The Option to Downscale Production in Recessionary Times

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Abstract

This paper values the option of a firm to downscale production in recessionary times. The regulator allows downscaling as long as the recession lasts. The end of the recession is modelled via a Poisson process. We show that the disinvestment timing is affected by two contrary effects. First, when the probability increases that the recession will be over soon, the firm will disinvest sooner, because after the recession is over the downscale option no longer exists. Second, the firm will downscale later because the expected future revenue stream increases when the recession will be over sooner. As a result the disinvestment trigger is non-monotonic in the intensity of the Poisson process. The second effect dominates when the recession is expected to last long. We also study the disinvestment problem in case there is a risk of bankruptcy due to lack of available funds in the credit market. It is shown that the optimal disinvestment threshold is the same as when there is no bankruptcy risk. This threshold, however, may not be appropriate due to the bankruptcy threshold.

1 Introduction

Standard recessions typically involve a period of reduced demand. The 2008/2009 recession was non-standard inasmuch as it also involved widespread dislocation in capital markets; even firms with healthy fundamentals had difficulty borrowing. The implication is that it became important for firms to have cash. As The Economist (2009) puts it:

"What about the current recession?...The most obvious winners are the established giants: market leaders that entered the recession with cash in their pockets and sound management systems under their belts."

As a reaction to the drop in demand firms can consider to downsize production.[1], which is the topic of this paper. In our framework the option to downscale production is available as long as the recession lasts. The motivation is that after the recession is over, it is much more diffcult to convince government and/or unions that a measure like cutting staff is a reasonable thing to do.

We consider two scenarios. First, we analyze the option to downscale production in a usual recession, i.e. firms are facing a situation where demand reduces over time. We find that the firm decides to downscale at the moment that the sum of the sunk cost of downscaling and the loss in revenue due to lower capacity is strictly lower than the production cost savings resulting from downscaling. This implies that a positive time interval exists on which the firm does not undertake the downscaling option, while at the same time expected cash flow resulting from the downscaling operation is strictly positive. This confirms, e.g., Dixit (1992), who argues that in the presence of significant sunk costs it is rational to persist and endure some amount of losses in the hope that industry profitability may improve. Uncertainty increases the value of the option to downscale. The larger the option value the more it is worthwhile to keep this option alive, which makes inertia optimal. In fact, downscaling is rational not when the net present value corresponding to the downscale operation turns positive, but when it exceeds the value of the downscale option (cf. O'Brien and Folta (2009)). The net present value of downscaling at the moment this option is exercised goes up when the economic environment becomes more uncertain.

We further find that the decision to downscale mainly depends on two contrary effects. First, as soon as the recession is over the firm loses the option to downscale. This makes that the firm will speed up the decision to downscale in a situation of an increased probability that the recession will be over within the next time interval. Second, when the probability that the recession will be over soon goes up, this implies that the firm's discounted revenue stream will go up too. Then it is less needed to downscale so that the firm will wait longer with exercising this option. If the recession is expected to last long, this second effect dominates, whereas the first effect dominates in case of an (expected) short length recession.

In the second scenario we consider a recession like the one that started in 2008. This implies that, next to the drop in demand, the firm also faces an environment in which capital markets do not function in the sense

[1]For example, on the site http://www.msnbc.msn.com/id/28519217/ it is stated: "A global economic downturn has hammered the auto industry in Japan and elsewhere, forcing carmakers to cut staff, lower production and delay new models."

that banks do not lend, and stock prices are so low that it is very expensive for firms to issue new equity. So, firms need to self-finance their operations, as is illustrated by the following quote from The Economist (2008):

"...the only option is to ride out the recession. But companies can do this only if they have enough liquidity..."

An implication for the firm is that it needs to build up a cash buffer to prevent bankruptcy for liquidity reasons. The effect on the downscaling decision is that a shortage of cash raises the firm's incentive to downscale. This happens because the optimal disinvestment decision does not change. However, a liquidity shortage may make it impossible for the firm to continue operations and force the firm to downscale in a last-ditch attempt to survive.

The paper is organized as follows. Section 2 analyzes the downscale decision in case of a recession with operating credit markets. Section 3 on the other hand analyzes the downscale option when credit markets cannot be accessed (a so-called "credit crunch") during a recession. Section 4 concludes.

