Reversibility and switching options values in the geological disposal of the radioactive waste*

Oana Ionescu† Sandrine Spaeter‡

May 17, 2011

Abstract

This article offers some economic insights for the debate on the reversible geological disposal of radioactive waste. Irreversibility due to large sunk costs, an important degree of flexibility and several sources of uncertainty are taken into account in the decision process relative to the radioactive waste repository. We draw up a stochastic model in a continuous time framework to study the decision problem of a decision-maker that must carry out a reversible repository project for the radioactive waste, with multiple disposal stages. We consider that the value of reversibility of a radioactive waste package is jointly affected by economic and technological uncertainty. They are modelized, first, by a 2-Dimensional Geometric Brownian Motion, and, second, by a Geometric Brownian Motion with a Poisson jump process. A numerical analysis and a sensitivity study of various parameters are also proposed.

Keywords: radioactive waste; reversibility; switching; real option theory.

JEL Classification: D81, Q40, Q50.

---

*ANDRA is aknowledged for financial support and for providing data. We thank Jean-Alain Héraud, Luis Aparicio, Marcel Boyer, Patrick Cohendet, Jean-Noël Dumont, Pierre Lasserre, Louis Londe, Nathalie Popiolek, Rodolphe Raffard and participants in the 50th Annual Conference of SCSE (Canadian Society of Economics) for helpful comments.

†PhD Candidate ANDRA and BETA, CNRS and Nancy-University. Email: oana.ionescu@univ-nancy2.fr

‡BETA, CNRS and University of Strasbourg. Email: spaeter@unistra.fr

1 Introduction

The motivation for this research relates to current concerns about the radioactive waste disposal in many countries that produce nuclear energy. Governments have progressively set up legal frameworks for the radioactive waste management. Recently, the research on waste disposal has led to the conclusion that for the highly radioactive waste (HLW), the disposal in deep geological layers (at depths between 250 m and 1000 m) is the best option for the safety of current and forthcoming populations and the protection of the environment. Consequently, this option is under investigation in several countries including France, Sweden, Finland, USA, etc.[1] Given different political, social and mainly geological conditions for disposal, projects of deep geological facilities of highly radioactive waste diverge from one country to another. As yet, no deep geological disposal facilities are in operation.[2] Thus, the waste material keeps continuing to be accumulated in storage facilities and kept under close and active monitoring.

Contrary to surface storage, deep geological repositories are passively safe and designed to provide the isolation from the human environment without future maintenance. However, an important advantage of surface storage is the ease of retrieving the radioactive material if necessary. This allows future generations to take different decisions concerning the existing radioactive waste. Though, recent research shows that a geological repository could technically be designed so that closure of the facility can be delayed for a period of several hundred years. In this period, the repository and the surrounding environment can be monitored if necessary, and the facility can be designed to allow for retrieval of the emplaced material if required.

To understand the main points at stake for reversibility, let us evoque the French case. It is currently considered as one of the most evoluated in the debate on reversible disposal[3]. In France, the Planning Act n° 2006-739 of 28 June 2006, institutes deep geological disposal

[1] These aspects were recently discussed at the International Conference on Reversibility and Retrievability, held in Reims, France, in December 2010. Experts from 16 NEA (Nuclear Energy Agency) member countries have participated in this conference.

[2] However, we can mention some important experimental laboratories like Forsmark in Sweden or Bure in eastern France. In France, the Government has authorized ANDRA (National Agency for the Radioactive Waste Management) to carry out geological investigations over an area of about 30 km^2 in order to site the geological disposal facility CIGEO (Industrial Center for Geological Disposal), which must accomodate HLW. The operational phase could start by 2025, if its licence is granted in 2015.

[3] A more detailed description of the French project can be found in Aparicio (2010).

as a norm. The Act also prescribes the fact that the repository must be reversible for a minimal period of one hundred years. The reversibility implies that at each step of decision different options are available: being able to retrieve the radioactive waste if the arrival of new information justifies it, to reevaluate the disposal process, to modify the system parameters or to continue on the same path. Hence the issue that the decision maker is facing in the management of the radioactive waste concerns different types of possible decisions at different dates. It makes appear different possible stages of retrievability of the waste packages, as illustrated on Figure 1.

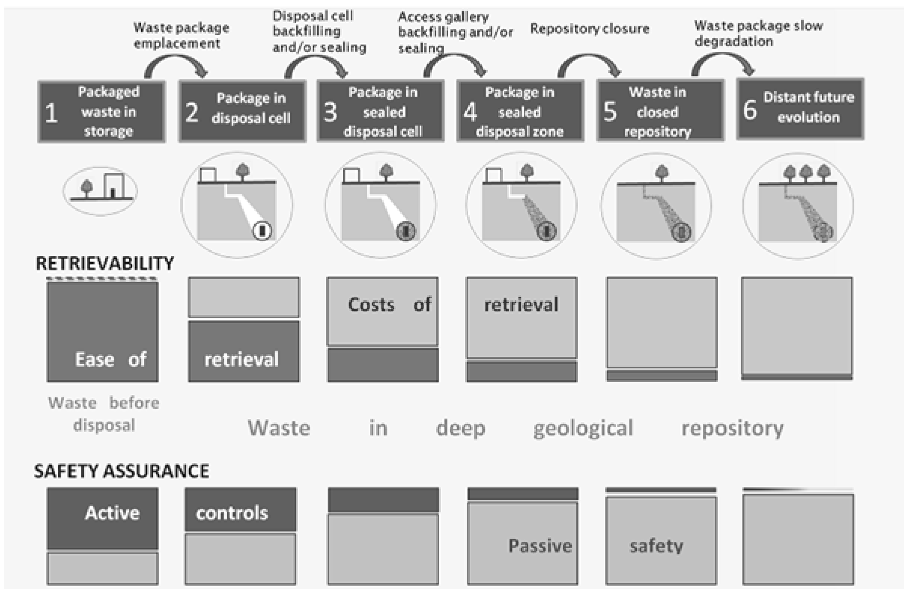

Figure 1: Lifecycle stages of the waste in a deep geological repository Source: ANDRA 2011

Reversibility is ensured by the existence of multiple disposal stages, with changing degrees of retrievability, passive safety and active controls for the waste packages in the deep geological repository. This ability to act on the disposal process itself provides flexibility by giving the decision-maker, but also the forthcoming generations, the possibility to change the repository design concept according to advances in research, to experience feedback and to technical progress. For instance, if a new technology, much safer or more space saving emerges or if a new use of waste is discovered in the future, the disposal process needs to be reevaluated and it may worth to retrieve the waste packages.

Therefore, the considerable amount of uncertainty, the arrival of new information in the future and the exceptional dimension of temporality are important aspects that must be taken into account in the decision process when defining the concept of reversibility. Among the most significant uncertainties related to the subject we can cite: the value of the radioactive waste, which may be affected by economic and technological factors, the long-term cost of deep disposal facilities during the operational phase, the additional costs implied by an eventual extraction of radioactive waste containers, some possible delays concerning the deep geological disposal in reason of a non-authorization to use a certain location (the starting date of deep storage could be postponed and consequently new costs of encapsulation and maintenance are to be supported).

Hence, there is a need for a better understanding of how uncertainty affects the decision process of a regulator through time. For instance, knowing the existence of possible, but uncertain changes in the retrieval value of the radioactive waste, should the decision-maker in charge of the radioactive waste management keep continuing to dispose of the packages on the current stage, or should she switch to a stage with a higher/lower degree of retrievability? What is the value associated with each of these possible alternatives? What are the optimal triggers values for which the option to switch is exercised? What are the main parameters that influence the value of these switching options?

In order to answer these questions, we propose a real options model in a continuous time framework. We consider two sources of uncertainty: the uncertain market value of radioactive materials contained in the waste packages and the uncertain evolution of technological progress in the nuclear waste management techniques. The value of the radioactive waste follows, first, a 2-Dimensional Geometric Brownian Motion process, and, second, a compound Geometric Brownian Motion and Poisson process. Given these underlying stochastic processes, our objective is to provide an explicit characterization of the value of the different options to switch from one disposal stage to another at each possible date. We also show how each option depends on future options: we deal with compound switching options and we use the real option theory as developed earlier by Myers (1976), Brennan and Schwartz (1985) and McDonald and Siegel (1986).

Since their work, there has been an increasing literature concerning applications of the real options approach to investments involving uncertainty and flexibility in the decision process. In particular, the valuation of benefits resulting from investments in flexible modes of production or technologies, is addressed by Kulatilaka (1993), Dixit and Pindyck (1994), Childs, Ott and Triantis (1998), Cortazar, Schwartz, and Casassus (2001)[4]. In most of these papers the value of flexibility is derived from the possibility for a firm to move in the future to an alternative mode of production, as a response to initially unforeseen changes in economic and technological aspects The decision to switch to a different alternative involves switching costs and new expected payoffs. Typically the value of the option to switch is defined as the difference between the expected benefits from investing in a flexible rather than a *rigid* technology.

Other studies that include technological uncertainty in the real option framework are those of Grenadier and Weiss (1997) and Farzin et al. (1998), which focus on the uncertainty in technological progress. They analyze the optimal time for a technology adoption with a stochastic innovation process for the profit and the arrival time. Grenadier and Weiss (1997) considers the technological adoption strategy of a firm given a sequence of stochastic technological innovations in the future. Upon arrival of a new technology, the firm may decide to adopt or not this new technology. This decision depends on its previous decision on current technology. The technological progress is modeled as a Geometric Brownian Motion process. Farzin et al. (1998) consider a continuous-time model where the uncertainty concerns the timing and importance of technological improvements. By including the technical change modeled with a Poisson process, they extend the model of Dixit and Pindyck (1994). They obtain that a higher degree of uncertainty leads to a delay in the technology adoption. Murto (2007) considers a revenue uncertainty, modeled as a Geometric Brownian Motion, and technology uncertainty with innovations arriving at a Poisson process rate. With innovations, the costs of the underlying investment decrease. The option to invest is exercised when the investment cost is sufficiently low and the output is sufficiently high. With the combined uncertainty, the overall effect is a delay in the investment.

Our paper contributes to the existing work by formulating a model with sequentially embedded options and by applying it to the case of radioactive waste management. The geological disposal of radioactive waste with a real option approach is also investigated in Gollier and Devezeaux de Lavergne (2001) and in Loubergé et al. (2001). The first paper highlights the idea that the value of reversibility is a real option that can be exercised by a future generation. Given a stochastic evolution of the value of raw materials contained in radioactive waste, the

[4] A more detailed survey of each of these contributions can be found in Heraud and Ionescu (2010).

authors analyze the costs and the benefits of the reversibility. They show that it is socially optimal to implement a reversible storage only if when the radioactive row material's value reaches a given threshold.

Loubergé et al.(2001) investigate the optimal timing to switch from surface storage to deep geological disposal of radioactive waste. They use a real options approach based on the minimization of different costs of the project. While surface storage entails high stable costs, deep disposal involves initial investment and random future expenses due to unanticipated future actions. However, they do not take into account the possibility to retrieve the waste once disposed of in deep repository.

Besides, it is only possible in the quoted papers to switch from the interim storage to the deep geological disposal. However, as seen in Figure 1, the reversibility as defined by the Nuclear Energy Agency implies that more than two stages should exist. Hence in our paper, we consider a more sophisticated model and we focus on the impact of future technological improvements on the option value of switching among several possible disposal stages. In particular, we analyze the interaction between the switching options at each date and we show how they are influenced by different stochastic processes.

