REAL OPTIONS IN PUBLIC PRIVATE PARTNERSHIP – CASE OF A TOLL ROAD CONCESSION

Frances F. Blank*, Tara K. N. Baidya†, Marco A. G. Dias‡

*Dept. of Industrial Engineering, PUC-Rio,
R. Toneleiros 30 / 504 - Rio de Janeiro, Brazil
Phone #: (55-21) 9766-2281
e-mail: frances.blank@gmail.com

† Dept. of Industrial Engineering, PUC-Rio,
Rua Marquês de São Vicente, 225 - Rio de Janeiro, Brazil
Phone #: (55-21) 3527-1284 or (55-21) 3527-1285
e-mail: baidya@ puc-rio.br

‡ Adjunct Professor Dept. of Industrial Engineering, PUC-Rio
& Senior Consultant, Petrobras/Exploration & Production
Rua Marquês de São Vicente, 225 - Rio de Janeiro, Brazil
Phone #: (55-21) 3527-1284 or (55-21) 3224-2164
e-mail: marcodias@petrobras.com.br

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ABSTRACT

Governments around the world have been encouraging private investments in infrastructure through Public Private Partnership (PPP) framework. In the transport sector, for example, project finance and PPP are largely used in toll road concessions. The PPP agreements may include subsidies, guarantees and other forms of support designed to reduce the risk to the private investor. Some real options can be identified in these structures and it is necessary to use the correct methodology to analyze project economic feasibility and risk allocation. Regarding the revenue risk in transport projects, different models of guarantees have been proposed. In Brazil, the 4th Line of the Metro of São Paulo is the first example of a PPP implementation and the mechanism used to mitigate the demand risk was based on minimum and maximum levels of demand. As an example of application, a hypothetical toll road concession is modeled and three real options are proposed and analyzed: a minimum traffic guarantee, a maximum traffic ceiling and an implicit option to abandon.

Keywords: Real options; public private partnership; PPP; project finance; toll road concession; government guarantees; option to abandon

INTRODUCTION

Recently Public Private Partnership (PPP) is being used as an important tool of financial engineering. The objective of this mechanism is to create conditions so that the private sector can participate in the construction and operation of public projects, which look infeasible in the first instance. Governments can make such projects viable by offering guarantees and subsidies under certain conditions and offering tax incentives under other conditions.

The government is likely to make better decisions about guarantees and subsidies when its advisers have an overview for judging if support is justified, when they know how to estimate the cost and when they evaluate carefully the costs and benefits in each situation (Irwin, 2007). In most cases, real options theory is required.

These guarantees and subsidies have been used in many countries, especially in transport concessions, but deciding on this course of action is not an easy task. One of these mechanisms is the guarantee involving minimum and maximum levels of traffic or revenue which has been proposed and valued by many authors in different ways, using analytical solutions, binomial tree methods and the Monte Carlo Simulation (Brandão and Cury, 2006; Galera, 2006; Brandão and Saraiva, 2007; Wibowo, 2004; Huang and Chou, 2005; Cheah and Liu, 2006; Chiara, Garvin and Vecer, 2007; Irwin, 2003; Irwin, 2007), as described in Blank (2008).

An additional benefit of the proposed guarantee is to minimize an implicit abandonment option. In high leveraged projects involving project finance structures, the concessionaire could decide to pay the debt service or to abandon the project in each period (Pollio, 1998). In this case, the government should look at the guarantee option additionally to minimize the probability of abandonment.
This work presents a composition of a minimum traffic guarantee and a maximum traffic limit with different bands of protection, based on a real PPP case in Brazil, and proposes the existence of an implicit right of abandonment by the sponsors.

LITERATURE REVIEW

Private participation in infrastructure projects has been sought by governments around the world, especially in emerging economies. These projects involve large amount of investment and are often limited recourse asset-based financed through project finance structures. Both parties, private and public sector, are concerned with the project viability. But while the private sector is primarily interested in the project profitability, the public sector is also pursuing social benefits.
The project can be financially attractive to private investors or not. Its profitability, and consequently its feasibility, is subject to some specific risks associated with infrastructure projects. Governments, concerned with the mitigation of these risks, may offer incentives as guarantees to attract private capital. Considering all these characteristics, the use of real options tools is important to assess the project’s correct value.
According to Pollio (1998), the use of project finance can be strategically explained within an option framework. Under this approach, the project completion risks are transferred from sponsors to lenders. An additional flexibility will affect the project value, since the sponsors have an option, and not the obligation, to repay the loan.
Real option theory is largely used in the literature on infrastructure projects, especially in transportation projects such as toll road concessions. Brandão (2002) values the options to abandon and to expand in the Via Dutra, a toll road concession in Brazil, using a discrete methodology. Garvin *et al* (2002) applied real option valuation on a model of the Dulles Greenway, a toll road in Virginia, USA, to incorporate the option of waiting to build the highway limited to five years. Bowe and Lee (2004) analyze the Taiwan High-Speed Rail project, the construction and operation of the rail system embodying multiple interacting flexibilities, involving the option to defer or postpone construction, the option to abandon early in the construction phase, the options to expand or to contract and the option to abandon or switch use at any time. Zhao, Sundararajan and Tseng (2004) model a highway system focusing on real options of expansion and rehabilitation. The valuation proposed requires quantitative models of uncertainties such as demand, costs and land availability. Wei-hua and Da-shuang (2004) propose a concession decision model with three real options embedded: the option to adjust concession price, the option to develop surrounding land and the option to expand capacity. The key risks considered in the decision process were demand risk, inflation risk, land price risk and completion risk.
When the profitability of the project is weak, governments have been using some mechanisms to mitigate risks that adversely impact the return to the private sector. The use of these instruments makes such projects feasible and attracts private capital.
In transport concessions, some benefits have real options characteristics. If no effort is made to correctly quantify them, governments may be providing an unnecessarily large subsidy or sponsors will be disregarding the project’s correct return (Chiara, Garvin and Vecer, 2007). The main risk factor is related to demand or traffic, which is difficult to estimate, and there

are some mechanisms that permit a mitigation and reallocation of this risk. Such instruments can be classified according to three criteria: the trigger variable chosen – which can be traffic, revenues or even IRR; the risk allocation between the parties, involving sometimes minimum and maximum target levels for the trigger variable; or the compensation mechanism adopted, including a subsidy or a change in contract length (Vassallo, 2006). Given these criteria, three main approaches adopted around the world should be especially mentioned. The first approach emphasizes the economic balance of the concession through the IRR, establishing usually acceptable levels for this variable. The second is based on guarantees of traffic or revenues, where the risk is shared between the government and the concessionaire, since minimum and maximum bands are usually considered. The third is related to the length of the contract which should match the moment when a target variable is achieved.

As examples of benefits, Rose (1998) and Alonso-Conde *et al* (2007) analyze the Melbourne CityLink Project, a toll road in Australia. Contractually, two agreements can be identified as interacting options embedded in the project. The first one is that the government has the right to terminate the project before the end of the concession term if investor's IRR is greater than a certain agreed value. The second is the option the investors have to defer the payment of the concession fee to the government under certain conditions. Wibowo (2004) takes an Indonesian toll road project as a case study to analyze the financial impact of different kind of guarantees provided by the government. The guarantees discussed are revenue, traffic, tariff, debt and maximum interest rate guarantees.

