

# Dynamic investment and capital structure under manager-shareholder conflict\*

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**Abstract:** This paper investigates the interactions between the investment and financing decisions of a firm under manager-shareholder conflicts arising from asymmetric information. In particular, we extend the manager-shareholder conflict problem in a real options model by incorporating debt financing. We show that manager-shareholder conflicts over investment policy increase the investment and default triggers and coupon payments, which lead to an increase in the value of debt and a decrease in the value of equity. Moreover, given the presence of manager-shareholder conflicts, debt financing increases investment and decreases total social welfare. As a result, there is a trade-off between the efficiency of investment and total social welfare with debt financing. These results fit well with the findings of previous empirical work in this area.

**Keywords:** Real options; debt financing; agency problem; asymmetric information.

**JEL classification:** G1; G3.

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# 1 Introduction

Modigliani and Miller (1958) argue that financing and investment decisions are completely separable in a perfectly competitive market. Since their seminal work, the corporate finance literature has studied the interaction between investment and financing decisions with various market frictions. In most modern corporations, for example, shareholders delegate the investment decision to managers, thereby taking advantage of their special skills and expertise. In this situation, asymmetric information is likely. Asymmetric information is the situation where managers privately observe a portion of the underlying state variable not observed by shareholders. Managers with private information thus have an incentive to provide false reports and then divert to themselves free cash flow. Consequently, asymmetric information leads to manager-shareholder conflicts.<sup>1</sup>

The real options model has become a standard framework for investment timing decisions in corporate finance. Dixit and Pindyck (1994) provide an excellent overview of the standard real options approach. In the standard real options model, however, there are two major limitations. First, the standard approach is within an all-equity financing framework. Second, there is no manager-shareholder conflict because the firm, by assumption, is owner-managed.

Several studies have already begun the task of *separately* incorporating either debt financing or manager-shareholder conflicts due to asymmetric information in the real options model. Dynamic models that allow for the interaction between investment and financing decisions include Brennan and Schwartz (1984), Mauer and Triantis (1994), and Sundaresan and Wang (2006, 2007). Alternatively, dynamic models under manager-shareholder conflicts resulting from asymmetric information include Bjerksund and Stensland (2000), Grenadier and Wang (2005), and Nishihara and Shibata (2008).<sup>2</sup> Under asymmetric information, shareholders must design a contract to provide incentives for managers to truthfully reveal private information.<sup>3</sup> The implied investment timing then differs significantly from that in the standard full information real options model. Al-

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<sup>1</sup>Starting with the seminal work of Myers and Majluf (1984), manager-shareholder conflicts resulting from asymmetric information have been widely discussed in static models.

<sup>2</sup>While these papers focus on manager-shareholder conflicts, similar problems arise between shareholder and bondholders. Mello and Parsons (1992), Leland (1998), Parrino and Weisbach (1999), Mauer and Ott (2000), Morellec (2001), Childs, Mauer, and Ott (2005), and Mauer and Sarkar (2005) examine shareholder-bondholder conflicts.

<sup>3</sup>In order to truthfully reveal private information, Bjerksund and Stensland (2000) and Grenadier and Wang (2005) assume that the shareholder gives a bonus incentive to the manager, while Shibata (2008) and Nishihara and Shibata (2008) assume that he/she gives a bonus incentive and/or imposes the penalty for false reporting.

though these strategies turn out to be suboptimal, they reduce the shareholders' losses resulting from asymmetric information. Without any incentive mechanism that induces the manager to reveal private information truthfully, shareholders suffer further distortions.

To our best knowledge, there has been little examination of debt financing over investment policy under manager-shareholder conflicts arising from asymmetric information. Recently, Morellec (2004) and Childs and Mauer (2008) examined the interaction between investment and financing decisions under manager-shareholder conflicts arising from managerial discretion, *not* asymmetric information.<sup>4</sup> Thus, we focus on the manager-shareholder conflicts arising from asymmetric information. We then explore several important questions. First, how does debt financing influence the manager's investment decision? Second, how does the manager's information advantage affect coupon payments, leverage and default decisions, and debt and equity values? Finally, how large are the agency costs with all-equity financing and debt financing?

Our paper examines the interactions between investment and financing decisions under manager-shareholder conflicts resulting from asymmetric information in a real options framework. In particular, we extend the manager-shareholder conflict problem in the real options model developed by Grenadier and Wang (2005), Shibata (2008), and Nishihara and Shibata (2008) by incorporating debt financing.

Our paper provides several important results. First, manager-shareholder conflicts over investment policy increase the investment and default triggers and coupon payments, leading to an increase in debt value and a decrease in equity value. These results are the same as those in the seminal work by Myers and Majluf (1984), and fit well with the findings of empirical studies (see Korajczyk, Lucas, and McDonald, 1991; Jung, Kim, and Stulz, 1996). Second, given the presence of manager-shareholder conflicts, debt financing leads to an increase in investment (i.e., a decrease in the investment trigger) and a decrease in total social welfare (i.e., an increase in the agency cost).<sup>5</sup> Thus, there is a trade-off in the efficiency of investment and total social welfare with debt financing. In addition, in our numerical simulations, the agency cost, defined as the total social loss resulting from asymmetric information, is decreasing with the volatility of market uncertainty. These results are also consistent with the findings in Myers and Majluf (1984). Third, an increase in the penalty for a manager's false report does *not necessarily* decrease the agency cost,

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<sup>4</sup>In Morellec (2004), manager-shareholder conflicts are introduced by considering that the manager derives utility from both retaining control and investing in new projects. In Childs and Mauer (2008), they result from managerial risk aversion.

<sup>5</sup>Under full information, debt financing leads to an increase in investment. See Sundaresan and Wang (2006, 2007).

although it *always* increases the equity (shareholder's) value.<sup>6</sup> These results imply that a shareholder's (individual) rationality does *not necessarily* lead to total social rationality. However, the no-agency solutions and values are approximated closely as the penalty is increased without limit (although unlimited penalties are only of theoretical interest). The results for unlimited penalties are the same as in the seminal static work by Baron and Besanko (1984).

The remainder of the paper is organized as follows. Section 2 describes the setup of the model and derives the solution in the no-agency (full information) problem. Section 3 formulates the optimization problem and provides the optimal contracts in the agency problem. Section 4 discusses the model implications of the agency problem. Section 5 concludes.

## 2 Full information

In this section, we begin with a description of the setup. We then provide the solution to the no-agency (full information) problem with all-equity and debt financing, respectively. Finally, we compare the solutions given in the no-agency problems with all-equity and debt financing.

### 2.1 Setup

Consider an owner (shareholder) with an option to invest in a single project. We assume that the owner delegates the investment decision to a manager. Throughout our analysis, all agents are assumed to be risk neutral and to maximize their expected payoff. At each investment time, the firm may issue a mixture of debt and equity to finance the cost of exercising the investment opportunity. Assume that debt has a tax advantage in that the firm faces a constant tax rate  $\tau > 0$  on income after servicing the interest payments on debt.

The investment yields an instantaneous cash flow  $QX(t)$ , where  $Q > 0$  is the constant quantity and  $X(t)$  is the price at time  $t$  given by the following geometric Brownian motion:

$$dX(t) = \mu X(t)dt + \sigma X(t)dz(t), \quad X(0) = x, \quad (1)$$

where  $z(t)$  denotes the standard Brownian motion defined on a probability space  $(\Omega, \mathcal{F}, \mathbb{Q})$  and  $\mu$  and  $\sigma$  are constant parameters. For convergence, we assume that  $r > \mu$  where  $r$  is a

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<sup>6</sup>In our model, private information is fully revealed at investment, in that the shareholder impose the penalty on the manager when the false report by manager is detected.

constant interest rate. The cash flow  $QX(t)$  can be regarded as the firm's earnings before interest and taxes if the firm were in operation. Let  $\Pi(x)$  denote the after-tax (all-equity financed) values of assets in place generated from exercising the investment opportunity. Under all-equity financing, the asset in place from exercising the investment opportunity is

$$\Pi(x) = \frac{1 - \tau}{r - \mu} Qx. \quad (2)$$

When investment is undertaken, a one-time cost expenditure  $I$  is paid. The investment cost,  $I$ , could take one of two possible values:  $I_1$  or  $I_2$  with  $I_2 > I_1 > 0$ . We denote  $\Delta I = I_2 - I_1$ . We assume that  $I_1$  represents a "lower cost" expenditure and  $I_2$  represents a "higher cost" expenditure. The probabilities of drawing  $I_1$  and  $I_2$  are equal to  $q$  and  $1 - q$ , respectively, i.e.,  $\mathbb{P}(I_1) = q$  and  $\mathbb{P}(I_2) = 1 - q$  where  $q \in (0, 1)$ .

