REAL INPUT-OUTPUT SWITCHING OPTIONS

Roger Adkins*

Bradford University School of Management

Dean Paxson**

Manchester Business School

Real Options Conference Turku

13 February 2011

JEL Classifications: C39, D81, G31

Keywords: Optional switching models, simultaneous equation solutions, input-output parameter value sensitivities.

Acknowledgements: We thank Alcino Azevedo, Jörg Dockdendorf, Michael Flanagan, Paul Johnson and Peter Oliver for helpful comments.

*Bradford University School of Management, Emm Lane, Bradford BD9 4JL, UK.

r.adkins@bradford.ac.uk,

** Manchester Business School, Booth Street West, Manchester, M15 6PB, UK.

dean.paxson@mbs.ac.uk,+44(0)1612756353.

REAL INPUT-OUTPUT SWITCHING OPTIONS

Abstract

We provide quasi-analytical solutions to real options embedded in flexible facilities which have stochastic inputs and outputs with switching costs. We show the facility value and the optimal switching input and output triggers, when there is the possibility for multiple switching between operating and not operating. The facility value and optimal shut down triggers are also calculated, when there is only a one-way switch between operating and suspension (which amounts to abandonment). An illustration is provided for a heavy crude oil field production which requires natural gas as an input, with shut-down and start-up switching costs. More general applications in the energy field include the operational spark spread with switching costs. It may be appropriate to extend these models to many manufacturing, distribution and transportation activities.

REAL INPUT-OUTPUT SWITCHING OPTIONS

Input-output switching options are often embedded in facilities, or situations, sometimes developed by an alert manager. For instance, in Manchester, when “trade was very slack; cottons could find no market” cotton mill owners Messrs. Carson “were no hurry about the business…the weekly drain of wages , useless in the present state of the market, was stopped”.[1]

We provide quasi-analytical solutions for basic two factor multiple switching option models, switching from an operating state with an option to suspend operations, or from a suspended state to an operating state, when both output price and input cost are stochastic. As a simplification, a single switch from operating to suspension (abandonment) is also considered.

1 GENERAL SWITCHING OPTIONS

When is the right time for an operator of a flexible facility to switch back and forth between possible outputs or inputs in order to maximise value when switching costs are taken into account? Which factors should be monitored in making these decisions? How much should an investor pay for such a flexible operating asset? What are the strategy implications for the operator, investor and possibly for policy makers?

Flexible production and processing facilities are typically more expensive to operate, and with a higher initial investment cost, than inflexible facilities. Obviously a flexible input-output facility, which might require an additional investment cost, might be idle at times. What is frequently misunderstood is that the additional option value through “operating flexibility” (Trigeorgis and Mason, 1987) may have significant value in uncertain markets, especially if there is less than perfect correlation between inputs and outputs. Examples of plausibly flexible input-output facilities include crude oil refineries producing gasoline and heating oil, soya bean refineries producing soya meal and oil, facilities producing ethanol from corn or sugar

[1] Elizabeth Gaskell, *Mary Barton, A Tale of Manchester Life*, 1848, page 52.

beets as inputs, and electricity generation (from coal, or natural gas). In the commodity futures markets, such synthetic facilities are termed the "crack", "crush", "agrifuel" and "spark" spreads, although these do not usually involve switching costs or other operating costs (apart from margin requirements and brokers fees and commissions). Perhaps less flexible "facilities" are chickens or hogs or cattle as outputs using corn as inputs, or human exercise as input and good health as output. The shutting down and restarting operations of these activities in these less flexible vehicles perhaps have hard-to-define switching costs.

The traditional approach to determine switching boundaries between two modes is to discount future cash flows and use Jevons-Marshallian present value triggers. This methodology does not fully capture the option value which may arise due to the uncertainty in future input or output prices. The value of waiting to gain more information on future price or cost developments, and consequently on the optimal switching triggers can be best viewed in a real options framework.

Conceptually, the switch between two volatile assets or commodities can be modelled as an exchange option as in Margrabe (1978) for European options. McDonald and Siegel (1986) and Paxson and Pinto (1995) model American perpetual exchange options, which are linear homogeneous in the underlying stochastic variables. Two-factor problems which are linear homogeneous, $V(\lambda \cdot x; \lambda \cdot y) = \lambda \cdot V(x; y)$, can typically be solved analytically by substitution of variables, so that the partial differential equation can be reduced to a one-factor differential equation. An example of this dimension reduction is the perpetual American spread put option in McDonald and Siegel (1985), who assume costless stopping and restarting possibilities.

He and Pindyck (1992) present an analytical model for flexible production capacity, where switching costs and product-specific operating costs are ignored, thereby eliminating the components which would lead to a non-linearity of the value function in the underlying processes. Brekke and Schieldrop (2000) also assume costless switching in their study on the value of operating flexibility between two stochastic input factors, in which they determine the optimal investment timing for a flexible technology in comparison to a technology that does not allow input switching.

Brennan and Schwartz (1985) consider switching states from idle to operating, operating to suspension, and then back, based on one stochastic factor. Paxson (2005) extends the solution for up to eight different state options, each with a distinct trigger, but for only one stochastic factor.

With constant switching cost, operating cost and multiple switching, the problem is no longer homogenous of degree one and the dimension reducing technique cannot be used. Adkins and Paxson (2011) present quasi-analytical solutions to input switching options, where two-factor functions are not homogeneous of degree one, and thus dimension reducing techniques are not available.

The next section presents real option models for an asset with switching opportunities between inputs and outputs with uncertain prices, taking into account switching costs (and allows for other operating costs). Section three illustrates application of these models to a case where the input price is linked to natural gas futures prices and the output is crude oil, a kind of reverse "crack" spread. Section four discusses some policy and strategy implications. Section five concludes, suggesting further applications, and reviews the model limitations.

2 Multiple Input-Output Switching

2.1 Assumptions

Consider a flexible facility which can be used to produce one output using one input, but by incurring a switching cost can be shut down, and by incurring another switching cost can be restarted. Assume the prices of the output, x and the input y, are stochastic and correlated and follow geometric Brownian motion:

$$dx = (\mu_x - \delta_x)x\,dt + \sigma_x\,x\,dz_x \tag{1}$$

$$dy = (\mu_y - \delta_y)y\,dt + \sigma_y\,y\,dz_y \tag{2}$$

with the notations:

μ Required return on the output/input

δ Convenience yield of the output/input

σ Volatility of the output/input

dz Wiener processes

ρ Correlation between the input and output prices: $dz_x\, dz_y / dt$

The instantaneous cash flow in the operating mode is the respective commodity price of the output less unit input plus any other operating cost, assuming production of one (equivalent) unit per annum. Any other operating costs (not shown in the examples) are per unit produced. A switching cost of S_{12} is incurred when switching from operating mode '1' to the non-operating mode '2', and S_{21} for switching back. The appropriate discount rate is r for non- stochastic elements, such as constant other operating costs.