Our results show that, generally, the uncertainty leads to a delay in the decision to retrieve the waste packages. It is shown that this delay is more pronounced in the GBM with positive jumps case, when the increased probability of important innovations that may arrive in the future, increases the value of the option to wait in order to be sure of the optimality of the retrieval. The decision-maker may prefer to retain flexibility in the deep geological repository in order to assimilate information about the evolution of the value of the radioactive waste packages. One of the main insights of our model is that the reversibility implied by the deep geological repository with multiple stages may act like a hedge against the randomness of the fluctuations of future values of waste packages. For instance, the decision-maker may switch earlier to a less reversible stage, bearing in mind that he has the option to reverse the decision for higher values in the future and also the subsequent option to switch to a stage with an even lower degree of retrievability, if the value of radioactive waste packages is sufficiently low.

We organize the paper into two sections in addition to this introduction. The second section presents, first, the model and, second, the determination of the optimal switching value for each stage of retrievability for radioactive waste disposal. In the third section we report the numerical results and conduct the sensitivity analysis.

2 The model

In this section we present our modeling framework for the decision problem in the reversible radioactive waste repository. As explained above, the reversibility of the repository of the radioactive waste with multiple disposal stages gives the decision-maker the option to change or postpone decisions at each, given, decision node. We therefore analyze the decisions to switch ultimate waste packages from a stage with a higher/lower degree of retrievability to one with a lower /higher degree of retrievability as a dichotomy of choices. We touch here one of the most important features of our model: the interdependences between multiple sequential switching options.

2.1 Assumptions

Consider a decision-maker in charge with the radioactive waste management. She must build a reversible repository involving multiple disposal stages with different degrees of retrievability for the waste packages. Such repository implies that the decision-maker can either adopt the initial disposal stage during the entire lifetime of the project or switch to another stage as soon as future changes in factors influencing the process makes it valuable, while keeping the option to switch back. This option to switch does not come for free, implying different switching costs. The decision-maker must then analyze the conditions under which the switching is valuable, given current features and uncertain future evolutions.

Let us assume that the repository displays three disposal stages, differentiated by the degree of retrievability. We denote with $S = \{s-1, s, s+1\}$ the discrete set of possible stages. Stage $s-1$ is the most retrievable (for instance, the surface storage), which means that the radioactive waste package is really easy to retrieve (in technical and monetary terms). Stage $s+1$ is the less retrievable (closure phase of the repository), while stage s is the intermediary stage (access gallery may be closed). For instance, going to a less retrievable stage means to make the individual waste packages more compact, more isolated by concrete barriers, etc. Returning to the preceding stage involves de-compacting and successively reopening the barriers, etc.

The horizon time for the decision process is finite and the final date is T.[5]

The operating costs associated with the radioactive waste packages on each stage and the switching costs among stages are known with certainty. Because of technical reasons, it is only possible to switch from one stage to one immediate neighbor stage. Moreover, operations related to the withdrawal of packages, like opening the gallery or a seal, are more expensive than operations that concern emplacement. Indeed, they are technically more complex and they often involve components that have already changed over the life cycle. This implies that the state of the packages and their structure may be different at the time of the retrieval, implying the setting up of new protection measures or additional equipment. Lastly, the more retrievable the current stage, the lower the cost of switching to an even more retrievable stage. These fair assumptions on the switching costs are formalized as followed:

Assumption 1 *The costs of switching from stage i to stage j are denoted as $c^{i,j}$ with $i = s-1, s, s+1$, $j = s-1, s, s+1$, and $|i-j| = 1$. For instance, they satisfy:*

$$c^{s,s-1} > c^{s,s+1} \tag{1}$$

In addition, each stage of retrievability implies different operating costs of radioactive waste packages. These costs are constant in time for a given stage of retrievability. It is fair o assume that the more reversible the stage, the higher the operating costs.

Assumption 2 *We denote as c^i, $i = s-1, s-s+1$ the operating cost induced by the maintenance of a waste package at stage i. This cost is due at each period and it satisfies:*

$$c^{i+1} < c^i \quad i = s-1, s. \tag{2}$$

Actually, the passivity of the repository which increases with stages represents the fundamental difference between deep disposal and storage. The observation and monitoring system is less intense as the development of the repository proceeds and along the various sealing stages. Thus, the maintenance costs decrease until the closure phase, characterized by total passivity.

Furthermore, we consider that the cash-flow (output) offered by a radioactive waste package differs according to the ease of retrieval. For example, a package disposed on the last stage

[5] T is sufficiently large to be able to consider several generations.

of retrievability (closure phase of the repository) provides a minimum value in terms of cash-flows, for it cannot be used rapidly. Therefore for each disposal stage, different output values are to be considered. We formalize this by introducing a so-called "index of retrievability", a proportionality factor, which captures the intrinsic properties of each disposal stage, which by their nature slow down or accelerate the use of the waste package. The ordering condition of the retrievability indices is:

$$\delta^{s-1} > \delta^{s} > \delta^{s+1} \quad \forall s \in S \tag{3}$$

In the model the strategic variable will be the value of a waste package at a given date t. We denote it as w_t and, as it will be defined in the next subsection, this value is affected through time by economic and technological uncertainties.

The decision-maker decides to either switch from the current retrievability stage to another, neighbor one, or to keep staying on the same path at date t with respect to the observed value of the package w_t: the higher (lower) this value, which encompasses the future expected values, the more the chance to switch to more (less) retrievable stage. Because switching and operating costs are different at each stage, the threshold of the value w_t that induces a switch to a more retrievable stage will be different from the one that induces a switch to a less retrievable one. Formally, we denote them $w_t^{s,s+1}$ and $w_t^{s,s-1}$ at stage s. At stage $s-1$ (respectively $s+1$) we only have a lower (upper) threshold $w_t^{s-1,s}$ (respectively $w_t^{s+1,s}$). For any value higher (lower) than the upper (lower) threshold, a switch to a more (less) retrievable stage occurs. At stage s (respectively $s-1$; $s+1$), no switch occurs at date t if $w_t^{s,s+1} < w_t < w_t^{s,s-1}$ (respectively if $w_t^{s-1,s} < w_t$; $w_t < w_t^{s+1,s}$).

In the following subsection, we analyze the decision-maker's problem for each of the three disposal stages and at each date using dynamic programming.

2.2 Valuing the switching options among disposal stages with a Geometric Brownian Motion

The future value of the radioactive waste packages is uncertain and we denote w_t the net present value of a package at date t. It depends on the expected, and discounted, values at future periods. New information can arrive over the lifetime of the project at each date t and consequently, the net present value w_t of a package is affected by two types of uncertainty:

the market uncertainty which represents variation in economic parameters related to the radioactive industrial sector (prices of materials contained in the ultimate radioactive waste or in the container itself that may be recovered), and technological uncertainty (the arrival of new technologies or the occurrence of unexpected drawbacks). Thus, the value of the radioactive waste package at each date is defined by the stochastic process $\{w_t\}_{t\geq 0}$. In this section, we assume that it follows a bi-dimensional Geometric Brownian Motion (GBM). This means that small technological changes occur in a continuous way, so that incremental technological innovation is considered. In the subsection 2.3., we introduce discrete and radical innovation or breakthrough, formalized by a Poisson process.

A detailed justification for a GBM for the market value of materials contained in the radioactive waste is described by Gollier and Devezeaux de Lavergne (2001). Our assumption of a GBM for the technological progress follows Grenadier and Weiss (1997) or Bethuyne (2001).

Let us define p_t as the price of radioactive materials on the market and θ_t the variable which represents the arrival of a new information concerning smooth technological changes. Here the technological progress in the radioactive field changes in a continuous way following a certain trendline (different technologies or equipment may improve with many but relative small increments). Thus, θ_t denotes the latest developed technology or the state of the current technological research in the radioactive waste field. The technological progress or, alternatively the state of research (gradual progress through knowledge), variates exogenously at a rate μ_θ. The strength at which exogenous random shocks react on this rate is represented by σ_θ. Also, the higher θ_t, the more significant the technological progress.

The following GBM processes are considered:

$$dp_t = \mu_p p_t dt + \sigma_p p_t d\varepsilon_p \tag{4}$$

$$d\theta_t = \mu_\theta \theta_t dt + \sigma_\theta \theta_t d\varepsilon_\theta \tag{5}$$

with $\sigma_\theta > 0$, $\sigma_p > 0$, $\mu_\theta \geq 0$, $\mu_p \geq 0$ and the initial conditions $p_0 = p \geq 0$, $\theta_0 = \theta \geq 0$.

The increments of standard Brownian processes are represented by $d\varepsilon_\theta$ and $d\varepsilon_p$, while σ_θ, σ_p are the instantaneous volatilities. The parameters μ_θ and μ_p represent the instantaneous expected returns. Moreover, for the sake of simplicity, we consider that the two stochastic processes are uncorrelated, $E(d\varepsilon_p d\varepsilon_\theta) = 0$. This means that the random shocks on each variable, that lead them to deviate from their expected trend, are due to independent events.

Hence the value w_t of a radioactive waste package at date t is explained by both processes (market uncertainty and technological uncertainty):

$$w_t = f(\theta_t, p_t) \tag{6}$$

Applying Itô's Lemma to the function $f(\theta_t, p_t)$ we obtain:

$$df(\theta_t, p_t) = dw_t = (\mu_\theta \theta_t f_\theta + \mu_p p_t f_p + \frac{1}{2}\sigma_\theta^2 \theta_t^2 f_{\theta\theta} + \frac{1}{2}\sigma_p^2 p_t^2 f_{pp})dt + f_\theta \theta_t \sigma_\theta d\varepsilon_\theta + f_p p_t \sigma_p d\varepsilon_p$$

where f_θ, f_p and $f_{\theta\theta}, f_{pp}$ are the partial derivatives of function $f(\theta_t, p_t)$.

Following McDonald and Siegel (1986) or Cortazar et al. (2000), we consider a multiplicative relation between θ_t and p_t: $w_t = \theta_t p_t$. This means that a given increase of any of these variables has a similar effect on the value of the reversible project. For future reference in the paper it is worth noticing that w_t follows a stochastic process, with a total drift $\mu_w = \mu_\theta + \mu_p$, volatility $\sigma_w = \sqrt{\sigma_\theta^2 + \sigma_p^2}$ and the standard Wiener process $\varepsilon_w = \dfrac{1}{\sigma_w}\left(\sigma_\theta d\varepsilon_\theta + \sigma_p d\varepsilon_p\right)$. Indeed with $f_\theta = p_t$, $f_p = \theta_t$ and $f_{\theta\theta} = f_{pp} = 0$ we have:

$$\begin{aligned} dw_t &= (\mu_\theta \theta_t p_t + \mu_p p_t \theta_t)dt + p_t \theta_t \sigma_\theta d\varepsilon_\theta + \theta_t p_t \sigma_p d\varepsilon_p \\ &= \left(\mu_\theta + \mu_p\right) w_t dt + \left(\sigma_\theta d\varepsilon_\theta + \sigma_p d\varepsilon_p\right) w_t \\ &= \mu_w w_t dt + \sigma_w w_t d\varepsilon_w \end{aligned} \tag{7}$$

with $w_0 = w$.

Besides, a variable with initial value w_0 that follows a GBM process displays the following expected value for some future date t:

$$E\left[w_t\right] = w_0 e^{\mu_w t} \tag{8}$$

Now we are able to analyze the decision-maker's problem. Her objective is to maximize the expected total value of the reversible disposal project over its lifetime, by choosing at each date the optimal disposal stage knowing the preceding one. She observes the current value of the radioactive waste package w_t and she must decide for a given stage $s \in S$ whether to continue on this stage or to switch to a less or a more retrievable one, given the uncertainty and the operational constraints.

