

# Let Managers Decide: Designing Contracts for Optimal Investment Decisions

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## **Abstract**

Real options literature commonly assumes that, either the investment opportunity is directly managed by the owners, or the managers are perfectly aligned with them. However, agency conflicts occur and managers reveal interests not totally in line with those of the shareholders. This may have a major impact on the value maximizing decisions, namely, on the optimal timing to invest. This paper contributes to the scarce literature that accounts for agency problems on the exercise of real options. We propose a model which allows to set an optimal contract structure in order to avoid inadequate actions from the manager. In our model, shareholders need not to follow the future evolution of project value drivers in order to guarantee optimal behavior. It is shown that even small deviations from the optimal compensation scheme may lead to highly sub-optimal decisions. Optimal contracts are also established for special situations, namely, to account for impatient managers, for non-proprietary (or non-monopolistic) real options, and also by considering the existence of effort costs for the manager.

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# 1 Introduction

In the recent decades, real options framework has become one of the fundamental foundations of investment evaluation. Laying roots in the options pricing theory of the seminal work of Black and Scholes (1973), it was coined by Myers (1977) in a reference of the growth opportunity on corporate assets. Later, Myers (1984) defended the relevance of real options to strengthen the interaction between finance and strategy in corporate environment, which Kester (1984) reiterated in the same year. Developments rapidly began in capital budgeting area, with groundwork papers such as Brennan and Schwartz (1985) where an investment opportunity was evaluated under a real options framework comprising commodity price uncertainty, and McDonald and Siegel (1986) who considers uncertainty in both projects cash-flows and costs. Other relevant works appeared since then, extending the concept and its applicability, and bridging with various fields. Find two excellent surveys in Dixit and Pindyck (1994) and Trigeorgis (1996).

In financial theory, optimal investment decisions are taken under the premise of firms market value maximization (Jensen (2001)). Usually, in an all-equity firm, value maximization is not deteriorated by inside determinants when owners have full control of their endogenous variables. When some undesirable factors arise, for instance high ownership dispersion (Berle and Means (1932)) or owners lack of expertise (Shleifer and Vishny (1997)), the shift of control decision to an exogenous entity (agent) is an inevitability which naturally raises the probability of value destruction if there is a misalign of targets between manager and owner. This is the preeminent agency dilemma, formally established and generalized in the seminal paper of Jensen and Meckling (1976), but which first academic observation can be found two hundred years earlier in Smith (1776).

In order to reduce agency conflict, literature proposes internal and external mechanisms. The first category comprises incentive contracts (Jensen and Smith Jr (1985)), insider ownership (Jensen and Meckling (1976)), existence of large investors (Shleifer and Vishny (1986)), board of administration (Fama and Jensen (1983)), the existence of debt and dividend policy, which reduces free cash-flows (Jensen (1986)). External mechanisms encompasses, for instance, managers reputation (Shleifer and Vishny (1997)), managers market competition (Fama (1980)), output market competition (Hart (1983)), takeovers market (Easterbrook and Fischel (1981)), monitoring by investment professionals (Chung and Jo (1996)) or legal framework (Shleifer and Vishny (1997)). Despite its relevance, these mechanisms have limitations, which several works have shown. We highlight Holmstrom (1982), Shleifer and Vishny (1997), Grossman and Hart (1980), Burkart (1995), Jensen (1993).

Ideally, shareholders would not need to have full information and control over manager if he defines a proper framework of pecuniary incentives, so that risk bearing and risk premium are shared with manager. Nevertheless, as Shleifer and Vishny (1997) notes, incentive contracts can create opportunities for self-dealing under contract negotiation inefficiency which will lead to misappropriation of firm's value to manager.

Investment timing decisions, when studied under a real options approach, usually tend to assume perfect aligning of interests between managers and shareholders, ignoring the impact of agency conflicts. Recently, this issue has kept the attention of some authors, generating bridging papers such as Grenadier and Wang (2005), which examines investment timing decision for a single project, where the owner delegates the investment decision to the manager. Manager behavior will account for asymmetric information and moral hazard, generating sub-optimal decisions that can be corrected through an optimal contract, aligning the incentives of owners and managers, and Nishihara and Shibata (2008) extends this model incorporating a relationship between an audit mechanism and bonus-incentives sensitive to managers deviated actions. Shibata and Nishihara (2010) extends these works by incorporating debt financing on investment expenditure.

Hori and Osano (2010) presents an agency model under a real options framework where managerial compensation is designed endogenously including a contingent claim on firms cash-flows using stock options.

Our primary purpose will be to relax the assumption that managers are perfectly aligned with owners (as assumed in the standard real options literature, e.g. Dixit and Pindyck (1994)) and to provide a perceptive but yet meaningful framework where a principal entity (a shareholder or a group of shareholders) owns an option, but for plausible reasons (i.e., incapacity, opportunity cost or control difficulty) need to hire a manager to supervise the option, to follow market conditions, and to take the investment decision. In order to avoid inadequate actions, a contract structure is defined, using a contingent element based on projects cash-flows, which optimal solution will maximize shareholders value, while transferring decision process to manager.