## 2.2 No-agency problem with all-equity financing

Let  $E_{Uk}(x)$  denote the equity value of the unlevered firm for  $I = I_k$  ( $k \in \{1, 2\}$ ). Here, the subscript "U" stands for the unlevered firm. The value  $E_{Uk}(x)$  is formulated as

$$E_{Uk}(x) = \sup_{T_{Uk}^i} \mathbb{E} \left[ \int_{T_{Uk}^i}^{+\infty} e^{-rt} (1 - \tau) QX(t) dt - e^{-rT_{Uk}^i} I_k \mid X(0) = x \right], \quad k \in \{1, 2\} \quad (3)$$

where  $T_{Uk}^i > 0$ . Here, the superscript "i" stands for the investment strategy, the operator  $\mathbb{E}[\cdot \mid X(0) = x]$  denotes the expectation operator given that  $X(0) = x$ , and  $T_{Uk}^i$  is the stopping time that the investment is exercised at trigger  $x_{Uk}^i$ , i.e.,  $T_{Uk}^i := \inf\{t \geq 0; X(t) \geq x_{Uk}^i\}$ . Throughout the paper, it is assumed that the current state variable  $X(0) = x$  is sufficiently low so that the investment is not undertaken immediately. Mathematically, we assume that  $T_{Uk}^i > 0$  and  $x < x_{Uk}^i$  for all  $k$ . Using standard arguments, we can formulate the no-agency problem with all-equity financing as follows:

$$\max_{x_{U1}^i, x_{U2}^i} \sum_{k=1}^2 \mathbb{P}(I_k) \left( \frac{x}{x_{Uk}^i} \right)^\beta (\Pi(x_{Uk}^i) - I_k), \quad (4)$$

where  $\beta$  is positive constant and strictly larger than 1, i.e.,

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1. \quad (5)$$

The mechanism under the no-agency problem with all-equity financing is defined by  $\mathcal{M}_U^* = (x_{Uk}^{i*})$  and is obtained by

$$x_{Uk}^{i*} = \frac{\beta}{\beta - 1} \frac{1}{\Pi(1)} I_k, \quad k \in \{1, 2\}, \quad (6)$$

where the superscript “\*” stands for the solution under the no-agency problem. The equity value at time 0 is obtained by

$$E_U^*(x) = \sum_{k=1}^2 \mathbb{P}(I_k) \left( \frac{x}{x_{Uk}^{i*}} \right)^\beta (\Pi(x_{Uk}^{i*}) - I_k). \quad (7)$$

The value given in (7) is used as a benchmark.<sup>7</sup>

### 2.3 No-agency problem with debt financing

In this subsection, we consider the investment strategy in the no-agency problem with debt financing. This is identical to the simple models in the Sundaresan and Wang (2006, 2007).

We assume that the firm issues debt with infinite maturity. This assumption, which is the same as that in Leland (1994), simplifies the analysis substantially and does not alter the key economic insights. Under debt financing, the firm must take into account the optimal capital structure at investment as well as the optimal default timing after investment.

Let  $T_{Lk}^i$  and  $T_k^d$  denote the investment and default timings of the levered firm for  $I = I_k$ , respectively. Here, the subscript “L” stands for the levered firm issuing debt, and the superscripts “i” and “d” stand for the investment and default strategies, respectively. Mathematically, these two timings are defined as  $T_{Lk}^i = \inf\{t \geq 0; X(t) \geq x_{Lk}^i\}$  and  $T_k^d = \inf\{t \geq T_{Lk}^i; X(t) \leq x_k^d\}$  where  $x_{Lk}^i$  and  $x_k^d$  denote the investment and default triggers, respectively.

We begin with deriving the optimal default timing backwards. The levered firm decides on the default timing to maximize equity value after investment,  $E_{Lk}(x_{Lk}^i)$ , defined as

$$E_{Lk}(x_{Lk}^i) = \sup_{T_k^d} \mathbb{E} \left[ \int_{T_{Lk}^i}^{T_k^d} e^{-r(t-T_{Lk}^i)} (1-\tau)(QX(t) - c_k) dt \middle| X(T_{Lk}^i) = x_{Lk}^i \right], \quad k \in \{1, 2\}, \quad (8)$$

where  $c_k$  denotes the coupon payment for  $I = I_k$ . As in Leland (1994),  $E_{Lk}(x_{Lk}^i)$  is obtained as

$$E_{Lk}(x_{Lk}^i) = \Pi(x_{Lk}^i) - \Pi(x_k^d) \left( \frac{x_{Lk}^i}{x_k^d} \right)^\gamma - (1-\tau) \frac{c_k}{r} \left( 1 - \left( \frac{x_{Lk}^i}{x_k^d} \right)^\gamma \right), \quad k \in \{1, 2\}, \quad (9)$$

where the optimal default trigger after investment is given by

$$x_k^d(c_k) = \frac{\gamma}{\gamma-1} \frac{r-\mu}{Q} \frac{c_k}{r}, \quad k \in \{1, 2\}, \quad (10)$$

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<sup>7</sup>The model in this subsection is the standard real options model developed by McDonald and Siegel (1986).

and  $\gamma$  is the negative constant, i.e.,

$$\gamma = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0. \quad (11)$$

Note that  $x_k^d$  is given as a linear function of  $c_k$ . In contrast, the debt value of the levered firm after investment,  $D_{Lk}(x_{Lk}^i)$ , is given as

$$D_{Lk}(x_{Lk}^i) = \mathbb{E} \left[ \int_{T_{Lk}^i}^{T_k^d} e^{-r(t-T_{Lk}^i)} c_k dt + (1-\alpha) e^{-r(T_k^d-T_{Lk}^i)} \Pi(x_k^d) \middle| X(T_{Lk}^i) = x_{Lk}^i \right] \quad (12)$$

$$= \frac{c_k}{r} \left( 1 - \left( \frac{x_{Lk}^i}{x_k^d} \right)^\gamma \right) + (1-\alpha) \Pi(x_k^d) \left( \frac{x_{Lk}^i}{x_k^d} \right)^\gamma, \quad k \in \{1, 2\}, \quad (13)$$

where  $\alpha \in (0, 1)$  is the bankruptcy cost. The sum of the equity and debt values is equal to the firm value,  $V_{Lk}(x_{Lk}^i) = E_{Lk}(x_{Lk}^i) + D_{Lk}(x_{Lk}^i)$ , as follows:

$$V_{Lk}(x_{Lk}^i) = \Pi(x_{Lk}^i) + \frac{\tau c_k}{r} \left( 1 - \left( \frac{x_{Lk}^i}{x_k^d} \right)^\gamma \right) - \alpha \Pi(x_k^d) \left( \frac{x_{Lk}^i}{x_k^d} \right)^\gamma, \quad k \in \{1, 2\}. \quad (14)$$

Here, the first term on the right-hand side in (14) represents the value of the unlevered firm, the second term is the tax shield, and the third term is the bankruptcy cost.

We then consider the coupon payment level decided optimally at the time of investment when debt is issued to maximize the capital structure of the firm. The coupon payment  $c_k$  is chosen to maximize firm value  $V_{Lk}(x_{Lk}^i)$ , i.e.,

$$c_k(x_{Lk}^i) = \frac{r}{r-\mu} \frac{\gamma-1}{\gamma} \frac{1}{h} Q x_{Lk}^i, \quad k \in \{1, 2\}, \quad (15)$$

where

$$h = \left( 1 - \gamma \left( 1 - \alpha + \frac{\alpha}{\tau} \right) \right)^{-1/\gamma} \geq 1. \quad (16)$$

Note that  $c_k$  is a linear function of  $x_{Lk}^i$ , and that  $x_k^d$  is a linear function of  $x_{Lk}^i$ , substituting (15) into (10). These results are the same as in representative capital structure models such as Leland (1994) where financing is made at the current time  $t = 0$ . That is, the coupon payment in (15) has been evaluated at time 0. On the other hand, we allow financing to be made at the endogenously decided time of investment  $T_{Lk}^i$ . We can then consider the optimal capital structure by endogenously deciding on the optimal investment strategy.

Finally, we consider the investment timing. Under debt financing, the equity value at time  $T_{Lk}^i$ , when the debt is issued, becomes

$$E_{Lk}(x_{Lk}^i) - (I_k - D_{Lk}(x_{Lk}^i)) = V_{Lk}(x_{Lk}^i) - I_k, \quad k \in \{1, 2\}.$$

It is straightforward that the fund from the bondholder,  $K_k > 0$  for  $I = I_k$ , is equal to the debt value at the time when the debt is issued, i.e.,  $K_k = D_{Lk}(x_{Lk}^i)$ . Thus, the shareholder's maximization problem is formulated as

$$\max_{x_{L1}^i, x_{L2}^i} \sum_{k=1}^2 \mathbb{P}(I_k) \left( \frac{x}{x_{Lk}^i} \right)^\beta \{V_{Lk}(x_{Lk}^i) - I_k\}. \quad (17)$$

Then, we have

$$x_{Lk}^{i*} = \frac{\beta}{\beta - 1} \frac{\psi}{\Pi(1)} I_k = \psi x_{Uk}^{i*}, \quad k \in \{1, 2\}, \quad (18)$$

where

$$\psi = \left( 1 + \frac{1}{h} \frac{\tau}{1 - \tau} \right)^{-1} \leq 1. \quad (19)$$

Clearly we have  $x_{Lk}^{i*} \leq x_{Uk}^{i*}$ . In addition,  $c_k^*$  and  $x_k^{d*}$  are obtained by substituting (18) into (15) and (10), i.e.,  $c_k^* = c_k(x_{Lk}^{i*})$  and  $x_k^{d*} = x_k^d(c_k^*)$ .

