Tax Convexity, Investment, and Capital Structure [*]

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Abstract

This paper examines the interaction between investment and financing decisions of a firm using a real options approach. The firm is endowed with a perpetual option to invest in a project at any time by incurring an irreversible investment cost at that instant. The amount of the irreversible investment cost is directly related to the intensity of investment that is endogenously chosen by the firm. The firm is subject to a convex corporate income tax schedule in which profits are taxed at a higher rate while losses are taxed at a lower rate. At the investment instant, the firm can finance the project by issuing debt and equity such that the optimal capital structure is determined by the trade-off between interest tax-shield benefits and bankruptcy costs of debt. We show that the optimal investment intensity of the levered firm is identical to that of the unlevered firm. While the prevalence of tax convexity does not seem to affect the firm's investment decisions, it lowers the firm's optimal default trigger and leverage ratio in a quantitatively significant manner. Our findings thus suggest that any distortionary effect arising from tax convexity on the firm's investment decisions is almost completely neutralized by the adjustment in the firm's optimal capital structure.

JEL classification: G31; G32; G33

Keywords: Capital structure; Investment intensity; Investment timing; Tax convexity

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Abstract

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1. Introduction

Since the seminal work of Modigliani and Miller (1958), a rich theoretical literature has shown that market imperfections such as taxes, bankruptcy costs, agency conflicts, and information asymmetries drive linkages between corporate investment and financing decisions (see, e.g., Myers, 1974, 1977; Myers and Majluf, 1984; Dotan and Ravid 1985; Dammon and Senbet 1988; Mauer and Triantis 1994; Childs et al. 2005; Mauer and Sarkar, 2005; Wong, 2010). Most of the work in this literature, however, ignores the fact that corporate income tax schedules are by and large progressive and convex (see, e.g., Graham and Smith, 1999; Sarkar and Goukasian, 2006). To examine whether the prevalence of tax convexity has any significant effects on corporate financing decisions, Sarkar (2008) incorporates tax convexity—defined by asymmetric tax treatments of profits and losses—into a standard real options model of capital structure *à la* Leland (1994) and Goldstein et al. (2001). Based on extensive numerical simulations, Sarkar (2008) finds that tax convexity indeed affects leverage ratios and default boundaries in a quantitatively significant manner, and thereby cannot be ignored in corporate financing decisions.

The results of Sarkar (2008), albeit intriguing, are silent about how corporate investment and financing decisions interact when tax convexity prevails. The purpose of this paper is to fill the gap by extending the analysis of Sarkar (2008) to the case wherein both corporate investment and financing decisions are endogenously determined. To this end, we develop a continuous-time model of an owner-managed firm. The firm is initially endowed with a perpetual option to invest in a project at any time by incurring an irreversible investment cost at that instant. The amount of the irreversible investment cost determines the intensity of investment, which is a choice variable of the firm. The project generates a stream of stochastic earnings that follow a lognormal diffusion process and increase with the intensity of investment, as in Capozza and Li (1994, 2002) and Bar-Ilan and Strange (1999). We follow Sarkar (2008) to model tax convexity by a piecewise linear tax schedule, where profits are taxed at a higher rate while losses are taxed at a lower rate.

The firm makes three decisions regarding the undertaking of the project: the timing, intensity, and financing of investment. The firm's investment timing decision is characterized by a threshold (the investment trigger) such that the project is undertaken at the first instant when the earnings from the project reach the investment trigger from below (see, e.g., McDonald and Siegel 1986; Dixit and Pindyck 1994). The firm's investment intensity decision affects the amount of the irreversible investment cost according to a known technology that exhibits decreasing returns to scale. At the investment instant, the firm makes its financing decision by issuing debt and equity, where the debt issued is perpetual with a constant coupon payment per unit time. The firm chooses the optimal coupon payment so as to trade off the interest tax-shields against the bankruptcy costs of debt. The firm also chooses the optimal time to default on the debt obligation, which is tantamount to finding a threshold (the default trigger) such that the value of equity vanishes at the first instant when the earnings from the project reach the default trigger from above (see, e.g., Leland, 1994; Goldstein et al., 2001; Morellec, 2001). Upon default, equity holders get nothing and debt holders receive the liquidation value.

As a benchmark, we consider the firm's optimal investment decisions when only equity financing is allowed. We show that the unlevered firm chooses the optimal investment intensity that equates the value of the firm per unit intensity of investment to the marginal cost of investment at the investment instant. This is the usual optimality condition that the marginal return on investment is equal to the marginal cost of investment at the optimum. On the other hand, the unlevered firm chooses the optimal investment trigger taking into account the opportunity cost arising from killing the investment option when the project is undertaken, which is captured by the option value multiple (see Abel et al., 1996). The optimal investment trigger as such equates the value of the firm at the investment instant to the investment cost augmented by the option value multiple. Combining these two optimality conditions implies that the optimal investment intensity is the one at which the marginal cost of investment is equal to the average cost augmented by the option value multiple.

When the firm is allowed to issue debt and choose the coupon payment optimally, we show that the optimal investment intensity of the levered firm is identical to that of the unlevered firm. To understand the intuition of this seemingly surprising result, we consider the case wherein the coupon payment is exogenously given and not a choice variable of the firm. If the coupon payment is fixed at an amount below (above) the optimal level, the marginal interest tax-shield benefit of debt must be greater (smaller) than the marginal bankruptcy cost of debt. An increase in the investment intensity renders the fixed coupon payment to be further away from (closer to) the optimal level, thereby reducing (enhancing) that the marginal return on investment. On the other hand, the higher (lower) marginal interest tax-shield benefit of debt as compared to the marginal bankruptcy cost of debt makes the opportunity cost of killing the investment option go down (up). The optimal investment intensity as such equates the marginal cost of investment to the average cost plus the decrease (increase) in the opportunity cost per unit intensity of investment augmented by the option value multiple. When the coupon payment is optimally chosen, there are no longer any adjustments to the value of the investment option per unit intensity of investment and to the marginal return of investment. Hence, the levered firm and the unlevered firm adopt the same optimal investment intensity that equates the marginal cost of investment to the average cost augmented by the option value multiple.[1]

To examine the economic significance of tax convexity on the firm's investment and financing decisions, we follow Sarkar (2008) to conduct numerical analysis with a set of reasonable parameter values. We find that making the corporate income tax schedule convex does not affect the optimal investment intensity and raises the optimal investment trigger insignificantly. The prevalence of tax convexity, however, has non-trivial adverse effect on both the optimal default trigger and the optimal leverage ratio. Our findings suggest that any distortionary effect arising from tax convexity on the firm's investment decisions is almost completely neutralized by the adjustment in the firm's optimal capital structure. Hence, we can conclude that tax convexity can be ignored in corporate investment decisions, but cannot be ignored in corporate financing decisions (see also Sarkar, 2008).

The rest of this paper is organized as follows. Section 2 delineates our continuous-time model of a firm that is endowed with a perpetual option to invest in a project under uncertainty. The firm is subject to a convex corporate income tax schedule. Section 3 derives the values of debt and equity of the firm at the investment instant. Section 4 examines the firm's optimal investment decisions in the benchmark case of all-equity financing. Section 5 characterizes the optimal investment and financing decisions of the firm. Section 6 numerically measures the economic significance of tax

[1] Wong (2010) derives the neutrality of debt in investment intensity when corporate income tax schedules are linear and symmetric, which is a special case of our findings under tax convexity.

convexity on the firm's investment and financing decisions. The final section concludes.

2. The model

Consider a risk-neutral, owner-managed firm that has monopoly access to a perpetual option to invest in a project.[2] Time is continuous and indexed by $t \in [0, \infty)$. The default-free term structure is flat with a known instantaneous rate of interest, $r > 0$.

The firm makes three decisions regarding the undertaking of the project: the timing, intensity, and financing of investment. The firm's investment intensity, $q \geq 0$, affects the stream of stochastic earnings before interest and taxes (EBIT), $\{qX_t : t \geq 0\}$, generated from the project, where $X_t > 0$ is a state variable specifying the project's EBIT at time t per unit intensity of investment. The stochastic process, $\{X_t : t \geq 0\}$, is governed by the following geometric Brownian motion:

$$\mathrm{d}X_t = \mu X_t \ \mathrm{d}t + \sigma X_t \ \mathrm{d}Z_t, \tag{1}$$

where $\mu < r$ and $\sigma > 0$ are constant parameters, and $\mathrm{d}Z_t$ is the increment of a standard Wiener process under the risk-neutral probability space, $(\Omega, \mathcal{F}, \mathcal{Q})$.[3] Eq. (1) implies that the growth rate of X_t is normally distributed with a mean, $\mu\Delta t$, and a variance, $\sigma^2\Delta t$, over a time interval, Δt. The initial value of the state variable, $X_0 > 0$, is known at $t = 0$.

To undertake the project at endogenously chosen time, $t \geq 0$, and intensity, $q \geq 0$, the firm has to incur an irreversible investment cost, $I(q)$, at that instant, where $I(0) \geq 0$, $I'(0) = 0$, and $I'(q) > 0$ and $I''(q) > 0$ for all $q > 0$.[4] We further assume that the elasticity of the investment cost with respect to the intensity of investment, $qI'(q)/I(q)$, is strictly increasing in q. It is well known that finding the optimal time to invest in the project is tantamount to finding a threshold value, X_I, of the state variable, X_t, such that the firm optimally exercises the investment option at the first instant when X_t reaches X_I from below (see, e.g., McDonald and Siegel, 1986; Dixit and Pindyck, 1994). We refer to X_I as the investment trigger, which is a choice variable of the firm. Let $T_I = \inf\{t \geq 0 : X_t = X_I\}$ be the (random) first passage time of the state variable, X_t, to reach the investment trigger, X_I, from below, starting off at $t = 0$.

At the investment instant, T_I, the firm can issue debt and equity to finance the investment cost, $I(q)$. The debt issued by the firm is perpetual in that debt holders receive a constant coupon

[2] The assumption of risk neutrality is innocuous as long as there are arbitrage-free and complete financial markets in which assets can be traded to span the state variable that determines the value of the firm.