This work differs from closed related literature, namely Grenadier and Wang (2005) and Hori and Osano (2010), by considering a compensation contract where: (1) the variable component is strictly contingent on the critical value and not through a stock options scheme; (2) the fixed component is continuous over time; but (3) this continuous fixed component differs whether manager is administering the option or running the active project. We distinguish from these authors by including a pre-exercising management continuous wage.

The rest of the paper is organized as follows. Section 2.1 presents the basic framework

where shareholder's optimal investment strategy and manager's optimal solution are derived, considering the incentives contract. Section 2.2 sets the equilibrium solution that aligns the managers interests with those of shareholders. Section 3 presents a comparative statics analysis and a numerical example. Section 4 sets the optimal solutions for some particular situations, namely, for impatient managers, for non-proprietary investment opportunities, and to account for the existence of effort costs for the manager. Section 5 concludes.

## 2 Model

In this section we derive the model, presenting the assumption, the main steps, and settling the optimal equilibrium for the compensation scheme. This optimal equilibrium will be compared with results one would obtain for a project directly managed by the owners.

### 2.1 Setup

A firm has an option to invest in a single project. The shareholders decide to hire a manager for running this investment opportunity; the agent will follow the market conditions and take the investment decision. The decision for professional management arises from restrictions that constrict owner's own actions. We assume that shareholders want to maximize their project value, although, they are limited by their own conditions such as lack of specific know-how, equity structure or simply a matter of opportunity cost, which will lead them to hire an agent to manage the option and to take optimal investment timing decision.

Since both stockholders and managers are rational players and utility maximizers, an issue of asymmetric information and control asymmetry (similar to hidden action of Grenadier and Wang (2005)) arises. In a such a context, the owners incapacity in controlling manager's effort and actions, implies that they can't fully control manager engagement. Therefore, stockholders won't be able to identify manager's decisions ex ante neither will be able to reset manager's actions after contractual establishment implicating that owners must properly design the contract before delegate project's control to manager.

In order to achieve our purpose we will use the standard contingent claims approach, as defined in Dixit and Pindyck (1994). Therefore, we start to define the present value of

cash flows as variable  $V(t)$ <sup>2</sup>, which follows a geometric Brownian motion (gBm) so that:

$$dV(t) = \alpha V(t)dt + \sigma V(t)dz \quad (1)$$

where  $V(0) > 0$ ,  $dz$  is the increment of the Wiener process,  $\alpha$  is the instantaneous conditional expected relative change in  $V$ , also known as drift.  $\alpha = r - \delta$  ( $r > \delta$ ), where  $r$  is the risk-free rate and  $\delta$  ( $> 0$ ) corresponds to the opportunity cost from deferring, and  $\sigma$  is the instantaneous conditional standard deviation. Additionally, shareholders and managers are assumed to be risk neutral players.

Similarly to a call option configuration, shareholders will pay an investment cost  $K_s$  (that after spent will be perceived to owners as a sunk cost) and, since manager gets a salary, owners have an additional wage cost.

This wage comprehends two different states. At first, manager will earn an option management fee  $w_i$ , corresponding to a continuous fixed wage for managing the idle project, i.e., while he watches market conditions and wait for the appropriate investment moment. While realistic, this is ignored by the related literature. Grenadier and Wang (2005) assumes, implicitly, the manager works for free prior exercising the options, and Hori and Osano (2010) considers a fixed global payment for the manager, which is independent from the time the project remains idle.

After exercising the option, the manager will earn a fixed continuous wage ( $w_a$ ) plus a value-sharing bonus ( $\phi V$ ) depending on the value of project cash flows. Note that  $\phi V$  is same as the present value of a portion  $\phi$  of each annual cash flow.

We define some assumptions concerning the labor market and manager's inflows, as follows:

**Assumption 1:** *Owners can't administer directly the option to invest. Also they are unable of properly observe some key value drivers (namely,  $V$ ,  $\sigma$ ,  $\alpha$ ), so the option becomes useless without a manager.*

**Assumption 2:** *Managers and options to invest aren't scarce, so that owners can always find a manager for running his projects, and managers can always find another investment opportunity needing to be managed.*

**Assumption 3:** *The parameter  $w_i$  represents the managers market price for running an idle project (meaning that the owners can't find a less expensive manager). Also, we assume that the fixed wage to manage the active project,  $w_a$ , is lower than  $w_i$ , so that manager's utility function integrates awaiting value. Note, however, that the lower fixed*

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<sup>2</sup>For convenience, in the remainder of this paper, we drop the reference to time and simply represent  $V(t)$  as  $V$ .

salary will be compensated by an appropriate value-sharing bonus.

