Optimal Investment Strategies for Product-Flexible and Dedicated Manufacturing Systems under Demand Uncertainty *

VERENA HAGSPIEL[1]

[1]*Department of Econometrics and Operations Research and CentER, Tilburg University*

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Abstract

This paper studies the optimal investment strategy of a firm having the managerial freedom to acquire both flexible or dedicated production capacity. Flexible Capacity is more expensive but allows the firm to switch costlessly between products and handle changes in relative volumes among products in a given product mix. Dedicated capacities restrict to manufacture one specific product but for lower acquisition cost. Specifically, I model the investment decision of a monopolistic firm selling two products in a market characterized by price-dependent and uncertain demand, in a continuous time setting. The paper takes a real option approach to consider optimal capacity investment decisions under uncertainty. Besides the timing of the investment, the firm can choose the optimal capacity level and is free to undergo investment in flexible or dedicated capacity. The sensitivity of the firm's optimal capacity investment considering flexible and dedicated capacity to key problem components is analyzed. The main focus is on the effect of demand variability - a key driver of flexibility - as well as substitutability and product profitability effects.

I find that if uncertainty goes up the firm invests later in higher capacity. Flexibility especially pays off when uncertainty is high, substitutability low, and profit levels between the two products are substantially different. In the flexible case, under high demand the firm just produces the most profitable product, if demand is low the firm produces both products to make total market demand bigger. In the dedicated case the firm invests in both capacities only if the substitutability rate is low and profitability

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of both products high enough. Otherwise, it restricts investment to one dedicated capacity for the more profitable product.

1 Introduction

Flexibility in manufacturing operations is becoming increasingly more important to industrial firms. Increasing market demand volatility, internationalization of markets and competition, as well as shorter product life cycles pose new challenges for companies. Since the investment cost in flexible capacity mostly exceeds the investment cost of dedicated capacity, firms need to know how much this flexibility is worth for them. This work will focus on the effect of demand volatility on firm's investment decisions and the value of product flexibility, which is widely cited as one of the most (if not the most) strategically important flexibility types (see for example Goyal and Netessine (2006), Jordan and Graves (1995)).

The automotive industry is a good example for an industry where manufacturers' decisions on investing in production capacity and on the optimal level of flexibility are critical. On one hand, expanding already installed capacity is very expensive (Andreou (1990)) and therefore, the installed capacity must be sufficient for the whole life cycle of the product and easy adaptable to new product lines. On the other hand, the profitability of the products are threatened by low utilization of capacity as well as under-capacity. Japanese carmarkers very early implemented the concept of flexibility, which constituted them a significant advantage compared to their US and European competitors that traditionally built plants that were dedicated to producing a single car model. Tardy, also the European and American car industry started to activate in terms of flexibility. The BMW group for example recently advertised that a new production factory in Leipzig added new production capacity with a high level of flexibility[1]. Despite this recent approach of European and US car manufacturers to strive for more manufacturing flexibility, Japanese companies still lead in this aspect, which is according to Goyal et al. (2006) "an advantage that is at least partially responsible for the increasing market share of the Japanese carmakers".

The investment in and management of flexible capacity has received significant attention in the operations literature. While early research in this field has focused on scenarios with exogenously given prices and static time models, recent papers extend this approach including responsive pricing and multi-stage decision problems. These multi-stage decision models are built up in the structure of: first invest in capacity, then receive additional information and finally exploit capacity optimally according to revealed information. While this structure shows a characteristic of real option models, the models are restricted to one-period models

[1]The BMW groups states that the production program includes not just the full range of cars of the BMW 1 Series (three-door, Coup and Convertible) but also the BMW X1. At the Volkswagen plant in Zwickau (Germany) Passat and Golf run off the same assembly line. Additional to the flexibility of the assembly line also the supply to the production line is fully flexible. This allows to change between the models and different interior equipment without any adaption costs neither time lags. See http://www.bmwgroup.com/d/nav/index.html?http://www.bmwgroup.com/d/0_0_www_bmwgroup_com/produktion/produktionsnetzwerk/produktionsstandorte/produktionsstandorte.shtml

not taking into account timing. This work applies a continuous time setting as done in the real options literature which allows to gain insight in the optimal timing of the firm's investment decision. The theory of real options explored flexibility mainly as it is related to the timing structure of capacity acquisition and did not deal with technologies that exhibit flexibility per se. This work presents a model which takes into account both aspects: flexibility in timing and investment in flexible capacity.

Specifically, this paper studies the investment decision of a monopolistic firm having the managerial freedom to acquire flexible or dedicated manufacturing capacity, in a continuous time setting. Flexible capacity allows the firm to manufacture all of its products with the same production facility while dedicated capacity restricts to one product. Since product flexibility has not been clearly define in the literature, for the purpose of this study product flexibility is defined as a system's ability to switch costlessly between products, and handle changes in relative volumes among products. Products differ in substitutability and profitability in the market. The firm wants to protect efficiently against uncertainty in demand for all of its products. It can choose the timing as well as the quantity of the investment and is free to invest in flexible or dedicated production capacity, choosing the one that leads to the highest expected profit.

I analyze the results with a specific focus on four cases of product combinations. The four cases differ regarding profitability and substitutability rate between the products: (1) The first case considers a product combination of two almost similarly profitable products with a low substitutability rate. The car models Passat and Golf of Volkswagen are an example for such a product combination. In the Volkswagen plant in Mosel these two car models are produced at an assembly line that is fully flexible in switching between the two models without adaption time lags nor costs. (2) National-brand manufacturers that additional to their brand product also produce private label products, are an example for firms that produce two good substitutable products for the same market while producing one product (their own manufacturer brand) is more profitable for them than manufacturing the second product for a private label retailer. This describes an example for the second case of a product combination analyzed with highly substitutable goods that substantially differ in profit. Danone, the famous French food-products corporation for example, produces a variation of their popular cream cheese dessert "Fruchtzwerge" also for the private label product "Desira" of Germany's biggest discounter Aldi[2]. (3) The third case depicts the scenario of a firm producing two products for the same market that are almost equally profitable and highly substitutable. For brand manufacturers with less successful national brands, so called B and C brands, the production of private labels can be almost as profitable as producing their own brand product. Concorp group, a Dutch confectionery company for example states publicly that they do produce for dual branding[3]. (4) The fourth case considers a firm selling

[2]See on p. 39 in 'Aldi - Welche Marke Steckt Dahinter? 100 Aldi-Top-Artikel und Ihre Prominenten Hersteller.' Muenchen: Suedwest Verlag by Schneider, Martina.

[3]Concorp group states on its website that they "...build brands and deliver private label concepts with added value in all segments of selected national and international confectionery markets." Concorp group produces candy foam and boiled sweets for three different brand names in its production site in Waddinxveen. See http //www.concorp.nl/international/pdf/concorp_international_overview.pdf

two products with a low substitutability rate and a substantial difference in profitability, in the same market. For an example think of a technology company producing a newly established touch screen mobile and a less successful obsolescent model in the same production facility.

This work focuses on the effect of demand variability - a key driver of flexibility - together with substitutability and product profitability effects. I show that in the flexible case, under high demand the firm just produces the most profitable product, if demand is low the firm produces both products to make total demand bigger. Comparing the optimal flexible investment strategies for the previously mentioned cases the firm selling two products with a low substitutability rate and high profitability difference invests in significantly higher capacity. Capacity size is growing more than proportionally with the uncertainty level. This confirms the intuition that a firm producing two almost equally profitable products with a low substitutability rate profits the most by the down size potential to increase the market size by producing both products. In the dedicated case the firm invests in both capacities if substitutability rate is low and profitability of both products high enough. In all other cases the firm decides to ignore demand for one product in the market and installs one dedicated capacity for the more profitable product. In this case the firm can just gain from the downside potential when demand levels are very low and therefore the negative effect of restriction to produce up to full capacity once it has installed capacity for both products, is dominating. For both, dedicated and flexible capacity investment, the result holds that the firm invests later in higher capacity if demand uncertainty increases. A result also obtained for production flexible capacity investment by Hagspiel et al. (2010) and Dangl (1999).

In order to study the value of flexibility, I consider as a benchmark the situation in which a firm relies on maximal two dedicated capacities rather than on one flexible capacity. Flexibility especially pays off when uncertainty is high, substitutability low, and profit levels between the two products are substantially different. In this case the flexible firm has the possibility to increase its total market demand if demand falls low by including the production of the less profitable second product. The dedicated firm on the other hand relinquishes production of one product completely by acquiring just one dedicated capacity for the more profitable product.

Two streams of literature are relevant to this study: the first considers the issue of product flexibility from an operations management perspective. The issue of resource flexibility has become a significant interest in the management science community beginning of the nineties, following the increasing viability of flexible, computer-controlled manufacturing systems. From the operational management literature this work is closely related to the relatively recent stream of papers about resource flexibility initiated by work of Fine and Freud (1989). Fine and Freud derive necessary and sufficient conditions for the acquisition of flexible capacity that are based on a two-stage convex quadratic program. In the first stage a technology investment decision is made. After observing demand realization, an optimal production decision is made at the second stage. Inspired by Fine and Freud, Van Mieghem presents closely related work that disproves Fine and Freud's claim that flexible capacity would not provide additional value when product demands are perfectly positively correlated. They show that in addition to its adaptability to demand mix changes, product-flexible technology provides another opportunity for revenue improvement through its ability to exploit differentials in price (margin) mix. He argues that product flexibility generates an option to produce and sell more of highly profitable products at the expense of less profitable products and show that this option can remain valuable even with perfectly positively correlated product demand.

