Independence of Capacity Ordering and Financial Subsidies to Risky Suppliers*

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Abstract

The risk of supply disruptions due to suppliers' financial problems plays a prominent role in manufacturers' risk portfolios. Even large suppliers (e.g. Delphi) could file for bankruptcy, and manufacturer's actions, such as financial subsidies to the suppliers, affect profoundly suppliers' financial health. Using a dynamic, stochastic, periodic-review model of the manufacturer's joint capacity reservation and financial subsidy decisions and a firm-value model of the supplier's financial state, this paper addresses the following questions: What is the optimal joint capacity ordering and financial subsidy policy for the manufacturer? Must subsidy and capacity ordering decisions be made jointly? How good are the recommendations from the traditional procurement models, which ignore the benefits of controlling the supplier's financial state through subsidies? The paper presents general assumptions that allow the manufacturer to make ordering decisions independently from subsidy decisions and investigates interactions between ordering and subsidy decisions when these assumptions are violated. Conditions are presented for the optimal subsidy policy to have a "subsidize-up-to" structure and for the optimal ordering decisions to be newsvendor fractiles.

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1 Introduction

In October 2005, Delphi Corporation, the largest auto-parts supplier in the United States, filed for Chapter 11 bankruptcy protection. According to Lester (2002), Delphi, which was ranked 63rd on the S&P 500 list, had \$17.1 billion in assets, \$22.2 billion in debt, and \$4.3 billion in unfunded pension liabilities. While the size of Delphi's bankruptcy is exceptional, it is just one example of a long series of recent supplier bankruptcies in the automotive and other industries.[1]

Following a supplier's bankruptcy, the probability of supply disruptions for the manufacturer increases, as labor strikes are more likely to occur during contract renegotiations and supplier bankruptcy reorganization. For example, following Delphi's bankruptcy, both Delphi's and GM's executives warned of possible supply disruptions (General Motors Corporation is Delphi's largest customer, accounting for 51% of Delphi's sales). Even without labor strikes, order fulfillment and product quality of the bankrupt supplier could be adversely affected, because bankrupt suppliers lose key personnel, forgo investments in manufacturing process improvements, and sacrifice quality control, either by choice or due to lack of financing (see Welch, 2005, for other examples of indirect bankruptcy costs). The bondholders of the bankrupt supplier, concerned primarily with recovering loans they extended to the firm, may try to liquidate the supplier's operations in order to monetize the supplier's assets or try to "hold up" the manufacturer for additional payments.[2]

How can a manufacturer deal with the supplier's deteriorating financial state and the resulting loss of the supplier's operational capacity? One answer to this question is offered by Ford Motor Company and its supplier, Visteon, which considered filing for bankruptcy in 2005. Visteon's bankruptcy was averted after Ford (which is the largest of Visteon's customers, accounting for 65% of Visteon's sales) agreed to pay between \$1.6 billion and \$1.8 billion to help with Visteon's restructuring (White, 2005).

Similar to Ford, General Motors subsidizes its ailing supplier, Delphi. However, unlike Ford, GM could not afford to buy Delphi out of bankruptcy completely, for this would have required a \$9 billion investment, a sum that would have forced GM itself into a perilous financial state, dramatically increasing GM's financing costs.[3]

[1] Collins & Aikman Corporation filed for bankruptcy protection on May 17, 2005, Universal Automotive Industries filed for bankruptcy protection on May 26, 2005, Dana Corporation filed for bankruptcy protection on March 3, 2006, Dura Automotive Systems filed for bankruptcy protection on October 10, 2006.

[2] A well-known "hold up" example is the UPF Thompson v. Land Rover case. UPF Thompson was a supplier of chassis to Land-Rover's Discovery model. After UPF Thompson's bankruptcy in December of 2001, KPMG, which was representing the claimholders of UPF Thompson, demanded a £35 million additional payment from Land Rover, threatening to stop supplying chassis otherwise. Although Land Rover objected to these demands, considering them blackmail, KPMG successfully argued in court that it is obligated to exploit, in the interests of the claimholders, every asset of UPF Thompson, including its strategic position with respect to Land-Rover (see Jennings, 2002). The case was settled outside of the courts for an undisclosed payment from Land Rover to KPMG.

[3] As the Wall Street Journal reported on January 12, 2007, General Motors bonds were rated Caa1 by Moody's, B- by Standard & Poor's, and B by Fitch. The yield on GM bonds, maturing in July 2023, was 8.98%. For comparison,

A number of interesting research questions arise from GM/Delphi, Ford/Visteon, and other examples. Specifically, what is the optimal joint capacity ordering and financial subsidy policy for a manufacturer? Must subsidy and capacity ordering decisions be made jointly? How good are the recommendations from the traditional procurement models, which ignore the benefits of controlling the supplier's financial state through subsidies?

To answer these questions I consider a dynamic, stochastic model of a supply chain, consisting of a manufacturer and its supplier, and that combines elements of firm-value financial default models and operational procurement models. The combined model is general as it imposes weak assumptions on the evolution of the supplier's financial state and allows general convex functions to represent the manufacturer's operating costs. For this general model I put forth conditions under which the manufacturer can make capacity reservation decisions independently of the financial subsidy decisions. This separation result is similar in spirit to Modigliani and Miller (1958) propositions that under certain conditions (specifically, perfect capital markets) one can separate investment and financing decisions. One consequence of the separation result, is that the optimal operational decisions in this paper can be the same as in the traditional procurement models. For instance, when operating costs have the newsvendor structure, the optimal order quantities satisfy the newsvendor fractile expressions.

The separation property does not mean that the subsidy decisions are irrelevant. On the contrary, an option to subsidize the supplier could add significant value to the manufacturer. I consider two examples for the use of subsidies: the subsidies are used by the supplier to reduce its current liabilities, and the subsidies are used to invest in assets. For the model where subsidies are invested in assets, the manufacturer's optimal policy has the "subsidize-up-to" structure. For the model where subsidies reduce liabilities, this structure need not be optimal, but one can still analyze the optimal subsidies. Having found the optimal policies, I investigate their properties, for example by studying comparative statics with respect to problem parameters. To illustrate how the analysis of this paper can be applied in practice I discuss Ford's decision to subsidize Visteon in 2005.

Finally, I study the interactions between subsidy and capacity ordering decisions when the separation assumptions do not hold.

To the best of my knowledge, this paper is the first attempt to model the relationship between the supplier's financial state and supplier's capabilities to fulfill manufacturer's order. This work is related to the growing body of research on disruption risk management in supply chains (see, for example, Vidal and Goetschalckx, 2000; Tomlin and Wang, 2005; Tomlin, 2006; Babich et al., 2007; Babich, 2006; Dada et al., 2007; Tomlin and Snyder, 2006; Snyder and Daskin, 2006; Yang et al.,

10-year treasury yield was 4.66% and 20-year treasury yield was 4.84%. Thus, GM must pay approximately 100% premium over the government rates to entice investors to lend GM money.

2008, and references therein), and the research on random yields and supplier random capacities, particularly a paper by Ciarrallo et al. (1994). In their paper, Ciarrallo et al. (1994) prove that for a dynamic, periodic-review inventory problem, where supply uncertainty is modeled as random supplier's capacity and random capacities are a sequence of i.i.d. random variables, a base-stock policy is optimal. This paper extends the main results in Ciarrallo et al. (1994) to the model with general interperiod operating cost functions, Markov process for supply uncertainty, and subsidy decisions that control the process for supply uncertainty. Because in this paper random capacity is driven by a Markov process (instead of i.i.d. random variables), analytically I can characterize a capacity reservation problem only (instead of an inventory problem, as in Ciarrallo et al. 1994). The model in this paper is much more complex than a typical random-yield model, because, in addition to determining the optimal ordering policy I control the dynamics of the distribution of supply disruptions over time. Even the simplest mathematical properties of the corresponding optimization problem, like convexity or unimodality are non-trivial to derive. Therefore, it is remarkable that there exists a set of assumptions that allows the decomposition of this complex problem into simpler ones and that leads to the optimal policies, which look deceivingly easy and similar to the well-known inventory policies.

In a related work, Swinney and Netessine (2008) study the choice between short-term and long-term contracts that the buyer offers to two suppliers in a two-period model, where suppliers could go bankrupt if their random production costs turn out to be too high. Swinney and Netessine (2008) show that the buyer prefers long-term contracts with the suppliers, because this arrangement allows the buyer to offer higher a price to the supplier in the first period, reducing the supplier's likelihood of default. This is an interesting insight, which can be interpreted in the context of this paper as that the subsidies to the suppliers can be used by the manufacturer to control supplier default risk. My model is different from the one by Swinney and Netessine (2008) in several dimensions. For example, I solve an N-period problem with general interperiod operating cost functions, general random capacity functions, the financial subsidies as decisions, and financial subsidy costs, but with only one supplier. I also employ a different model for the supplier's financial defaults.

The supplier's financial state model in this paper belongs to the family of firm-value financial defaults models. Firm-value (also called Merton) models are based on the seminal work by Black and Scholes (1973) and Merton (1974), who considered corporate debt as a contingent claim on the value of the firm. Because these models focus on the economic reasons for bankruptcies, firm-value models are well-suited for studies of the interaction between lenders and borrowers, effect of taxes and bankruptcy costs, determination of the optimal capital structure (see, for example, Leland, 1994; Leland and Toft, 1996; Briys and de Varenne, 1997), and as in this paper, of interactions between the supplier's financial state and its effective capacity.

The remainder of the paper is organized as follows. Model and assumptions are presented in Section 2, followed by the derivation of the optimal policy in Section 3. In Sections 4 effects of codependence between supply and demand shocks on the optimal policies are investigated. In Section 5 I discuss effects of inventory on the optimal policies. Model with proportional random yield is considered in Section 6. Effects of up-front production costs are studied in Section 7. The paper concludes with a discussion of managerial implications, model limitations, and possible extensions in Section 8. Technical results, proofs, and a Ford vs. Visteon example are presented in the Appendix.

2 Model and Assumptions

Consider a dynamic, periodic-review, symmetric-information model of a manufacturer reserving production capacity with a risky supplier.[4] The number of periods in the planning horizon is N. The uncertainty in the final-product market is represented by $\{D_n\}_{n=1}^N$ — a sequence of i.i.d. random variables with p.d.f. f_D and c.d.f. F_D. One can think of the final-product market uncertainty as the demand for the product (the term we[5] will use) or the market size. Let r be the risk-free interest rate in the economy.[6] Then $\alpha = e^{-r\Delta}$, where Δ is the length of one period, represents the one-period discount rate. Assume that either conditions for the existence of the unique pricing measure are satisfied (see Pliska, 1997) or that the manufacturer is risk-neutral.

The timing of events within each period is as follows: at the beginning of a period the manufacturer observes the supplier's state (which will be described shortly), the level of its own inventory, x, and decides on how much, z, capacity to reserve with the supplier and how much, Θ, to pay the supplier immediately (details of payments will be discussed shortly). By the end of the period the uncertainty about the supplier's capacity is realized and capacity, Y, becomes available to the manufacturer. The manufacturer pays for the realized capacity and uses it to meet demand, D, incurring operating costs $\mathcal{M}(x+Y, D)$ (examples of operating costs will be discussed shortly).

The supplier may incur upfront costs based on the order it received (the upfront costs will be denoted $c_0(z)$) and it must pay costs contingent on the realized capacity (the contingent costs will be denoted $c(Y)$, where function $c(\cdot)$ is increasing and convex). The supplier accepts the contract

[4] Because products suppliers provide are often customized, the possibility of the manufacturer changing suppliers during the planning horizon will not be considered. One can attribute the restriction of finite (or countable) decision points to human-resource constraints, which limit how closely the suppliers can be monitored. For example, in the automotive industry a manufacturer could have thousands of suppliers and only a few supplier-account managers. These human-resource constraints could have been to blame, in part, for Land Rover's management overlooking UPF Thompson's bankruptcy. According to Lester (2002), the purchasing director of Land Rover had over 900 contracts to supervise.

[5] Here and in the sequel 'we' refers to the readers and the author.

[6] One could have assumed that the risk-free rate follows a stochastic process (e.g. a Gaussian diffusion as in Briys and de Varenne 1997) But this, more general, model does not generate new insights, while making calculations and exposition more tedious.

with the manufacturer if the manufacturer covers both upfront and contingent costs at the time they are incurred. Furthermore, the manufacturer may decide to pay the supplier an additional amount $\theta \geq \underline{\theta}$ at the beginning of a period. This additional amount could represent the benefits to the supplier from loans extended at favorable terms, or guarantees of future orders, or payments to the supplier for reserving capacity. For the latter, $\theta = wz$, where w is the capacity reservation price. Because the manufacturer determines both the price, w, and the order quantity, z, one can use a single variable θ in the analysis (that is the manufacturer need not inflate its order, z, because increasing price, w, is easier). In general, I will refer to all kinds of additional payments represented by θ as *subsidies* (even if some of them are not subsidies literally). Parameter $\underline{\theta}$ is the minimum profit the supplier requires when working with the manufacturer and its value reflects the market power of the supplier. For the ease of exposition $\underline{\theta} = 0$, but this assumption does not affect the qualitative insights of this paper. In summary, at the beginning of each period the manufacturer pays $\Theta = c_0(z) + \theta$ to the supplier, and at the end of the period the manufacturer pays $c(Y)$ to the supplier.

I do not explicitly model the supplier's decisions (because the resulting general equilibrium model, connecting the supplier's decisions with the process for its financial state and with the manufacturer's decisions is not tractable). But to alleviate concerns that the supplier could "take the money and run", note that the supplier is paid its production cost after it delivers capacity to the manufacturer. Furthermore, the manufacturer may have visibility into supplier's actions (e.g. GM or Ford have close relationships with their suppliers Delphi and Visteon) and the threat of losing future business is a significant deterrent for the supplier. Similarly, the threat of losing a supplier prevents the manufacturer from reneging on its payments.