**Assumption 4:** *Manager can only broke contract before option exercise and this only happens if the option becomes worthless. In this case manager will earn a fixed compensation  $\frac{w_i}{r}$ .*

**Assumption 5:** *After establishing the compensation scheme, renegotiations are not allowed.*

**Assumption 6:** *For the owners, value-sharing bonus are less expensive than monitoring costs.*

Shareholders will pay a wage to manager whether he exercises the option or not, so that their option value will have to consider that component. Setting  $S(V)$  as shareholders' option value,  $V_s$  as theirs optimal exercising value (the trigger), then we have the following ordinary differential equation (o.d.e.):

$$\frac{1}{2}\sigma^2V^2S''(V) + (r - \delta)VS'(V) - rS(V) - w_i = 0 \quad (2)$$

$S(V)$  must satisfy the appropriate boundary conditions that ensure that shareholders will choose the optimal investment decision. Therefore, we have the following conditions, which the first is the value matching condition, the second is the smooth-pasting condition and the third is an absorption barrier:

$$S(V_s) = V_s - K_s - \left(\frac{w_a}{r} + \phi V_s\right) \quad (3)$$

$$S'(V_s) = 1 - \phi \quad (4)$$

$$S(0) = -\frac{w_i}{r} \quad (5)$$

The last condition arises from Assumption 4, which implies that, since manager is the only player with full information, he will choose to leave the firm receiving a compensation  $\frac{w_i}{r}$ . Note that if he does so, he won't earn more than the market equilibrium wage, since the additional bonus hypothesis dies, so for self-fulfillment reasons, manager will chase another projects. Additionally, this condition represents the sunk cost that shareholders have for maintaining the option alive.

The general solution consists of a homogenous component and a particular solution that, satisfying the last boundary condition, will take the form:

$$S(V) = AV^\beta - \frac{w_i}{r} \quad (6)$$

In order to find the owner's option value and the critical value  $V_s$  we use the remaining

boundary conditions (3) and (4). Substituting and rearranging we have:

$$S(V) = \begin{cases} \left(\frac{V}{V_s}\right)^\beta \frac{1}{\beta-1} \left(K_s - \frac{w_i - w_a}{r}\right) - \frac{w_i}{r} & \text{for } V < V_s \\ (1-\phi)V - K_s - \frac{w_a}{r} & \text{for } V \geq V_s \end{cases} \quad (7)$$

and the trigger:

$$V_s = \frac{\beta}{\beta-1} \frac{1}{1-\phi} \left(K_s - \frac{w_i - w_a}{r}\right) \quad (8)$$

where:

$$\beta = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \quad (9)$$

Equation (7) is the value function for shareholders however shareholders don't actually know their function's value drivers (Assumption 1) and, consequently, the concrete value will depend of manager's choice, which can possibly be misaligned with owner's optimal result. Since owners propose a partial contingent payment, a comparable option is implicitly given to manager though, inversely to shareholders, managers will earn a wage. So we need to estimate manager's value function  $M(V)$ , and the critical value  $V_m$ , by solving the following o.d.e.:

$$\frac{1}{2}\sigma^2 V^2 M''(V) + (r-\delta)V M'(V) - rM(V) + w_i = 0 \quad (10)$$

respecting boundary conditions:

$$M(V_m) = \frac{w_a}{r} + \phi V_m \quad (11)$$

$$M'(V_m) = \phi \quad (12)$$

$$M(0) = \frac{w_i}{r} \quad (13)$$

As mentioned before, the first two conditions are fundamental to ensure that the optimal decision is taken and the last one is an absorption barrier related to  $V$ 's stochastic nature. The general solution of equation (10) is:

$$M(V) = AV^\beta + \frac{w_i}{r} \quad (14)$$

In which results the following solution for  $V_m$  and  $M(V)$ , where  $\beta$  is given by equation

(9):

$$V_m = \frac{\beta}{\beta - 1} \frac{1}{\phi} \frac{w_i - w_a}{r} \quad (15)$$

and

$$M(V) = \begin{cases} \left(\frac{V}{V_m}\right)^\beta \frac{1}{\beta - 1} \frac{w_i - w_a}{r} + \frac{w_i}{r} & \text{for } V < V_m \\ \frac{w_a}{r} + \phi V & \text{for } V \geq V_m \end{cases} \quad (16)$$

## 2.2 Optimal wage settling

Since  $w_i$  is exogenous (Assumption 3), shareholders can only influence manager's decision using  $w_a$  and  $\phi$ . By interacting  $w_a$  with  $w_i$  (with  $w_a < w_i$ ) owners can define manager's opportunity cost for implementing the project, but they can only get manager's to truly align his target with theirs through  $\phi$  so that residual claims are shared between players. In order to determine the optimal  $\phi$  (i.e.,  $\phi^*$ ), individual optimal decisions must be aligned, so that  $V_m$  must equal  $V_s$ , which results:

$$\phi^* = \frac{K_m}{K_s} \quad (17)$$

where:

$$K_m = \frac{w_i - w_a}{r} \quad (18)$$

Equation (17) shows that, in the presence of information asymmetry, value-sharing component enforces optimal decisions, which will only depend on the relation between the opportunity cost for the manager ( $K_m$ ) and the opportunity cost for the shareholders ( $K_s$ ). Therefore, possibly surprising, shareholders can build an optimal contract scheme ignoring the project key value drivers, and by only knowing  $K_s$ ,  $w_i$ ,  $w_a$  and  $r$  (which is consistent with Assumption 1).