Most closely related to my work are two recently published papers of Chod & Rudi (2005) and Bish & Wang (2004), who study the resource investment decision of two-product, price setting firm that operates in a monopolistic setting. Chod & and Rudi look at the effect of demand variability and demand correlation on the optimal flexible resource investment decision and show that expected profit is increasing in variability and decreasing in the correlation of normally distributed demand. Bish & Wang's model is more general than that one of Chod & and Rudi in allowing the firm to invest in flexible and dedicated resources at the same time but they do not include cross-price effects. Both previously mentioned papers present two-stage models that allows them to gain insight in the optimal resource size and allocation but deprives the timing aspect of investment decisions. Unlike these papers, I focus on an economic environment where uncertainty in demand of the two products arises from one single market. The investment decision is made facing uncertainty in the general economic situation. This becomes especially interesting for an industry that just recently witnessed one of the biggest economic crises in history that affected the whole global market.

Applying a continuous time setting allows me to gain insight in the optimal timing of the firm's investment strategy. For evaluating investment decisions that have the following three characteristics: (1) the investment considered is irreversible, (2) their is uncertainty about future rewards and (3) a leeway about timing of investment, the theory of real options is used to evaluate such investment decisions. But real options theory explores flexibility mainly as it related to the timing structure of capacity or information acquisition or commitment of resources: that means that the firm looses flexibility when it makes an irreversible commitment. Most papers do not deal with technologies that exhibit flexibility per se. Now that more and more firms undergo investment in flexible capacity because it appears for them to be a necessary tool to hedge against highly volatile demand, it is important to develop these models further with a special attention to include the ability of flexible capacity. This paper wants to take a crucial step in this direction. I explicitly consider the use of a flexible (product-) technology.

Though there are a few real option papers that deal with investments in flexible capacity. These papers consider product flexible capacity by evaluating the option to switch between different products but do not look at capacity that can handle more products at the same time. Early approaches have been presented by Kulatilaka (1988) and Triantis and Hodder (1989). Triantis and Hodder evaluate product(-mix) flexibility based on option principles. Kulatilaka (1988) applies option pricing principles to the same problem using a stochastic dynamic programming formulation that includes clostly switching between modes of operation. Andreou (1990) published a more applied study associated with the General Motors Research Laboratories that focuses on the economic evaluation of product flexibility. He presents a financial model for calculating the dollar value of flexible plant capacity for two products under conditions of uncertain market demand. While these papers do evaluate investment in flexible technology based on option theory, the do not include the timing decision and capacity choice.

This work was inspired by the increasing interest in development of the real options theory regarding technological flexibility shown by the operations and production management sector . Bengtsson (2001) presented a work that relates the real options literature to manufacturing flexibility from an industrial engineering/production management perspective. He refers to product flexibility as one of the flexibility types that have not been treated as real options yet. While his work addresses a wide range of manufacturing flexibility, Bengtsson and Olhager (2002) use real options theory to evaluate one specific type, i.e. product-mix flexibility, in a real case analysis. Their main focus is on solving for the value of a production system with multiple products which is applied to real case data, while the timing or capacity size decisions are not considered. Furthermore, there is an increasing number of real data cases and empirical analysis in this area. Two recent papers are for example Goyal et al. (2006) or Fleischmann et al. (2006), both focusing on the automotive industry.

The paper is organized as follows. The next section presents the general model and solves the optimization problems for the flexible capacity and dedicated capacity case. The optimal investment triggers for size and time of investment are derived. The first part of Section 3 analyzes the capacity and timing decision for flexible capacity investment and shows how investment timing and size are affected by demand uncertainty. The second part concentrates on analyzing investment in dedicated production capacity. Section 4 studies the optimal investment strategy of a firm having the option to choose between flexible and dedicated capacity investment and quantifies the value of flexibility. Section 5 concludes.

2 Model

Consider a firm that produces two products, indicated by product A and B. The firm has to decide about the optimal capacity investment. This involves three decisions: when to invest, the size of the capacity and in which type of capacity to invest. The firm can invest in maximal two dedicated capacities, each of which can produce only one product, or in a more expensive, flexible production capacity, which can produce both products.

The firm is uncertain about future demand where the inverse demand function are assumed to be linear. The inverse demand functions for the two products are given by

$$p_A(\theta, q_A, q_B) = \theta - q_A - \gamma q_B,$$
$$p_B(\theta, q_B, q_A) = \alpha\theta - q_B - \gamma q_A,$$

where the demand intercept θ follows the geometric Brownian motion

$$d\theta_t = \mu\theta_t dt + \sigma\theta_t dW_t. \tag{1}$$

In this expression μ is a constant representing the trend, σ is the uncertainty parameter and dW_t is the increment of a Wiener process implying that it is independently and normally distributed with mean 0 and variance dt. I often refer to the uncertainty in demand intercepts simply as "demand uncertainty" in this paper. $\gamma \in (-1, 1)$ is the product substitutability parameter, and $\gamma > 0$ ($\gamma < 0$) signifies that the products are substitutes (complements). Since products made by the same flexible resource tend not to be complements, most applications are characterized by a nonnegative γ. Therefore, this work will focus on the case of the products being substitutes. The two products are assumed to be sold in the same market. Product A is the more profitable product in this market, i.e. $\alpha < 1$. $\alpha \in (0, 1)$ is referred to as the profitability parameter of product B. Denote production quantity of product A (B) at time time by $q_{t,A}$ ($q_{t,B}$). From now on I drop the time subscript whenever there can be no misunderstanding. For simplicity variable production costs are not considered yet. It follows that the profit flow is defined by

$$\Pi(\theta) = \max_{q_A, q_B} [p_A q_A + p_B q_B].$$

For simplicity, variable cost of production are not included in the model. Total production output, i.e. $q = q_A + q_B$, is restricted to be up to full capacity. This means that the firm utilizes all of its available resources after investment. The flexible capacity is denoted by K_F and the dedicated capacities by K_{D_A} and K_{D_B}, respectively. The investment cost are sunk and assumed to be linear (for the same assumption see for example, Fine & Freud (1990), Van Mieghem (1998) or Chod & Rudi (2005)). Let c_i denote the unit cost of investing in resource K_i, $i = F, D_A, D_B$, where $c_{D_A}, c_{D_B} < c_F$. After investment the firm always produces up to full capacity, a constraint also referred to as capacity clearance.

2.1 Flexible Capacity

Consider a firm that has to decide about investment in flexible capacity. Flexible capacity allows it to produce both products, A and B, on the same production line. It decides about the optimal time to invest and the optimal capacity size invested in, considering that it has to produce always up to full capacity after the moment of investment. The optimal output rate for the two products q_A^* and q_B^*, respectively, is determined by maximizing the profit flow considering the capacity clearance constraint ($q_A + q_B = K_F$) and the upper and lower boundaries for each of the two output rates, $0 \leq q_B, q_A \leq K_F$. This gives

$$q_A^* = \begin{cases} \frac{\theta(1-\alpha)}{4(1-\gamma)} + \frac{K_F}{2} & \text{for } \theta < \frac{2(1-\gamma)}{(1-\alpha)} K_F \\ K_F & \text{for } \theta > \frac{2(1-\gamma)}{(1-\alpha)} K_F \end{cases}$$

$$q_B^* = \begin{cases} -\frac{\theta(1-\alpha)}{4(1-\gamma)} + \frac{K_F}{2} & \text{for } \theta < \frac{2(1-\gamma)}{(1-\alpha)} K_F \\ 0 & \text{for } \theta > \frac{2(1-\gamma)}{(1-\alpha)} K_F \end{cases}$$

where we denote the boundary $\frac{2(1-\gamma)}{(1-\alpha)} K_F$ as $\hat{\theta}$. For low demand, $\theta \in [0, \hat{\theta})$, the firm will produce both products . If demand increases, i.e. $\theta \in [\hat{\theta}, \infty)$, the firm will switch to use full capacity K_F for production of the more profitable product A and suspend production of product B. Expressions (2) and (2) imply that the profit flow is given by

$$\Pi(\theta) = \begin{cases} \frac{(1-\alpha)^2}{8(1-\gamma)}\theta^2 + \frac{(1+\alpha)}{2}\theta K_F - \frac{(1+\gamma)}{2}K_F^2 & \text{for } \theta < \frac{2(1-\gamma)}{(1-\alpha)}K_F \\ (\theta - K_F)\, K_F & \text{for } \theta > \frac{2(1-\gamma)}{(1-\alpha)}K_F \end{cases} \qquad (2)$$

In order to find the value of this investment project ($V(\theta, K_F)$), the dynamic programming approach is applied. Then this value function must satisfy the Bellman equation

$$V(\theta, K) = \pi(\theta, K)dt + E\left[V(\theta + d\theta, K)e^{-rdt}\right],$$

where r is the (constant) discount rate. Applying Ito's Lemma, substituting and rewriting leads to the differential equation (see, e.g., Dixit and Pindyck (1994))

$$\frac{1}{2}\sigma^2\theta^2\frac{\partial^2 V}{\partial\theta^2} + \mu\theta\frac{\partial V}{\partial\theta} - rV + \Pi(\theta) = 0.$$

Solving this equation for $V(\theta, K)$, considering that we have two different regions, and ruling out bubble solutions, we get the following value of the project:

$$V(\theta) = \begin{cases} A_1\theta^{\beta_1} + a_1\theta^2 + a_2\theta K_F + a_3 K_F^2 & \text{for } \theta < \frac{2(1-\gamma)}{(1-\alpha)}K_F \\ B_2\theta^{\beta_2} + \frac{\theta K_F}{r-\mu} - \frac{K_F^2}{r} & \text{for } \theta > \frac{2(1-\gamma)}{(1-\alpha)}K_F \end{cases} \qquad (3)$$

with $a_1 = \frac{(1-\alpha)^2}{8(1-\gamma)(r-2\mu-\sigma^2)}$, $a_2 = \frac{(1+\alpha)}{2(r-\mu)}$ and $a_3 = -\frac{(1+\gamma)}{2r}$. $V(\theta)$ must be continuously differentiable across the boundary $\theta_K = \frac{2(1-\gamma)}{(1-\alpha)}K_F$.