The financial state of the supplier at the beginning of period n is denoted by A_n. The financial state transition rule is $A_{n+1} = T_A(A_n, \theta_n, B_n)$, where $\{B_n\}_{n=1}^N$ are i.i.d. random variables, each with a c.d.f. F_B and a p.d.f. f_B, which has infinite support. The transition function, T_A, depends on the subsidy amount, θ, rather than the total payment, $\Theta = c_0(z) + \theta$. Because $c_0(z)$ payment covers the supplier's cash outflow of the same amount, the net cash flow to the supplier at the beginning of a period equals the subsidy, θ. Similarly, the contingent payment, $c(Y)$, at the end of a period covers supplier's corresponding costs. Thus, if the manufacturer would like to control the evolution of the supplier's financial state, it can do so through subsidies, θ. The joint distribution of B_n and D_n is given by a c.d.f. F_{BD}.

An example of a model of the supplier's financial state and the transition rule, T_A, is based on the celebrated firm-value (also called structural or Merton) family of models for pricing credit risk. In such models, the supplier's financial state, A, is the asset value of the supplier firm. During each period (after subsidies are received) the assets of the supplier evolve due to cash flows from its businesses (other than the business with the manufacturer). A typical assumption in the credit-risk literature is that assets follow a Geometric Brownian Motion (GBM) process (under a risk-neutral measure), between the times when cash flows from the manufacturer arrive:

$$\frac{dA(\tau)}{A(\tau)} = \mu d\tau + \sigma dW(\tau). \tag{1}$$

In (1) μ could be different from the risk-free rate, r, because of the dividends paid to the claimholders on the supplier firm. Random shock, B, represents the percentage change in the asset value, $A(\Delta)/A(0)$, each period. Under the GBM assumption B is log-normally distributed with parameters $\left((\mu - \frac{1}{2}\sigma^2)\Delta, \sigma\sqrt{\Delta}\right)$ (recall that Δ is the length of each period). The GBM process for the assets, besides being well-studied and accepted in finance literature, facilitates calculations of the optimal policies in this paper. One could relax the GBM assumption (see Section 8) without losing the qualitative insights of this paper, but I will retain it for expositional reasons.

Continuing with the example based on credit-risk models, the supplier has debt on its books. The amounts and maturities of debt are public knowledge at time 0.[7] The payments on debt are F_n and are due at the end of period $n = 1, ..., N$. This assumption corresponds to either a perpetual loan with interest payments, F_n, or a loan which is retired every period and replaced with a new loan with a face value of F_n.

The supplier uses subsidies, θ, either to increase its asset level at the beginning of a period (say period n) from A_n to $A_n + \theta$, or to reduce immediate liabilities from F_n to $F_n - \alpha^{-1}\theta$ (factor α^{-1} reflects the fact that liabilities are reduced at the end of the period). In this paper we will examine the consequences of both modeling assumptions and will refer to them as *asset-investment model* and *liabilities-reduction model*. For the asset-investment model, the transition rule for the supplier's financial state is $A_{n+1} = T_A(A_n, \theta_n, B_n) = (A_n + \theta)B_n$. One can think of asset investments as impulse controls which change the level of the GBM process (1) at given times. These impulse controls connect the supplier's financial health with the manufacturer's operational decisions (as we will see, the optimal subsidies depend on the optimal capacity reservation decisions). For the liabilities-reduction model, the transition rule for the supplier's financial state is $A_{n+1} = T_A(A_n, \theta_n, B_n) = A_n B_n$.

Similar to Merton (1974), I model the supplier's financial viability by the ratio of assets to liabilities at the of the period, called the distance to bankruptcy, that is by $\xi_n = \xi_n(A_n, \theta_n, B_n) = \frac{(A_n+\theta_n)B_n}{F_n}$ for the asset-investment model and $\xi_n = \xi_n(A_n, \theta_n, B_n) = \frac{A_n B_n}{F_n - \alpha^{-1}\theta_n}$ for the liabilities-reduction model. In general, for any interpretation of the supplier's financial state, the distance to bankruptcy, ξ_n, depends on the subsidy, θ_n, the current financial state, A_n, and the shock to the financial state, B_n, through $\xi_n = \xi_n(A_n, \theta_n, B_n)$. It is natural to assume that $\xi_n(A, \cdot, B)$ is

[7] A number of empirical studies show that many firms are slow to change their debt level even if they deviate from the "optimal" debt/equity ratio (see Fama and French, 2002; Welch, 2004; Leary and Roberts, 2005).

increasing for all A and B, i.e. higher subsidies increase the distance to bankruptcy for any initial financial state A and shock B. I will also assume that $\xi_n(A, \theta, \cdot)$ is increasing for all A and θ, i.e. higher values of shocks B result in greater distance to bankruptcy.

As was discussed in the introduction, the supplier's financial state may affect labor availability, its output rate, quality of products, tardiness in order fulfilment, etc. In this paper the model of supply risk to the manufacturer is an uncertain supplier's capacity, Y_n, which is a function of the order quantity, z_n, and the supplier's distance to bankruptcy, ξ_n, that is $Y_n = Y_n(z_n, \xi_n)$. Details of modeling random capacity are presented in Section 3.

The cost to the manufacturer of paying $\Theta = c_0(z) + \theta$ up-front to the supplier is $\phi(\Theta)$, where function ϕ satisfies $\phi(0) = 0$, $\phi'(\theta) \geq 1$, $\phi''(\theta) \geq 0$. One could think that as the manufacturer pays the supplier, it worsens its own financial state. The manufacturer's credit ratings could be downgraded, leading to higher financing costs. A special case of the cost function ϕ is $\phi(\theta) = \theta$. Although, only positive subsidies will be allowed, for technical reasons, it will be convenient to define ϕ on all real numbers as follows: $\phi(s) = 0$, for all $s \leq 0$.

As was mentioned earlier, function $\mathcal{M}(x + y, d)$ represents the manufacturer's operating costs (minus revenues). In this expression y is the supplier's capacity available to the manufacturer during a period, x is the inventory, and d is the realization of demand. In addition, define the expected operating cost function $M(y) = E[\mathcal{M}(y, D)]$, where expectation is with respect to the distribution of demand D, conditional on the value of $Y = y - x$. In this paper we will need the expected operating costs function, M, to be convex. The general forms of functions $\mathcal{M}$ and M are sufficient to derive the majority of results. This is important, because this affords us flexibility to incorporate features of a particular practical problem into the operating cost function. For example, one can consider a model for $\mathcal{M}$ where the manufacturer buys from a spot market as a recourse against a supply shortage. In building intuition about the properties of the operating cost functions it is useful to consider three special cases: newsvendor model, linear demand model, and iso-elastic demand model.

$$\text{Newsvendor:} \quad \mathcal{M}(y, d) = h(y - d)^+ + p(d - y)^+, \qquad M(y) = hE(y - D)^+ + pE(D - y)^+, \tag{2a}$$

$$\text{Linear:} \quad \mathcal{M}(y, d) = -p(d - gy)y, \qquad M(y) = -p(E[D] - gy)y, \tag{2b}$$

$$\text{Iso-elastic:} \quad \mathcal{M}(y, d) = -pdy^{-g}y, \qquad M(y) = -pE[D]y^{-g}y. \tag{2c}$$

Parameters h and p represent marginal overage and underage costs for the newsvendor model and p represents unit revenue for the other models. One could also think of p as the expected per unit recourse cost in case of a supply shortage. Parameter g is the price sensitivity. For the iso-elastic demand model, $0 < g < 1$.

The manufacturer may maintain raw materials inventory as a hedge against future supply disruptions. The transition rule for inventory is $x_{n+1} = T_x(x_n + Y_n - D_n)$, where T_x is a non-decreasing function.

To summarize, the manufacturer solves the following sequential optimization model:

$$\min E \sum_{n=1}^{N} \alpha^{n-1} \Big\{ \phi \left[c_0(z_n) + \theta_n \right] + c[Y_n(z_n, \xi_n)] + \mathcal{M}[x_n + Y_n(z_n, \xi_n), D_n] \Big\}, \tag{3a}$$

s.t.

$$z_n \geq 0, \quad \theta_n \geq 0, \quad \xi_n = \xi_n(A_n, \theta_n, B_n), \tag{3b}$$

$$A_{n+1} = T_A(A_n, \theta_n, B_n), \quad n = 1, ..., N, \tag{3c}$$

$$x_{n+1} = T_x[x_n + Y_n(z_n, \xi_n) - D_n], \quad n = 1, ..., N. \tag{3d}$$

Throughout this paper I will assume that all functions are sufficiently smooth (so that their derivatives are integrable) in order for the integration and differentiation operators to be interchangeable (Folland, 1984, Chapter 2).

3 Conditions for Independence of Ordering Decisions from Subsidy Decisions

As was mentioned in the introduction, the analysis of random-yield models (similar to (3)) is non-trivial and leads to complex optimal policies (e.g., see Henig and Gerchak, 1990; Parlar et al., 1995). The problem in this paper is even more difficult than a typical random-yield problem, because we would like to know not only the optimal ordering policy, given a process for random supply, but also the optimal policy for controlling the evolution of the random supply process over time. In this section I will describe a set of assumptions that will enable us to find the optimal capacity ordering policy separately from finding the optimal random-supply-process control policy, thus affording analytical solutions to our complex problem. In the following sections I will discuss consequences of relaxing these assumptions.

Assumption 1. *Demand and supply shocks are independent (i.e. D_n and B_n are independent).*

This assumption is reasonable for many problems. Recall the interpretation of supply shocks, B_n, as generated by the supplier's businesses other than its business with the manufacturer. If the manufacturer and these other businesses are not in the same industry, then demands (and hence payments to the supplier) that the manufacturer and other businesses observe can be independent. The relaxation of the *independence* assumption is considered in Section 4.

Assumption 2. *The manufacturer does not carry inventory (i.e. state variable $x_n = 0$).*

In some settings this assumption is justified. For example, one could think that the products are short-lived. Every period in the model represents the lifecycle of a product and inventory of obsolete products is not valuable. At the same time the manufacturer maintains the long-term relationship with the supplier. The relaxation of the *no inventory* assumption is discussed in Section 5. Without inventory, one can combine contingent ordering costs $c(y)$ and operating costs $\mathcal{M}(y,d)$ as follows: $\mathcal{L}(y,d) \stackrel{\text{def}}{=} c(y) + \mathcal{M}(y,d)$. Taking expectation with respect to demand, D, $L(y) \stackrel{\text{def}}{=} c(y) + M(y)$.

Assumption 3. *The supplier delivers capacity* $Y(z,\xi) = \min[z, K(\xi)]$

Function K is the supplier's *effective capacity* and its argument is the supplier's distance to bankruptcy, ξ. The random capacity Assumption 3 not only leads to the desirable mathematical properties of the manufacturer's model, but also it is more appropriate for the automotive industry examples, which motivated this research. An alternative, proportional random-yield assumption, $Y(z,\xi) = z\min[1, K(\xi)]$, is discussed in Section 6.

I will assume that the supplier effective capacity function is

$$K(\xi) = k \min(\xi, 1), \tag{4}$$

where k is the regular capacity level of the supplier. If the supplier is financially viable (i.e. $\xi \geq 1$) then it can provide the manufacturer up to its regular production capacity k. If the supplier is experiencing financial difficulties (i.e. $\xi < 1$) the supplier's capacity is proportionately reduced. In general, for a strictly increasing, concave function $g : \mathbb{R}_+ \to \mathbb{R}_+$, (where $\mathbb{R}_+$ is the set of positive real numbers), and $K(\xi) = k \min[g(\xi), 1]$, the qualitative results of this paper will remain true (although mathematical expressions will change), but we will focus on expression (4) for simplicity. Section 8 contains further discussion of the modeling choice for the effective capacity, K, and its effects on the analysis.

Assumption 4. *The supplier's capacity costs depend only on the realized capacity* ($c_0(\cdot) \equiv 0$).

The costs of capacity to the supplier comes from the cost of its production assets (e.g. number of workers it employs). Therefore, one could argue that the costs to the supplier are more likely to depend on the realized capacity, Y (which is a function of supplier's assets), than on the order, z, the manufacturer placed. Assumptions of payments contingent only on the quantity delivered have appeared in many earlier random-yield models (e.g., Anupindi and Akella, 1993; Ciarrallo et al., 1994; Dada et al., 2007). Implications of relaxing Assumption 4 are presented in Section 7.

With these assumptions, problem (3) becomes

$$\min E \sum_{n=1}^{N} \alpha^{n-1} \Big\{ \phi(\theta_n) + \mathcal{L}[Y(z_n, \xi_n), D_n] \Big\}, \tag{5a}$$

s.t.

$$z_n \geq 0, \quad \theta_n \geq 0, \quad \xi_n = \xi_n(A_n, \theta_n, B_n), \tag{5b}$$

$$Y(z, \xi) = \min[z, K(\xi)], \tag{5c}$$

$$K(\xi) = k \min(\xi, 1), \tag{5d}$$

$$A_{n+1} = T_A(A_n, \theta_n, B_n), \quad n = 1, ..., N. \tag{5e}$$

To solve optimization problem (5) it is useful to rewrite it in a DP recursion form. Let $V_n(A)$ be the optimal cost to the manufacturer starting operations in period n when the supplier's financial state is A. Then, $V_{N+1}(A) = 0$, and for $n = 1, ..., N$

$$V_n(A) = \min_{z \geq 0, \theta \geq 0} \{\phi(\theta) + EL[Y(z, \xi_n)] + \alpha EV_{n+1}[T_A(A, \theta, B)]\}, \tag{6}$$

where B has the same distribution as B_1 and $\xi_n = \xi_n(A, \theta, B)$. It will be convenient to define $J_n(A, z, \theta) = \phi(\theta) + EL[Y(z, \xi_n)] + \alpha EV_{n+1}[T_A(A, \theta, B)]$. Using this function, recursion equation (6) can be rewritten as $V_n(A) = \min_{z \geq 0, \theta \geq 0} J_n(A, z, \theta)$.

3.1 Optimal Ordering Policy

In the next two subsections we will find the optimal capacity ordering and financial subsidy policies for problem (6). The following proposition describes the optimal ordering policy. All proofs are in the Appendix.

