Incorporating Managerial Information into Valuation of Early Stage Investments *

Sebastian Jaimungal† and Yuri Lawryshyn‡

Abstract

Real options analysis (ROA) is widely recognized as a superior method for valuing projects with managerial flexibilities. Yet, its adoption remains limited due to varied difficulties in its implementation. In this work, we propose a real options approach that utilizes managerial cash-flow estimates to value early stage project investments. Our model is based on the assumption that managers can provide pessimistic, likely and optimistic sales and gross margin percent estimates. We introduce a market sector indicator, which is assumed to be correlated to a tradeable market index, which drives the project's sales estimates. Another indicator, assumed partially correlated to the sales indicator drives the gross margin percent estimates. In this way we can model a cash-flow process that is partially correlated to a traded market index. This provides the mechanism for valuing real options of the cash-flow in a financially consistent manner. The method requires minimal subjective input of model parameters and is very easy to implement, based on simple managerial estimate.

Keywords: Real Options; Managerial Information; Cash-Flow Replication; Project Valuation

1 Introduction

Real option analysis (ROA) has been recognized as a superior method to quantify the value of real-world investment opportunities where managerial flexibility can influence their worth, as compared to standard net present value (NPV) and discounted cash-flow (DCF) analysis. ROA builds on the seminal work of Black and Scholes (1973) on financial option valuation. Shortly after, Myers (1977) correctly recognized that both financial options and project decisions are exercised after uncertainties are resolved. Early techniques therefore applied the Black-Scholes equation directly to value put and call options on tangible assets (see, for example, Brennan and Schwartz (1985)). Since this early connection, ROA has been popularized by business publications and valuation texts (Copeland and Tufano (2004), Trigeorgis (1996)), and in the last decade has transitioned from academic circles to heightened industry attention.

As will be discussed below, a number of practical and theoretical approaches for real option valuation have been proposed in the literature, yet industry's adoption of real option valuation is limited, primarily due to the inherent complexity of the models (Block (2007)). A number of leading practical approaches, some of which have been embraced by industry, lack financial rigor

*This work was partially supported by research grants from NSERC of Canada.

†Department of Statistics, University of Toronto, e-mail: sebastian.jaimungal@utoronto.ca

‡Department of Chemical Engineering and Applied Chemistry, University of Toronto, e-mail: yuri.lawryshyn@utoronto.ca

while many theoretical approaches are not practically implementable. In this paper, our aim is to develop a practical, easily implementable method that is consistent with financial theory. The approach is developed for the valuation of R&D-type real option investments, but is not limited to this type only. From a practical perspective, the methodology can be implemented based on simple managerial derived cash-flow estimates, and requires minimal subjectivity with respect to parameter estimation. From a theoretical perspective, the method properly accounts for both market (systematic) and private (idiosyncratic) risks, and ensures that cash-flows from one period to the next are correlated in a consistent manner.

The work presented in this paper is an extension of an earlier real options method developed by the authors (see Jaimungal and Lawryshyn (2010)). Our previous work assumed that future cash-flow estimates are provided by the manager in the form of a probability density function (PDF) at each time period. As was discussed, the PDF can simply be triangular (representing typical, optimistic and pessimistic scenarios), normal, log-normal, or any other continuous density. Second, we assumed that there exists a *market sector indicator* that uniquely determines the cash-flow for each time period and that this indicator is a Markov process. The market sector indicator can be thought of as market size or other such value. Third, we assumed that there exists a tradable asset whose returns are correlated to the market sector indicator. While this assumption may seem somewhat restrictive, it is likely that in many market sectors it is possible to identify some form of market sector indicator for which historical data exists and whose correlation to a traded asset/index could readily be determined. One of the key ingredients of our original approach is that the process for the market sector indicator determines the managerial estimated cash-flows, thus ensuring that the cash-flows from one time period to the next are consistently correlated. A second key ingredient is that an appropriate risk-neutral measure is introduced through the minimal martingale measure[1] (MMM) (Föllmer and Schweizer (1991)), thus ensuring consistency with financial theory in dealing with market and private risk, and eliminating the need for subjective estimates of the appropriate discount factor typically required in a DCF calculation.

As discussed in our original work, the advantages of the proposed approach include both practical and theoretical aspects and are listed here:

1. *The approach utilizes managerial cash-flow estimates.* In practice, managers are often expected to supply likely, optimistic and pessimistic cash-flow estimates when valuing a potential project. Our method relies on these estimates to formulate the option valuation.
2. *The approach requires little subjectivity with respect to parameter estimation.* Aside from the cash-flow estimates, the other key parameters required for the valuation will depend on the models chosen for the the market sector indicator process and the traded asset process. However, in many cases these parameters can be estimated based on historical data. Furthermore, in our approach, most of those parameters do not affect the project's valuation nor do they affect the value of the (European) option to invest.
3. *The approach provides a* ***missing link*** *between practical estimation and theoretical frameworks.* As will be discussed further, many traditional / theoretical frameworks presented in the literature bring fruitful insights regarding managerial decision making in the real options context or unique ways to value special cases of real options problems; however, these methods do not provide a mechanism to link real cash-flow estimates to the models. The approach pre-