As a result, the mechanism,  $\mathcal{M}_L^* = (x_{Lk}^{i*}, x_k^{d*}, c_k^*)$ , turns out to be

$$(x_{Lk}^{i*}, x_k^{d*}, c_k^*) = \left( \psi x_{Uk}^{i*}, \frac{x_{Lk}^{i*}}{h}, \zeta I_k \right), \quad k \in \{1, 2\},$$

where

$$\zeta = \frac{\gamma - 1}{\gamma} \frac{\beta}{\beta - 1} \frac{r}{1 - \tau} \left( h + \frac{\tau}{1 - \tau} \right)^{-1}. \quad (20)$$

Note that all the solutions,  $x_{Lk}^{i*}$ ,  $x_k^{d*}$ , and  $c_k^*$ , are linear functions of  $I_k$  ( $k \in \{1, 2\}$ ). Naturally, the investment trigger is greater than the default trigger, i.e.,  $x_{Lk}^{i*} > x_k^{d*}$  due to  $h > 1$ .

At time  $T_{Lk}^{i*} := \inf\{t \geq 0; X(t) \geq x_{Lk}^{i*}\}$ , the equity, debt, and firm values are obtained by

$$E_{Lk}(x_{Lk}^{i*}) = \Pi(x_{Lk}^{i*}) - \Pi(x_k^{d*})h^\gamma - (1 - \tau)(1 - h^\gamma) \frac{c_k^*}{r}, \quad (21)$$

$$D_{Lk}(x_{Lk}^{i*}) = \frac{c_k^*}{r}(1 - h^\gamma) + (1 - \alpha)h^\gamma x_k^{d*}, \quad (22)$$

and

$$V_{Lk}(x_{Lk}^{i*}) = E_{Lk}(x_{Lk}^{i*}) + D_{Lk}(x_{Lk}^{i*}) = \psi^{-1} \Pi(x_{Lk}^{i*}), \quad (23)$$

respectively, where  $k \in \{1, 2\}$ . The leverage and credit spread at the investment time,  $L_k(x_{Lk}^{i*})$  and  $cs_k(x_{Lk}^{i*})$ , are given by

$$L_k(x_{Lk}^{i*}) = \frac{D_{Lk}(x_{Lk}^{i*})}{V_{Lk}(x_{Lk}^{i*})} = \frac{\gamma - 1}{\gamma} \frac{\psi}{1 - \tau} \frac{1}{h} (1 - \xi), \quad (24)$$



and

$$cs_k(x_{Lk}^{i*}) = \frac{c_k^*}{D_{Lk}(x_{Lk}^{i*})} - r = r \frac{\xi}{1 - \xi}, \quad (25)$$

where

$$\xi = \left(1 - (1 - \alpha)(1 - \tau) \frac{\gamma}{\gamma - 1}\right) h^\gamma, \quad (26)$$

respectively. Here, we have  $0 < \xi < 1$  where we have used the fact that  $h > 1$ ,  $\gamma < 0$ ,  $0 \leq \alpha < 1$ , and  $0 \leq \tau < 1$ .

The equity value at time 0,  $E_L^*(x)$ , is obtained as

$$\begin{aligned} E_L^*(x) &= \sum_{k=1}^2 \mathbb{P}(I_k) \left(\frac{x}{x_{Lk}^{i*}}\right)^\beta \{V_{Lk}(x_{Lk}^{i*}) - I_k\} \\ &= \psi^{-\beta} E_U^*(x). \end{aligned} \quad (27)$$

Here we have  $E_L^*(x) \geq E_U^*(x)$  due to  $\psi \leq 1$  and  $\beta > 1$ . Moreover, since the debt value at time 0,  $D_L^*(x)$ , is equal to zero,<sup>8</sup> the firm value at time 0,  $V_L^*(x)$ , is equal to the equity value, i.e.,  $V_L^*(x) = E_L^*(x)$ .

### 3 Asymmetric information

In this section, we begin with a description of the contract under asymmetric information. We then provide the solution to the agency (asymmetric information) problem with all-equity and debt financing, respectively. Finally, we show that the no-agency solutions and values are approximated closely as the penalty for a manager's false report is increased without limit (although unlimited penalties are only of theoretical interest).

#### 3.1 Contracts under agency problems

In this section, we examine the investment strategy under manager-shareholder conflicts due to asymmetric information.

We assume that the price,  $X(t)$ , is observed by both the shareholder and the manager. However, the one-time investment cost,  $I$ , is privately observed only by the manager. Immediately after making a contract with the shareholder at time 0, the manager observes

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<sup>8</sup>Let  $K_k$  denote the amount financed from the bondholder. The debt value before investment is

$$D_L^*(x) = \sum_{k=1}^2 \mathbb{P}(I_k) \left(\frac{x}{x_{Lk}^{i*}}\right)^\beta (D_{Lk}(x_{Lk}^{i*}) - K_k) = 0, \quad (28)$$

where we have used the fact that  $D_{Lk}(x_{Lk}^{i*}) = K_k$ .

whether the cost is of  $I_1$  (lower cost) or  $I_2$  (higher cost). However, the shareholder cannot observe the realized value of  $I$ . Therefore, the shareholder must induce the manager to reveal private information truthfully at the time when the manager undertakes the investment; otherwise, the shareholder suffers further losses.<sup>9</sup>

We assume that the shareholder uses two methods to ensure the manager truthfully reveals the private information he/she holds. One is a bonus incentive and the other is audit technology.<sup>10</sup> The shareholder designs a contract at time zero that gives the bonus incentive and audits the manager at the time of investment.<sup>11</sup> Thus, the contract is specified for any trigger that the investment will be undertaken.

The audit technology allows the owner at a cost to verify the state announced by the manager, and to impose the penalty  $P \geq 0$  on the manager for cheating when a false report is detected. Here,  $P \geq 0$  is positive constant and given exogenously. We assume that the manager is detected with probability  $p_j(\tilde{I}_k)$ , which may be contingent on a reported  $\tilde{I}_k$  if the owner incurs a cost  $g(p_j(\tilde{I}_k))$  with  $g(0) = 0$ ,  $g' > 0$ ,  $g'' > 0$ , and  $\lim_{p_j(\tilde{I}_k) \uparrow 1} g(p_j(\tilde{I}_k)) = +\infty$ .

Since there are only two possible values of the cost  $I$ , there can be at most two pairs consisting of three components (investment trigger, bonus, and probability of an audit) in the agency problem with all-equity financing. That is, the contract in the agency problem with all-equity financing is modeled as a mechanism,

$$\mathcal{M}_U^{**} = (x_U^i(\tilde{I}_k), w_U(\tilde{I}_k), p_U(\tilde{I}_k)), \quad k \in \{1, 2\},$$

which may be contingent on a reported value  $\tilde{I}_k$ . Here,  $x_U^i(\tilde{I}_k)$  is the investment trigger,  $w_U(\tilde{I}_k)$  is the bonus to the manager,  $p_U(\tilde{I}_k)$  is the probability of an audit, and the superscript “\*\*” stands for the optimum in the agency problem. Because the revelation principle ensures that the manager reveals a true  $I_k$  as private information, we make no distinction between a reported  $\tilde{I}_k$  and a true  $I_k$ .<sup>12</sup> Thus, we can drop the suffix tilde on the reported  $\tilde{I}$  and simply write the contract as  $(x_{Uk}^i, w_{Uk}, p_{Uk})$ .

Similarly, on the other hand, the contract in the agency problem with debt financing,  $\mathcal{M}_L^{**}$ , is defined as

$$\mathcal{M}_L^{**} = (x_{Lk}^i, w_{Lk}, p_{Lk}, x_k^d, c_k), \quad k \in \{1, 2\}.$$

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<sup>9</sup>Note that the manager could attempt to report  $I_2$  when the realized value is  $I_1$ , while he/she would not attempt to report  $I_1$  when  $I = I_2$ . In equilibrium, the manager will report the true value to the shareholder.

<sup>10</sup>Giving a bonus incentive to the manager can be viewed as a *carrot*, while auditing and fining the manager for false reporting can be considered as a *stick*.

<sup>11</sup>Renegotiation between the shareholder and the manager is not allowed. While commitment may cause *ex post* inefficiency in investment timing, it increases the *ex ante* owner’s option value.

<sup>12</sup>See Mas-Colell, Whinston, and Green (1995) for the revelation principle.

Note that  $\mathcal{M}_L^{**}$  consists of five components (investment trigger, bonus, probability of an audit, default trigger, and coupon payment).

### 3.2 Agency problem with all-equity financing

In this subsection, we provide the solution to the agency problem with all-equity financing. The shareholder's optimization problem is to maximize the equity value via  $\mathcal{M}_U^{**}$ , i.e.,

$$\max_{x_{U1}^i, w_{U1}, p_{U1}, x_{U2}^i, w_{U2}, p_{U2}} \sum_{k=1}^2 \mathbb{P}(I_k) \left( \frac{x}{x_{Uk}^i} \right)^\beta \{ \Pi(x_{Uk}^i) - I_k - w_{Uk} - g(p_{Uk}) \},$$

subject to

$$\left( \frac{x}{x_{U1}^i} \right)^\beta w_{U1} \geq \left( \frac{x}{x_{U2}^i} \right)^\beta (w_{U2} + \Delta I - p_{U2}P), \quad (29)$$

$$\left( \frac{x}{x_{U2}^i} \right)^\beta w_{U2} \geq \left( \frac{x}{x_{U1}^i} \right)^\beta (w_{U1} - \Delta I - p_{U1}P), \quad (30)$$

$$q \left( \frac{x}{x_{U1}^i} \right)^\beta w_{U1} + (1-q) \left( \frac{x}{x_{U2}^i} \right)^\beta w_{U2} \geq 0, \quad (31)$$

$$w_{Uk} \geq 0, \quad k \in \{1, 2\}, \quad (32)$$

$$p_{Uk} \geq 0, \quad k \in \{1, 2\}. \quad (33)$$

Constraints (29) and (30) are the *ex post* incentive compatibility constraints for the manager under states  $I_1$  and  $I_2$ , respectively. Consider, for example, constraint (29). The manager's payoff in state  $I_1$  is  $(x/x_{U1}^i)^\beta w_{U1}$  if he/she tells the truth, but it is  $(x/x_{U2}^i)^\beta (w_{U2} + \Delta I - p_{U2}P)$  if he/she instead claims that it is state  $I_2$ . Thus, he/she tells the truth if (29) is satisfied. Constraint (30) follows similarly.