Further assumptions are the lifetime of the facility is infinite, and the company is not restricted in the shut down/start up choice because of selling or buying commitments. Moreover, the typical assumptions of real options theory apply, with interest rates, convenience yields, volatilities and correlation constant over time.

2.2 Quasi-analytical Solution for Multiple Input-Output Switching

The asset value with opportunities to continuously switch between an operating mode and a suspended mode (when both inputs and outputs are stochastic) is given by the present value of perpetual cash flows in the current operating mode plus the option to switch to the alternative mode. Let V_1 be the asset value in operating mode '1', producing output x at input cost y, and V_2 the asset value in a suspension mode '2'. The switching options depend on the two correlated stochastic variables x and y, and so do the asset value functions, which are defined by the following partial differential equations:

$$\frac{1}{2}\sigma_X^2 x^2 \frac{\partial^2 V_1}{\partial x^2} + \frac{1}{2}\sigma_Y^2 y^2 \frac{\partial^2 V_1}{\partial y^2} + \rho\sigma_X\sigma_Y xy \frac{\partial^2 V_1}{\partial x \partial y} + (r-\delta_X)x\frac{\partial V_1}{\partial x} + (r-\delta_Y)y\frac{\partial V_1}{\partial y} - rV_1 + (x-y) = 0 \quad (3)$$

$$\frac{1}{2}\sigma_X^2 x^2 \frac{\partial^2 V_2}{\partial x^2} + \frac{1}{2}\sigma_Y^2 y^2 \frac{\partial^2 V_2}{\partial y^2} + \rho\sigma_X\sigma_Y xy \frac{\partial^2 V_2}{\partial x \partial y} + (r-\delta_X)x\frac{\partial V_2}{\partial x} + (r-\delta_Y)y\frac{\partial V_2}{\partial y} - rV_2 = 0 \quad (4)$$

The operating mode has the production income (x-y) and the option to suspend; the suspension mode has only the option to re-start operations. For stochastic outputs and inputs, the partial differential equations are satisfied by the following general solutions:

$$V_1(x, y) = \frac{x}{\delta_x} - \frac{y}{\delta_y} + A x^{\beta_{11}} y^{\beta_{12}} \tag{5}$$

where β_{11} and β_{12} satisfy the characteristic root equation

$$\tfrac{1}{2}\sigma_x^2\beta_{11}(\beta_{11}-1)+\tfrac{1}{2}\sigma_y^2\beta_{12}(\beta_{12}-1)+\rho\sigma_x\sigma_y\beta_{11}\beta_{12}+\beta_{11}(r-\delta_x)+\beta_{12}(r-\delta_y)-r=0, \tag{6}$$

and

$$V_2(x, y) = B x^{\beta_{21}} y^{\beta_{22}} \tag{7}$$

where β_{21} and β_{22} satisfy the characteristic root equation

$$\tfrac{1}{2}\sigma_x^2\beta_{21}(\beta_{21}-1)+\tfrac{1}{2}\sigma_y^2\beta_{22}(\beta_{22}-1)+\rho\sigma_x\sigma_y\beta_{21}\beta_{22}+\beta_{21}(r-\delta_x)+\beta_{22}(r-\delta_y)-r=0 \tag{8}$$

The characteristic root equation (6) is solved by combinations of β_{11} and β_{12} forming an ellipse of such form that β_{11} could be positive or negative and β_{12} could be positive or negative. The same is true for equation (8). Since the option to switch from operating to suspension decreases with x and increases with y, β_{11} must be negative and β_{12} positive. Likewise, β_{21} must be positive and β_{22} negative. Switching between the operating and suspension modes always depends on the level of both x and y. At the switching points (x_{12}, y_{12}) (shut down) and (x_{21}, y_{21}) (start up), the asset value in the current mode must be equal to the asset value in the alternative mode net of switching costs. These value matching conditions are stated formally below:

$$V_1(x_{12}, y_{12}) = V_2(x_{12}, y_{12}) - S_{12})$$
$$V_2(x_{21}, y_{21}) = V_1(x_{21}, y_{21}) - S_{21})$$

$$A x_{12}^{\beta_{11}} y_{12}^{\beta_{12}} + \frac{x_{12}}{\delta_x} - \frac{y_{12}}{\delta_y} = B x_{12}^{\beta_{21}} y_{12}^{\beta_{22}} - S_{12} \tag{9}$$

$$B x_{21}^{\beta_{21}} y_{21}^{\beta_{22}} = A x_{21}^{\beta_{11}} y_{21}^{\beta_{12}} + \frac{x_{21}}{\delta_x} - \frac{y_{21}}{\delta_y} - S_{21} \tag{10}$$

Furthermore, smooth pasting conditions hold at the boundaries:

$$\beta_{11} A x_{12}^{\beta_{11}-1} y_{12}^{\beta_{12}} + \frac{1}{\delta_x} = \beta_{21} B x_{12}^{\beta_{21}-1} y_{12}^{\beta_{22}} \tag{11}$$

$$\beta_{12} A x_{12}^{\beta_{11}} y_{12}^{\beta_{12}-1} - \frac{1}{\delta_y} = \beta_{22} B x_{12}^{\beta_{21}} y_{12}^{\beta_{22}-1} \tag{12}$$

$$\beta_{21} B x_{21}^{\beta_{21}-1} y_{21} = \beta_{11} A x_{21}^{\beta_{11}-1} y_{21}^{\beta_{12}} + \frac{1}{\delta_x} \tag{13}$$

$$\beta_{22} B x_{21}^{\beta_{21}} y_{21}^{\beta_{22}-1} = \beta_{12} A x_{21}^{\beta_{11}} y_{21}^{\beta_{12}-1} - \frac{1}{\delta_y} \tag{14}$$

There are only 8 equations, (6) and (8-14), for 10 unknowns, β_{11}, β_{12}, β_{21}, β_{22}, A, B, x_{12}, y_{12}, x_{21}, y_{21}, so there is no completely analytical solution. Yet, for every value of x, there has to be a corresponding value of y when switching should occur, (x_{12}, y_{12}) and (x_{21}, y_{21}). So a quasi-analytical solution can be found by assuming values for x, which then solves the set of simultaneous equations for all remaining variables, given that $x = x_{12} = x_{21}$. This procedure is repeated for many values of x, providing the corresponding option values and the switching boundaries.