[1] As we discuss in Section 4.1, the risk-neutral MMM is a particular risk-neutral measure which produces variance minimizing hedges.

sented here provides a mechanism to link many traditional frameworks, where the underlying process driving the cash-flows or project value is based on a Markov process, such as geometric Brownian motion (GBM) or a mean reverting process, to managerial cash-flow estimates. For example, our method can be used with entry / exit and switching frameworks such as those proposed by Sodal, Koekebakker, and Aadland (2008) and Lin (2009), to value multistage investment opportunities as proposed by Berk, Green, and Naik (2004), or to account for managerial risk aversion as proposed by Henderson (2007).

4. *The approach is theoretically consistent.* We emphasize this advantage, again, because a number of practical real options approaches fail in this respect. For example, the popular Market Asset Disclaimer (MAD) approach proposed by Copeland and Antikarov (2001) and other approaches, such as the one proposed by Datar and Mathews (2004), require an artificial mechanism to link cash-flows from one period to the next. Because our approach links the managerial cash-flow estimates through a GBM, or some other Markov process, the correlation of cash-flows from one period to the next is captured appropriately. Recall though that the project value is not being modeled directly, rather we model the underlying sector indicator which drives the project value. Furthermore, through the risk-neutral MMM, our approach provides a theoretical mechanism to properly account for both market and private risk.

In this work, we extend our model by assuming that it is the sales, again provided by management, that are driven by the market sector indicator. Furthermore, we allow for gross margin percent values to also be stochastic and estimated by managers, and these are assumed to be partially correlated (based on managerial experience / estimates) to the sales estimates. The methodology provides a natural extension to our original method, and provides the framework for practical implementation.

2 Real Options in Practice

Despite the theoretical appeal and managerial awareness of real option analysis, a number of surveys (see Block (2007), Hartmann and Hassan (2006), Truong, Partington, and Peat (2008), Bennouna, Meredith, and Marchant (2010)) have shown a limited application of ROA in industry. A number of them indicate that complexity of the tools led to a lack of acceptance of ROA by decision-makers. Many ROA approaches have been proposed for practitioners and Borison (2005) categorizes the five main approaches, namely, the Classical, the Subjective, the Market Asset Disclaimer, the Revised Classical and the Integrated. Furthermore, the author provides a succinct description of each, discussing pros and cons. A short discussion of each of the five approaches' strengths and weaknesses follows.

The Classical Approach (see Amram and Kulatilaka (1999)) relies on the assumption that the value of the real project cash-flows can be closely replicated by a *known* traded asset. The market parameters of the traded asset can then be used directly to value options associated with the project. The main strength of the Classical approach is its objectivity. However, Borison (2005) contends that the main issue with this approach is the assumption that the project's value can be replicated by a traded asset. Arguably, this contention would be true for DCF methods as well. Clearly, the volatility, and therefore risk, of the stock price of a company, which likely consists of multiple projects, is going to be less than the volatility / risk of a single project. Another issue is actually finding a traded asset that can reasonably replicate the project value. Thus, while the method is objective by design, the ability to find a replicating traded asset is questionable.

The Subjective approach (see Luehrman (1997), Luehrman (1998a) and Luehrman (1998b)) also assumes that the real project cash-flow value can be replicated by a traded asset, but relies on purely subjective assumptions regarding the traded asset's market parameter values. While this approach circumvents the difficulty of finding a traded asset to be used in the replication, as Borison (2005) emphasizes, the subjective nature of this approach is its inherent shortcoming.

To strengthen its appeal and gain acceptance from academics and practitioners, Copeland and Antikarov (2001) introduced the Market Asset Disclaimer (MAD) method/approach. Brandao and Dyer (2005) expand on the MAD approach with the implementation of decision trees. The main strength of the MAD approach is its ease of implementation, requiring only a basic understanding of the binomial option valuation method, and yet it provides a mechanism to account for managerial uncertainty in estimating the project cash-flow volatility. The MAD approach is even more tractable when implemented in a decision tree setting. A number of practitioners have utilized the MAD method (see for example Pendharkar (2010)), however, the theoretical basis for the model is debated. Borison (2005) highlights two main issues with the MAD approach; one with respect to the subjectivity of inputs and the other with respect to the geometric Brownian motion (GBM) assumption placed on the project value. Further, this approach leads to consistently increasing real (call) option value as volatility increases, which contradicts the observations of Oriani and Sobrero (2008). We discuss this last point further in Section 5.

