Markov Perfect Equilibrium, Discrete Choice Model and the Innovator's Dilemma under the Customers' Preferences Uncertainty

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Abstract

This paper investigates the equilibrium investment policies of two different firms under customers' preferences uncertainty. The incumbent firm, which owns a superior old technology, produces merchandise that can satisfy current customers at the beginning of the investment game. The startup firm, which possesses an inferior old technology, does not capture the customers' satisfaction but it has a possibility to cultivate a new technology that can attract the customers in the future if the customers' preferences are changed. We consider two types of equilibria in our valuation model. The first one is a price equilibrium at each time point derived from the Bertrand competition. To represent customers' diversity and products differentiation we use a discrete choice model. The other one is a Markov perfect equilibrium where each firm have options to invest either in the old technology or in the new technology depending on customers' preferences which are modeled as a Markov process.

Keywords: The Innovator's Dilemma; Discrete Choice Model; Markov perfect Equilibrium

1 Introduction

In competitive markets once a company acquires her competitive advantage over other firms she makes various strategic decisions to protect her competitive advantage. Thus, it is reasonable to conclude that an entrant firm has a difficulty in entering and growing in the existing market from the relatively inferior position. On the other hand, we observe many cases where leader firms that has established a dominant position in a market fails to respond correctly to the market change or technology change to lose their dominant positions. Examples are Sears in the retail market, a battle in the computer market from mainframe, minicomputer to the personal desktop, and hard disk markets with different sizes[1].

For a leader firm that has established a dominant position in one market what is important to protect her dominant position under the uncertain environment? For an entrant firm, on the other hand, what is necessary to successfully enter the market against the initial inferior position? There are many empirical studies that analyze common factors for protecting the

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[1] See Christensen[2000].

competitive advantages or essential causes for losing the dominant positions in the management literature. For example, Finkelstein[2003] points out that bureaucratic behaviors of the management, conceit, poor project planning, poor governance against new technology, misconception, are main causes of the management failure. He concludes that most of the failure are attributed to the irrationality and inability of the management.

Christensen[2000], on the other hand, assets that the rational management might be a cause of the failure in a different environment. He distinguishes disruptive technology from sustaining technology and insists that management must change her strategic decision in response to the type of the technology. He shows that good management, which is the very reason they succeed in dealing with sustaining technology, is the most powerful cause for their failure in dealing with the disruptive technology. Because they listen to the existing customers, which paradoxically leads to ignoring potential new customers. Furthermore, he points out that the pace of the technology the firm develops often outstrips what the customers need in the market.

It is difficult to analyze the ex post fact optimality under uncertainty and competition after the incident. The main purpose of this paper is to investigate the equilibrium strategies ex ante for leader firm and entrant firm under uncertainty and examine the relation between the rationality and ex post results of the competition under uncertainty.

Motivating above argument this paper examines the dynamics of the technology investment game between the incumbent firm, which leads the industry with a superior old technology, and the startup firm, which newly enters the same market with a new technology. The equilibrium strategies for both firms can be derived with a use of the real option analysis and the game theory.

In the paper, we focus on two firms producing similar merchandise that has more than two different functions such as a basic function, size, capacity, speed and so on. Each customers selects a product to maximize the utility by evaluating the level or quality of each function embedded in the product in addition to its price. The level of each function depends on the history of each firm's investment decision. To incorporate this aspect into the model, we consider the investment game where two firms compete to invest either in the old technology or in the new technology to improve one of the two different functions embedded in the products in addition to the price competition. We model the customers' diversification, namely we assume a random utility model. In other words, we adopt a discrete choice model to describe the product differentiation and to analyze the equilibrium price of the product, market share and revenue for each firm.

We consider customers' preference uncertainty with respect to the functions embedded in the products. We assume that the customers' relative preferences change over time. It is well known in the marketing literature that customers first focus on the basic function of a new product but once the basic function becomes satisfactory, their interest moves on to another function such as its design. We assume in the model that the relative preference of the new function against the old function follows a discrete Markov process and formalize the dynamic investment competition for both incumbent firm and the startup firm.

We consider two firms that have different levels of two technologies which are used to improve the level of the corresponding function. An incumbent firm, which has an advantage with respect to the old technology, establishes a dominant position in the market since the old technology can create the higher level of the first function that are preferred by the current customers. The other firm called a startup firm is an entrant of the market with a new technology which can improve the second function that is not preferred by the current customers. Thus, the market share of the startup firm is low[2]. There are potential customers who choose to buy products by evaluating the levels of both functions embedded in the products. We assume that each customer has a random utility and chooses a product to maximize the utility. The random utility model can be justified as the following interpretations. Each customer is supposed to have a deterministic utility function. However, we can imperfectly observe the characteristics influencing the individual's choice and have only imperfect knowledge of the utility function. The utility function can be divided into two parts;one reflects the known, observable part and the other is a utility that can not be observable, which can be modeled as a random variable. For further discussion of the random utility see Anderson, Palma and Thisse[1992].

At the beginning of the investment game we consider, most customers prefer the first function created by the old technology where the incumbent firm has an advantage, to the second function created by the new technology. The future relative preference between the first function and the second function is uncertain. Each firm can invest either in the old technology or in the new technology to improve the level of the corresponding function. The probability of success is known and thus, at each time point, each firm selects either technology to level up the corresponding function.

