Assessing Alternative R&D Investment Projects under Uncertainty*

Motoh Tsujimura†
Faculty of Economics, Ryukoku University

Abstract

In this paper, we consider that there are two kinds of technologies distinguished by the level of productivity and their associated costs, such that one technology increases quantity of output more than alternative technology. The firm must then decide which technology to invest and when to invest the chosen R&D project to maximize the profit. To solve the firm's problem, we formulate it as an optimal stopping problem.

Keywords: alternative technology; R&D; real options; optimal stopping

1 Introduction

Research and development (R&D) investment plays an important role of increasing firm's performance. See, for example, Ettlie (1998) and references there in. When a firm conducts an R&D investment, the firm faces uncertainty of success of R&D investment and future cash flow after success of R&D investment. Due to flexibility of decision-making of investment, it is able for the firm to wait R&D investment. That is, the firm has an option to invest R&D project. See, for example, Paxson (2003) for more detail. Then, it is important to examine investment timing of R&D project.

In this paper, we consider that there are two kinds of technologies distinguished by the level of productivity and their associated costs, such that one technology increases quantity of output more than alternative technology. The firm must then decide which technology to invest and when to invest the chosen R&D project to maximize the profit. To solve the firm's problem, we formulate it as an optimal stopping problem.

Our analysis differs and relates to previous work in several respects. For instance, while Shih and Hung (2008) investigates R&D investment problem when the firm has just a single technology option, we analyze the outcomes when the firm has two technology options. We refer to the former as the single R&D investment project and the latter as the alternative R&D projects. In related work, Décamps, Mariotti and Villeneuve (2006) examine the investment

*This research was supported in part by the Grant of Trust Companies Association of Japan and the Grant-in-Aid for Scientific Research (No. 20510149) of the Japan Society for the Promotion of Science.
†Corresponding author
Address: 67 Tsukamoto-cho, Fukakusa Fushimi-ku, Kyoto 612-8577 JAPAN
Phone: +81-75-645-8460; E-mail: tsujimura@econ.ryukoku.ac.jp

decision problem of two alternative projects without technological innovation. They then show the value of the flexibility when the agent can choose between the alternative projects.

The rest of the paper is organized as follows. Section 2 describes the firm's single R&D investment problem. Section 3 examines alternative R&D investment problem. Next, we presents the numerical analysis in Section 4. Lastly, Section 5 concludes the paper.

2 Single R&D Investment

Suppose that a risk neutral firm is considering investing R&D projetct as in Shih and Hung (2008). The level of the technology progresses by investing in R&D projetct. We assume that there are two technology options. One is higher level technology than the other. The former is labeled by H and the latter is labeled by L. In this section, we assume that the firm has either R&D project H or L available as R&D project options, but not both. If the firm choseses project i $(i = \{H, L\})$, it generates a net cash flow $a^i X_t$, where a^i represents the level of technology with $a^H > a^L$ and X_t represents the dynamics of the net cash flow. We assume that the process of X_t is governed by the following stochastic differential equation:

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}W_t, \quad X_0 = x \ (> 0), \tag{2.1}$$

where μ $(\in \mathbb{R})$ and σ (> 0) are constants. W_t is a standard Brownian motion on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\}_{t\geq 0})$, where $\mathcal{F}_t$ is generated by W_t in $\mathbb{R}$, i.e., $\mathcal{F}_t = \sigma(W_s, s \leq t)$.

We assume that the success of the R&D investment depends on a Poisson distribution with mean arrival rate λ. $1/\lambda$ is the expected time until the new technology is available during a time interval of infinitesimal length $\mathrm{d}t$. Let θ_t be governed by the following Poisson process:

$$\mathrm{d}\theta_t = \begin{cases} 1, & \text{w.p. } \lambda(I^i)\mathrm{d}t, \\ 0, & \text{w.p. } 1 - \lambda(I^i)\mathrm{d}t, \end{cases} \tag{2.2}$$

where I^i is the R&D investment with $I^H > I^L$. As in Shih and Hung (2008), we assume that the mean arrival rate is given by:

$$\lambda(I) = b^i I^i, \tag{2.3}$$

where b^i is a constant with $b^i I^i \in (0, 1)$ and $0 < b^H < b^L$.