The benefit to be focused on this paper is the minimum demand guarantee or, in a toll road project, minimum traffic or revenue guarantee. Irwin (2003) examines some types of support provided by governments, including guarantees of risks not under the government's control, such as the risk of future demand for public services provided. Such guarantees are similar to put options and should be correctly valued using option-pricing techniques, as the government of Colombia did in the mid 1990s to value this option in the case of the El Cortijo–El Vino toll road. He also proposes the valuation of revenue guarantees using the concept of the market price of revenue risk. Chiara, Garvin and Vecer (2007) propose a new approach for revenue guarantees, considering that the exercise dates are determined during the operational phase. Huang and Chou (2005) value a minimum revenue guarantee, an option to abandon during the construction phase and the interaction among them, using an analytical method. They use Taiwan High Speed Rail Project as a numerical case. Cheah and Liu (2006) analyze the minimum revenue guarantee in Malaysia-Singapore Second Crossing, but the option was modeled on the cash flow. In this case, they also considered the payment to the government of revenue in excess if cash flow lies above a certain level. Brandão and Cury (2006) propose a hybrid model to BR-163, a Brazilian toll road, incorporating a minimum revenue guarantee and also the payment of revenue in excess if traffic is above a certain level. They use a discrete method and the options are modeled also on the cash flow. Brandão and Saraiva (2007) analyze the same project but the options are modeled directly on traffic levels and valued using the Monte Carlo simulation. Galera (2006) develops an analytical model to price different real options for highways concessions in Spain. He also values a minimum and a maximum traffic level option, using Black, Scholes and Merton's formula. To estimate the parameters, he used a historical series of existing concessions in Spain. Such as Irwin (2003), Brandão and Saraiva (2007) and Galera (2006) also use the concept of market price of the underlying asset (traffic or revenue).

PPP IN BRAZIL

In Brazil, the relevance of PPP is related to the infrastructure deterioration and the scarcity of public resources to invest. The Brazilian Federal Law 11079, approved in 2004, defines PPP as a supported concession (Brazil, 2004). There is also the local legislation regulating the PPP in the states.

The 4th Line of the Metro of São Paulo is the first example of a PPP implementation in Brazil. A contract was signed in November 2006 with a consortium led by CCR – Companhia de Concessões Rodoviárias, a toll road company in Brazil and one of the major private toll road concession groups in Latin America. The project involves a 30-year concession to operate a 12.8 km stretch of subway in São Paulo, the biggest city in Brazil. The investment made by the consortium will be US$ 340 million.

The mechanism used to mitigate the demand risk in the above mentioned PPP is based on minimum and maximum levels of demand. There is a range of demand without protection (up to ±10% of the projected demand). Then there will be two bands of protection (the first between ±10% and ±20% of the projected demand and the second after ±20% of the projected demand, limited to ±40% of the projected demand). There are two lower levels – or floors – and two upper levels – or ceilings – for the traffic involving payments from the government to the concessionaire or from the concessionaire to the government (SÃO PAULO, 2006)

Considering the same fee for all consumers, the mechanism can be described as following. Let D_i be the real demand in period *i*; $\overline{D}_i$ be the projected demand in period *i* and p the tariff for the consumer. Then:

- If the real demand lies between 90% and 110% of the projected demand, there will be neither subsidy nor taxation.

- If the real demand lies between 80% and 90% of the projected demand, the revenue will be adjusted by the following formula:

  $$Md = [0.6\ (0.9\overline{D}_i - D_i)]p \qquad (1)$$

  In this range, the government gives a protection of 60%. The revenue will be complemented by 60% of what lacks for 90% of the projected demand.

- If the real demand lies below 80% of the projected demand, the revenue will be adjusted by the following formula:

  $$Md = \{0.06\overline{D}_i + [0.9\ (0.8\overline{D}_i - D_i)]\}p \qquad (2)$$

  In this range, the government gives a protection of 90%. The revenue will be complemented by 90% of what lacks for 80% of the projected demand, considering the previous level.

- If the real demand lies between 110% and 120% of the projected demand, the revenue will be adjusted by the following formula:

  $$Md = -[0.6\ (D_i - 1.1\overline{D}_i)]p \qquad (3)$$

  In this range, the concessionaire pays the government 60% of what exceeds 110% of the projected demand.

- If the real demand lies above 120% of the projected demand, the revenue will be adjusted by the following formula:

  $$Md = -\{0.06\overline{D}_i + [0.9\ (D_i - 1.2\overline{D}_i)]\}p \qquad (4)$$

In this range, the concessionaire pays the government 90% of what exceeds 120% of the projected demand, considering the previous level.

- If the real demand lies below 60% or above 140% of the projected demand, the economic balance should be re-established.

Considering hypothetical demand, the situation can be represented as in Figure 1.

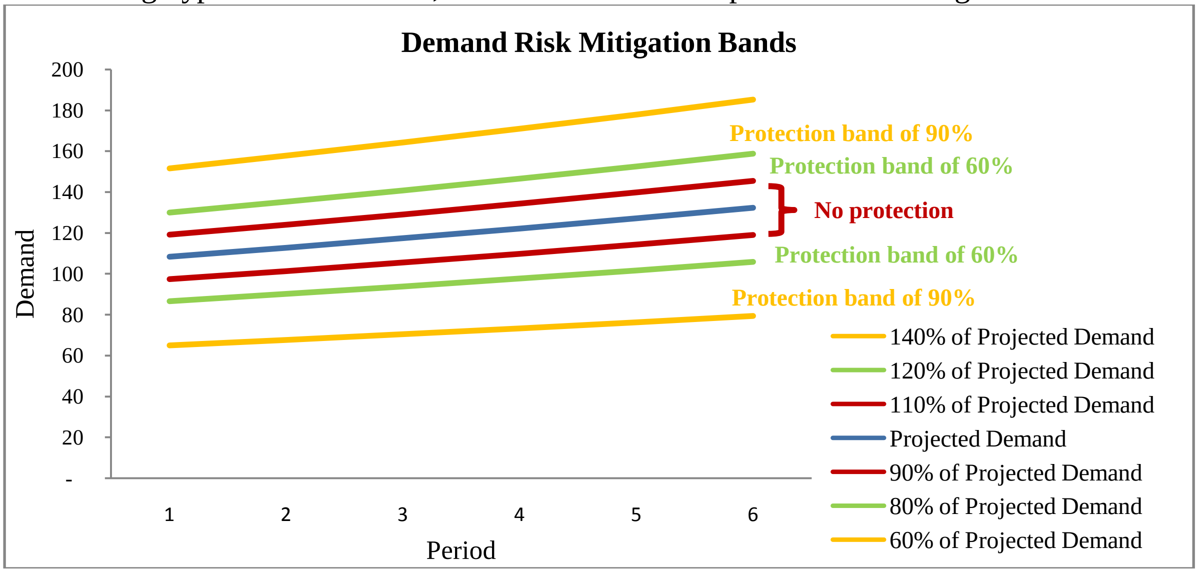


Figure 1: Demand Risk Mitigation Bands

Such conditions can be modeled as a composition of put and call options.
If demand lies below 90% of projected demand, the concessionaire has two puts that can be simultaneously exercised depending on real demand. Their payoffs in each period are:

$$Put1: Md_1 = [0.6\,max(0.9\overline{D}_i - D_i, 0)]p \quad (5)$$
$$Put2: Md_2 = [0.3\,max(0.8\overline{D}_i - D_i, 0)]p \quad (6)$$