Using the fact that $V(\theta, K_F)$ must be continuously differentiable across the boundary θ_K one can derive the constants A_1 and B_2:

$$\begin{aligned} A_1(K_F) &= K_F^{2-\beta_1}\frac{1}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_1}(1-\gamma)\left[\frac{(2-\beta_2)}{2(r-2\mu-\sigma^2)} - \frac{(1-\beta_2)}{(r-\mu)} - \frac{\beta_2}{2r}\right] \\ B_2(K_F) &= K_F^{2-\beta_2}\frac{1}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_2}(1-\gamma)\left[\frac{(2-\beta_1)}{2(r-2\mu-\sigma^2)} - \frac{(1-\beta_1)}{r-\mu} - \frac{\beta_1}{2r}\right] \end{aligned}$$

Corollary 1 in Appendix A shows that A_1 is negative for all parameter values and B_2 positive.

The value of the investment project in the region $\theta < \theta_K$ consists of four terms where the last three terms constitute the cash flow generated by the sales. The first term $A_1(K_F)\,\theta^{\beta_1}$, which is negative, corrects for the fact that in an mathematically optimal case the production quantity of product B would turn negative for $\theta > \hat{\theta}$. Economically this does not make sense and therefore the output quantity is constrained by $q_B^* \geq 0$. The absolute value of this term decreases with θ.

In the region $\theta \geq \hat{\theta}$, demand is that large that the firm uses all of its installed capacity to produce the more profitable product A. This generates a discounted cash flow stream that is reflected in the second and third term of the value of the investment project associated with this region. The first term, $B_2(K_F)\,\theta^{\beta_2}$, describes the option value that accounts for the additional possibility that in case demand decreases the company can switch its production back to two products and therefore gains revenue. This option value is decreasing for large θ.

Knowing the value of the project, $V(\theta, K_F)$, one is able to derive the optimal investment strategy. In general the procedure is as follows. First, the optimal capacity choice $K_F^*(\theta)$ is determined for a given level of θ by setting the marginal value of the project equal to the marginal investment costs c_F. Second, one derives the optimal investment threshold θ^*. For this demand level θ^* it holds that the firm is indifferent between investment and waiting with investment. Investment (waiting) is optimal for a θ being larger (lower) than θ^*.

Investment can take place either in region I or in region II. Investing while $\theta < \hat{\theta}$ means that the firm uses the capacity invested in, to produce both products right after the investment has been undertaken, while investing in region $\theta \geq \hat{\theta}$ implies that the full capacity level is used to produce only product A

The following proposition provides equations that implicitly determine the threshold θ_F^* and the corresponding capacity level $K_F^*(\theta^*)$ in each of the two cases. The optimal investment decision corresponds to the case that provides the largest expected value of the investment project.

Proposition 1 *Concerning the firm's investment policy there are two possibilities:*

1. *Given that the firm produces a positive amount of both products right after the investment moment, the optimal capacity level $K_F^*(\theta)$ is implicitly determined by*

$$\frac{\partial A_1}{\partial K_F}\theta^{\beta_1} + a_2\theta + 2a_3K_F^*(\theta) - c_F = 0 \tag{4}$$

In case the obtained $K^*(\theta)$ is such that from the resulting production quantity it follows that it is not an interior solution, the optimal capacity is replaced by the boundary solution

$$K^*(\theta) = \theta\frac{(1-\alpha)}{2(1-\gamma)}. \tag{5}$$

If the solution of equation (4) is negative, the optimal capacity is set to zero, i.e.

$$K^*(\theta) = 0. \tag{6}$$

The investment threshold θ^* is implicitly determined by

$$a_1(\beta_1 - 2)\theta^{*2} + a_2(\beta_1 - 1)\theta^* K_F^*(\theta^*) + \beta_1 a_3 K_F^*(\theta^*)^2 - \beta_1 c_F K_F^*(\theta^*) = 0 \tag{7}$$

2. *Given that the firm uses full capacity to produce the more profitable products A right after the investment moment, the optimal capacity level $K_F^*(\theta)$ is implicitly determined by*

$$\frac{\partial B_2}{\partial K_F}\theta^{\beta_2} + \frac{\theta}{r-\mu} - \frac{2}{r}K_F^*(\theta) - c_F = 0. \tag{8}$$

In case the obtained $K^*(\theta)$ does not constitute an interior solution the optimal capacity is replaced by the boundary solution (5) If the solution of (8) is negative the optimal capacity is set to zero.

The investment threshold θ^* *is implicitly determined by*

$$B_2\theta^{*\beta_2}(\beta_1-\beta_2)+\frac{\theta^* K_F^*(\theta^*)}{r-\mu}(\beta_1-1)-\beta_1\frac{K_F^*(\theta^*)^2}{r}-\beta_1 c_F K_F^*(\theta^*)=0 \tag{9}$$

Out of these two possibilities the firm chooses the one that gives the highest expected value of the project discounted back to an initial demand intercept level θ_0*, which is given by* $\left(\frac{\theta_0}{\theta_F^*}\right)^{\beta_1} V\left(\theta_F^*, K_F^*(\theta_F^*)\right)$ *.*

2.2 Dedicated Capacity

The difference with the previous section is that the firm has to decide about optimal investment in dedicated capacities. Dedicated capacity can satisfy only one product. It is assumed that the firm invests at the same time in both capacity levels, provided that the firm wants to produce both products. The firm has to decide when to invest and in how much capacity. The firm has the following two investment options: it can invest in two dedicated production capacities, each of which is restricted to produce one of the two products A and B, respectively. The second option applies if the production of product B is not profitable enough in this market. In that case it is optimal for the firm to ignore demand for product B in the market and invest in just one dedicated production facility for product A. The unit cost of capacity are equal for product A and product B, i.e. $c_{D_A}=c_{D_B}=c_D$. The firm will choose for option with the highest expected project value. After investment the firm has to produce up to full capacity forever.

For the latter case the profit of the firm is given by

$$\Pi=(\theta-q_A)q_A. \tag{10}$$

Considering the capacity clearance constraint, $q_A=K_{D,A}$, the profit flow can be rewritten as a function of dedicated capacity $K_{D,A}$:

$$\Pi=(\theta-K_{D,A})K_{D,A}.$$

Familiar steps lead to the following value of the investment project:

$$V(\theta,K_{D,A})=\frac{\theta K_{D,A}}{r-\mu}-\frac{K_{D,A}^2}{r}.$$

In case that the firm invests at the same time in two dedicated production facilities, for product A and product B, respectively, the profit of the firm $\Pi=p_Aq_A+p_Bq_B$ is maximize w.r.t to the output rates q_A and q_B. Considering the capacity clearance constraints for each product respectively, $q_A=K_{D,A}$ and $q_B=K_{D,B}$, implies that the profit flow is given by

$$\Pi(\theta,K_{D,A},K_{D,B})=(\theta-K_{D,A})K_{D,A}+(\alpha\theta-K_{D,B})K_{D,B}-2\gamma K_{D,A}K_{D,B}.$$

Familiar steps lead to the following project value

$$V(\theta,K_{D,A},K_{D,B})=\frac{\theta}{r-\mu}\left[K_{D,A}+\alpha K_{D,B}\right]-\frac{K_{D,A}^2+2\gamma K_{D,A}K_{D,B}+K_{D,B}^2}{r}.$$

The optimal capacity level for every relevant value of θ is derived by maximizing the project value minus investment cost $c_D(K_{D,A}+K_{D,B})$ for a given demand intercept level θ. In case the obtained $K_{D,i}$ $(i = A, B)$ is negative, the optimal capacity is replaced by the boundary solution $K^*_{D,i}(\theta) = 0$. Knowing the optimal capacity level for all relevant demand levels, the optimal investment threshold θ^* is derived. The following proposition provides the expressions for threshold θ^* and the corresponding capacity level $K^*_D(\theta^*)$ in each of the two cases. The optimal investment decision corresponds to the investment strategy that provides the largest expected project value for the firm.