The spread between the two switching boundaries can be viewed in term of the wedges, defined below.

$$\frac{y_{12}}{\delta_y} \cdot \Omega_{y_{12}} - \frac{x_{12}}{\delta_x} \cdot \Omega_{x_{12}} = S_{12} \tag{15}$$

$$\frac{x_{21}}{\delta_x} \cdot \Omega_{x_{21}} - \frac{y_{21}}{\delta_y} \cdot \Omega_{y_{21}} = S_{21} \tag{16}$$

The Marshallian rule is satisfied when all wedges (Ω_{x12}, Ω_{x21}, Ω_{y12}, Ω_{y21}) are equal to one. The wedges for the real option model are:

$$\Omega_{x_{12}} = \Omega_{x_{21}} = 1 - \frac{\beta_{12} - \beta_{22}}{\beta_{12}\beta_{21} - \beta_{11}\beta_{22}} \tag{17}$$

$$\Omega_{y_{12}} = \Omega_{y_{21}} = 1 - \frac{\beta_{21} - \beta_{11}}{\beta_{12}\beta_{21} - \beta_{11}\beta_{22}} \tag{18}$$

Since β_{12} and β_{21} are positive and β_{11} and β_{22} are negative and the denominator of (17) and (18) needs to be positive to justify the option values, the wedges are less than one. This demonstrates that the switching hysteresis (band of inaction, no switching) is larger than suggested by the Marshallian rule.

2.2 Single Switch

The solution for the asset value with a one-way switching option from the above model with multiple switching is straight-forward. This one-way switch constitutes an abandonment option, where the switching cost is the abandonment cost. The asset value V_{1S} is given by (5) with the characteristic root equation (22), and V_{2S} is given by (7) with B=0, thereby eliminating the option to switch back. Applying the same solution procedure as before, a quasi-analytical solution is obtained.

$$A\, x_{12}^{\beta_{11}}\, y_{12}^{\beta_{12}} + \frac{x_{12}}{\delta_x} - \frac{y_{12}}{\delta_y} + S_{12} = 0 \qquad (19)$$

Furthermore, smooth pasting conditions hold at the boundaries:

$$\beta_{11} A x_{12}^{\beta_{11}-1}\, y_{12}^{\beta_{12}} + \frac{1}{\delta_x} = 0 \qquad (20)$$

$$\beta_{12}\, A\, x_{12}^{\beta_{11}}\, y_{12}^{\beta_{12}-1} - \frac{1}{\delta_y} = 0 \qquad (21)$$

where β_{11} and β_{12} satisfy the characteristic root equation

$$\tfrac{1}{2}\sigma_x^2\beta_{11}(\beta_{11}-1)+\tfrac{1}{2}\sigma_y^2\beta_{12}(\beta_{12}-1)+\rho\sigma_x\sigma_y\beta_{11}\beta_{12}+\beta_{11}(r-\delta_x)+\beta_{12}(r-\delta_y)-r=0, \qquad (22)$$

The characteristic root equation (22) together with value matching condition (19) and smooth pasting conditions (20) and (21) represents the system of 4 equations, while there are 5 unknowns, β_{11}, β_{12}, A, x_{12}, y_{12}.

2.4 Numerical Illustrations

Figure 1 shows the simultaneous solution of the ten equations, assuming $x_{12}=x_{21}$, and deriving the trigger for cost $y_{12}>x_{12}$ that would justify suspension, and the trigger for cost $y_{21}<x_{21}$ that would justify re-starting operations.

The two switching boundaries can be viewed in terms of the wedges from EQ 18 (.0495) and EQ 17 (.0543).

From EQ 15 (150.21/.04)*.0495-(100/.04)*.0543=50, the shut down cost.

From EQ 16 (100/.04)*.0543-(53.18/.04)*.0495=70, the restart up cost.

Figure 1

MULTIPLE INPUT-OUTPUT SWITCHING OPTION			
	INPUT		
PRICE	x	100	
COST	y	50	
Convenience yield of x	δ_x	0.04	
Convenience yield of y	δ_y	0.04	
Volatility of x	σ_x	0.40	
Volatility of y	σ_y	0.30	
Correlation x with y	ρ	0.50	
Risk-free interest rate	r	0.05	
Switching cost from x to y	S_{12}	50	
Switching cost from y to x	S_{21}	70	
Switching boundary OP to SHUT	**x_{12}**	**100**	
Switching boundary SHUT to OP	**x_{21}**	**100**	
	SOLUTION		OPTION
Asset value in operating mode	$V_1(x,y)$	1718.27	468.27
Asset value in shut down mode	$V_2(x,y)$	1649.16	1649.16
	A	10.17	
	B	9.76	
Switching boundary OP to SHUT	**$y_{12}(x)$**	**150.21**	
Switching boundary SHUT to OP	**$y_{21}(x)$**	**53.18**	
Solution quadrant	β_{11}	-0.40116	
Solution quadrant	β_{12}	1.45116	
Solution quadrant	β_{21}	1.42289	
Solution quadrant	β_{22}	-0.36360	

The switching input cost boundaries y_{12} and y_{21} are derived in the solution of EQS 6, 8-14, assuming $x_{12}=x_{21}=100$. The asset values are derived by inserting the solutions for A, B, β_{11},β_{12}, β_{21} and β_{22} into EQS 5 and 7, assuming current x=100, y=50.

If the output price is 100, input price 50, the operating PV is 1250. The option to shut down is worth 468.27, but the facility should not be shut down unless the input price exceeds 150. If shut down, and the output price is 100, operations should not be restarted until the input price is less than 53, so if y=50 this mode value is academic. Of course, all of these triggers and values change as the eight parameter values change, and as x_{12} and x_{21} change.

Figure 2 shows the sensitivity of the spreads to changes in the output price volatility. Note that slope of the effective partial derivative of the spread between the (higher) input price which justifies shut down and the (lower) input price which justifies re-start up increases the higher the standard (x=100) output volatility. The hysteresis (band of inaction, not shutting down if operating, and not restarting if shut down) increases with increases in the volatility of either input or output prices.