When the firm invests R&D project i, the firm's expected discounted profit $J^i(x; \tau_S^i)$ is given by:

$$J^i(x; \tau_S^i) = \mathbb{E}\left[\mathrm{e}^{-r\tau_S^i} \left(\int_{\tau_S^i}^{\infty} \mathrm{e}^{-\lambda(I^i)(t-\tau_S^i)} \lambda(I^i) \mathrm{e}^{-r(t-\tau_S^i)} a^i X_t \mathrm{d}t - I^i \right)\right], \tag{2.4}$$

where r $(> \mu)$ is a discount rate, $\tau_S^i \in \mathcal{T}$ is the investment time of the R&D project i, and $\mathcal{T}$ is the set of all admissible investment times. Therefore, the firm's problem is to maximize the expected discounted profit over $\mathcal{T}$:

$$V^i(x) = \sup_{\tau_S^i \in \mathcal{T}} J(x; \tau_S^i) = J(x; \tau_S^{i*}), \tag{2.5}$$

where V^i is the value function and τ_S^{i*} is the optimal timing to invest the R&D project i.

The firm's problem (2.5) is formulated as an optimal stopping problem. As is well known, optimal stopping problems are solved by variational inequalities. See, for example, Hu and Øksendal (1998), Dupuis and Wang (2002), Øksendal (2003).

Let $G^i(X_t)$ be given by:

$$
\begin{aligned}
G^i(X_t) &= \int_t^\infty \mathrm{e}^{-\lambda(I^i)(s-t)}\lambda(I^i)\mathrm{e}^{-r(s-t)}a^i X_s \mathrm{d}s - I^i, \\
&= \frac{b^i I^i a^i X_t}{r + b^i I^i - \mu} - I^i.
\end{aligned} \tag{2.6}
$$

The region where the firm has not invested R&D project i is defined by:

$$
\mathcal{C}_S^i = \{x; V^i(x) > G^i(x)\}. \tag{2.7}
$$

That is, $\mathcal{C}_S^i$ is the continuation region and yields the timing of investing the R&D project i, τ_S^i, given by:

$$
\tau_S^i = \inf\{t > 0; X_t \notin \mathcal{C}_S^i\}. \tag{2.8}
$$

We now define the variational inequalities of the firm's problem (2.5).

Definition 2.1 (Variational Inequalities). *The following relations are the variational inequalities of the firm's problem (2.5):*

$$
\mathcal{L}V^i(x) \leq 0, \tag{2.9}
$$

$$
V^i(x) \geq G^i(x), \tag{2.10}
$$

$$
\mathcal{L}V^i(x)[V^i(x) - G^i(x)] = 0, \tag{2.11}
$$

where $\mathcal{L}$ is the differential operator defined by:

$$
\mathcal{L} := \frac{1}{2}\sigma^2 x^2 \frac{\mathrm{d}^2}{\mathrm{d}x^2} + \mu x \frac{\mathrm{d}}{\mathrm{d}x} - r. \tag{2.12}
$$

(2.12) is the complementary condition and can be rewritten as follows. If $x \in \mathcal{C}_S^i$, we then have:

$$
\mathcal{L}V^i(x) = 0. \tag{2.13}
$$

Alternatively, if $x \notin \mathcal{C}_S^i$, we have:

$$
V^i(x) - G^i(x) = 0. \tag{2.14}
$$

Next, we investigate whether the value function is a solution to the variational inequalities. From the formulation of the firm's problem (2.5), we conjecture the optimal R&D investment strategy as follows. If the process of $X = \{X_t\}_{t\geq 0}$ reaches some threshold x_S^i, the firm invests R&D project i, and otherwise does not. Thus, the optimal timing of investing the R&D project i is given by:

$$
\tau_S^i = \inf\{t > 0; X_t \geq x_S^i\}. \tag{2.15}
$$

The variational inequalities imply that (2.13) holds for $x < x_S^i$. We conjecture a solution to (2.13) is:

$$
V^i(x) = B_{S1}^i x^{\beta_1} + B_{S2}^i x^{\beta_2}, \tag{2.16}
$$

where B^i_{S1} and B^i_{S2} are unknowns to be determined. β_1 and β_2 are the solutions to the following characteristic equation:

$$\frac{1}{2}\sigma^2\beta(\beta-1)+\mu\beta-r=0, \tag{2.17}$$

and are calculated as:

$$\begin{aligned}\beta_1&=\frac{1}{2}-\frac{\mu}{\sigma^2}+\sqrt{\left(\frac{\mu}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}>1,\\ \beta_2&=\frac{1}{2}-\frac{\mu}{\sigma^2}-\sqrt{\left(\frac{\mu}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2r}{\sigma^2}}<0.\end{aligned} \tag{2.18}$$