2 Downscaling in a Recession with Working Credit Markets

Consider a firm that faces the inverse demand function

$$P(Q,Y) = YQ^{-\frac{1}{\gamma}},$$

where Q is the quantity produced, $\gamma > 1$ the price elasticity, and Y follows a geometric Brownian motion. As long as the recession lasts we have that the drift of Y is negative $\mu_R < 0$ and from the moment that the recession is over we have that the drift returns to its normal positive level $\mu_N > 0$. The volatility is equal to σ and is not affected by the recession. This implies that during the recession it holds that

$$dY = \mu_R Y dt + \sigma Y dz,$$

where dz is the increment of a Wiener process. When the recession is over we have

$$dY = \mu_N Y dt + \sigma Y dz.$$

The end of the recession is modelled as the arrival of a Poisson process with rate λ. During the recession the firm has the option to downscale its operation from Q_0 to $Q_1\,(< Q_0)$ at a cost of I. This option is only available during the recession, because only then the regulator allows it to lay off workers. The firm considers downscaling especially when revenue falls below production costs, where we assume constant unit costs of production being equal to C. The lower the revenue becomes, the more attractive it is to exercise the downscaling option, implying that downscaling is especially attractive for low values of Y.

The profit of the firm is equal to

$$\pi(Q,Y) = (P(Q,Y) - C)\,Q.$$

At the moment that recession is over the expected value of the firm equals

$$V(Q,Y) = E\left[\int_0^\infty \pi(Q,Y)\exp(-rt)\,dt\right] = \frac{YQ^{1-\frac{1}{\gamma}}}{r-\mu_N} - \frac{CQ}{r}.$$

To analyze the decision to downscale we first determine the value of the firm after downscaling, while the recession is still ongoing. Denoting this value by $V_1(Y)$, we prove in Appendix A that

$$V_1(Y) = \frac{r+\lambda-\mu_N}{r+\lambda-\mu_R}\frac{Q_1^{1-\frac{1}{\gamma}}Y}{r-\mu_N} - \frac{CQ_1}{r}. \tag{1}$$

The first term of $V_1(Y)$ stands for the expected discounted revenue stream after downscaling, while the second term is equal to the discounted stream of production costs while producing the amount of Q_1 forever. Note that if there were no recession, i.e. $\mu_R = \mu_N$, the expected discounted revenue stream is just the instantaneous revenue divided by the discount rate net from the revenue growth rate μ_N. The presence of the recession corrects this expected discounted revenue stream in a negative direction by multiplication with the term $\frac{r+\lambda-\mu_N}{r+\lambda-\mu_R}$, which is less than one. Note that this "*revenue correction factor*" is increasing in μ_R, because the recession is less severe when μ_R is large. The *revenue correction factor* is also increasing in λ, because the higher the probability that the recession will be over within a given amount of time, the less the revenue stream needs to be corrected for the presence of the recession.

Next we determine the value of the firm during the recession, but then before downscaling. Denoting this value by V_0, we show in Appendix B that it is given by

$$V_0(Y) = B_2 Y^{\beta_2} + \frac{r+\lambda-\mu_N}{r+\lambda-\mu_R}\frac{Q_0^{1-\frac{1}{\gamma}}Y}{r-\mu_N} - \frac{CQ_0}{r}.$$

Analogous to expression (1), the last two terms of $V_0(Y)$ relate to the discounted revenue and cost stream, but now for a production level of Q_0. The first term, with unknown constant B_2, and β_2 being the negative root of the quadratic polynomial

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \mu_R\beta - (r+\lambda) = 0, \tag{2}$$

is the value of the option to downscale. Since downscaling becomes more profitable for lower values of Y, this option value is decreasing in Y.

Employing the value matching and smooth pasting conditions

$$\begin{aligned} V_0(Y^*) &= V_1(Y^*) - I, \\ \left.\frac{\partial V_0(Y)}{\partial Y}\right|_{Y=Y^*} &= \left.\frac{\partial V_1(Y)}{\partial Y}\right|_{Y=Y^*}, \end{aligned}$$

we find the downscale threshold level

$$Y^* = \frac{\beta_2}{\beta_2-1}\frac{r+\lambda-\mu_R}{r+\lambda-\mu_N}\frac{C(Q_0-Q_1)-rI}{Q_0^{1-\frac{1}{\gamma}} - Q_1^{1-\frac{1}{\gamma}}}\frac{r-\mu_N}{r}. \tag{3}$$

At Y^* the firm is indifferent between downscaling or waiting with downscaling. For Y smaller than Y^* it is optimal to downscale immediately, while otherwise it is optimal to wait with downscaling.