[3] The assumption that $\mu < r$ is needed to ensure that the value of the firm is finite.

[4] We allow for $I(0) > 0$ to account for some fixed set-up costs that are required to initiate the project. The strict convexity of $I(q)$ implies that the project exhibits decreasing returns to scale.

payment, $C > 0$, per unit time until default occurs, where C is a choice variable of the firm. The coupon payments to debt holders are tax-deductible so that the taxable income is $qX_t - C$ per unit time. As in Sarkar (2008), the firm pays corporate income taxes at a constant tax rate, $\tau \in (0, 1)$, if the taxable income is positive, i.e., if $qX_t > C$, and at another lower rate, $\theta\tau$, if the taxable income is negative, i.e., if $qX_t < C$, where $\theta \in [0, 1]$ is a constant. In the extreme case that $\theta = 0$, the corporate income tax schedule is convex and asymmetric with no loss-offset provisions. In the other extreme case that $\theta = 1$, the corporate income tax schedule becomes linear and symmetric with full loss-offset provisions. In general, a smaller value of θ signifies higher tax convexity. We can, therefore, interpret θ as a parameter that inversely gauges the degree of tax convexity in the corporate income tax schedule.

Equity holders have limited liability and thus the option to default on their debt obligations. The optimal policy for equity holders is to default at the first instant when the value of equity vanishes, which is equivalent to solving the default trigger, X_D, at which the value of equity vanishes as the state variable, X_t, reaches X_D the first time from above (see, e.g., Leland, 1994; Goldstein et al., 2001; Morellec, 2001).[5] Let $T_D = \inf\{t \geq T_I : X_t = X_D\}$ be the (random) first passage time at which the default trigger, X_D, is reached from above, starting off at the investment instant, T_I.

At the default instant, T_D, the firm is immediately liquidated and absolute priority is enforced. Following Mello and Parsons (1992) and Morellec (2001), we assume that, after default, the new owners continue to employ the asset in its current use to yield the unlevered value, $V^U(q, X_D)$:

$$V^U(q, X_D) = \mathrm{E}_{\mathcal{Q}}^{X_D}\left[\int_{T_D}^{\infty} e^{-r(t-T_D)}(1-\tau)qX_t \,\mathrm{d}t\right] = (1-\tau)\left(\frac{qX_D}{r-\mu}\right), \tag{2}$$

where $\mathrm{E}_{\mathcal{Q}}^{X_D}(\cdot)$ is the expectation operator with respect to the risk-neutral probability measure, $\mathcal{Q}$, conditional on X_D. The liquidation value of the firm at the default instant, T_D, is then given by $(1-b)V^U(q, X_D)$, where $b \in [0, 1]$ is a parameter gauging the severity of bankruptcy costs.[6] Since absolute priority is enforced, equity holders get nothing and debt holders receive the liquidation value upon default.

We summarize the firm's investment and financing decisions by a triple, (q, X_I, C), that specifies the investment intensity, q, the investment trigger, X_I, and the coupon payment, C. We solve the firm's decision problems by using backward induction and proceed in two steps. The first step is to derive the values of debt and equity after the firm has undertaken the project, fixing the investment intensity at $q > 0$ and the coupon payment at $C > 0$. Denote $X_D(q, C)$ as the default trigger

[5] This stock-based definition of default implies that it is optimal for equity holders to inject capital into the firm as long as the firm has positive economic net worth, and that the firm is insolvent on a flow basis at the default instant.

[6] Even when $b = 0$, the firm will not entirely finance by issuing debt. Too much leverage risks bankruptcy with the concomitant losses of the tax deductibility of coupon payments, thereby imposing limits on the usage of debt in the absence of bankruptcy costs (see Brennan and Schwartz, 1978; Leland, 1994).

endogenously chosen by equity holders. For all $X \in [X_D(q,C), C/q]$, the firm is currently suffering from negative taxable income and the value of equity is given by $E_0(q,X,C)$. On the other hand, for all $X \geq C/q$, the firm is currently receiving positive taxable income and the value of equity is given by $E_1(q,X,C)$. The value of debt is given by $D(q,X,C)$ for all $X \geq X_D(q,C)$.

The second step uses the arbitrage-free values of debt and equity to solve the firm's optimal investment and financing decisions at $t = 0$. Specifically, the firm issues perpetual debt to raise $D(q, X_I, C)$ at the investment instant, T_I. The difference, $I(q) - D(q, X_I, C)$, is raised from (paid to if negative) equity holders. The firm makes its investment and financing decisions, (q, X_I, C), so as to maximize the value of equity, $F^L(X_0)$, at $t = 0$ (i.e., the value of the investment option):

$$F^L(X_0) = \max_{q>0, X_I>X_0, C>0} \begin{cases} \mathrm{E}_{\mathcal{Q}}^{X_0}\left\{e^{-rT_I}[D(q,X_I,C) + E_0(q,X_I,C) - I(q)]\right\} & \text{if } C \geq qX_I, \\ \mathrm{E}_{\mathcal{Q}}^{X_0}\left\{e^{-rT_I}[D(q,X_I,C) + E_1(q,X_I,C) - I(q)]\right\} & \text{if } C < qX_I, \end{cases} \tag{3}$$

where $\mathrm{E}_{\mathcal{Q}}^{X_0}(\cdot)$ is the expectation operator with respect to the risk-neutral probability measure, $\mathcal{Q}$, conditional on X_0.

3. Valuation of debt and equity

In this section, we derive the values of debt and equity after the firm has undertaken the project, fixing the investment intensity at $q > 0$ and the coupon payment at $C > 0$.

Using the standard arbitrage arguments (see, e.g., Dixit and Pindyck, 1994), the value of debt, $D(q,X,C)$, must satisfy the following ordinary differential equation:

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 D(q,X,C)}{\partial X^2} + \mu X \frac{\partial D(q,X,C)}{\partial X} - rD(q,X,C) + C = 0, \tag{4}$$

for all $X \geq X_D(q,C)$, subject to the following two boundary conditions:

$$\lim_{X\to\infty} \frac{D(q,X,C)}{X} < \infty, \tag{5}$$

and

$$D[q, X_D(C), C] = (1-b)V^U[q, X_D(q,C)]. \tag{6}$$

Eq. (5) rules out speculative bubbles as X approaches infinity. Eq. (6) is the value-matching condition such that the value of debt is equal to the liquidation of the firm, $(1-b)V^U[q, X_D(q,C)]$, when X reaches the default trigger, $X_D(q,C)$.

Likewise, using the standard arbitrage arguments (see, e.g., Dixit and Pindyck, 1994), the value of equity when the current taxable income is negative, $E_0(q, X, C)$, and that when the current taxable income is positive, $E_1(q, X, C)$, must satisfy the following two ordinary differential equations:

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 E_0(q,X,C)}{\partial X^2} + \mu X \frac{\partial E_0(q,X,C)}{\partial X} - rE_0(q,X,C) + (1-\theta\tau)(qX - C) = 0, \tag{7}$$

for all $X \in [X_D(q,C), C/q]$, and

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 E_1(q,X,C)}{\partial X^2} + \mu X \frac{\partial E_1(q,X,C)}{\partial X} - rE_1(q,X,C) + (1-\tau)(qX - C) = 0, \tag{8}$$

for all $X \geq C/q$, respectively, subject to the following five boundary conditions:

$$\lim_{X\to\infty} \frac{E_1(q,X,C)}{X} < \infty, \tag{9}$$

$$E_0[q, X_D(q,C), C] = 0, \tag{10}$$

$$\left.\frac{\partial E_0(q,X,C)}{\partial X}\right|_{X=X_D(q,C)} = 0, \tag{11}$$

$$E_0(q, C/q, C) = E_1(q, C/q, C), \tag{12}$$

and

$$\left.\frac{\partial E_0(q,X,C)}{\partial X}\right|_{X=C/q} = \left.\frac{\partial E_1(q,X,C)}{\partial X}\right|_{X=C/q}. \tag{13}$$

Eq. (9) rules out speculative bubbles as X approaches infinity. Eq. (10) is the value-matching condition such that the value of equity vanishes when X reaches the default trigger, $X_D(q,C)$. Eq. (11) is the smooth-pasting condition such that the default trigger, $X_D(q,C)$, maximizes the value of equity. Eqs. (12) and (13) hold because the geometric Brownian motion described in Eq. (1) can freely diffuse across $X = C/q$, thereby rendering the equity value function to be continuously differentiable at $X = C/q$.[7]

Define the following two constants, α and β, that are derived from the fundamental quadratic equation, $\frac{1}{2}\sigma^2 y(y-1) + \mu y - r = 0$:[8]

$$\alpha = \frac{\mu}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 0, \tag{14}$$

[7] See Dixit (1993) for a heuristic argument and Karatzas and Shreve (1988) for a rigorous proof.

[8] That is, $-\alpha$ and β are the negative root and the positive root of the fundamental quadratic equation, $\frac{1}{2}\sigma^2 y(y-1) + \mu y - r = 0$, respectively.

and

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1. \tag{15}$$

Solving Eqs. (4), (7), and (8) subject to the given boundary conditions yields our first proposition.