If x goes to 0, the firm does not make a profit. Then, we obtain the following boundary condition of the firm's problem:

$$\lim_{x\to 0}V^i(x)=0. \tag{2.19}$$

It follows from (2.16) and (2.19) that we put $B^i_{S2}=0$. Then, the value function is divided by the level of x as follows:

$$V^i(x)=\begin{cases}B^i_{S1}x^{\beta_1}, & x<x^i_S,\\ G^i(x), & x\geq x^i_S.\end{cases} \tag{2.20}$$

I^i is endogenously determined as a function of x by

$$I^{i*}(x)=\arg\max\left\{G^i(x);I^i(x)\right\}=\arg\max\left\{\frac{b^iI^i(x)a^ix}{r+b^iI^i(x)-\mu}-I^i(x);I^i(x)\right\}. \tag{2.21}$$

It follows from (2.21) that I^{i*} satisfies the following condition:

$$\frac{b^ia^ix(r-\mu)}{(r+b^iI^i(x)-\mu)^2}=1. \tag{2.22}$$

The unknown B^i_{S1} and threshold x^i_S are calculated by the following simultaneous equations:

$$B^i_{S1}x^{\beta_1}=\frac{b^iI^i(x)a^ix}{r+b^iI^i(x)-\mu}-I^i(x), \tag{2.23}$$

$$\beta_1B^i_{S1}x^{\beta_1-1}=\frac{\mathrm{d}}{\mathrm{d}x}\left[\max\left[\frac{b^iI^i(x)a^ix}{r+b^iI^i(x)-\mu}-I^i(x)\right]\right]. \tag{2.24}$$

These respective equations are well known as the value-matching and smooth-pasting conditions.

Using the envelope theorem, (2.24) can be expressed as

$$\beta_1B^i_{S1}x^{\beta_1-1}=\frac{b^iI^i(x)a^i}{r+b^iI^i(x)-\mu}. \tag{2.25}$$

Then, we obtain that:

$$I^{i*}=\frac{(r-\mu)}{(\beta_1-1)b^i}, \tag{2.26}$$

$$x^i_S=\left(\frac{\beta_1}{\beta_1-1}\right)^2\frac{(r-\mu)}{a^ib^i}, \tag{2.27}$$

$$B^i_{S1}=\beta_1^{-2}\left(\frac{\beta_1}{\beta_1-1}\right)^{2(1-\beta_1)}(a^i)^{\beta_1}\left(\frac{r-\mu}{b^i}\right)^{1-\beta_1}. \tag{2.28}$$

3 Alternative R&D Investments

In this section, we consider that the firm has two R&D project options and assume that the firm invests either R&D project H or L. We follow the framework in Décamps, Mariotti and Villeneuve (2006) who investigate the choice problem between two alternative investment projects. Let τ_A be the timing of investing project H or L given by:

$$\tau_A = \min\left[\tau_A^H, \tau_A^L\right], \tag{3.1}$$

where τ_A^i $(i = \{H, L\})$ is the timing of investing the R&D project i where the firm has two R&D project options. Then, the firm's expected discounted profit J is:

$$\begin{aligned} J(x;\tau_A) = \mathbb{E}\Bigg[& \mathbf{1}_{\{\tau_A^H \le \tau_A^L\}} \mathrm{e}^{-r\tau_A^H} \left(\int_{\tau_A^H}^{\infty} \mathrm{e}^{-\lambda(I^H)(t-\tau_A^H)} \lambda(I^H) \mathrm{e}^{-r(t-\tau_A^H)} a^H X_t \mathrm{d}t - I^H \right) \\ & + \mathbf{1}_{\{\tau_A^H > \tau_A^L\}} \mathrm{e}^{-r\tau_A^L} \left(\int_{\tau_A^L}^{\infty} \mathrm{e}^{-\lambda(I^L)(t-\tau_A^L)} \lambda(I^L) \mathrm{e}^{-r(t-\tau_A^L)} a^L X_t \mathrm{d}t - I^L \right) \Bigg] \\ = \mathbb{E}\Bigg[& \mathbf{1}_{\{\tau_A^H \le \tau_A^L\}} \mathrm{e}^{-r\tau_A^H} G^H(X_{\tau_A^H}) + \mathbf{1}_{\{\tau_A^H > \tau_A^L\}} \mathrm{e}^{-r\tau_A^L} G^L(X_{\tau_A^L}) \Bigg]. \end{aligned} \tag{3.2}$$