We denote by V_0 the expected discounted value at $t = 0$ of the reversible disposal project until the final date T, for one radioactive waste package, with $c^{i,j}$ the switching costs and

$\pi^{i,j}$ the cumulative probabilities[6] of reaching the threshold values. Finally, the maximization program is:

$$\underset{\{j\in S\}}{Max} V_0(\widetilde{w}) = \int_0^T \left[\sum_{i\in S}\sum_{j\in S} \pi^{i,j} (\delta^j E(\widetilde{w}) - c^j) \right] e^{-rt} dt - \sum_{i\in S}\sum_{j\in S} \pi^{i,j} c^{i,j} e^{-rt^{i,j}} \tag{9}$$

with $c^{s,s} = 0$, $t^{i,j}-$ the dates for switching from stage i to stage j, and r the discount rate with $r > \mu_w$ by assumption[7].

To solve this program we must take into account all the options associated with each possible switch between stages, which in turn are determined by the threshold values of the radioactive waste package. In particular, the net present value of the reversible disposal project for one radioactive waste package stored on the intermediary stage s at the date t, namely $V_t^s(w_t)$, is determined by taking into account that once the packages are on the disposal stage s, it may be optimal to switch back to the stage $s-1$, if the value w_t becomes large enough and go further to the stage $s+1$, if w_t is sufficiently low:

$$V_t^s(w_t) = \begin{cases} v_t^s + F^{s,s+1}(w_t) + F^{s,s-1}(w_t), & \text{if } \; w_t^{s,s+1} \leq w_t \leq w_t^{s,s-1} \\ v_t^{s+1} + F^{s+1,s}(w_t) - c^{s,s+1}, & \text{if } w_t \leq w_t^{s,s+1} \\ v_t^{s-1} + F^{s-1,s}(w_t) - c^{s,s-1}, & \text{if } w_t \geq w_t^{s,s-1} \end{cases} \tag{10}$$

$F^{s,s\pm1}(w_t)$ represents the value of the option to switch from the disposal stage s to the stage $s\pm1 \in S$. Moreover, v_t^s is the net expected value of the project if the waste package were staying on stage $s \in S$ until the terminal date T. It is defined as following[8]:

$$v_t^s = E\int_t^T (w_\tau \times \delta^s - c^s) e^{-r\tau} d\tau \tag{11}$$

Furthermore, from the dynamic programming principle, the decision-maker takes the decision to switch or not to switch at date t i such a manner that the option value satisfies:

$$F^{s,s\pm1}(w_t) = \arg\max \left(e^{-rdt} E_t \left[F^{s,s\pm1}(w_{t+1}) \right] \right) \tag{12}$$

[6] The expression for the cumulative probabilities are given in Appendix 1.

[7] The restriction $r > \mu_w$, commonly use in real options models, is necessary to ensure that there is a strictly positive opportunity cost of holding the option, so that it will not be held indefinitely.

[8] In Appendix 2 we show that this gives

$$v_t^s = \frac{w_0 \times \delta^s}{r - \mu_w}(1 - e^{-(r-\mu_w)(T-t)}) - \frac{c^s}{r}(1 - e^{-r(T-t)}).$$

The decision-maker's objective at each date t is to maximize the sum of current cash flow from not exercising the option (zero) and the discounted value of having the option at the next date. Thus, she maximizes her return from holding the option, which is just equal to the option's capital appreciation. According to Bellman's principle, we have: [9]

$$rF^{s,s\pm1}(w_t)dt = E\left[dF^{s,s\pm1}(w_t)\right] \tag{13}$$

From Ito's Lemma, we obtain the following differential equation:

$$\frac{1}{2}\sigma^2\left(w_t\right)^2 F_{ww}^{s,s\pm1}(w_t) + \mu_w w_t F_w^{s,s\pm1}(w_t) - rF^{s,s\pm1}\left(w_t\right) = 0 \tag{14}$$

with the following general solution, where D is the option value of switching to a less retrievable stage and U the option value of switching to a more retrievable stage:

$$F^{s,s\pm1}\left(w_t\right) = Dw_t^{\alpha} + Uw_t^{\beta} \tag{15}$$

where $\alpha < 0$ and $\beta > 1$ are the solution of the following characteristic equation[10]:

$$\frac{1}{2}\sigma_w^2\beta(\beta-1) + \mu_w\beta - r = 0 \tag{16}$$

There is no clear economic meaning for parameters α and β. However, they can be considered as a factor describing the distance between the deterministic case and the uncertainty, given their explicit dependence on the drift and the volatility of the GBM. Hence, they allow to study the dependence of the threshold for the radioactive waste packages on the parameters of the stochastic process.

Proposition 1 *If the radioactive waste package is disposed on the intermediary disposal stage s at the date t, by combining both opportunities to switch upward and downward on the retrievability scale, we obtain the following expression for the net present value of the project:*

$$V_t^s = \begin{cases} v_t^s + D_{s+1}w_t^{\alpha} + U_{s-1}w_t^{\beta} & w_t^{s,s+1} \leq w_t \leq w_t^{s,s-1} \\ v_t^{s-1} + D_s w_t^{\alpha} + U_{s-2}w_t^{\beta} - c^{s,s-1}, & w_t \geq w_t^{s,s-1} \\ v_t^{s+1} + D_{s+2}w_t^{\alpha} + U_s w_t^{\beta} - c^{s,s+1}, & w_t \leq w_t^{s,s+1} \end{cases} \tag{17}$$

[9] See Dixit and Pindyck (1994) for a detailed description.

[10] See Appendix 3 for the derivation of the characteristic equation.

with $D_{s+1}w_t^{\alpha}$ *the value of the option for the downward switch to the stage* $s+1 \in S$

with $U_{s-1}w_t^{\beta}$ *the value of the option for the upward switch to the stage* $s-1 \in S$

Proof. See Appendix 4 ■

Now we must find $w^{s,s\pm1}$and $w^{s\pm1,s}$, the trigger values of the stochastic process $\{w_t\}_{t=1,..,T}$ at which it becomes optimal to switch from one stage to another one. We also need to determine $D_s, U_s, D_{s+1}, U_{s-1}$,the coefficients of the switching options values.

For each switching point, we apply the optimality conditions of value-matching (18) and (19), and smooth pasting (20) and (21). The value matching condition reflects the fact that for a given trigger value, the value of project before the switch must equal the value of the project after the switch, minus the switching cost. The smooth pasting conditions are first order conditions. [11]

$$\begin{aligned} V^s(w_t^{s,s\pm1}) &= V^{s\pm1}(w_t^{s,s\pm1}) - c^{s,s\pm1} && (18) \\ V^{s\pm1}(w_t^{s\pm1,s}) &= V^s\left(w_t^{s\pm1,s}\right) - c^{s\pm1,s} && (19) \end{aligned}$$

and

$$\left[V^s(w_t^{s,s\pm1})\right]' = \left[V^{s\pm1}(w_t^{s,s\pm1}) - c^{s,s\pm1}\right]' \quad (20)$$

$$\left[V^{s\pm1}(w_t^{s\pm1,s})\right]' = \left[V^s\left(w_t^{s\pm1,s}\right) - c^{s\pm1,s}\right]' \quad (21)$$

For the case with three available disposal stages, we obtain a system of eight equations non-linear in threshold values, with eight unknown variables: the option values coefficients for the downward switch and upward switch, respectively $D_s, D_{s+1}, U_s, U_{s-1}$, and the trigger values $w_t^{s,s-1}$, $w_t^{s,s+1}$, $w_t^{s-1,s}$, $w_t^{s+1,s}$. Since generally, closed-form analytical solutions cannot be obtained for this non-linear system, we perform a numerical analysis in the third section[12].

Given that the optimal strategy of switching from one disposal stage to another is based on the threshold values and option coefficients, their identification permits the decision-maker to better understand and evaluate each disposal scenario for the radioactive waste.

Because at each stage the decision-maker may go further to a less (more) reversible stage or to continue on the same stage, the reversible project of geological disposal involves a series of

[11] More detailed explanation of these conditions can be found in appendix C of chapter 4 in Dixit and Pindyck (1994).

[12] An extended form of the system of equations (18) to (21) can be found in Appendix 4.

compound options (options on options) which may create follow-up opportunities and interactions. For example, realizing an earlier real option (such as closing the galleries of access) can change the value of future options for the retrieval of waste packages. This type of interactions between various options involved in the reversible disposal of radioactive waste are important in the valuation of the project. They need to be valued together because their combined value may differ from their separate values.

2.3 Switching options with a compound Geometric Brownian Motion and Poisson process

In this subsection, the market price p of the waste packages keeps following a GBM. Nevertheless, we consider here discrete technological changes contrary to the preceding subsection. The technological progress in the nuclear sector is no longer a smooth process, but rather a process where radical innovations can occur ponctually. For instance, think about innovation programs such as the transmutation of the high level waste or the regeneration of fissile material. This property is formalized by assuming that the technological variable follows a Poisson jump process.

Finally, the value w of the radioactive waste package follows a combined GBM and Poisson jump process. Technological changes are "events" of a Poisson process and between technological jumps, the radioactive waste value is assumed to evolve according to a GBM, reflecting the economic conditions.

A change in technological progress causes the value w to jump by some percentage $\theta > 0$ or $-\theta < 0$ of the current level, other things being equal. A jump θ implies improvement in research, while a jump $-\theta$ implies drawbacks of the technological progress. The probability of a technological change during any short period of time dt is λdt, and the probability of no change is $1 - \lambda dt$, where λ represents the mean arrival rate of an event during the interval dt.

We denote with γ the probability of a positive jump once a jump occurs. Hence the Poisson process q is such that:

$$dq = \begin{cases} 0 \text{ with probability } 1 - \lambda dt \\ \theta \text{ with probability } \gamma\lambda dt \\ -\theta \text{ with probability } \ (1-\gamma)\lambda dt \end{cases} \tag{22}$$

And finally, w follows the stochastic process characterized by:

$$dw = \mu w dt + \sigma w d\varepsilon + w dq \tag{23}$$

where dq is assumed to be independent of $d\varepsilon$: $E(dq d\varepsilon) = 0$.

As shown by Merton (1976), a variable that follows a jump-diffusion process has the following expected value:

$$E\left[w_t\right] = w_0 \exp\left[\mu + \gamma\lambda\theta - (1-\gamma)\lambda\theta\right] \tag{24}$$

This means that a positive λ leads to different expected percentage changes in the value of w in each period (positive or negative, depending on θ).