Constraints (31) and (32) are the *ex ante* participation constraint and the *ex post* limited-liability constraints, respectively. The nonnegative bonus incentive  $w_{Uk}$  ensures that the manager makes an agreement about employment. For example, if  $w_{U2} < 0$ , then the manager would refuse the contract on learning that  $I = I_2$ .

Constraints (33) are obvious, where  $p_{Uk}$  is the probability of an audit. Here,  $p_{Uk} < 1$  in (33) is automatically satisfied. This statement is shown by  $\lim_{p_{Uk} \uparrow 1} g(p_{Uk}) = +\infty$  and  $g'(p_{Uk}) > 0$ .

In summary, there are seven inequality constraints (29) to (33) on the agency problem with all-equity financing. Fortunately, we can simplify the problem in the following three steps. First, (31) is automatically satisfied, because (32) implies (31). Second, the manager in state  $I_2$  does not have the incentive to tell a lie as a manager in state  $I_1$ . This is because the manager in state  $I_2$  suffers a loss in making such a false announcement. Thus, (30) is automatically satisfied,  $p_{U1}^{**} = 0$  and  $w_{U2}^{**} = 0$  are obtained in optimum. Finally,

(29) is binding, because the shareholder can increase his/her residual payoff by reducing the bonus incentive  $w_{U1}$  paid to the manager as much as possible. Thus, we only need to take into consideration the three constraints.

Consequently, the shareholder's optimization problem is simplified as

$$\begin{aligned} \max_{x_{U1}^i, x_{U2}^i, w_{U1}, p_{U2}} \quad & q \left( \frac{x}{x_{U1}^i} \right)^\beta \{ \Pi(x_{U1}^i) - I_1 - w_{U1} \} \\ & + (1 - q) \left( \frac{x}{x_{U2}^i} \right)^\beta \{ \Pi(x_{U2}^i) - I_2 - g(p_{U2}) \}, \end{aligned} \quad (34)$$

subject to

$$\left( \frac{x}{x_{U1}^i} \right)^\beta w_{U1} = \left( \frac{x}{x_{U2}^i} \right)^\beta (\Delta I - p_{U2}P), \quad w_{U1} \geq 0, \quad p_{U2} \geq 0. \quad (35)$$

Then, the optimal contract  $\mathcal{M}_U^{**}$  is obtained by

$$\begin{aligned} (x_{U1}^{i**}, w_{U1}^{**}, p_{U1}^{**}) &= \left( x_{U1}^{i*}, \left( \frac{x_{U1}^{i*}}{x_{U2}^{i**}} \right)^\beta (\Delta I - p_{U2}^{**}P), 0 \right) \\ (x_{U2}^{i**}, w_{U2}^{**}, p_{U2}^{**}) &= \left( \frac{\beta}{\beta - 1} \frac{1}{\Pi(1)} I_{U2}^{**}, 0, p_{U2}^{**} \right), \end{aligned}$$

where

$$I_{U2}^{**} = (I_2 + g(p_{U2}^{**}) + \frac{q}{1-q}(\Delta I - p_{U2}^{**}P)), \quad (36)$$

and

$$p_{U2}^{**} = \begin{cases} 0, & \text{if } 0 \leq P \leq \frac{q}{1-q}g'(0), \\ g'^{-1}(\frac{1-q}{q}P), & \text{if } \frac{q}{1-q}g'(0) \leq P < \max\{\Delta I, \frac{1-q}{q}g'(\frac{\Delta I}{P})\}, \\ \frac{\Delta I}{P}, & \text{otherwise.} \end{cases} \quad (37)$$

Note that  $w_{U1}^{**} > 0$  and  $p_{U2}^{**} = 0$  if  $0 \leq P \leq \frac{q}{1-q}g'(0)$ ,  $w_{U1}^{**} > 0$  and  $p_{U2}^{**} > 0$  if  $\frac{q}{1-q}g'(0) \leq P < \max\{\Delta I, \frac{1-q}{q}g'(\frac{\Delta I}{P})\}$ ,  $w_{U1}^{**} = 0$  and  $p_{U2}^{**} \geq 0$  otherwise. These regions are respectively defined as *bonus-only region*, *combination region*, and *audit-only region*. The solution for the bonus-only region is exactly the same as in Grenadier and Wang (2005).<sup>13</sup>

There are four important properties in  $\mathcal{M}_U^{**}$ . The first property of the solution is that  $x_{U1}^{i*} = x_{U1}^{i**}$  and  $x_{U2}^* \leq x_{U2}^{**}$  due to

$$x_{U2}^{i**} = x_{U2}^{i*} + \frac{\beta}{\beta - 1} \frac{1}{\Pi(1)} \left( g(p_{U2}^{**}) + \frac{q}{1-q}(\Delta I - p_{U2}^{**}P) \right),$$

where  $g(p_{U2}^{**}) \geq 0$  and  $(\Delta I - p_{U2}^{**}P) \geq 0$ . In particular, we obtain

$$x_{U2}^* < x_{U2}^{**}, \quad (38)$$

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<sup>13</sup>In the Grenadier and Wang (2005) model, the shareholder induces the manager to truthfully reveal private information by giving the manager only the bonus incentive.

for  $P < +\infty$ . The reason is that at least one of  $w_{U1}^{**} \geq 0$  (i.e.,  $(\Delta I - p_{U2}^{**}P) \geq 0$ ) and  $p_{U2}^{**} \geq 0$  (i.e.,  $g(p_{U2}^{**}) \geq 0$ ) for  $P < +\infty$  is strictly positive. It is less costly for the shareholder to distort  $x_{U2}^{i**}$  away from  $x_{U2}^{i*}$  than to distort  $x_{U1}^{i**}$  away from  $x_{U1}^{i*}$  in equilibrium.

The second property of the solution is that  $(\Delta I - p_{U2}^{**}P) > w_{U1}^{**} \geq 0$ . Here,  $(\Delta I - p_{U2}^{**}P)$  can be regarded as the *information rent* for the manager in state  $I_1$ . The owner gives the manager in state  $I_1$  a portion of the information rent. Importantly, note that the information rent is decreasing with  $P$ . This result corresponds to the remarkable result for unlimited penalties (see Subsection 3.4).

The third property of the solution is that  $p_{U2}^{**}$  is increasing and concave with  $P$  in the combination region, while  $p_{U2}^{**}$  is decreasing and convex with  $P$  in the audit-only region. The first statement is straightforward because  $g(p_{U2})$  is increasing and convex with  $p_{U2}$ . The second statement is shown by  $p_{U2}^{**} = \frac{\Delta I}{P}$  in optimum.

The fourth property of the solution is that an increase in the penalty  $P$  changes the contract  $\mathcal{M}_U^{**}$  from the bonus-only region to the audit-only region via the combination region. This property is intuitive as follows. For example, as  $P$  becomes larger, audit technology is more readily available. The shareholder then tends to take into consideration the audit technology rather than the bonus incentive.

The equity and manager's values at time 0,  $E_U^{**}(x)$  and  $M_U^{**}(x)$ , are obtained by

$$E_U^{**}(x) = q \left( \frac{x}{x_{U1}^{i*}} \right)^\beta \{ \Pi(x_{U1}^{i*}) - I_1 \} + (1 - q) \left( \frac{x}{x_{U2}^{i**}} \right)^\beta \{ \Pi(x_{U2}^{i**}) - I_2^{**} \}, \quad (39)$$

$$M_U^{**}(x) = q \left( \frac{x}{x_{U2}^{i**}} \right)^\beta (\Delta I - p_{U2}^{**}P), \quad (40)$$

respectively. The sum of these two values at time 0,  $E_U^{**}(x) + M_U^{**}(x)$ , is

$$E_U^{**}(x) + M_U^{**}(x) = q \left( \frac{x}{x_{U1}^{i*}} \right)^\beta \{ \Pi(x_{U1}^{i*}) - I_1 \} + (1 - q) \left( \frac{x}{x_{U2}^{i**}} \right)^\beta \{ \Pi(x_{U2}^{i**}) - I_2 - g(p_{U2}^{**}) \}. \quad (41)$$

The value given in (41) represents the value of total social welfare. Here, we have

$$E_U^*(x) \geq E_U^{**}(x) + M_U^{**}(x). \quad (42)$$

where we have used the fact that  $x_{U2}^{i**} \geq x_{U2}^{i*} = \operatorname{argmax}_{x_{U2}^i} (x/x_{U2}^i)^\beta \{ \Pi(x_{U2}^i) - I_2 \}$ . In particular, the inequality (42) is strictly positive for  $P < +\infty$ . This is because  $x_{L2}^{i**} > x_{L2}^{i*}$  for  $P < +\infty$ . Thus we can define the agency cost  $AC_U(x)$  by

$$\begin{aligned} AC_U(x) &:= E_U^*(x) - E_U^{**}(x) - M_U^{**}(x) \\ &= (1 - q) \left[ \left( \frac{x}{x_{U2}^{i*}} \right)^\beta \{ \Pi(x_{U2}^{i*}) - I_2 \} - \left( \frac{x}{x_{U2}^{i**}} \right)^\beta \{ \Pi(x_{U2}^{i**}) - I_2 - g(p_{U2}^{**}) \} \right] \geq 0. \end{aligned} \quad (43)$$

In particular, as in the value of total social welfare, we have  $AC_U(x) > 0$  for  $P < +\infty$ . The manager-shareholder conflicts lead to a distortion in total social welfare.