Considering traditional model of investment under contingent claims, as it appears in Dixit and Pindyck (1994), optimal trigger is defined as:

$$V^* = \frac{\beta}{\beta - 1} K_s \quad (19)$$

Our model shows that no-agent critical value will be shared between owners and manager. This means that their share of value will equal no-agent critical value excluding the share demanded by agent under his terms. Substituting (8) and (15) into (19) and

arranging we have:

$$V^* = (1 - \phi)V_s + \phi V_m \quad (20)$$

Additionally, under optimal contract definition ( $\phi^*$ ), the agent chooses  $V_m$  which equals  $V_s$  and  $V^*$ . Therefore, shareholders will ensure that manager's critical value is also  $V^*$  implying that no value is misappropriated, since manager will only have the sharing-value bonus that owners are willing to give him and that they find acceptable as a reward for manager's loyalty and activity.

### 3 Comparative Statics and Numerical Example

#### 3.1 Comparative Statics

Recall  $\phi^*$  as presented in equation (17), which is a function of  $w_i$ ,  $w_a$ ,  $r$ , and  $K_s$ . Taking the derivatives:

$$\frac{\partial \phi^*}{\partial w_i} > 0 \quad (21)$$

$$\frac{\partial \phi^*}{\partial w_a} < 0 \quad (22)$$

$$\frac{\partial \phi^*}{\partial r} < 0 \quad (23)$$

$$\frac{\partial \phi^*}{\partial K} < 0 \quad (24)$$

we find that the optimal value-sharing rate ( $\phi^*$ ) is positively related with the wage the manager receives prior the project implementation – the higher (lower)  $w_i$ , the higher (lower) the opportunity cost for the manager that comes from launching the project, so the higher (lower) the value-sharing that ensures optimal decision –, and negatively related with wage post implementation – the higher (lower)  $w_a$ , the lower (higher) the opportunity cost for the manager.  $\phi$  is also negatively related with the interest-rate and the investment cost.

It is particularly relevant to observe the opposite effects that changes in  $\phi$  produces on the optimal triggers  $V_s$  and  $V_m$ :

$$\frac{\partial V_s(\phi)}{\partial \phi} > 0 \quad (25)$$

$$\frac{\partial V_m(\phi)}{\partial \phi} < 0 \quad (26)$$

the higher (lower) the value shared with the manager the higher (lower) the shareholders' trigger, meaning later optimal investment for them, and, on contrary, the higher (lower) the values for  $\phi$  the lower the manager's trigger, meaning sooner optimal investment. This is due to the opposite effect of  $\phi$  on the value of the option to wait and defer the project implementation. Any deviation from critical  $\phi$  creates misaligning between manager and the shareholders and potentially leads to a suboptimal decision taken by the manager, when the shareholders interests are concerned. As we can see from the numerical example below, even a small deviation from  $\phi^*$  has a significant impact on the manager trigger, and, in the final, on the timing for the project implementation.

Additionally, by fixing  $w_i$  and  $w_a$  ( $w_i > w_a$ ), we find that:

$$V_s \rightarrow \frac{\beta}{\beta-1} \left( K_s + \frac{w_a - w_i}{r} \right) < V^* \quad \text{as } \phi \rightarrow 0 \quad (27)$$

$$V_m \rightarrow +\infty \quad \text{as } \phi \rightarrow 0 \quad (28)$$

$$V_s \rightarrow +\infty \quad \text{as } \phi \rightarrow 1 \quad (29)$$

$$V_m \rightarrow \frac{\beta}{\beta-1} \frac{w_i - w_a}{r} \quad \text{as } \phi \rightarrow 1 \quad (30)$$

and by fixing  $\phi$ , and remember that  $\phi \in (0, 1)$ , we see that:

$$V_s \rightarrow \frac{\beta}{\beta-1} \frac{1}{1-\phi} K_s > V^* \quad \text{as } w_a \rightarrow w_i \quad (31)$$

$$V_m \rightarrow 0 \quad \text{as } w_a \rightarrow w_i \quad (32)$$

Equations (27) and (28) shows that when there's no variable compensation for the manager, the optimal trigger for the shareholders is lower than the one appearing in the standard real options approach (this is due to the savings in wages that reduces the *net* investment cost). However, if that happens, the manager will never launch the project, since he will not give up the higher fixed salary for managing the option, to just receive lower one for managing the active project. Naturally, in equilibrium, this reduction must be compensated by a positive (and adequate) value-sharing rate. Equations (29) and (30) indicates that as the value-sharing rate tends to its maximum value, it will be never optimal for the shareholder to sunk the investment cost  $K$ , and the trigger for the manager tends to its minimum value, where its investment cost corresponds to  $\frac{w_i - w_a}{r}$ . Finally, equations (31) and (32) shows that in absence of wage savings, the shareholders optimal trigger will be higher than that of the project if managed directly by them, and also that for the manager it will be optimal to invest immediately.

### 3.2 Numerical Example

Let us now present a numerical example. Assume a firm holding the option to invest in a given project. The shareholders decide for professional management, and so they hire a manager for running this investment opportunity. The agent major tasks are to monitor the option's key value-drivers and, at some point in time, take the decision to invest. As we said, the shareholders are interested in designing a contract with the right compensation for the manager, using a contingent element based on projects cash-flows, which guaranties that the option to invest is exercised at the optimal moment, from their point of view. In our model, this is done by defining a critical  $\phi$ , for a given  $w_a$ , that aligns both the agent and principle interests.

