

Option to Acquire, LBOs and Debt Ratio in a Growing Industry

Makoto Goto*

May 17, 2010

Abstract

In this paper, we investigate LBO in a growing industry where the target company has a growth option. Especially, we focus the bidder's option to acquire the target company. An important setting is that the optimal timing is determined under the capital constraint. As our main results, we show that a growth option leads to delay in LBO, high leverage and low risk, and that default risk has the opposite sensitivity before/after growth.

Keywords: M&A; LBO; Growth options

JEL classification: D81; D92; G32; G34

1 Introduction

M&A is one of the most important topics for all public firms. Especially, leveraged buyouts (LBOs) is increasing in recent years rapidly. LBOs need small capital due to debt financing and are used in management buyouts (MBOs). Goto et al. (2009) investigated the mechanism of LBO by using takeover frameworks of Lambrecht and Myers (2007) in a declining industry. They focus the bidder's option to acquire the target company, and find the optimal timing and expending capital, moreover the optimal capital structure of the new company.

Due to Lambrecht and Myers (2007), M&A can be divided into the following two types:

1. a type of seeking a synergy effect and a growth opportunity,
2. a type of seeking effectiveness by dismissal, integration and disinvestment.

Goto et al. (2009) is categorized to the second type. Our focus is a growing industry where the target company has a growth option, so our type is the first one.

M&A is widely investigated from the view point of both practical and theoretical aspects. We orient a theoretical analysis, especially using a real options approach.

*Graduate School of Economics and Business Administration, Hokkaido University. Address: Kita 9, Nishi 7, Kita-ku, Sapporo 060-0809, Japan; E-mail: goto@econ.hokudai.ac.jp

Existing literature using a real options approach has the following studies. Shleifer and Vishny (2003) claim misvalue in stock markets causes takeover. Rhodes-Kropf and Viswanathan (2004) show market bias leads to correlation between takeover action and market estimation. Lambrecht (2004) provides an M&A model motivated scale economy in a growing industry. Morellec and Zhdanov (2005) analyze the roll of multi-bidders and imperfect information on takeover action. Lambrecht and Myers (2007) provides a real options model of takeover and disinvestment in a declining industry. Leland (2007), Lambrecht and Myers (2008) and Tian et al. (2008) investigate M&A using debts.

Goto et al. (2009) focused LBO in a declining industry and show that uncertainty leads to delay in LBO, and that LBO leads to junk bonds, usual leverage and low risk. Key points in this work are following. We focus LBO in a growing industry where the target company has a growth option. And we investigate the impact of growth options on LBO. As our main results, we show that a growth option leads to delay in LBO, high leverage and low risk, and that default risk has the opposite sensitivity before/after growth.

The rest of the paper is organized as follows. Section 2 describes the model settings. Section 3 derive the value functions. Next, we present numerical analyses in Section 4. Lastly, Section 5 concludes the paper.

2 The model

We consider the target company whose capital is all equity. X_t is the EBIT of target company:

$$dX_t = \alpha X_t dt + \sigma X_t dW_t, \quad (1)$$

where α is the instantaneous expected growth rate of X_t , $\sigma (> 0)$ is the instantaneous volatility of X_t , and W_t is a standard Brownian motion. The target company is assumed to have an opportunity to expand its business to γ times by paying out the cost K . In other words, the target company has a growth option. We have the value of the target company:

$$V_T(x) = \sup_{t_T \in \mathcal{T}} \mathbb{E} \left[\int_t^{t_T} e^{-r(s-t)} (1-\tau) X_s ds + \int_{t_T}^{\infty} e^{-r(s-t)} (1-\tau) \gamma X_s ds - e^{-r(t_T-t)} K \middle| X_t = x \right], \quad (2)$$

where r is the discount rate, τ is the tax rate, \mathcal{T} denotes the collection of admissible stopping times in $[t, \infty)$. The growth time is

$$t_T = \inf\{t > 0 : X_t \geq X_T\}, \quad (3)$$

where X_T is the growth threshold.