We show in Appendix 5 that the expected time until w takes a Poisson jump over the project horizon T is given by:

$$E(\tau) = \frac{1}{\lambda} - T \times e^{-\lambda T} - \frac{e^{-\lambda T}}{\lambda} \tag{25}$$

Still we show in Appendix 6 that the expected value $v^s(w_t)$ of the project for a radioactive waste package that would be disposed on a given and unchanged stage s from date t to date T, is:

$$\begin{aligned} v^s(w_t) &= E\left[\int_t^T (\delta^s \times w_\tau - c^s) e^{-r\tau} d\tau\right] \\ &= \frac{\delta^s \times w_0}{r - (\mu + \gamma\lambda\theta - (1-\gamma)\lambda\theta)}(1 - e^{-(r-\mu-\gamma\lambda\theta+(1-\gamma)\lambda\theta)(T-t)}) - \frac{c^s}{r}(1 - e^{-r(T-t)}) \end{aligned} \tag{26}$$

Finally, applying Ito's Lemma[13] to the combined GBM and poisson stochastic process leads to:[14]

$$\frac{1}{2}\sigma^2 w_t F_{ww}^{s,s\pm1}(w_t) + \mu w_t F_w^{s,s\pm1}(w_t) + \lambda F^{s,s\pm1}\left[(1+\theta)w_t\right] - (\lambda + r)F\left(w_t\right) = 0 \tag{27}$$

It has the following general solution

$$F_{ww}^{s,s\pm1}\left(w_t\right) = D w_t^\alpha + U w_t^\beta, \tag{28}$$

[13] As shown in Appendix 7, the second term from the standard Ito's Lemma contains a part due to GBM and another one due to the Poisson Process.

$$dF^{s,s\pm1}(w) = \frac{\partial F^{s,s\pm1}(w)}{\partial t} dt + \frac{\partial F^{s,s\pm1}(w)}{\partial w} dw$$

[14] See Appendix 7 for the derivation of the characteristic equation and its solutions.

where D and U are constants to be determined, and α and β are the roots of the characteristic equation:

$$\frac{1}{2}\sigma^2\beta\left(\beta-1\right)+\mu\beta+\lambda\left(1+\gamma\theta-(1-\gamma)\theta\right)^{\beta}-\left(\lambda+r\right)=0 \tag{29}$$

The solution for α and β must be solved numerically.

This is done in the next section where our intention is to compare this more realistically stochastic process with the smooth evolution for the value of the radioactive waste. We expect different results, given that the switching thresholds will differ not only through the effects of λ, but also through the new solution for α and β, which depend now on $\sigma, \mu, \lambda, \theta$ and r.

3 Numerical analysis with French data and discussion

This section provides some simulations that permit a deeper analysis of the characteristics of the decision-maker's optimal strategy. We consider both cases studied below (GBM and combined GBM/Poisson). We study how the dynamic stochastic model reacts to changes in key parameter values. We also focus on the impact of a change in the trigger values on the decision to switch.

3.1 French Data and trigger values

The numerical analysis is performed by building on costs data provided by the French Agency in charge with the radioactive waste management (ANDRA).

First let us state that $T = 100$ years. It corresponds to the reversibility period as stated in the Planning Act No. 2006-739 of 28 June 2006 concerning the sustainable management of radioactive materials and waste. Second, given our available data, we consider the first three retrievability stages concerning the geological disposal of Figure 1, with the intermediary stage $s = 2$. The values used below for operating and switching costs for the highly radioactive waste (HLW), unknown today, were estimated relatively to the costs for intermediary level waste(ILW), normalized at 1.[15]The switching cost $c^{1,2}$ induced, for instance, by the addition of new protective barriers around the waste emplacement cell , is set to 0.6. The cost of closing

[15] Costs estimations were conducted by ANDRA for the near-surface disposal (at depths of tens of meters) of the ILW, which is currently in operation at Centre de l'Aube in France.

the storage area (the access gallery remains open), $c^{2,3}$, equals 2. The switching costs when going from stage 3 to stage 2 are about 5 times higher than those prevailing when switching from stage 2 to stage 1: we assume that $c^{3,2} = 15$ and $c^{2,1} = 3$. In addition, the operating costs are $c^1 = 6$; $c^2 = 4$; $c^3 = 2$. All these costs are considered for 1 m^3 of radioactive waste package. These cost values justify the assumptions made in subsection 2.1. The retrieval of a waste package is more expensive than its emplacement in the geological area and the operating costs are higher for stages with a higher degree of retrievability. The risk-free interest rate, r, is set to 0.03 for the base case[16].

The deterministic case, with no uncertainty ($\mu_w = \sigma_w = 0$), will serve as a benchmark. It permits us to obtain some initial values for the triggers.

Concerning the 2-Dimensional GBM case we use, as Gollier et al.(2001) and Loubergé et al. (2001), the following parameters that describe the stochastic trend of the value of a 1 m^3 of radioactive waste package: $\mu_p = 0.01,\ \mu_\theta = 0,\ \mu_w = 0.01,\ \sigma_p = 0.03,\ \sigma_\theta = 0.07,\ \sigma_w = 0.076$.

Two additional assumptions must be made when dealing with the combined GBM and Poisson stochastic process . First, we consider $\lambda = 0.002$ (important innovations or drawbacks are rare). Second the probability of a positive jump is assumed to be very high, $\gamma = 0.8$, and θ is equal to 0.8, because we would like to examine the decision process behavior after the arrival of important news.

Table 1 summarizes the ranges of trigger values obtained for each of both considered cases. Obviously, they are dependent upon the parameters assumptions outlined above. The range obtained for the thresholds values $w^{1,2}$, $w^{2,1}$, $w^{3,2}$ and $w^{2,3}$ is mainly explained by the interaction between switching costs and thresholds, which is intuitive. For instance, if the retrieval cost $w^{2,3}$ is large, the value of the option to switch is large and the value of radioactive waste for which the retrievability is justified must be high. Each switching cost affects its corresponding threshold.

When uncertainty is introduced, we observe that the required trigger values for switching to a more (respectively less) reversible stage of disposal increase (respectively decrease), compared to the deterministic case. One can also observe that the difference between $w^{2,1}$ and $w^{2,3}$

[16] The future costs of deep geological disposal are to be supported by waste producers. Thus, the cost of waste disposal is discounted in terms of accounting standards applied by producers. For example, the discount rate used is 3% net, equivalent to 5% yield and 2% inflation.

Trigger values	NPV $\mu_w = 0$ $\sigma_w = 0$	2D-GBM $\mu_w = 0.01$ $\sigma_w = 0.076$	GBM+Poisson (positive jumps) $\lambda = 0.002$ $\gamma = 0.8$	GBM+Poisson (negative jumps) $\lambda = 0.002$ $\gamma = 0.2$
$w^{1,2}$	1.9811	1.7370	1.7815	1.6489
$w^{2,1}$	2.0947	2.6743	2.7083	2.5762
$w^{2,3}$	1.9369	1.4706	1.4963	1.4121
$w^{3,2}$	2.4736	3.2743	3.2864	3.1953

Table 1: Trigger values of the radioactive waste package

increases with uncertainty (higher in the GBM-Poisson and 2-Dimensional GBM cases than in the NPV case). Indeed the decision-maker shall wait on an intermediary disposal stage for more extreme differences in downward and upward trigger values. In the context of our example the retrieval of waste packages becomes optimal more "rapidly" when the value w of the package is stationary and deterministic (NPV model). Then follows the GBM-Poisson case with negative jumps, the 2-Dimensional GBM case, and finally the GBM-Poisson case with positive jumps. The economic explanation is as follows.

For instance, an increase in the uncertainty (this means an increase in the volatility) increases the chance that the decision maker learn in the future that the value w may fall enough to make the switch to a more reversible stage sub-optimal. This creates an opportunity cost of switching early to a more reversible stage and the value of the option to switch is equal to this opportunity cost. Hence in order to avoid this cost, it will be in the interest of the decision maker to keep the waste package a longer time on a less reversible stage. This explains the higher threshold for the retrieval, compared to the deterministic case. Inversely, the decision maker will be more reluctant to switch to a less reversible stage, if the value of the radioactive waste package rises enough to justify the retrieval.

For the GBM with positive jumps case, the decision-maker switches earlier to a less reversible stage and later to a more reversible stage. In this case, the probability of positive jumps affects positively the expected change in the value of the waste package (as can be seen from equation (24))and also the instantaneous variance of changes. This increases the value of the opportunity to switch to a more reversible stage, and thus increases the opportunity cost of switching earlier than waiting. The decision-maker prefers to retain the option until the value of the waste package has reached a level that makes optimal the switching to a more reversible stage. The higher the probability of technological progress in the future, the higher the needed threshold for optimal retrieval. Moreover, given the flexibility of the whole project, the decision-maker keeps in mind the fact that he has also the option to switch to a less reversible stage if the value of the package is sufficiently low. When positive jumps are important, the threshold value for switching to a less reversible stage is higher, and thus the option value of waiting is smaller relative to the value of waiting for the upward switch. Actually, one of the main insights of our model is that reversibility implied by the deep geological disposal may cover the risk of fluctuations of future values of waste packages, since some decisions are reversible (for example, the decision of partial backfilling may be made by bearing in mind the reversibility).

For the GBM with negative jumps, the decision-maker will have some incentives to hold a longer time the option to switch to a less reversible stage, while switching early to a more reversible stage. The higher the probability of smaller values for the waste package, the lower the threshold that satisfies the optimality of switching to a less reversible stage.

Table 2 presents the results for the coefficients of switching options values. The highest ones are those obtained with the GBM-Poisson model with positive jumps.

Option values coefficients	2D-GBM $\mu_w = 0.01$ $\sigma_w = 0.076$	GBM+Poisson (positive jumps) $\lambda = 0.002$ $\gamma = 0.8$	GBM+Poisson (negative jumps) $\lambda = 0.002$ $\lambda = 0.2$
D_2	142.6414	200.0801	98.9484
U_1	51.5029	65.4380	39.1740
D_3	5.6681	6.7611	4.5001
U_2	10.2392	12.3299	7.9998

Table 2: Option values coefficients

We note that the coefficient for the option to switch down from stage 1 to 2 (D_2) is significantly larger than the others. This is because the value of this first option to switch to a less reversible stage includes the option to switch further to the subsequent stage or to reverse decisions at the optimal time (compound options).

Figure 2 shows the different options in the 2-D GBM case. Given our data, the most significant option is the option to start the switching of the radioactive waste from surface storage to deep geological disposal. The value of the options to switch up increases with the value of the waste package. As expected, the option value to dispose the waste packages on less reversible stages increase for low values of radioactive waste packages and decreases for high values. In addition, the option to retrieve the waste from the last disposal stage is more important than the option to retrieve the package when it is disposed on the intermediary stage for all levels of waste package value, given that $c^{3,2}>c^{2,1}$ by a large amount. Let us mention that the convexity of options functions is due to the values of parameters α and β, the roots of the fundamental quadratic equation (16).

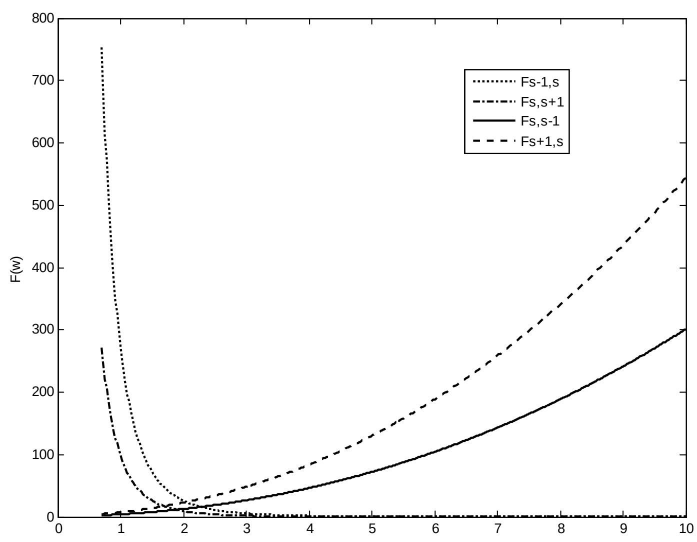

Figure 2: 2-D GBM case-Values of downward and upward switching options

Figure 3 depicts the evolution of switching options values for the GBM-Poisson (positive jumps).[17] Compared to the 2-D GBM, the options values to switch to a more reversible stage

[17] See Appendix 8 for illustrations in GBM-Poisson (negative jumps) case.

increase more rapidly with the value of waste packages. This result holds because of the opportunities of important technological innovation or unexpected jumps in the market price of the radioactive materials.