Consider the values for the parameters presented in Table 1:

Parameter	Value	Description
$K_s$	\$1,000	Investment cost
$r$	0.05	Risk-free interest rate
$\sigma$	0.20	Instantaneous volatility
$\delta$	0.03	Dividend-yield
$w_i$	\$5	Fixed wage for managing the idle project
$w_a$	\$2	Fixed wage for managing the active project
$\phi$	—	Value-sharing rate

**Table 1:** The base case parameters.

Based on equations (8), (15), (17), and (19) we find the following output values:

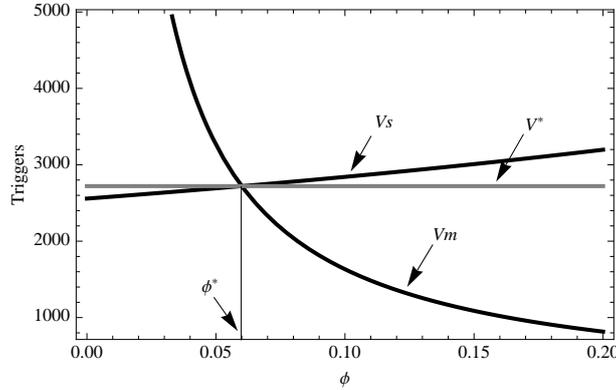
Output	Value
$\phi^*$	0.06
$V_s = V_m = V^*$	\$2,720.8

**Table 2:** The output values for the parameters presented in Table 1.

The shareholder will offer the manager a value-sharing rate of 0.06, as the price to ensure optimal behavior. In this case, both the owners and the manager share the same trigger, and the project will be launched optimally when  $V$  hits \$2,720.8.

Figure 1 shows the impact of different levels of  $\phi$  on the optimal triggers for the shareholders ( $V_s$ ) and for manager ( $V_m$ ), and also the optimal trigger for the project if managed directly by the owners ( $V^*$ ). All the three functions met at  $\phi = \phi^*$ . While  $V_s$  presents a small sensitivity to  $\phi$ ,  $V_m$  reveals to be highly sensitive to this parameter. The conclusion

is straightforward: if manager behave according his own interest, even small deviations from  $\phi^*$  will imply a significant sub-optimal decision for the shareholders. Establishing a  $\phi < \phi^*$  the project will be launched too late, and a  $\phi > \phi^*$  the implementation will be taken too soon.



**Figure 1:** The trigger for different levels of  $\phi$ . The other parameters are according Table 1.

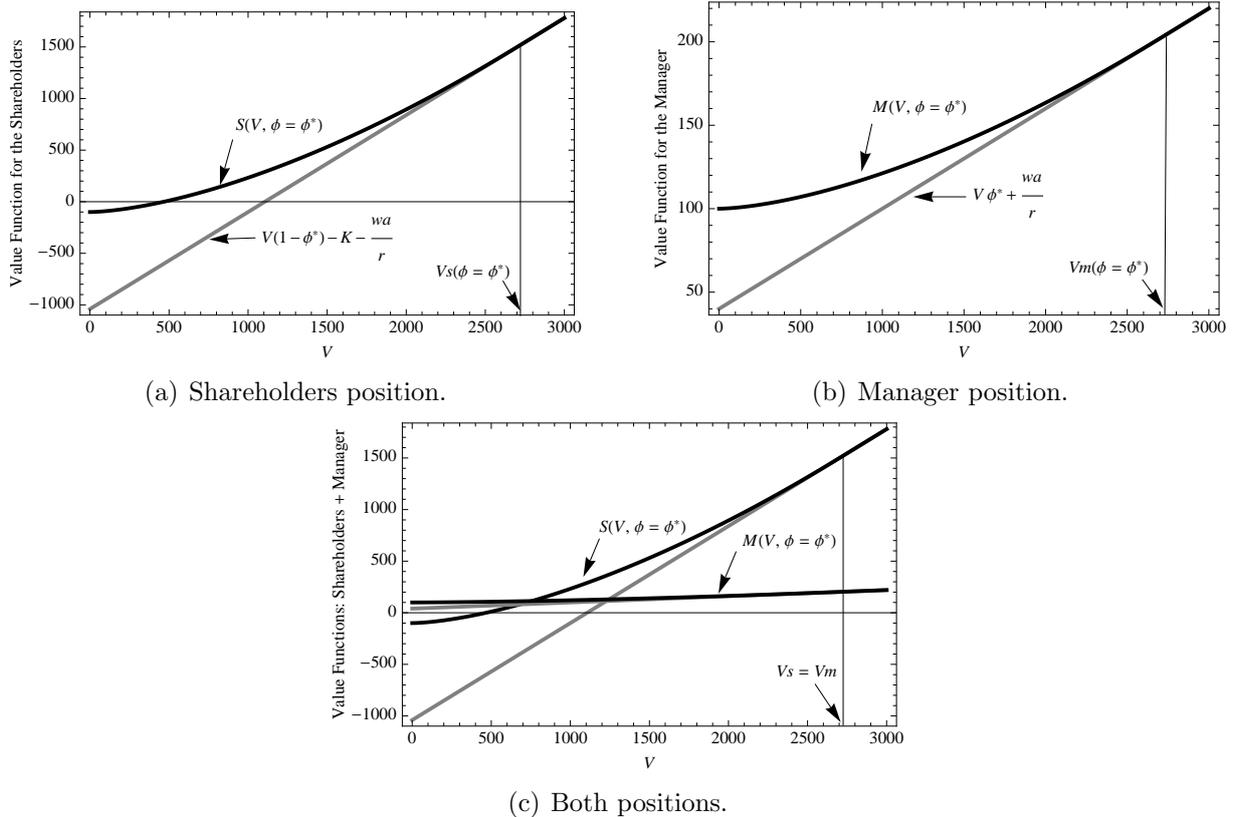
Figures 2(a) and 2(b) shows the value functions for the shareholders and for the manager, with the standard appearance. The optimal decision for both will be maintaining the options alive until  $V$  hits  $V_s$  and  $V_m$ , respectively. Note, however, that  $S(V)$  presents an odd negative region due to the wage that the owners pays the agent for managing the option. For this reason, there's a region where  $M(V)$  dominates  $S(V)$ , as one can see from Figure 2(c).