If demand lies above 110% of projected demand, the government has two calls that can be simultaneously exercised depending on the real demand. Their payoffs in each period are:

$$Call\ 1: Md_1 = -[0.6\,max(D_i - 1.1\overline{D}_i, 0)]p \quad (7)$$
$$Call\ 2: Md_2 = -[0.3\,max(D_i - 1.2\overline{D}_i, 0)]p \quad (8)$$

CASE OF TOLL ROAD CONCESSION

Based on this case study, a model is created to evaluate a toll road concession. The hypothetical project involves a PPP for a 25-year toll road concession in Brazil. Regarding the traffic risk, to make the concession interesting to the private sector, the government offers a minimum traffic guarantee. The structure involves *project finance* and a high percentage of the initial investment is financed by third parties.

METHODOLOGY

In the literature on toll road projects, many authors model the risk variable as a Geometric Brownian Movement (Rose, 1998; Irwin, 2003; Huang e Chou, 2005; Wei-hua e Da-shuang, 2006; Galera, 2006; Irwin, 2007). Although the movement can be more complex according to other authors (Chiara, Garvin and Vecer, 2007; Garvin and Cheah, 2004; Brandão and Cury, 2006; Brandão and Saraiva, 2007; Zhao, Sundararajan and Tseng, 2004), in this work the traffic is modeled as Geometric Brownian Movement because it permits different methods of analysis. Besides, using simulation methods, it is easy to extend to other movements.

$$\frac{d\theta}{\theta} = \alpha dt + \sigma dz \qquad (9)$$

where θ is the traffic
α is the expected drift
σ is the volatility
dz is a Wiener process

MINIMUM AND MAXIMUM TRAFFIC LEVEL OPTIONS

Considering only one level of minimum and maximum traffic, representing the minimum traffic guarantee and the maximum traffic limit, the options can be represented in figure 2 and modeled as follows.

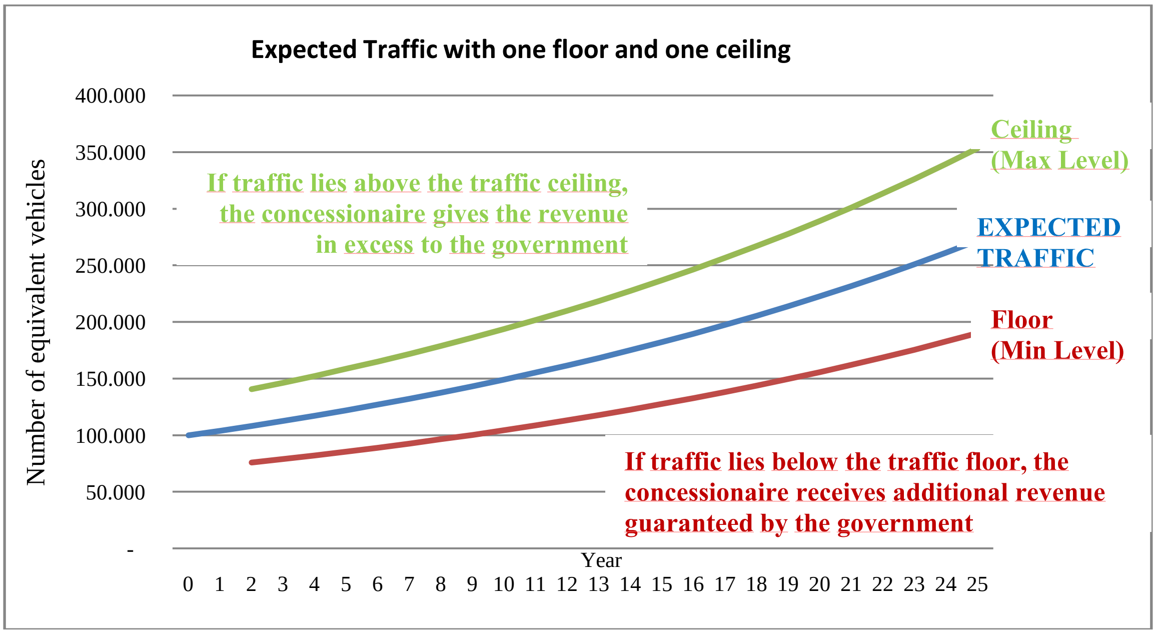


Figure 2: Project with one floor and one ceiling

Let θ_i be the real traffic and $\overline{\theta_i}$ the expected traffic in year *i* (in equivalent vehicles per day[1]). Let a_1 be a percentage below 100% and b_1 a percentage above 100%, based on the expected traffic and representing the minimum traffic level and the maximum traffic level respectively. Let y_1 be a percentage corresponding to the part of revenue that will be received or paid by the concessionaire. Let τ be the direct revenue tax fee and p the toll fee. Considering continuous operation (365 days per year), the put's and call's payoff for each year during the concession term can be defined as:

$$\textit{Put 1}: \quad GR_t = [y_1 \max(a_1\overline{\theta}_t - \theta_t, 0)].365.(1-\tau).p \qquad (10)$$

$$\textit{Call 1}: \quad RR_t = -[y_1\, max(\theta_t - b_1\overline{\theta}_t, 0)].365.(1-\tau).p \qquad (11)$$

[1] Equivalent vehicles is a standardized number of vehicles equivalent to two axel automobile (Brandão and Saraiva, 2007)

The options' values were calculated for different symmetric combinations of minimum and maximum traffic levels and different percentages of protection, based on the parameters used in the studied PPP. The options are both modeled directly on the same underlying asset, the traffic level. The options are mutually exclusive, but they exist simultaneously at each period of the concession term. Two methods are proposed and compared: an analytical method (Galera, 2006) and a Monte Carlo simulation method (Brandão and Saraiva, 2007). Although the application of the analytical method seems to be simple, there is a taxation problem regarding income tax, as explained in Blank (2008). The Monte Carlo simulation method bypasses this problem.

