occur until the end of the project in the 5.0 Mty mode (calculated at t=0 and brought up to time t through the risk free interest rate) and the PV of the CAPEX expected to occur until the end of the project in the 3.5 Mty mode (calculated at t=0 and brought up to time t through the risk free interest rate).

The payoff of the option at expiration date is given by the following equation:

$$Payoff_T = \begin{cases} ((PV_{5.0} - PV_{3.5}) - X_T) & if \quad (PV_{5.0} - PV_{3.5}) > X_T \\ 0 & otherwise \end{cases}$$

If the difference between the PV of the expanded project ($PV_{5.0}$) and the PV of the 3.5 Mty project ($PV_{3.5}$) is greater than the expansion cost ($X_T$), the project must be expanded, generating a payoff equal to $((PV_{5.0} - PV_{3.5}) - X_T)$. Otherwise, the current extraction rate is kept, generating a zero payoff.

This procedure is executed recursively, from the expiration date of the option (T) to the current date (t=0). At each node prior to the expiration date, an optimality test is made, comparing the value obtained by exercising the option at that node ($Payoff_t$) with the value of keeping the option alive, computed as a weighted average – where the probabilities p and (1-p) are multiplied by the upward payoff ($Payoff_{t+1}^u$) and downward payoff ($Payoff_{t+1}^d$) at the successive year (t+1) – discounted at the risk free interest rate (r). At each node, the following calculation is executed:

$$Max\ (Payoff_t\ ;\ \frac{p * Payoff_{t+1}^u + (1-p) * Payoff_{t+1}^d}{\exp(r)})$$

**b) Option to verticalize**

The flexibility to verticalize has also been priced as an American call option, in which the exercise price changes at each time period ($X_t$) and are equal to the incremental investments needed to verticalize the project at a given point in time. The exercise prices ($X_t$) at time t have been determined by the difference between the PV of the CAPEX expected to occur from the opening date of the project until the end of the project in the 3.5 Mty verticalized mode (calculated at t=0 and brought up to time t through the risk free rate) and the PV of the CAPEX expected to occur until the end of the project in the 3.5 Mty non-verticalized mode (calculated at t=0 and brought up to time t through the risk free interest rate).

The payoff of the option at expiration date is given by the following equation:

$$Payoff_T = \begin{cases} ((PV_{Chemical\ plant} - PV_{3.5}) - X_T) & if\ (PV_{Chemical\ plant} - PV_{3.5}) > X_T \\ 0 & otherwise \end{cases}$$

If the difference between the PV of the verticalized project ($PV_{Chemical\ plant}$) and the PV of the non-verticalized project ($PV_{3.5}$) is greater than the verticalization cost ($X_T$),the project must be verticalized, generating a payoff equal to $(PV_{Chemical\ plant} - PV_{3.5} - X_T)$ Otherwise, the current operating mode is kept, generating a zero payoff.

The same recursive procedure described above is used to compute an optimality test at each node prior to the expiration date:

$$Max\ (Payoff_t\ ;\ \frac{p * Payoff_{t+1}^{u} + (1-p) * Payoff_{t+1}^{d}}{\exp(r)})$$

Repeating the step above recursively until the first node of the lattice, the value of the option to verticalize is obtained.

## IV. Results and conclusion

Using a time to maturity equal to 15 years, the following values have been obtained for the option to expand and the option to verticalize:

| Flexibility | Value of the Option (MUS$) |
| --- | --- |
| Option to Expand from 3.5 to 5.0 Mty | 33.036 |
| Option to Verticalize | 248.678 |

The binomial lattices created for each real option studied is presented below. The analysis of the lattice provides indication about the moments and situations in which the option must be exercised.