Proposition 1. *Fixing the investment intensity at $q > 0$ and the coupon payment at $C > 0$, the firm's default trigger, $X_D(q, C)$, is the unique solution to the following equation:*

$$(1-\theta)\left(\frac{r+\mu\alpha}{r-\mu}\right)\frac{\tau C}{r} - (1-\theta\tau)(\alpha+1)\left[\frac{qX_D(q,C)}{r-\mu} - \left(\frac{\alpha}{\alpha+1}\right)\frac{C}{r}\right]\left[\frac{C}{qX_D(q,C)}\right]^{\beta} = 0. \tag{16}$$

The value of debt, $D(q, X, C)$, after the firm has undertaken the project is given by

$$D(q,X,C) = \frac{C}{r} - \left[\frac{C}{r} - (1-b)(1-\tau)\frac{qX_D(q,C)}{r-\mu}\right]\left[\frac{X_D(q,C)}{X}\right]^{\alpha}, \tag{17}$$

for all $X \geq X_D(q, C)$. The value of equity when the firm is currently suffering from negative taxable income, $E_0(q, X, C)$, is given by

$$E_0(q,X,C) = (1-\theta\tau)\left(\frac{qX}{r-\mu} - \frac{C}{r}\right) - \left(\frac{1-\theta}{\beta-1}\right)\left(\frac{\alpha}{\alpha+\beta}\right)\frac{\tau C}{r}\left(\frac{qX}{C}\right)^{\beta}$$
$$+(1-\theta\tau)\left(\frac{\beta-1}{\alpha+\beta}\right)\left[\left(\frac{\beta}{\beta-1}\right)\frac{C}{r} - \frac{qX_D(q,C)}{r-\mu}\right]\left[\frac{X_D(q,C)}{X}\right]^{\alpha}, \tag{18}$$

for all $X \in [X_D(q, C), C/q]$. The value of equity when the firm is currently receiving positive taxable income, $E_1(q, X, C)$, is given by

$$E_1(q,X,C) = (1-\tau)\left(\frac{qX}{r-\mu} - \frac{C}{r}\right) - \left(\frac{1-\theta}{\alpha+1}\right)\left(\frac{\beta}{\alpha+\beta}\right)\frac{\tau C}{r}\left(\frac{C}{qX}\right)^{\alpha}$$
$$+(1-\theta\tau)\left(\frac{\beta-1}{\alpha+\beta}\right)\left[\left(\frac{\beta}{\beta-1}\right)\frac{C}{r} - \frac{qX_D(q,C)}{r-\mu}\right]\left[\frac{X_D(q,C)}{X}\right]^{\alpha}, \tag{19}$$

for all $X \geq C/q$.

Proof. See Appendix A. □

When the corporate tax schedule is linear and symmetric, i.e., when $\theta = 1$, Eq. (16) reduces to

$$qX_D(q,C) = \left(\frac{\alpha}{\alpha+1}\right)\left(\frac{r-\mu}{r}\right)C < C, \tag{20}$$

so that the firm is insolvent on a flow basis at the default instant. In this case, Eqs. (18) and (19) imply that

$$E_0(q,X,C) = E_1(q,X,C) = (1-\tau)\left(\frac{qX}{r-\mu} - \frac{C}{r}\right) + \left(\frac{1-\tau}{\alpha+1}\right)\frac{C}{r}\left[\frac{X_D(q,C)}{X}\right]^{\alpha}. \tag{21}$$

where we have used Eq. (20). Using Eq. (20), we can write Eq. (21) as

$$\begin{aligned} E_0(q,X,C) &= E_1(q,X,C) \\ &= (1-\tau)\left(\frac{qX}{r-\mu} - \frac{C}{r}\right) + (1-\tau)\left[\frac{C}{r} - \frac{qX_D(q,C)}{r-\mu}\right]\left[\frac{X_D(q,C)}{X}\right]^{\alpha}. \end{aligned} \tag{22}$$

The first term on the right-hand side of Eq. (22) is the value of equity if default is forbidden. The second term captures the value of the default option, where $(1-\tau)[C/r - qX_D(q,C)/(r-\mu)]$ is the value of the savings from contributing additional equity capital to cover subsequent shortfalls at the instant when the default option is exercised, discounted to the present by the stochastic discount factor, $[X_D(q,C)/X]^{\alpha}$.

When the corporate tax schedule is convex and asymmetric, i.e., when $\theta \in [0,1)$, Eq. (16) implies that[9]

$$\left(\frac{\alpha}{\alpha+1}\right)\left(\frac{r-\mu}{r}\right)C < qX_D(q,C) < C. \tag{23}$$

Inspection of Eqs. (20) and (23) reveals that the firm raises its default trigger as the corporate income tax schedule becomes convex and asymmetric. Tax convexity thus induces the firm to default on its debt earlier. At the default instant, the firm remains insolvent on a flow basis. We can write Eq. (18) as

$$\begin{aligned} E_0(q,X,C) &= (1-\theta\tau)\left(\frac{qX}{r-\mu} - \frac{C}{r}\right) + (1-\theta\tau)\left[\frac{C}{r} - \frac{qX_D(q,C)}{r-\mu}\right]\Phi_1(q,X,C) \\ &\quad + \left[E_1(q,C/q,C) - (1-\theta\tau)\left(\frac{\mu}{r-\mu}\right)\frac{C}{r}\right]\Phi_2(q,X,C), \end{aligned} \tag{24}$$

for all $X \in [X_D(q,C), C/q]$, where

$$\Phi_1(q,X,C) = \left\{\left[\frac{X_D(q,C)}{X}\right]^{\alpha} - \left[\frac{qX_D(q,C)}{C}\right]^{\alpha}\left(\frac{qX}{C}\right)^{\beta}\right\}\Big/\left\{1 - \left[\frac{qX_D(q,C)}{C}\right]^{\alpha+\beta}\right\}, \tag{25}$$

and

$$\Phi_2(q,X,C) = \left\{\left(\frac{qX}{C}\right)^{\beta} - \left[\frac{qX_D(q,C)}{C}\right]^{\beta}\left[\frac{X_D(q,C)}{X}\right]^{\alpha}\right\}\Big/\left\{1 - \left[\frac{qX_D(q,C)}{C}\right]^{\alpha+\beta}\right\}. \tag{26}$$

[9] See Appendix A for the derivation.

We can interpret $\Phi_1(q, X, C)$ in Eq. (25) as the stochastic discount factor for one dollar received at the first instant when X reaches $X_D(q, C)$ from above, conditional on $X \in [X_D(q, C), C/q]$. Likewise, we can interpret $\Phi_2(q, X, C)$ in Eq. (26) as the stochastic discount factor for one dollar received at the first instant when X reaches C/q from below, conditional on $X \in [X_D(q, C), C/q]$. The first term on the right-hand side of Eq. (24) is the value of equity when the current taxable income is negative if default is forbidden. The second term captures the value of the default option, where $(1 - \theta\tau)[C/r - qX_D(q, C)/(r - \mu)]$ is the value of the savings from contributing additional equity capital to cover subsequent shortfalls at the instant when the default option is exercised, discounted to the present by the stochastic discount factor, $\Phi_1(q, X, C)$. The last term captures the option value of entering into a higher tax rate bracket, where $E_1(q, C/q, C) - (1 - \theta\tau)\mu C/r(r - \mu)$ is the change in equity value at the instant when the firm starts to receive positive taxable income, discounted to the present by the stochastic discount factor, $\Phi_2(q, X, C)$. We can write Eq. (19) as

$$E_1(q, X, C) = (1 - \tau)\left(\frac{qX}{r - \mu} - \frac{C}{r}\right) + \left[E_0(q, C/q, C) - (1 - \tau)\left(\frac{\mu}{r - \mu}\right)\frac{C}{r}\right]\left(\frac{C}{qX}\right)^{\alpha}, \qquad (27)$$

for all $X \geq C/q$. The first term on the right-hand side of Eq. (27) is the value of equity when the current taxable income is positive if default is forbidden. The second term captures the option value of entering into a lower tax rate bracket, where $E_0(q, C/q, C) - (1 - \tau)\mu C/r(r - \mu)$ is the change in equity value at the instant when the firm starts to suffer from negative taxable income, discounted to the present by the stochastic discount factor, $(C/qX)^{\alpha}$.

4. Benchmark case of all-equity financing

In this section, we consider a benchmark wherein the firm is restricted to finance the project with equity only (i.e., $C \equiv 0$). This is the case studied by Capozza and Li (1994, 2002) and Bar-Ilan and Strange (1999).

The value of the unlevered firm at $t = 0$, $F^U(X_0)$, is given by

$$F^U(X_0) = \max_{q>0, X_I>X_0} \mathrm{E}_{\mathcal{Q}}^{X_0}\left\{e^{-rT_I}[V^U(q, X_I) - I(q)]\right\}, \qquad (28)$$

where T_I is the investment instant, $\mathrm{E}_{\mathcal{Q}}^{X_0}(\cdot)$ is the expectation operator with respect to the risk-neutral probability measure, $\mathcal{Q}$, conditional on X_0, and $V^U(q, X_I)$ is defined in Eq. (2) with X_D replaced by X_I and T_D replaced by T_I. It is well-known (see, e.g., Karatzas and Shreve, 1988; Dixit and Pindyck, 1994) that

$$\mathrm{E}_{\mathcal{Q}}^{X_0}\left(e^{-rT_I}\right) = \left(\frac{X_0}{X_I}\right)^{\beta}, \qquad (29)$$

if $X_I > X_0$, where $\beta > 1$ is defined in Eq. (15). Substituting Eqs. (2) and (29) into Eq. (28) yields

$$F^U(X_0) = \max_{q>0, X_I>X_0} \left[(1-\tau)\left(\frac{qX_I}{r-\mu}\right) - I(q)\right]\left(\frac{X_0}{X_I}\right)^{\beta}. \tag{30}$$

The first-order conditions for the optimization problem on the right-hand side of Eq. (30) are given by

$$(1-\tau)\left(\frac{X_I^U}{r-\mu}\right) = I'(q^U), \tag{31}$$

and

$$(1-\tau)\left(\frac{q^U X_I^U}{r-\mu}\right) = \left(\frac{\beta}{\beta-1}\right)I(q^U), \tag{32}$$

where q^U and X_I^U are the optimal investment intensity and trigger of the unlevered firm, respectively.

Solving Eqs. (31) and (32) yields the following proposition.