Proposition 1. *In every period, for any subsidy decision, θ, the optimal order quantity, z, in problem (6) is the minimum between regular capacity level, k, and the solution of the following equation:*

$$L'(z) = 0. \tag{7}$$

From Proposition 1, for the newsvendor model (2a) with linear costs $c(y) = cy$, the optimal order quantity satisfies the newsvendor's critical fractile expression $\Pr[D \leq z] = \frac{p-c}{p+h}$. Proposition 1 yields results in non-newsvendor settings as well (which are presented in the Appendix). The main takeaway from Proposition 1 is that, given the model assumptions, particularly the assumption regarding the supplier effective capacity (Assumption 3), the optimal capacity order does not depend on the subsidy amount. This is great news from the manager's point of view, because the existing models, which ignore financial subsidies, generate the optimal order quantities nevertheless. The research takeaway from this proposition is that we can eliminate one of the decision variables (order quantity) from the optimization problem (6). One should not interpret the statement of Proposition 1 as the irrelevance of financial subsidies. Although order quantities do not depend on financial subsidies, the manufacturer's objective function does. In the next subsection we will find the optimal financial subsidy policy, which minimizes the manufacturer's cost.

It is reasonable to assume that, unless the supplier is in financial trouble, the supplier has sufficient capacity to satisfy the manufacturer's optimal order. In other words, if z^0 is the solution of (7), then the regular supplier's capacity $k \geq z^0$. I will assume for the remainder of the paper that this is the case, and therefore the optimal order quantity from Proposition 1 is z^0. Consequently, $Y(z^*, \xi) = \min[z^0, k\min(\xi, 1)] = \min[z^0, k\xi]$, and one can focus on the effective capacity defined as $K(\xi) = k\xi$, rather than (5d).

3.2 Optimal Subsidy Policy

Knowing that the optimal order quantity, z^*, satisfies (7), problem (6) is simplified to $V_{N+1}(A) = 0$, and for $n = 1, ..., N$

$$V_n(A) = \min_{\theta \geq 0} \left\{ \phi(\theta) + EL[Y(z^*, \xi_n)] + \alpha E V_{n+1}[T_A(A, \theta, B)] \right\}, \tag{8}$$

where $\xi_n = \xi_n(A, \theta, B)$, $Y(z, \xi) = \min(z, K(\xi))$ and $K(\xi) = k\xi$. Properties of this optimization problem and its solution depend on the properties of functions $\psi_n(\theta) = EL[Y(z^*, \xi_n)]$.

Lemma 1. *If for all n, A, and B function $\xi_n(A, \cdot, B)$ is increasing, then $\psi_n(\theta) = EL[Y(z^*, \xi_n)]$, where $\xi_n = \xi_n(A_n, \theta, B_n)$, is decreasing in θ. If function $\xi_n(A, \cdot, B)$ is concave, then function $\psi_n(\cdot)$ is convex.*

For the models where the supplier's financial state is the supplier's asset value, function $\xi_n(A, \cdot, B)$ is increasing. Therefore, for those models $\psi_n(\cdot)$ is decreasing. Furthermore, for the asset-investment model, function $\xi_n(A, \cdot, B)$ is concave and therefore $\psi_n(\cdot)$ is convex. Unfortunately, for the liabilities-reduction model, $\xi_n(A, \cdot, B)$ is convex and therefore $\psi_n(\cdot)$ need not be convex.

3.3 Optimal Subsidy Policy: Asset-Investment Model

For the asset-investment model, transition rule is $A_{n+1} = T_A(A_n, \theta, B_n) = (A_n + \theta)B_n$, and the distance to bankruptcy is $\xi_n = \xi_n(A_n, \theta, B_n) = \frac{(A_n+\theta)B_n}{F_n}$. It will be convenient to define a new decision variable: the supplier's *asset level immediately after receiving financial subsidy*, $a = A + \theta$. Using this variable, the transition function and the distance to bankruptcy are redefined as $A_{n+1} = T_A(a, B_n) = aB_n$ and $\xi_n = \xi_n(a, B_n) = \frac{aB_n}{F_n}$. Dynamic program (8) is as follows. $V_{N+1}(A) = 0$, and for $n = 1, ..., N$

$$\begin{aligned} V_n(A) &= \min_{a \geq A} H_n(A, a), \\ H_n(A, a) &= \phi(a - A) + EL[Y(z^*, \xi_n)] + \alpha E V_{n+1}(aB). \end{aligned} \tag{9}$$

Modifying Lemma 1 to accommodate the new decision variable, one can verify that $\psi_n(a) = EL[Y(z^*, \xi_n(a, B))]$ is decreasing and convex in a, which leads to the familiar "up-to" structure of the policy.

Proposition 2. *For all n, functions V_n and H_n (defined in (9)) are convex and the subsidy policy has a "subsidize-up-to" structure. Specifically, there exist target levels, S_n, such that the optimal level of the supplier's assets after financial subsidies is $a_n^* = \max(S_n, A_n)$. Furthermore, if $\phi(x)$ is linear, then target levels, S_n, do not depend on the current level of the supplier's assets, A_n.*

The subsidize-up-to policy resembles base-stock inventory policy. In spite of that, the optimality of the subsidize-up-to policy is not obvious. Lemma 1 and the modeling assumptions that lead to convex function ψ_n are needed in the proof of Proposition 2. To further appreciate the importance of the result, consider that for the other natural model of how subsidies are used — the liabilities-reduction model — subsidize-up-to policy does not have to be optimal. Still, as the next subsection shows one can find the optimal policy for that model as well.

3.4 Optimal Subsidy Policy: Liabilities-Reduction Model

For the liabilities-reduction model, transition rule is $A_{n+1} = T_A(A_n, \theta, B_n) = A_n B_n$, and the distance to bankruptcy is $\xi_n = \xi_n(A_n, \theta, B_n) = \frac{A_n B_n}{F_n - \alpha^{-1}\theta}$, which is a convex function of θ. This model is myopic with respect to the subsidy decisions. Therefore, even though convexity results from Lemma 1 cannot be applied, one can still identify sufficient conditions for the quasi-convexity of the manufacturer's optimization problem (Proposition 3 below). Instead of the specific form (i.e. $\xi_n(A_n, \theta, B_n) = A_n B_n/(F_n - \alpha^{-1}\theta)$) of the distance to bankruptcy let's consider two more general representations $\xi_n(A_n, \theta, B_n) = q_n(A_n, \theta)X_n(B_n)$ and $\xi_n(A_n, \theta, B_n) = q_n(A_n, \theta) + X_n(B_n)$. Function q_n in these expressions can be thought of as the distance to bankruptcy without random shock to the supplier's assets. To be consistent with earlier assumptions, function $q_n(A, \cdot)$ must be increasing for all A. For the former representation it is also necessary that $X_n(B) \geq 0$ for all B.

Proposition 3. *Suppose that for $n = 1, ..., N$, $\xi_n(A_n, \theta, B_n) = q_n(A_n, \theta)X_n(B_n)$ or $\xi_n(A_n, \theta, B_n) = q_n(A_n, \theta) + X_n(B_n)$. If $\frac{\phi'(\cdot)}{q_n'(A, \cdot)}$ is increasing for all A, then the manufacturer's problem (8) is quasi-convex and the optimal subsidy amount, θ_n^*, is the maximum of zero and the solution of equation*

$$-k\frac{\partial q_n(A_n, \theta)}{\partial \theta} E\left[X_n(B_n) L'(k\xi_n) 1_{\{k\xi_n \leq z^*\}}\right] = \phi'(\theta), \tag{10}$$

when $\xi_n = \xi_n(A_n, \theta, B_n) = q_n(A_n, \theta)X_n(B_n)$ or equation

$$-k\frac{\partial q_n(A_n, \theta)}{\partial \theta} E\left[L'(k\xi_n) 1_{\{k\xi_n \leq z^*\}}\right] = \phi'(\theta), \tag{11}$$

when $\xi_n = \xi_n(A_n, \theta, B_n) = q_n(A_n, \theta) + X_n(B_n)$.

The main insight from this proposition is that, if ratio $\frac{\phi'(\cdot)}{q_n'(A, \cdot)}$ (the marginal subsidy costs over the marginal supplier's distance to bankruptcy function) is increasing for all A, then to find the optimal subsidy amount in any period, given the initial value of the supplier's assets, A, the manufacturer must solve equation (10) or equation (11). In addition, first order conditions (10) and (11) define conditions when the manufacturer will not offer subsidies:

$$-k\frac{\partial q_n(A_n,0)}{\partial\theta}E\left\{X_n(B_n)L'[k\xi_n(A_n,0,B_n)]1_{\{k\xi_n(A_n,0,B_n)\le z^*\}}\right\}<\phi'(0) \tag{12a}$$

$$-k\frac{\partial q_n(A_n,0)}{\partial\theta}E\left\{L'[k\xi_n(A_n,0,B_n)]1_{\{k\xi_n(A_n,0,B_n)\le z^*\}}\right\}<\phi'(0) \tag{12b}$$

To prove Proposition 3 it is critical to assume that function $q_n(A,\cdot)$ is increasing, but the specific definition of function q_n is not essential (one example is $q_n(A,\theta)=A/(F_n-\alpha^{-1}\theta)$). Given an increasing function q_n, one can find function ϕ, which satisfies $\phi(0)=0$, $\phi'(\theta)\ge 1$, $\phi''(\theta)\ge 0$, and the sufficient condition for quasi-convexity: i.e. $\frac{\phi'(\cdot)}{q'(A,\cdot)}$ is increasing. For example, if $F_n=F$ for all n and $q_n(A,\theta)=A/(F-\alpha^{-1}\theta)$, the financial cost function could be chosen as follows $\phi(\theta)=\frac{\mathcal{A}}{\mathcal{F}-\alpha^{-1}\theta}-\frac{\mathcal{A}}{\mathcal{F}}$ for $\mathcal{A}\ge\mathcal{F}^2$ and $\mathcal{F}\le F$. This cost function ϕ satisfies all of the conditions above.

The first order conditions (10) and (11) may appear, at first glance, to be difficult to compute and analyze. But with GBM distribution (1) on the evolution of the supplier's assets and independence of the demand and supply shocks, these conditions require only slightly more computation effort[8] than the Black-Scholes formula for option prices. The expressions for the first order condition (10) for the specific inter-period operating cost functions are presented in the Appendix.

3.5 Model and Solution Properties, Comparatives Statics

In this subsection we will discuss the properties of the model and the solution. First, let's consider the relationship between the target asset level and the supplier's assets at the beginning of a period.

Proposition 4. *If the cost of financial subsidies, ϕ, is a strictly convex function, then, for the asset-investment model (9),*

(i) *The optimal subsidize-up-to level, S_n, and the optimal asset level, a_n^*, increase in the current value of the supplier's assets, A, for any period n.*

(ii) *Value functions V_n are decreasing for all n.*

Next, we will study the dependence of the optimal decisions on problem parameters for the specific forms of the manufacturer's operating cost models. The following lemma will be useful in proving Proposition 5.

Lemma 2. *Let π be some parameter that affects operating cost function L. If $\frac{\partial^2}{\partial y\partial\pi}L(y;\pi)\le 0$ then, for the asset-investment model (9), for all n, $V_n(A;\pi)$ is submodular and the optimal subsidize-up-to level S_n is increasing in π.*

[8] Additional effort is needed to find the root of a non-linear equation numerically.

Proposition 5.

For the asset-investment model (9) and linear capacity costs, $c(y) = cy$:

(i) *For the newsvendor model (2a), the optimal subsidize-up-to levels, S_n, increase in the shortage penalty, p, decrease in the capacity cost, c, and decrease in the holding cost, h. If the demand is normally distributed ($D_n = \mu_D + \sigma_D Z_n$, where $Z_n \sim \mathcal{N}(0,1)$), then the optimal subsidize-up-to levels, S_n, increase in the mean demand, μ_D. If the demand is normally distributed and $\frac{p-c}{p+h} \leq \frac{1}{2}$, the optimal subsidize-up-to levels, S_n, decrease in the standard deviation, σ_D, of the demand.*

(ii) *For the linear-demand model (2b), the optimal subsidize-up-to levels, S_n, increase in the expected demand, $E[D]$, decrease in the capacity cost, c, and increase in the price sensitivity, g.*

(iii) *For the iso-elastic demand model (2c), the optimal subsidize-up-to levels, S_n, increase in the expected demand, $E[D]$, decrease in the capacity cost, c, and increase in the revenue, p.*

To derive the next property of the optimal solution, I will focus on a single-period newsvendor model (or the newsvendor model where the supplier uses financial subsidies to reduce current liabilities) and take advantage of equation (28) (see Appendix) that describes the optimal subsidy amounts.

Proposition 6. *For a single-period newsvendor model, if $\frac{\phi'(\cdot)}{q'(A,\cdot)}$ is increasing and*

$$\left(\mu - \frac{\sigma^2}{2}\right)\Delta \geq \ln\left(\frac{z^*}{kq(A,0)}\right), \tag{13}$$

then the optimal subsidy amount, θ^*, increases in the volatility of the supplier's assets, σ.

Inequality (13) is a sufficient condition on the values of the supplier's asset volatility, σ, the drift of the supplier's assets, μ, the optimal order quantity, z^*, and the supplier production capacity function prior to random shock, q. Roughly speaking, this condition says that the volatility, σ, is not too large relative to the drift, μ. Then Proposition 6 states that the optimal subsidy to the supplier is increasing in the volatility of the supplier's assets, σ. The following numerical example supplement the analytical statements in this section.