Proposition 2 *Concerning the firm's investment policy for dedicated capacity there are two possibilities:*

1. *Given that it is optimal for the firm to invest in dedicated production capacity for both products the optimal capacity levels for product A and B, respectively are given by*

$$K^*_{D,A}(\theta) = \begin{cases} 0 & \text{for } \theta < \hat{\theta_A} \\ \frac{\theta r}{2(r-\mu)}\frac{(1-\alpha\gamma)}{1-\gamma^2} - \frac{c_D r}{2(1+\gamma)} & \text{for } \theta > \hat{\theta_A} \end{cases}$$

$$K^*_{D,B}(\theta) = \begin{cases} 0 & \text{for } \theta < \hat{\theta_B} \\ \frac{\theta r}{2(r-\mu)}\frac{(\alpha-\gamma)}{1-\gamma^2} - \frac{c_D r}{2(1+\gamma)} & \text{for } \theta > \hat{\theta_B} \end{cases}$$

where $\hat{\theta_A} = \frac{c_D(r-\mu)(1-\gamma)}{(1-\alpha\gamma)}$ *and* $\hat{\theta_B} = \frac{c_D(r-\mu)(1-\gamma)}{(\alpha-\gamma)}$. *Total optimal dedicated capacity is given by*

$$K^*_D(\theta) = \begin{cases} 0 & \text{for } \theta < \hat{\theta_A} \\ \frac{\theta r}{2(r-\mu)}\frac{(1-\alpha\gamma)}{1-\gamma^2} - \frac{c_D r}{2(1+\gamma)} & \text{for } \hat{\theta_A} < \theta < \hat{\theta_B} \\ \theta\frac{r}{2(r-\mu)}\frac{(1+\alpha)}{(1+\gamma)} - c_D\frac{r}{(1+\gamma)} & \text{for } \hat{\theta_B} < \theta \end{cases} \tag{11}$$

The investment thresholds are given by

$$\theta^*_D = \frac{\beta_1 c_D(r-\mu)(1+\gamma-2\gamma^2)}{\beta_1(1+\alpha\gamma) - 2(1+(\beta_1-1)\gamma^2)} \tag{12}$$

for region $\hat{\theta_A} < \theta < \hat{\theta_B}$ and

$$\theta^*_{D_1} = c_D\left(\frac{\beta_1-1}{\beta_1-2}\right)\left[\frac{(1+\alpha^2-2\alpha\gamma)}{4(1-\gamma)(1+\alpha)(r-\mu)}\right] + \frac{r}{(1+\gamma)(r-\mu)}\sqrt{(\beta_1-1)^2(1+\alpha)^2 - 2\beta_1(\beta_1-2)\frac{1+\alpha^2-2\alpha\gamma}{1-\gamma}}$$

$$\theta^*_{D_2} = c_D\left(\frac{\beta_1-1}{\beta_1-2}\right)\left[\frac{(1+\alpha^2-2\alpha\gamma)}{4(1-\gamma)(1+\alpha)(r-\mu)}\right] - \frac{r}{(1+\gamma)(r-\mu)}\sqrt{(\beta_1-1)^2(1+\alpha)^2 - 2\beta_1(\beta_1-2)\frac{1+\alpha^2-2\alpha\gamma}{1-\gamma}}$$

for region $\theta > \hat{\theta^B}$.

2. *Given that it is more profitable for the firm to invest in just one production capacity for product A ignoring demand for product B, the optimal capacity level for product A is given by*

$$K^*_{D,A}(\theta) = \frac{r}{2(r-\mu)}\theta - \frac{r}{2}c_D. \tag{13}$$

The investment threshold is determined by

$$\theta^*_D = \left(\frac{\beta_1}{\beta_1 - 2}\right)(r - mu)c_D.$$

Out of these choices the firm chooses the one that gives the highest expected value of the project discounted back to an initial demand intercept level θ_0*, which is given by* $\left(\frac{\theta_0}{\theta^*_D}\right)^{\beta_1} V\left(\theta^*_D, K^*_D(\theta^*_D)\right)$ *.*

3 Results

This section presents results for investment in flexible and dedicated capacity independently. The optimal investment strategy of a firm considering both flexible and dedicated capacity, will be analyzed in Section 4.

3.1 Flexible Capacity Investment

As shown in section 3, the flexible firm can either invest in the θ-region I, i.e. $\theta \in [0, \frac{2(1-\gamma)}{1-\alpha}K_F)$, where the firm sets an upper bound for output at the moment of investment and uses this capacity to produce both products right after the moment of investment, or invest in the second θ-region, i.e. $\theta \in [\frac{2(1-\gamma)}{1-\alpha}K_F, \infty)$. Investing in region II means that the firm invests in flexible capacity that is used up to full extend for production of the more profitable product A. Once investment has been made the firm is flexible to adapt the relative production volumes among products to the changing demand level. Facing low demand it will make use of the downside potential to produce both products in order to increase total market size. For high demand levels the firm will use the full available capacity to produce the more profitable product A.

Figure 1 shows an example of two good substitutable products with substitutability parameter $\gamma = 0.8$, assuming a substantial difference in the profitability of the two products. Product B is much less profitable than product A with a profitability parameter of value $\alpha = 0.2$. The other parameter values assumed are $\mu = 0.02$, $\sigma = 0.1$, $r = 0.1$ and $c_F = 100$. Solving equations (4) and (8) the optimal capacity choice for the two regions is derived. Comparing the expected values of the investment project for the two regions it can be concluded that it is optimal to invest in the second region at the investment trigger $\theta^* = 21.147$, provided that the initial θ-value lies below this θ^*. In particular, the firm invests immediately if the current value of θ exceeds θ^*, while otherwise it waits with investment until θ becomes equal to θ^*. The optimal size of acquired capacity is $K^*(\theta^*) = 8.22$. After the investment the firm can adapt the relative production volume among products according to changing demand to receive the highest possible profit. For this specific numerical example the firm will continue using full capacity to produce just product A unless demand drops drastically, i.e. below a θ-bound of $\hat{\theta}_K = 4.11$. Since the two products are good substitutes in the market but producing product B results in significantly less profit for the firm, it is optimal for a wide range of

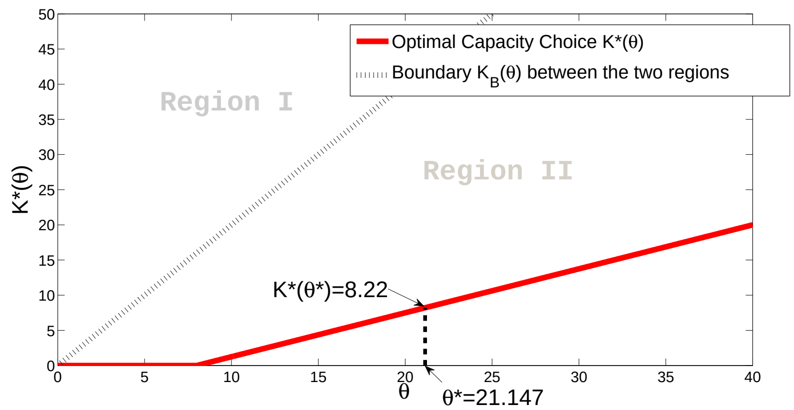


Figure 1: **Case:** $\alpha = 0.2$**,** $\gamma = 0.8$ — Optimal Investment Capacity as a function of demand intercept θ. Region I constitutes the region where the capacity level for a specific demand realization is used to produce a relative volume of both products. Region II describes the $(K(\theta, \theta))$ area where it is optimal for the firm to use full capacity for the production of the more profitable product A. (Parameter values: $r = 0.1$, $\mu = 0.02$, $\sigma = 0.1$ and $c_F = 100$)

demand realization to keep producing just one product, i.e. the more profitable product A, with flexible capacity. Just for very low demand realizations the firm can gain profit from the downside potential to avoid overcapacity by increasing total market size including demand for product B.