Proposition 2. *The unlevered firm's optimal investment intensity, q^U, is the unique solution to*

$$I'(q^U) = \left(\frac{\beta}{\beta-1}\right)\frac{I(q^U)}{q^U}, \tag{33}$$

and the unlevered firm's optimal investment trigger, X_I^U, is given by

$$X_I^U = \left(\frac{r-\mu}{1-\tau}\right)\left(\frac{\beta}{\beta-1}\right)\frac{I(q^U)}{q^U}. \tag{34}$$

The value of the unlevered firm, $F^U(X_0)$, at $t=0$ is given by

$$F^U(X_0) = \left(\frac{\beta}{\beta-1} - 1\right)I(q^U)\left(\frac{X_0}{X_I^U}\right)^{\beta}. \tag{35}$$

Proof. Dividing Eq. (32) by q^U and substituting the resulting equation into Eq. (31) yields Eq. (33). The uniqueness of q^U follows from the fact that $qI'(q)/I(q)$ is strictly increasing in q. Rearranging terms of Eq. (32) yields Eq. (34). Substituting Eq. (32) into Eq. (30) with $q = q^U$ and $X_I = X_I^U$ yields Eq. (35). □

To see the intuition of Proposition 2, we use Eq. (2) to write Eqs. (31) and (32) as

$$\frac{V^U(q^U, X_I^U)}{q^U} = I'(q^U), \tag{36}$$

and

$$V^U(q^U, X_I^U) - I(q^U) = \left(\frac{\beta}{\beta - 1} - 1\right) I(q^U), \tag{37}$$

respectively. Eq. (36) states that the optimal investment intensity, q^U, equates the value of the unlevered firm per unit intensity of investment, $V^U(q^U, X_I^U)/q^U$, to the marginal cost of investment, $I'(q^U)$, at the investment instant. This is the usual optimality condition that the marginal return on investment is equal to the marginal cost of investment at the optimum. Eq. (37) states that the optimal investment trigger, X_I^U, is the one at which the the net present value of the project, $V^U(q^U, X_I^U) - I(q^U)$, is equal to the value of the investment option, $I(q^U)/(\beta - 1)$. The literature on irreversible investment under uncertainty refers to the expression, $\beta/(\beta - 1) > 1$, as the option value multiple (see Abel et al., 1996). It measures the wedge between the value of the project at the investment instant, $V^U(q^U, X_I^U)$, and the investment cost, $I(q^U)$, which captures the opportunity cost arising from killing the investment option when the project is undertaken, as is evident from Eq. (37). Combining Eqs. (36) and (37) yields Eq. (33). That is, at the optimal investment intensity, q^U, the marginal cost of investment, $I'(q^U)$, is equal to the average cost, $I(q^U)/q^U$, augmented by the option value multiple, $\beta/(\beta - 1)$.

5. Optimal investment and financing decisions

In this section, we solve the firm's optimal investment and financing decisions by using the arbitrage-free values of debt and equity derived in Section 3. Let $V^L(q, X_I, C)$ be the value of the levered firm at the investment instant, T_I:

$$V^L(q, X_I, C) = \begin{cases} D(q, X_I, C) + E_0(q, X_I, C) & \text{if } C \geq qX_I, \\ D(q, X_I, C) + E_1(q, X_I, C) & \text{if } C < qX_I, \end{cases} \tag{38}$$

where $D(q, X_I, C)$, $E_0(q, X_I, C)$, and $E_1(q, X_I, C)$ are given by Eqs. (17), (18), and (19), respectively, with $X = X_I$.

Substituting Eqs. (29) and (38) into Eq. (3) yields the value of the levered firm, $F^L(X_0)$, at $t = 0$:

$$F^L(X_0) = \max_{q>0, X_I>X_0, C>0} [V^L(q, X_I, C) - I(q)] \left(\frac{X_0}{X_I}\right)^\beta. \tag{39}$$

The first-order conditions for the optimization problem on the right-hand side of Eq. (39) imply the following three optimality conditions:[10]

$$\frac{V^L(q^L, X_I^L, C^L)}{q^L} = I'(q^L), \tag{40}$$

$$V^L(q^L, X_I^L, C^L) - I(q^L) = \left(\frac{\beta}{\beta - 1} - 1\right) I(q^L), \tag{41}$$

and

$$\left.\frac{\partial V^L(q, X_I, C)}{\partial C}\right|_{q=q^L, X_I=X_I^L, C=C^L} = 0, \tag{42}$$

where q^L, X_I^L, and C^L are the optimal investment intensity, investment trigger, and coupon payment of the levered firm, respectively.

Solving Eqs. (40), (41), and (42) yields the following proposition.

Proposition 3. *Irrespective of whether the corporate income tax schedule is linear or convex, the optimal investment intensity of the levered firm, q^L, is identical to that of the unlevered firm, q^U. Furthermore, the optimal investment trigger of the levered firm, X_I^L, is strictly less than that of the unlevered firm, X_I^U. The value of the levered firm, $F^L(X_0)$, at $t = 0$ is given by*

$$F^L(X_0) = F^U(X_0) + \left[\left(\frac{X_I^U}{X_I^L}\right)^\beta - 1\right] F^U(X_0). \tag{43}$$

Proof. Subtracting Eq. (40) from Eq. (41) yields

$$I'(q^L) = \left(\frac{\beta}{\beta - 1}\right) \frac{I(q^L)}{q^L}. \tag{44}$$

Since $qI'(q)/I(q)$ is strictly increasing in q, Eqs. (33) and (44) imply that $q^L = q^U$. Substituting Eqs. (35) and (41) into Eq. (39) and using the fact that $q^L = q^U$ yields Eq. (43). Since the optimal choice of the unlevered firm is feasible to, but is not chosen by, the levered firm ($C^L > 0$), it must be the case that $F^L(X_0) > F^U(X_0)$. It then follows from Eq. (43) that $X_I^L < X_I^U$. □

The intuition of Proposition 3 is as follows. The optimality conditions for the levered firm's optimal investment decision, Eqs. (40) and (41), take on the same form as those for the unlevered firm's optimal investment decision, Eqs. (36) and (37). The levered firm's optimal investment intensity, q^I, as such is characterized by the one that equates the marginal cost of investment,

[10]See Appendix B for the derivation of Eqs. (40) and (41).

$I'(q^I)$, to the average cost, $I(q^I)/q^I$, augmented by the option value multiple, $\beta/(\beta-1)$, i.e., by Eq. (44). Since the elasticity of the investment cost with respect to the intensity of investment, $qI'(q)/I(q)$, is strictly increasing in q, Eqs. (33) and (44) imply that $q^L = q^U$, thereby rendering the neutrality of debt in investment intensity. Wong (2010) derives such a neutrality result when the corporate income tax schedule is linear and symmetric. Proposition 3 thus extends Wong's (2010) result to the more general setting wherein the corporate income tax schedule is allowed to be convex and asymmetric.

Since $q^U = q^L$, Eqs. (37) and (41) imply that the net present value of the project to the unlevered firm at the investment instant, $T_I^U = \inf\{t \geq 0 : X_t = X_I^U\}$, is exactly the same as that to the levered firm at the investment instant, $T_I^L = \inf\{t \geq 0 : X_t = X_I^L\}$. Owning to the interest tax-shield benefits of debt, the levered firm is induced to lower the optimal investment trigger as compared to the unlevered firm, i.e., $X_I^L < X_I^U$, so as to invest in the project earlier.[11] The value of the levered firm, $F^L(X_0)$, at $t = 0$ as such exceeds that of the unlevered firm, $F^U(X_0)$, by the value of the interest tax-shield benefits of debt (net of bankruptcy costs), which is given by the second term on the right-hand side of Eq. (43).

Proposition 3 shows that debt is neutral to investment intensity if the firm can choose its capital structure optimally. It is of interest to see whether such a neutrality result remains intact if the coupon payment, $C > 0$, is exogenously given and not a choice variable of the firm. In this case, the first-order conditions for the optimization problem on the right-hand side of Eq. (39) imply the following pair of optimality conditions:[12]

$$\frac{1}{q(C)}\left\{V^L[q(C), X_I(C), C] - \frac{\partial V^L[q(C), X_I(C), C]}{\partial C}\right\} = I'[q(C)], \tag{45}$$

and

$$V^L[q(C), X_I(C), C] - I[q(C)] = \left(\frac{\beta}{\beta - 1} - 1\right)\left\{I[q(C)] - \frac{\partial V^L[q(C), X_I(C), C]}{\partial C}\right\}, \tag{46}$$

where $q(C)$ and $X_I(C)$ are the optimal investment intensity and trigger of the levered firm, respectively, for a fixed value of C.

Solving Eqs. (45) and (46) yields the following proposition.

Proposition 4. *For an exogenously given coupon payment, $C > 0$, the optimal investment intensity of the levered firm, $q(C)$, is less than, equal to, or greater than that of the unlevered firm, q^U, if C*

[11] It is well known (see, e.g., Sarkar 2000; Shackleton and Wojakowski 2002; Wong 2007) that the expected time to exercise the investment option (the investment timing) is given by $\mathrm{E}_{\mathcal{Q}}^{X_0}(T_I) = \ln(X_I/X_0)/(\mu - \sigma^2/2)$, whenever $\mu > \sigma^2/2$. Hence, the investment trigger and the investment timing are positively related.

[12] See Appendix C for the derivation of Eqs. (45) and (46).

is less than, equal to, or greater than the optimal coupon payment, C^L, respectively.