The Revised Classic approach assumes that certain real projects derive their inherent value from exogenous market factors (market risk) and should be treated in the Classical approach, whereas other projects derive value mostly from endogenous factors (private / company specific risk) and should be treated with classical decision analysis theory (see Dixit and Pindyck (1994)). Clearly, projects that are highly dependent on commodity prices should be valued using the Classical approach, whereas certain new product development initiatives, such as blockbuster drug development, will likely succeed or fail based on internal factors and the degree of success of the project will be influenced, only to a small degree, by market factors. As Borison (2005) argues, the main issue with the Revised Classic approach is its "all or nothing" nature, which does not reflect the realities of most projects. Furthermore, in the case of a project dominated by endogenous factors, it is unclear what discount factor to use. Theoretically, the correct factor should be the risk-free rate, however the weighted average cost of capital (WACC) is often used in decision tree analysis (DTA).

The Integrated approach recognizes that most projects consist of a combination of market (systematic / exogenous) and private (idiosyncratic / endogenous) risk factors (see Smith and Nau (1995)). The approach provides a mechanism to value the market risk of a project through hedging with appropriate tradeable assets while private risk is valued by discounting expected values at the risk-free rate. Properly applied, the Integrated approach is consistent with financial theory. As Borison (2005) notes, the Integrated approach requires more work and is more difficult to explain, but is the only approach that accounts for the fact that most corporate investments have both market and private risk.

A number of methods have been proposed that utilize the Integrated approach. For example, Berk, Green, and Naik (2004) provide a valuation model of a multi-stage investment project, typical of R&D projects, that includes a parameter that captures the correlation between the project cash-flow process and a tradeable index or stock. Unfortunately, the entire cash-flow process is modeled with a single GBM, a practice that is common in academic real options literature, which makes it impractical to apply to actual managerial estimates. Chen, Zhang, and Lai (2009) introduce an

Integrated framework for the valuation of IT projects where the cash-flow uncertainty related to market factors is separated from private uncertainty cash-flows and each is modeled separately. Real option value is determined by applying the Classical approach to the market risk related cash-flows and this value is added to the NPV results obtained from analysis of the private risk cash-flows. Similar approaches, where market and private risk is separated, have been proposed by others (see for example Shockley (2006) and Luo, Sheu, and Hu (2008)).

A very simple practical approach for the valuation of R&D investments was proposed by Datar and Mathews (2004) (see also Mathews and Datar (2007) and Mathews (2009)). The method relies on utilizing Monte Carlo simulation to determine the real option value based on simple probability distribution cash-flow projections. The major contribution of the approach is that it relies on cash-flow scenarios which managers are comfortable in projecting; namely, it relies on pessimistic, likely and optimistic forecasts. Collan, Fullér, and Mezei (2009) extend the Datar-Mathews method in a practical fuzzy numbers context. However, these methods do not differentiate between market and private risk, nor do they provide a financially consistent framework for correlating cash-flows from one period to the next.

As mentioned previously, we aim to present a model that is practical to implement, requiring limited detail regarding cash-flow estimates with minimal subjective estimation of market parameters, but that is consistent with financial theory, properly accounting for market and private risk, and ensuring that the cash-flows are correlated appropriately among the time periods. We begin by presenting a method to replicate cash-flow estimates through the introduction of a market sector indicator process.

3 Replicating Cash-Flow Distributions

When managers think about investing in projects, they typically have in mind a cash-flow associated with three scenarios: (i) the most likely scenario (ii) the optimistic scenario and (iii) the pessimistic scenario. Datar and Mathews (2004) proposed a methodology based on such scenarios and we formulate our approach in a similar manner. An example of the three cash-flow scenarios is shown in Figure 1. The three scenarios may be derived through Monte Carlo simulations representing the technical risk inherent in the project, the corporations potential market share, the market value of the end product and so on. Regardless of how the manager comes to this cash-flow distribution, one of our main goals is to provide a consistent dynamic model which leads to a project cash-flow possessing any distribution which a manager provides. Our analysis is not limited to the triangular distribution shown in the example, although this distribution is, perhaps, the simplest form that managers employ widely.

We assume there is a traded market index I_t which the manager can invest in. This market index is, for simplicity, assumed to be a geometric Brownian motion (GBM) and satisfies the SDE

$$\frac{dI_t}{I_t} = \mu\, dt + \sigma\; dB_t, \tag{1}$$

where B_t is a standard Brownian motion under the real-world measure $\mathbb{P}$. Furthermore, we introduce an underlying observable, but not tradable, process X_t which drives the sales S_t generated from the project. This underlying process can be thought of as a *market sales sector indicator*. For simplicity,