Expression (3) can be rewritten into

$$\frac{r+\lambda-\mu_N}{r+\lambda-\mu_R}\frac{Y^*\left(Q_0^{1-\frac{1}{\gamma}}-Q_1^{1-\frac{1}{\gamma}}\right)}{r-\mu_N}+\frac{\beta_2}{\beta_2-1}I=\frac{\beta_2}{\beta_2-1}\frac{C\left(Q_0-Q_1\right)}{r}.$$

If for the moment we put the fraction $\frac{\beta_2}{\beta_2-1}$ equal to one, the above equality says that the firm chooses for downscaling at the moment that the expected revenue loss due to downscaling plus the cost of downscaling I, is equal to the cost reduction achieved by downscaling. In other words, downscaling takes place at the moment that the net present value of downscaling equals zero. However, since β_2 is negative, the fraction $\frac{\beta_2}{\beta_2-1}$ is lower than one so that the net present value of downscaling is strictly positive at the moment that downscaling takes place. i.e. the threshold level Y^* falls below the level of Y at which the costs and benefits of downscaling match. This implies that during some time interval of positive length the firm accepts losses in the hope that industry profitability may improve. Dixit (1992) argues that this is perfectly rational in the presence of sunk costs. The reason is that there is a value of waiting associated with irreversible decisions in an uncertain environment (see, e.g., Dixit and Pindyck (1994)). As McGrath et al. (2004, p. 99) put it: "under uncertainty, it is rational to keep options open, to hesitate when uncertainty is beyond one's ability to influence it". As uncertainty increases, the distribution of possible future outcomes widens, the potential for significant profit improvement increases, and downscaling becomes less attractive. The options logic is pertinent here because the firm can always reduce the downside losses by downscaling later if conditions deteriorate (O'Brien and Folta (2009)). This is mathematically confirmed by the fact that if uncertainty goes up, the fraction $\frac{\beta_2}{\beta_2-1}$ goes down, so that the firm waits longer in the sense that downscaling will take place at a lower threshold value Y^*.

In Figure 1 the downscale threshold Y^* is plotted as function of λ. From this figure we conclude that the disinvestment trigger is non-monotonic in the intensity of the Poisson process. This is because there are two effects at work here. On the one hand, if λ goes up the recession is expected to be over sooner, and after the recession is over the firm loses the option to downscale. This implies that the firm will downscale sooner, i.e. Y^* goes up with λ. This "*impatience effect*" is quantified in (2) where we see that an increase in λ has the same effect as an increase in the discount rate r. We conclude that this effect works through β_2, i.e. an increase of λ raises β_2, which in turn raises the fraction $\frac{\beta_2}{\beta_2-1}$ and thus also Y^*.

On the other hand, if λ goes up the *revenue correction factor* $\frac{r+\lambda-\mu_N}{r+\lambda-\mu_R}$ goes up too, which is also for the reason that the recession is expected to last shorter. This makes that downscaling is less needed, so the firm waits longer with downscaling implying that Y^* decreases. Note that for larger values of λ the effect of an increase of λ on the *revenue correction factor* is smaller, which explains that the *impatience effect* dominates for large values of λ so that Y^* goes up there.

Let us now assume that the downscale option is still present after the recession is over. In Figure 2 we compare the optimal disinvestment triggers in the two different models, where Y^*_{all} stands for the threshold level of downscaling in case the downscale option does not vanish after the recession is over. We conclude that in this case the firm will disinvest later, i.e. $Y^*_{all} < Y^*$. The reason is that the *impatience effect*, and