Proof. At the optimal coupon payment, C^L, we have $q(C^L) = q^L$ and $X_I(C^L) = X_I^L$. When $C = C^L$, Eq. (42) implies that Eqs. (45) and (46) reduce to Eqs. (40) and (41), respectively. Dividing Eq. (46) by $q(C)$ and subtracting the resulting equation from Eq. (45) yields

$$I'[q(C)] = \left(\frac{\beta}{\beta - 1}\right)\frac{1}{q(C)}\left\{I[q(C)] - \frac{\partial V^L[q(C), X_I(C), C]}{\partial C}\right\}. \tag{47}$$

It then follows from Eq. (42) and the second order conditions for the optimization problem on the right-hand side of Eq. (39) that the right-hand side of Eq. (47) is positive, zero, or negative, depending on whether C is less than, equal to, or greater than C^L, respectively. Since $qI'(q)/I(q)$ is strictly increasing in q, Eqs. (33) and (47) imply that $q(C)$ is less than, equal to, or greater than q^U, depending on whether C is less than, equal to, or greater than C^L, respectively. □

The intuition of Proposition 4 is as follows. Inspection of Eqs. (45) and (46) reveals that we need to take the effect of the exogenously given coupon payment, C, on the value of the levered firm, $V^L[q(C), X_I(C), C]$, into account when analyzing the firm's optimal investment decisions, which is captured by

$$\frac{\partial V^L[q(C), X_I(C), C]}{\partial C} > (<)\ 0 \quad \text{if} \quad C < (>)\ C^L, \tag{48}$$

where the inequality follows from Eq. (42) and the second order conditions for the optimization problem on the right-hand side of Eq. (39). Eq. (48) implies that the marginal interest tax-shield benefit of debt is greater (smaller) than the marginal bankruptcy cost of debt should C be smaller (greater) than the optimal coupon payment, C^L. An increase in the investment intensity renders C to be further away from (closer to) C^L if $C < (>)\ C^L$, thereby implying that the marginal return on investment is reduced (enhanced) by an amount, $[1/q(C)] \times \partial V^L[q(C), X_I(C), C]/\partial C$, as is evident from the left-hand side of Eq. (45). The right-hand of Eq. (46) is the value of the investment option at the investment instant, which gauges the opportunity cost when the project is undertaken. If $C < (>)\ C^L$, the marginal interest tax-shield benefit of debt is greater (smaller) than the marginal bankruptcy cost of debt, thereby implying that the opportunity cost of killing the investment option goes down (up) by an amount, $[1/(\beta - 1)] \times \partial V^L[q(C), X_I(C), C]/\partial C$, as is evident from the right-hand side of Eq. (46). Since the adjustment to the value of the investment option per unit intensity of investment and that to the marginal return of investment are not the same if C differs from C^L, the firm's optimal investment intensity is no longer determined by the optimality condition in the benchmark case of all-equity financing. Specifically, if $C < (>)\ C^L$, the optimal investment intensity, $q(C)$, is the one that equates the marginal cost of investment, $I'[q(C)]$, to the average cost,

$I[q(C)]/q(C)$, plus the decrease (increase) in the opportunity cost per unit intensity of investment, $-[1/q(C)] \times \partial V^L[q(C), X_I(C), C]/\partial C$, augmented by the option value multiple, $\beta/(\beta-1)$, thereby implying that $q(C) < (>)\ q^U$. The inflexibility of the coupon payment as such renders debt to be non-neutral to investment intensity.

6. Numerical analysis

To gain more insight into the economic significance of tax convexity on the firm's optimal investment and financing decisions, we have to rely on numerical analysis. To this end, we set the investment cost function equal to $I(q) = 10 + q^4$. Following Sarkar (2008), we adopt the following parameter values: the annualized riskless rate of interest, r, is 8%; the corporate income tax rate, τ, is 15%; the bankruptcy cost parameter, b, is 30%; and the state variable, X_t, takes on the initial value, $X_0 = 1$, with the annualized growth rate, $\mu = 2\%$, and the annualized standard deviation, $\sigma = 30\%$.[13] Table 1 reports our numerical results for three different values of the tax convexity parameter ($\theta = 1$, 0.4, and 0), and for six different coupon payments ($C = 0$, 1, 3, 5, 7, and 9).

(Insert Table 1 here)

The first row in Table 1 reports the optimal investment intensity and trigger of the unlevered firm, $q^U = 2.0551$ and $X_I^U = 2.4508$, respectively. As is evident from Table 1, higher tax convexity (i.e., a smaller value of θ) increases the optimal investment intensity, $q(C)$, the optimal investment trigger, $X_I(C)$, and the optimal default trigger, $X_D[q(C), C]$, but decreases the optimal leverage ratio, $D[q(C), X_I(C), C]/V^L[q(C), X_I(C), C]$, for all $C = 1$, 3, 5, 7, and 9. Making the corporate income tax schedule more convex reduces the marginal interest tax-shield benefit of debt and thus decreases $\partial V^L[q(C), X_I(C), C]/\partial C$. It then follows that the firm is induced to invest later by raising $X_I(C)$, to default earlier by lifting $X_D[q(C), C]$, and to rely less on debt by lowering $D[q(C), X_I(C), C]/V^L[q(C), X_I(C), C]$, in response to an increase in the degree of tax convexity. From Eq. (47) and the fact that $qI'(q)/I(q)$ is strictly increasing in q, it must be true that $q(c)$ increases as the corporate income tax schedule becomes more convex. Table 1 shows that $q(C)$ and $X_I(C)$ are smaller than q^U and X_I^U, respectively, for all $C = 1$ and 3, and that $q(C)$ and $X_I(C)$ are greater than q^U and X_I^U, respectively, for all $C = 5$, 7, and 9. Furthermore, for all $C = 1$, 3, 5, and 7, we have $C < q(C)X_I(C)$ so that the firm optimally invests in the project at the instant when the EBIT, $q(C)X_I(C)$, exceeds the exogenously given coupon payment, C. When $C = 9$, we have

[13] Sarkar (2008) uses a growth rate of X_t equal to $12\% - 6\% = 6\%$, where 12% is the required rate of return on X_t based on the true probability density and 6% is the payout rate. Under the risk-neutrality density, the growth rate of X_t becomes $8\% - 6\% = 2\%$, where 8% is the riskless rate of interest.

$C < q(C)X_I(C)$ if $\theta = 0$, and $C > q(C)X_I(C)$ if $\theta = 0.4$ and 1. In the latter case, the firm optimally invests in the project at the instant when the EBIT is not enough to cover the high coupon payment, particularly when the degree of tax convexity is not too high.

To examine the economic significance of tax convexity on the firm's investment and financing decisions, we calculate the maximum percentage change in each of the four key variables when $\theta = 0$ to the corresponding one when $\theta = 1$. For the optimal investment intensity, $q(C)$, the maximum percentage change ranges from 0.26% when $C = 1$ to 2.22% when $C = 9$, implying that tax convexity does not seem to affect $q(C)$ in any significant manner. For the optimal investment trigger, $X_I(C)$, the maximum percentage change ranges from 0.70% when $C = 1$ to 7.07% when $C = 9$, which is economically significant only when C is unreasonably large. For the optimal default trigger, $X_D[q(C), C]$, the maximum percentage change ranges from 3.18% when $C = 9$ to 5.18% when $C = 1$, whereas for the optimal leverage ratio, $D[q(C), X_I(C), C]/V^L[q(C), X_I(C), C]$, the maximum percentage change ranges from -1.01% when $C = 1$ to -5.67% when $C = 9$. Tax convexity seems to have rather limited effect on either variable. We can thus conclude that making the corporate income tax schedule more convex is unlikely to significantly affect the firm's investment and financing decisions when the coupon payment is exogenously given.

(Insert Table 2 here)

Table 2 reports our numerical results for six different values of the tax convexity parameter ($\theta = 1$, 0.8, 0.6, 0.4, 0.2, and 0), where the firm is allowed to choose its coupon payment optimally. As is shown in Table 2, the optimal coupon payment, C^L, ranges from 3.0167 when $\theta = 0$ to 3.4815 when $\theta = 1$. The findings that the optimal investment intensity, $q(C)$, is smaller than that of the unlevered firm, q^U, for all $C = 1$ and 3, and that $q(C)$ is greater than q^U for all $C = 5$, 7, and 9 in Table 1 are thus consistent with the results in Proposition 4. Table 2 shows that the optimal investment intensity, q^L, is 2.0551, which is invariant to the degree of tax convexity and equal to q^U. The optimal investment trigger, X_I^L, ranges from 2.3341 when $\theta = 1$ to 2.3497 when $\theta = 0$, thereby implying that X_I^L is less than the optimal investment trigger of the unlevered firm, X_I^U. These findings are consistent with the results in Proposition 3. The maximum percentage change in X_I^L when $\theta = 0$ to that when $\theta = 1$ is only 0.67%, implying that tax convexity has at most trivial effect on X_I^L. We can thus conclude that any distortionary effect arising from tax convexity on the firm's investment decisions is almost completely neutralized by the adjustment in the firm's optimal capital structure.

Inspection of Table 2 reveals that higher tax convexity (i.e., a smaller value of θ) decreases, rather than increases, the optimal default trigger, $X_D(q^L, C^L)$, which is contradictory to the findings in Table 1. The reason for the difference is that the firm adjusts the optimal coupon payment, C^L, downward in response to an increase in the degree of tax convexity, as is evident from Table 2. This makes default less likely, thereby inducing the firm to lower the optimal default trigger. The maximum percentage change in $X_D(q^L, C^L)$ when $\theta = 0$ to that when $\theta = 1$ is -8.61%. For a reasonable increase in tax convexity from $\theta = 1$ to $\theta = 0.4$, the percentage change in $X_D(q^L, C^L)$ is -5.64%, which remains economically significant. The optimal leverage ratio, $D(q^L, X_I^L, C^L)/V^L(q^L, X_I^L, C^L)$, ranges from 44.69% when $\theta = 0$ to 50.27% when $\theta = 1$, which is consistent with the average leverage ratio of 44% for US firms documented in Rajan and Zingales (1995). The maximum percentage change in $D(q^L, X_I^L, C^L)/V^L(q^L, X_I^L, C^L)$ when $\theta = 0$ to that when $\theta = 1$ is -11.10%. For a reasonable increase in tax convexity from $\theta = 1$ to $\theta = 0.4$, the percentage change in $D(q^L, X_I^L, C^L)/V^L(q^L, X_I^L, C^L)$ is -7.18%, which is by all means economically significant. Hence, if the firm is allowed to choose the coupon payment optimally, tax convexity has non-trivial effect on both the optimal default trigger and the optimal leverage ratio. Sarkar (2008) reaches a similar conclusion that tax convexity cannot be ignored in corporate financing decisions when capital structure is endogenously determined.

7. Conclusion

In this paper, we have examined the interaction between investment and financing decisions of a risk-neutral, owner-managed firm using a real options approach. The firm is endowed with a perpetual option to invest in a project at any time by incurring an irreversible investment cost at that instant. The amount of the irreversible investment cost determines the intensity of investment with decreasing returns to scale. The project generates a stream of stochastic earnings that follow a lognormal diffusion process and increase with the intensity of investment. Following Sarkar (2008), we have assumed that the firm is subject to a convex corporate income tax schedule in which profits are taxed at a higher rate while losses are taxed at a lower rate. The firm can finance the project by issuing debt and equity at the investment instant, where the firm chooses its optimal capital structure by trading off between interest tax-shield benefits and bankruptcy costs of debt as in Leland (1994) and Goldstein et al. (2001).