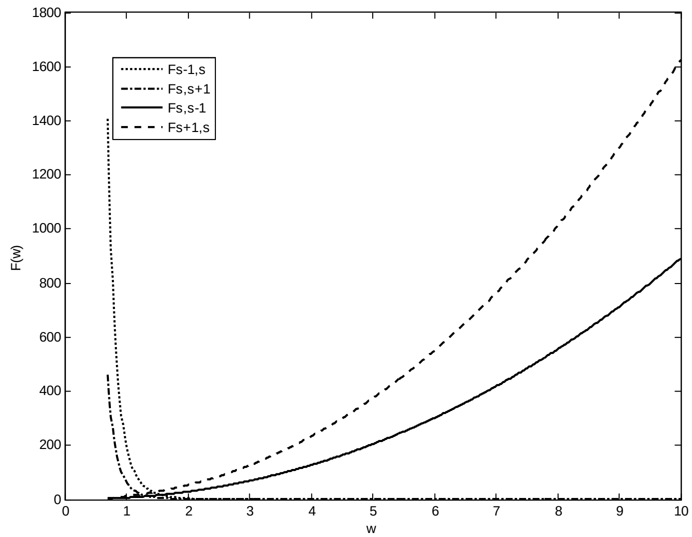

Figure 3: Switching options values in GBM-Poisson (positive jumps) case

In both cases, for the intermediary stage s, we can see that for high values of waste package, the value of the option to retrieve the waste package relative to the option to transfer it on a less reversible stage is more important. Given the flexibility of the project, the decision-maker must consider these options simultaneously.

3.2 Comparative static for the 2-Dimensional GBM model

In this subsection we analyze the evolution of the option values and the trigger values in the 2-Dimensional GBM case when parameters $r, \sigma_p, \sigma_\theta, \mu_p$,$\mu_\theta$ and switching costs vary. The base case to which all the values are compared was described in Subsection 3.1.

Figure 4 displays a forecast of the evolution of the radioactive waste package value w, with a starting value of 0.1, a drift parameter of 0.01 and volatility of 0.07. The current social and economic conditions permit us to consider a nearly zero value for the ultimate waste. The x-axis represents the time-scale (100 years) and the y-axis represents the GBM paths for w.

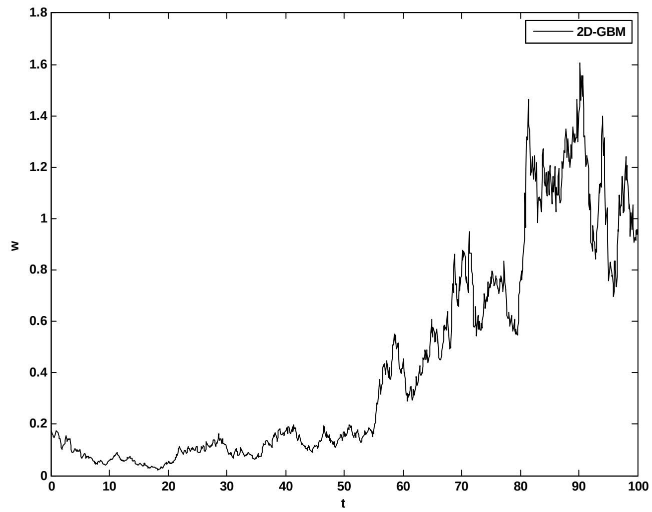

Figure 4: Sample path of 2D-GBM process for the value of radioactive waste package

We observe that certain thresholds values may be attained, and consequently important decisions will have to be taken in the future. This can justify the decision of the French decision-maker to consider a sufficiently large period of reversibility (minimum 100 years). Also, given that in the near future the value of radioactive waste package is low, the decision-maker has more incentives to realize today the switch of waste packages to the reversible deep facilities.

Figure 5 presents the evolution of the value of radioactive waste package with respect to variations in the discount rate, the volatility and the drift parameter.

An increase in the discount rate lowers both threshold values for upward and downward switching options. This means that when the discount rate is high, that is the opportunity cost of waiting is high, the decision-maker is more willing to exercise the option of switching the radioactive waste package on disposal stages with a higher degree of retrievability. As shown by Gollier (2007), for the minimal period of reversibility of storage of 100 years, different discount rates may be applied. For a period of time inferior to 30 years, the decision-maker may apply a higher discount rate, but for periods beyond 30 years, r must be very low (1%, 2%). We consider an intermediate rate, i.e 3%, as taken by assumption by the radioactive waste producers to evaluate their provisions.

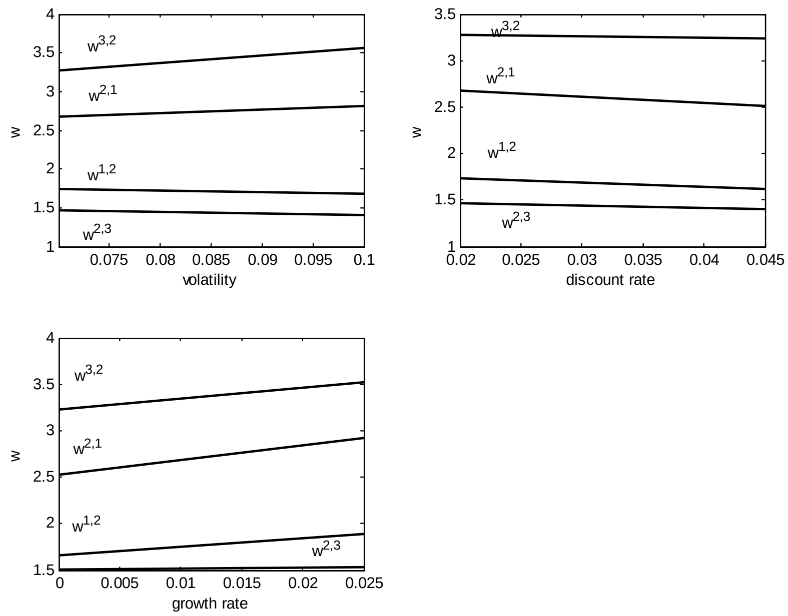

Figure 5: 2-D GBM-Evolution of the thresholds with respect to volatility, discount rate and growth rate

Concerning the volatility parameter, we obtain that $w^{2,1}$and $w^{3,2}$are quite sensitive to variations in volatility. The thresholds for the upward switch on the retrievability scale increase with uncertainty and the thresholds for the downward switch decrease. On one hand, the decision-maker will be less willing to retrieve the waste packages for a higher volatility of the waste package value. On the other hand, if the waste package is disposed on a stage with a higher degree of retrievability, the decision-maker is more reluctant to switch the packages on a less reversible stage. This result is highlighted in the real options theory.

Finally, a higher value of the growth parameter induce an increase of all thresholds. Obviously, the higher the growth parameter, the higher the chance that the value of radioactive package be high in the future. Thus, the decision-maker will be more reluctant to switch to a more reversible stage and more willing to go to a less reversible stage.

Figure 6 displays the relation between the threshold values and the switching costs. The impact of the cost of switching to a more reversible stage, $c^{2,1}$ on the corresponding threshold, $w^{2,1}$ is obvious. If it increases, the decision to switch up becomes more costly and the threshold

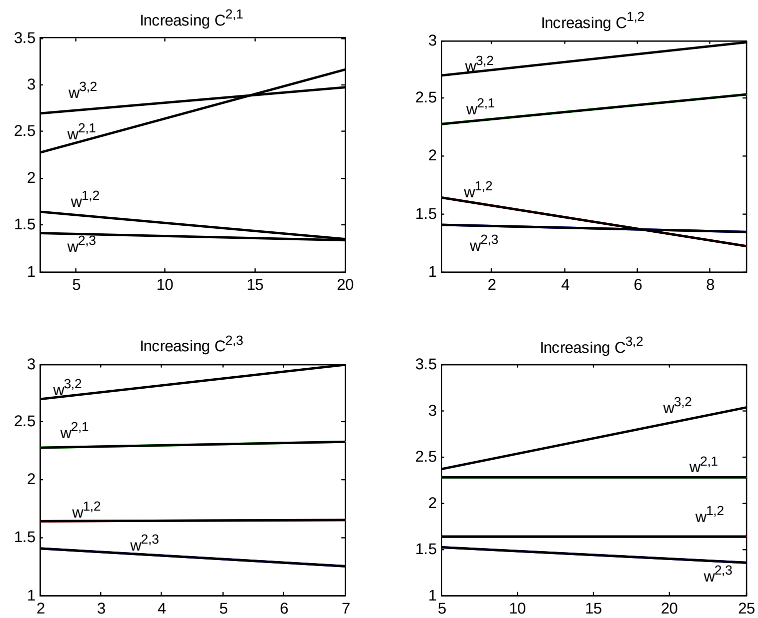

Figure 6: 2-D GBM -Thresholds values as functions of switching cost

for retrieval increases. On the other hand, the impact of the switching cost $c^{2,1}$ on the threshold $w^{2,3}$ is hard to observe on the figure. Nevertheless, it seems intuitive that an increase in the cost of switching to retrieval induces a decrease of the threshold relative to a switch to a less reversible stage. The decision-maker switches to a stage with a lower degree of retrievability less likely if she will have to pay large switching costs in order to retrieve the packages whenever the value of the radioactive waste increases in the future. The impact of the switching cost $c^{1,2}$ on the threshold $w^{2,1}$ also needs further explanation. If the value of the waste package falls in the near future, the decision-maker is reluctant to lose the value of the option to remain on the intermediary stage, and thus avoiding to support again large costs for switching to stages where the ease of retrieval is small.

We also obtain that for large values of $c^{2,1}$, the threshold for switching from the intermediary stage 2 to stage 1 becomes higher than the threshold $w^{3,2}$ that signals the retrieval from the last disposal stage. The relation between thresholds and corresponding switching costs just mentioned above explains this changing order. Consequently, for large values of $c^{2,1}$ the decision to completely retrieve the waste is delayed. In addition, when the switching cost for complete retrieval is large, the figure shows that it may be optimal to accelerate the closure phase of the repository, given that the threshold concerned, $w^{2,3}$, is higher than the threshold $w^{1,2}$. Similar effects can be induced from an increase of $c^{2,3}$ and $c^{3,2}$ on the thresholds $w^{2,3}$ and $w^{3,2}$.

3.3 Comparative static for the Compound GBM and Poisson case

Let us consider that the value of the radioactive waste package, w, follows a combined GBM-Poisson process, as described by equation (23). The parameter λ is the mean arrival rate of jumps in the value of waste packages. As λ increases, the time gap between jumps falls, so that for any given value of θ, a larger λ leads to more frequent jumps. As stated in Table 1, the threshold values are higher than those in the 2-D GBM and NPV cases for positive jumps and lower for negative, large, jumps.

Figure 7 displays both processes for comparison. As in the 2-D GBM case, because of the nearly zero today value of waste packages and the fact that in the nuclear field radical innovation are rare (this is captured by a low value of λ), it takes a very long time to reach the thresholds during the reversibility period of 100 years. However, opportunities to revise decisions may also appear. For example, if research and development provide ways to reduce the degree of difficulty of retrieval or changes affecting long term safety previous steps may be reevaluated.