Example 1. For the first numerical example, let's consider a single-period newsvendor model, with the parameter values: period length $\Delta = 1$, drift of the supplier's asset process $\mu = 0.05$, base-case volatility of the supplier's asset process $\sigma = 0.3$, initial level of the supplier's assets $A(0) = 1000$, level of the supplier's liabilities $F = 1000$, regular supplier's capacity $k = 5500$. Supplier applies financial subsidies to reduce its liabilities. The manufacturer faces normally distributed demand with a mean $\mu_D = 5000$ and standard deviation $\sigma_D = 1000$. The profit margin for the manufacturer is $r - c = 0.5$, holding cost is $h = 0.5$, penalty for shortage $p = 1$. The manufacturer's cost of financial subsidies is $\phi(\theta) = \frac{\mathcal{A}}{\mathcal{F}-\theta} - \frac{\mathcal{A}}{\mathcal{F}}$ where $\mathcal{A} = \mathcal{F}^2$, $\mathcal{F} = 1000$. In this example parameter values are chosen so that the manufacturer's orders contribute significantly to the supplier's business and the supplier is close to bankruptcy at the beginning of the planning horizon.[9]

Figure 1 illustrates how the optimal subsidy amount, θ^*, depends on the parameters of the stochastic process describing the evolution of the supplier's assets. The left panel of this figure shows the dependence of θ^* on the volatility of the supplier's assets, σ. As we know from Proposition 6, for small σ, θ^* increases in σ. The left panel of Figure 1 also shows that if the supplier's assets are too volatile, the manufacturer will start reducing the subsidy amount. The right panel of Figure 1 shows the dependence of the optimal subsidy amount on the initial supplier's asset level, $A(0)$. As one can expect, when the initial supplier's asset level is already high, the manufacturer will not offer any subsidies. When the initial supplier's asset level is too low, the manufacturer will not offer any subsidies either, because benefits of subsidies do not justify the cost of subsidies, $\phi(\theta)$.

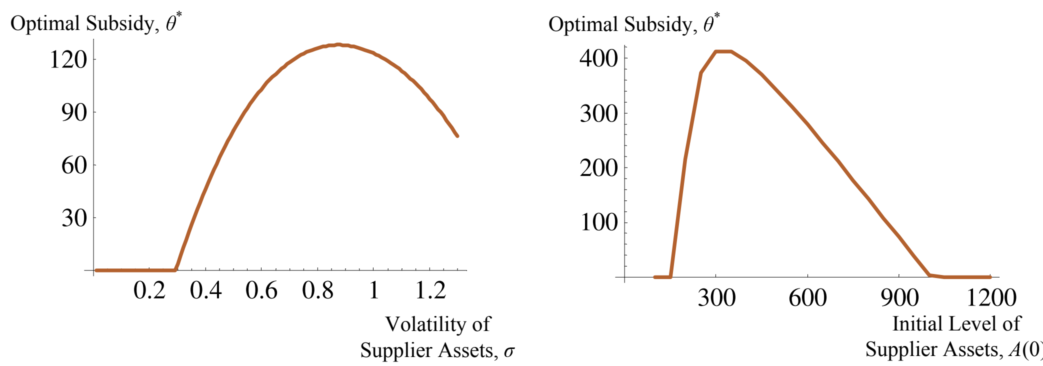

Figure 1: The optimal subsidy amount, θ^*, as a function of supplier parameters: asset volatility, σ, and initial asset level, $A(0)$ (see Example 1 for the numerical values of other model parameters)

Figure 2 illustrates the dependence of the optimal subsidy amount, θ^*, on the demand parameters: mean, μ_D, and standard deviation, σ_D. The graph in the left panel of Figure 2 is consistent with the statement of Proposition 5 applied to the one-period problem, that is, the optimal subsidy amount increases in the mean demand. From the right panel of Figure 2 we see that, for this numerical example, the optimal subsidy amount decreases in the demand standard deviation, σ_D. As we know from Proposition 5, if $\frac{p}{p+h} \leq \frac{1}{2}$ the optimal subsidy decreases in σ_D. There are numerical examples (not presented here), where $\frac{p}{p+h} > \frac{1}{2}$ and the optimal subsidy is non-monotone in σ_D.

The benefits to the manufacturer from subsidizing the supplier could be significant, as Figure 3 illustrates. This figure shows the percent improvement in the manufacturer's objective function

[9] For all numerical examples, I chose to present only the results that are the most interesting and select parameter values that lead to results that are both representative and interesting.

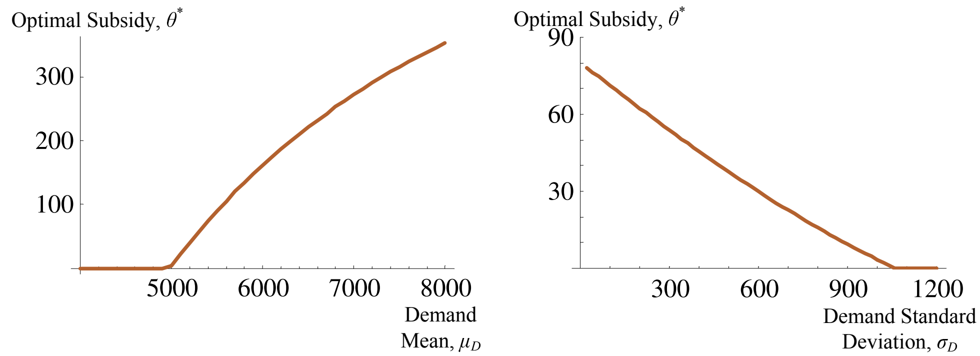

Figure 2: The optimal subsidy amount, θ^*, as a function of demand parameters: mean, μ_D, and standard deviation, σ_D (see Example 1 for the numerical values of other model parameters)

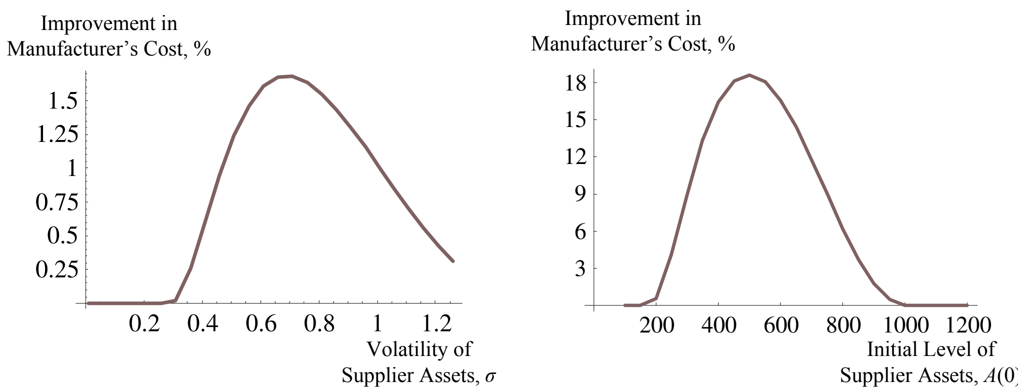

Figure 3: Percent improvement in manufacturer's cost if subsidies are given to the supplier as a function of the volatility of the supplier's assets σ and supplier initial asset level $A(0)$ (see Example 1 for the numerical values of other model parameters)

(in this case, newsvendor objective function) as a result of giving the supplier the optimal subsidy instead of zero subsidy. The magnitude of benefits for a particular real-life problem will depend on the values of the problem parameters, such as operating cost function, L, and subsidy cost function, ϕ. However, it is important to observe that paying suppliers only their marginal production cost can be suboptimal for the manufacturer, especially when the supplier is in or close to bankruptcy. This concludes the first numerical example.

I also applied results of this section to analyze the decision of Ford to subsidize Visteon in 2005. The discussion of Ford vs. Visteon example is presented in the Appendix.

4 Dependent Supply and Demand Shocks

A condition necessary for the separation of the capacity ordering policy from the subsidy policy is the independence between the manufacturer's demand, D, and the shocks, B, to the supplier's state, A. One may argue that if all supplier's customers (including the manufacturer) are in the same industry, then the demand and the shocks could be positively correlated. Alternatively, it can also happen that as the demand for the manufacturer's products increases the demand for the products of other firms decreases (along with cash flows to the supplier). For instance, the manufacturers and the other firms could be competing in the same final product market. This would lead to negative correlated between the demand and the shocks. Finally, if the supplier's customer portfolio is well diversified, then the demand and the shocks can be independent. We have studied the latter situation in earlier sections. In this section we investigate the effect of correlation. Assumptions 2, 3, and 4 continue to hold. Without the separation between ordering and subsidy decisions the analysis of problem (3) is more complex and to glean insights into problem properties we will use a numerical study.

Example 2. Consider a single-period newsvendor model where the supplier uses subsidies to invest in its assets, with the following parameter values: period length $\Delta = 1$, drift of the supplier's asset process $\mu = 0.05$, initial level of the supplier's assets $A(0) = 50$, level of the supplier's liability $F = 100$, regular supplier's capacity $k = 130$. The manufacturer faces normally distributed demand with mean $\mu_D = 100$ and standard deviation $\sigma_D = 20$. The overage cost is $h = 1$, the underage cost is $p = 1.2$. The manufacturer's cost of financial subsidies is $\phi(\theta) = \theta$. As the measure of correlation in this example I use linear correlation coefficient, that is $\rho \stackrel{\text{def}}{=} corr[D, dW(\Delta)] = \frac{Cov[D,dW(\Delta)]}{\sqrt{Var[D]}\sqrt{Var[dW(\Delta)]}}$, where $dW(\Delta)$ is the shock to the supplier's assets (equation (1)) over the planning horizon Δ.

Figure 4 illustrates the effect of correlation, ρ, and the volatility of the supplier's assets, σ, on the order quantity, the target level of the supplier's assets, and the optimal objective value. Observe that as the correlation between demand and supply shocks increases, the optimal order quantity increases and the objective value decreases (the objective value represents the manufacturer's costs). Intuitively, as the correlation increases, high demand is more likely to be matched by the positive shock to the supplier's assets and the manufacturer is more likely to meet the demand with the supplier's capacity. Although Figure 4 shows that the target supplier's asset levels increase in correlation, other numerical experiments showed that this relationship could be non-monotone.

From Figure 4, the optimal order quantity, z^*, is decreasing in the volatility of the supplier's assets, σ, except when supply and demand shocks are independent, $\rho = 0$ (the case studied in the previous section). Target asset level is decreasing in the volatility and the costs of the manufacturer are increasing. These observations are consistent with the properties of the optimal solution discussed in Section 3.5. This concludes the example.

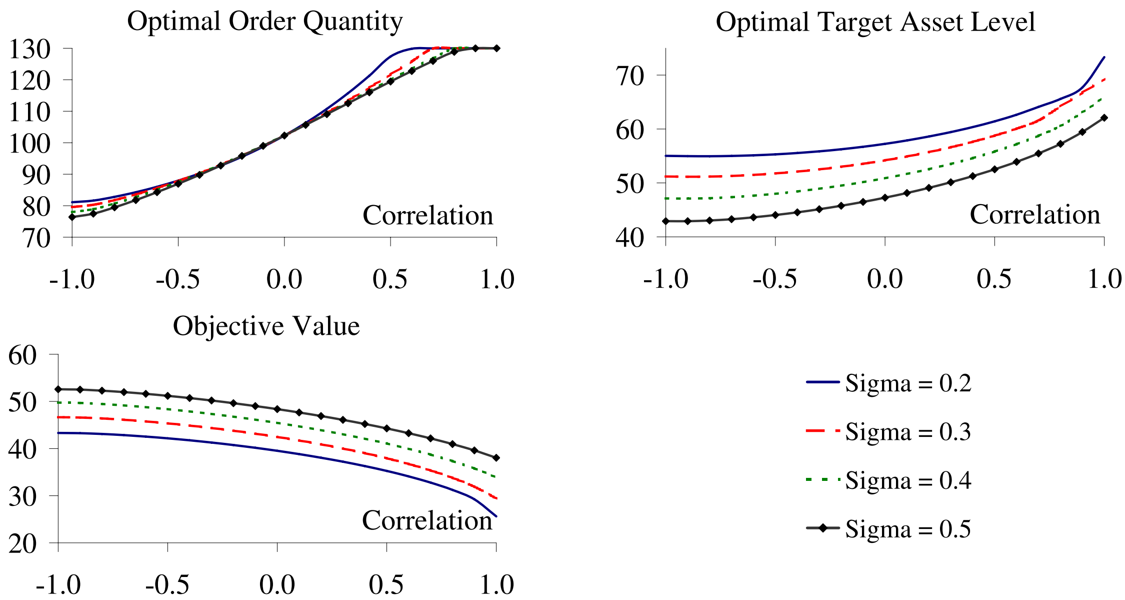

Figure 4: The effect of correlation between manufacturer's demand and the supplier's asset shocks. Top left panel: the optimal order quantity of the manufacturer. Top right panel: the optimal target level of the supplier's assets after subsidy. Bottom panel: the optimal objective value of the manufacturer. The x-axes of each picture measures correlation. Sigma refers to the volatility of the supplier's assets, σ.