Within our real options model, we have established a neutrality result of debt in investment intensity in that the optimal investment intensity of the levered firm is identical to that of the unlevered firm. Furthermore, we have numerically measured the economic significance of tax convexity on the firm's investment and financing decisions. While the prevalence of tax convexity does not seem to affect the firm's investment decisions, it lowers the firm's optimal default trigger and leverage ratio in a quantitatively significant manner. Hence, we can conclude that tax convexity can be ignored in corporate investment decisions, but cannot be ignored in corporate financing decisions (see also Sarkar, 2008).

Appendix

A. *Proof of Proposition 1*

The solution to Eq. (4) is given by

$$D(q, X, C) = \frac{C}{r} + K_1 X^{-\alpha} + K_2 X^{\beta}, \tag{A.1}$$

for all $X \geq X_D(q, C)$, where K_1 and K_2 are constants to be determined by Eqs. (5) and (6), and α and β are given by Eqs. (14) and (15), respectively. Since $\beta > 1$, it follows from Eqs. (5) and (A.1) that $K_2 = 0$. Solving K_1 from Eq. (6) and Eq. (A.1) with $K_2 = 0$ yields

$$K_1 = \left\{(1-b)V^U[q, X_D(q, C)] - \frac{C}{r}\right\} X_D(q, C)^{\alpha}. \tag{A.2}$$

Substituting Eqs. (2) and (A.2) into Eq. (A.1) with $K_2 = 0$ yields Eq. (17).

The solutions to Eqs. (7) and (8) are given by

$$E_0(q, X, C) = (1 - \theta\tau)\left(\frac{qX}{r-\mu} - \frac{C}{r}\right) + K_3 X^{-\alpha} + K_4 X^{\beta}, \tag{A.3}$$

for all $X \in [X_D(q, C), C/q]$, and

$$E_1(q, X, C) = (1 - \tau)\left(\frac{qX}{r-\mu} - \frac{C}{r}\right) + K_5 X^{-\alpha} + K_6 X^{\beta}, \tag{A.4}$$

for all $X \geq C/q$, where K_3 to K_6 are constants to be determined by Eqs. (9) to (13). Since $\beta > 1$, it follows from Eqs. (9) and (A.4) that $K_6 = 0$. Using Eq. (A.3), we can write Eqs. (10) and (11) as

$$(1 - \theta\tau)\left[\frac{qX_D(q, C)}{r-\mu} - \frac{C}{r}\right] + K_3 X_D(q, C)^{-\alpha} + K_4 X_D(q, C)^{\beta} = 0, \tag{A.5}$$

and

$$(1 - \theta\tau)\frac{q}{r-\mu} - \alpha K_3 X_D(q, C)^{-\alpha-1} + \beta K_4 X_D(q, C)^{\beta-1} = 0, \tag{A.6}$$

respectively. Multiplying β to Eq. (A.5) and $X_D(q,C)$ to Eq. (A.6), and subtracting the resulting equations yields

$$K_3 = (1-\theta\tau)\left(\frac{\beta-1}{\alpha+\beta}\right)\left[\left(\frac{\beta}{\beta-1}\right)\frac{C}{r} - \frac{qX_D(q,C)}{r-\mu}\right]X_D(q,C)^{\alpha}. \tag{A.7}$$

Multiplying α to Eq. (A.5) and $X_D(q,C)$ to Eq. (A.6), and adding the resulting equations yields

$$K_4 = (1-\theta\tau)\left(\frac{\alpha+1}{\alpha+\beta}\right)\left[\left(\frac{\alpha}{\alpha+1}\right)\frac{C}{r} - \frac{qX_D(q,C)}{r-\mu}\right]X_D(q,C)^{-\beta}. \tag{A.8}$$

Using Eq. (A.3) and Eq. (A.4) with $K_6 = 0$, we can write Eqs. (12) and (13) as

$$(1-\theta\tau)\left(\frac{\mu}{r-\mu}\right)\frac{C}{r} + K_3\left(\frac{C}{q}\right)^{-\alpha} + K_4\left(\frac{C}{q}\right)^{\beta} = (1-\tau)\left(\frac{\mu}{r-\mu}\right)\frac{C}{r} + K_5\left(\frac{C}{q}\right)^{-\alpha}, \tag{A.9}$$

and

$$(1-\theta\tau)\frac{q}{r-\mu} - \alpha K_3\left(\frac{C}{q}\right)^{-\alpha-1} + \beta K_4\left(\frac{C}{q}\right)^{\beta-1} = (1-\tau)\frac{q}{r-\mu} - \alpha K_5\left(\frac{C}{q}\right)^{-\alpha-1}. \tag{A.10}$$

Multiplying α to Eq. (A.9) and C/q to Eq. (A.10), and adding the resulting equations yields

$$(1-\theta)\left(\frac{r+\mu\alpha}{r-\mu}\right)\frac{\tau C}{r} + (\alpha+\beta)K_4\left(\frac{C}{q}\right)^{\beta} = 0. \tag{A.11}$$

Substituting Eq. (A.8) into Eq. (A.11) yields Eq. (16).

Since $-\alpha$ and β are the negative and positive roots of the fundamental quadratic equation, $\frac{1}{2}\sigma^2 y(y-1) + \mu y - r = 0$, we have

$$r + \mu\alpha = \frac{1}{2}\sigma^2\alpha(\alpha+1), \tag{A.12}$$

and

$$r - \mu\beta = \frac{1}{2}\sigma^2\beta(\beta-1). \tag{A.13}$$

From Eqs. (15) and (14), we have

$$\alpha\beta = \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2} - \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 = \frac{2r}{\sigma^2}, \tag{A.14}$$

and

$$(\alpha+1)(\beta-1) = \alpha\beta + \beta - \alpha - 1 = \frac{2(r-\mu)}{\sigma^2}. \tag{A.15}$$

Substituting Eqs. (A.12), (A.14), and (A.15) into Eq. (16) yields

$$(1-\theta)\frac{\tau C}{\beta} - (1-\theta\tau)\left[qX_D(q,C) - \left(\frac{\beta-1}{\beta}\right)C\right]\left[\frac{C}{qX_D(q,C)}\right]^{\beta} = 0. \tag{A.16}$$

Define the following function:

$$G(X) = (1-\theta)\frac{\tau C}{\beta} - (1-\theta\tau)\left[qX - \left(\frac{\beta-1}{\beta}\right)C\right]\left(\frac{C}{qX}\right)^{\beta}. \tag{A.17}$$

Differentiating Eq. (A.17) with respect to X yields

$$G'(X) = (1-\theta\tau)(\beta-1)\left(\frac{qX-C}{X}\right)\left(\frac{C}{qX}\right)^{\beta}, \tag{A.18}$$

and

$$G''(X) = (1-\theta\tau)(\beta-1)\left[\frac{(\beta+1)C-\beta qX}{X^2}\right]\left(\frac{C}{qX}\right)^{\beta}. \tag{A.19}$$

Eq. (A.19) implies that $G(X)$ is strictly convex (concave) for all $X < (>)\ C(\beta+1)/\beta q$. Eq. (A.17) implies that $G(X)$ approaches infinity (zero) as X goes to zero (infinity). Eq. (A.18) then implies that $G(X)$ attains a local minimum at $X = C/q$ and a local maximum as X approaches infinity. Evaluating Eq. (A.17) at $X = C(\beta-1)/\beta q$ yields

$$G\left[\left(\frac{\beta-1}{\beta}\right)\frac{C}{q}\right] = (1-\theta)\frac{\tau C}{\beta} \geq 0, \tag{A.20}$$

since $\theta \in [0,1]$, where the equality holds only when $\theta = 1$. Evaluating Eq. (A.17) at $X = C/q$ yields

$$G\left(\frac{C}{q}\right) = (1-\theta)\frac{\tau C}{\beta} - (1-\theta\tau)\frac{C}{\beta}\left(\frac{C}{C}\right)^{\beta} = -(1-\tau)\frac{C}{\beta} < 0, \tag{A.21}$$

since $\tau \in (0,1)$. Hence, the default trigger, $X_D(q,C)$, is uniquely determined by Eq. (16).

Using Eqs. (A.11), (A.12), and (A.15), we have

$$K_4 = -\left(\frac{1-\theta}{\beta-1}\right)\left(\frac{\alpha}{\alpha+\beta}\right)\frac{\tau C}{r}\left(\frac{C}{q}\right)^{-\beta}. \tag{A.22}$$

Substituting Eqs. (A.7) and (A.22) into Eq. (A.3) yields Eq. (18). Multiplying β to Eq. (A.9) and C/q to Eq. (A.10), and subtracting the resulting equations yields

$$(1-\theta)\left(\frac{r-\mu\beta}{r-\mu}\right)\frac{\tau C}{r} - (\alpha+\beta)(K_3-K_5)\left(\frac{C}{q}\right)^{-\alpha} = 0. \tag{A.23}$$

Substituting (A.13) and (A.15) into Eq. (A.23) yields

$$K_5 = K_3 - \left(\frac{1-\theta}{\alpha+1}\right)\left(\frac{\beta}{\alpha+\beta}\right)\frac{\tau C}{r}\left(\frac{C}{q}\right)^{\alpha}. \tag{A.24}$$

Substituting Eqs. (A.7) and (A.24) into Eq. (A.4) with $K_6 = 0$ yields Eq. (19).