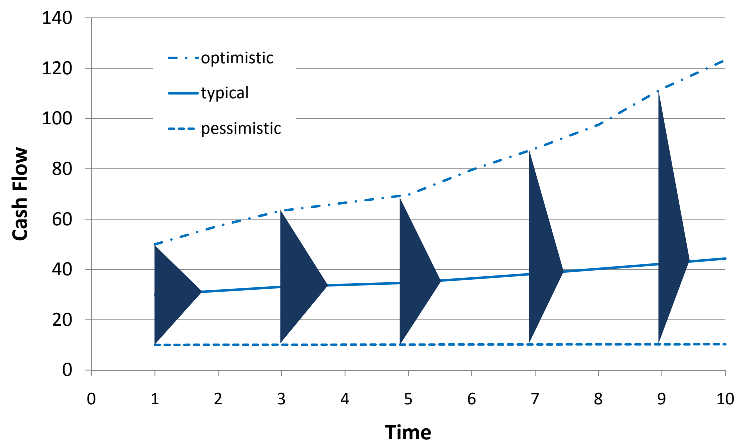


Figure 1: An example of typical, optimistic and pessimistic cash-flow streams provided by a manager.

we assume that the sales sector indicator is a standard Brownian motion (BM)

$$dX_t = \rho_{SI}\, dB_t + \sqrt{1-\rho_{SI}^2}\, dW_t^S \tag{2}$$

where W_t^S is a standard Brownian motion under the real-world measure $\mathbb{P}$ independent of B_t. We emphasize here that from a European real option perspective, choosing a Brownian motion instead of a GBM will have no bearing on the results of the valuation. As well, we introduce a second underlying observable, but not tradable, process Y_t which drives the gross margin percent (GM%), M_t, that the product generates. We will assume that this process is correlated with the process driving sales X_t with correlation ρ_{SM} and specifically write,

$$dY_t = \rho_{SM}\, dX_t + \sqrt{1-\rho_{SM}^2}\, dW_t^M \tag{3}$$

where W_t^M is a standard Brownian motion under the real-world measure $\mathbb{P}$ independent of B_t and W_t^S. As such, the implied correlation between the traded market index and the margin indicator is $\rho_{IM} = \rho_{SI}\rho_{SM}$.

We remark that although it is quite common in the real options literature to assume project values to be GBMs, as GBM captures the essential source of uncertainty inherent in valuation, here, we instead use a BM to drive cashflows. These cash-flows are not the project values, instead, the BMs drive the sales and the GM%, and the cash-flows are given as a product of sales and GM% $V_k = S_k M_k$ at times T_k ($k = 1, \ldots, n$). Further, the sales and GM% are determined from their respective indicators such that

$$S_k = \varphi_k^S(X_{T_k}^S) \qquad \text{and} \qquad M_k = \varphi_k^M(X_{T_k}^S) \tag{4}$$

for some collection of functions φ_k^S and φ_k^M. One of our goals is to find explicit forms for these functions such that the manager specified distributions for sales and GM% are matched exactly.

Under these assumptions the project value can be viewed as a strip of European contingent claims on the sector indicator with payoff functions

$$v_k = (1-\kappa_k)\varphi_k^S\varphi_k^M - \alpha_k \tag{5}$$

at time T_k, where α_k and κ_k are fixed and variable deterministic cost components of the cash-flow at time T_k. When the indicators are high, the project value will be high, and when the indicators are low, the the project value will be low. Furthermore, since the cash-flows are all driven by the same underlying indicators, and the indicators are dependent on their past, there is a natural correlation between cash-flows induced by the path dependence in the indicators. If the indicators are high at the time of one cash-flow, resulting in a large cash-flow, then the probability of a large cash-flow at the next time-step is also high. This is a very desirable feature which has clear economic grounding. Contrastingly, in a number of practical approaches, the cash-flow distributions are typically assumed to be independent or correlation is introduced in a rather adhoc manner (as in the Datar and Mathews (2004) approach).

Focusing on a single sales or GM% distribution, our task is to determine φ^S and φ^M such that at the cash-flow date T, S_k and M_k possess the manager specified distribution $F^*(s)$ and $G^*(m)$ – we use an asterisk to remind the reader that these distributions are provided by the manager. This requirement can be restated as, find φ^S and φ^M such that

$$\mathbb{P}(S_T < s) = \mathbb{P}(\varphi^S(X_T) < s) = F^*(s) \qquad \text{and} \qquad \mathbb{P}(M_T < m) = \mathbb{P}(\varphi^M(Y_T) < v) = G^*(m)\,. \quad (6)$$

These matching conditions are of the same form for both sales and GM%, so we will focus only sales as the result for GM% is identical. The matching can be visualized as in Figure 2 for the case of a triangular managerial distribution.