Therefore, the firm's problem is to choose the timing of investing either R&D project H or L to maximize their expected total discounted profit J:

$$V(x) = \sup_{\tau_A \in \mathcal{T}} J(x;\tau_A) = J(x;\tau_A^*). \tag{3.3}$$

From (2.7) the region where the firm invests neither R&D project H nor L is defined by:

$$\mathcal{C}_A = \{x; V(x) > \max[G^H(x), G^L(x)]\}. \tag{3.4}$$

That is, $\mathcal{C}_A$ is the continuation region. Then, τ_A is given by:

$$\tau_A = \inf\{t > 0; X_t \notin \mathcal{C}_A\}. \tag{3.5}$$

As in Section 2, the firm's problem is formulated as an optimal stopping problem and is solved via the variational inequalities. The variational inequalities of the firm's problem (3.3) are as follows:

$$\mathcal{L}V(x) \le 0, \tag{3.6}$$

$$V(x) \ge \max[G^H(x), G^L(x)], \tag{3.7}$$

$$\mathcal{L}V(x)\left[V(x) - \max\left[G^H(x), G^L(x)\right]\right] = 0. \tag{3.8}$$

Let $\tilde{x}$ be the value of X such that $G^H(x) = G^L(x)$. Then, $\tilde{x}$ is calculated as:

$$\tilde{x} = (I^H - I^L)\left[\frac{b^H I^H h^H}{r + b^H I^H - \mu} - \frac{b^L I^L h^L}{r + b^L I^L - \mu}\right]^{-1}. \tag{3.9}$$

The value function V smoothly pastes neither the function G^H nor the function G^L at $x = \tilde{x}$. Then, we obtain the following result.

Proposition 3.1. *When the state variable is $\tilde{x}$, the firm invests neither R&D project.*

Décamps, Mariotti and Villeneuve (2006) provide a rigorous treatment in their Proposition 2.2. Furthermore, Décamps, Mariotti and Villeneuve (2006) obtain the following result in their Theorem 2.1.

Theorem 3.1. *Assume that:*

$$\left(\frac{\beta_1}{a^H}\right)^{-\beta_1}\left(\frac{r-\mu}{(\beta_1-1)b^H}\right)^{1-\beta_1} < \left(\frac{\beta_1}{a^L}\right)^{-\beta_1}\left(\frac{r-\mu}{(\beta_1-1)b^L}\right)^{1-\beta_1}. \tag{3.10}$$

Let x_A^i $(i=\{H,L\})$ be the threshold of investing the R&D project i when the firm has two R&D project options. The timing of investing R&D project L, τ_A^L, is given by:

$$\tau_A^L = \inf\{t>0; x_S^L \le X_t \le x_A^L\}. \tag{3.11}$$

Conversely, the timing of investing R&D project H, τ_A^H, is given by:

$$\tau_A^H = \inf\{t>0; X_t \ge x_A^H\}. \tag{3.12}$$

The continuation region $\mathcal{C}_A$ is redefined by:

$$\mathcal{C}_A = \{x; x < x_S^L,\ x_A^L < x < x_A^H\}. \tag{3.13}$$

From (3.11), the region where the R&D project L is invested is defined by:

$$\mathcal{I}_L = \{x; x_S^L \le x \le x_A^L\}. \tag{3.14}$$

Similarly, from (3.12), the region where the R&D project H is invested is defined by:

$$\mathcal{I}_H = \{x; x \ge x_A^H\}. \tag{3.15}$$

The continuation region $\mathcal{C}_A$ is divided into two regions. The first region is defined by:

$$\mathcal{C}_{AL} = \{x; x < x_S^L\}, \tag{3.16}$$

where $\mathcal{C}_{AL}$ is the continuation region when the firm has only the R&D project L. The second region is defined by:

$$\mathcal{C}_{ALH} = \{x; x_A^L < x < x_A^H\}. \tag{3.17}$$

This region arises from the flexibility where the firm can choose between the R&D project L and H.