B. *Derivation of Eqs. (40) and (41)*

Differentiating Eq. (16) with respect to q yields

$$\frac{\partial X_D(q,C)}{\partial q} = -\frac{X_D(q,C)}{q}. \tag{A.25}$$

Differentiating Eq. (16) with respect to C yields

$$\frac{\partial X_D(q,C)}{\partial C} = \frac{X_D(q,C)}{C}. \tag{A.26}$$

Suppose first that $C^L \geq q^L X_I^L$. In this case, the first-order conditions for the optimization problem on the right-hand side of Eq. (39) are given by

$$\begin{aligned}
&(1-\theta\tau)\left(\frac{q^L X_I^L}{r-\mu}\right) + \alpha\left[\frac{C^L}{r} - (1-b)(1-\tau)\frac{q^L X_D(q^L,C^L)}{r-\mu}\right]\left[\frac{X_D(q^L,C^L)}{X_I^L}\right]^{\alpha} \\
&-(1-\theta\tau)\alpha\left(\frac{\beta-1}{\alpha+\beta}\right)\left[\left(\frac{\beta}{\beta-1}\right)\frac{C^L}{r} - \frac{q^L X_D(q^L,C^L)}{r-\mu}\right]\left[\frac{X_D(q^L,C^L)}{X_I^L}\right]^{\alpha} \\
&-(1-\theta)\left(\frac{\beta}{\beta-1}\right)\left(\frac{\alpha}{\alpha+\beta}\right)\frac{\tau C^L}{r}\left(\frac{q^L X_I^L}{C^L}\right)^{\beta} - q^L I'(q^L) = 0,
\end{aligned} \tag{A.27}$$

$$\begin{aligned}
&(1-\theta\tau)\left(\frac{q^L X_I^L}{r-\mu}\right) - \left(\frac{\alpha+\beta}{\beta-1}\right)\left[\frac{C^L}{r} - (1-b)(1-\tau)\frac{q^L X_D(q^L,C^L)}{r-\mu}\right]\left[\frac{X_D(q^L,C^L)}{X_I^L}\right]^{\alpha} \\
&+(1-\theta\tau)\left[\left(\frac{\beta}{\beta-1}\right)\frac{C^L}{r} - \frac{q^L X_D(q^L,C^L)}{r-\mu}\right]\left[\frac{X_D(q^L,C^L)}{X_I^L}\right]^{\alpha} \\
&+\left(\frac{\beta}{\beta-1}\right)\left[\frac{\theta\tau C^L}{r} - I(q^L)\right] = 0,
\end{aligned} \tag{A.28}$$

and

$$\frac{\theta\tau C^L}{r} - (\alpha+1)\left[\frac{C^L}{r} - (1-b)(1-\tau)\frac{q^L X_D(q^L,C^L)}{r-\mu}\right]\left[\frac{X_D(q^L,C^L)}{X_I^L}\right]^{\alpha}$$

$$+(1-\theta\tau)(\alpha+1)\left(\frac{\beta-1}{\alpha+\beta}\right)\left[\left(\frac{\beta}{\beta-1}\right)\frac{C^L}{r}-\frac{q^L X_D(q^L,C^L)}{r-\mu}\right]\left[\frac{X_D(q^L,C^L)}{X_I^L}\right]^{\alpha}$$

$$+(1-\theta)\left(\frac{\alpha}{\alpha+\beta}\right)\frac{\tau C^L}{r}\left(\frac{q^L X_I^L}{C^L}\right)^{\beta}=0, \qquad (A.29)$$

where we have used Eqs. (A.25) and (A.26). Multiplying $\alpha/(\alpha+1)$ to Eq. (A.29) and adding the resulting equation to Eq. (A.27) yields

$$(1-\theta\tau)\left(\frac{q^L X_I^L}{r-\mu}\right)+\left(\frac{\alpha}{\alpha+1}\right)\frac{\tau C^L}{r}\left[\theta-\left(\frac{1-\theta}{\beta-1}\right)\left(\frac{q^L X_I^L}{C^L}\right)^{\beta}\right]=q^L I'(q^L). \qquad (A.30)$$

Multiplying $(\alpha+\beta)/(\beta-1)(\alpha+1)$ to Eq. (A.29) and subtracting the resulting equation from Eq. (A.28) yields

$$(1-\theta\tau)\left(\frac{q^L X_I^L}{r-\mu}\right)+\left(\frac{\alpha}{\alpha+1}\right)\frac{\tau C^L}{r}\left[\theta-\left(\frac{1-\theta}{\beta-1}\right)\left(\frac{q^L X_I^L}{C^L}\right)^{\beta}\right]=\left(\frac{\beta}{\beta-1}\right)I(q^L). \qquad (A.31)$$

Using Eqs. (17), (18), (38), and (A.29), we have

$$V^L(q^L,X_I^L,C^L)=(1-\theta\tau)\left(\frac{q^L X_I^L}{r-\mu}\right)+\left(\frac{\alpha}{\alpha+1}\right)\frac{\tau C^L}{r}\left[\theta-\left(\frac{1-\theta}{\beta-1}\right)\left(\frac{q^L X_I^L}{C^L}\right)^{\beta}\right]. \qquad (A.32)$$

Substituting Eq. (A.32) into Eqs. (A.30) and (A.31) yields Eq. (40) and Eq. (41), respectively.

Suppose now that $C^L < q^L X_I^L$. In this case, the first-order conditions for the optimization problem on the right-hand side of Eq. (39) are given by

$$(1-\tau)\left(\frac{q^L X_I^L}{r-\mu}\right)+\alpha\left[\frac{C^L}{r}-(1-b)(1-\tau)\frac{q^L X_D(q^L,C^L)}{r-\mu}\right]\left[\frac{X_D(q^L,C^L)}{X_I^L}\right]^{\alpha}$$

$$-(1-\theta\tau)\alpha\left(\frac{\beta-1}{\alpha+\beta}\right)\left[\left(\frac{\beta}{\beta-1}\right)\frac{C^L}{r}-\frac{q^L X_D(q^L,C^L)}{r-\mu}\right]\left[\frac{X_D(q^L,C^L)}{X_I^L}\right]^{\alpha}$$

$$+(1-\theta)\left(\frac{\alpha}{\alpha+1}\right)\left(\frac{\beta}{\alpha+\beta}\right)\frac{\tau C^L}{r}\left(\frac{C^L}{q^L X_I^L}\right)^{\alpha}-q^L I'(q^L)=0, \qquad (A.33)$$

$$(1-\tau)\left(\frac{q^L X_I^L}{r-\mu}\right)-\left(\frac{\alpha+\beta}{\beta-1}\right)\left[\frac{C^L}{r}-(1-b)(1-\tau)\frac{q^L X_D(q^L,C^L)}{r-\mu}\right]\left[\frac{X_D(q^L,C^L)}{X_I^L}\right]^{\alpha}$$

$$+(1-\theta\tau)\left[\left(\frac{\beta}{\beta-1}\right)\frac{C^L}{r}-\frac{q^L X_D(q^L,C^L)}{r-\mu}\right]\left[\frac{X_D(q^L,C^L)}{X_I^L}\right]^{\alpha}$$

$$-\left(\frac{1-\theta}{\alpha+1}\right)\left(\frac{\beta}{\beta-1}\right)\frac{\tau C^L}{r}\left(\frac{C^L}{q^L X_I^L}\right)^{\alpha}+\left(\frac{\beta}{\beta-1}\right)\left[\frac{\tau C^L}{r}-I(q^L)\right]=0, \qquad (A.34)$$

and

$$\frac{\tau C^L}{r} - (\alpha+1)\left[\frac{C^L}{r} - (1-b)(1-\tau)\frac{q^L X_D(q^L, C^L)}{r-\mu}\right]\left[\frac{X_D(q^L, C^L)}{X_I^L}\right]^\alpha$$

$$+(1-\theta\tau)(\alpha+1)\left(\frac{\beta-1}{\alpha+\beta}\right)\left[\left(\frac{\beta}{\beta-1}\right)\frac{C^L}{r} - \frac{q^L X_D(q^L, C^L)}{r-\mu}\right]\left[\frac{X_D(q^L, C^L)}{X_I^L}\right]^\alpha$$

$$-(1-\theta)\left(\frac{\beta}{\alpha+\beta}\right)\frac{\tau C^L}{r}\left(\frac{C^L}{q^L X_I^L}\right)^\alpha = 0, \tag{A.35}$$

where we have used Eqs. (A.25) and (A.26). Multiplying $\alpha/(\alpha+1)$ to Eq. (A.35) and adding the resulting equation to Eq. (A.33) yields

$$(1-\tau)\left(\frac{q^L X_I^L}{r-\mu}\right) + \left(\frac{\alpha}{\alpha+1}\right)\frac{\tau C^L}{r} = q^L I'(q^L). \tag{A.36}$$

Multiplying $(\alpha+\beta)/(\beta-1)(\alpha+1)$ to Eq. (A.35) and subtracting the resulting equation from Eq. (A.34) yields

$$(1-\tau)\left(\frac{q^L X_I^L}{r-\mu}\right) + \left(\frac{\alpha}{\alpha+1}\right)\frac{\tau C^L}{r} = \left(\frac{\beta}{\beta-1}\right)I(q^L). \tag{A.37}$$

Using Eqs. (17), (18), (38), and (A.29), we have

$$V^L(q^L, X_I^L, C^L) = (1-\tau)\left(\frac{q^L X_I^L}{r-\mu}\right) + \left(\frac{\alpha}{\alpha+1}\right)\frac{\tau C^L}{r}. \tag{A.38}$$

Substituting Eq. (A.38) into Eqs. (A.36) and (A.37) yields Eq. (40) and Eq. (41), respectively.