Monte Carlo simulation method

Risk-neutral Monte Carlo Simulation was used to evaluate the options. The GBM discretization[2] is given by:

$$\theta_{t+\Delta t} = \theta_t e^{\left(\alpha - \lambda\sigma - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}} \qquad (12)$$

where λ is the market price of risk of traffic. This parameter can be estimated by (Hull, 2006)[3]:

$$\lambda = \frac{\rho_{\theta,m}}{\sigma_m}(\mu_m - r) \qquad (13)$$

where $\rho_{\theta,m}$ is the correlation between traffic changes and a market index returns
μ_m is the expected return of a market index
σ_m the volatility of a market index

Simulating the traffic and the cash flows year by year, it is possible to calculate the project's original NPV without any options and the project's NPV in the presence of guarantees in every year of the concession. The value added by the options is given by:

$$Value\ Added = NPV\ with\ options - original\ NPV \qquad (14)$$

Analytical method

Galera (2006) studies different real options to value highways concessions in Spain. A solution based on an analytical method, using Black, Scholes and Merton's formula, is proposed. Since the traffic is a GBM, based on his model, the put equation which represents the revenue to be received in each period could be defined as

$$GR_i(t=0) = 365.(1-\tau).p.y_1\left[a_1\overline{\theta}_i e^{-rt}\, N(-d_2) - \theta_0 e^{(\alpha-\lambda\sigma-r)t} N(-d_1)\right] \qquad (15)$$

[2] $\varepsilon \sim N(0,1)$.
[3] According to Hull (2006), the underlying variable is not necessarily an investment asset. It could even be a temperature measure. In this case, the traffic is the underlying variable. This risk-neutral model, as in Brandão and Saraiva (2007) and Galera (2006), was based in the methodology proposed by Hull (2006). This methodology is also used by Irwin (2003).

$$where\ d_1 = \frac{ln\left(\frac{\theta_0}{a_1\overline{\theta}_i}\right) + \left(\alpha - \lambda\sigma + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}\ \ and\ d_2 = \frac{ln\left(\frac{\theta_0}{a_1\overline{\theta}_i}\right) + \left(\alpha - \lambda\sigma - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

r is the risk free rate
$\overline{\theta}_i$ is the daily average traffic level in year *i*
θ_0 is the initial expected daily average traffic level

Similarly, the solution for the call equation which represents the revenue in excess to be paid in each period could be defined as

$$RR_t(t=0) = -365.(1-\tau).p.y_1\left[\theta_0 e^{(\alpha-\lambda\sigma-r)t}N(d_1) - b_1\overline{\theta}_i e^{-rt}\ N(d_2)\right] \qquad (16)$$

$$where\ d_1 = \frac{ln\left(\frac{\theta_0}{b_1\overline{\theta}_i}\right) + \left(\alpha - \lambda\sigma + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}\ and\ d_2 = \frac{ln\left(\frac{\theta_0}{b_1\overline{\theta}_i}\right) + \left(\alpha - \lambda\sigma - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

Let *n* be the concession term given in years. The value added by the compounded options to the NPV, including revenue in excess to be paid from the concessionaire to the government and additional revenue to be received by the concessionaire from the government is given by:

$$Value\ Added = \sum_{i=1}^{n} GR_i(t=0) + RR_i(t=0) \qquad (17)$$

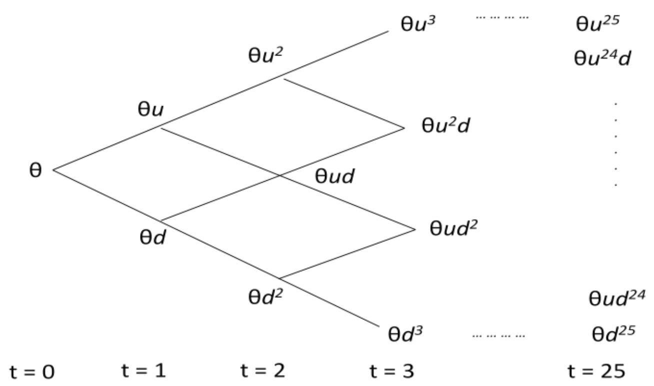
ABANDONMENT OPTION

Pollio (1998) proposes the real options approach for evaluating project finance structures with limited recourse. In this structure, the sponsors have an additional flexibility given by an implicit right of abandonment at each repayment date. Its payoff can be defined as

$$max(project\ value - the\ value\ of\ outstanding\ debt, 0) \qquad (18)$$

This means that the project involves a loan with a series of options. The borrower – or the sponsors – will only exercise it if project equity is nil or negative. Since the project finance structure results in a responsibility limitation to the sponsors, they could decide whether or not to invest more whenever the cash flow is negative. This right of abandonment can be represented by a put option. To value it, the methodology used involves backward optimization. A Cox, Ross and Rubinstein tree can be built to represent the traffic evolution, as in figure 3. Since the traffic follows a GBM and assuming that for each node the traffic θ_i^s in the period *i* and state s can increase to $u\theta_i^s$ or decrease to $d\theta_i^s$ in the following period, the parameters used were as follows[4]:

[4] McDonald (2006), p. 347, 359



$$u = e^{\sigma\sqrt{\Delta t}} \qquad (19)$$

$$d = \frac{1}{u} = e^{-\sigma\sqrt{\Delta t}} \qquad (20)$$

$$q = \frac{e^{(r-\delta)\Delta t} - d}{u - d} \qquad (21)$$

Figure 3: Cox, Ross and Rubinstein Binomial Tree

where q is the risk-neutral probability of traffic increasing
r is the risk free rate
σ is the traffic volatility
δ is the traffic "convenience fee"[5]

Based on the traffic tree, a cash flow tree is also built. It is possible then to calculate the project value going backwards from the last period of the cash flow tree. For each node, considering the implicit abandonment option, the NPV[6] can be written as:

$$NPV_i^s = max\left(CF_i^s + \frac{1}{(1+r)}\left[q.NPV_{i+1}^u + (1-q)NPV_{i+1}^d\right], 0\right) \qquad (22)$$

where NPV_i^s is the NPV in period $t = i$ and state s
CF_i^s is the cash flow in period $t = i$ and state s
NPV_{i+1}^u is the NPV in period $t = i+1$ and state u
NPV_{i+1}^d is the NPV in period $t = i+1$ and state d

Based on the NPV tree, it is possible to identify an abandonment region including a set of nodes. The decision rule can be represented by a traffic value which corresponds, approximately, to the first state node where the abandonment is exercised in each period. It is given by the highest traffic for which the abandonment exercise is the optimal decision in each period – here called traffic threshold. With traffic threshold for each period, the threshold curve is complete and defines the abandonment region during the whole concession term.

Considering interaction among options, the threshold curve changes when minimum and maximum traffic level options exist. In this case, the cash flow trees should be rebuilt since there is additional revenue to be received or revenue in excess to be paid in each node. In the presence of multiple real options in a project, the interaction among them influences their values (Trigeorgis, 1996). The minimum and maximum traffic level options may lose value when the implicit abandonment option is considered.

[5] The modified drift used in risk-neutral simulation is given by $\alpha - \lambda\sigma = r - \delta$ (Hull, 2006)

[6] In this model, the abandonment cost equals to zero, but it can be redesigned to include additional costs, as abandonment fees.