5 Inventory and Backorders

In this section we will relax Assumption 2 by introducing inventory and backorders. Assumptions 1, 3, and 4 continue to hold. Problem (3) becomes:

$$\min E \sum_{n=1}^{N} \alpha^{n-1} \Big\{ \phi(\theta_n) + c[Y(z_n, \xi_n)] + \mathcal{M}[x_n + Y(z_n, \xi_n), D_n] \Big\}, \tag{14a}$$

s.t.

$$z_n \geq 0, \quad \theta_n \geq 0, \quad \xi_n = \xi_n(A_n, \theta_n, B_n), \tag{14b}$$

$$Y(z, \xi) = \min[z, K(\xi)], \quad K(\xi) = k \min(\xi, 1), \tag{14c}$$

$$A_{n+1} = T_A(A_n, \theta_n, B_n), \quad n = 1, ..., N, \tag{14d}$$

$$x_{n+1} = x_n + Y(z_n, \xi_n) - D_n, \quad n = 1, ..., N. \tag{14e}$$

Equation (14e) represents the inventory transition rule between periods (i.e., $T_x(s) = s$). It is convenient to combine ordering and inventory cost terms in one expression. For that I need to make additional assumptions that the contingent order cost is linear, $c(y) = cy$, and that at the end of the last period inventory can salvaged at the same price as it was bought initially. Then a rearrangement of terms in the objective of optimization problem (14), as in Heyman and Sobel (2004, Chapter 3), leads to the desired representation. Specifically, the objective function of problem

(14) contains

$$\sum_{n=1}^{N} \alpha^{n-1} \left[cY_n + \mathcal{M}(x_n + Y_n, D_n) \right] - cx_{N+1}. \tag{15}$$

This can be written as

$$\sum_{n=1}^{N} \alpha^{n-1} \left[c(x_n + Y_n) - cx_n + \mathcal{M}(x_n + Y_n, D_n) \right] - cx_{N+1}. \tag{16}$$

Using transition rule (14e) and collecting similar terms one obtains

$$\sum_{n=1}^{N} \alpha^{n-1} \left[c(1-\alpha)(x_n + Y_n) + \mathcal{M}(x_n + Y_n, D_n) \right] + consts. \tag{17}$$

Thus, as before one can define the operating cost function, $\mathcal{L}(y,d) = c(1-\alpha)y + \mathcal{M}(y,d)$, which combines both ordering cost and inventory cost. Let's redefine DP recursion to accommodate inventory state variable x: $V_{N+1}(x, A) = 0$, and for $n = 1, ..., N$:

$$V_n(x, A) = \min_{z \geq 0, \theta \geq 0} \left\{ \phi(\theta) + EL[x + Y(z, \xi_n)] + \alpha E V_{n+1}(x_{n+1}, A_{n+1}) \right\} \tag{18a}$$

s.t.

$$\xi_n = \xi_n(A, \theta, B), \quad Y(z, \xi) = \min[z, K(\xi)], \quad K(\xi) = k \min(\xi, 1), \tag{18b}$$

$$A_{n+1} = T_A(A, \theta, B), \quad n = 1, ..., N, \tag{18c}$$

$$x_{n+1} = x_n + Y(z, \xi_n) - D_n, \quad n = 1, ..., N. \tag{18d}$$

As a corollary to Proposition 5 observe that for the last period, N, the optimal target supplier's asset level, S_N, is decreasing in the inventory level, x_N (assuming that $x_N < E[D_N]$). Intuitively, the more inventory the manufacturer has at the beginning of period N, the more demand it is guaranteed to satisfy, and the less it relies on subsidies to assure the supply. Similar intuition applies to other periods as well, however, a rigorous proof of this statement is challenging.

Because the introduction of inventory variable, x, in problem (18) interferes with the separation of subsidy and ordering policies, deriving analytical properties of the optimization problem and the solution (as we did in Section 3.5) is difficult. To glean insights into interactions of inventory investments and subsidies we continue with a numerical study.

Example 3. Let's consider a two-period newsvendor model where the supplier uses subsidies to invest in its assets, with the following parameter values: period length $\Delta = 1$, drift of the supplier's asset process $\mu = 0$, initial level of the supplier's assets $A(0) = 200$, levels of the supplier's liabilities $F = 500$, demand every period is deterministic $D = 100$, and regular supplier's capacity $k = 110$. The underage cost is $p = 15$. The manufacturer's cost of financial subsidies is $\phi(\theta) = \theta$. The discount factor is $\alpha = e^{-r\Delta}$, where $r = 0.01$. Initial inventory $x_1 = 0$.

Figure 5 demonstrates how inventory investments and subsidy decisions depend on the supplier's asset volatility, σ, and the overage cost, h. The shaded portion of the tables correspond to the cases in which the manufacturer invests in inventory. As one would expect, an inventory investment is more likely for lower values of the overage cost.

Optimal Order Quantity

Sigma \ h	1	1.5	2	2.5	3
0.25	110	100	100	100	100
0.3	110	109	100	100	100
0.35	110	110	110	100	100
0.4	110	110	110	110	100
0.45	110	110	110	110	110
0.5	110	110	110	110	110

Target Supplier Asset Level

Sigma \ h	1	1.5	2	2.5	3
0.25	586	624	624	624	624
0.3	604	605	664	664	664
0.35	637	637	637	708	708
0.4	679	679	679	679	754
0.45	723	723	723	723	723
0.5	770	770	770	770	770

Figure 5: Inventory investments and subsidies as functions of supplier's asset volatility (σ) and overage cost (h). Left panel: the optimal order quantity, z_1; Right panel: the optimal target supplier's asset level, S_1. Shaded portion corresponds to the cases in which the manufacturer invests in inventory. Supplier's asset volatility ranges from 0.25 to 0.5. The overage cost ranges from 1 to 3.

An inventory investment is more likely as the supplier's asset volatility increases. Intuitively, an inventory investment is a safe way for the manufacturer to manage its costs in the second period. In contrast, an investment in the supplier's assets through subsidies, while improving the supplier's status in the first period, might not have significant consequences in the second period, because the supplier's assets fluctuate significantly. Thus, the greater the supplier's asset volatility, the more the manufacturer prefers using inventory as a risk-mitigation tool.

Finally, inventory and subsidies appear to be substitutes. From the right panel of Figure 5, when the manufacturer stops using inventory as a risk-mitigation tool (e.g. as the overage cost increases), the manufacturer raises the supplier's optimal asset level.[10] This concludes the example.

6 Proportional Random Yield

An assumption frequently used to describe supply uncertainty is that of a proportional random yield. Using notation in this paper, with a proportional random yield the capacity available to the manufacturer is $Y(z, \xi) = y(\xi)z$, where y is a random number between zero and one. Proportional random yields are typically used to model uncertainties in a production process (e.g. in a semiconductor industry), but one could also imagine that in case of a shortage the supplier might decide to allocate available capacity proportionately to the size of the order, z[11] (although as I will argue

[10] A comparison of the subsidy levels for the model in this section and the model in the previous sections will reflect not only the flexibility to carry inventory, but also additional burden of backorders affecting costs in several periods. Because these effects are difficult to separate, the comparison of the two models is omitted.

[11] I am grateful to the anonymous referee for offering this motivation.

at the end of this section the supplier has better ways of rationing capacity). As we will see in this section, in the proportional yield model the separation of the financial subsidy and capacity ordering decisions does not hold. Intuitively, a manufacturer, which does not not provide significant subsidy to the supplier, might still get the required number of parts by artificially inflating its order quantity (similarly to the rationing game that leads to the bullwhip effect in Lee et al. 1997).

Using previously defined notation, I will assume that the proportional random yield, y, depends on the supplier's distance to bankruptcy at the end of the period, ξ, and define $y(\xi) = \min(\xi, 1)$. Assumptions 1, 2, and 4 continue to hold. Optimization problem (3) becomes:

$$\min E \sum_{n=1}^{N} \alpha^{n-1} \Big\{ \phi(\theta_n) + \mathcal{L}[Y(z_n, \xi_n), D_n] \Big\}, \tag{19a}$$

s.t.

$$z_n \geq 0, \quad \theta_n \geq 0, \quad \xi_n = \xi_n(A_n, \theta_n, B_n) \tag{19b}$$

$$Y(z, \xi) = y(\xi) z, \tag{19c}$$

$$y(\xi) = \min(\xi, 1), \tag{19d}$$

$$A_{n+1} = T_A(A_n, \theta_n, B_n), \quad n = 1, ..., N. \tag{19e}$$

Unfortunately, as the previous work shows (e.g. Henig and Gerchak 1990) proportional random-yield models do not possess convenient mathematical properties. For the optimization problem (19), interperiod costs are not convex or even quasi-convex in decision variables z and θ. Thus, analytical solution of this problem, particularly if the problem is dynamic, is difficult to obtain.

The following numerical example (and analysis) shows how capacity ordering decisions depend on the supplier's financial state in the model with the proportional random yield.

Example 4. Consider a single-period newsvendor model where the supplier uses subsidies to invest in its assets. Assume that demand is deterministic and consider two cases: $D = 100$ and $D = 500$. Other parameters are the same as in Example 2. Figure 6 shows the optimal capacity order and target supplier's asset level as functions of the supplier's asset volatility, σ. Left panel corresponds to demand $D = 100$ and right panel corresponds to demand $D = 500$. The left vertical axes measure capacity orders. The right vertical axes measure target supplier's asset levels.

For demand $D = 100$ case, it is optimal to keep the assets at the current level $A = 50$, but inflate the order quantity. As the supplier's asset volatility increases, similar to the observation in Example 2, the optimal capacity order quantity decreases. Thus, as our intuition suggested, the capacity orders depend on the state of the supplier's assets.

For demand $D = 500$ case, observe that both the order quantity and the target supplier's asset level may change as the volatility of the supplier's assets increases. Mathematical intuition

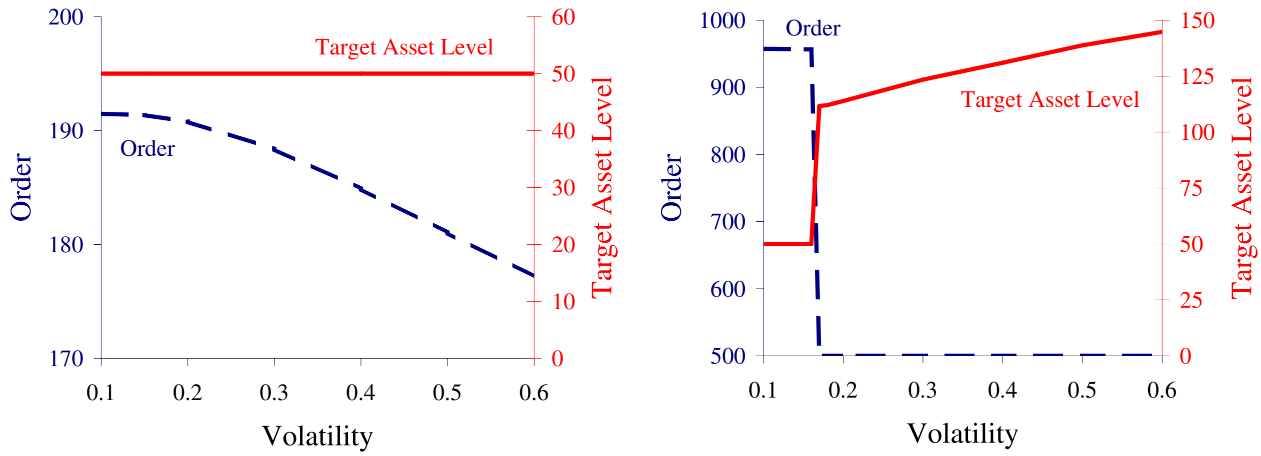

Figure 6: Optimal capacity order and target supplier's asset level as a function of volatility in the model with proportional random yield. Left panel: demand is 100 units. Right panel: demand is 500 units.

regarding what might be happening is as follows. Consider the first order derivatives of the objective function, $J(z, \theta) = \phi(\theta) + E[L(yz)]$, for the one-period model (19).

$$\frac{\partial J(z,\theta)}{\partial z} = E[L'(yz)y] \tag{20a}$$

$$\frac{\partial J(z,\theta)}{\partial \theta} = \phi'(\theta) + zE\left[L'(yz)\frac{dy}{d\theta}\right] \tag{20b}$$

If $\frac{\partial J(z,0)}{\partial \theta} < 0$, the manufacturer would like to give subsidies to the supplier. The first term of $\frac{\partial J(z,\theta)}{\partial \theta}$ does not depend on z. Informally, the second term is larger, in absolute value, for larger values of the multiplier z. At the optimum larger demand will correspond to larger values of z and if $E\left[L'(yz)\frac{dy}{d\theta}\right] < 0$ the second term will offset the first term $\phi'(\theta) \geq 1$ leading to positive optimal subsidies when demand $D = 500$. Also observe that the total quantity delivered, yz, can be increased (stochastically) either by increasing capacity order quantity, z, or by higher subsidies, θ. Thus, the two decisions are substitutable – greater subsidy corresponds to reduced order quantity. This concludes the example.

Finally, I would like to argue that a rationing game based on the orders placed is not the best approach for a supplier to allocate its sparse capacity. The supplier knows such rationing game would lead to artificially inflated orders from the manufacturers and to the bullwhip effect. Furthermore, the benefit to the supplier does not increase in the order size. The benefit to the supplier is what we call subsidy from the manufacturer — the value the manufacturer shares with the supplier beyond the supplier just breaking even. Thus, a supplier's best customers are the ones who pay the most, and if a supplier would like to reward its best customers it should allocate sparse capacity not based on order sizes, but based on the subsidies. We already modeled such allocation in the original model in this paper.

7 Upfront Costs

In this section we will relax the zero upfront costs assumption and, instead, assume that upfront costs are $c_0(z)$, where $c_0'(\cdot) > 0$ and $c_0''(\cdot) > 0$. Assumptions 1,2, and 3 continue to hold. The DP recursion is $V_{N+1}(A) = 0$, and for $n = 1, ..., N$

$$V_n(A) = \min_{z \geq 0, \theta \geq 0} \{\phi[c_0(z) + \theta] + EL[Y(z, \xi_n)] + \alpha EV_{n+1}[T_A(A, \theta, B)]\}, \tag{21}$$

where $\xi_n = \xi_n(A, \theta, B)$. As before, it will be convenient to define $J_n(A, z, \theta) = \phi[c_0(z) + \theta] + EL[Y(z, \xi_n)] + \alpha EV_{n+1}[T_A(A, \theta, B)]$. Intuitively, with the addition of upfront costs the manufacturer will order less than it would for problem (6). Let z^0 be the optimal capacity order for the problem without upfront costs and let's continue assuming that regular capacity is large enough, that is $k > z^0$. According to Proposition 1, z^0 satisfies $L'(z) = 0$.

Lemma 3. *For any subsidy level, θ, and the initial supplier's asset level, A, the minimizer $z^* = \arg\min_{z \geq 0} J_n(A, z, \theta)$ satisfies $z^* \leq z^0$. If functions ϕ and c_0 are strictly increasing, then $z^* < z^0$.*

The takeaway from Lemma 3 is that one can restrict the range of order quantities to be $z \leq z^0$. This restriction facilitates further analysis. For example, consider how the choice of the order quantity, z, and the subsidy amount, θ, interact in problem (21). If function ϕ is strictly convex, then in term $\phi[c_0(z) + \theta]$, order quantity, z, and subsidy, θ, are substitutes: to minimize costs we trade off increasing z against increasing θ. On the other hand, in term $EL[Y(z, \xi)]$, order quantity, z, and subsidy, θ, are compliments: to minimize costs we prefer to increase both z and θ. To prove the latter recall that $Y(z, \xi) = \min(z, k\xi)$; $\xi(A, \theta, B)$ is increasing in θ; thus, to increase $Y(z, \xi)$ we need to increase z and θ; and for $y < z^0$, $L(y)$ is decreasing. In general, whether order quantity, z, and subsidy, θ, are compliments or substitutes depends on which of the effects dominates.