From the variational inequalities (3.6)–(3.8), for $x \in \mathcal{C}_A$ we have:

$$\mathcal{L}V(x) = 0, \tag{3.18}$$

For $x < x_S^L$, when x reaches x_S^L, the firm invests the R&D projects L. Then, we have V given by (2.20). For $x_A^L < x < x_A^H$, when x reaches x_A^L before x_A^H, the firm invests the R&D project L. Alternatively, when x reaches x_A^H before x_A^L, the firm invests the R&D project H. Thus, the firm has two types of flexibility in this region. The value function is then:

$$V(x) = B_{AH}x^{\beta_1} + B_{AL}x^{\beta_2}, \tag{3.19}$$

where B_{AH} and B_{AL} are unknowns to be determined. The first term on the right-hand side is the value of the flexibility from where the agent chooses the timing of investing the R&D project H. The second term is the value of the flexibility from where the agent chooses the R&D project L. Then, V is divided by the level of x as follows:

$$V(x) = \begin{cases} B^L_{S1} x^{\beta_1}, & x < x^L_S, \\ G^L(x), & x^L_S \le x \le x^L_A, \\ B_{AH} x^{\beta_1} + B_{AL} x^{\beta_2}, & x^L_A < x < x^H_A, \\ G^H(x), & x \ge x^H_A. \end{cases} \tag{3.20}$$

As in Section 2, we have to determine the unknowns: B_{AH}, B_{AL} and the thresholds: x^L_A, x^H_A. These are calculated using simultaneous equations:

$$B_{AH} x^{\beta_1} + B_{AL} x^{\beta_2} = \frac{b^L I^{L*} a^L x}{r + b^L I^{L*} - \mu} - I^{L*}, \tag{3.21}$$

$$B_{AH} x^{\beta_1} + B_{AL} x^{\beta_2} = \frac{b^H I^{H*} a^H x}{r + b^H I^{H*} - \mu} - I^{H*}, \tag{3.22}$$

$$\beta_1 B_{AH} x^{\beta_1 - 1} + \beta_2 B_{AL} x^{\beta_2 - 1} = \frac{b^L I^{L*} a^L}{r + b^L I^{L*} - \mu}, \tag{3.23}$$

$$\beta_1 B_{AH} x^{\beta_1 - 1} + \beta_2 B_{AL} x^{\beta_2} = \frac{b^H I^{H*} a^H}{r + b^H I^{H*} - \mu}. \tag{3.24}$$

Unfortunately, as we cannot analytically derive these thresholds, in the following section we numerically calculate their values.

4 Numerical Analysis

In this section, we will numerically calculate the thresholds: x^L_S, x^H_S, x^L_A, and x^H_A, and R&D investments: I^L and I^H. Furthermore, we investigate the effects of changes in the parameters on the thresholds and R&D investments. The basic parameter values are set out as follows: $r = 0.06$, $\mu = 0.01$, $\sigma = 0.2$, $a^H = 2$, $a^L = 1$, $b^H = 0.02$, and $b^L = 0.1$.

The value function V where the firm has two R&D project options is illustrated in Figure 1. The threshold values are calculated as $x^L_S = 2.000$, $x^H_S = 5.000$, $x^L_A = 2.891$, $x^H_A = 5.185$ in the base case. The indifference value of the shift variable is $\tilde{x} = 4.000$.

We provide the results of the comparative static analysis of the thresholds and the R&D investments in Figures 2–9. Figure 2 shows that the continuation region $\mathcal{C}_A$ is increasing in the volatility of the cash flow, σ. The investing regions $\mathcal{I}_L$ and $\mathcal{I}_H$ are both decreasing in σ. Figure 7 indicates the amounts of R&D investment of both projects, I^L and I^H, are increasing in σ. These results imply that the incentive to wait for new information of business environment becomes strong as uncertainty of future cash flow increases. Then, the amounts of investments are respectively increase as uncertainty of future cash flow increases.