Then the bidder establishes a special purpose vehicle (SPV) to acquire the target by capitalizing $I > 0$. The SPV issues a corporate bond whose value is D and coupon payment is c . The bond is a nonrecourse loan. The SPV acquires the target's stock and they merge into a new subsidiary company of the bidder at time t_B . And we need the following capital constraint:

$$V_T(X_{t_B}) = I + D. \quad (4)$$

3 The Value Functions

We have the value of the target company analytically:

$$V_T(x) = \begin{cases} A_T x^{\beta_1} + \frac{(1-\tau)x}{r-\alpha}, & \text{for } x < X_T, \\ \frac{(1-\tau)\gamma x}{r-\alpha}, & \text{for } x \geq X_T, \end{cases} \quad (5)$$

$$A_T = \left(\frac{(1-\tau)(\gamma-1)X_T}{r-\alpha} - K \right) X_T^{-\beta_1}, \quad (6)$$

$$X_T = \frac{\beta_1}{\beta_1-1} \frac{r-\alpha}{(1-\tau)(\gamma-1)} K, \quad (7)$$

where β_1 is the positive root of the following characteristic equation:

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r = 0. \quad (8)$$

Next the value of the new subsidiary company consists of the equity and debt values:

$$V_N(x) = E_N(x) + D_N(x). \quad (9)$$

$E_N(x)$ is the equity value of the new company which has to choose the growth time t_G and the default time before and after growth t_D^1 and t_D^2 :

$$E_N(x) = \sup_{t_G, t_D^1, t_D^2 \in \mathcal{T}} \mathbb{E} \left[\mathbf{1}_{\{t_D^1 < t_G\}} \int_t^{t_D^1} e^{-r(s-t)} (1-\tau)(X_s - c) ds \right. \\ \left. + \mathbf{1}_{\{t_D^1 \geq t_G\}} \left(\int_t^{t_G} e^{-r(s-t)} (1-\tau)(X_s - c) ds + \int_{t_G}^{t_D^2} e^{-r(s-t)} (1-\tau)(\gamma X_s - c) ds \right. \right. \\ \left. \left. - e^{-r(t_G-t)} K \right) \middle| X_t = x \right]. \quad (10)$$

Coupon payment c is determined by the capital constraint:

$$V_T = I + D. \quad (11)$$

We can solve Equation (10) by dividing into two parts at growth time:

$$E_N(x) = \begin{cases} 0, & \text{for } x \leq X_D^1, \\ A_G x^{\beta_1} + A_D^1 x^{\beta_2} + \frac{(1-\tau)x}{r-\alpha} - \frac{(1-\tau)c}{r}, & \text{for } X_D^1 < x < X_G, \\ E_N^2(x) - K, & \text{for } x \geq X_G, \end{cases} \quad (12)$$

$$E_N^2(x) = \begin{cases} 0, & \text{for } x \leq X_D^2, \\ A_D^2 x^{\beta_2} + \frac{(1-\tau)\gamma x}{r-\alpha} - \frac{(1-\tau)c}{r}, & \text{for } x > X_D^2, \end{cases} \quad (13)$$

where β_2 is the negative root of the characteristic equation (8), X_G is the growth threshold and X_D^1 and X_D^2 are the default thresholds before and after growth, respectively:

$$t_G = \inf\{t > 0 : X_t \geq X_G\}, \quad (14)$$

$$t_D^1 = \inf\{t > 0 : X_t \leq X_D^1\}, \quad (15)$$

$$t_D^2 = \inf\{t > t_G : X_t \leq X_D^2\}. \quad (16)$$

Although A_D^2 and X_D^2 are found analytically:

$$A_D^2 = \left(\frac{(1-\tau)c}{r} - \frac{(1-\tau)\gamma X_D^2}{r-\alpha} \right) (X_D^2)^{-\beta_2}, \quad (17)$$

$$X_D^2 = \frac{\beta_2}{\beta_2 - 1} \frac{r - \alpha}{\gamma r} c, \quad (18)$$

A_G , A_D^1 , X_G and X_D^1 are found numerically.