### 3.3 Agency problem with debt financing

In this subsection, we provide the solution to the agency problem with debt financing. The optimization problem is to maximize the equity value with debt financing, i.e.,  $E_{Lk}(x) - (I_k - D_{Lk}(x)) = V_{Lk}(x) - I_k$ . Thus, it is defined to maximize the firm value via  $\mathcal{M}_L^{**}$ , i.e.,

$$\max_{x_{L1}^i, w_{L1}, p_{L1}, x_{L2}^i, w_{L2}, p_{L2}} \sum_{k=1}^2 \mathbb{P}(I_k) \left( \frac{x}{x_{Lk}^i} \right)^\beta \{V_{Lk}(x_{Lk}^i) - I_1 - w_{Lk} - g(p_{Lk})\},$$

subject to

$$\left( \frac{x}{x_{L1}^i} \right)^\beta w_{L1} \geq \left( \frac{x}{x_{L2}^i} \right)^\beta (w_{L2} + \Delta I - p_{L2}P), \quad (44)$$

$$\left( \frac{x}{x_{L2}^i} \right)^\beta w_{L2} \geq \left( \frac{x}{x_{L1}^i} \right)^\beta (w_{L1} - \Delta I - p_{L1}P), \quad (45)$$

$$q \left( \frac{x}{x_{L1}^i} \right)^\beta w_{L1} + (1-q) \left( \frac{x}{x_{L2}^i} \right)^\beta w_{L2} \geq 0, \quad (46)$$

$$w_{Lk} \geq 0, \quad k \in \{1, 2\}, \quad (47)$$

$$p_{Lk} \geq 0, \quad k \in \{1, 2\}. \quad (48)$$

These constraints are the same as those in the agency problem with all-equity financing.

Then, the mechanism  $\mathcal{M}_L^{**}$  turns out to be

$$\begin{aligned} (x_{L1}^{i**}, w_{L1}^{**}, p_{L1}^{**}, x_1^{d**}, c_1^{**}) &= \left( x_{L1}^{i*}, \left( \frac{x_{L1}^{i*}}{x_{L2}^{i**}} \right)^\beta (\Delta I - p_{L2}^{**}P), 0, x_1^{d*}, c_1^* \right) \\ (x_{L2}^{i**}, w_{L2}^{**}, p_{L2}^{**}, x_2^{d**}, c_2^{**}) &= \left( \frac{\beta}{\beta - 1} \frac{\psi}{\Pi(1)} I_{L2}^{**}, 0, p_{L2}^{**}, \frac{x_{L2}^{i**}}{h}, \zeta I_{L2}^{**} \right), \end{aligned} \quad (49)$$

where  $I_{L2}^{**}$  is defined by

$$I_{L2}^{**} = (I_2 + g(p_{L2}^{**}) + \frac{q}{1-q}(\Delta I - p_{L2}^{**}P)), \quad (50)$$

and  $p_{L2}^{**}$  is obtained by

$$p_{L2}^{**} = \begin{cases} 0, & \text{if } 0 \leq P \leq \frac{q}{1-q}g'(0), \\ g'^{-1}\left(\frac{1-q}{q}P\right), & \text{if } \frac{q}{1-q}c'(0) \leq P < \max\{\Delta I, \frac{1-q}{q}g'\left(\frac{\Delta I}{P}\right)\}, \\ \frac{\Delta I}{P}, & \text{otherwise.} \end{cases} \quad (51)$$

Here,  $c_k^{**}$  and  $x_k^{d**}$  are obtained by  $c_k^{**} = c_k(x_{Lk}^{i**})$  and  $x_k^{d**} = x_k^d(c_k^{**})$ , respectively. Note that  $w_{L1}^{**} = w_{U2}^{**}$  and  $p_{L2}^{**} = p_{U2}^{**}$  for any  $P$ . Here, there are exactly the same four important properties as the agency problem with all-equity financing.

Importantly,  $x_{L2}^{i**}$ ,  $c_2^{**}$ , and  $x_2^{d**}$  are rewritten as

$$x_{L2}^{i**} = x_{L2}^{i*} + \frac{\beta}{\beta - 1} \frac{\psi}{\Pi(1)} \left( g(p_{L2}^{**}) + \frac{q}{1-q}(\Delta I - p_{L2}^{**}P) \right), \quad (52)$$

$$c_2^{**} = c_2^* + \zeta \left( g(p_{L2}^{**}) + \frac{q}{1-q}(\Delta I - p_{L2}^{**}P) \right), \quad (53)$$

and

$$x_2^{d**} = x_2^{d*} + h^{-1} \frac{\beta}{\beta - 1} \frac{\psi}{\Pi(1)} \left( g(p_{L2}^{**}) + \frac{q}{1 - q} (\Delta I - p_{L2}^{**} P) \right), \quad (54)$$

respectively. These results imply

$$x_{L2}^{i**} \geq x_{L2}^{i*}, \quad x_2^{d**} \geq x_2^{d*}, \quad c_2^{**} \geq c_2^*, \quad (55)$$

due to  $g(p_{L2}^{**}) \geq 0$  and  $(\Delta I - p_{L2}^{**} P) \geq 0$ . In particular, these inequalities given in (55) are strictly positive for  $P < +\infty$ .<sup>14</sup> Thus, manager-shareholder conflicts increase the investment trigger, coupon payment, and default trigger. That is, the manager-shareholder conflicts lead to a distortion in the investment and default timing and coupon payment decisions.

We begin with a comparison of the *ex post* values. The equity, debt, and firm values at time  $T_{L2}^{i**} := \inf\{t \geq 0; X(t) \geq x_{L2}^{i**}\}$ ,  $E_{L2}(x_{L2}^{i**})$ ,  $D_{L2}(x_{L2}^{i**})$ , and  $V_{L2}(x_{L2}^{i**})$ , are obtained by

$$E_{L2}(x_{L2}^{i**}) = \Pi(x_{L2}^{i**}) - \Pi(x_2^{d**}) h^\gamma - (1 - \tau)(1 - h^\gamma) \frac{c_2^{**}}{r}, \quad (56)$$

$$D_{L2}(x_{L2}^{i**}) = \frac{c_2^{**}}{r} (1 - h^\gamma) + (1 - \alpha) h^\gamma x_2^{d**}, \quad (57)$$

and

$$V_{L2}(x_{L2}^{i**}) = E_{L2}(x_{L2}^{i**}) + D_{L2}(x_{L2}^{i**}) = \psi^{-1} \Pi(x_{L2}^{i**}), \quad (58)$$

respectively.<sup>15</sup> From (55), we have

$$E_{L2}(x_{L2}^{i**}) \leq E_{L2}(x_{L2}^{i*}), \quad D_{L2}(x_{L2}^{i**}) \geq D_{L2}(x_{L2}^{i*}), \quad V_{L2}(x_{L2}^{i**}) \geq V_{L2}(x_{L2}^{i*}).$$

In particular,  $E_{L2}(x_{L2}^{i**}) < E_{L2}(x_{L2}^{i*})$  and  $D_{L2}(x_{L2}^{i**}) > D_{L2}(x_{L2}^{i*})$  for  $P < +\infty$ . That is, manager-shareholder conflicts decrease the equity value at time  $T_{L2}^{i**}$ , while they increase the debt and firm values. These results are the same as those in the seminal work of Myers and Majluf (1984),<sup>16</sup> and fit well with the findings of empirical studies (see Korajczyk, Lucas, and McDonald, 1991; Jung, Kim, and Stulz, 1996).

In addition, the leverage and credit spread at time  $T_{L2}^{i**}$  are given by

$$L_2(x_{L2}^{i**}) = \frac{D_{L2}(x_{L2}^{i**})}{V_{L2}(x_{L2}^{i**})} = \frac{\gamma - 1}{\gamma} \frac{\psi}{1 - \tau} \frac{1}{h} (1 - \xi), \quad (59)$$

<sup>14</sup>As for the agency problem with all-equity financing, at least one of  $g(p_{L2}^{**}) \geq 0$  and  $(\Delta I - p_{L2}^{**} P) \geq 0$  is strictly positive for  $P < +\infty$ .

<sup>15</sup>Since  $x_{L1}^{i**} = x_{L1}^{i*}$ ,  $E_{L1}(x_{L1}^{i**})$ ,  $D_{L1}(x_{L1}^{i**})$ , and  $V_{L1}(x_{L1}^{i**})$ , are given in (21), (22) and (23).

<sup>16</sup>Myers and Majluf (1984) argue that firms whose managers have superior information should go to the bond market for external capital.

and

$$cs_2(x_{L2}^{i**}) = \frac{c_2^{**}}{D_{L2}(x_{L2}^{i**})} - r = r \frac{\xi}{1 - \xi}. \quad (60)$$

That is,  $L_2(x_{L2}^{i**}) = L_2(x_{L2}^{i*})$  and  $cs_2(x_{L2}^{i**}) = cs_2(x_{L2}^{i*})$ . Consequently, the leverage and credit spread are exactly the same between the no-agency and agency problems.