Choosing a relatively low substitutability parameter ($\gamma = 0.2$) but a high profitability for product B ($\alpha = 0.8$) increases the value of this downside potential for the firm significantly. In fact, it is optimal for the firm to invest in capacity at investment threshold $\theta_F^* = 22.669$. For this parameter choice the investment moment lays in region II which means that the firm uses purchased capacity to produce both products at the moment of investment. Figure 2 (which illustrates this example) shows the optimal capacity choice as a function demand intercept θ. The figure shows that unlike the previous example, the capacity function $K^*(\theta)$ switches at $\hat{\theta_S} = 9.902$ from optimal investment in region II to optimal investment in region I. Compared to the previous example flexible capacity is very valuable for a firm selling two almost equally profitable products with a low substitutability rate. The firm purchases significantly higher capacity $K^*(\theta^*) = 12.94$. Figure 3 illustrates the advantage of flexible production capacity for a firm selling two almost equally profitable products with a low substitutability rate by means of the following numerical example. The upper plot of Figure 3 shows a simulation of the demand intercept θ with a drift rate of $\mu = 0.02$ and volatility $\sigma = 0.1$ for a time period of 10 years, i.e. $t \in [0, 10]$. The firm will invest as soon as the demand intercept hits the value $\theta_F^* = 22.669$ for the first time, which is (for this specific simulation) after 1.6 years. The second plot (Figure 3) shows the optimal production decision from the moment of investment on. Flexible capacity allows the firm to adapt the relative production volume of the two products, relatively, in order to obtain the highest possible profit facing its capacity constraint of $K_F^*(\theta^*) = 12.94$. For low demand the firm uses full

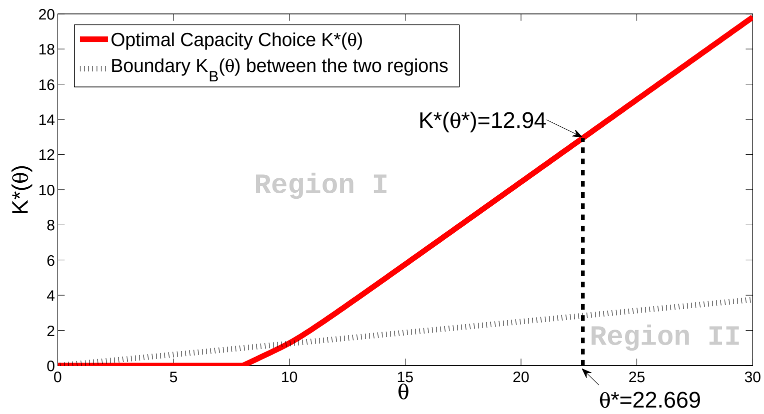
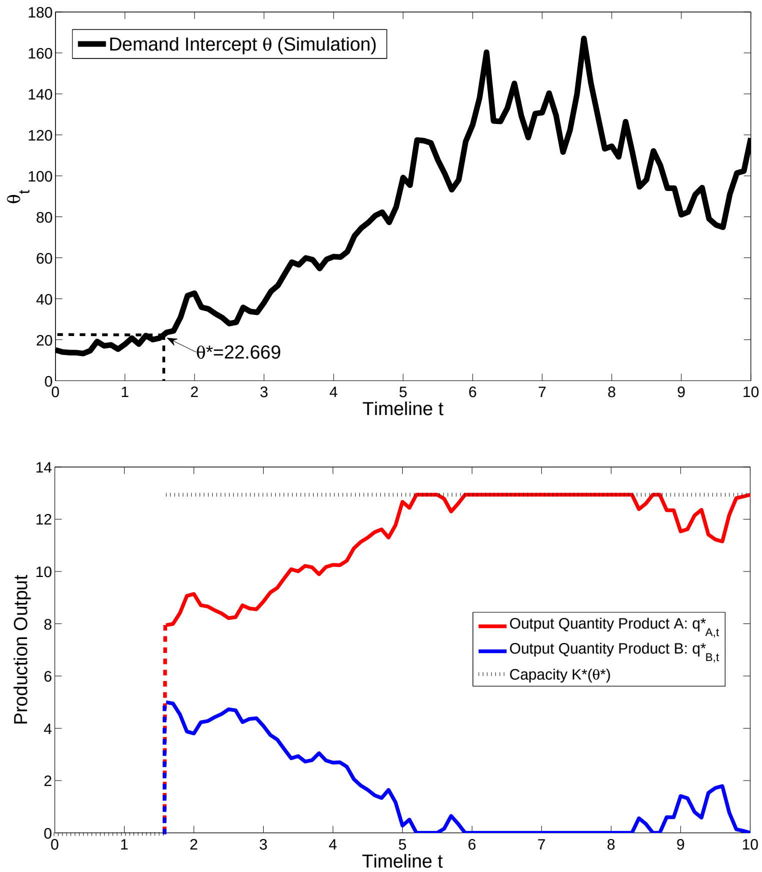


Figure 2: **Case:** $\alpha = 0.8$**,** $\gamma = 0.2$ — Optimal Investment Capacity as a function of demand intercept θ (Parameter values: $r = 0.1$, $\mu = 0.02$, $\sigma = 0.1$ and $c_F = 100$)

capacity to produce both products. If demand raises above a certain threshold, i.e. when demand intercept reaches the level $\hat{\theta} = 103.52$, it is most profitable to suspend production of product B and use full capacity (i.e. $K_F^*(\theta^*) = 12.94$) to produce just product A. Being able to adapt relative production volume optimally across the two products allows the firm to avoid over- as well as under capacity for a wide range of possible demand intercepts.

Subsequently I analyze the effect of demand variability on the flexible investment decision. Four specific "extreme" cases that arise from different combinations of product profitability and substitutability are compared: The first case considers a product combination of two almost similar profitable products with low substitutability rate, indicated as 'Case: H - L'. For the numerical example the parameter values $\alpha = 0.9$ and $\gamma = 0.1$ are chosen. 'Case H - H' represents a product combination of two highly substitutable and almost equally profitable products. The numerical parameter values are $\alpha = 0.9$ and $\gamma = 0.9$. The third case, indicated with 'Case: L - H', represents the setting of a firm producing two products that are highly substitutable but one product is significantly less profitable than the other (parameter values $\alpha = 0.1$ and $\gamma = 09$). And last but not least the case of two products with low substitutability rate ($\alpha = 0.1$) and significant difference in profitability ($\gamma = 0.1$) between the products is considered. This case is referred to as 'Case: L - L'.

The magnitude of the impact of demand variability on the optimal capacity size and investment threshold are illustrated in Figure 4. This figure is based on the values $r = 0.1$, $\mu = 0.02$ and $c_F = 100$ which forms the base case for numerical illustrations throughout the rest of the paper. It confirms the widely accepted result that higher uncertainty increases capacity size but delays investment. When uncertainty goes up, a higher demand level is needed before it is optimal to invest. This effect is partly caused by the fact that capacity increases with uncertainty, and partly due to the real options result that in a more uncertain economic environment the firm has a higher incentive to wait for more information before undertaking the investment



A **B**

Figure 3: Production Time Line for a simulation result of demand intercept θ. Panel A: Simulated demand intercept process θ_t plotted against time line. Panel B: Optimal production output of product A and B, respectively over time period $t \in [0, 10]$. (Parameter values: $r = 0.1$, $\mu = 0.02$, $\sigma = 0.1$ and $c_F = 100$)

(see Dixit and Pindyck (1994)).

Figure 4 shows that the capacity size is higher for the firm selling two products with a low substitutability rate and high profitability difference. The difference in capacity size between 'Case: L- H' and the other cases gets more significant for higher uncertainty. First the difference in capacity of the 4 cases is not so large while profit is already 50% higher (see results in Table3). While difference in profit remains approximately constant for increasing the difference in capacity is growing more than proportionally. This confirms my intuition that the high capacity size result is not just driven by the general argument of 'investing later in more capacity' but strengthened by the high value of product flexible capacity for firm producing two almost equally profitable products with a low substitutability rate which increases the willingness to invest in a high capacity level. See Panel A of Figure 3 that shows the high demand range in which the firm will benefit from product-flexible capacity by producing both products for a wide demand range ($\theta \in [0, \hat{\theta})$)and suspend production of product B only facing extremely high demand.

Table 1 shows the effect of profitability on the optimal investment strategy keeping the substitutability rate constant and low ($\gamma = 0.1$). Observe that the optimal capacity size and the expected profit of the project are increasing in profitability of product B. The non monotonic effect on the investment threshold is striking. For a graphical illustration of the non-monotonic behavior of investment timing see Figure 5. This result is driven by two contrary effects: on one hand the firm invests later in more capacity while on the other hand higher value of the project would lead the firm to invest earlier. The capacity effect is stronger for low profitability parameters while the effect of higher project value dominates for cases of two almost equally profitable products.

Figure 5 shows the optimal investment thresholds for the situation when the substitutability rate of the two products is low and the profitability of product B changes from low (0.1) to high (0.9). The optimal investment threshold is increasing in α for low value of α and decreasing for high values of α. This effect is stronger for environment with high demand volatility. Capacity size and expected profit are both increasing in α.

3.2 Dedicated Capacity Investment

Deriving the optimal investment thresholds for dedicated capacity, it is surprising that for most cases the firm will decide to purchase just one dedicated capacity for the more profitable product and fully ignores demand for product B. Table 2 shows the optimal investment thresholds for a specific parameter choice, comparing the previously introduced four cases. For the case of almost equally profitable products with a low substitutability rate the firm purchases significantly more dedicated capacity for product A than for the (slightly) less profitable product B. For all other cases the firm would commit itself at the moment of investment to have just one capacity for product A at its disposal forever. To built intuition for this result, note that in the three latter cases demand would have to fall very low so that the firm can actually gain from the possibility to make the total market size bigger with producing the second (less profitable) product B.