Given the growth strategy X_G and the default strategy X_D^1 and X_D^2 , the value of the debt issued to acquire the target, i.e., the debt passed to the new company $D_N(x)$ ($= D$) is calculated. Here we assume investment cost to growth K is expended by equity. Therefore, debt holders have no concern with the investment, so that coupon payment c never change before/after the investment. We have

$$D_N(x) = \mathbb{E} \left[\int_t^{t_D^1 \wedge t_D^2} e^{-r(s-t)} c ds + \mathbf{1}_{\{t_D^1 < t_G\}} e^{-(t_D^1-t)} (1-\theta) \frac{(1-\tau)X_{t_D^1}}{r-\alpha} + \mathbf{1}_{\{t_D^1 \geq t_G\}} e^{-(t_D^2-t)} (1-\theta) \frac{(1-\tau)\gamma X_{t_D^2}}{r-\alpha} \Big| X_t = x \right], \quad (19)$$

where θ is the default cost (LGD). Dividing into two parts at growth time again, we have

$$D_N(x) = \begin{cases} (1-\theta) \frac{(1-\tau)x}{r-\alpha}, & \text{for } x \leq X_D^1, \\ B_G x^{\beta_1} + B_D^1 x^{\beta_2} + \frac{c}{r}, & \text{for } X_D^1 < x < X_G, \\ D_N^2(x), & \text{for } x \geq X_G, \end{cases} \quad (20)$$

$$D_N^2(x) = \begin{cases} (1-\theta) \frac{(1-\tau)\gamma x}{r-\alpha}, & \text{for } x \leq X_D^2, \\ B_D^2 x^{\beta_2} + \frac{c}{r}, & \text{for } x > X_D^2, \end{cases} \quad (21)$$

where

$$B_D^2 = \left((1-\theta) \frac{(1-\tau)\gamma X_D^2}{r-\alpha} - \frac{c}{r} \right) (X_D^2)^{-\beta_2}, \quad (22)$$

$$B_G = (B_D^2 - B_D^1) X_G^{\beta_2 - \beta_1}, \quad (23)$$

and B_D^1 is a complicated closed form.

Then total value of the new company is following:

$$V_N(x) = E_N(x) + D_N(x), \quad (24)$$

$$= \begin{cases} (1 - \theta) \frac{(1 - \tau)x}{r - \alpha}, & \text{for } x \leq X_D^1, \\ (A_G + B_G)x^{\beta_1} + (A_D^1 + B_D^1)x^{\beta_2} + \frac{(1 - \tau)x}{r - \alpha} + \frac{\tau c}{r}, & \text{for } X_D^1 < x < X_G, \\ V_N^2(x) - K, & \text{for } x \geq X_G, \end{cases} \quad (25)$$

and the value after growth is

$$V_N^2(x) = \begin{cases} (1 - \theta) \frac{(1 - \tau)\gamma x}{r - \alpha}, & \text{for } x \leq X_D^2, \\ (A_D^2 + B_D^2)x^{\beta_2} + \frac{(1 - \tau)\gamma x}{r - \alpha} + \frac{\tau c}{r}, & \text{for } x > X_D^2. \end{cases} \quad (26)$$

We can interpret four terms of Equation (25) as the growth option, the default cost, the earning profit and the tax benefit, in order.