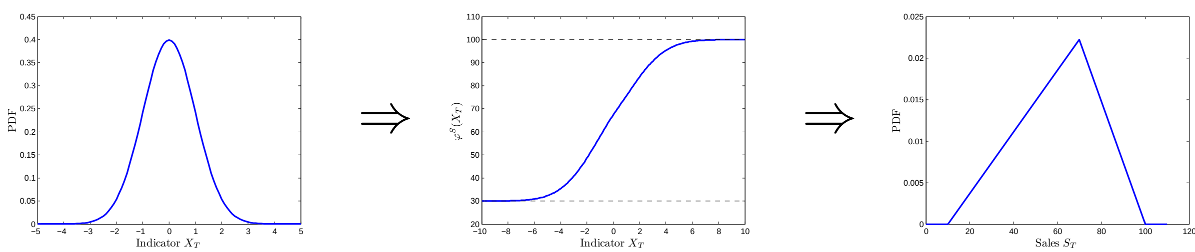


Figure 2: The underlying pdf $f_{X_T}(x)$ is mapped through the value function $\varphi^S(x)$ to match the triangular sales distribution.

It is not difficult to see that if $\varphi^S(x)$ is assumed invertible and the sales distribution $F^*(v)$ is invertible, then the solution is unique. However, invertibility is by no means necessary. Nonetheless, we restrict our analysis to this case as it leads to sound economically meaningful results. Note that the probability matching equation (6) can be also be interpreted as a quantile matching relationship where φ acts as a probability distortion function. Following Jaimungal and Lawryshyn (2010), who set up the problem with GBM and geometric mean-reverting indicators, we have the following the following result.

Proposition 1 The Replicating Payoff. *The payoff function $\varphi^S(x)$ which produces the manager specified distribution $F^*(v)$ for the sales at time T, when the underlying driving uncertainty X_t is a BM, is given by*

$$\varphi^S(x) = F^{*-1}\left(\Phi\left(\frac{x - x_0}{\sqrt{T}}\right)\right), \quad (7)$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function.

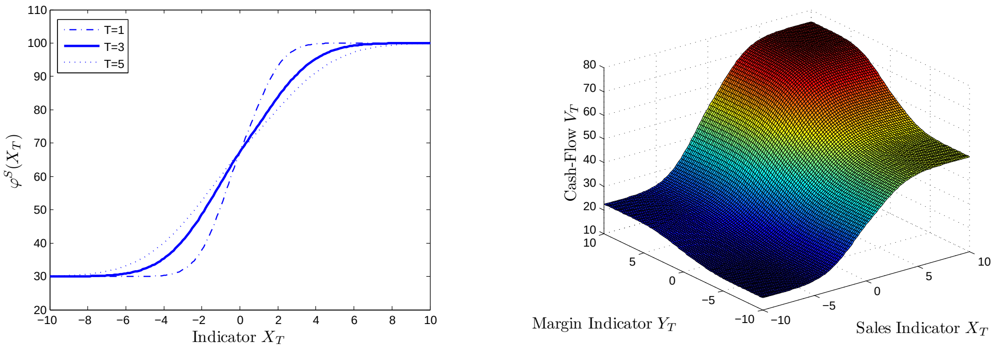


Figure 3: Left panel: The replicating payoff function of a triangular distribution for a BM sales indicator at three different maturities. Right panel: The cash-flow generated at $T = 3$ as a function of the sales and margin indicators. The triangular distribution parameters for sales are 100, 70, and 30 and for margin are 0.5, 0.6 and 0.75, with no fixed and variable costs.

If the estimated sales distribution has compact support, then the payoff function $\varphi^S(S)$ will be bounded above and below. For example, if F^* is a triangular distribution , then φ^S is bounded above and below by v_+ (maximum sales) and v_- (minimum sales), respectively, and there is a single point x_* at which the curvature changes sign. The left panel in Figure 3 shows the replicating payoff for three different maturities. Each of these payoffs produce the exact same triangular distribution, however they produce the distribution at three different maturity dates. In the right panel of Figure 3, the cash-flow V_T is shown as a function of the sales and margin indicators (X_T, Y_T).

The cash-flow matching described here can be embedded into many practical real-option valuation questions ranging from an irreversible investment in a project at a fixed future, to the timing option to invest in a project, or abandon a project, and so on. As well, theoretical approaches can be layered on top of our cash-flow matching. We, however, focus our attention to the case of irreversible investment in a project.

4 Option Valuation

In this section, we address how our framework can be used to value an option to invest in a project. Again, the formulation here follows closely that of Jaimungal and Lawryshyn (2010). In this formulation, as discussed above, the assumption is that managers will provide sales and GM% estimates at a sequence of dates. To provide some context to the following discussion, an example of sales and GM% estimates for a potential project are given in Table 1. Note that we have included deterministic fixed and variable components.