Despite our focus on the key parameter  $\phi$ , the compensation scheme presented in this paper consists in a mix between  $\phi$  and  $w_a$ . Accordingly, there is a critical pair of  $\phi$ 's and  $w_a$ 's (i.e., there is a critical  $w_a$  for a given  $\phi$ , or a critical  $\phi$  for a given  $w_a$ ) that ensures optimal behavior for the manager. Figure 3 pictures this combination: the optimal pairs of  $\phi$ 's and  $w_a$ 's correspond to the interception of the two surfaces.

Figure 4 shows the manager value function for different levels of  $w_a$ . The higher (lower) the wage for managing the active project, the lower (higher) critical value-sharing rate. So what shareholders should do is to find the optimal scheme trading-off the fixed and variable compensation components.

## 4 Particular Situations

In this section three particular situations are analyzed. In 4.1 we consider the existence of impatient managers, in 4.2 we account for existence of a non-monopolistic project, and,



**Figure 2:** The value functions for the shareholders and for manager. The parameters are according Table 1, and  $\phi = \phi^* = 0.06$ .

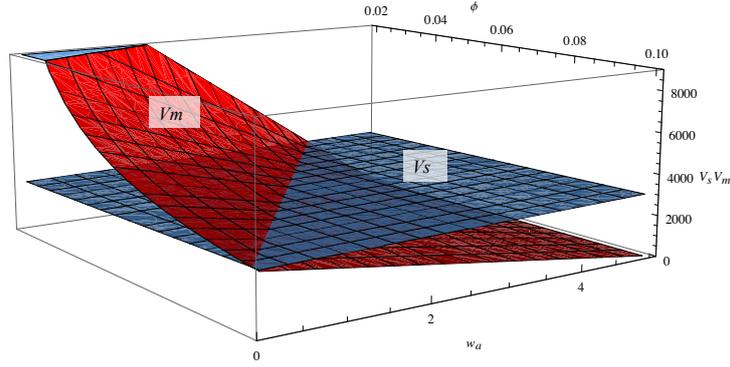
finally, in 4.3 we extend the model to account for the existence of effort costs for the manager. In all this cases, the impact on compensation is analyzed, and we show the solution for designing the optimal compensation schemes.

## 4.1 Impatient managers

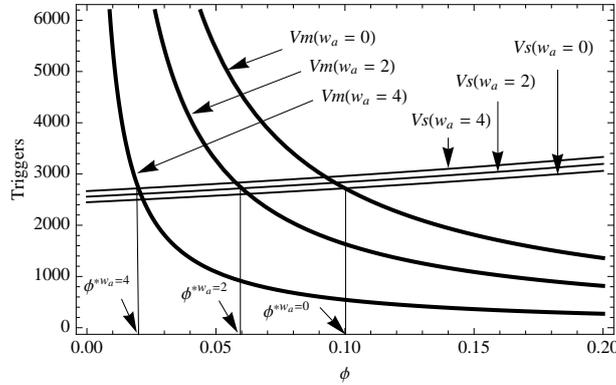
Until now we have assumed that both shareholders and managers value the project cash-flows identically. However, as pointed out by Grenadier and Wang (2005), managers can be more impatient than shareholders. They present several reasons to justify impatient managers: short-term preferences, empire building, greater perquisites consumption and reputation.

We follow Grenadier and Wang (2005), and account the manager's greater impatience by increasing the discount rate from  $r$  to  $r + \xi$ . This produces different payoffs valuation for the owners and for the manager.

The optimal value-sharing rate for an impatient manager ( $\phi_\xi^*$ ) is as follows (details on



**Figure 3:** The triggers  $V_s$  and  $V_m$  for different levels of  $\phi$  and  $w_a$ . The other parameters are according Table 1.