Figure 3 shows that the continuation region for the R&D projects L and H, $\mathcal{C}_{ALH}$, is increasing in the technology parameter for the R&D project H, a^H. Recall that the continuation region for the R&D project L, $\mathcal{C}_L$, is independent of a^H. Then, the continuation region $\mathcal{C}_A$ is increasing in a^H. The investing region of the R&D project H is increasing in a^H, while the investing region of the R&D project L is decreasing in a^H. These results mean that the higher the cash flow from the project H is, it is easy for the project H to be chosen when the technology parameter of the project L does not change. Note that, from (2.26), I^H and I^L are independent of a^H.

Figure 4 shows that the continuation region $\mathcal{C}_A$ is decreasing in the technology parameter for the R&D project L, a^L. The investing region of the R&D project L is increasing in a^L, while the investing region of the R&D project H is decreasing in a^L. These results mean that the higher the cash flow from the project L is, it is easy for the project L to be chosen when the technology parameter of the project H does not change. Note that, from (2.26), I^L and I^H are also independent of a^L.

Figure 5 shows that the continuation region for the R&D projects L and H, $\mathcal{C}_{ALH}$, is decreasing in the parameter which determines the mean arrival rate of the R&D project H, b^H. Recall that the continuation region for the R&D project L, $\mathcal{C}_L$, is independent of b^H. Then, the continuation region $\mathcal{C}_A$ is decreasing in b^H. The investing region of the R&D project H is increasing in b^H, while the investing region of the R&D project L is decreasing in b^H. Figure 8 indicates the amount of R&D investment of the project H, I^H, is decreasing in b^H. Note that, from (2.26), I^L is independent of b^H. These results mean that the higher the mean arrival rate of the R&D project H is, it is easy for the project H to be chosen when the mean arrival rate of the R&D project L does not change. Decreasing the threshold x_A^H lowers the cash flow of the project, the amount of investment for the project H is decrease as b^H increases.

Figure 6 shows that the continuation region $\mathcal{C}_A$ is decreasing in b^L. The investing region of the R&D project L is increasing in b^L, while the investing region of the R&D project H is decreasing in b^L. Figure 9 indicates the amount of R&D investment of the project L, I^L, is decreasing in b^L. Note that, from (2.26), I^H is also independent of b^L. These results mean that the higher the mean arrival rate of the R&D project L is, it is easy for the project L to be chosen when the mean arrival rate of the R&D project H does not change. Decreasing the threshold x_S^L lowers the cash flow of the project, the amount of investment for the project L is decrease as b^L increases.

5 Conclusion

In this paper we examined a firm's R&D investment problem with alternative R&D project. To solve it, we formulate it as an optimal stopping problem. We first investigate the single R&D project and obtain the closed form of the threshold. Next, we investigate the alternative R&D projects. Unfortunately, the thresholds of the projects are not explicitly derived. Therefore, we conduct numerical and comparative static analysis. The representative findings indicate that the continuation region increases in volatility of cash flow, that is, with uncertainty, while the R&D project investing regions decrease in volatility. The amounts of investments of both projects increase as uncertainty increases.

To conclude the paper, we suggest a number of possible extensions for our model. First, to future work we defer examination of finite time horizon. Second, in this paper, we assume that the dynamics of cash flow are given by a one stochastic differential equation. It is able to assume different dynamics of cash flow for each project. Third, firms are in competition in actual business environment. We then consider two competing firms problem formulates as a game theoretic real options model as in Huisman (2001). We leave these topics for future research.

References

Décamps, J.-P., T. Mariotti and S. Villeneuve (2006), Irreversible Investment in Alternative Projects, *Economic Theory*, **28**, pp. 425–448.

Dupuis, P. and H. Wang (2002), Optimal Stopping with Random Intervention Times, *Advances in Applied Probability*, **34**, pp. 141–157.

Ettlie, J.E. (1998), R&D and Global Manufacturing Performance, *Management Science*, **44**, pp. 1–11.

Hu, Y. and B. Øksendal (1998), The Optimal Time to Invest when the Price Processes are Geometric Brownian Motions, *Finance and Stochastics*, **2**, pp. 295–310.

Huisman, K.J.M. (2001), *Technology Investment: A Game Theoretical Real Options Approach*, Kluwer Academic Publishers, Boston.

Øksendal, B. (2003), *Stochastic Differential Equations: An Introduction with Applications*, 6th ed., Springer-Verlag, New York.

Paxson, D.A. (2003), Selective Review of Real R&D Options Literature, in Paxson, D.A. (ed.), *Real R&D Options*, Butterworth-Heinemann, pp. 291–232.