In realistic projects, upon investment, a manager will receive not just a single uncertain cash-flow at a fixed time T, but rather, a stream of uncertain cash-flows $v_1, v_2, \ldots, v_k$ at a sequence of dates $T_1, T_2, \ldots, T_n$. To value the option, the manager must provide a distributional assumption on the set of future cash-flows. In our original work, it was assumed that managers would provide the cash-flow distribution. Here, we provide a more practical framework where it is assumed the manager will provide sales and GM% estimates, from which the cash-flows can be determined, as per Equation 5. On the date T_0 of investment in the project, the project value V_{T_0} is the discounted expectation of the uncertain cash-flows under a risk-neutral measure $\mathbb{Q}$ – we will elaborate on how to choose the risk-neutral measure and its relationship to the real-world measure below. Moreover, the project value on this date becomes a function of the prevailing market sector indicator:

$$\begin{aligned} V_{T_0}(S_{T_0}, M_{T_0}) &= \sum_{k=1}^{n} e^{-r(T_k - T_0)}\, \mathbb{E}^{\mathbb{Q}}\left[v_k \mid S_{T_0}, M_{T_0} \right] \\ &= \sum_{k=1}^{n} e^{-r(T_k - T_0)}\, \mathbb{E}^{\mathbb{Q}}\left[\varphi_k(S_{T_k}) \mid S_{T_0}, M_{T_0} \right] . \end{aligned} \tag{8}$$

Here the notation $\mathbb{E}^{\mathbb{Q}}[\cdot]$ represents expectation with respect to the risk-neutral probability measure $\mathbb{Q}$. This expression shows that the project value can be viewed as a strip of European options on the sector indicator. Finally, the value, at time $t < T_0$, of the real option to invest in the project at a cost of K is given by the expectation

$$RO_t(S, M) = e^{-r(T_0 - t)}\, \mathbb{E}^{\mathbb{Q}}\left[\left(V_{T_0}(S_{T_0}, M_{T_0}) - K\right)_+ \middle| S_t = S,\, M_t = M \right]. \tag{9}$$

This expression shows that the real option to invest in the option is a compound option on the sector indicator. Our remaining task is to compute the expectations appearing in (8) and (9). These computations will require an understanding of the distribution of the individual cash-flows v_k under the risk-neutral measure. In the next subsection we derive the risk-neutral distribution under the assumption that the agent wishes to minimize the variance of a potential hedge.

We note that some approaches (such as decision tree analysis) instead use real-world probabilities and discount cash-flows at the weighted average cost of capital (WACC). Our methodology also applies to such valuation procedures, and, if a manager wishes to use them, the risk-neutral measure discussion below can be skipped, the rate r should be replaced with the WACC, and the risk-neutral distributions replaced with the real-world ones.

4.1 The Risk-Neutral Measure

If valuation is carried out using the real-world probabilities and the distribution of the future cash-flow is assumed triangular, then, our pricing results will be similar to those in Datar and Mathews (2004)[2]. However, this approach is not entirely consistent with economic theory. The main reason is that the real-world pricing measure should not be used. Furthermore, since the underlying source of uncertainty – the sector indicator – which drives the project value is not a traded asset, the market is incomplete. Jaimungal and Lawryshyn (2010) show that an appropriate risk-neutral measure $\mathbb{Q}$, corresponding to a variance minimizing hedge, is one under which we have the following dynamics

$$dX_t = \widehat{\nu}\, dt + \rho_{SI}\, d\widehat{B}_t + \sqrt{1 - \rho_{SI}^2}\, d\widehat{W}_t^S, \tag{10}$$

$$dY_t = \widehat{\gamma}\, dt + \rho_{SI}\rho_{SM} d\widehat{B}_t + \rho_{SM}\sqrt{1 - \rho_{SI}^2}\, d\widehat{W}_t^S + \sqrt{1 - \rho_{SM}^2}\, d\widehat{W}_t^M, \tag{11}$$

$$\frac{dI_t}{I_t} = r\, dt + \sigma\, d\widehat{B}_t \tag{12}$$

where $\widehat{B}_t$, $\widehat{W}_t^S$ and $\widehat{W}_t^M$ are standard uncorrelated Brownian motions under the risk-neutral $\mathbb{Q}$ and the risk-neutral drift of the indicators are

$$\widehat{\nu} = -\rho_{SI}\, \frac{\mu - r}{\sigma} \qquad \text{and} \qquad \widehat{\gamma} = -\rho_{SI}\rho_{SM}\, \frac{\mu - r}{\sigma} \tag{13}$$

[2]The results will not be exact because, as discussed previously, our method provides for a natural correlation among the cash-flows whereas Datar and Mathews utilize an adhoc approach.

Notice that the drift of the indicators are precisely the CAPM drift of an asset correlated to the market index.

4.2 Computing the Option Value

It is now straight forward to determine the project value at the time of investment. As discussed previously, the project value is given by the expectations appearing in (8), therefore we can either simulate the indicator processes X_t and Y_t, build a simple two-dimensional tree for valuation purposes, apply numerical integration, or formulate the problem as a partial differential equation (PDE). For simplicity, the PDE approach is chosen here.