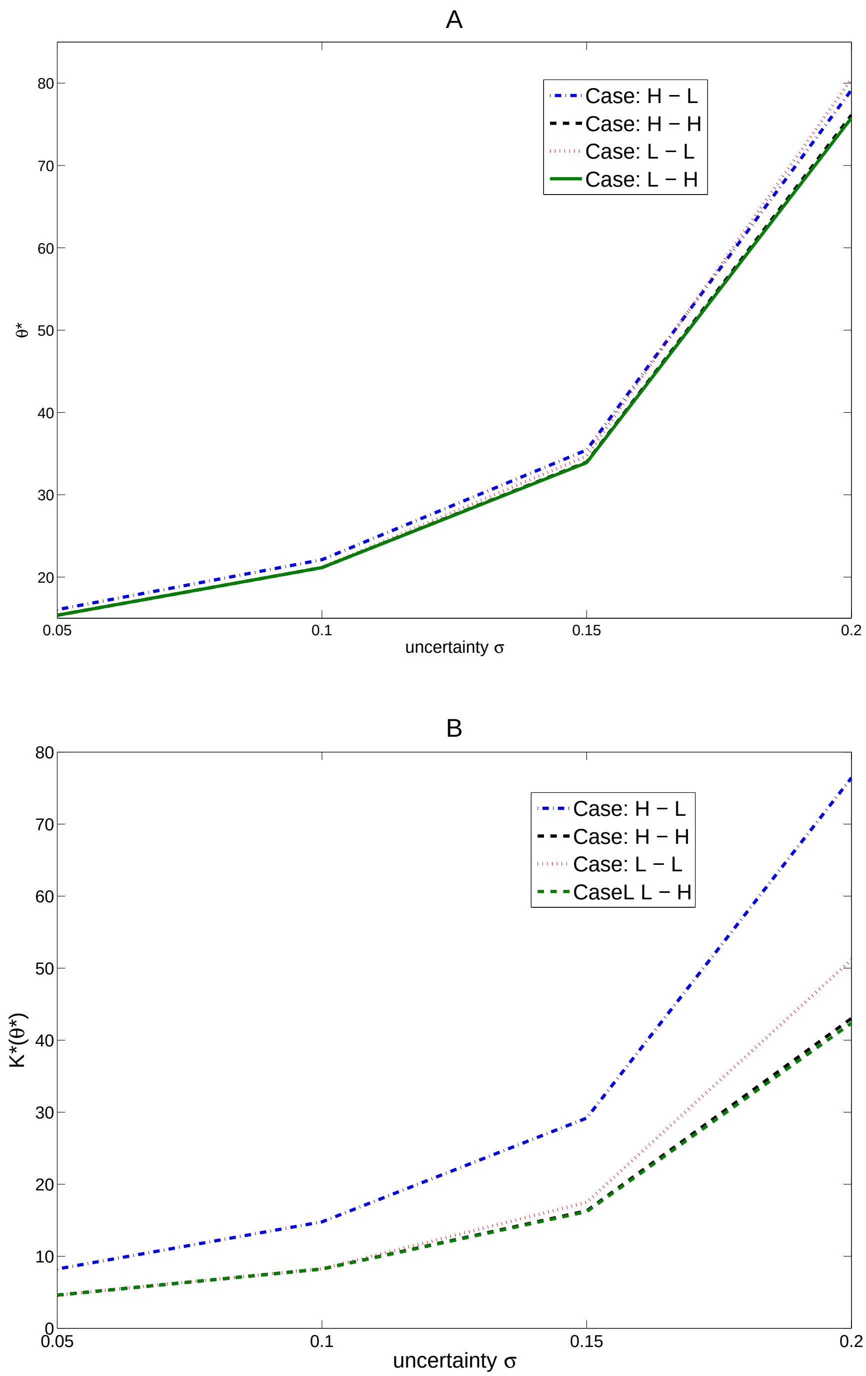


Figure 4: **Optimal Investment Strategy** comparing the following Cases: '**Case H - L**': $\alpha = 0.1$ and $\gamma = 0.9$. '**Case H - H**': $\alpha = 0.9$ and $\gamma = 0.9$. '**Case L - L**': $\alpha = 0.1$ and $\gamma = 0.1$. '**Case L - H**':$\alpha = 0.1$ and $\gamma = 0.9$. ***Panel A:*** Optimal Investment Threshold θ^* as a function of demand volatility σ; ***Panel B:*** Optimal Capacities invested in $K^*(\theta^*)$, as a function of volatility σ. (Parameter values: $r = 0.1$, $\mu = 0.02$ and $c_F = 100$)

Table 1: Shows the optimal investment strategy for changing profitability parameter α and fixed substitutability parameter $\gamma = 0.1$. The expected profit is discounted back to an initial demand value $\theta_0 = 10$ for reasons of comparison. The left Panel shows the results for the case of uncertainty $\sigma = 0.1$, the right one for $\sigma = 0.2$. (Parameter Values: $r = 0.1$, $\mu = 0.02$ and $c_F = 100$)

α	γ	θ_F	K_F^*	region	Π_F	θ_F	K_F^*	region	Π_F
0.1	0.1	21.1933	8.28394	region II	60.7534	80.5503	51.1684	region I	204.144
0.2	0.1	21.2484	8.36382	region II	60.8294	81.9064	54.132	region I	208.054
0.3	0.1	21.4191	8.61164	region I	61.0404	83.2303	57.6358	region I	213.719
0.4	0.1	21.8966	9.35158	region I	61.746	84.2842	61.4848	region I	221.666
0.5	0.1	22.5608	10.6113	region I	63.6416	84.8279	65.4131	region I	232.462
0.6	0.1	23.058	12.0578	region I	67.4197	84.6654	69.1242	region I	246.706
0.7	0.1	23.1514	13.3313	region I	73.5772	83.6784	72.3382	region I	265.036
0.8	0.1	22.8103	14.2512	region I	82.5011	81.8363	74.8276	region I	288.148
0.9	0.1	22.1084	14.7773	region I	94.5956	79.1807	76.4309	region I	316.854

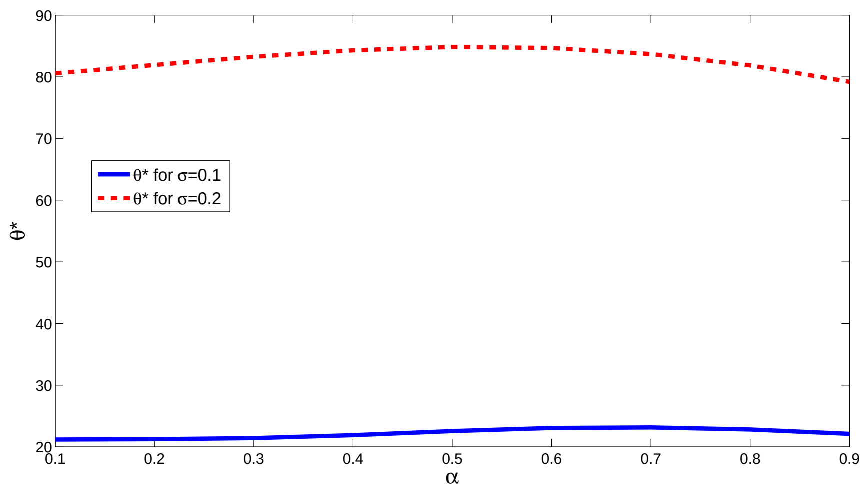


Figure 5: **Optimal Investment Threshold** as a function of profitability parameter α (Parameter values: $\gamma = 0.1$, $r = 0.1$, $\mu = 0.02$ and $c_F = 100$)

Table 2: Investment Strategies of Dedicated Capacity investment for the previously introduced cases (description see figure 4). (Parameter values: $L = 0.1$, $H = 0.9$, $r = 0.1$, $\mu = 0.02$ and $c_D = 100$)

CASE: $\alpha = H$**,** $\gamma = L$

σ	θ^*	$K^*_{D,A}(\theta^*)$	$K^*_{D,B}(\theta^*)$	$K^*_D(\theta^*)$
0.05	16.0587	4.6802	3.56501	8.24522
0.1	22.1392	8.17342	6.63597	14.8094
0.15	35.5252	15.8636	13.3966	29.2602
0.2	79.4644	41.1065	35.5881	76.6946

CASE: $\alpha = H$**,** $\gamma = H$**; Case:** $\alpha = L$**,** $\gamma = L$**; and CASE:** $\alpha = L$**,** $\gamma = H$

σ	θ^*	$K^*_{D,A}(\theta^*)$
0.05	15.3644	4.60274
0.1	21.1472	8.21699
0.15	33.8987	16.1867
0.2	75.7771	42.3607

For most demand intercept realizations it can gain highest profit satisfying just the demand for product A. The threat of possible overcapacity of product B dominates the value of the downside potential to increase total market demand by producing both products for low demand realizations.

Only the firm that faces a product combination of two similarly profitable products with a low substitutability rate can profit from this downside potential at a wide range of demand.

4 Value of Flexibility

One of the main objectives of this paper is to quantify the value of flexible capacity. In order to derive the flexibility value, the situation in which a firm relies on maximal two dedicated capacities rather than on one flexible capacity is considered as a benchmark. The two optimal investment strategies for flexible and dedicated capacity investment are compared assuming that the unit investment cost of flexible and dedicated capacity are equally high, i.e. $c_D = c_F =: c$. The value of flexibility is therefore given by the difference in expected profit of the flexible and dedicated capacity investment strategies.