Shih, P-T. and W. Hung, 2008. The Overall of Volatility on Investment, *Economics Letters*, **99**, pp. 324–327.

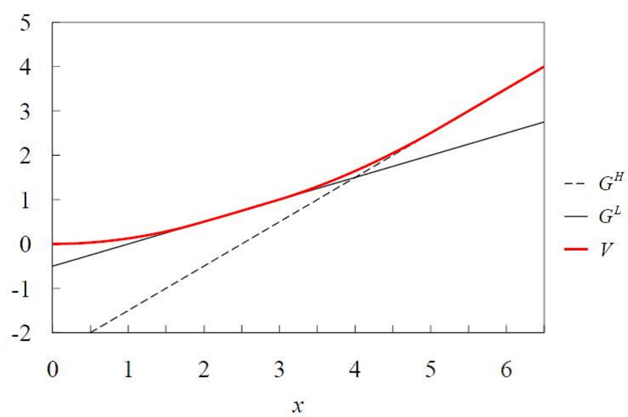

Figure 1: Value function of alternative R&D projects.

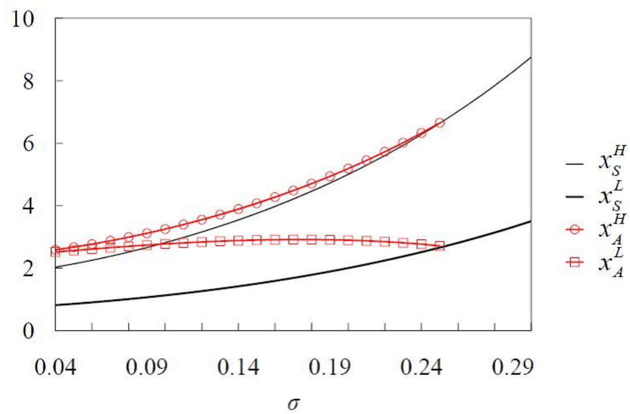

Figure 2: Comparative statics of thresholds with respect to σ.

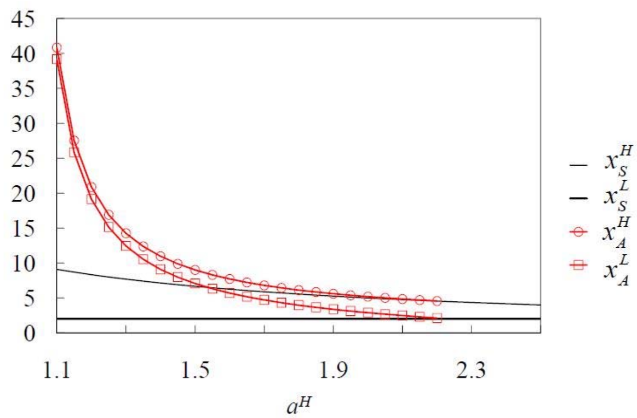

Figure 3: Comparative statics of thresholds with respect to a^H.

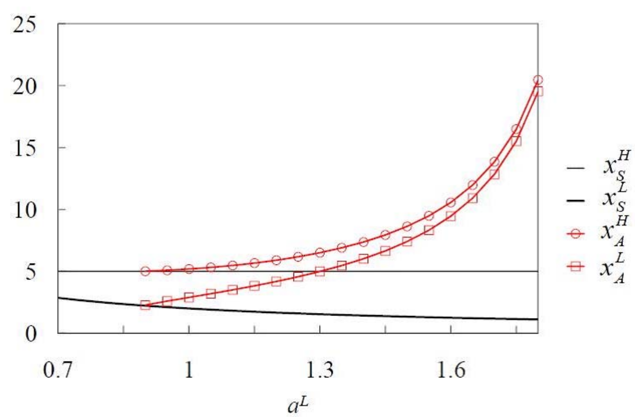

Figure 4: Comparative statics of thresholds with respect to a^L.