Given the traffic threshold curves, it is also possible to calculate the probability and the average time of abandonment in each situation, with or without the minimum and maximum traffic level options, using real Monte Carlo simulation. The results are given by:

$$Probability\ of\ Abandonment = \frac{number\ of\ interactions\ where\ abandonment\ is\ excercised}{total\ of\ interactions} \quad (23)$$

$$Average\ Time\ of\ Abandonment = \frac{\sum periods\ when\ abandonment\ is\ exercised}{number\ of\ interactions\ where\ abandonment\ is\ excercised} \quad (24)$$

When abandonment is considered, the guarantee option has an additional important benefit. It becomes interesting not only from the sponsors' point of view, but also from the lenders'. Besides increasing the expected project value and decreasing the sponsor's risk, the guarantee can be designed to reduce the probability of abandonment, and consequently the risk to the lenders.

PARAMETERS AND RESULTS

The GBM parameters for the traffic were estimated based on data available for other Brazilian toll road concessions[7]. The operating revenues start only by the 2nd year. The relevant parameters are listed below:

p	R$ 5,50	Tariff
τ	14%	Direct taxes
N	25 years	Concession term
θ_0[8]	100.000	Initial expected daily average traffic level
α	4% p.a.	Traffic drift
σ	10% p.a.	Traffic volatility
Inv	R$ 1.000 MM	Initial investment (50% in year 0 and 50% in year 1)
Loan	R$ 700 MM	Loan principal (50% in year 0 and 50% in year 1) with 2-years of delayed payment
r	6% p.a.	Risk-free rate
i	8% p.a.	Loan rate
n_2	15 years	Loan term
OC_1	R$ 30 MM	Annual operating costs in year 1
OC_2	R$ 60 MM	Annual operating costs from year 2 to year 25
MC_1	R$ 50 MM	Annual maintenance costs from year 2 to year 9
MC_2	R$ 70 MM	Annual maintenance costs from year 10 to year 18
MC_3	R$ 90 MM	Annual maintenance costs from year 19 to year 25
n_3	15 years	Investment depreciation term
IR	34%	Income Tax

[7] The concession data used to estimate traffic parameters refers to toll roads managed by CCR and OHL Brasil, two companies listed on the São Paulo Stock Exchange (Bovespa).

[8] There is no traffic in year 0 and year 1. θ_0 is a reference value to estimate traffic in the following years. The expected traffic values for each year were calculated based on the GBM.

$\rho_{\theta,m}$	0,40	Correlation between ABCR Index changes and a IBovespa returns based on a period from 1Q2000 and 2Q2007[9]
μ_m	12% p.a.	expected IBovespa return
σ_m	25% p.a.	volatility of IBovespa
λ	0.096	market price of risk of traffic
δ	2.96% p.a.	traffic "convenience fee"
u	1.1052	traffic increase factor (binomial tree)
d	0.9048	traffic decrease factor (binomial tree)
q	69.74%	risk-neutral probability (binomial tree)

Table 1: Parameters

Using @Risk software to simulate cash flows, the expected NPV for the original project without any options was R$ 70.5 MM. The value added by symmetric combinations of minimum and maximum traffic levels and different percentages of protection using both methods are presented below:

Value added by min / max traffic options using analytical method (R$ 000)

	y_1 (Percentage of revenue to be paid or received)					
a_1 / b_1 (Min / Max traffic levels as percentages of expected traffic)	**50%**	**60%**	**70%**	**80%**	**90%**	**100%**
50% / 150%	(4,914)	(5,896)	(6,879)	(7,862)	(8,845)	(9,827)
60% / 140%	7,837	9,404	10,971	12,539	14,106	15,674
70% / 130%	32,088	38,506	44,923	51,341	57,759	64,176
80% / 120%	69,600	83,520	97,440	111,360	125,280	139,200
90% / 110%	119,764	143,717	167,670	191,623	215,576	239,529

Table 2: Value added by options using analytical method

Value added by min / max traffic options using simulation (R$ 000) and respective differences between both methods

	y_1											
a_1 / b_1	**50%**		**60%**		**70%**		**80%**		**90%**		**100%**	
50% / 150%	(2,285)	53%	(698)	88%	(2,986)	57%	1,251	116%	(2,023)	77%	(2,991)	70%
60% / 140%	8,990	15%	12,166	29%	12,647	15%	15,089	20%	17,722	26%	17,970	15%
70% / 130%	29,567	-8%	37,118	-4%	41,322	-8%	50,364	-2%	53,823	-7%	59,581	-7%
80% / 120%	63,394	-9%	74,735	-11%	86,058	-12%	98,282	-12%	109,855	-12%	118,942	-15%
90% / 110%	107,739	-10%	126,364	-12%	146,176	-13%	163,975	-14%	182,744	-15%	199,310	-17%

Table 3: Value added by options using simulation method

The values of table 3 can be graphically represented as:

[9] IBovespa(São Paulo Stock Exchange Index) was considered the market index. ABCR Index was chosen to represent the traffic. This index is calculated by Brazilian Roads Concessionaires Association (ABCR) and a consulting firm in Brazil.

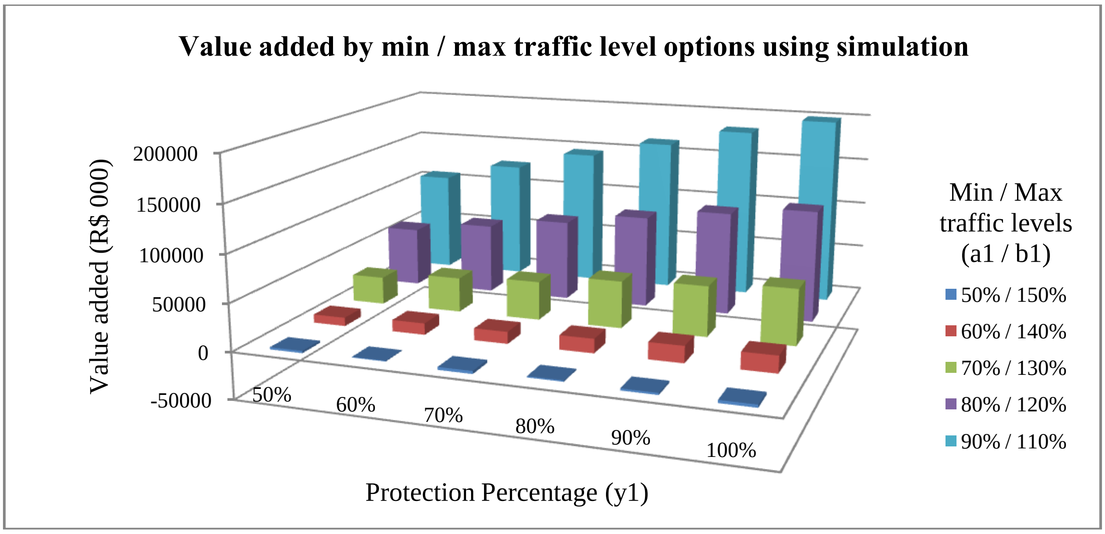


Figure 4: Value added by options using simulation methods

The difference between the values obtained using both methods can be explained by how income tax is treated. In the analytical model, the options' premium is calculated as a net revenue and directly added to the original project NPV; alternatively, in the simulation model, the options' premium is based on net profit in each year, after income tax. When using the simulation model, income tax treatment is correct, because the additional and exceeded revenue (from minimum and maximum traffic level, respectively) impacts the profit and, consequently, income tax and the cash flow in each period. Comparing both methods, the simulation one should be preferred.[10]

Regarding the simulation method results, the value added in each year considering different symmetric combinations of minimum / maximum traffic level options are presented in Figure 4. It can be negative during some years of the concession, depending on the minimum and maximum traffic levels. The lower the minimum guaranteed level – and the higher the symmetric maximum traffic level – the longer the period of negative premium is. For the first years, the maximum level options exceed the minimum level options.