We then compare the *ex ante* values before the investment is undertaken. The equity and manager's values at time 0,  $E_L^*(x)$  and  $M_L^{**}(x)$ , are

$$\begin{aligned} E_L^{**}(x) &= q \left( \frac{x}{x_{L1}^{i*}} \right)^\beta \{V_{L1}(x_{L1}^{i*}) - I_1\} + (1 - q) \left( \frac{x}{x_{L2}^{i**}} \right)^\beta \{V_{L2}(x_{L2}^{i**}) - I_{L2}^{**}\} \\ &= \psi^{-\beta} E_U^*(x), \end{aligned} \quad (61)$$

and

$$\begin{aligned} M_L^{**}(x) &= q \left( \frac{x}{x_{L2}^{i**}} \right)^\beta (\Delta I - p_{L2}^{**} P), \\ &= \psi^{-\beta} M_U^{**}(x), \end{aligned} \quad (62)$$

respectively. The value of total social welfare is

$$\begin{aligned} E_L^{**}(x) + M_L^{**}(x) &= q \left( \frac{x}{x_{L1}^{i*}} \right)^\beta \{V_{L1}(x_{L1}^{i*}) - I_1\} + (1 - q) \left( \frac{x}{x_{L2}^{i**}} \right)^\beta \{V_{L2}(x_{L2}^{i**}) - I_2 - g(p_{L2}^{**})\} \\ &= \psi^{-\beta} (E_U^{**}(x) + M_U^{**}(x)). \end{aligned} \quad (63)$$

Most importantly, we obtain

$$E_L^*(x) \geq E_L^{**}(x) + M_L^{**}(x), \quad (64)$$

where we have used the fact that  $x_{L2}^{i**} \geq x_{L2}^{i*} = \operatorname{argmax}_{x_{L2}^i} (x/x_{L2}^i)^\beta \{V_{L2}(x_{L2}^i) - I_2\}$ . In particular, the inequality (64) is strictly positive for  $P < +\infty$ . The reason is that  $x_{L2}^{i**} > x_{L2}^{i*}$  for  $P < +\infty$ . Thus we can define the agency cost,  $AC_L(x)$ , by

$$\begin{aligned} AC_L(x) &:= E_L^*(x) - E_L^{**}(x) - M_L^{**}(x) \\ &= \psi^{-\beta} AC_U(x) \geq 0. \end{aligned} \quad (65)$$

In particular, this inequality is strictly positive for  $P < +\infty$ . Even with debt financing, the manager-shareholder conflicts lead to a distortion in total social welfare.

Finally, the results given in (61), (62), and (65) mean that  $E_L^{**}(x) \geq E_U^{**}(x)$ ,  $M_L^{**}(x) \geq M_U^{**}(x)$ , and  $AC_L(x) \geq AC_U(x)$ . In Subsection 4.2, we will examine these further characteristics.



### 3.4 Unlimited penalties

We have already seen that the manager-shareholder conflict problems always lead to a distortion in total social welfare, i.e.,  $AC_j(x) > 0$  for  $P < +\infty$  ( $j \in \{U, L\}$ ). This is because penalties are, in practice, limited. Although unlimited penalties are of theoretical interest, we examine the effect of unlimited penalties on the optimal solutions and values in the agency problems with all-equity and debt financing. Thus, we can see that there is a relationship between the no-agency and agency problems.

As  $P \uparrow +\infty$ , we have

$$p_{j2}^{**} \rightarrow 0, \quad x_{j2}^{i**} \downarrow x_{j2}^{i*}, \quad x_2^{d**} \downarrow x_2^{d*}, \quad c_2^{**} \downarrow c_2^*, \quad w_{j1}^{**} \downarrow w_{j1}^*, \quad (66)$$

and

$$E_j^{**}(x) \uparrow E_j^*(x), \quad M_j^{**}(x) \downarrow 0, \quad AC_j(x) \rightarrow 0, \quad (67)$$

for all  $j$  ( $j \in \{U, L\}$ ). Here,  $x_{j2}^{i**}$ ,  $x_2^{d**}$ , and  $c_2^{**}$  are monotonically decreasing with  $P$ .<sup>17</sup> It is interesting that  $AC_j(x)$  is *not* monotonically decreasing with  $P$  although  $E_j^{**}(x)$  and  $M_j^{**}(x)$  are monotonically increasing and decreasing with  $P$ , respectively.<sup>18</sup>

Consequently, as the penalty for detecting a manager's false report goes to infinity, the solutions and values in the agency problems converge to those in the no-agency problems.<sup>19</sup> These are the same results as in the static work of Baron and Besanko (1984, Proposition 4).

## 4 Model implications

In this section, we analyze several of the more important implications of the model. We begin with an investigation of the impact of manager-shareholder conflicts on the investment and default triggers, coupon payments, and equity and debt values. We then consider the costs and benefits of debt financing in the agency problems. In addition, we consider asset substitution between the shareholder and the manager. Finally, we investigate the stock price reaction to investment.

In the numerical examples, we define the cost function for an audit by

$$g(p_{jk}) = \eta \frac{p_{jk}}{1 - p_{jk}}, \quad k \in \{1, 2\}, j \in \{U, L\}, \quad (68)$$

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<sup>17</sup>See Figures 1 and 2 in the following section.

<sup>18</sup>See Figures 4, 5, and 6 in the following section.

<sup>19</sup>As a result, our solutions in the agency problem with all-equity financing include those in the two related papers, McDonald and Siegel (1986) and Grenadier and Wang (2005). On the other hand, the solutions in the agency problem with debt financing include those in the simple version of the Sundaresan and Wang (2006, 2007) models.

where  $\eta > 0$  is a positive constant.<sup>20</sup> The parameters are given by  $q = 0.5$ ,  $\mu = 0.03$ ,  $r = 0.07$ ,  $Q = 1$ ,  $\sigma = 0.2$ ,  $I_1 = 50$ ,  $I_2 = 80$ ,  $\eta = 20$ ,  $\tau = 0.4$ ,  $\alpha = 0.4$ , and  $x = 1$ .

#### 4.1 Effects of manager-shareholder conflicts

We begin with a description of the solution in the agency problems with all-equity and debt financing. We then consider the *ex ante* values in the agency problems.

Figures 1 and 2 demonstrate the investment triggers ( $x_{U2}^{il}, x_{L2}^{il}$ ) and the default triggers and the coupon payments ( $c_2^l, x_2^{dl}$ ) with the penalty  $P$ , respectively ( $l \in \{*, **\}$ ). In the agency problems with all-equity and debt financing, the bonus-only region is  $0 \leq P \leq 20$ , the combination region is  $20 \leq P \leq 66.67$ , and the audit-only region is  $P \geq 66.67$ . For  $P < +\infty$ , we have  $x_{U2}^{i**} > x_{U2}^{i*}$ ,  $x_{L2}^{i**} > x_{L2}^{i*}$ ,  $x_2^{d**} > x_2^{d*}$ , and  $c_2^{**} > c_2^*$ . In particular,  $x_{U2}^{i**}$ ,  $x_{L2}^{i**}$ ,  $x_2^{d**}$ , and  $c_2^{**}$  are monotonically decreasing with  $P$ , and converge to  $x_{U2}^{i*}$ ,  $x_{L2}^{i*}$ ,  $x_2^{d*}$ , and  $c_2^*$ , respectively, as  $P$  goes to infinity. That is, as  $P \uparrow +\infty$ , we obtain  $x_{U2}^{i**} \downarrow x_{U2}^{i*}$ ,  $x_{L2}^{i**} \downarrow x_{L2}^{i*}$ ,  $x_2^{d**} \downarrow x_2^{d*}$ , and  $c_2^{**} \downarrow c_2^*$ .

[Insert Figures 1 and 2 about here]

Figure 3 depicts the bonus incentive and the probability for an audit ( $w_{j1}^{**}, p_{j2}^{**}$ ) ( $j \in \{U, L\}$ ). Note that  $w_{U1}^{**} = w_{L1}^{**}$  and  $p_{U2}^{**} = p_{L2}^{**}$ . On the one hand,  $w_{j1}^{**}$  is monotonically decreasing with  $P$ . On the other hand,  $p_{j2}^{**}$  is constant at zero in the bonus-only region, increasing and concave with  $P$  in the combination region, and decreasing and convex with  $P$  in the audit-only region.

[Insert Figure 3 about here]

Figure 4 depicts the manager's values ( $M_U^{**}(x), M_L^{**}(x)$ ) with the penalty  $P$ . We can see  $M_L^{**}(x) > M_U^{**}(x)$ . Moreover,  $M_U^{**}(x)$  and  $M_L^{**}(x)$  are monotonically decreasing with  $P$ , and converge to zero.

[Insert Figure 4 about here]

Figure 5 depicts the equity values ( $E_U^l(x), E_L^l(x)$ ) with  $P$  ( $l \in \{*, **\}$ ). Recall that  $E_U^*(x) > E_U^{**}(x)$  and  $E_L^*(x) > E_L^{**}(x)$  for  $P < +\infty$ . The agency problem decreases the equity value. However,  $E_U^{**}(x)$  and  $E_L^{**}(x)$  are monotonically increasing with  $P$ , and converge to  $E_U^*(x)$  and  $E_L^*(x)$  as  $P$  goes to infinity. That is, we have  $E_U^{**}(x) \uparrow E_U^*(x)$  and  $E_L^{**}(x) \uparrow E_L^*(x)$  as  $P \uparrow +\infty$ .