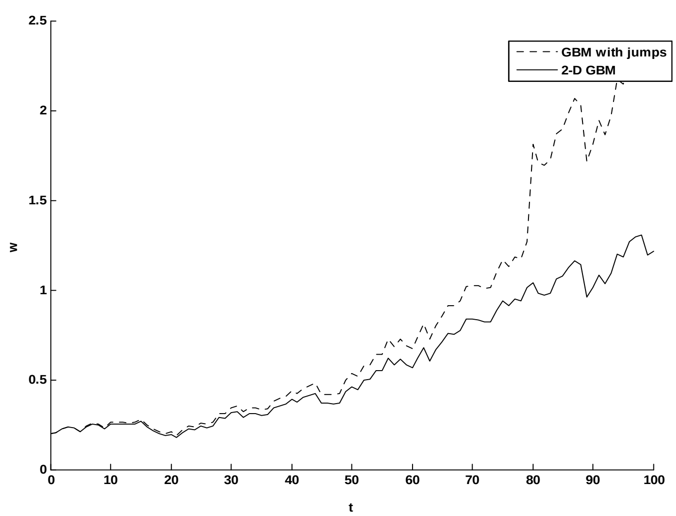

Figure 7: Sample path of GBM-Poisson process for the value of radioactive waste package

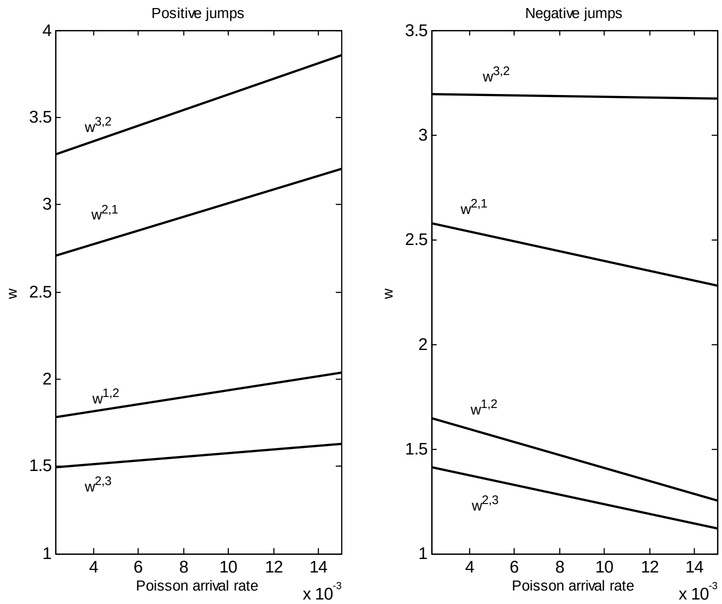

Figure 8: Variation of thresholds with respect to lambda

Figure 8 shows how the critical waste package values vary with λ for negative and positive values of θ. A higher λ with a positive (negative) θ implies higher (lower) thresholds values. This is due to the fact that an increase in λ leads to a larger expected change in w at each period (recall equation (24)). This tends to increase (respectively decrease) the opportunity cost of switching to a more reversible stage for $\theta > 0$ (respectively $\theta < 0$).

Figure 9 depicts the impact of different values of r, μ and σ on the thresholds among stages when $\theta > 0$.[18] The relationship is similar to the one found in the 2-D GBM model as the waste package value still variates between jumps according to a GBM.

Figure 10 represents the variation of the switching thresholds with respect to the transition costs. We observe a similar evolution following an increase of the switching cost $c^{2,1}$. Nevertheless, an increase in emplacement costs $c^{1,2}$ and $c^{2,3}$, induce less sensible thresholds values of retrieval, $w^{2,1}$ and $w^{3,2}$. Moreover, the threshold $w^{3,2}$ is much more sensitive to an increase of $c^{3,2}$ than that of $c^{2,3}$. The economic reasoning behind these interactions described in the previous subsection still holds.

[18] The comparative statics for the GBM-Poisson process with negative jumps is available upon request.

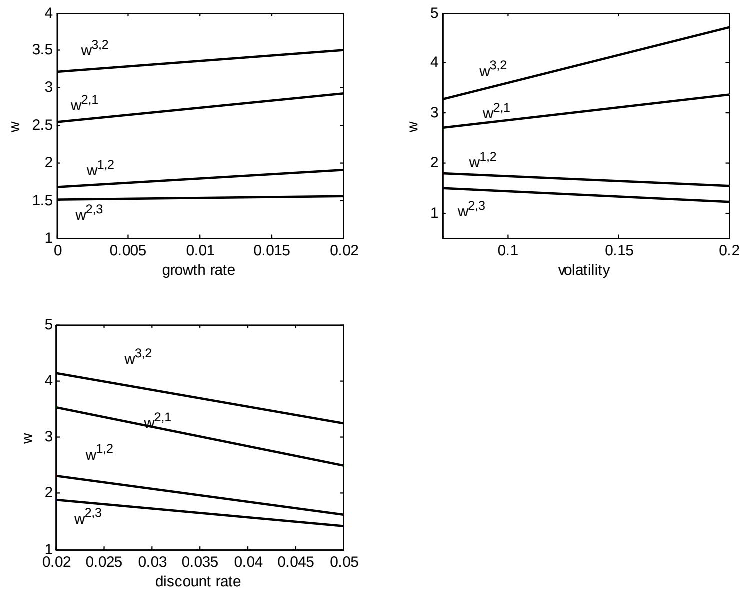

Figure 9: Thresholds with respect to μ, σ and r, in the GBM-Poisson case with $\theta > 0$.

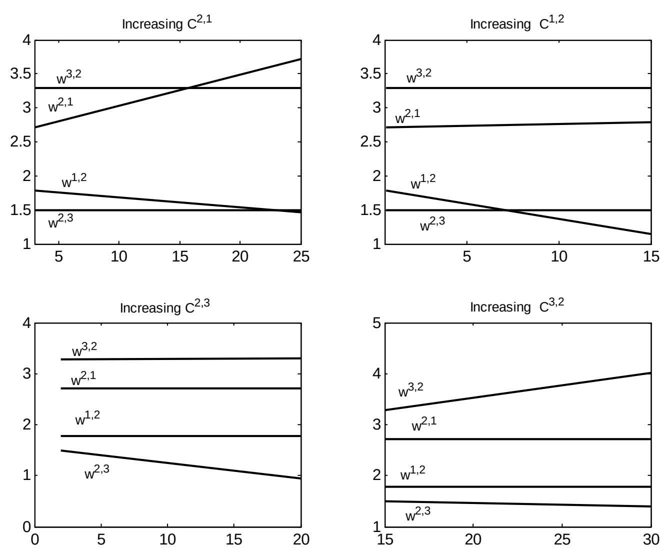

Figure 10: Evolution of thresholds as function of switching costs with GBM-Poisson (positive jumps)

Both cases show that the decision-maker delays her switching decisions as the uncertainty (volatility) on the value of the radioactive waste increases. The possibility of jumps in the waste package value considered in the second model, was divided into two cases: one in which the jumps were expected to be positive (associated with radical innovations in the technology of disposal), and one in which they were expected to be negative (associated with failures in the technology). When drawbacks are considered, the decision-maker exercises the option to retrieve the waste sooner compared to the case where only technological progress is considered. If the probability of important innovations increases, then the retrieval of waste packages is calling to a highest threshold and it sees the longest delay before the decision-maker is convinced of the optimality of the retrieval.

APPENDIX

1.Cumulative probabilities of switching $\pi^{i,j}(i,j \in S)$

At each date the decision-maker must calculate the probability of meeting the conditions for switch. She knows only the normal cumulative density functions $\Omega(w_t^{s,s+1}), \Omega(w_t^{s,s-1})$. The different cumulative probabilities associated to the GBM process of w_t are defined as following:[19]

$$\begin{aligned}
\pi^{s,s+1} &= \Pr(w_t \le w_t^{s,s+1}) = \Omega(\frac{\ln \frac{w_t^{s,s+1}}{w_t} - \left(\mu_w - \frac{1}{2}\sigma_w^2\right) t}{\sigma_w \sqrt{t}}) + \\
&\left(\frac{w_t}{w_t^{s,s+1}}\right)^{\frac{2\left(\mu_w - \frac{1}{2}\sigma_w^2\right)}{\sigma_w^2}} \times \Omega\left(\frac{\ln \frac{w_t}{w_t^{s,s+1}} + \left(\mu_w - \frac{1}{2}\sigma_w^2\right) t}{\sigma_w \sqrt{t}}\right) \\
\pi^{s,s-1} &= \Pr(w_t \ge w_t^{s,s-1}) = \Omega(\frac{\ln \frac{w_t}{w_t^{s,s-1}} + \left(\mu_w - \frac{1}{2}\sigma_w^2\right) t}{\sigma_w \sqrt{t}}) + \\
&\left(\frac{w_t^{s,s-1}}{w_t}\right)^{\frac{2\left(\mu_w - \frac{1}{2}\sigma_w^2\right)}{\sigma_w^2}} \times \Omega(\frac{\ln \frac{w_t}{w_t^{s,s-1}} - \left(\mu_w - \frac{1}{2}\sigma_w^2\right) t}{\sigma_w \sqrt{t}}) \\
\pi^{s} &= \Pr(w_t^{s,s+1} \le w_t \le w_t^{s,s-1}) = \exp\left(\frac{2\left(\mu_w - \frac{1}{2}\sigma_w^2\right)}{\sigma_w^4} \ln \frac{w_t^{s,s-1}}{w_t}\right) \times \\
&\Omega\left(\frac{\ln \frac{w_t^{s,s+1}}{w_t} - 2\ln \frac{w_t^{s,s-1}}{w_t} - \left(\mu_w - \frac{1}{2}\sigma_w^2\right) t}{\sigma_w^2 \sqrt{t}}\right)
\end{aligned}$$

[19] For the determination of cumulative probabilities, see Harrison (1985).