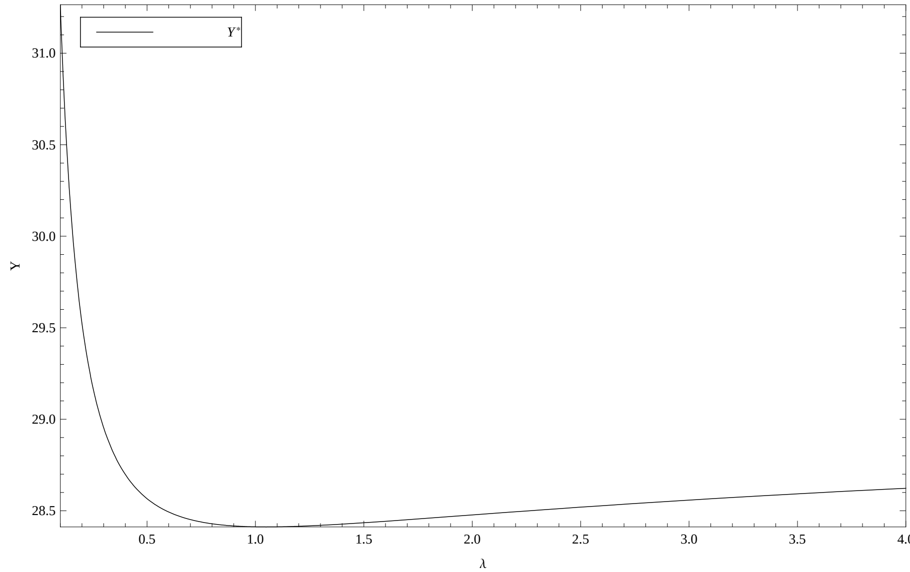

Figure 1: Optimal downscale level Y^* as function of λ. Parameter values are $I = 20$, $r = 0.04$, $\sigma = 0.1$, $\mu_r = -0.04$, $\mu_N = 0.02$, $Q_0 = 1$, $Q_1 = 0.8$, $C = 35$, and $\gamma = 2$.

thus the corresponding upward pressure on Y^*, has disappeared. Hence, only the *revenue correction factor* effect is at work here, which at the same time explains that Y^*_{all} is decreasing with λ.

3 Downscaling in a Recession with Credit Crunch

One of the defining features of the credit crunch recession is that liquidity is hard to get by. This results in many firms going bankrupt, not because they are insolvent, but because they are illiquid.

This section develops an extension of the model presented in the previous section by assuming that the firm has no access to short-term capital and that the firm has no access to sufficient liquidity to continue production if losses are made. This implies that the firm goes bankrupt as soon as the profit stream of the firm gets below an exogenously determined threshold B. This assumption is made to ensure analytical tractability of the model. That is, at production level Q_k, $k = 0, 1$, the firms goes bankrupt as soon as

$$\pi_k(Y) \leq B \iff Y \leq Y^k \equiv BQ_k^{1/\gamma - 1} + CQ_k^{1/\gamma}.$$

The possibility of bankruptcy extends into the period after the firm has downscaled, but before the recession is over. This implies that the present value of downscaling has to include a correction for the possibility of bankruptcy. It is shown in Appendix C that the present value $F_1(\cdot)$ equals

$$F_1(y) = B_2 Y^{\beta_2} + \frac{r + \lambda - \mu_N}{r + \lambda - \mu_R} \frac{Q_1^{1-1/\gamma}}{r - \mu_N} - \frac{CQ_1}{r},$$