Figure 2

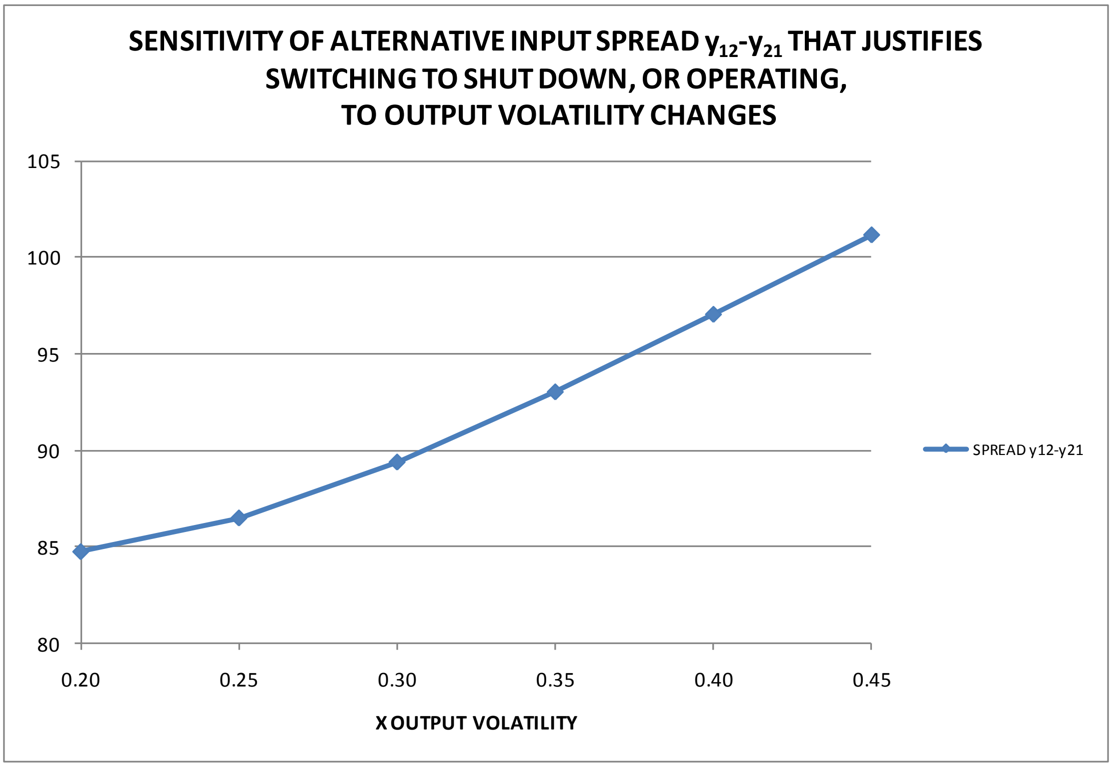


Figure 3

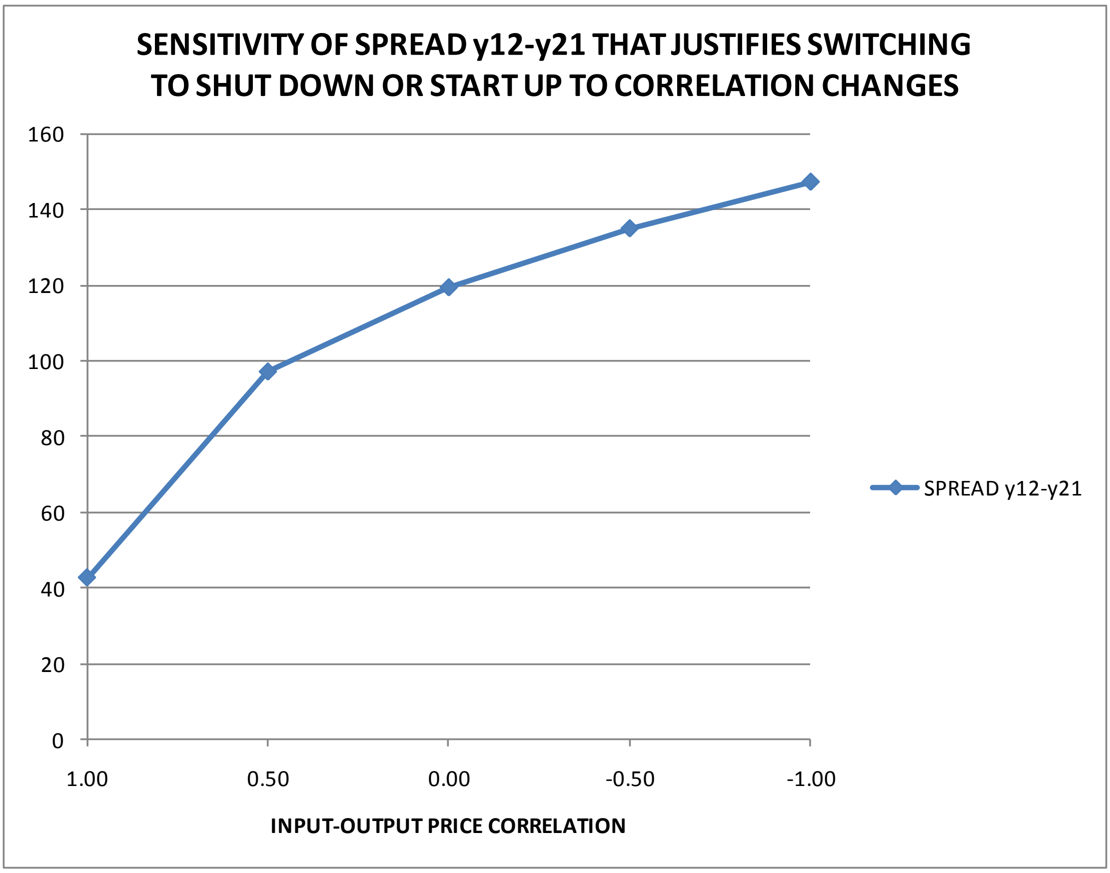
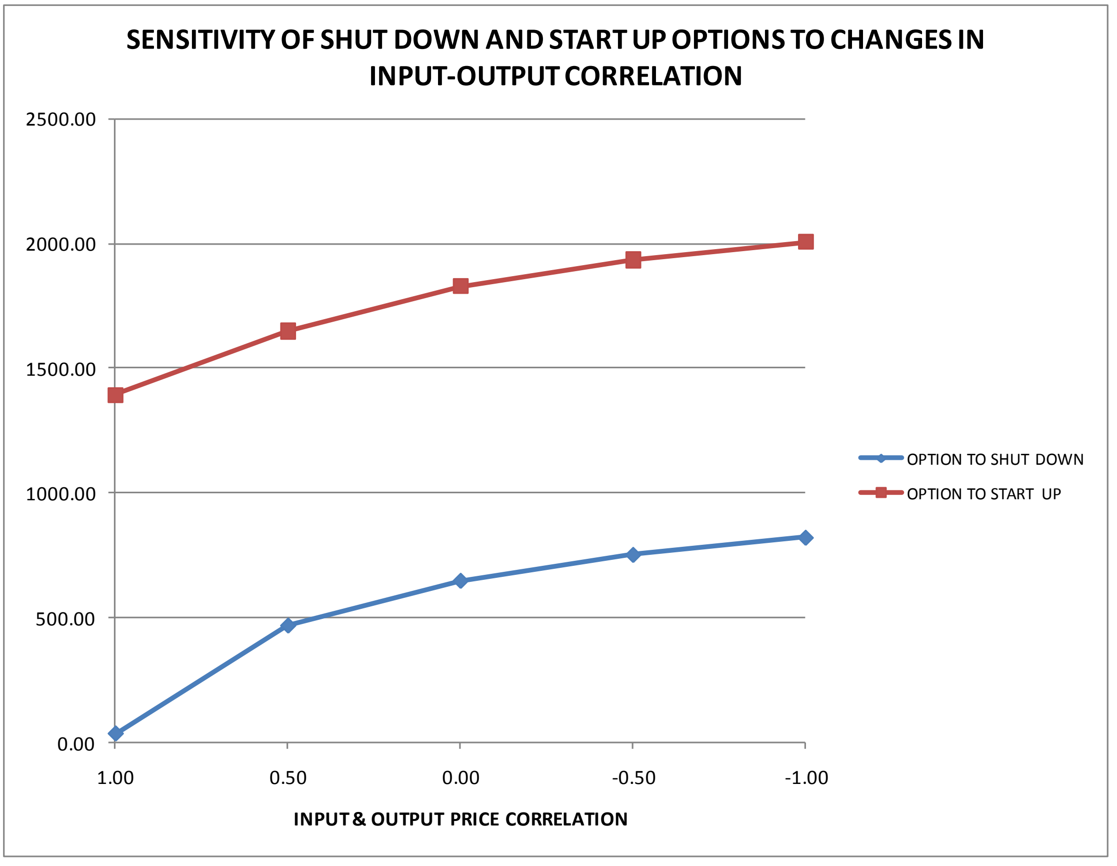