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<sup>20</sup>This function satisfies  $c(0) = 0$ ,  $c' > 0$ ,  $c'' > 0$ , and  $\lim_{p_{jk} \uparrow 1} c(p_{jk}) = +\infty$ .

[Insert Figure 5 about here]

Figure 6 demonstrates the agency costs,  $AC_U(x)$  and  $AC_L(x)$ , with  $P$ . We can see  $AC_j(x) > 0$  for  $P < +\infty$ , and  $AC_j(x) \rightarrow 0$  as  $P \uparrow +\infty$  ( $j \in \{U, L\}$ ). Interestingly,  $AC_j(x)$  is *not* monotonically decreasing with  $P$ . Two conflicting effects cause this phenomenon: the first is that an increase in  $P$  decreases the manager's value, the second is that an increase in  $P$  increases the equity value.

[Insert Figure 6 about here]

## 4.2 Costs and benefits of debt financing

We begin by investigating the benefits of debt financing under manager-shareholder conflict.

First, we compare the investment triggers and values with all-equity and debt financing. Because we can rewrite  $x_{L2}^{i**}$  as  $x_{L2}^{i**} = \psi x_{U2}^{i**}$ , we obtain

$$x_{L2}^{i**} \leq x_{U2}^{i**}. \quad (69)$$

Recall also that  $E_L^{**}(x) = \psi^{-\beta} E_U^{**}(x)$ . From this, we have

$$E_L^{**}(x) \geq E_U^{**}(x). \quad (70)$$

where we have used the fact that  $\psi \leq 1$  and  $\beta > 1$ . These results imply that debt financing enables the shareholder to increase investment (i.e., decrease the investment trigger) and equity value under manager-shareholder conflicts. This finding can be regarded as a benefit of debt financing.

[Insert Figure 7 about here]

Figure 7 demonstrates the firm value in  $I = I_2$  in the agency problem. We can see  $x_{L2}^{i**} = 14.00 < 18.66 = x_{U2}^{i**}$ ,  $V_{L2}(x_{L2}^{i**}) = 170.97 = \Pi(x_{U2}^{i**})$ , and  $(x/x_{L2}^{i**})^\beta \{V_{L2}(x_{L2}^{i**}) - I_{L2}^{**}\} > (x/x_{U2}^{i**})^\beta \{\Pi(x_{U2}^{i**}) - I_{U2}^{**}\}$ . In addition, we can see that  $V_{L2}(x)$  is concave in  $x$ , while  $\Pi(x)$  is linear in  $x$ . These results imply that  $E_L^{**}(x) > E_U^{**}(x)$  where  $x < x_{L1}^*$ .

We then consider the costs of debt financing under manager-stockholder conflict. Recall that  $AC_L(x) = \psi^{-\beta} AC_U(x)$  from (65). Given that  $\psi \leq 1$  and  $\beta > 1$ , we have

$$AC_L(x) \geq AC_U(x).$$

That is, debt financing increases the agency cost. This result corresponds to the costs of debt financing. Importantly, we have  $AC_L(x) > AC_U(x)$  although  $M_L^{**}(x) > M_U^{**}(x)$  and

$E_L^{**}(x) > E_U^{**}(x)$ . These results imply that the distortion in total social welfare increases with debt financing although both the shareholder and the manager prefer debt financing to all-equity financing. That is, individual rationality does *not necessarily* lead to total social rationality.

Therefore, there are benefits and costs of debt financing under manager-shareholder conflict. In other words, there is a trade-off in the efficiency of the investment trigger and total social welfare when issuing debt for investment under manager-shareholder conflict.

Moreover, we consider the effect on agency costs of market volatility  $\sigma$ . Figure 8 demonstrates the agency costs with  $\sigma$ . Here, an increase in  $\sigma$  has an ambiguous effect on the agency cost,  $AC_j(x)$ . It is interesting that the agency cost with debt financing is decreasing with  $\sigma$ , while the agency cost with all-equity financing is unimodal with  $\sigma$ .<sup>21</sup> The numerical finding that the agency cost is decreasing with market volatility is consistent with the finding obtained in Myers and Majluf (1984).<sup>22</sup>

[Insert Figure 8 about here]

### 4.3 Asset substitution

The notion that manager-shareholder conflicts affect wealth transfer is now widely accepted. We already know that the shareholder must give the manager in  $I = I_1$  the portion of his/her information rent for  $P$  in the bonus-only and combination regions (i.e.,  $0 \leq P \leq \max\{\Delta I, \frac{1-q}{q}g'(\frac{\Delta I}{P})\}$ ). In such a case, asset substitution arises. Interestingly, however, an increase in market volatility  $\sigma$  may reduce the magnitude of this problem.

We consider the effect on the shareholder's and manager's values with  $\sigma$ . An increase in  $\sigma$  increases the shareholder's value,  $E_j^{**}$ , while it has an ambiguous effect on the manager's value,  $M_j^{**}$  ( $j \in \{U, L\}$ ). If the inequalities,

$$\left| \log\left(\frac{x}{x_{U2}^{**}}\right) \right| < \left| \frac{1}{\beta - 1} \right|, \quad (71)$$

$$\left| \log\left(\frac{x}{x_{L2}^{i**}}\right) \frac{\partial \beta}{\partial \sigma} + (-\beta)(x_{L2}^{i**})^{-1} \frac{x_{L2}^{i**}}{\partial \psi} \frac{\partial \psi}{\partial h} \frac{\partial h}{\partial \gamma} \frac{\partial \gamma}{\partial \sigma} \right| > \left| \frac{1}{\beta - 1} \frac{\partial \beta}{\partial \sigma} \right|, \quad (72)$$

are satisfied,<sup>23</sup> then an increase in  $\sigma$  decreases  $M_U^{**}$  and  $M_L^{**}$ . Under these conditions (71) and (72), there is a wealth transfer from the manager to the shareholder with increasing market volatility  $\sigma$ .

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<sup>21</sup>The fact that the agency cost is unimodal with  $\sigma$  is the same finding as in Shibata (2008).

<sup>22</sup>They indicate that the loss decreases when the market's uncertainty about the value of assets in place is reduced.

<sup>23</sup>These inequalities are obtained by differentiating  $M_j^{**}$  with  $\sigma$ .

Figure 9 depicts asset substitution in the agency problems. Suppose that  $P = 30$  (i.e., the solution and value are obtained in the combination region). We can see that wealth transfers emerge if  $\sigma$  is more than 0.34 and 0.41 in the agency problems with debt financing and with all-equity financing, respectively.

[Insert Figure 9 about here]

#### 4.4 Stock price reaction

We now consider the stock price reaction to the manager's investment decision. The stock prices in the agency problem with all-equity and debt-financing are the shareholder's option values, (39) and (63), respectively.

Prior to the point at which  $X_t$  reaches the trigger  $x_{j1}^{i*}$  ( $j \in \{U, L\}$ ), the market does not know the true value of  $I$ . The market believes that  $I = I_1$  with probability  $q$  and  $I = I_2$  with probability  $1 - q$ .

Once  $X_t$  hits the trigger  $x_{j1}^{i*}$ , the private information is fully revealed. The manager's investment decision signals  $I = I_1$  to the market. If the manager undertakes the investment at  $x_{j1}^{i*}$ , then the stock price instantly jumps up to

$$E_{j1}(x_{j1}^{i*}) = V_{j1}(x_{j1}^{i*}) - I_1 - w_{j1}^{**}, \quad j \in \{U, L\}.$$

Otherwise, the market then recognizes  $I = I_2$ . The stock price instantly jumps down to

$$E_{j2}(x_{j1}^{i*}) = \left(\frac{x_{j1}^{i*}}{x_{j2}^{i**}}\right)^\beta (V_{j2}(x_{j1}^{i**}) - I_2 - c(p_{j2}^{**})), \quad j \in \{U, L\}.$$

Figure 10 demonstrates the stock price reaction to investment. Suppose  $P = 30$ . Then the stock price with debt financing is 63.08 just prior to  $x_{L1}^{i*} = 6.42$ . If the manager undertakes the investment at  $x_{L1}^{i*}$ , the stock price jumps up to 71.59. Otherwise, the stock price jumps down to 54.56.<sup>24</sup>

[Insert Figure 10 about here]

## 5 Concluding remarks

This paper extends the agency problem arising from manager-shareholder conflicts in a real options model by incorporating debt financing. Manager-shareholder conflicts over

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<sup>24</sup>Suppose the agency problem is with all-equity financing. Then, the stock price is 87.70 just prior to  $x_{L1}^{i*} = 8.56$ . If the manager undertakes the investment at  $x_{L1}^{i*}$ , the stock price jumps up to 102.20. Otherwise, the stock price jumps down to 73.19

investment policy increase the investment and default triggers and coupon payments that lead to an increase in debt value and a decrease in equity value. Compared with an all-equity financing framework, with debt financing investment timing is earlier while the agency cost is larger. Thus, there is a trade-off in the efficiency of investment and total social welfare. Our results fit well with the findings of previous work.