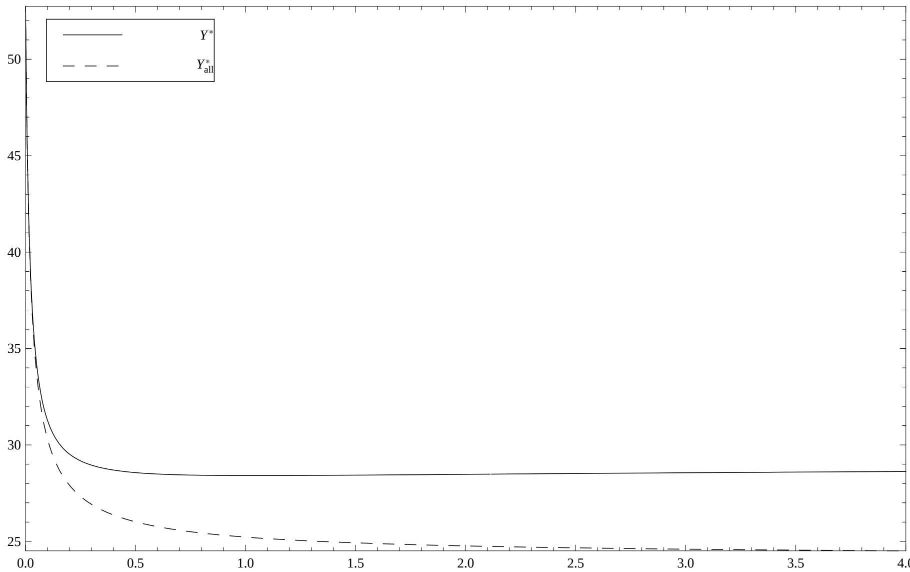

Figure 2: Optimal downscale level Y^* as function of λ and the optimal downscale level Y^*_{all} in case the downscale option is not lost when the recession ends. Parameter values are $I = 20$, $r = 0.04$, $\sigma = 0.1$, $\mu_r = -0.04$, $\mu_N = 0.02$, $Q_0 = 1$, $Q_1 = 0.8$, $C = 35$, and $\gamma = 2$.

where

$$B_2 = \frac{CQ_1}{r}(Y^1)^{-\beta_2} - \frac{r+\lambda-\mu_N}{r+\lambda-\mu_R}\frac{Q_1^{1-1/\gamma}}{r-\mu_N}(Y^1)^{1-\beta_2}.$$

The continuation value of the option, $F_0(Y)$, can be derived just as before. Thus, it equals

$$F_0(Y) = A_1 Y^{\beta_1} + A_2 Y^{\beta_2} + \frac{r+\lambda-\mu_N}{r+\lambda-\mu_R}\frac{Q_0^{1-1/\gamma}}{r-\mu_N} - \frac{CQ_1}{r}.$$

Ruling out speculative bubbles we set $A_1 = 0$. The optimal downscaling threshold Y^{**} is now obtained by using the familiar value-matching and smooth pasting conditions,

$$F_0(Y^{**}) = F_1(Y^{**}) - I, \quad \text{and} \quad \left.\frac{\partial F_0(Y)}{\partial Y}\right|_{Y=Y^{**}} = \left.\frac{\partial F_1(Y)}{\partial Y}\right|_{Y=Y^{**}}. \tag{4}$$

It is easy to see that this leads to the threshold

$$Y^{**} = \frac{\beta_2}{\beta_2 - 1}\frac{r+\lambda-\mu_R}{r+\lambda-\mu_N}\frac{r-\mu_N}{Q_1^{1-1/\gamma} - Q_0^{1-1/\gamma}}\Big(\frac{C(Q_1-Q_0)}{r} + I\Big). \tag{5}$$

Note that this threshold is the same as the threshold in the case that there is no risk of bankruptcy. The option value, however, is lower because the constant A_1 is lower in the case with bankruptcy risk. The effect here is the same as the familiar result from Leahy (1993) that the investment threshold in monopoly and perfect competition are equal.

Obviously this threshold only applies as long as $Y^* > Y^0$. Otherwise the firm has to downscale production at the sub-optimal threshold Y^0 in a last-ditch attempt to prevent bankruptcy. So,

$$Y^{**} = \max\{Y^*, Y^0\}. \tag{6}$$

Note that if $Y^* < Y^0$, the value of the option to downscale production reduces. After all, the constant A_2 is not determined by the value-matching and smooth-pasting conditions (4), but by the boundary condition $F_0(Y^0) = F_1(Y^0) - I$. In order to distinguish between these two cases let A_2^U denote the unrestricted case, where the constant is determined by the value-matching and smooth-pasting conditions. In the case where $Y^* < Y^0$, let the constant be denoted by A_2^R. So,

$$F_0(Y) = \frac{r+\lambda-\mu_N}{r+\lambda-\mu_R}\frac{Q_0^{1-1/\gamma}}{r-\mu_N} - \frac{CQ_1}{r} + \begin{cases} A_2^U Y^{\beta_2} & \text{if } Y^* \geq Y^0, \\ A_2^R Y^{\beta_2} & \text{if } Y^* < Y^0. \end{cases}$$

4 Conclusion

In this paper we have analysed the decision of a firm to downscale production in recessionary times. We have derived the optimal trigger and have shown that it is non-monotonic in the expected duration of the recession. This non-monotonicity arises due to the presence of two opposing effects. Firstly, there is a revenue correction effect, which decreases in the expected duration of the recession. This effect is due to the fact that the expected growth rate of the firm's profits is lower during the recession than in normal times. It enters the optimal policy inversely, so that an increase in the expected duration of the recession leads to earlier disinvestment. The second effect is an impatience effect. which is due to the fact that the expected duration of the recession inversely enters the discount rate. That is, an increase in the expected duration of the recession leads to a lower discount rate and, thus, to a higher value of the option to disinvest and, thus, later disinvestment. It turns out that neither of these two effects dominates.