Figure 3 shows the sensitivity of the spread between input prices justifying shut down and restart up to changes in the correlation between input and output prices. Even if input and output prices are perfectly correlated, a shut down is not justified until the input price exceeds 112 (indicating a negative present value of operating when output is 100), and a re-start up is not justified until the input price is less than 70 (indicating a strong positive present value of operating when output is 100) due to the irrecoverable switching costs.

Figure 4 shows that the value of the options to shut down and restart is very sensitive to the correlation between input and output prices.

Figure 4



Even for perfectly correlated input and output prices, there is a small value in shut down and start up options, as both prices move while the assumed switching costs remain constant. For negatively correlated input and output prices, of course, the option to shut down is very valuable. It is interesting that the slopes of the sensitivity curves are sharply rising from complete to partial correlation, but tend to flatten as the degree of negative correlation increases. Note that under high correlation, the option value to start up may be academic.

Figure 5

SINGLE IN-OUT SWITCH OPTION

	INPUT			
OUTPUT PRICE	x	100		
INPUT COST	y	50		
Convenience yield of x	δ_x	0.04		
Convenience yield of y	δ_y	0.04		
Volatility of x	σ_x	0.40		
Volatility of y	σ_y	0.30		
Correlation x with y	ρ	0.50		
Risk-free interest rate	r	0.05		
Switching cost from x to y	S_{12}	50		
Switching boundary OP to SHUT	x_{12}	100		
	SOLUTION		OPTION	OPERATING
Asset value in operating mode	$V_1(x,y)$	1630.84	380.84	1250.00
	A	9.89		
Switching boundary OP to SHUT	$y_{12}(x)$	**337.04**		
Solution quadrant	β_{11}	-0.42545	*must be negative*	
Solution quadrant	β_{12}	1.43396	*must be positive*	

The switching input cost boundary y_{12} is derived in the solution of EQS 19-22 assuming x_{12}=100. The asset value is derived by inserting the solutions for A, β_{11} and β_{12} into EQ 5.

Figure 5 shows that the single switching boundary is 224% greater than for multiple switching, with similar parameter values. The asset value V_{1S} is 5% lower compared to multiple switching, because the option to shut without any re-starting is almost 20% lower than the multiple switching option.

Figure 6

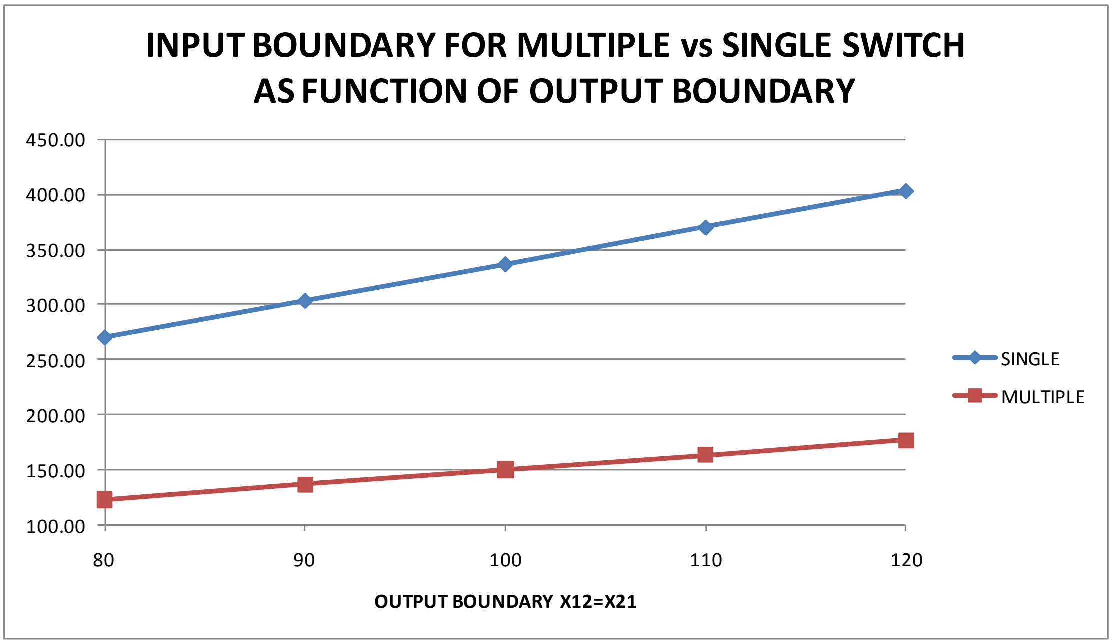


Figure 6 shows that the output boundary for a single switch increases at a slightly higher rate over the input boundary compared to a multiple switch[2].