In order to compare two investment strategies that have different optimal moments of investment, one needs to compare the discounted expected project values. Assuming the optimal investment thresholds derived in Section 2, the expression of the value of flexibility is equal to:

$$V_f = \left(\frac{\theta_0}{\theta^*_F}\right)^{\beta_1} V\left(\theta^*_F, K^*_F(\theta^*_F)\right) - \left(\frac{\theta_0}{\theta^*_D}\right)^{\beta_1} V\left(\theta^*_D, K^*_D(\theta^*_D)\right), \tag{14}$$

where the expected project values are discounted back to the beginning of the considered time period with the initial demand intercept θ_0.

Table ?? shows the results of the value of flexibility (V_f) for the numerical example presented in the previous section. The discounted expected project values, denoted by Π_i for $i = D, F$ are discounted back to an initial demand level of $\theta_0 = 10$. Furthermore, the relation of expected profit of dedicated investment to flexible investment, i.e. $\frac{\Pi_D}{\Pi_F}$ is given.

Table ?? shows that the value of flexibility is most significant if uncertainty is high, substitutability low, and profit levels between the two products are substantially different (see Case $\alpha = L$, $\gamma = L$). Assuming demand uncertainty of $\sigma = 0.2$, substitutability parameter γ equal to 0.1 and low profitability of product B compared to product A, the value of flexibility is substantially higher than for the other cases. Flexibility especially pays off in this case because it allows the firm to avoid over-capacity by increasing the market size including the less profitable product in production for cases of low demand, while a dedicated firm restricts itself in the optimal case to just one capacity for the more profitable product. As the optimal investment capacity for the flexible firm is significantly higher ($K_F^* = 51.17$) than for the dedicated firm ($K_D^* = 42.36$), the flexible firm is additionally less threatened by under-capacity in high demand periods.

5 Conclusions

This paper considers the timing and capacity choice of a firm facing stochastic demand. Two types of capacity investment are distinguished. The flexible capacity investment allows the firm to produce both products with the same production facility, while investment in dedicated capacity investment restricts the firm to produce just one product by purchased dedicated production facility. The firm makes three decisions: choice of investment time, choice of capacity, and type of capacity investment, i.e. flexible or dedicated. Concerning the timing and capacity decision I develop implicit solutions, which are investigated numerically. I show that for both flexible and dedicated capacity investment, the firm invests later in higher capacity if demand uncertainty increases. Flexibility especially pays off when uncertainty is high, substitutability low, and profit levels between the two products are substantially different. In the flexible case, under high demand the firm just produces the most profitable product, if demand is low the firm produces both products to increase total demand. In the dedicated case the firm invests in both capacities if the substitutability rate is low and profitability of both products high enough. Otherwise the firm will ignore demand for the less profitable product in the market and install just one dedicated capacity.

Numerous extensions of this model deserve further analysis. This includes analyzing different cost structures, asymmetric demand curves, and different demand functions. While in this paper I consider that the firm can investment at one moment in time in either flexible or dedicated capacity, I am currently in the process of extending this model to allow for multiple investments. Specifically, this means that the firm is free to either undergo investment in one step at a single freely chosen point of time (dedicated and flexible

Table 3: **Optimal Profits** of flexible and dedicated investment, respectively, discounted back to the initial demand level $\theta_0 = 10$ comparing four cases (description of these cases can be found in caption of Figure 4). Π_F denotes the discounted expected profit of the flexible capacity investment and Π_D the discounted expected profit for dedicated investment. Expression V_f denotes the **Value of Flexibility** . (Parameter values: $L = 0.1$, $H = 0.9$, $r = 0.1$, $\mu = 0.02$ and $c = 100$)

CASE: $\alpha = H$**,** $\gamma = L$

σ	θ_F^*	K_F^*	region	Π_F	θ_D^*	$K_{D,A}^*$	$K_{D,B}^*$	K_D^*	Π_D	V_f	$\frac{\Pi_D}{\Pi_F}$
0.05	16.0461	8.23173	I	52.6722	16.0587	4.6802	3.56501	8.24522	52.5854	0.0868	99.8352%
0.1	22.1084	14.7773	I	94.5956	22.1392	8.17342	6.63597	14.8094	94.3769	0.2187	99.7688%
0.15	35.4425	29.178	I	172.058	35.5252	15.8636	13.3966	29.2602	171.493	0.565	99.6716%
0.2	79.1807	76.4309	I	316.854	79.4644	41.1065	35.5881	76.6946	315.413	1.441	99.5452%

CASE: $\alpha = H$**,** $\gamma = H$

σ	θ_F^*	K_F^*	region	Π_F	θ_D^*	$K_{D,A}^*$	$K_{D,B}^*$	K_D^*	Π_D	V_f	$\frac{\Pi_D}{\Pi_F}$
0.05	15.3644	4.60274	II	35.3001	15.3644	4.60274	0	4.60274	35.3001	0	100%
0.1	21.1521	8.22414	II	60.6937	21.1472	8.21699	0	8.21699	60.6864	0.0073	99.988%
0.15	33.9694	16.3011	II	107.423	33.8987	16.1867	0	16.1867	107.253	0.17	99.8417%
0.2	76.1413	43.022	I	194.666	75.7771	42.3607	0	42.3607	193.74	0.926	99.5243%

CASE: $\alpha = L$**,** $\gamma = L$

σ	θ_F^*	K_F^*	region	Π_F	θ_D^*	$K_{D,A}^*$	$K_{D,B}^*$	K_D^*	Π_D	V_f	$\frac{\Pi_D}{\Pi_F}$
0.05	15.3644	4.60274	II	35.3001	15.3644	4.60274	0	4.60274	35.3001	0	100%
0.1	21.1933	8.28394	II	60.7534	21.1472	8.21699	0	8.21699	60.6864	0.067	99.8897%
0.15	34.7053	17.4918	I	108.981	33.8987	16.1867	0	16.1867	107.253	*1.728*	98.4144
0.2	80.5503	51.1684	I	204.144	75.7771	42.3607	0	42.3607	193.74	**10.404**	94.9036%

CASE: $\alpha = L$**,** $\gamma = H$

σ	θ_F^*	K_F^*	region	Π_F	θ_D^*	$K_{D,A}^*$	$K_{D,B}^*$	K_D^*	Π_D	V_f	$\frac{\Pi_D}{\Pi_F}$
0.05	15.3644	4.60274	II	35.3001	15.3644	4.60274	0	4.60274	35.3001	0	100%
0.1	21.1472	8.21699	II	60.6864	21.1472	8.21699	0	8.21699	60.6864	0	100%
0.15	33.8988	16.1868	II	107.253	33.8987	16.1867	0	16.1867	107.253	0	100%
0.2	75.7797	42.3654	II	193.747	75.7771	42.3607	0	42.3607	193.74	0.007	99.9964%

capacity) or in incremental steps at different points in time. Proceeding stepwise gives additional flexibility as the firm can respond to resolving uncertainty by choosing the investment timing individually for each step.

A FLEXIBLE Capacity

$$V(\theta) = \begin{cases} A_1\theta^{\beta_1} + a_1\theta^2 + a_2\theta K_F + a_3 K_F^2 & \text{for } \theta < \frac{2(1-\gamma)}{(1-\alpha)}K_F \\ B_2\theta^{\beta_2} + \frac{\theta K_F}{r-\mu} - \frac{K_F^2}{r} & \text{for } \theta > \frac{2(1-\gamma)}{(1-\alpha)}K_F \end{cases} \tag{15}$$

In order to check the second order condition for the optimal capacities I derive

$$\frac{\partial V(\theta, K_F)}{\partial K_F^2} = \begin{cases} \frac{\partial^2 A_1}{\partial K_F^2}\theta^{\beta_1} + 2a_3 & \text{for } \theta < \frac{2(1-\gamma)}{(1-\alpha)}K_F \\ \frac{\partial^2 B_2}{\partial K_F^2}\theta^{\beta_2} - \frac{2}{r} & \text{for } \theta > \frac{2(1-\gamma)}{(1-\alpha)}K_F \end{cases} \tag{16}$$

Sign of the parameters:

Corollary 1 *The constant A_1 is negative for all parameter choices:*

$$A_1 = K_F^{2-\beta_1}\frac{1}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_1}(1-\gamma)\left[\frac{(2-\beta_2)}{2(r-2\mu-\sigma^2)} - \frac{(1-\beta_2)}{(r-\mu)} - \frac{\beta_2}{2r}\right]$$

$$K_F^{2-\beta_1} > 0$$
$$\frac{1}{\beta_2-\beta_1} < 0$$
$$\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_1}(1-\gamma) > 0$$

$$\begin{aligned}\left[\frac{(2-\beta_2)}{2(r-2\mu-\sigma^2)} - \frac{(1-\beta_2)}{(r-\mu)} - \frac{\beta_2}{2r}\right] &= \\ 2r(\mu+\sigma^2) + (-1)\beta_2(2\mu^2+\mu\sigma^2+r\sigma^2) &> 0\end{aligned}$$