We also considered a situation which may be particularly relevant in the 2008/2009 recession, namely the addition of frozen credit markets ("credit crunch"). We have modelled this by imposing an exogenous lower boundary of the profit stream that, if breached, automatically leads to downscaling if the firm hasn't done so yet, or bankruptcy if it has. The idea behind this formulation is that the better the cash position of the firm, the lower this bankruptcy threshold will be. It turns out that if the firm can take an optimal decision it will do so at the same time as when bankruptcy is not a concern. This happens because the possibility of bankruptcy lowers the present value of downscaling and the option value of downscaling by the same amount.

Appendix

A Value of the Firm after Downscaling during Recession

After the firm has downscaled and the recession is still ongoing the value of the firm, denoted by V_1, has to satisfy the following Bellman equation

$$rV_1(Y) = \pi(Q_1, Y) + \lim_{dt\downarrow 0}\frac{E[dV_1(Y)]}{dt}.$$

We have that

$$E\left[dV_1\left(Y\right)\right] = \lambda dt\left(F\left(Q_1,Y\right)-V_1\left(Y\right)\right)+\left(1-\lambda dt\right)\left(\mu_R Y\frac{\partial V_1\left(Y\right)}{\partial Y}dt+\frac{1}{2}\sigma^2Y^2\frac{\partial^2 V_1\left(Y\right)}{\partial Y^2}dt\right)+o\left(dt\right).$$

Substitution and rewriting gives

$$\left(r+\lambda\right)V_1\left(Y\right)=\left(YQ_1^{-\frac{1}{\gamma}}-C\right)Q_1+\lambda F\left(Q_1,Y\right)+\mu_R Y\frac{\partial V_1\left(Y\right)}{\partial Y}+\frac{1}{2}\sigma^2Y^2\frac{\partial^2 V_1\left(Y\right)}{\partial Y^2}. \tag{A.1}$$

The solution of the homogeneous equation is equal to

$$V_1\left(Y\right)=A_1Y^{\beta_1}+A_2Y^{\beta_2},$$

where β_1 is the positive root and β_2 the negative root of

$$\frac{1}{2}\sigma^2\beta\left(\beta-1\right)+\mu_R\beta-\left(r+\lambda\right)=0.$$

Since $V_1\left(0\right)=0$ we have that $A_2=0$. Furthermore, we have that $A_1=0$ as we rule out speculative bubbles. A particular solution for (A.1) is given by

$$V_1\left(Y\right)=aY+b.$$

Substitution results in

$$\left(r+\lambda\right)\left(aY+b\right)=\left(YQ_1^{-\frac{1}{\gamma}}-C\right)Q_1+\lambda\left(\frac{YQ_1^{1-\frac{1}{\theta}}}{r-\mu_N}-\frac{CQ_1}{r}\right)+\mu_R Ya,$$

solving gives

$$\begin{aligned} a &= \frac{r+\lambda-\mu_N}{r+\lambda-\mu_R}\frac{Q_1^{1-\frac{1}{\gamma}}}{r-\mu_N}, \\ b &= -\frac{CQ_1}{r}. \end{aligned}$$

Thus

$$V_1\left(Y\right)=\frac{r+\lambda-\mu_N}{r+\lambda-\mu_R}\frac{Q_1^{1-\frac{1}{\gamma}}Y}{r-\mu_N}-\frac{CQ_1}{r}.$$