3. ILLUSTRATED REVERSE "CRACK" SPREAD FLEXIBILITY

3.1 Heavy Crude Oil Field in South America[3]

Consider a heavy crude oil field ("HCOF"), with a high viscosity, where hot steam has to be injected into the reservoir to reduce the viscosity, allowing the oil to flow through the permeable sands, and providing some drive of the reservoir, as well as easier flow through a pipeline. The steam is produced in boilers fuelled by natural gas. Suppose there is an arrangement that the HCOF owner is required to sell any crude oil produced to the government at the equivalent WTI crude oil futures price (nearby contract each end month) less a $4 discount for heavy oil, and pay the government 1.2 times the BTU equivalent of the NYMEX nearby natural gas futures price (NG) each end month for the gas used in the production.

Petroleum engineers believe that the HCOF would have a production profile of 100,000 bbls. per annum, with a very slow decline. It is accepted that production can be shut down at any time for an expenditure of around $20 per bbl. capacity or re-started at any time at approximately the same expense. Around 6 MCF (thousand cubic feet) of gas are required to produce one barrel of oil, which happens to be the BTU equivalent.

3.2 Petroleum Prices

As of January 2007, the current nearby quotation for WTI crude oil on NYMEX is around $54.51. The historical volatility over the last twenty-one years of end month nearby WTI futures is 29% per annum. Natural gas futures on NYMEX are around $5.70 per MCF, with a historical volatility of 37% (also reflecting seasonality). The correlation between gas and oil futures is around 16%, also influenced by seasonality.

---

[2] Note the input boundaries are nearly a linear function of the output boundaries in this example, indicating an easy approximate rule for calculating some switching boundaries.

[3] While this illustration is based on an actual field in South America, some details have been altered or simplified.

The equivalent convenience yield of WTI (riskless interest rate less the 21 year average drift) is .0088 and .0064 for NG.

Figure 7

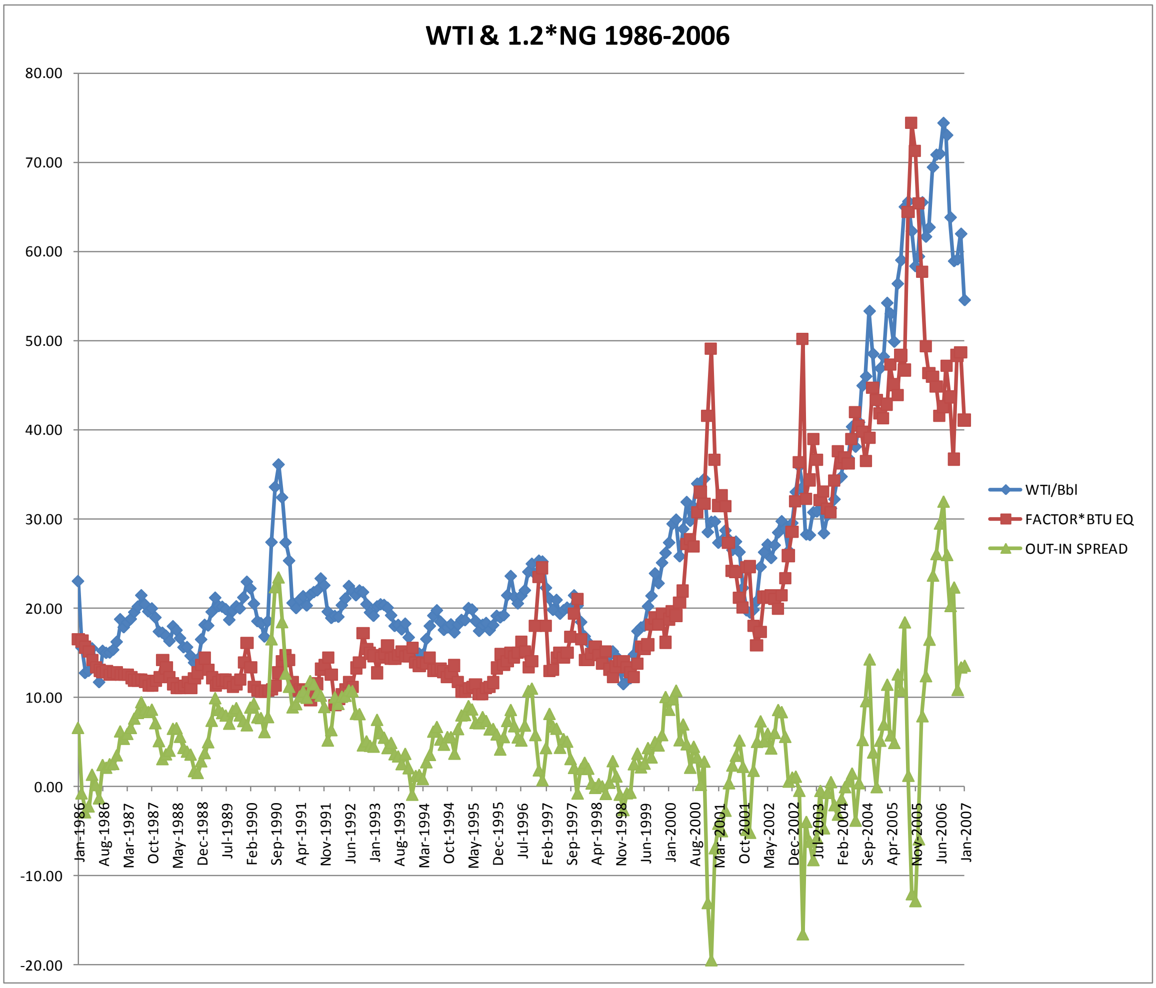


Figure 7 shows that usually (and in January 2007) WTI is higher than 1.2 times the NG BTU equivalent, but there have been times such as in the northern winter of 2000, 2002 and 2004 when the output price less the input price is negative. Since seasonal demand for crude oil products in South America tends to be the opposite of that in North America, possibly it would be appropriate if production is stopped during such northern winter periods.

3.3 Switching Boundaries and Asset Values

Using the specified parameter values in the multiple input and output switching model, Figure 8 shows the derived switching triggers and switching option values.