Finally, we consider the bidder's option to acquire the target company. After acquisition by expending the capital to establish SPV I , the bidder gets the equity of the new company:

$$F_B(x) = \sup_{t_B \in \mathcal{T}} \mathbb{E} [e^{-rt_B} (E_N(X_{t_B}) - I)], \quad (27)$$

$$= \begin{cases} A_B x^{\beta_1}, & \text{for } x < X_B, \\ A_G x^{\beta_1} + A_D^1 x^{\beta_2} + \frac{(1 - \tau)x}{r - \alpha} - \frac{(1 - \tau)c}{r} - I, & \text{for } x \geq X_B, \end{cases} \quad (28)$$

where t_B is the acquiring time and X_B is the acquiring threshold:

$$t_B = \inf\{t > 0 : X_t \geq X_B\}. \quad (29)$$

Although A_B has a closed form:

$$A_B = \left(A_G X_B^{\beta_1} + A_D^1 X_B^{\beta_2} + \frac{(1 - \tau)X_B}{r - \alpha} - \frac{(1 - \tau)c}{r} - I \right) X_B^{-\beta_1}, \quad (30)$$

X_B is found numerically. The capital constraint must hold at the acquiring time:

$$V_T(X_B) = I + D(X_B; c). \quad (31)$$

4 Numerical Analyses

In this section, we use the basic parameter values in Table 1 for the numerical calculation. Figures 1 and 2 are the value functions of the new company and the value of the bidder's option, respectively, for the basic parameter.

Table 1: Parameter Values		
parameter		value
volatility	σ	0.15
expected growth rate	α	0.02
discount rate	r	0.1
effective tax rate	τ	0.6
scale parameter	γ	1.5
investment cost	K	5
loss given default	θ	0.5
expended capital	I	2