First, we define $G_t(X,Y) \equiv RO_t(S(X), M(Y))$. Applying Ito's lemma, and standard techniques, the PDE becomes,

$$rG = \frac{\partial G}{\partial t} + \widehat{\nu}\frac{\partial G}{\partial x} + \widehat{\gamma}\frac{\partial G}{\partial y} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2} + \frac{1}{2}\frac{\partial^2 G}{\partial y^2} + \rho_{SM}\frac{\partial^2 G}{\partial x \partial y}. \tag{14}$$

The boundary condition at $t = T_n$ is given by $G(X_{T_n}, Y_{T_n}) = v_n$ (see Equation 5). A simple explicit routine is then applied recursively to solve Equation 14 such that at each cash-flow date, T_k, v_k is added to $G(X_{T_k}, Y_{T_k})$. At the option strike date, T_0, we calculate $(G(X_{T_0}, Y_{T_0}) - K)_+$.

To illustrate the methodology, consider a project for which a manager has provided the sales, GM%, fixed costs and variable costs estimates as in Table 1.

Table 1: Example sales, GM%, fixed costs and variable costs scenarios for a project which costs $50 to invest in at year 2.

Scenario	End of Year Sales / Margin						
	3	4	5	6	7	8	9
Optimistic	80	116	153	177	223	268	314
	(50%)	(60%)	(65%)	(60%)	(60%)	(55%)	(55%)
Most Likely	52	62	74	77	89	104	122
	(30%)	(40%)	(40%)	(40%)	(35%)	(35%)	(35%)
Pessimistic	20	23	24	18	20	20	22
	(20%)	(20%)	(20%)	(20%)	(15%)	(10%)	(10%)
SG&A*	10%	5%	5%	5%	5%	5%	5%
Fixed Costs	30	25	20	20	20	20	20

* Sales / General and Administrative Costs

The project can be invested in at the end of year 2 at a cost of $50. In Figure 4 we show how the value of the option to invest in the project varies as a function of the two indicators. The value of the project is then determined by determining $G(X_0, Y_0)$. For this example, $r = 0.05$, $\mu = 0.1$, $\rho_{SI} = 0.7$, $\rho_{SM} = 0.3$ and the real option value of the project was determined to be $6.7.

As we proposed in Jaimungal and Lawryshyn (2010), the correlation between the *market sector indicator*, or, in this case, the *sales sector indicator*, and the traded index can likely be determined based on historical data. For example, estimates of market size can be correlated to a given traded market index. However, it is likely that managers may struggle to estimate the correlation between sales and GM%. One approach to overcome this issue would be to look at historical data of sales versus GM%. The impact on option value for varying correlations, ρ_{SI} and ρ_{SM}, is presented in Figure 5. As can be seen, the real option value is not very sensitive to varying ρ_{SM}. Thus, managers do not need to expend much effort in determining this parameter. Ultimately, the most important parameters, besides sales and cost estimates, are the market parameters μ, σ and ρ_{SM} – the exact same parameters that are of greatest importance in standard DCF methods that rely on CAPM. Thus, we make the argument, that the method presented here requires no more estimation of parameters than what is normally done in DCF analysis, yet the method allows managers to properly value potential real options as well as properly value projects with uncertain cash flows, for the case where $K = 0$. We feel our method is superior to the DCF method even when optionally is not considered.

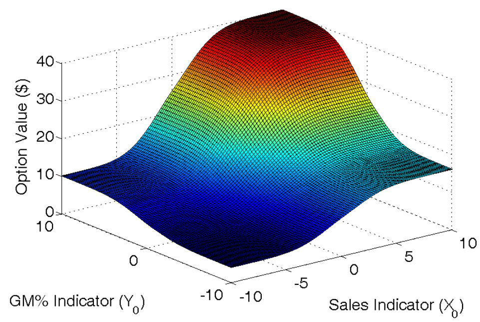


Figure 4: Value of the real option to invest in the project generating sales and margin in Table 1.

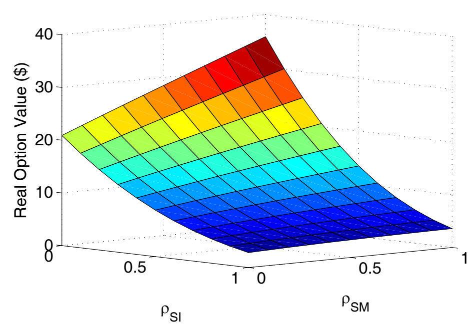
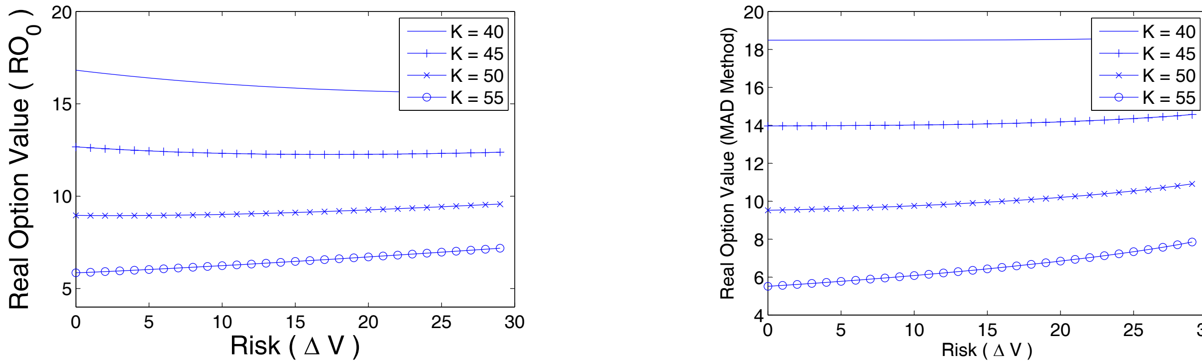