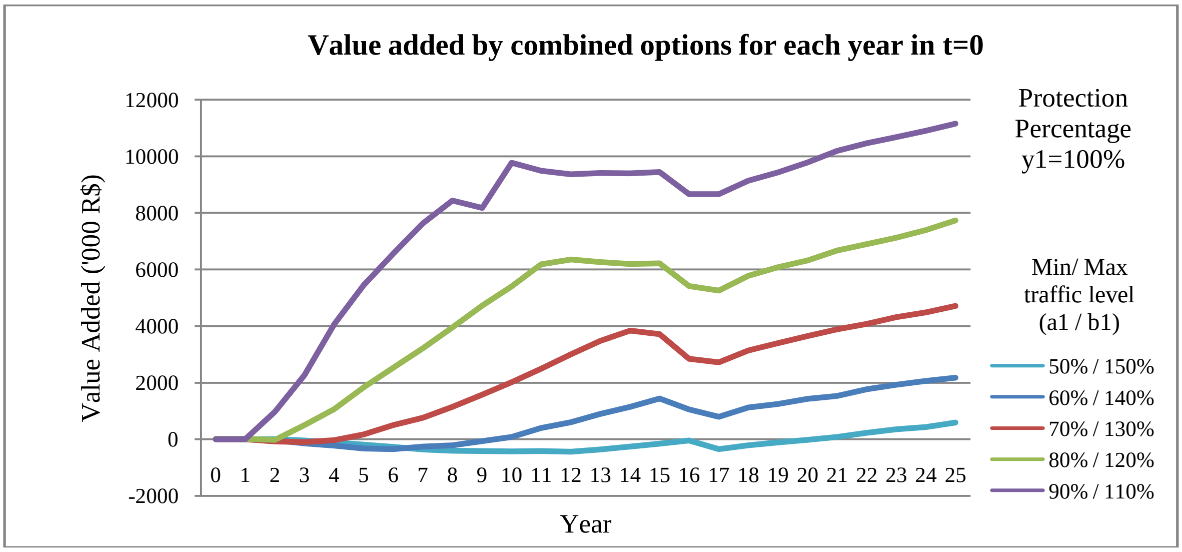


Figure 5: Value added by combined options for each year in t=0

[10] If the income tax is zero, the simulation results converge to the analytical results.

The total value added by the minimum and maximum traffic options to the expected NPV can be very high or even negative depending on the minimum guaranteed traffic level and the correspondent maximum traffic level. The government should choose an optimal combination regarding the return to sponsors and its own risk exposure.
Under the strategic project finance structure, when the implicit abandonment option is considered, there are other factors that can influence the government's decision about guarantee options. In this case, the sponsor will decide optimally to keep managing the project or to abandon it at each repayment date. This option adds value to the project and interacts with the minimum and maximum traffic level options previously analyzed.
According to the methodology, in the absence of minimum and maximum traffic level options, the original threshold curve and the abandonment region can be graphically represented as follows.

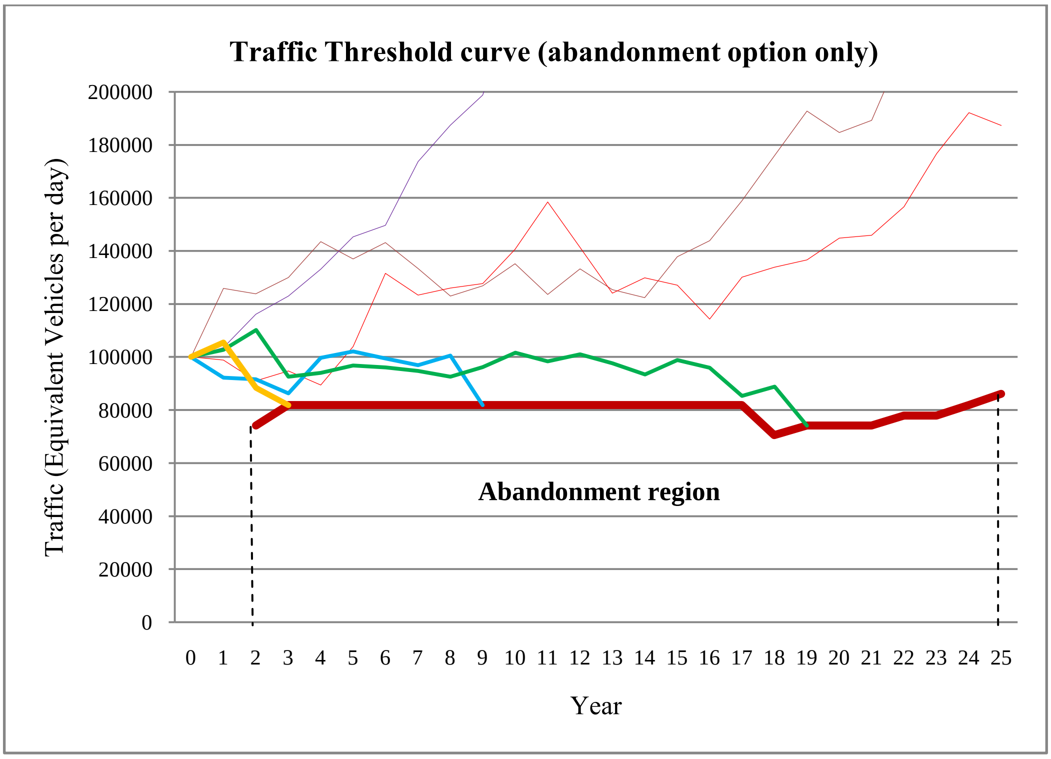


Figure 6: Original Traffic threshold curve

The line that limits the original abandonment region in figure 5 is the original traffic threshold curve (when no other option is considered in the project). Random paths represent the stochastic traffic. When any path hits the threshold curve, the process stops and the project is abandoned.
When minimum and maximum traffic level options are added to the model, new threshold curves are obtained. Different situations can be proposed to analyze how the options interact.
Considering again the symmetric combinations of minimum and maximum traffic levels (given by the percentages a_1 / b_1 over the expected traffic in each period) and 100% percentage of protection (y_1 = 100%), the threshold curves are graphically represented as follows in figure 6:

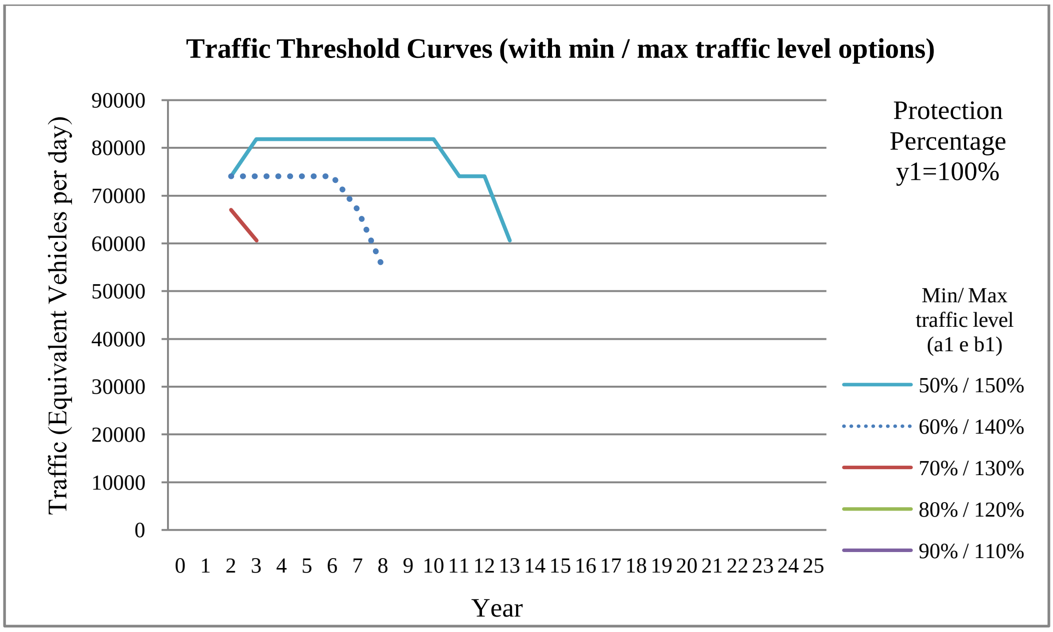


Figure 7: Traffic threshold curves (with min / max traffic level options and 100% protection)

In this case, if the traffic floor is 80% or 90% of the expected traffic, there is no threshold curve, and consequently abandonment is never optimal. Considering the other floors of 50%, 60% and 70% of the expected traffic, the correspondent traffic threshold curves involve only a few years in the beginning of the concession term.
Considering a percentage of protection of 50%, the threshold curves are as follows in figure 7.

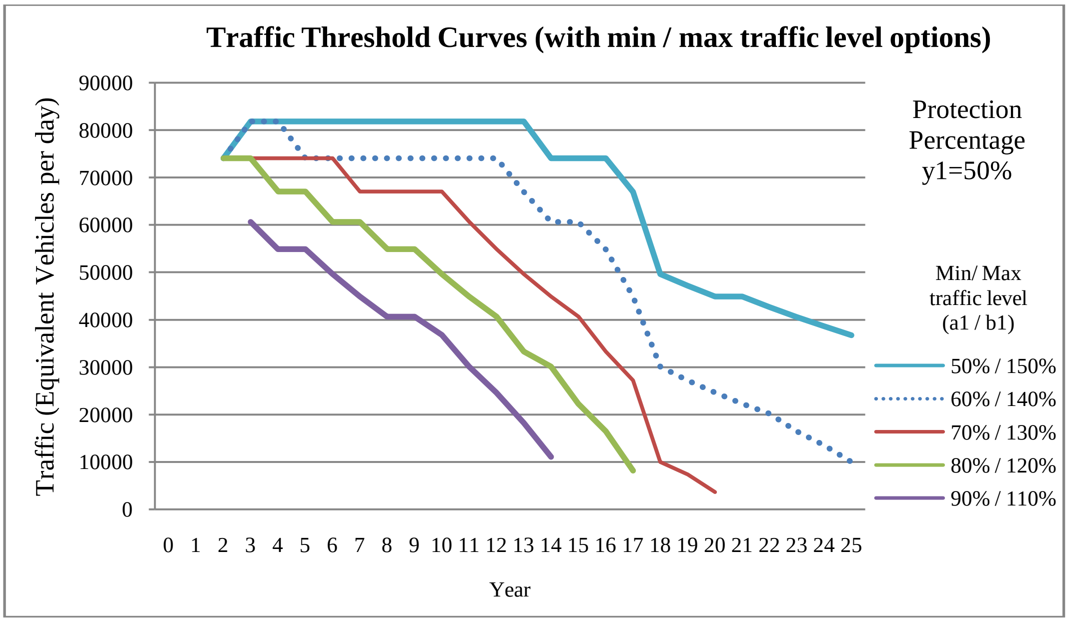


Figure 8: Traffic threshold curves (with min / max traffic level options and 50% of protection)

In this case, for all symmetric combinations of floors and ceilings, the threshold curves exist. However, as the floor becomes lower, the abandonment becomes possible in the last years of the concession term. Besides this, the threshold traffic values for the first years becomes higher, increasing the probability of abandonment, as expected.

Using @Risk software, when the abandonment option is considered (without minimum and maximum traffic level options), the expected project NPV is R$ 104.2 MM. Comparing it to the original expected NPV, the value added by the abandonment option is:

$$Value\ added\ by\ the\ abandonment\ option = 104.2MM - 70.5MM$$
$$= R\$\ 33.7\ MM$$

When minimum and maximum traffic level options are also included in the model, the options interact and their values changes. Considering for example the situations with protection percentage of 100% (y_1=100%) with different symmetric options of minimum and maximum traffic, the comparative results are[11]:

Min / Max traffic level (a_1/b_1) with 100% of protection (y_1=100%)	**Without abandonment option (R$ 000)**		**With abandonment option (R$ 000)**	
	NPV	**Value added by all the options**	**NPV**	**Value added by all the options**
50% / 150%	66,987	(2,991)	84,232	14,254
60% / 140%	87,891	17,970	92,729	22,808
70% / 130%	129,766	59,581	129,662	59,477
80% / 120%	189,535	118,942	189,535	118,942
90% / 110%	269,629	199,310	269,629	199,310

Table 4: VPL and value added with and without abandonment options

For higher levels of guaranteed traffic (higher minimum traffic level options), as 80% and 90% of the expected traffic, the abandonment option is worthless, since it is never exercised – as expected in figure 6. As the guarantee decreases, represented by a decrease in the floor level, the abandonment option becomes more relevant and the total value added by the existing options becomes higher.

But when considered together with the abandonment option, the guarantees options have a strategic importance. From the government's point of view, it is possible to design a guarantee which minimizes the probability of abandonment and consequently political and social problems. On the other hand, the guarantees lower the default risk to the lenders. This means that loan interest rate could even be reduced and the project made more attractive.

Based on the threshold curves, it is possible to calculate the probability of abandonment. In the original project, when only implicit abandonment option is considered, the probability of abandonment is 14.93% and the average time is 7.22 years. When minimum and maximum traffic level options are also considered, the results are presented as follows:

[11] The value added by the options was calculated through simulation using $VPL\ with\ options - original\ VPL\ (without\ options)$.