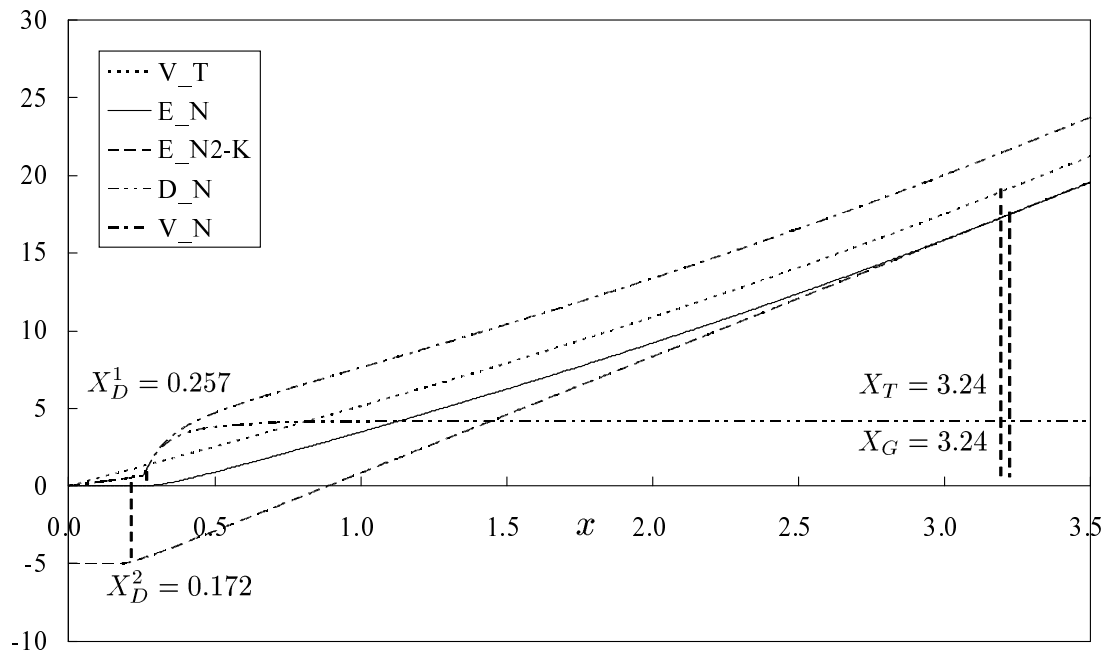


Figure 1: The value functions of the new company for basic parameter values

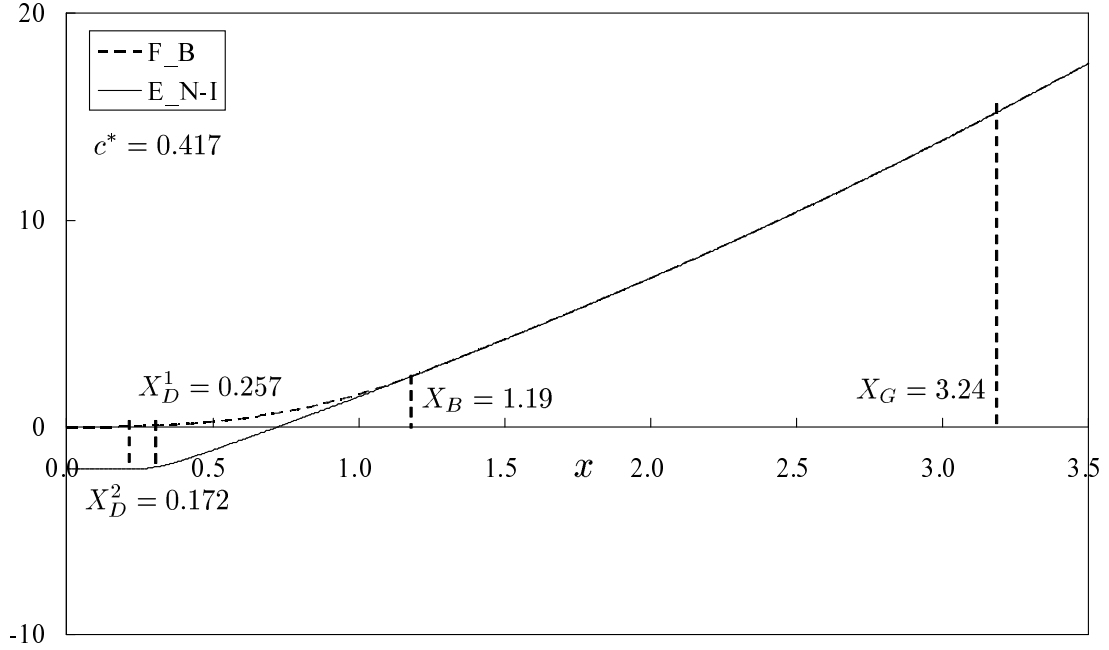


Figure 2: The value of the bidder's option for basic parameter values

Next we provide comparative statics with respect to some parameters. We consider the non-option model for comparison. Non-option means that the target company has no opportunity to expand its business. Setting $\gamma = 1$ and $K = 0$ in Equations (2), (10) and (19) mathematically, we have

$$\bar{V}_T(x) = \frac{(1-\tau)x}{r-\alpha}, \quad (32)$$

$$\bar{E}_N(x) = \begin{cases} 0, & \text{for } x \leq \bar{X}_D, \\ \left(\frac{(1-\tau)c}{r} - \frac{(1-\tau)\bar{X}_D}{r-\alpha} \right) x^{\beta_2} + \frac{(1-\tau)x}{r-\alpha} - \frac{(1-\tau)c}{r}, & \text{for } x > \bar{X}_D, \end{cases} \quad (33)$$

$$\bar{D}_N(x) = \begin{cases} (1-\theta) \frac{(1-\tau)x}{r-\alpha}, & \text{for } x \leq \bar{X}_D, \\ \left((1-\theta) \frac{(1-\tau)\bar{X}_D}{r-\alpha} - \frac{c}{r} \right) x^{\beta_2} + \frac{c}{r}, & \text{for } x > \bar{X}_D, \end{cases} \quad (34)$$