Figure 5: Value of the real option for varying ρ_{SI} and ρ_{SM}.

5 Sensitivity to Risk

When a manager is faced with the option to invest in two cash-flows, each with the same expected return (under the real-measure) but different volatilities, should he/she pick the cash-flow with higher or lower volatility? Standard real options approaches always result in choosing the more volatile cash-flow since (call) option values increase with volatility. However, there is an important ingredient missing in these approaches – they all distill down to assuming that the present value of the cash-flow is a tradable asset. This assumption is tacitly false[3], and in our approach we address

[3]In the case of the MAD approach, this assumption would not necessarily be false if one discounted the simulated cash flows with a discount rate that appropriately matched the risk of the project – i.e. a more volatile cash flow return distribution would be calculated using an appropriately higher discount rate than a less volatile one. However,



(a) Real Option Value (RO_0) (b) Real Option Value – MAD Approach

Figure 6: The sensitivity of the value of the real option on the risk ΔV for several investment costs, K, for our approach (a) and the MAD approach (b). The model parameters are as follows $S_0 = 50$, $\nu = 10\%$, $\eta = 20\%$, $\mu = 8\%$, $\sigma = 10\%$, $\rho = 0.5$ and $r = 5\%$.

this issue by using the sector indicator(s) to drive the cash-flow, which, itself, is not traded, but it is correlated to the tradable index. Here, we demonstrate with a single cash-flow, that increased volatility may result in higher or lower option values depending on the cost of investing as well as on the correlation between the sector indicator and the traded index.

Consider a triangular distribution with attachment points $30-\Delta V$, 70 and $100+\Delta V$. For every ΔV, these distributions all have the same expected value of $\frac{200}{3}$, however, as ΔV increases, the risk of the cash-flow increases. In Figure 6(a) we show how the value of the option to invest at $T_0 = 2$ in the cash-flow paid at $T_1 = 3$ depends on the investment cost K. As the figure shows, increasing the risk of the cash-flow sometimes first decreases the value of option before it begins to increase – this is quite distinct from other approaches. The underlying reason why the option value may decrease with increased risk is that, although the real-world expectations of each of these cash-flows are equal, the risk-neutral expectations are not. And, it is the risk-neutral expectation which determines the true value of the cash-flow, i.e. the project value. As a result, the project value is in fact a decreasing function of the cash-flow risk (when the $\mathbb{P}$-expectations are the same). Thus, for certain strike levels, this lower project value induces a lower option value, however, as the investment cost increases, the increased risk (and therefore upside potential) takes over the decrease in the project value, i.e. optionality wins out over the project value itself. Put another way, when the investment cost is zero (i.e. $K = 0$), there is no optionality and one expects that cash-flows would decrease in value as their uncertainty (volatility) increases. However, as the investment cost increases, the value of optionality grows and one eventually expects an increasing value with increased uncertainty.

The option value as calculated by applying the MAD method is presented in Figure 6(b). Since the MAD method treats the underlying value of the project as if it were traded, as do many other real options approaches, increasing project uncertainty will always lead to increasing value. As discussed above, this behavior is inconsistent with intuition. Contrastingly, but now consistent with intuition, our proposed approach will always exhibit decreasing and then increasing real option value as risk increases for $\rho > 0$ and nominal investment costs.

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the usual MAD approach is to use the same discount rate for all simulations.

6 Conclusions

This work builds on our previous proposed methodology and presents a real options approach that is practical to implement, requires limited detail regarding cash-flow estimates and minimal subjective estimation of market parameters, is consistent with financial theory, properly accounts for market and private risks, and ensures that the cash-flows are correlated appropriately among the time periods. Through the introduction of the *sales sector indicator* and the *GM% sector indicator*, we developed an approach where any managerial cash-flow can be properly valued by applying the minimum martingale measure. Thus, our approach is capable of accounting for both market and private risk. Furthermore, the matching approach ensures that cash-flows from one period to the next are correlated in a consistent manner. Many theoretical real options frameworks assume a simple stochastic process for the cash-flow value or the cash-flows themselves, and these processes cannot be used with real managerial cash-flow estimates. Our approach provides academics and practitioners with a method to bridge the gap between theoretical frameworks and practical estimation.

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