2. Proof of Equation (11)

Knowing that $E\left[w_t\right] = w_0 e^{\mu_w t}$,we write the expectation factor as follows:

$$v^s(w_t) = E\left[\int\limits_t^T (w_\tau \times \delta^s - c^s) e^{-r\tau} d\tau\right] = E\left[\delta^s \times \int\limits_t^T w_\tau e^{-r\tau} d\tau\right] - E\left[\int\limits_t^T c^s e^{-r\tau} d\tau\right]$$

$$E\left[\int\limits_t^T w_\tau e^{-r\tau} d\tau\right] = \int\limits_t^T w_0 e^{\mu_w \tau} e^{-r\tau} d\tau == \int\limits_t^T w_0 e^{(\mu_w - r)\tau} d\tau = \frac{w_0}{r - \mu_w}(1 - e^{-(r-\mu_w)(T-t)})$$

$$E\left[\int\limits_t^T c^s e^{-r\tau} d\tau\right] = \frac{c^s}{r}(1 - e^{-r(T-t)}),\ given \int\limits_t^T x e^{-r\tau} d\tau = x\frac{1 - e^{-rT}}{r}$$

$$v^s(w_t) = \frac{w_0 \times \delta^s}{r - \mu_w}(1 - e^{-(r-\mu_w)(T-t)}) - \frac{c^s}{r}(1 - e^{-r(T-t)})$$

3. Proof of Equation (16)

Until the adoption time the option to switch has no return, so the only return from having the option is the expected value $E\left[dF^{s,s\pm1}(w_t)\right]$ which according to Bellman principle, must equate the expected return on exercising the option. [20]

$$rF^{s,s\pm1}(w_t)dt = E\left[dF^{s,s\pm1}(w_t)\right] \tag{A.1}$$

We apply Itô's lemma for the last equation and we obtain:

$$rF(w_t) = \frac{E\left[dF^{s,s\pm1}(w_t)\right]}{dt} = \mu_w w_t F_w^{s,s\pm1}(w_t) + \frac{1}{2}\sigma_w^2 \left(w_t\right)^2 F_{ww}^{s,s\pm1}(w_t)$$

The following differential equation, which is satisfied by the option value, is derived from Bellman principle:

$$\mu_w w_t F_w^{s,s\pm1}(w_t) + \frac{1}{2}\sigma^2 \left(w_t\right)^2 F_{ww}^{s,s\pm1}(w_t) - rF^{s,s\pm1}(w_t) = 0 \tag{A.2}$$

and has the associated general solution:

$$F^{s,s\pm1}(w_t) = Dw_t^\alpha + Uw_t^\beta \tag{A.3}$$

where $\alpha < 0$ and $\beta > 1$ are the solutions of the quadratic equation

$$\frac{1}{2}\sigma_w^2 \beta(\beta - 1) + \mu_w \beta - r = 0 \tag{A.4}$$

[20] See Dixit and Pindyck (1994) for a detailed description.

$$\alpha,\beta=\frac{\left[-(\mu_w-\frac{1}{2}\sigma_w^2)\pm\sqrt{(\mu_w-\frac{1}{2}\sigma_w^2)^2+2\sigma_w^2 r}\right]}{\sigma_w^2}$$

When $F^{s,s+1}(w_t)=Uw_t^{\beta}$, the partial derivatives are: $F_w^{s,s+1}(w_t)=\beta Uw_t^{\beta-1}$and $F_{ww}^{s,s+1}(w_t)=\beta\left(\beta-1\right)Uw_t^{\beta-2}$. We substitute them in equation (A.2) and we obtain:

$$\mu_w\beta Uw_t^{\beta}+\frac{1}{2}\sigma\beta\left(\beta-1\right)Uw_t^{\beta}-rUw_t^{\beta}=0$$
$$\left[\frac{1}{2}\sigma^2\beta\left(\beta-1\right)+\mu_w\beta-r\right]Uw_t^{\beta}=0$$

Thus, in order for $F^{s,s+1}(w_t)=Uw_t^{\beta}$ to be a solution for the differential equation (A.2), the equation (A.4) must hold.

4. Proof of Proposition 1

For the intermediary stage s, the value of the reversible project is:

$$V_t^s(w_t)=\begin{cases} v_t^s+F^{s,s+1}(w_t)+F^{s,s-1}(w_t), & \text{if } w_t^{s,s+1}\leq w_t\leq w_t^{s,s-1}\\ v_t^{s+1}+F^{s+1,s}(w_t)-c^{s,s+1}, & \text{if } w_t\leq w_t^{s,s+1}\\ v_t^{s-1}+F^{s-1,s}(w_t)-c^{s,s-1}, & \text{if } w_t\geq w_t^{s,s-1}\end{cases}\tag{A.5}$$

For the disposal stage with the highest degree of retrievability, we have $F^{s,s-1}=D_s w_t^{\alpha}$, given that $U_{s-2}=0$ and for the last disposal stage , $s+1$, we have $F^{s+1,s}=U_s w_t^{\beta}$, given that $D_{s+2}=0$

Then the value of the project for each of the three stages can be written as follows:

$$V_t^{s-1}=\begin{cases} E(\int_t^T\delta^{s-1}\times w_\tau-c^{s-1})e^{-r\tau}d\tau+D_s w_t^{\alpha}, & w_t\geq w_t^{s-1,s}\\ E(\int_t^T\delta^{s}\times w_\tau-c^{s})e^{-r\tau}d\tau+D_{s+1}w_t^{\alpha}+U_{s-1}w_t^{\beta}-c^{s-1,s}, & w_t\leq w_t^{s-1,s}\end{cases}$$

$$V_t^{s}=\begin{cases} E(\int_t^T\delta^{s-1}\times w_\tau-c^{s-1})e^{-r\tau}d\tau+D_s w_t^{\alpha}+U_{s-2}w_t^{\beta}-c^{s,s-1}, & w_t\geq w_t^{s,s-1}\\ E(\int_t^T\delta^{s+1}\times w_\tau-c^{s+1})e^{-r\tau}d\tau+D_{s+2}w_t^{\alpha}+U_s w_t^{\beta}-c^{s,s+1}, & w_t\leq w_t^{s,s+1}\\ E(\int_t^T\delta^{s}\times w_\tau-c^{s})e^{-r\tau}d\tau+D_{s+1}w_t^{\alpha}+U_{s-1}w_t^{\beta} & w_t^{s,s+1}\leq w_t\leq w_t^{s,s-1}\end{cases}$$

$$V_t^{s+1}=\begin{cases} E(\int_t^T\delta^{s-1}\times w_\tau-c^{s+1})e^{-r\tau}d\tau+U_s w_t^{\beta}, & w_t\leq w_t^{s+1,s}\\ E(\int_t^T\delta^{s}\times w_\tau-c^{s})e^{-r\tau}d\tau+D_{s+1}w_t^{\alpha}+U_{s-1}w_t^{\beta}-c^{s+1,s}\ , & w_t\geq w_t^{s+1,s}\end{cases}$$

We write analytically the expression for the value matching conditions associated to the switches to disposal stages $s \pm 1 \in S$, when the radioactive waste is on the stage $s \in S$:

$$v^{s-1}(w_t^{s-1,s}) + D_s(w_t^{s-1,s})^\alpha = v^s(w_t^{s-1,s}) + D_{s+1}(w_t^{s-1,s})^\alpha + U_{s-1}(w_t^{s-1,s})^\beta \; - c^{s-1,s}$$

$$v^s\left(w_t^{s,s-1}\right) + D_{s+1}\left(w_t^{s,s-1}\right)^\alpha + U_{s-1}\left(w_t^{s,s-1}\right)^\beta = v^{s-1}\left(w_t^{s,s-1}\right) + D_s\left(w_t^{s,s-1}\right)^\alpha - c^{s,s-1}$$

$$v^s\left(w_t^{s,s+1}\right) + D_{s+1}\left(w_t^{s,s+1}\right)^\alpha + U_{s-1}\left(w_t^{s,s+1}\right)^\beta = v^{s+1}\left(w_t^{s,s+1}\right) + U_s\left(w_t^{s,s+1}\right)^\beta - c^{s,s+1}$$

$$v^{s+1}(w_t^{s+1,s}) + U_s(w_t^{s+1,s})^\beta = v^s(w_t^{s+1,s}) + D_{s+1}(w_t^{s+1,s})^\alpha + U_{s-1}(w_t^{s+1,s})^\beta \; - c^{s+1,s}$$

And making some arrangements, we obtain:

$$(D_s - D_{s+1})\left(w_t^{s,s-1}\right)^\alpha - U_{s-1}\left(w_t^{s,s-1}\right)^\beta =$$

$$= \frac{w_t^{s,s-1}(\delta^s - \delta^{s-1})}{r - \mu_w}(1 - e^{-(r-\mu_w)(T-t)}) + \frac{c^{s-1} - c^s}{r}(1 - e^{-r(T-t)}) + c^{s,s-1}$$

$$D_{s+1}\left(w_t^{s,s+1}\right)^\alpha + (U_{s-1} - U_s)\left(w_t^{s,s+1}\right)^\beta =$$

$$= \frac{w_t^{s,s+1}(\delta^{s+1} - \delta^s)}{r - \mu_w}(1 - e^{-(r-\mu_w)(T-t)}) + \frac{c^s - c^{s+1}}{r}(1 - e^{-r(T-t)}) - c^{s,s+1}$$

$$(D_s - D_{s+1})\,(w_t^{s-1,s})^\alpha - U_{s-1}(w_t^{s-1,s})^\beta =$$

$$= \frac{w_t^{s-1,s}(\delta^s - \delta^{s-1})}{r - \mu_w}(1 - e^{-(r-\mu_w)(T-t)}) + \frac{c^{s-1} - c^s}{r}(1 - e^{-r(T-t)}) - c^{s-1,s}$$

$$D_{s+1}(w_t^{s+1,s})^\alpha + (U_{s-1} - U_s)\,(w_t^{s+1,s})^\beta =$$

$$= \frac{w_t^{s+1,s}(\delta^s - \delta^{s+1})}{r - \mu_w}(1 - e^{-(r-\mu_w)(T-t)}) + \frac{c^{s+1} - c^s}{r}(1 - e^{-r(T-t)}) + c^{s+1,s}$$

And the smooth pasting conditions are:

$$(D_s - D_{s+1})\,\alpha\left(w_t^{s,s-1}\right)^{\alpha-1} - \beta U_{s-1}\left(w_t^{s,s-1}\right)^{\beta-1} = \frac{\delta^s - \delta^{s-1}}{r - \mu_w} \times (1 - e^{-(r-\mu_w)(T-t)})$$

$$D_{s+1}\alpha\left(w_t^{s,s+1}\right)^{\alpha-1} + (U_{s-1} - U_s)\,\beta\left(w_t^{s,s+1}\right)^{\beta-1} = \frac{\delta^{s+1} - \delta^s}{r - \mu_w} \times (1 - e^{-(r-\mu_w)(T-t)})$$

$$(D_s - D_{s+1})\,\alpha(w_t^{s-1,s})^{\alpha-1} - U_{s-1}\beta(w_t^{s-1,s})^{\beta-1} = \frac{\delta^s - \delta^{s-1}}{r - \mu_w} \times (1 - e^{-(r-\mu_w)(T-t)})$$

$$D_{s+1}\alpha(w_t^{s+1,s})^{\alpha-1} + (U_{s-1} - U_s)\,\beta(w_t^{s+1,s})^{\beta-1} = \frac{\delta^s - \delta^{s+1}}{r - \mu_w} \times (1 - e^{-(r-\mu_w)(T-t)})$$

5. Proof of Equation (25)

To determine $E(\tau)$ we use the fact that the probability of no event occurs in interval $(0, \tau)$ is $e^{-\lambda\tau}$.Therefore the probability that an event occurs on the short interval $(\tau, \tau + d\tau)$ is $\tau e^{-\lambda\tau}\lambda d\tau$. Then we have:

$$
\begin{aligned}
E(\tau) &= \int_0^T \tau e^{-\lambda\tau}\lambda d\tau = \lambda\int_0^T \tau e^{-\lambda\tau} d\tau \text{ We integrate by parts:} \\
E(\tau) &= \lambda\left\{\left[\tau\frac{e^{-\lambda\tau}}{-\lambda}\right]_0^T - \int_0^T \frac{e^{-\lambda\tau}}{-\lambda}d\tau\right\} \\
E(\tau) &= \lambda\left\{\left[\tau\frac{e^{-\lambda\tau}}{-\lambda}\right]_0^T - \left[\frac{e^{-\lambda\tau}}{\lambda^2}\right]_0^T\right\} \\
E(\tau) &= E(\tau) = \frac{1}{\lambda} - T\times e^{-\lambda T} - \frac{e^{-\lambda T}}{\lambda}
\end{aligned}
$$