Figure 8

		OPTIMAL SWITCHING BOUNDARIES			
Switching boundary OP to SHUT	x_{12}	30	40	50	60
Switching boundary OP to SHUT	y_{12} (x)	51.74	65.80	**79.50**	92.93
Switching boundary SHUT to OP	y_{21} (x)	15.99	22.78	**29.80**	36.99
Switching boundary SHUT to OP	x_{21}	30	40	50	60

SWITCHING MODEL VALUES	OPERATING PRESENT VALUE			-577.69	
Asset value in operating mode	V1(x,y)	4929.69	4920.49	4915.66	4912.88
Asset value in shut down mode	V2(x,y)	4920.30	4908.39	4902.68	4899.70

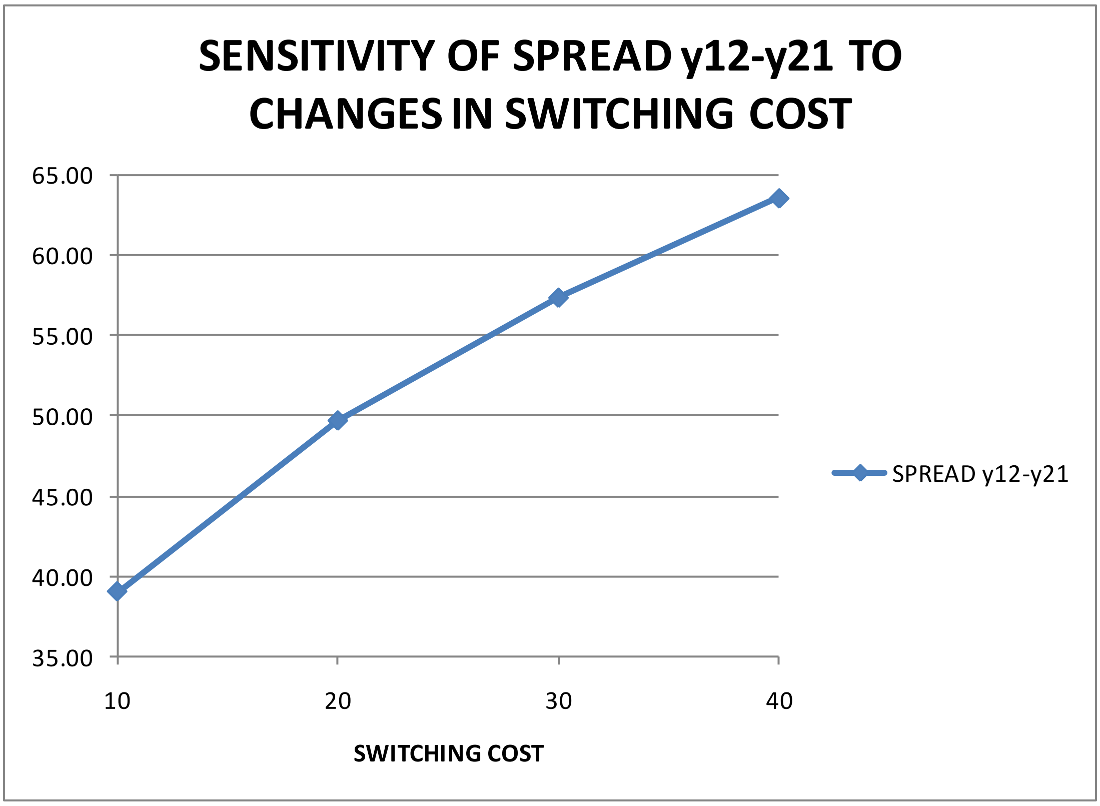
WEDGE y12=y21	EQ 18	0.0071	0.0059	0.0051	0.0046
WEDGE x12=x21	EQ 17	0.0111	0.0090	0.0077	0.0068
y12	EQ15	51.7363	65.8037	79.5018	92.9332
y21	EQ16	15.9921	22.7816	29.8008	36.9928

The switching input cost boundaries y_{12} and y_{21} are derived in the solution of EQS 6, 8-14, assuming $x_{12}=x_{21}=50$. The asset values are derived by inserting the solutions for A, B, β_{11},β_{12}, β_{21} and β_{22} into EQS 5 and 7. The wedges are derived from inserting the solutions for β_{11},β_{12}, β_{21} and β_{22} into EQS 18 and 17, and the boundaries y_{12} and $y_{21}$are shown to be consistent with EQS 15 and 16.

At the current net price for WTI of around $50, and 1.2 times NG of around $40, the calculated value of perpetual production is -$578 per bbl. capacity, because the equivalent convenience yield for oil exceeds that for NG. Yet a shut down of production would not be justified until NG price increases to above $79.50. Once shut down, a restart of production would be justified if the NG price fell to below $29.80. The values of the option to shut down and restart are very significant. In a present value world of certainty, with convenience yields equal to the riskless interest rate, the value of perpetual production ($50-$40)/.05=$200 per annual barrel of production (or times 100,000 bbls. per annum) equals $20 million. The HCOF asset value considering multiple switching options is worth $4916 per annual barrel (or times 100,000) equals $491.6 million, or almost 25 times the static present value estimate.

Figure 9 shows the sensitivity of the spread between input prices that justifies shutting down and restarting up to changes in the switching cost (assumed to be equal for both modes). An increase in switching cost results in a significant increase in the spread, indicating hysteresis, which levels off with higher levels of switching costs. Interestingly, there is not a substantial decrease in the option value of switching as switching costs increase, indicating that lowering switching costs at the expense of higher investment costs may not be worthwhile.

Figure 9

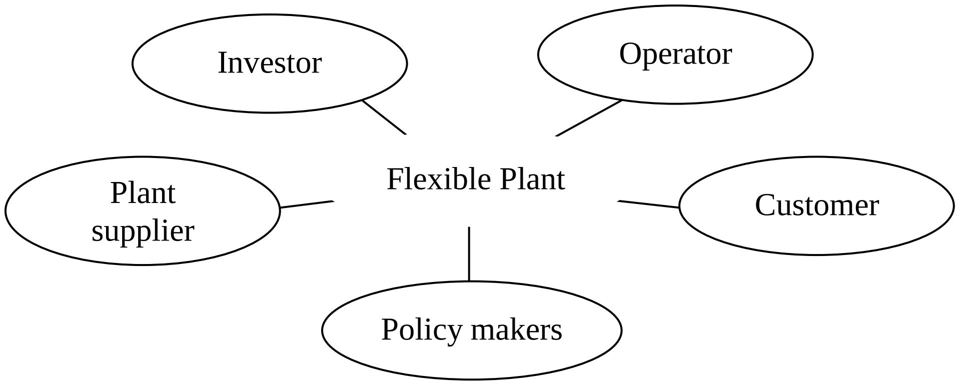


4 Policy and Strategy Implications

There are a number of stakeholders shown in Figure 10 whose best decisions should be based on these INPUT-OUTPUT switching models.

We have provided answers to the question of when an operator of a flexible facility should switch back and forth between possible outputs and inputs in order to maximise value when switching costs are taken into account. Eight factors should be monitored in making these decisions, in addition to the current input and output prices. An investor would not pay more than the combined current production and option value for such a flexible operating asset. There are several strategy implications for the operator, investor and possibly for policy makers.