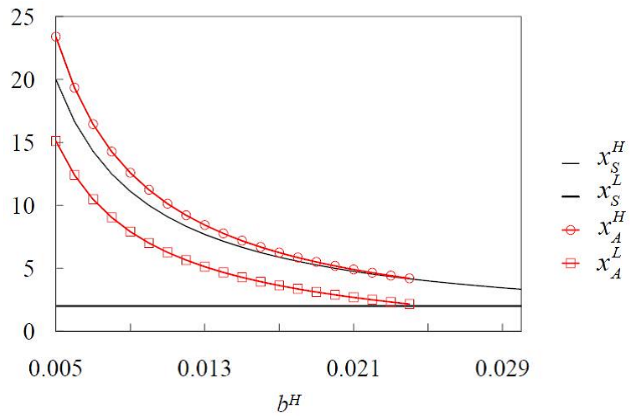

Figure 5: Comparative statics of thresholds with respect to b^H.

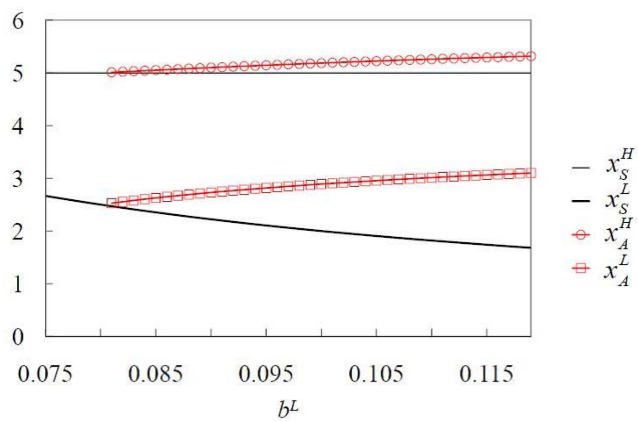

Figure 6: Comparative statics of thresholds with respect to b^L.

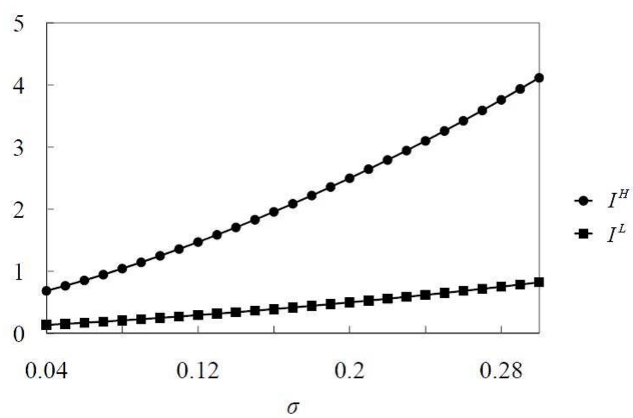

Figure 7: Comparative statics of R&D investments with respect to σ.

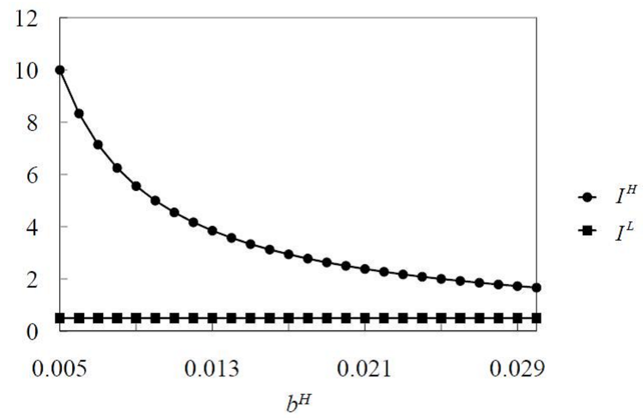

Figure 8: Comparative statics of R&D investments with respect to b^H.

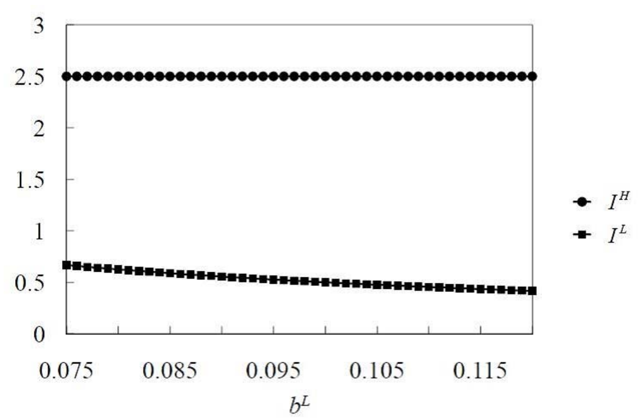

Figure 9: Comparative statics of R&D investments with respect to b^L.