6. Proof of Equation (26)

Knowing that $E\left[w_t\right] = w_0 e^{[\mu+\gamma\lambda\theta-(1-\gamma)\lambda\theta]t}$,we write the expectation factor as follows:

$$
v^s(w_t) = E\left[\int_t^T (\delta^s \times w_\tau - c^s)e^{-r\tau}d\tau\right] = E\left[\delta^s\times\int_t^T w_\tau e^{-r\tau}d\tau\right] - E\left[\int_t^T c^s e^{-r\tau}d\tau\right]
$$

$$
E\left[\int_t^T w_\tau e^{-r\tau}d\tau\right] = \int_t^T w_0 e^{[\mu+\gamma\lambda\theta-(1-\gamma)\lambda\theta]\tau}e^{-r\tau}d\tau = \int_t^T w_0 e^{[\mu+\gamma\lambda\theta-(1-\gamma)\lambda\theta-r]\tau}d\tau
$$

$$
= \frac{w_0}{r-(\mu+\gamma\lambda\theta-(1-\gamma)\lambda\theta)}(1-e^{-(r-\mu-\gamma\lambda\theta+(1-\gamma)\lambda\theta)(T-t)})
$$

$$
v^s(w_t) = \frac{\delta^s\times w_0}{r-(\mu+\gamma\lambda\theta-(1-\gamma)\lambda\theta)}(1-e^{-(r-\mu-\gamma\lambda\theta+(1-\gamma)\lambda\theta)(T-t)}) - \frac{c^s}{r}(1-e^{-r(T-t)})
$$

7. Proof of Equation (29)

We rewrite the Bellman equation:

$$
rF^{s,s\pm1}(w_t)dt = E\left[dF^{s,s\pm1}(w_t)\right]
$$

Next we expand the right hand side of the Bellman equation as follows:

$$
dF^{s,s\pm1}(w) = \frac{\partial F^{s,s\pm1}(w)}{\partial t}dt + \frac{\partial F^{s,s\pm1}(w)}{\partial w}dw
$$

>From the stochastic process GBM+Poisson jump, $dw = \mu w dt + \sigma w d\varepsilon + w dq$ and because $F^{s,s\pm1}(w)$ is not a function depending explicitly on time, we have $\frac{\partial F(w)}{\partial t} = 0$.

The second term $\frac{\partial F^{s,s\pm1}(w)}{\partial w}dw$ can be separated in two parts, one due to the geometric Brownian motion component $(\mu w dt + \sigma w d\varepsilon)$, for which we can apply the standard version of Ito's Lemma, and the second due to the jump process wdq. Thus,

-for the GBM part:

$$dF^{s,s\pm1}(w) = \frac{1}{2}F_{ww}^{s,s\pm1}(w)dw^2 + F_w^{s,s\pm1}(w)dw$$

-for the Poisson jump part:

$$dF^{s,s\pm1}(w) = F^{s,s\pm1}(w + \gamma\theta w - (1-\gamma)\theta w) - F^{s,s\pm1}(w) = F^{s,s\pm1}((1 + \gamma\lambda\theta - (1-\gamma)\lambda\theta)w) - F^{s,s\pm1}(w)$$

$$E\left[dF^{s,s\pm1}(w)\right] = \underbrace{\left[dF^{s,s\pm1}(w)\right]}_{GBM} + \underbrace{\lambda dt\left[dF^{s,s\pm1}(w)\right]}_{Poisson}$$

$$= \left[F_w^{s,s\pm1}E(dw) + \frac{1}{2}F_{ww}^{s,s\pm1}E(dw^2)\right] + \lambda dt\left[F^{s,s\pm1}(w + \gamma\theta w - (1-\gamma)\theta w) - F^{s,s\pm1}(w)\right]$$

$$= \left[F_w^{s,s\pm1}\mu w dt + \frac{1}{2}F_{ww}^{s,s\pm1}\sigma^2 w^2 dt\right] + \lambda dt\left[F^{s,s\pm1}(1 + \gamma\lambda\theta - (1-\gamma)\lambda\theta)w) - F^{s,s\pm1}(w)\right]$$

$$E\left[dF(w)\right] = \frac{1}{2}F_{ww}^{s,s\pm1}\sigma^2 w^2 dt + F_w^{s,s\pm1}\mu w dt + \lambda dt\left[F^{s,s\pm1}(1 + \gamma\lambda\theta - (1-\gamma)\lambda\theta)w) - F^{s,s\pm1}(w)\right]$$

We substitute in the Bellman equation:

$$rF^{s,s\pm1}(w)dt = \frac{1}{2}F_{ww}^{s,s\pm1}(w)\sigma^2 w^2 dt + F_w^{s,s\pm1}(w)\mu w dt + \lambda dt\left[F^{s,s\pm1}(1 + \gamma\lambda\theta - (1-\gamma)\lambda\theta)w) - F^{s,s\pm1}(w)\right]$$

$$rF^{s,s\pm1}(w) = \frac{1}{2}F_{ww}^{s,s\pm1}(w)\sigma^2 w^2 + F_w^{s,s\pm1}(w)\mu w + \lambda\left[F^{s,s\pm1}(1 + \gamma\lambda\theta - (1-\gamma)\lambda\theta)w) - F^{s,s\pm1}(w)\right]$$

$$\frac{1}{2}F_{ww}^{s,s\pm1}(w)\sigma^2 w^2 + F_w^{s,s\pm1}(w)\mu w - (r+\lambda)F^{s,s\pm1}(w) + \lambda F^{s,s\pm1}(1 + \gamma\lambda\theta - (1-\gamma)\lambda\theta)w) = 0$$

admitting the general solution:

$$F^{s,s\pm1}(w) = Dw^\alpha + Uw^\beta$$

Starting from the solution of the differential equation, we calculate the derivatives of $F(w)$ and we replace them in the equation:

$$F(w) = Dw^\beta; F_w(w) = \beta Dw^{\beta-1}; F_{ww}(w) = \beta\left(\beta-1\right)Dw^{\beta-2}$$

$$\frac{1}{2}\left[\beta\left(\beta-1\right)Dw^{\beta-2}\right]\sigma^2 w^2 + \left[\beta Dw^{\beta-1}\right]\mu w - (r+\lambda)Dw^\beta + \lambda D\left[(1 + \gamma\lambda\theta - (1-\gamma)\lambda\theta)w)\right]^\beta = 0$$

$$Dw^\beta\left[\frac{1}{2}\left[\beta\left(\beta-1\right)\right]\sigma^2 + \beta\mu + \lambda\left(1 + \gamma\lambda\theta - (1-\gamma)\lambda\theta\right)^\beta - (r+\lambda)\right] = 0$$

Thus, β is the solution of the equation:

$$\frac{1}{2}\left[\beta\left(\beta-1\right)\right]\sigma^2 + \beta\mu + \lambda\left(1 + \gamma\lambda\theta - (1-\gamma)\lambda\theta\right)^\beta - (r+\lambda) = 0$$

8. Switching options values in GBM-Poisson (negative jumps) case

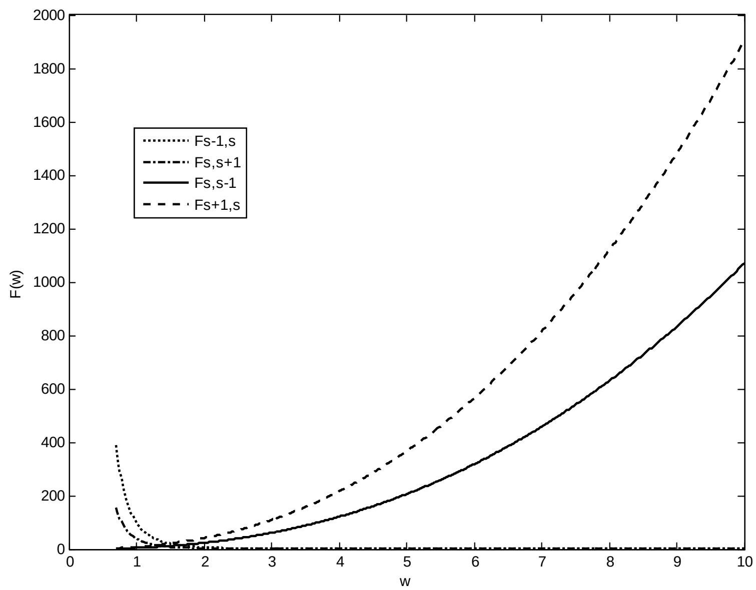

Figure 11: Switching options values in GBM-Poisson (negative jumps) case

References

[1] Aparicio, L.,et al (ed.)(2010), "Making nuclear waste governable-Deep underground disposal and the challenge of reversibility", Springer, ANDRA.

[2] Bethuyne, G., (2001), "The timing of technology adoption by a cost-minimizing firm", Journal of Economics, vol.76, no.2, p.123-154.

[3] Brennan, M.J., Schwartz E.S.(1985),"Evaluating Natural Resource Investment", Journal of Business, vol.58, no 2, p.135-157.

[4] Childs, P.D., S.H. Ott, and A.J. Triantis. (1998), "Capital Budgeting for Interrelated Projects: A Real Options Approach", Journal of Financial and Quantitative Analysis 33.3, p. 305-334.

[5] Cortazar, G., Schwartz E.S., Casassus J. (2001) "Optimal exploration investments under price and geological-technical uncertainty: a real options model",R & D Management, 31, p. 181-189.

[6] Dixit, A.K. , Pindyck, R.S. (1994) ,"Investment under Uncertainty", Princeton University Press, Princeton, NJ.

[7] Dixit, A.K.(1989) , "Entry and exit decisions under uncertainty", Journal of Political Economy, p. 620–638.

[8] Farzin, Y., Huisman, K. J. M., Kort, P. M. (1998), "Optimal timing of technology adoption", Journal of Economic Dynamics and Control 22, p. 779-799.

[9] Gollier, C. et Devezeaux de Lavergne, J-G. (2001), "Analyse quantitative de la réversibilité du stockage des déchets nucléaires : une valorisation des déchets", Economie et Prévision, no149, p.1-13.

[10] Gollier, C., (2007), "The consumption-based determinants of the term structure of discount rates", Mathematics and Financial Economics, 1 (2), p. 81-102.

[11] Grenadier, S., Weiss, A.M. (1997),"Investment in Technological Innovations: An Options Pricing Approach".Journal of Financial Economics, 44, p. 397-416.

[12] Harrison, M.J. (1985)" Brownian Motion and Stochastic Flow Systems", John Wiley & Sons.

[13] Heraud, J.A.,Ionescu, O. (2010), "The French Framework of Nuclear Waste Repository: the contribution of economic analysis", Working paper BETA.

[14] Kulatilaka, N. (1993),"The value of flexibility: The case of a dual-fuel industrial steam boiler",Financial Management 22(3), p. 271-280.

[15] Loubergé, H., Chesney, M., Villeneuve, S.(2001), "Long-term risk management of radioactive waste: a real option approach", Journal of Economic Dynamics & Control”, no 27, p. 5.

[16] McDonald, R. and Siegel, D. (1986) "The Value of Waiting to Invest", Quarterly Journal of Economics, 10, November, p. 707 – 727.

[17] Merton, R. C. (1976), "Option Pricing When Underlying Stock Returns are Discontinuous", Journal of Financial Economics, 3, p.125-144.

[18] Murto, P. (2007), "Timing of Investment under Technological and Revenue-Related Uncertainties", Journal of Economic Dynamics and Control, 31, p. 1473–1497.

[19] Myers, S.C. (1977),"Determinants of Corporate Borrowing", Journal of Financial Economics, no.5, November, p.147-175.

[20] Schwartz, E.S. , Trigeorgis, L. (2001), "Real Options and Investment under Uncertainty", MIT Press, Cambridge, MA.

[21] Trigeorgis, L. (1999), "Real Options: Managerial Flexibility and Strategy in Resource allocation", The MIT Press, Cambridge, Massachusetts.