$$\bar{V}_N(x) = \begin{cases} (1-\theta) \frac{(1-\tau)x}{r-\alpha}, & \text{for } x \leq \bar{X}_D, \\ - \left(\theta \frac{(1-\tau)\bar{X}_D}{r-\alpha} + \frac{\tau c}{r} \right) x^{\beta_2} + \frac{(1-\tau)x}{r-\alpha} + \frac{\tau c}{r}, & \text{for } x > \bar{X}_D, \end{cases} \quad (35)$$

$$\bar{F}_B(x) = \begin{cases} (\bar{E}_N(\bar{X}_B) - I)x^{\beta_1}, & \text{for } x < \bar{X}_B, \\ \bar{E}_N(x) - I, & \text{for } x \geq \bar{X}_B. \end{cases} \quad (36)$$

Table 2: Leverage ratio w.r.t. I

I	Growth option model		Non-option model	
	c^*	ratio	c^*	ratio
1.0	0.19	0.47	0.18	0.46
1.5	0.29	0.47	0.26	0.46
2.0	0.42	0.48	0.35	0.46
2.5	0.57	0.49	0.44	0.46
3.0	0.77	0.50	0.53	0.46
3.5	1.08	0.52	0.62	0.46
	+	+	+	0

Table 3: Risk w.r.t. I

I	Growth option model					Non-option model	
	X_B	X_G	AtoD	AtoG	GtoD	X_B	AtoD
1.0	0.56	3.24	0.45	2.67	3.16	0.55	0.44
1.5	0.86	3.24	0.68	2.37	3.12	0.83	0.66
2.0	1.19	3.24	0.93	2.05	3.06	1.10	0.88
2.5	1.54	3.24	1.19	1.69	3.00	1.38	1.11
3.0	1.96	3.24	1.49	1.27	2.92	1.65	1.33
3.5	2.53	3.24	1.87	0.71	2.79	1.93	1.55
	+	0	+	-	-	+	+

Although the default threshold \bar{X}_D has a closed form:

$$\bar{X}_D = \frac{\beta_2}{\beta_2 - 1} \frac{r - \alpha}{r} c, \quad (37)$$

the acquiring threshold \bar{X}_B is found numerically.

We analyze leverage ratio and risk. The former is defined by D_N/V_N and the latter by the difference between thresholds, such as acquisition and default (AtoD): $X_B - X_D^1$ ($\bar{X}_B - \bar{X}_D$ for non-option model), acquisition and growth (AtoG): $X_G - X_B$ and growth and default (GtoD): $X_G - X_D^2$. Tables 2, 4 and 6 illustrate comparative statics of leverage ratio with respect to I , σ and α , respectively. Tables 3, 5 and 7 illustrate comparative statics of risk with respect to I , σ and α , respectively. Tables 8 and 9 illustrate comparative statics of both leverage ratio and risk with respect to γ and K , respectively.

The difference of results between our model and non-option model (Goto et al., 2009) is as follows. First, our model with a growth option has higher coupon and leverage ratio than non-option model. This result shows the fact that lenders prefer to lend more money to business with more growth opportunities. The difference becomes pronounced when I , σ and α are large. Next, the difference between acquisition and default thresholds (AtoD) show that our model has less risk than non-option model. This result also becomes pronounced when parameter values are large.

Table 4: Leverage ratio w.r.t. σ

	Growth option model		Non-option model	
σ	c^*	ratio	c^*	ratio
0.05	0.13	0.32	0.13	0.32
0.10	0.23	0.40	0.22	0.40
0.15	0.42	0.48	0.35	0.46
0.20	1.52	0.58	0.61	0.52
	+	+	+	+

Table 5: Risk w.r.t. σ

	Growth option model					Non-option model	
σ	X_B	X_G	AtoD	AtoG	GtoD	X_B	AtoD
0.05	0.67	2.63	0.57	1.96	2.56	0.67	0.56
0.10	0.84	2.90	0.69	2.06	2.80	0.83	0.68
0.15	1.19	3.24	0.93	2.05	3.06	1.10	0.88
0.20	2.84	3.67	2.04	0.83	3.11	1.59	1.26
	+	+	+	-	+	+	+