Proposition 7. *Restrict the order quantity space to $z \in [0, z^0]$ and consider problem (21). If ϕ is linear, then $\frac{\partial^2}{\partial z \partial \theta} J_n(A, z, \theta) \leq 0$ for all A.*

With restriction $z \leq z^0$ one can prove that for the asset-investment model, optimization problem (21) has convex structure. As was done for Proposition 2, define a new decision variable, $a = A + \theta$ — supplier's asset level — and redefine transition and distance to bankruptcy functions: $T_A(a, B) = aB$, $\xi_n = \xi_n(a, B) = \frac{a}{F_n} B$. The problem becomes:

$$V_n(A) = \min_{0 \leq z \leq z^0, A \leq a} \{\phi[c_0(z) + a - A] + EL[Y(z, \xi_n)] + \alpha EV_{n+1}(aB)\}. \tag{22}$$

For this model, $J_n(A, z, a) = \phi[c_0(z) + a - A] + EL[Y(z, \xi_n)] + \alpha EV_{n+1}(aB)$.

Proposition 8. *Restrict the order quantity space to $z \in [0, z^0]$. Assume functions ϕ and c_0 are strictly increasing and consider problem (22). For all n, functions J_n and V_n are convex. The optimal order quantity, z_n^*, and supplier's asset level, a_n^*, satisfy the following KKT conditions:*

$$\phi'[c_0(z) + a - A_n]c_0'(z) + \Pr[z \leq k\xi_n]L'(z) = \lambda_n, \tag{23a}$$

$$\lambda_n z = 0, \quad \lambda_n \geq 0, \quad z \geq 0, \tag{23b}$$

$$\phi'[c_0(z) + a - A_n] + E\left[L'(k\xi_n)k\frac{\partial \xi_n}{\partial a}1_{\{z>k\xi_n\}}\right] + \alpha E[V_{n+1}'(aB_n)B_n] = \mu_n, \tag{23c}$$

$$\mu_n(a - A_n) = 0, \quad \mu_n \geq 0, \quad a \geq A_n. \tag{23d}$$

8 Conclusions, Limitations, and Extensions

Many manufacturing firms are facing supply risks due to the deteriorating financial health of their suppliers. Motivated by real-life examples, this paper studies the use of subsidies (in every form) as a way of controlling supply risk and answers a number of important questions: what is the optimal joint capacity ordering and financial subsidy policy for a manufacturer? Must subsidy and capacity ordering decisions be made jointly? How good are the recommendations from the traditional procurement models, which ignore the benefits of controlling the supplier's financial state through subsidies?

An important contribution of this work is a set of assumptions under which (for an otherwise very general model) the optimal ordering decisions do not depend on the subsidy decisions. Consequently, if the managers believe that these assumptions describe their business situation accurately, they can continue using the existing decision support systems for the procurement decisions — an important managerial takeaway. A research takeaway is the approach for solving a complex, dynamic, random-yield problem. The problem considered in this paper is more difficult than a typical random-yield model, because we not only decide on the optimal ordering policy, but also control the dynamics of the supply disruptions distribution over time. Proving even the simplest properties for the optimization problem in this paper (even for a one-period problem), like convexity or unimodality, is non-trivial. Therefore, the set of assumptions, that allows one to decompose the problem and to derive the optimal policies is very useful.

The separation result does not mean that subsidies to the suppliers are irrelevant. On the contrary, the value of the option to give subsidies could be very significant. I consider two examples of the effects of subsidies on the supplier's financial state. For the model where subsidies are used to reduce the supplier's liabilities, the manufacturer's dynamic problem can be decomposed into a sequence of one-period optimization problems, whose solutions I specify. For the model where subsidies are used to increase the supplier's assets, I prove that a "subsidize-up-to" policy is optimal for the manufacturer. That is, every period, the manufacturer has a target level to which it should try to bring the supplier's assets by offering financial subsidies. The paper presents conditions for the manufacturer to share supply-chain profits with the supplier via subsidies. Traditionally, in supply-chain models, a firm must have either market power or private information to receive its part of the supply chain profit. This paper suggests that even when suppliers have no market power and no private information, they can earn positive profits.

I derived a number of comparative statics results, which offer economic intuition for decision making. For instance, the optimal subsidy amount and the subsidy target level increase in the expected final-product demand. Thus, when the manufacturer introduces new products (and demand is expected to grow), it should support its suppliers. On the other hand, demand volatility for new products could be high and I show that subsidies should decrease (when the overage costs are higher than the underage cost). Which one of these effects will prevail depends on the parameter values.

The analysis shows that the optimal subsidies increase in the shortfall penalty and the sensitivity of market price to the quantity. One can interpret shortfall penalty as the cost of losing customers to a competitor. Similarly, the sensitivity of price to quantity may be an outcome of oligopolistic competition. Thus, if there are relatively few firms in the market (again, this is true for products at the early stages of their life-cycles; alternatively, this could apply to industries with high entry costs, such as automotive or aerospace), subsidies to the suppliers should be higher than in markets with many firms (e.g. mature markets).

A relationship between the supplier's asset volatility and subsidies is non-monotone. As long as the volatility is not too great (the paper provides a rigorous statement) an increase in the volatility results in greater subsidies. However, if the volatility is already high, further increases will result in lower subsidies. In practice, asset volatility tends to increase when a firm experiences financial difficulties. The observation in this paper tells the managers to support suppliers that are experiencing relatively small financial difficulties, but limit support to suppliers that are in deep trouble.

The results of this paper are applied to analyze Ford's decision to subsidize Visteon in 2005 (see Appendix). The optimal policies implied by the model are close to the actual payments made by Ford to Visteon.

I study the consequences of relaxing assumptions that lead to the separation of ordering and subsidy decisions: independence between supply and demand shocks, no inventory, random capacity, and zero upfront costs. The optimal order quantities increase in the correlation between supply and demand shocks (because the manufacturer takes advantage of the higher demand and the corresponding higher effective supplier capacity). For the model with inventory and backorders, I find that the manufacturer relies on inventory investments as a risk-mitigation tool when the volatility of the supplier's assets is high and uses subsidies when this volatility is low. For the model with proportional random yield, order quantities and subsidies are substitutes as the manufacturer can assure supply either by generous payments to the supplier or by artificially inflating orders. Finally, the optimal order quantities in the model with upfront costs are lower than those in the main model and whether order quantities and subsidies are substitutes or complements depends on the convexity of the manufacturer's cost function. The insights on the effect other problem parameters are consistent with the results from the main model.

A number of generalizations would not change the main insights discussed in this paper. (*i*) If the risk-free interest rate, $r(t)$, follows a stochastic process (e.g., a Gaussian diffusion as in Briys and de Varenne, 1997), the problem becomes computationally more complex, but the insights derived from the current model do not change. (*ii*) One can also make the process for the supplier's assets (1) more general by introducing time-dependent parameters or jumps (as in Zhou, 1997). Two complications arise if one uses a jump-diffusion process to describe the evolution of the supplier's assets. First, a question needs to be addressed whether the "jump" risk is priced by the markets. Second, some of the proofs rely on variable B (shock to the supplier's assets) being an absolutely continuous random variable and would have to be modified to allow for jumps (which can be done). (*iii*) One can add jumps in the supplier's assets between periods (e.g. a random fraction of the supplier's assets is lost). (*iv*) Alternatively, one can assume that the supplier's asset level at the beginning of a period is an increasing concave function of the supplier's assets at the end of the previous period (this could reflect, for example, bankruptcy costs eroding the supplier's asset value). I believe that the results on separation of ordering decisions from subsidy decisions and the structure of the optimal subsidy policy will continue to hold. (*v*) Finally, one can assume that there are other random shocks affecting the supplier's capacity, in addition to the shocks to the supplier's assets. Specifically, the effective supplier's capacity (given by (5d)) can be defined as $K(\xi) = k \min[\xi C, 1]$ where C represents random shocks to the supplier's capacity due to factors independent of the asset-related shock B.

Several reasonable assumptions can be made regarding the *effective capacity* function, K. For example, one could assume that production capacity drops to 0 in case of the supplier's bankruptcy ($\xi < 1$). Specifically, $K(\xi) = k1_{\{\xi \geq 1\}}$, where k is the *regular capacity* level of the supplier. More generally, one could assume that in case of a bankruptcy a random fraction, ν, of the production capacity is lost. That is, $K(\xi) = k\left(1 - \nu 1_{\{\xi < 1\}}\right)$, where ν is independent of shocks to the supplier's financial state, B_n. This model is similar to the random recovery models of corporate-debt defaults. These alternative assumptions are less attractive in practice than the assumptions I employed (because one needs to justify why the loss in the capacity is independent of the magnitude of the drop in the financial health of the supplier). They also less convenient for mathematical analysis. While they allow for the order decision z^* to be independent of the subsidy decision, θ, they make function $EL[Y(z^*, \xi)]$ non-convex in θ, which interferes with the proof of Proposition 2.

Although answers derived using the model in this paper are close to empirical observations, one needs to be careful in applying these results directly, because several important considerations were not accounted for. For example, I did not consider strategic benefits to the manufacturer from offering subsidies to the suppliers. The manufacturer may own stock in the supplier firm and would benefit if the value of that stock did not drop. In addition, as part of the payment arrangements, the supplier may transfer some of its assets to the manufacturer, as was the case with Visteon. If these side benefits are direct, that is their value can be priced, this value can be easily added to the calculations of the optimal subsidy amounts. On the other hand, it is much more difficult to model and price indirect costs of financial subsidies, such as agency costs. For example, it is difficult to price the consequences of the supplier's altering its behavior either by exerting less effort or by investing in riskier projects in anticipation of a bailout from the manufacturer.

The effects of competition among suppliers and among manufacturers, supplier selection problems, the use of diversification, models of market entry and exit, dynamic games, are left, along with agency problems, for future research.

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A Appendix. Technical Results.

A.1 Optimal Ordering Policy

Corollary 1 (Corollary to Proposition 1). *For the linear-demand model (2b) with linear costs, $c(y) = cy$, the optimal order quantity is*

$$z = \frac{E[D] - c/p}{2g}. \tag{24}$$

For the iso-elastic demand model (2c) with linear costs, $c(y) = cy$, the optimal order quantity is

$$z = \left(\frac{E[D](1-g)}{c/p} \right)^{\frac{1}{g}}. \tag{25}$$

Proposition 9. *For the newsvendor model (2a) with the deterministic demand, D, and linear costs, $c(y) = cy$, the optimal order quantity satisfies $z = D$.*

A.2 Optimal Subsidy Policy: Liabilities Reduction Model

In this section we will assume that $\xi_n = \xi_n(A_n\theta, B_n) = q_n(A_n, \theta)B_n$, where $q_n(A, \theta) = \frac{A}{F_n - \alpha^{-1}\theta}$ and that the ordering costs are $c(y) = cy$. $\mathcal{N}$ is the standard normal c.d.f.

Corollary 2 (Corollary to Proposition 3). *For the newsvendor model (2a), for any asset level A, the first order condition (10) becomes*

$$\phi'(\theta) + k\frac{\partial q_n(A,\theta)}{\partial \theta} E\left[\left(c + h1_{\{k\xi_n \geq D\}} - p1_{\{k\xi_n < D\}}\right) 1_{\{z^* \geq k\xi_n\}} B_n \right] = 0. \tag{26}$$

If the demand is a continuous random variable, then the first order condition can be written as

$$\phi'(\theta) - (p+h)k\frac{\partial q_n(A,\theta)}{\partial \theta} E\left[B_n 1_{\{k\xi_n \leq D \leq z^*\}} \right] = 0 \tag{27}$$

or, equivalently,

$$\phi'(\theta) - k\frac{\partial q_n(A,\theta)}{\partial \theta} e^{\mu\Delta}(p+h) \int_0^{z^*} \mathcal{N}\left[d\left(\frac{x}{kq_n(A,\theta)} \right) \right] f_D(x)\, dx = 0, \tag{28}$$

$$d(x) = \frac{\ln(x) - \left(\mu + \frac{\sigma^2}{2}\right)\Delta}{\sigma\sqrt{\Delta}}. \tag{29}$$

Corollary 3 (Corollary to Proposition 3). *For the linear demand model (2b), the first order condition (10) becomes*

$$\phi'(\theta) + k\frac{\partial q_n(A,\theta)}{\partial \theta} E\left\{ B_n[c - pED + 2pgk\xi_n] 1_{\{k\xi_n \leq z^*\}} \right\} = 0 \tag{30}$$

or, equivalently,

$$\phi'(\theta) + k\frac{\partial q_n(A,\theta)}{\partial\theta}\Bigg\{(c - pED)e^{\mu\Delta}\mathcal{N}\left[d\left(\frac{z^*}{kq_n(A,\theta)}\right)\right] + \\ 2pgkq_n(A,\theta)e^{(2\mu+\sigma^2)\Delta}\mathcal{N}\left[d\left(\frac{z^*}{kq_n(A,\theta)}\right) - \sigma\sqrt{\Delta}\right]\Bigg\} = 0, \tag{31}$$

where function d is defined in (29).

Corollary 4 (Corollary to Proposition 3). *For the iso-elastic demand model (2c), the first order condition (10) becomes*

$$\phi'(\theta) + k\frac{\partial q_n(A,\theta)}{\partial\theta}E\left\{B[c - pED(1-g)(k\xi_n)^{-g}]1_{\{k\xi_n \le z^*\}}\right\} = 0 \tag{32}$$

or, equivalently,

$$\phi'(\theta) + k\frac{\partial q_n(A,\theta)}{\partial\theta}\Bigg\{ce^{\mu\Delta}\mathcal{N}\left[d\left(\frac{z^*}{kq_n(A,\theta)}\right)\right] - \\ pED(1-g)(kq_n(A,\theta))^{-g}e^{(1-g)\left(\mu - \frac{g\sigma^2}{2}\right)\Delta}\mathcal{N}\left[d\left(\frac{z^*}{kq_n(A,\theta)}\right) + g\sigma\sqrt{\Delta}\right]\Bigg\} = 0, \tag{33}$$

where function d is defined in (29).