Some extensions of the model would prove interesting. For example, since liquidation is costly for all agents in our model, it would be interesting to include strategic debt service. After investment is undertaken with debt financing, the shareholder (or manager) would renegotiate with the bondholder to renege on the contractual coupon payments when the firm is close to or in financial distress. There would then be another agency conflict with bondholders. Thus, it would be interesting to consider the implied investment triggers under two different agency conflicts.

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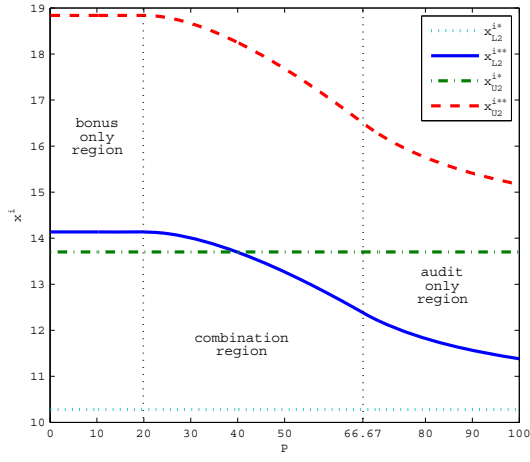


Figure 1:  $x_2^i$  with  $P$

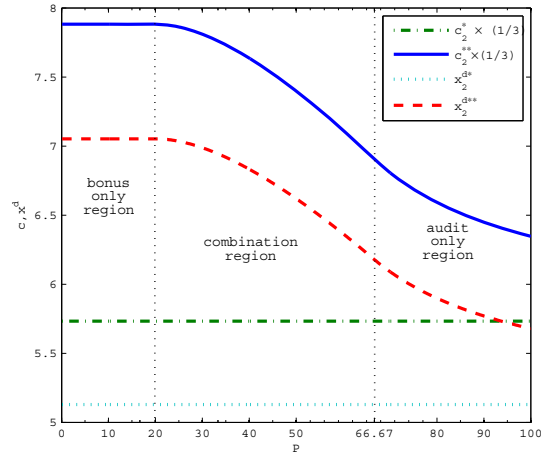


Figure 2:  $c_2$  and  $x_2^d$  with  $P$

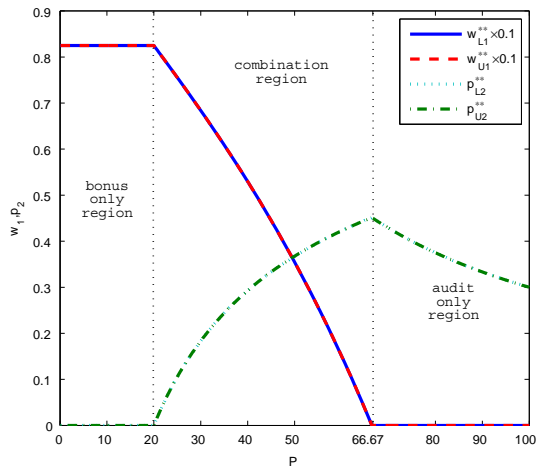


Figure 3:  $w_1$  and  $p_2$  with  $P$

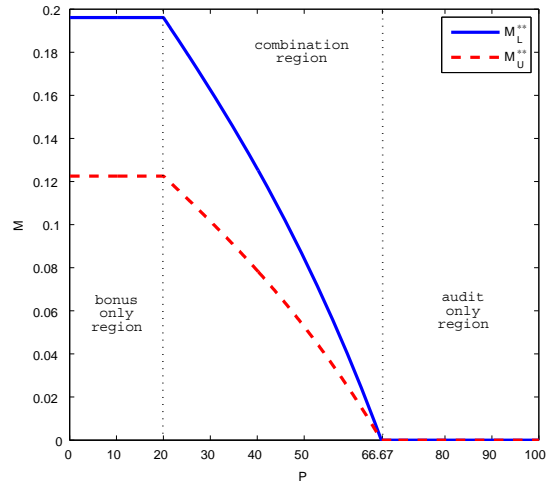


Figure 4:  $M_j^{**}$  with  $P$

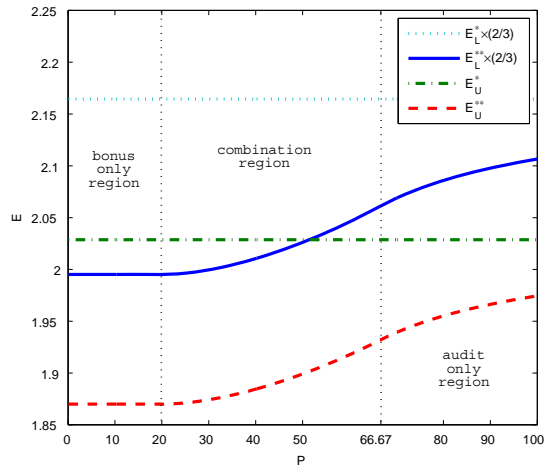


Figure 5:  $E_j^l(x)$  with  $P$

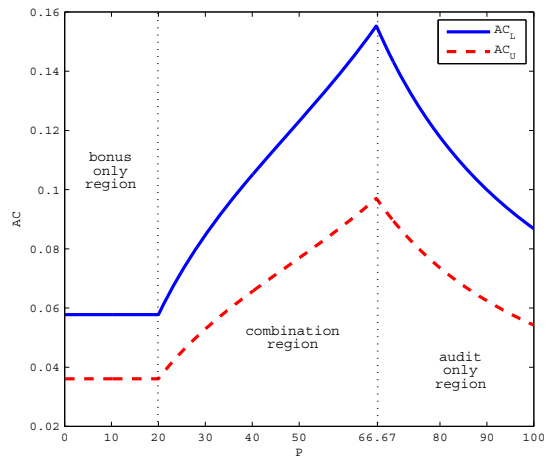


Figure 6:  $AC_j$  with  $P$

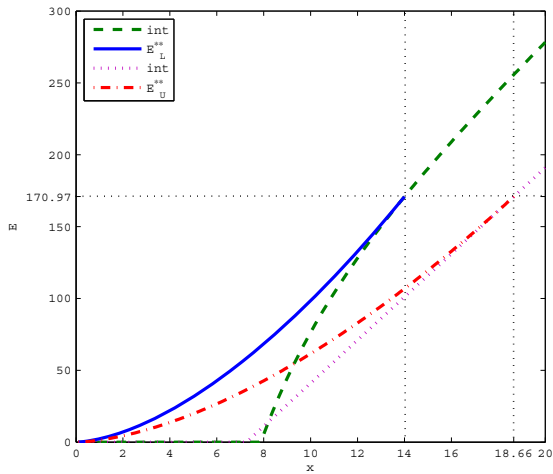


Figure 7: values with  $x$  equity value  $E_j$

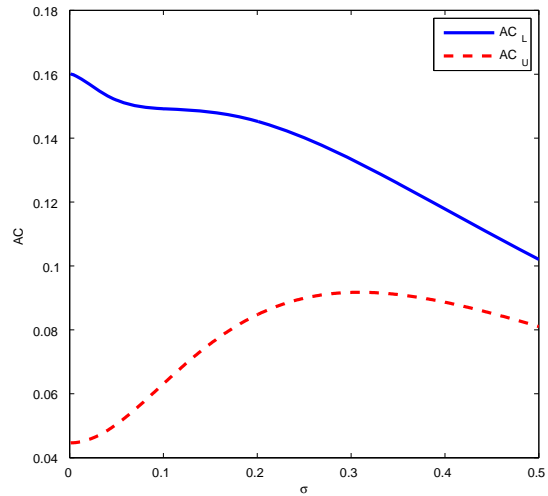


Figure 8:  $AC_j$  with  $\sigma$

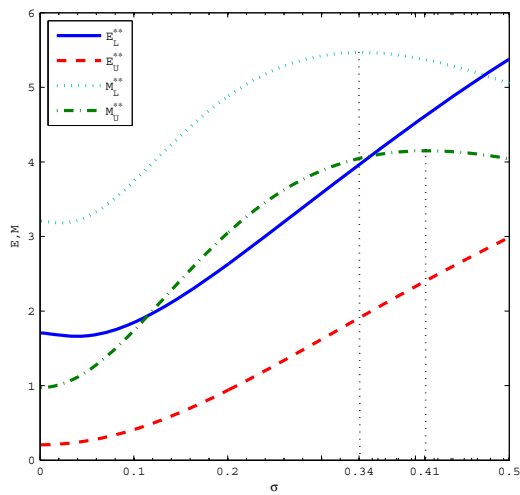


Figure 9:  $E_j^{**}$ ,  $M_j^{**}$  with  $\sigma$

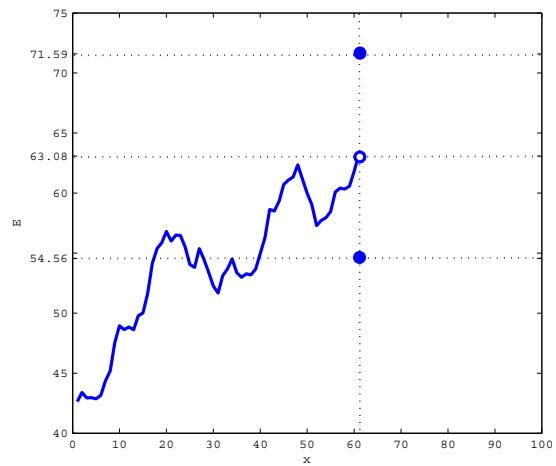


Figure 10: Price Reaction