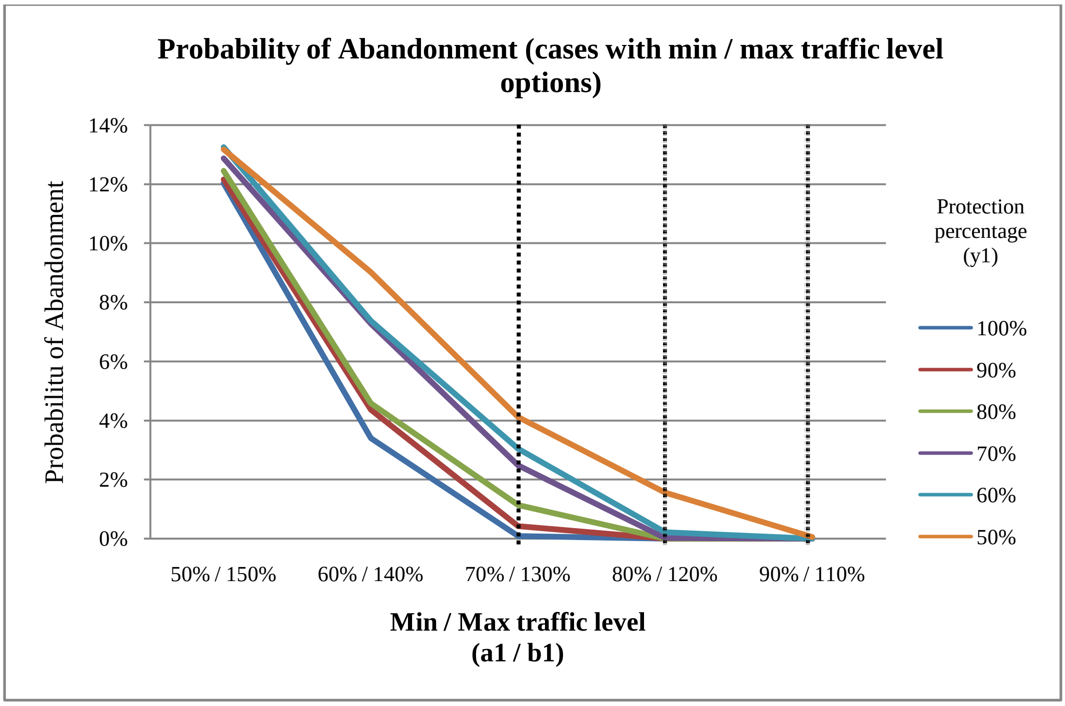


Figure 9: Probability of abandonment

As the protection percentage increases, the probability of abandonment decreases for all guaranteed traffic levels. Considering the floor level from 70% to 90% (and symmetric ceilings, respectively), the probability becomes much lower than the original 14.93% for all protection percentages analyzed.

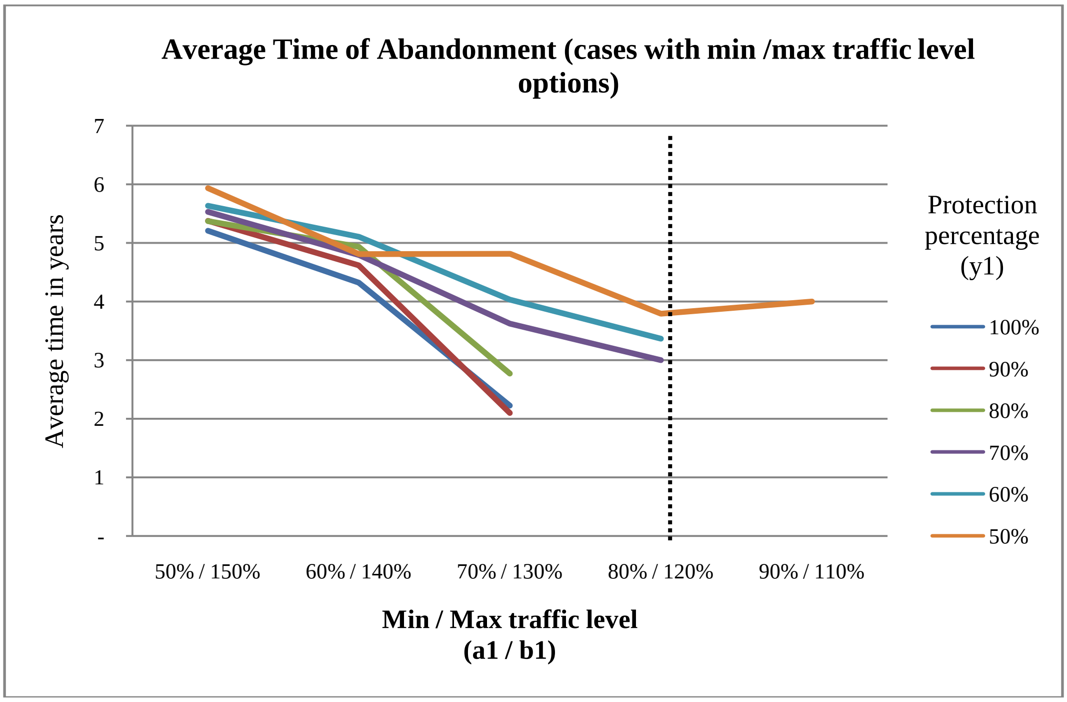


Figure 10: Average time of abandonment

The average time of abandonment occurs by the 6th year in all situations. Since the abandonment is likely to happen in the first years of the concession term, the government could review the traffic projections and limit the payments of guarantees. The lenders would be injured because of the default, giving rise to a renegotiation.

Using a combination of floor, ceiling and protection percentage, the government can choose the guarantee level it wants to offer. Three objectives should be considered: the concession should be attractive to private capital; the probability of abandonment can be limited to desired level; the government can minimize its risk exposure. Non-symmetric combinations of traffic floors and ceilings and more than one level of traffic floor and ceiling (as in the PPP of Metro of SP) can be also studied and compared.

CONCLUSIONS

The support mechanisms applied to public infrastructure projects to attract private capital can be very sophisticated. They should be designed considering benefits and risk exposure, and the correct valuation may require financial tools such as real options theory. These instruments with options characteristics used by governments or even embedded flexibilities identified in a project can add value and mitigate and reallocate risks, reducing the risk to the private investor and making the project more attractive.

Two methods were proposed to value the combination of minimum and maximum traffic level options in a toll road concession, based on Galera (2006) and Brandão and Saraiva (2007). The analytical method was shown to be incomplete compared to the simulation method, and the results can present considerable differences. In the analytical method, the present payoffs' values are added directly to the original project's NPV (without options), disregarding the effect of income tax. The correct valuation of the options involved is important since the feasibility of the project may depend on it. In both methods, the use of the market price of traffic risk estimated through existing toll-road concessions was an important step.

An additional benefit from the support mechanisms such as the guarantee proposed is to minimize the probability of abandonment. In highly leveraged projects involving project finance structures, the concessionaire could decide to pay the debt service or to abandon the project in each period. In this case, the government should additionally look at the guarantee option to minimize the probability of abandonment, since it could cause even social problems. Government should choose the optimal combination of minimum and maximum traffic levels, avoiding a high guarantee, but keeping expected return to sponsors and lowering the probability of abandonment. Besides, when abandonment is considered, the guarantee turns to be interesting also to lenders, since it reduces the risk of default.

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