The constant B_2 is positive for all parameter choices:

$$B_2 = K_F^{2-\beta_2}\frac{1}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_2}(1-\gamma)\left[\frac{(2-\beta_1)}{2(r-2\mu-\sigma^2)} - \frac{(1-\beta_1)}{r-\mu} - \frac{\beta_1}{2r}\right]$$

$$\begin{aligned}\left[\frac{(2-\beta_1)}{2(r-2\mu-\sigma^2)} - \frac{(1-\beta_1)}{(r-\mu)} - \frac{\beta_1}{2r}\right] &= \\ 2r(\mu+\sigma^2) + (-1)\beta_1(2\mu^2+\mu\sigma^2+r\sigma^2) &=\end{aligned}$$

still part missing First order derivative of A_1 and B_2 w.r.t K_F:

$$\begin{aligned}\frac{\partial A_1}{\partial K_F} &= K_F^{1-\beta_1}\frac{(2-\beta_1)}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_1}(1-\gamma)\left[\frac{(2-\beta_2)}{2(r-2\mu-\sigma^2)} - \frac{(1-\beta_2)}{(r-\mu)} - \frac{\beta_2}{2r}\right] \\ \frac{\partial B_2}{\partial K_F} &= K_F^{1-\beta_2}\frac{(2-\beta_2)}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_2}(1-\gamma)\left[\frac{(2-\beta_1)}{2(r-2\mu-\sigma^2)} - \frac{(1-\beta_1)}{r-\mu} - \frac{\beta_1}{2r}\right]\end{aligned}$$

Second order derivative of A_1 and B_2 w.r.t K_F:

$$\begin{aligned}
\frac{\partial^2 A_1}{\partial K_F^2} &= K_F^{-\beta_1}\frac{(1-\beta_1)(2-\beta_1)}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_1}(1-\gamma)\left[\frac{(2-\beta_2)}{2(r-2\mu-\sigma^2)}-\frac{(1-\beta_2)}{(r-\mu)}-\frac{\beta_2}{2r}\right] \\
\frac{\partial^2 B_2}{\partial K_F^2} &= K_F^{-\beta_2}\frac{(1-\beta_2)(2-\beta_2)}{\beta_2-\beta_1}\left[\frac{2(1-\gamma)}{1-\alpha}\right]^{-\beta_2}(1-\gamma)\left[\frac{(2-\beta_1)}{2(r-2\mu-\sigma^2)}-\frac{(1-\beta_1)}{r-\mu}-\frac{\beta_1}{2r}\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial A_1}{\partial K_F} &= K_F^{1-\beta_1}\left(\frac{2-\beta_1}{\beta_2-\beta_1}\right)\left[(2-\beta_2)a_1P_K^{2-\beta_1}+(1-\beta_2)P_K^{1-\beta_1}\left(a_2-\frac{1}{r-\mu}\right)-\beta_2P_K^{-\beta_1}\left(a_3+\frac{1}{r}\right)\right] \\
\frac{\partial B_2}{\partial K_F} &= K_F^{1-\beta_2}\left(\frac{2-\beta_2}{\beta_2-\beta_1}\right)\left[(2-\beta_1)a_1P_K^{2-\beta_2}+(1-\beta_1)P_K^{1-\beta_2}\left(a_2-\frac{1}{r-\mu}\right)-\beta_1P_K^{-\beta_2}\left(a_3+\frac{1}{r}\right)\right]
\end{aligned}$$

with $P_K = \frac{2(1-\gamma)}{(1-\alpha)}$

$$\begin{aligned}
\frac{\partial^2 A_1}{\partial K_F^2} &= K_F^{-\beta_1}\left(\frac{(1-\beta_1)(2-\beta_1)}{\beta_2-\beta_1}\right)\left[(2-\beta_2)a_1P_K^{2-\beta_1}+(1-\beta_2)P_K^{1-\beta_1}\left(a_2-\frac{1}{r-\mu}\right)-\beta_2P_K^{-\beta_1}\left(a_3+\frac{1}{r}\right)\right] \\
\frac{\partial^2 B_2}{\partial K_F^2} &= K_F^{-\beta_2}\left(\frac{(1-\beta_2)(2-\beta_2)}{\beta_2-\beta_1}\right)\left[(2-\beta_1)a_1P_K^{2-\beta_2}+(1-\beta_1)P_K^{1-\beta_2}\left(a_2-\frac{1}{r-\mu}\right)-\beta_1P_K^{-\beta_2}\left(a_3+\frac{1}{r}\right)\right]
\end{aligned}$$

Derive the optimal investment threshold assuming Kbound is the optimal capacity choice for both regions:

B Dedicated Capacity

B.1 One Capacity Case

$$\frac{\partial(V(\theta, K_{D,A}) - c_D K_{D,A})}{\partial K_{D,A}} = 0 \tag{17}$$

resulting in

$$K^*_{D,A}(\theta) = \frac{r}{2(r-\mu)}\left[\theta - (r-\mu)c_D\right] \tag{18}$$

$$V(\theta, K^*_{D,A}) - c_D K^*_{D,A} = \frac{r}{4(r-\mu)^2}\theta^2 - \frac{r}{2(r-\mu)}c_D\theta + \frac{r}{4}c_D^2 \tag{19}$$

The value of the option is

$$F(\theta) = A_1\theta^{\beta_1} \tag{20}$$

and value matching and smooth pasting results in the following investment threshold

$$\theta^* = \left(\frac{\beta_1}{\beta_1-1}\right)(r-\mu)\left[\frac{K_D}{r} + c_D\right] \tag{21}$$

Combining equation (18) and (21) gives

$$\theta^*_D = (\frac{\beta_1}{\beta_1-2})(r-\mu)c_D$$

and

$$K^*_D(\theta^*) = \frac{1}{(\beta_1-2)}rc_D \tag{22}$$

For product B we can derive the following optimal investment decisions

$$\Pi(\theta) = \alpha\theta(1 - K_{D,B})K_{D,B} \tag{23}$$

$$V(\theta, K_{D,B}) = \frac{\alpha\theta(1 - K_{D,B})K_{D,B}}{r - \mu} \tag{24}$$

$$K^*_{D,B} = \frac{\alpha\theta - c_D(r - \mu)}{2\alpha\theta} \tag{25}$$

$$\theta^* = \frac{(r - \mu)}{\alpha} c_D \left(\frac{\beta_1 + 1}{\beta_1 - 1}\right) \tag{26}$$

$$\theta^*_{1,2} = c_D \left(\frac{\beta_1 - 1}{\beta_1 - 2}\right) \left[\frac{(1 + \alpha^2 - 2\alpha\gamma)}{4(1 - \gamma)(1 + \alpha)(r - \mu)}\right] + / - \frac{r}{(1 + \gamma)(r - \mu)} \sqrt{(\beta_1 - 1)^2(1 + \alpha)^2 - 2\beta_1(\beta_1 - 2)\frac{(1 + \alpha^2 - 2\alpha\gamma)}{(1 - \gamma)}} \tag{27}$$

When is $\theta^*_{1,2} > \hat{\theta_B}$? I should show here that for $\theta < \hat{\theta_B}$ the case of one dedicated capacity investment is always optimal.

B.2 Two Capacities Case

Proof of Proposition 2: Solving the threshold equation for region $\hat{\theta_A} < \theta < \hat{\theta_B}$ gives the two results

$$\theta^*_{1,1} = \frac{c_D(r - \mu)(1 - \gamma)}{(1 - \alpha\gamma)}$$

$$\theta^*_{1,2} = \frac{\beta_1 c_D(r - \mu)(1 + \gamma - 2\gamma^2)}{\beta_1(1 + \alpha\gamma) - 2(1 + \gamma^2(\beta_1 - 1))}$$

$\theta^*_{1,1} = \hat{\theta_A}$ and therefore not within the boundaries of the considered region. Therefore, $\theta^*_{1,2}$ is the unique optimal investment threshold for this region.

Solving the threshold equation assuming that we are in region $\theta > \hat{\theta_B}$ gives the following two threshold results:

$$\theta^*_{D_{2,1}} = c_D \left(\frac{\beta_1 - 1}{\beta_1 - 2}\right) \left[\frac{(1 + \alpha^2 - 2\alpha\gamma)}{4(1 - \gamma)(1 + \alpha)(r - \mu)}\right] + \frac{r}{(1 + \gamma)(r - \mu)} \sqrt{(\beta_1 - 1)^2(1 + \alpha)^2 - 2\beta_1(\beta_1 - 2)\frac{1 + \alpha^2 - 2\alpha\gamma}{1 - \gamma}}$$

$$\theta^*_{D_{2,2}} = c_D \left(\frac{\beta_1 - 1}{\beta_1 - 2}\right) \left[\frac{(1 + \alpha^2 - 2\alpha\gamma)}{4(1 - \gamma)(1 + \alpha)(r - \mu)}\right] - \frac{r}{(1 + \gamma)(r - \mu)} \sqrt{(\beta_1 - 1)^2(1 + \alpha)^2 - 2\beta_1(\beta_1 - 2)\frac{1 + \alpha^2 - 2\alpha\gamma}{1 - \gamma}}$$

C Expected Present Value

The formula for $E[e^{-rT}]$, when θ follows the geometric Brownian motion (1), and T is the random first time the process reaches a fixed level $\hat{\theta}$ starting from the general initial position θ_0, is given by

$$E[e^{-rT}] = \left(\frac{\theta_0}{\hat{\theta}}\right)^{\beta_1} \tag{28}$$

where T is the (random) first time when process θ reaches θ^*. See e.g. Dixit and Pindyck (1994) for further explanation.

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