C. Derivation of Eqs. (45) and (46)

Suppose first that $C \geq q(C)X_I(C)$. In this case, the first-order conditions for the optimization problem on the right-hand side of Eq. (39) are given by

$$(1-\theta\tau)\left[\frac{q(C)X_I(C)}{r-\mu}\right] + \alpha\left\{\frac{C}{r} - (1-b)(1-\tau)\frac{q(C)X_D[q(C), C]}{r-\mu}\right\}\left\{\frac{X_D[q(C), C]}{X_I(C)}\right\}^\alpha$$

$$-(1-\theta\tau)\alpha\left(\frac{\beta-1}{\alpha+\beta}\right)\left\{\left(\frac{\beta}{\beta-1}\right)\frac{C}{r} - \frac{q(C)X_D[q(C), C]}{r-\mu}\right\}\left\{\frac{X_D[q(C), C]}{X_I(C)}\right\}^\alpha$$

$$-(1-\theta)\left(\frac{\beta}{\beta-1}\right)\left(\frac{\alpha}{\alpha+\beta}\right)\frac{\tau C}{r}\left[\frac{q(C)X_I(C)}{C}\right]^\beta - q(C)I'[q(C)] = 0, \tag{A.39}$$

and

$$(1-\theta\tau)\left[\frac{q(C)X_I(C)}{r-\mu}\right]-\left(\frac{\alpha+\beta}{\beta-1}\right)\left\{\frac{C}{r}-(1-b)(1-\tau)\frac{q(C)X_D[q(C),C]}{r-\mu}\right\}\left\{\frac{X_D[q(C),C]}{X_I(C)}\right\}^{\alpha}$$
$$+(1-\theta\tau)\left[\left(\frac{\beta}{\beta-1}\right)\frac{C}{r}-\frac{q(C)X_D[q(C),C]}{r-\mu}\right\}\left\{\frac{X_D[q(C),C]}{X_I(C)}\right\}^{\alpha}$$
$$+\left(\frac{\beta}{\beta-1}\right)\left\{\frac{\theta\tau C}{r}-I[q(C)]\right\}=0. \tag{A.40}$$

Using the Envelope Theorem to differentiate $V^L[q(C),X_I(C),C]$ with respect to C yields

$$\frac{\partial V^L[q(C),X_I(C),C]}{\partial C}$$
$$=\frac{\theta\tau C}{r}-(\alpha+1)\left\{\frac{C}{r}-(1-b)(1-\tau)\frac{q(C)X_D[q(C),C]}{r-\mu}\right\}\left\{\frac{X_D[q(C),C]}{X_I(C)}\right\}^{\alpha}$$
$$+(1-\theta\tau)(\alpha+1)\left(\frac{\beta-1}{\alpha+\beta}\right)\left\{\left(\frac{\beta}{\beta-1}\right)\frac{C}{r}-\frac{q(C)X_D[q(C),C]}{r-\mu}\right\}\left\{\frac{X_D[q(C),C]}{X_I(C)}\right\}^{\alpha}$$
$$+(1-\theta)\left(\frac{\alpha}{\alpha+\beta}\right)\frac{\tau C}{r}\left[\frac{q(C)X_I(C)}{C}\right]^{\beta}, \tag{A.41}$$

where we have used Eqs. (17) and (18). Substituting Eqs. (17) and (18) with $q=q(C)$ and $X_I=X_I(C)$ and Eq. (A.41) into Eqs. (A.39) and (A.40) yields Eqs. (45) and (46), respectively.

Suppose now that $C<q(C)X_I(C)$. In this case, the first-order conditions for the optimization problem on the right-hand side of Eq. (39) are given by

$$(1-\tau)\left[\frac{q(C)X_I(C)}{r-\mu}\right]+\alpha\left\{\frac{C}{r}-(1-b)(1-\tau)\frac{q(C)X_D[q(C),C]}{r-\mu}\right\}\left\{\frac{X_D[q(C),C]}{X_I(C)}\right\}^{\alpha}$$
$$-(1-\theta\tau)\alpha\left(\frac{\beta-1}{\alpha+\beta}\right)\left\{\left(\frac{\beta}{\beta-1}\right)\frac{C}{r}-\frac{q(C)X_D[q(C),C]}{r-\mu}\right\}\left\{\frac{X_D[q(C),C]}{X_I(C)}\right\}^{\alpha}$$
$$+(1-\theta)\left(\frac{\alpha}{\alpha+1}\right)\left(\frac{\beta}{\alpha+\beta}\right)\frac{\tau C}{r}\left[\frac{C}{q(C)X_I(C)}\right]^{\alpha}-q(C)I'[q(C)]=0, \tag{A.42}$$

and

$$(1-\tau)\left[\frac{q(C)X_I(C)}{r-\mu}\right]-\left(\frac{\alpha+\beta}{\beta-1}\right)\left[\frac{C}{r}-(1-b)(1-\tau)\frac{q(C)X_D[q(C),C]}{r-\mu}\right\}\left\{\frac{X_D[q(C),C]}{X_I(C)}\right\}^{\alpha}$$
$$+(1-\theta\tau)\left\{\left(\frac{\beta}{\beta-1}\right)\frac{C}{r}-\frac{q(C)X_D[q(C),C]}{r-\mu}\right\}\left\{\frac{X_D[q(C),C]}{X_I(C)}\right\}^{\alpha}$$

$$-\left(\frac{1-\theta}{\alpha+1}\right)\left(\frac{\beta}{\beta-1}\right)\frac{\tau C}{r}\left[\frac{C}{q(C)X_I(C)}\right]^{\alpha}+\left(\frac{\beta}{\beta-1}\right)\left\{\frac{\tau C}{r}-I[q(C)]\right\}=0, \tag{A.43}$$

Using the Envelope Theorem to differentiate $V^L[q(C), X_I(C), C]$ with respect to C yields

$$\begin{aligned}
&\frac{\partial V^L[q(C), X_I(C), C]}{\partial C}\\
&=\frac{\tau C}{r}-(\alpha+1)\left\{\frac{C}{r}-(1-b)(1-\tau)\frac{q(C)X_D[q(C),C]}{r-\mu}\right\}\left\{\frac{X_D[q(C),C]}{X_I(C)}\right\}^{\alpha}\\
&+(1-\theta\tau)(\alpha+1)\left(\frac{\beta-1}{\alpha+\beta}\right)\left\{\left(\frac{\beta}{\beta-1}\right)\frac{C}{r}-\frac{q(C)X_D[q(C),C]}{r-\mu}\right\}\left\{\frac{X_D[q(C),C]}{X_I(C)}\right\}^{\alpha}\\
&-(1-\theta)\left(\frac{\beta}{\alpha+\beta}\right)\frac{\tau C}{r}\left[\frac{C}{q(C)X_I(C)}\right]^{\alpha},
\end{aligned} \tag{A.44}$$

where we have used Eqs. (17) and (19). Substituting Eqs. (17) and (19) with $q = q(C)$ and $X_I = X_I(C)$ and Eq. (A.44) into Eqs. (A.42) and (A.43) yields Eqs. (45) and (46), respectively.

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Table 1. Behavior of the firm facing the exogenously given coupon payment for different degrees of tax convexity

Coupon payment C	Tax convexity θ	Investment intensity $q(C)$	Investment trigger $X_I(C)$	Default trigger $X_D[q(C), C]$	Leverage ratio $\frac{D[q(C),X_I(C),C]}{V^L[q(C),X_I(C),C]}$
0	$[0,1]$	2.0551	2.4508	0	0
1	1	1.9828	2.1911	0.1968	0.1879
1	0.4	1.9860	2.2006	0.2031	0.1867
1	0	1.9879	2.2064	0.2070	0.1860
3	1	2.0251	2.2533	0.5780	0.4654
3	0.4	2.0434	2.3121	0.5921	0.4527
3	0	2.0540	2.3466	0.6010	0.4456
5	1	2.1576	2.6398	0.9041	0.5796
5	0.4	2.1840	2.7374	0.9234	0.5600
5	0	2.1991	2.7943	0.9356	0.5489
7	1	2.2868	3.0801	1.1943	0.6330
7	0.4	2.3176	3.2091	1.2182	0.6105
7	0	2.3352	3.2844	1.2334	0.5978
9	1	2.4020	3.5228	1.4618	0.6632
9	0.4	2.4359	3.6800	1.4902	0.6392
9	0	2.4553	3.7717	1.5083	0.6256

The risk-neutral, owner-managed firm has an option to invest in a project. The firm's investment decisions are characterized by the investment trigger, X_I, at which the investment option is exercised, and by the investment intensity, q, according to the investment cost function, $I(q) = 10 + q^4$. At the investment instant, the firm raise the amount, $D(q, X_I, C)$, from debt holders subject to the exogenously given coupon payment, $C \geq 0$, and the rest, $I(q) - D(q, X_I, C)$, from equity holders. The parameter values are as follows: the riskless rate of interest, r, is 8%; the corporate income tax rate is either τ or $\theta\tau$, depending on whether the taxable income is positive or negative, respectively, where $\tau = 15\%$ and $\theta \in [0, 1]$; the bankruptcy cost parameter, b, is 30%; and the state variable, X_t, takes on the initial value, $X_0 = 1$, with the annualized growth rate, $\mu = 2\%$, and the annualized standard deviation, $\sigma = 30\%$.

Table 2. Behavior of the firm for different degrees of tax convexity

Tax convexity	Investment intensity	Investment trigger	Coupon payment	Default trigger	Leverage ratio
θ	q^L	X_I^L	C^L	$X_D(q^L, C^L)$	$\frac{D(q^L, X_I^L, C^L)}{V^L(q^L, X_I^L, C^L)}$
1	2.0551	2.3341	3.4815	0.6609	0.5027
0.8	2.0551	2.3379	3.3702	0.6472	0.4896
0.6	2.0551	2.3412	3.2692	0.6348	0.4776
0.4	2.0551	2.3443	3.1775	0.6236	0.4666
0.2	2.0551	2.3471	3.0936	0.6134	0.4564
0	2.0551	2.3497	3.0167	0.6040	0.4469

The risk-neutral, owner-managed firm has an option to invest in a project. The firm's investment decisions are characterized by the investment trigger, X_I, at which the investment option is exercised, and by the investment intensity, q, according to the investment cost function, $I(q) = 10 + q^4$. At the investment instant, the firm has to choose the coupon payment, C, to raise the amount, $D(q, X_I, C)$, from debt holders, and the rest, $I(q) - D(q, X_I, C)$, from equity holders. The parameter values are as follows: the riskless rate of interest, r, is 8%; the corporate income tax rate is either τ or $\theta\tau$, depending on whether the taxable income is positive or negative, respectively, where $\tau = 15\%$ and $\theta \in [0, 1]$; the bankruptcy cost parameter, b, is 30%; and the state variable, X_t, takes on the initial value, $X_0 = 1$, with the annualized growth rate, $\mu = 2\%$, and the annualized standard deviation, $\sigma = 30\%$.