**Figure 4:** The triggers  $V_s$  and  $V_m$  for different levels of  $\phi$  and  $w_a$ . The other parameters are according Table 1.

derivation appear in Appendix):

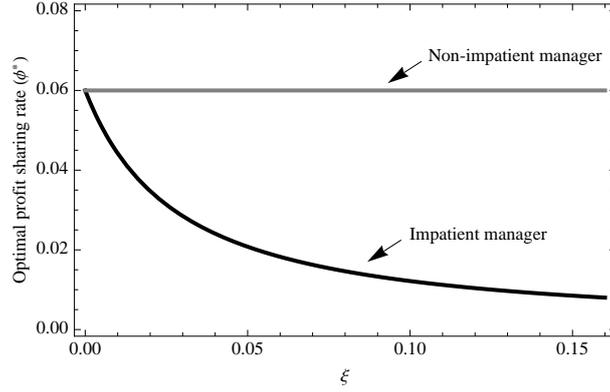
$$\phi_\xi^* = \frac{r(w_i - w_a)(\beta - 1)\gamma}{rK\beta(\gamma - 1)(r + \xi) - (w_a - w_i)[r(\beta - \gamma) - \beta(\gamma - 1)\xi]} \quad (33)$$

where  $\beta$  is as defined in (9), and  $\gamma = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2(r + \xi)}{\sigma^2}}$ . It can be shown that  $\phi_\xi^* < \phi^*$  for any  $\xi > 0$ , which means the impatient manager will demand less value-sharing for aligning its interests with the owners'.

The impact of  $\xi$  on the optimal value-sharing rate is illustrated in Figure 5.

## 4.2 Non-monopolistic projects

Consider now the investment opportunity is not proprietary, and, by the entrance of a rival, the option to invest can suddenly disappear. This random catastrophic event can



**Figure 5:** The optimal value-sharing rate. The other parameters are according Table 1.

be modeled in a well-known way, by including a parameter  $\lambda$  ( $> 0$ ) in the o.d.e. that must be followed by the owners value function (where  $\lambda$  is a Poisson rate of arrival)<sup>3</sup>:

$$\frac{1}{2}\sigma^2V^2S''(V) + (r - \delta)V S'(V) - (r + \lambda)S(V) - w_i = 0 \quad (34)$$

Following the standard procedures, the solution comes:

$$V_s^\lambda = \frac{\eta}{\eta - 1} \frac{1}{1 - \phi} \left( K_s - \frac{w_i - w_a}{r} \right) \quad (35)$$

where:

$$\eta = \frac{1}{2} - \frac{(r - \delta)}{\sigma^2} + \sqrt{\left( \frac{r - \delta}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2(r + \lambda)}{\sigma^2}} > 1 \quad (36)$$

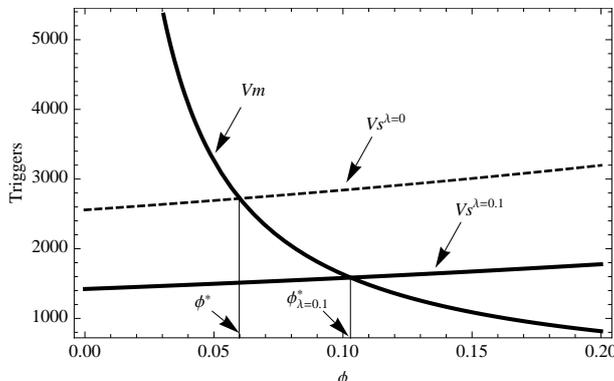
Since, for any positive  $\lambda$ ,  $\eta > \beta$  (where  $\beta$  is given by equation 9), we have  $V_s^\lambda < V_s$ . This means that, under the fear of preemption, the shareholders will be interested to invest earlier.

According to Assumption 2 this catastrophic event don't impact the manager value function nor his optimal trigger, which remains as presented in equation (15). Under this setting, the value-sharing rate becomes (see the proof in the Appendix):

$$\phi_\lambda^* = \frac{(w_i - w_a)\beta(\eta - 1)(r + \lambda)}{r [(w_a - w_i)(\beta - \eta) + Kr(\beta - 1)\eta] + \lambda [w_a(\beta - \eta) + w_i\beta(\eta - 1) + Kr(\beta - 1)\eta]} \quad (37)$$

<sup>3</sup>At the beginning the o.d.e. takes the form  $\frac{1}{2}\sigma^2V^2S''(V) + (r - \delta)V S'(V) - rS(V) + \lambda[0 - S(V)] - w_i = 0$ , where  $\lambda[0 - S(V)]$  captures the expected loss resulting from the entrance of a rival firm, which makes the option value drop from  $S(V)$  to 0.

It can be shown that  $\phi_\lambda^* > \phi^*$ , meaning that the owners will need to share more value with the manager in order to align both interest. In fact, and as we can see from Figure 6, the only way the owners have to induce early investment is to give value to managers.



**Figure 6:** The optimal value-sharing rate for non-proprietary option.  $\lambda = 0.1$ , and the other parameters are according Table 1.