<u>Option to expand from 3.5 to 5.0 Mty (MUS$)</u>

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | $0 |
| | | | | | | | | | | | | | | $0 | |
| | | | | | | | | | | | | | $0 | | $0 |
| | | | | | | | | | | | | $31.385 | | $0 | |
| | | | | | | | | | | | $90.199 | | $0 | | $0 |
| | | | | | | | | | | $135.804 | | $0 | | $0 | |
| | | | | | | | | | $200.596 | | $0 | | $0 | | $0 |
| | | | | | | | | $249.741 | | $0 | | $0 | | $0 | |
| | | | | | | | $287.234 | | $0 | | $0 | | $0 | | $0 |
| | Option to Expand (3,5Mt to 5Mt) | | | | | $293.602 | | $23.613 | | $0 | | $0 | | $0 | |
| | | | | | $259.595 | | $59.083 | | $0 | | $0 | | $0 | | $0 |
| | | | | $175.294 | | $72.268 | | $0 | | $0 | | $0 | | $0 | |
| | | | $116.986 | | $56.754 | | $0 | | $0 | | $0 | | $0 | | $0 |
| | | $77.330 | | $34.181 | | $0 | | $0 | | $0 | | $0 | | $0 | |
| | $50.713 | | $20.586 | | $0 | | $0 | | $0 | | $0 | | $0 | | $0 |
| **$33.036** | | $12.399 | | $0 | | $0 | | $0 | | $0 | | $0 | | $0 | |
| | $7.467 | | $0 | | $0 | | $0 | | $0 | | $0 | | $0 | | $0 |
| | | $0 | | $0 | | $0 | | $0 | | $0 | | $0 | | $0 | |
| | | | $0 | | $0 | | $0 | | $0 | | $0 | | $0 | | $0 |
| | | | | $0 | | $0 | | $0 | | $0 | | $0 | | $0 | |
| | | | | | $0 | | $0 | | $0 | | $0 | | $0 | | $0 |
| | | | | | | $0 | | $0 | | $0 | | $0 | | $0 | |
| | | | | | | | $0 | | $0 | | $0 | | $0 | | $0 |
| | | | | | | | | $0 | | $0 | | $0 | | $0 | |
| | | | | | | | | | $0 | | $0 | | $0 | | $0 |
| | | | | | | | | | | $0 | | $0 | | $0 | |
| | | | | | | | | | | | $0 | | $0 | | $0 |
| | | | | | | | | | | | | $0 | | $0 | |
| | | | | | | | | | | | | | $0 | | $0 |
| | | | | | | | | | | | | | | $0 | |
| | | | | | | | | | | | | | | | $0 |

## Option to verticalize from 3.5 Mty to 3.5 Mty with Verticalized Plant (MUS$)

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | $25.665.123 |
| | | | | | | | | | | | | | | $22.036.928 | |
| | | | | | | | | | | | | | $18.767.333 | | $13.074.303 |
| | | | | | | | | | | | | $15.856.145 | | $11.093.801 | |
| | | | | | | | | | | | $13.282.530 | | $9.315.180 | | $6.045.712 |
| | | | | | | | | | | $11.047.698 | | $7.737.371 | | $4.997.240 | |
| | | | | | | | | | $9.091.752 | | $6.348.001 | | $4.060.920 | | $2.138.485 |
| | | | | | | | | $7.372.190 | | $5.146.760 | | $3.235.554 | | $1.616.810 | |
| | | | | | | | $5.674.039 | | $4.097.846 | | $2.513.811 | | $1.155.814 | | $0 |
| | Option (3,5M to 3,5M Verticalized) | | | | | $3.991.585 | | $3.179.138 | | $1.894.891 | | $754.484 | | $0 | |
| | | | | | $2.630.117 | | $2.270.799 | | $1.357.280 | | $408.519 | | $0 | | $0 |
| | | | | $1.694.816 | | $1.373.412 | | $889.878 | | $209.773 | | $0 | | $0 | |
| | | | $1.071.883 | | $814.557 | | $480.326 | | $107.718 | | $0 | | $0 | | $0 |
| | | $667.243 | | $477.010 | | $258.650 | | $55.313 | | $0 | | $0 | | $0 | |
| | $409.747 | | $276.442 | | $138.980 | | $28.403 | | $0 | | $0 | | $0 | | $0 |
| **$248.678** | | $158.814 | | $74.531 | | $14.585 | | $0 | | $0 | | $0 | | $0 | |
| | $90.562 | | $39.897 | | $7.489 | | $0 | | $0 | | $0 | | $0 | | $0 |
| | | $21.321 | | $3.846 | | $0 | | $0 | | $0 | | $0 | | $0 | |
| | | | $1.975 | | $0 | | $0 | | $0 | | $0 | | $0 | | $0 |
| | | | | $0 | | $0 | | $0 | | $0 | | $0 | | $0 | |
| | | | | | $0 | | $0 | | $0 | | $0 | | $0 | | $0 |
| | | | | | | $0 | | $0 | | $0 | | $0 | | $0 | |
| | | | | | | | $0 | | $0 | | $0 | | $0 | | $0 |
| | | | | | | | | $0 | | $0 | | $0 | | $0 | |
| | | | | | | | | | $0 | | $0 | | $0 | | $0 |
| | | | | | | | | | | $0 | | $0 | | $0 | |
| | | | | | | | | | | | $0 | | $0 | | $0 |
| | | | | | | | | | | | | $0 | | $0 | |
| | | | | | | | | | | | | | $0 | | $0 |
| | | | | | | | | | | | | | | $0 | |
| | | | | | | | | | | | | | | | $0 |

The results obtained indicate that the option to expand and the option to verticalize represent substantial source of value for the project. They also suggest that the project should be initiated following the assumptions of Alternative 1, with both options alive (not exercised).

Before a final decision is taken, additional studies and simulations will be carried on. The valuation of the option to verticalize, starting the exploitation of the mine according to Alternative 2, is already in progress.

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