B Value of the Firm before Downscaling during Recession

Denote the value of the firm before the downscaling and before the recession is over by V_0. The value function in this period has to satisfy

$$rV_0\left(Y\right)=\pi\left(Q_0,Y\right)+\lim_{dt\downarrow 0}\frac{E\left[dV_0\left(Y\right)\right]}{dt},$$

where

$$E\left[dV_0\left(Y\right)\right] = \lambda dt\left(F\left(Q_0,Y\right)-V_0\left(Y\right)\right)+\left(1-\lambda dt\right)\left(\mu_R Y\frac{\partial V_0\left(Y\right)}{\partial Y}dt+\frac{1}{2}\sigma^2Y^2\frac{\partial^2 V_0\left(Y\right)}{\partial Y^2}dt\right)+o\left(dt\right).$$

Substitution gives

$$(r+\lambda)\, V_0\,(Y) = \left(YQ_0^{-\frac{1}{\gamma}} - C\right) Q_0 + \lambda F\,(Q_0, Y) + \mu_R Y \frac{\partial V_0\,(Y)}{\partial Y} + \frac{1}{2}\sigma^2 Y^2 \frac{\partial^2 V_0\,(Y)}{\partial Y^2}.$$

The solution is given by

$$V_0\,(Y) = B_1 Y^{\beta_1} + B_2 Y^{\beta_2} + \frac{r+\lambda-\mu_N}{r+\lambda-\mu_R} \frac{Q_0^{1-\frac{1}{\gamma}} Y}{r-\mu_N} - \frac{CQ_0}{r}.$$

We have that $B_1 = 0$ as we rule out speculative bubbles, i.e. the option to downscale will be worthless whenever Y takes extremely large values. Concluding we have that

$$V_0\,(Y) = B_2 Y^{\beta_2} + \frac{r+\lambda-\mu_N}{r+\lambda-\mu_R} \frac{Q_0^{1-\frac{1}{\gamma}} Y}{r-\mu_N} - \frac{CQ_0}{r}.$$

C Value of the Firm after Downscaling during Credit Crunch

The firm goes bankrupt as soon as $\pi(Q_1, Y_t) < B$, where B is exogenously given. Suppose that the recession is still ongoing and that Y is such that $\pi(Q_1, Y_t) > B$. In a small interval of time dt, the firm will not go bankrupt. Also, with probability λdt the recession ends and the firm's value will be

$$\mathbb{E}\Big[\int_0^\infty \pi(Q_1, Y_t) dt\Big] = \frac{Q_1^{1-1/\gamma} Y_t}{r-\mu_N} - \frac{C}{r}.$$

So, the present value of the firm after downscaling, denoted by $F(\cdot)$, can be defined recursively as

$$\begin{aligned} F(Y_t) &= \lambda dt\Big(\frac{Q_1^{1-1/\gamma} Y_t}{r-\mu_N} - \frac{C}{r}\Big) + (1-\lambda dt)(1-r dt)\mathbb{E}[F(Y_t + dY)|\mathscr{F}_t] \\ \iff \quad (r+\lambda)F(Y_t)dt &= \lambda dt\Big(\frac{Q_1^{1-1/\gamma} Y_t}{r-\mu_N} - \frac{C}{r}\Big) + \mathbb{E}[dF|\mathscr{F}_t] + o(dt). \end{aligned}$$

After applying Ito's lemma to compute the expectation, rearranging and taking the limit $dt \downarrow 0$ this leads to the differential equation

$$\frac{1}{2}\sigma^2 Y^2 F''(Y) + \mu_N Y F(Y) - (r+\lambda)F(Y) + \lambda\Big(\frac{Q_1^{1-1/\gamma} Y_t}{r-\mu_N} - \frac{C}{r}\Big) = 0,$$

which has the solution

$$F(Y) = B_1 Y^{\beta_1} + B_2 Y^{\beta_2} + \frac{r+\lambda-\mu_N}{r+\lambda-\mu_R} \frac{Q_1^{1-1/\gamma} Y_t}{r-\mu_N} - \frac{C}{r},$$

for unknown constants B_1 and B_2.

Ruling out speculative bubbles we set $B_1 = 0$. The second boundary condition is that the present value of the firm is zero once it goes bankrupt, i.e. $F(Y^1) = 0$. This implies that

$$B_2 = \frac{CQ_1}{r}(Y^1)^{-\beta_2} - \frac{r+\lambda-\mu_N}{r+\lambda-\mu_R} \frac{Q_1^{1-1/\gamma}}{r-\mu_N}(Y^1)^{1-\beta_2}.$$

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