Table 6: Leverage ratio w.r.t. α

	Growth option model		Non-option model	
α	c^*	ratio	c^*	ratio
0.00	0.19	0.37	0.18	0.37
0.01	0.26	0.42	0.25	0.41
0.02	0.42	0.48	0.35	0.46
0.03	1.02	0.56	0.56	0.51
	+	+	+	+

Table 7: Risk w.r.t. α

	Growth option model					Non-option model	
α	X_B	X_G	AtoD	AtoG	GtoD	X_B	AtoD
0.00	0.96	3.49	0.82	2.53	3.40	0.95	0.82
0.01	1.02	3.35	0.85	2.33	3.24	1.00	0.83
0.02	1.19	3.24	0.93	2.05	3.06	1.10	0.88
0.03	1.91	3.14	1.35	1.23	2.76	1.33	1.02
	+	-	+	-	-	+	+

Table 8: Leverage ratio and Risk w.r.t. γ

γ	c^*	ratio	X_B	X_G	AtoD	AtoG	GtoD
1.1	0.35	0.46	1.10	16.18	0.89	15.08	15.98
1.3	0.37	0.47	1.12	5.39	0.89	4.27	5.22
1.5	0.42	0.48	1.19	3.24	0.93	2.05	3.06
1.7	0.61	0.52	1.43	2.31	1.06	0.89	2.09
	+	+	+	-	+	-	-

Table 9: Leverage ratio and Risk w.r.t. K

K	c^*	ratio	X_B	X_G	AtoD	AtoG	GtoD
5	0.42	0.48	1.19	3.24	0.93	2.05	3.06
10	0.37	0.47	1.13	6.47	0.90	5.35	6.32
15	0.36	0.46	1.11	9.71	0.89	8.59	9.56
20	0.36	0.46	1.11	12.95	0.89	11.84	12.80
	-	-	-	+	-	+	+

5 Conclusion

In this paper, we have investigated the bidder's option to acquire the target company in LBO. While Goto et al. (2009) considered a declining industry, we assume that target has a growth option. An important setting is that the optimal timing is determined under the capital constraint. As our main result, we find that a growth option leads to delay in LBO, high leverage and low risk. And when the growth option is easily to exercised, the default risk is low before growth and high after growth except for the impact of uncertainty. For future works, we will compare with optimal debt issuing as Goto et al. (2009) investigated. Another topic is analysis of levered target.

References

- Goto, M., Takashima, R. and Tsujimura, M. (2009). A real options model of debt ratio in leveraged buyouts. The 4th National Conference on Real Options.
- Lambrecht, B. M. (2004). The timing and terms of mergers motivated by economies of scale. *Journal of Financial Economics*, **72**, 41–62.
- Lambrecht, B. M. and Myers, S. C. (2007). A theory of takeovers and disinvestment. *Journal of Finance*, **62**, 809–845.
- Lambrecht, B. M. and Myers, S. C. (2008). Debt and managerial rents in a real-options model of the firm. *Journal of Financial Economics*, forthcoming.
- Leland, H. E. (2007). Financial synergies and the optimal scope of the firm: Im-

plications for mergers, spinoffs, and structured finance. *Journal of Finance*, **49**, 1213–1252.

Morellec, E. and Zhdanov, A. (2005). The dynamics of mergers and acquisitions. *Journal of Financial Economics*, **77**, 649–672.

Rhodes-Kropf, M. and Viswanathan, S. (2004). Market valuation and merger waves. *Journal of Finance*, **59**, 2685–2718.

Shleifer, A. and Vishny, R. W. (2003). Stock market driven acquisitions. *Journal of Financial Economics*, **70**, 295–311.

Tian, Y., Nishihara, M. and Shibata, T. (2008). Financial synergies in M&A. Research Paper Series, No.55, Tokyo Metropolitan University.