Using numerical algorithms for finding equation roots, one can quickly compute the optimal subsidy amounts from expressions (28), (31), and (33).

For the newsvendor model with deterministic demand, the expressions for the optimal subsidy amounts and the corresponding interperiod cost are even simpler, as presented in the following:

Proposition 10. *Suppose that the demand, D_n, for every period $n = 1, ..., N$ is deterministic, and operating costs are from the newsvendor model. Then the optimal order quantity, $z_n^* = D_n$. If $\frac{\phi'(\cdot)}{q'(A,\cdot)}$ is increasing, the manufacturer's problem (8) is quasi-convex and the optimal subsidy amount, θ^*, is the maximum of zero and the solution of the following equation*

$$pk\frac{\partial q_n(A,\theta)}{\partial\theta}E[B_n 1_{\{k\xi_n \le D_n\}}] = \phi'(\theta) \tag{34}$$

or, equivalently,

$$pk\frac{\partial q_n(A,\theta)}{\partial\theta}e^{\mu\Delta}\mathcal{N}\left[d\left(\frac{D_n}{kq_n(A,\theta)}\right)\right] = \phi'(\theta), \tag{35}$$

where function d is defined in (29). The optimal interperiod cost is

$$\phi(\theta^*) + pE\left[D_n - kq_n(A,\theta^*)B\right]^+ = \\ = \phi(\theta^*) + p\left\{D_n\mathcal{N}\left[d\left(\frac{D_n}{kq_n(A,\theta^*)}\right) + \sigma\sqrt{\Delta}\right] - e^{\mu\Delta}kq_n\left(A,\theta^*\right)\mathcal{N}\left[d\left(\frac{D_n}{kq_n(A,\theta^*)}\right)\right]\right\}. \tag{36}$$

For the newsvendor model with deterministic demand, because the optimal capacity order is $z^* = D_n$ parameter h does not affect the solution.

B Appendix. Proofs.

Proof of Proposition 1.

For any θ, n, and A,

$$\frac{\partial J_n(A,z,\theta)}{\partial z} = \frac{\partial}{\partial z} EL[\min(z,K)] = E\left[L'(z)1_{\{z\leq K\}}\right] = L'(z)E\left[1_{\{z\leq K\}}\right]. \tag{37}$$

When $z > k$, $E\left[1_{\{z\leq K\}}\right] = 0$ and, therefore, $\frac{\partial J_n(A,z,\theta)}{\partial z} = 0$. For $z \leq k$, $E\left[1_{\{z\leq K\}}\right]$ is decreasing in z and because $1 - F_B(z) > 0$ for all $z \geq 0$, $E\left[1_{\{z\leq K\}}\right] = E\left[1_{\{z\leq k\xi\}}\right] > 0$. Let $\hat{z}$ be the solution of $L'(z) = 0$. If $\hat{z} \geq k$, because function L is convex, for all $z < k$, $\frac{\partial J_n(A,z,\theta)}{\partial z} < 0$. Therefore, the optimal order quantity is $z^* = k$. If $\hat{z} < k$, compute the second derivative of J_n with respect to z obtaining

$$\left.\frac{\partial^2 J_n(A,z,\theta)}{\partial z^2}\right|_{z=\hat{z}} = L''(\hat{z})E\left[1_{\{\hat{z}\leq K\}}\right] > 0. \tag{38}$$

Therefore, function $J_n(A,\cdot,\theta)$ is unimodal, with the lowest value at $z^* = \hat{z}$. ∎

Proof of Lemma 1.

In the following I will omit subindex n. Using $Y(z,\xi) = \min(z, K(\xi))$, $K(\xi) = k\xi$, and $\xi = \xi(A,\theta,B)$, the first derivative of function ψ is

$$\psi'(\theta) = E\left[L'(k\xi)k\frac{\partial \xi}{\partial \theta}1_{\{k\xi<z^*\}}\right]. \tag{39}$$

Because function L is convex with the minimum at z^*, for any $x < z^*$, $L'(x) \leq 0$. Therefore, if ξ is increasing in θ, then $\psi'(\theta) \leq 0$. The second derivative of function ψ is

$$\begin{aligned}\psi''(\theta) = E&\left\{L''(k\xi)\left(k\frac{\partial \xi}{\partial \theta}\right)^2 1_{\{k\xi<z^*\}}\right\} + \\ E\left\{L'(k\xi)k\frac{\partial^2 \xi}{\partial \theta^2}1_{\{k\xi<z^*\}}\right\} &+ E\left\{L'(k\xi)k\frac{\partial \xi}{\partial \theta}\left[-\frac{\partial \xi_B^{-1}(A,\theta,z/k)}{\partial \theta}\right]\delta_{\{k\xi=z^*\}}\right\},\end{aligned} \tag{40}$$

where δ is the Dirac's delta function and $\xi_B^{-1}(A,\theta,\cdot)$ is the inverse of function $\xi(A,\theta,\cdot)$, which is assumed to be increasing. Because function L is convex, the first term in the above expression is positive. If $\xi(A,\cdot,b)$ is concave for all A and b, the second term in the above expression is positive as well. The third term is equal to zero, because $L(z^*) = 0$. Therefore, $\psi''(\theta) \geq 0$. ∎

Proof of Proposition 2.

The proof follows the standard inductive approach. First, observe that $V_{N+1}(A) = 0$ is convex. Then, assume that V_{n+1} is convex. Consequently, $EV_{n+1}(aB)$ is convex in a. From Lemma 1, $\psi_n(a) = EL[Y(z^*,\xi_n)]$ is convex in a. By assumption, ϕ is convex. Therefore, $H_n(A,\cdot)$ is convex for all A. It follows that there exists a unique target level, S_n, to which one would like to bring the supplier's assets, if possible. This target level, in general, depends on the current level of the supplier's assets, A, due to the term $\phi(a - A)$.

Because variables A and a appear together only in the term $\phi(a - A)$, function H_n is jointly convex in (A, a). From Proposition B-4 in Heyman and Sobel (2004), it follows that function V_n is convex. This concludes the inductive step. ■

Proof of Proposition 3.

Consider $\xi_n(A, \theta, B) = q_n(A, \theta)X_n(B_n)$. For any period n, the first derivative of the objective function with respect to θ is (I omit subindex n)

$$\phi'(\theta) + k\frac{q(A,\theta)}{\partial\theta} E\left\{X(B)L'[k\xi(A,\theta,B)]1_{\{k\xi(A,\theta,B)\leq z^*\}}\right\}. \tag{41}$$

The first order condition is

$$-kE\left\{X(B)L'[k\xi(A,\theta,B)]1_{\{k\xi(A,\theta,B)\leq z^*\}}\right\} = \phi'(\theta)/\left(\frac{\partial q(A,\theta)}{\partial\theta}\right). \tag{42}$$

Left-hand side of this equation decreases in θ, because L is convex, $X(B) > 0$, and $q(A, \cdot)$ is increasing. Therefore, to guarantee the uniqueness of the solution of the first order condition, it is sufficient that $\frac{\phi'(\cdot)}{q'(A,\cdot)}$ be increasing. The proof for $\xi_n(A, \theta, B) = q_n(A, \theta) + X_n(B_n)$ is similar. ■

Proof of Proposition 4.

Part (i). Using definition (9) observe that $\frac{\partial^2 H_n(A,a)}{\partial A \partial a} = \frac{\partial^2 \phi(a-A)}{\partial A \partial a} < 0$. Therefore, S_n, which minimizes convex function $H_n(A, \cdot)$ increases in A. So does $a_n^* = \max(S_n, A)$.

Part (ii). By induction, suppose that $V_k(\cdot)$ is decreasing for all $k > n$. Let $A_1 \leq A_2$. Consider

$$V_n(A) = \begin{cases} H_n(A, S_n) & \text{if } A < S_n, \\ H_n(A, A) & \text{otherwise,} \end{cases} \tag{43}$$

where function H_n is defined in (9). If $S_n(A_1) \leq A_1$ and $S_n(A_2) \leq A_2$ (note that, from Proposition 4, $S_n(A_1) \leq S_n(A_2)$), from the induction assumption and Lemma 1, it follows that $V_n(A_1) = H_n(A_1, A_1) > H_n(A_2, A_2) = V_n(A_2)$.

If $A_1 \leq S_n(A_1)$ and $A_2 \leq S_n(A_2)$, then $V_n(A_1) = H_n(A_1, S_n(A_1)) \geq H_n(A_2, S_n(A_1)) \geq H_n(A_2, S_n(A_2)) = V_n(A_2)$.

If $S_n(A_1) \leq A_1$ and $A_2 \leq S_n(A_2)$, then $V_n(A_1) = H_n(A_1, A_1) \geq H_n(A_2, A_2) \geq H_n(A_2, S_n(A_2)) = V_n(A_2)$.

If $A_1 \leq S_n(A_1)$ and $S_n(A_2) \leq A_2$, then $V_n(A_1) = H_n(A_1, S_n(A_1)) \geq H_n(A_2, S_n(A_1)) \geq H_n(A_2, A_2) = V_n(A_2)$. ■

Proof of Lemma 2.

Let's rewrite DP recursion equation (9), accounting explicitly for the effect of parameter π:

$$V_n(A;\pi) = \min_{a\geq A} \{\phi(a-A) + EL[\min(z^*(\pi), k\xi_n);\pi] + \alpha EV_{n+1}(aB;\pi)\}. \tag{44}$$

Define $H_n(A,a;\pi) = \phi(a-A) + EL[\min(z^*(\pi), k\xi_n);\pi] + \alpha EV_{n+1}(aB;\pi)$. By induction, $V_{n+1}(A;\pi)$ is submodular. Then

$$\frac{\partial}{\partial a}H_n(A,a;\pi) = \phi'(a-A) + E\left[\frac{\partial}{\partial y}L(k\xi_n;\pi)1_{\{k\xi_n\leq z^*(\pi)\}}k\frac{\partial \xi_n}{\partial a}\right] + \alpha E\left[B\frac{\partial}{\partial A}V_{n+1}(aB;\pi)\right].$$

$$\begin{aligned}\frac{\partial^2}{\partial a\partial\pi}H_n(A,a;\pi) = &\underbrace{E\left[\frac{\partial^2}{\partial y\partial\pi}L(k\xi_n;\pi)1_{\{k\xi_n\leq z^*(\pi)\}}\frac{\partial \xi_n}{\partial a}\right]}_{\leq 0} + \\ &\underbrace{E\left[\frac{\partial}{\partial y}L(k\xi_n;\pi)\delta_{\{k\xi_n = z^*(\pi)\}}\frac{d}{d\pi}z^*(\pi)\frac{\partial \xi_n}{\partial a}\right]}_{=0} + \underbrace{\alpha E\left[B\frac{\partial^2}{\partial A\partial\pi}V_{n+1}(aB;\pi)\right]}_{\leq 0} \leq 0.\end{aligned}$$

Therefore, the optimal subsidize-up-to supplier's asset level, S_n, increases in π.

In addition, observe that $\frac{\partial^2}{\partial A\partial\pi}H_n(A,a;\pi) = 0$ and $\frac{\partial^2}{\partial A\partial\pi}H_n(A,a;\pi) = -\phi''(a-A) \leq 0$. Because $a \geq A$ is a lattice, it follows from Theorem 8-3 in Heyman and Sobel (2004) that $V_n(A;\pi)$ is submodular and the induction step is complete. ■

Proof of Proposition 5.

For the newsvendor model (2a), $\frac{\partial^2}{\partial y\partial p}L \leq 0$, $\frac{\partial^2}{\partial y\partial(-h)}L \leq 0$, and, for normal demand, $\frac{\partial^2}{\partial y\partial\mu_D}L \leq 0$. Therefore, from Lemma (2), it follows that S_n is increasing in p, decreasing in h, and, for normal demand, increasing in μ_D. Proofs of statements for the linear and iso-elastic demand models are similar.

A slightly more involved analysis is needed to prove that S_n decreases in σ_D. Observe that for the newsvendor model with normal demand, $L(y;\sigma_D) = cy + E[h(y-\mu_D-\sigma_D\epsilon)^+ + p(\mu_D+\sigma_D\epsilon-y)^+]$, where ϵ is the standard normal random variable. Function $L(\,\cdot\,;\sigma_D)$ need to be defined only for $y \leq z^*$, because it will have an input of $\min(z^*, k\xi)$. Because $\frac{p-c}{p+h} \leq \frac{1}{2}$, $z^* = \mu_D + \sigma_D\mathcal{N}^{-1}\left(\frac{p-c}{p+h}\right) \leq \mu_D$. Therefore,

$$\frac{\partial^2}{\partial y\partial\sigma_D}L(y;\sigma_D) = -(h+p)\cdot n\left(\frac{y-\mu_D}{\sigma_D}\right)\cdot\frac{y-\mu_D}{\sigma_D^2} \geq 0. \tag{45}$$

In the expression above, n is the standard normal p.d.f. ■

Proof of Proposition 6.