Figure 10



Investors

As shown in Figures 1, 5 and 8, the real option value of these flexible facilities is substantially greater than the present value of current production (= inflexible facilities), at the current assumed input and output price levels, and parameter values. Note the focus of alert investors is on choosing the appropriate model and on forecasting input and output price volatilities and correlations. A myopic investment analyst using net present values will probably undervalue flexible plants. External analysts may not have access to plant operating or switching costs, or indeed knowledge of any flexibility inherent in existing facilities, due conceivably to inadequate accounting disclosures, not currently required by accounting standard setting committees. Of course, realistic analysts may doubt that the chief option managers of flexible facilities will be aware of the potential optionality, or indeed make switches at appropriate times, so the Marshallian values might reflect a realistic allowance for management shortfalls.

Chief Real Options Manager

The alert chief options manager ("COM") is aware of input and output switching opportunities, the amount of switching costs, and periodically observes input and output prices, convenience yields (or proxies), updates expected volatilities and correlations, and so updates Figure 8 appropriately. Observed current spreads between input/output prices are compared to the updated triggers for switching. Naturally part of the appropriate compensation for the COM should be based on awareness of these opportunities, and performance in making actual input and output switches at appropriate times.

Originally, the COM would have calculated the value of a flexible plant V_1 or V_2, compared to an inflexible facility, which also indicates the warranted extra investment cost for facility flexibility. It would not be difficult to consider trade-offs for any deterministic lower efficiency due to the flexibility capacity.

Plant Suppliers

Originally, petroleum engineers and suppliers of facilities to the COM would have calculated the value of a flexible plant V_1 or V_2, with the capacity of shutting down and restarting at reasonable switching costs compared to an inflexible facility, which also indicates the warranted extra investment price that could be charged for facility flexibility. With the illustrated parameter values in Figures 1 and 5, a hypothetical multiple switch facility in the operating mode is worth only some 5% more than a single switch plant, but much more than an inflexible facility. In designing flexible facilities, it would not be difficult to consider trade-offs for any lower efficiency due to the flexibility capacity, or reduction of switching costs, against increased investment costs.

Customers

Output customers such as the government in the HCOF example may be aware of the intensions of such a producer to shut down and restart possibly with the opposite of the seasonality observed in North American markets for natural gas. Oil customers in North America would not appreciate stopping production during periods of heavy demand (but gas suppliers might offer a discount for reducing quantities consumed during winter months). So both counterparties might seek long-term agreements mitigating the shifts in output and input prices implied in using real option approaches for operating flexible facilities.

Policy Makers

Taxpayers beware. There will be national producers without flexible facilities, or not aware of needing to change output prices, and input sources, as the economic environment changes. Those producers priced out of the market will seek government barriers for other producers, or input/output subsidies as conditions change.

5 Conclusion and Further Applications and Limitations

Flexibility between outputs and inputs is particularly relevant in volatile commodity markets, or where free trade allows new entrants, cheaper inputs, or more valuable outputs. There are many applications for substitute outputs, substitute inputs, or alternative inputs and outputs. Dockendorf and Paxson (2011) examine further processed chemical products as essentially output alternatives. They note alternative uses of other types of facilities, such as multiuse sports or entertainment or educational facilities, transportation vehicles for passengers or cargo, rotating agricultural crops, and solar energy used for electricity or water desalination. Adkins and Paxson (2011) note there are numerous energy switching opportunities, such as palm or rape oil in biodiesel production, gas-oil-hydro-coal in electricity generation, that are reciprocal energy input switching options.

There are several examples of combinations of stochastic output and input prices, such as the “crack” spread for gasoline-heating oil as outputs for crude oil refineries, the “crush” spread for soya meal and soya oil as outputs for soya bean refineries, and “biofuel” spread for ethanol as the output for corn processing facilities. It may be appropriate to extend these stochastic input and output models to many manufacturing, distribution and transportation activities. Traditional models have considered labor and capital as inputs for manufacturing outputs. Distribution involves buying goods as inputs to a possible chain of outputs (outlets) with possibly a variable mark-up. Transportation offers the output of stochastic tariffs or tickets with the stochastic input of labor, fuel and capital.

But extensions of our general two stochastic factor models to some of these activities will be limited by the simplifying assumptions required, such as constant interest rates, convenience yields, volatilities, switching costs and other operating costs. A critical assumption is the constant correlation (and ignoring gas price seasonality) between oil and gas prices, which is not consistent with the time varying positive and negative spreads observed over the last twenty years. The impact on switching boundaries and on asset values of partial or imperfect hedging of the stochastic input and/or output prices has not been considered. Also, the effects of the entry, exit and strategies of other competing operators on input and output prices, and possible stochastic quantity of production, have been ignored. The alternative modes, operating and suspension, are rather limited. Extensions to capacity expansion or contraction or abandonment or, where appropriate, alternative uses, could be interesting.

References

Adkins, R. and D. Paxson. 2011. Reciprocal Energy-switching Options. *Journal of Energy Markets* 4(1), 1-25.

Brekke, K.A. and B. Schieldrop. 2000. Investment in Flexible Technologies under Uncertainty. In *Project Flexibility, Agency and Competition*, eds. M. Brennan and L. Trigeorgis. Oxford: Oxford University Press, 34-49.

Brennan, M. J. and E. S. Schwartz. 1985. Evaluating Natural Resource Investments. *Journal of Business* 58, 135-157.

Dockendorf, J. and D. Paxson. 2011. Continuous Rainbow Options on Commodity Outputs: What is the Value of Switching Facilities? *European Journal of Finance*, forthcoming.

He, H. and R.S. Pindyck. 1992. Investments in Flexible Production Capacity. *Journal of Economic Dynamics and Control* 16, 575-599.

Margrabe, W. 1978. The Value of an Option to Exchange One Asset for Another. *Journal of Finance* 33, 177-186.

McDonald, R. and D. Siegel. 1985. Investment and the Valuation of Firms When There is an Option to Shut Down. *International Economic Review* 26, 331-349.

______________________. 1986. The Value of Waiting to Invest. *The Quarterly Journal of Economics* 101, 707-728.

Paxson, D. 2005. Multiple State Property Options. *Journal of Real Estate Finance and Economics* 30, 341-368.

Paxson, D. and H. Pinto. 2005. Rivalry under Price and Quantity Uncertainty. *Review of Financial Economics* 14, 209-224.

Trigeorgis, L. and S. P. Mason. 1987. Valuing Managerial Flexibility. *Midland Corporate Finance Journal* 5, 14-21.