### 4.3 The inclusion of management effort costs

Supposing manager exerts effort in both states of his value function such that, when he is just managing the option, analyzing market conditions and studying the optimal moment to invest, he spends a continuous effort cost  $e_i$ , but when the project turns to be active, he increases his effort cost to  $e_a$ , representing all the diligences in order to implement the project and to effectively manage the business<sup>4</sup>. It is intuitive to think that managing the implemented project demands more effort than managing the option, so we assume that  $e_a > e_i$ .

Using equation (10), we restate o.d.e. considering this setting:

$$\frac{1}{2}\sigma^2V^2M''(V) + (r - \delta)VM'(V) - rM(V) + w_i - e_i = 0 \quad (38)$$

respecting boundary conditions:

$$M(V_m^e) = \frac{w_a}{r} + \phi V_m^e - \frac{e_a}{r} \quad (39)$$

$$M(V_m^e) = \phi \quad (40)$$

$$M(0) = \frac{w_i - e_i}{r} \quad (41)$$

<sup>4</sup>Grenadier and Wang (2005) also consider positive effort costs but in a different context. The manager, at time zero, incurs in an effort cost in order to increase the likelihood for a higher quality project.

So that the solution of  $M(V)$  and  $V_m^e$  (the optimal trigger under the existence of effort costs) are, respectively:

$$M(V) = \begin{cases} \left(\frac{V}{V_m}\right)^\beta \frac{1}{\beta-1} \left[ \frac{w_i - w_a}{r} + \frac{e_a - e_i}{r} \right] + \frac{w_i - e_i}{r} & \text{for } V < V_m^e \\ \frac{w_a - e_a}{r} + \phi V & \text{for } V \geq V_m^e \end{cases} \quad (42)$$

and

$$V_m^e = \frac{\beta}{\beta-1} \frac{1}{\phi} \left[ \frac{w_i - w_a}{r} + \frac{e_a - e_i}{r} \right] \quad (43)$$

Since shareholders' value function doesn't incorporate manager's effort it remains the same as in equation (7). But this occurrence has an implicit problem concerning misaligning of targets and expectations, because now we have a different optimal value-sharing bonus rate ( $\phi_e^*$ ), such that when  $V_s = V_m^e$ :

$$\phi_e^* = \frac{\frac{w_i - w_a}{r} + \frac{e_a - e_i}{r}}{K_s} \quad (44)$$

representing:

$$E_m = \frac{e_a - e_i}{r} \quad (45)$$

we get:

$$\phi_e^* = \frac{K_m + E_m}{K_s} = \phi^* + \frac{E_m}{K_s} \quad (46)$$

which results that  $\phi^* < \phi_e^*$ .

Note that  $\phi^* = \phi_e^*$  if  $e_i = e_a$ . This way, we show that if an increment of effort costs occurs (by the reasons we presented earlier) and if this increment is ignored, shareholders will give manager a lower value-sharing component and, consequently, a less valuable contract, failing to provide the proper aligning interests.

## 5 Conclusions

This paper overcomes the common assumption which implies that, either the investment opportunity is directly managed by the owners, or the managers are perfectly aligned with them. However, agency conflicts occur and managers reveal interests not totally in line

with those of the shareholders. This may have a major impact on the value maximizing decisions, namely, on the optimal timing to invest. Despite of this problem, literature that accounts for agency conflicts on the exercise of real options appears to be scarce.

In this context, we propose an optimal contract scheme (incorporating fixed wages and value-sharing bonus) in order to avoid inadequate actions from the manager. In our model, shareholders need not to follow the future evolution of project value drivers in order to guarantee optimal behavior. It was shown that even small deviations from the optimal compensation scheme may lead to highly suboptimal decisions.

In the end, optimal contracts were also established for special situations, namely, to account for impatient managers, for non-proprietary (or non-monopolistic) real options, and also by considering the existence of effort costs for the manager.

## Appendix

**Proof of equation (33):** The value function for the shareholders stands as in equation (7). The value function for the impatient manager must satisfy the o.d.e.:  $\frac{1}{2}\sigma^2x^2M''(V)+(r-\delta)xM'(V)+w_i=(r+\xi)M(V)$ , see Grenadier and Wang (2005) for the arguments. Following standard procedures the trigger comes  $V_m^\xi = \frac{\gamma}{\gamma-1} \frac{1}{\phi} \frac{w_i-w_a}{r+\xi}$ , where  $\gamma = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{(-\frac{1}{2} + \frac{\alpha}{\sigma^2})^2 + \frac{2(r+\xi)}{\sigma^2}}$ , while the trigger for the owners remains:  $V_s = \frac{\beta}{\beta-1} \frac{1}{1-\phi} (K_s + \frac{w_a-w_i}{r})$ . Both triggers met for  $\phi = \phi_\xi^*$ :  $V_m^\xi(\phi_\xi^*) = V_s(\phi_\xi^*)$ . Solve for  $\phi_\xi^*$ . **Proof of equation (37):** The new trigger for the owners appears in equation (35), the triggers for the manager remains as in (15). Both triggers met for  $\phi = \phi_\lambda^*$ :  $V_m(\phi_\lambda^*) = V_s^\lambda(\phi_\lambda^*)$ . Solve for  $\phi_\lambda^*$ .

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