Rewrite equation (28) as follows:

$$\int_0^{z^*} \mathcal{N}\left[d\left(\frac{x}{kq(A,\theta)}\right)\right] f_D(x)\,dx = \frac{\phi'(\theta)}{kq'_\theta(A,\theta)e^{\mu\Delta}(p+h)}. \tag{46}$$

Recognizing that θ, which satisfies this equation, is a function of σ, take the derivative of both sides of this equation with respect to σ.

$$\begin{aligned}&\int_0^{z^*} n\left[d\left(\frac{x}{kq(A,\theta)}\right)\right]\frac{\left[\left(\mu-\frac{\sigma^2}{2}\right)\Delta-\ln\left(\frac{x}{kq(A,\theta)}\right)\right]kq(A,\theta)-\sigma kq'_\theta(A,\theta)\theta'}{kq(A,\theta)\sigma^2\sqrt{\Delta}}f_D(x)\,dx=\\&\frac{\partial}{\partial\theta}\left(\frac{\phi'(\theta)}{kq'_\theta(A,\theta)e^{\mu\Delta}(p+h)}\right)\theta',\end{aligned}\tag{47}$$

where n is the p.d.f. of the standard normal random variable. Combining together terms with θ', we obtain

$$\begin{aligned}&\int_0^{z^*} n\left[d\left(\frac{x}{kq(A,\theta)}\right)\right]\frac{\left[\left(\mu-\frac{\sigma^2}{2}\right)\Delta-\ln\left(\frac{x}{kq(A,\theta)}\right)\right]}{\sigma^2\sqrt{\Delta}}f_D(x)\,dx=\\&\left[\int_0^{z^*} n\left[d\left(\frac{x}{kq(A,\theta)}\right)\right]\frac{kq'_\theta(A,\theta)}{kq(A,\theta)\sigma\sqrt{\Delta}}f_D(x)\,dx+\frac{\partial}{\partial\theta}\left(\frac{\phi'(\theta)}{kq'_\theta(A,\theta)e^{\mu\Delta}(p+h)}\right)\right]\theta'.\end{aligned}\tag{48}$$

Because q and $\frac{\phi'(\cdot)}{q'(A,\cdot)}$ are increasing in θ, the coefficient in front of θ' on the right hand side of the equation above is positive. The expression on the left hand side is positive if $\left(\mu-\frac{\sigma^2}{2}\right)\Delta\geq\ln\left(\frac{z^*}{kq(A,0)}\right)$, because $\xi\leq z^*$ and q is increasing in θ. ■

Proof of Lemma 3.

Derivative of J_n with respect to z is

$$\frac{\partial J_n(A,z,\theta)}{\partial z}=\phi'(c_0(z)+\theta)c_0'(z)+L'(z)\Pr[z\leq k\xi]\tag{49}$$

Because ϕ and c_0 are increasing, and $L'(z)>0$ for any $z>z^0$, it follows that $\frac{\partial J_n(A,z,\theta)}{\partial z}>0$ for any $z\geq z^0$. ■

Proof of Proposition 8.

Observe that $V_{N+1}(A)=0$ is convex. By induction, suppose V_{n+1} is convex. Then $EV_{n+1}(aB_n)$ is convex in (A,z,a).

Observe that $\min(z,k\xi_n)$ is concave in z and a. Function L is convex and for $y<z^0$, $L(y)$ is decreasing. Therefore, $EL(\min(z,k\xi_n))$ is convex in z and a. It follows that $J_n(A,z,a)=\phi[a-A+c_0(z)]+EL(\min(z,k\xi_n))+\alpha EV_{n+1}(aB_n)$ is convex in (A,z,a). Set $a\geq A$ and $0\leq z\leq z^0$ is convex. Therefore, by Proposition B-4 in Heyman and Sobel (2004), it follows that function $V_n(A)$ is convex. This concludes the inductive step.

From Lemma 3 we know that $z^*<z^0$ for all a. Therefore one needs to be concerned only with boundaries $z=0$ and $a=A$. KKT conditions (23) follow. ■

C Appendix. Ford vs. Visteon

To illustrate how models, discussed in this paper, can be applied in a realistic setting, in this section I study Ford's decision to give subsidy to Visteon in 2005. As was discussed in the introduction, Visteon Corporation considered filing for bankruptcy in May 2005. However, the bankruptcy was averted on May 24th after Ford Motor Company, which is Visteon's largest customer, promised to pay Visteon between \$1.6 and \$1.8 billion dollars and take over 24 of Visteon's plants, inventory of parts, and 17,400 workers (see White, 2005). Not all of this money was to be paid at once. Ford paid \$300 million for inventory immediately and lent Visteon \$250 million to meet its short-term debt payments. The rest of the money was to be paid from an escrow account over time, while Visteon undergoes restructuring, at approximately \$500 million per year.

To apply our models, we need to estimate the customer demand, the operating cost function, $\mathcal{L}$, and subsidy costs, ϕ, for Ford. We also need to calibrate the stochastic process for Visteon's assets as well as Visteon's production capacity.

Let's begin with Ford's operating cost function. Assume that a duration of the decision period $\Delta = 1$ year. Competition in the automotive industry is intense. Therefore, it is reasonable to assume that Ford is a price-taker, rather than a price-setter and use a newsvendor model (2a) for the Ford's operational cash flows. According to Wikipedia, Ford manufactured 6,418,000 cars in 2005. We have no other information about the demand distribution. For simplicity, let's assume that the demand for Ford for one year is deterministic and equal to $D = 6.4$ million cars. Based on the income statement for Ford for 2005, the gross profit (= revenues − costs of goods sold) was approximately \$5,000 per car. The shortage penalty, $\hat{p}$, which reflects goodwill losses, expediting and outsourcing costs in case the supply does not meet the demand, is difficult for us, Ford outsiders, to estimate (Ford managers may have some beliefs about it). Let's assume that a shortage penalty offsets tax charged on gross profit and $p = \$5,000$. Holding cost does not play a role in the deterministic demand model.

Next let's turn to estimating Visteon's parameters. In the middle of May 2005, when Ford executives were contemplating a subsidy to Visteon, 12-month LIBOR rates were quoted at 3.74%. I will use 3.7% (continuously compounded) as both the risk-free rate, r, and the drift, μ, of the supplier's asset process. To compute A and σ I use an approach proposed in Bharath and Shumway (2004). Specifically,

$$D = F, \quad A = E + D, \quad \sigma_D = 0.05 + 0.25\sigma_E, \quad \sigma = \frac{E}{E+D}\sigma_E + \frac{D}{E+D}\sigma_D. \tag{50}$$

From the 2004 annual report, the face value of Visteon's current liabilities (due within a year), as of December 31, 2004, was $F = \$3.84$ billion (Ford executives must have had a more precise estimate of Visteon's liabilities in May 2005). The stock prices for Visteon in the middle of May were between

\$3.50 and \$5 per share. As of December 31, 2004, there were 125 millon shares outstanding. This translates, approximately, to \$0.5 billion of total equity value. The implied volatility of Visteon's equity is $\sigma_E = 55\%$ (based on option prices in January 16, 2007). Thus,

$$D = F = \$3.84 \text{ billion}, \quad A = \$4.34 \text{ billion}, \tag{51a}$$

$$\sigma_D = 0.05 + 0.25\sigma_E = 0.05 + 0.25 \times 0.55 \approx 0.19, \tag{51b}$$

$$\sigma = \frac{E}{E+D}\sigma_E + \frac{D}{E+D}\sigma_D = \frac{0.5}{4.34}0.55 + \frac{3.84}{4.34}0.19 \approx 0.23. \tag{51c}$$

The regular capacity level $k = D$ (that is, unless Visteon is bankrupt, it should be able to meet Ford's deterministic demand).

Example 5. First, consider the model where a subsidy from Ford is used by Visteon to reduce its liabilities. I will model the cost of subsidies to Ford as $\phi(\theta) = \frac{\mathcal{A}}{\mathcal{F}-\theta} - \frac{\mathcal{A}}{\mathcal{F}}$, where $\mathcal{F} = F = 3.84$ billion and $\mathcal{A} = \mathcal{F}^2$.

Using these numbers we compute that, according to our model, the optimal subsidy from Ford to Visteon in 2005 should be $\theta^* = \$393$ million. This number is well below the total subsidy of \$1.6 – \$1.8 billion that Ford promised Visteon. There are several reasons for this discrepancy.

First, one can only guess the magnitude of the shortage penalty or Visteon's asset volatility (as well as other parameters) in our model. When making the decision to bail out Visteon, Ford managers had a much better idea how much the shortage would cost the company. Also, I computed implied volatility based on 2007 data. It could have been different in 2005. Figure 7 illustrates sensitivity of the optimal subsidy to the shortage penalty, p, and supplier's asset volatility, σ. Figure 8 illustrates the sensitivity of the optimal subsidy to the initial level of the supplier's assets, $A(0)$, and the supplier's liabilities, F. When drawing the graph for the sensitivity to the supplier's liabilities, I assumed that $A(0) = F + \$0.5$ billion. Futhermore, one can only guess the functional form of the operating costs. Fortunately, the majority of the results in this paper are true for any convex function L, representing the operating costs. Therefore, one could easily recompute the optimal subsidy if a more realistic operating cost function were available.

Second, the amount \$1.6 – \$1.8 billion is paid over a number of years, and I computed the optimal subsidy amount for the first year only. The optimal subsidy of \$393 million from our model is comparable with payments made by Ford to Visteon in 2005 only. Subsidy amounts for subsequent years will be computed in our model based on supplier's asset levels A_n at the beginning of each year.

Third, the only benefit to the manufacturer in our model from subsidizing its supplier is a more financially stable supplier. In real life, in addition to improving Visteon's financial condition, Ford received some inventory, 24 plants, and other assets in return for the subsidy, which would inflate the subsidy amount Ford is willing the give. This concludes the example.

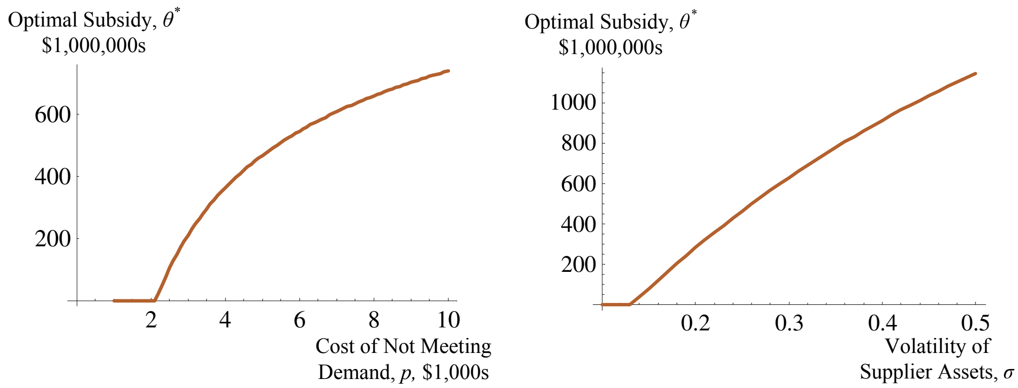

Figure 7: Sensitivity of the optimal subsidy, θ^*, to shortage penalty p and supplier's asset volatility σ (see Ford vs. Visteon example for the numerical values of other model parameters)

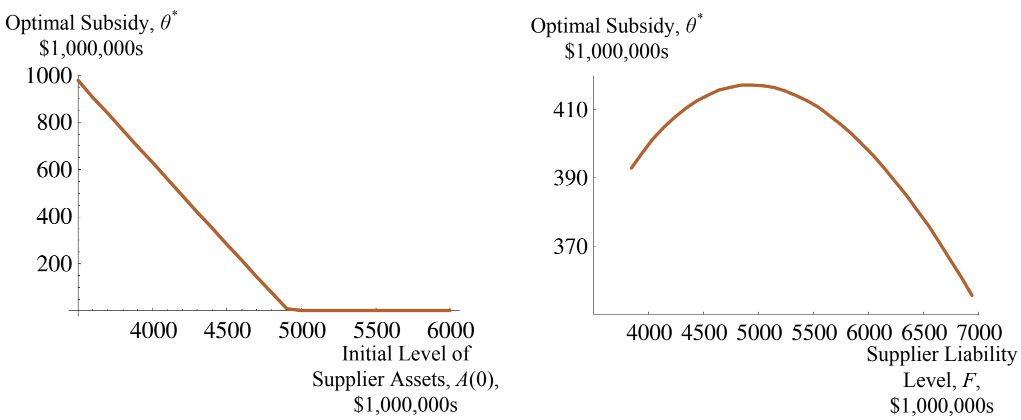

Figure 8: Sensitivity of the optimal subsidy, θ^*, to the initial level of the supplier's assets $A(0)$ and the supplier's liabilities F. For the figure of sensitivity to the supplier's liabilities it was assumed that $A(0) = F + \$0.5$ billion (see Ford vs. Visteon example for the numerical values of other model parameters)

Example 6. Next, let's consider a model where the supplier's assets are increased by the manufacturer's subsidies. Let the subsidy cost be $\phi(\theta) = \theta$.

Model (9) has several desirable numerical properties. First, the expected operating costs $EL[Y(z^*, \xi)]$ depend only on the action a and can be pre-computed. Second, the value function depends on action, a, only, which speeds up the computations of the expected future value. Third, with the linear assumption on the subsidy function ϕ, the optimization problem every period does not depend on the state, A, of the problem (according to Proposition 2). In particular, the target asset levels, S_n, do not depend on the state, A. Fourth, when solving optimization problem each period, one can expedite the search for the optimal action using monotonicity results from Proposition 4.

Suppose that the length of the planning horizon for Ford is 5 years. Figure 9 presents target asset levels, S_n, for Visteon's assets for $n = 1, ..., 5$ and for different values of supplier's asset volatility,

σ. Figure 9 shows that the target supplier's asset levels decrease as the manufacturer approaches

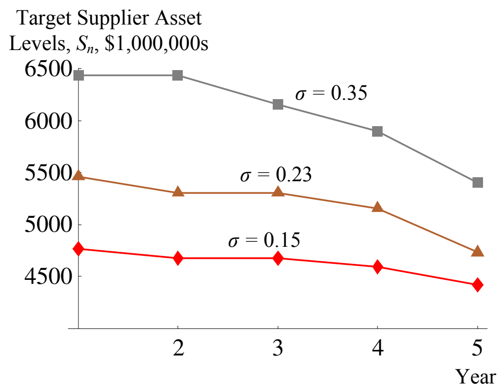

Figure 9: Target supplier's asset levels, S_n, as function of volatility of the supplier's assets, σ and year, n (see Ford vs. Visteon example for the numerical values of other model parameters)

the end of the five-year planning horizon. As we have seen already in the one-period model, higher volatility may result in higher subsidies. The middle curve in the figure, corresponding to $\sigma = 0.23$ is the base-case for the model where the supplier uses subsidies to reduce current liabilities. For this base-case model, the subsidy amounts, computed as the difference between target levels S_1 and the initial supplier's assets, $A(0)$, is approximately \$1.2 billion. The larger subsidy amount, compared with the number from the previous example, is due to the assumption that Ford's cost of paying financial subsidies is linear, $\phi(\theta) = \theta$. This concludes the example.