The incumbent firm and the startup firm compete with each other in two different ways. The first one is a Bertrand competition in which the firms maximize their revenues at each time point given the customers' relative preference. The second is a dynamic investment game under the relative preferences' uncertainty where each firm chooses one of the technologies to improve the level of the function, which is formalized as Markov perfect game.

Schivardi and Schneider[2008] analyze a similar dynamic investment game between the incumbent firm and the startup firm. They consider an uncertainty with respect to the success of the technology investment. Their model assumes that the success of the disruptive technology investment is unknown and it is estimated by learning from their investment history. The model in this paper, on the other hand, mainly focuses on the presence of two functions embedded in the product under the customers' preference uncertainty, which is mentioned in Christensen[2000].

Pages and McGuire[1994] discuss a numerical approach to solve Markov perfect Nash equilibrium. The Markov perfect Nash game is general enough to be applied to many business problems. They provide a simple algorithm to solve this problem.

In this paper we can derive the ex ante project values for both incumbent and startup firms and their equilibrium strategies when customers' preferences are changed in the future. Furthermore, with a help of the Monte Carlo method we generate a couple of sample paths to describe the customers' preference and examine the equilibrium strategies along with these samples. According to our numerical examples we observe the following. First, the project value of the incumbent firm is usually larger than that of the startup firm which reflects our assumption that the incumbent firm has an advantage with respect to the old technology over the startup firm at the beginning of the investment game. Second, in comparing the market share or the revenue at the end of the game it is often the case where the startup firm could dominate the incumbent firm against the initial disadvantage, which clearly indicates the fact in our model the rational decision making does not always protect the incumbent firm from

[2]The old technology corresponds to the sustaining technology, while the new one corresponds to the disruptive technology defined by Christensen[[2000].

the defeat by the startup firm in the presence of customers preferences uncertainty.

The rest of the paper is organized as follows. Section 2 develops a dynamic investment model as a Markov perfect game. In section 3 we illustrate several numerical examples and analyze equilibrium strategies, as well as the project values for both firms. Concluding Remarks are in Section 4.

2 Model

This section develops a valuation model that are motivated by the introduction in section 1. We assume that the lifetime of the project considered in this paper is finite and discrete;i.e., define discrete time points as $t = 0, 1, \ldots, T$. Consider two different firms which denote $k = I, S$. The firm denoted by I is called incumbent firm that has a dominant old technology used for creating the first function in the product while the firm denoted by S is called a startup firm that produce similar merchandise with different level of functions. In the model we focus on two types of technologies($j = old, new$) that are used for the similar products. The old technology supports the first function embedded in the products which is highly evaluated by the current customers. The new technology, on the other hand, can improve the second function, which is not paid attention by the current customers, but it could become more valuable when the customers' preferences are changed in the future.

Let $d_k^j(t), k = I, S, j = old, new$ denote the level of technology j for firm k at time t. In the model a larger $d_k^j(t)$ implies high level or high quality of the function created by the technology j. At each discrete time point each firm can invest either in the new technology or in the old technology that improve the level of the corresponding function embedded in the products by paying the amount of K_k^j. Due to the competitive environment of this market we assume that both firms must keep investing at each time period.

The level of the function is improved by the degree of Δd when the technology investment succeeds while the level is not changed when it fails. Let q_k be the probability of success for firm $k's$ technology investment. Each firm k decides the price of its products $p_k(t)$ at each time t and sell them to the customers. The variable cost of a product is denoted by c_k. Suppose there are N potential customers in the market and each customer $n, n = 1, \ldots, N$ possesses the following random utility U_k^n toward firm $k's$ products, which depends on its price and the level of the functions, namely

$$U_k^n(t) = y^n(t) - p_k(t) + a_k(t) + \varepsilon_k^n(t), k = I, S, \tag{1}$$

where y^n is a reservation price for customer n, a_k represents the utility that values each function embedded in the products and ε_k^n is an idiosyncratic utility that represent each customer's personal preference. In our model ε_k^n is iid random variable under double exponential law with parameter μ and η. The probability density function of the double exponential distribution is given by

$$f(x) = \exp\left[-\exp\left(-\frac{x-\eta}{\mu}\right)\right]. \tag{2}$$

Its expectation $E[\varepsilon_k^n]$ and variance $V[\varepsilon_k^n]$ are known, respectively, as

$$E[\varepsilon_k^n] = \mu\gamma + \eta, \tag{3}$$

$$V[\varepsilon_k^n] = \frac{\mu^2\pi^2}{6}, \tag{4}$$

where $n = 1, \ldots, N, k = I, S$. Note parameter γ indicates Euler constant,i.e., $\gamma \approx 0.577$. a_k is dependent of the levels of both functions embedded in the products. For each firm $k = I, S$ we assume

$$a_k (t) = d_k^{old} (t) + \theta (t) d_k^{new} (t), k = I, S, \tag{5}$$

where $\theta (t)$ represents a relative preference of the second function created by the new technology over that by the old technology. We assume that $\theta (t)$ follows a discrete Markov process in general. Let us denote $\Theta (t)$ be a state space of the Markov process at time t. For simplicity, we denote $\Theta (t + 1, \theta (t))$ be a state space at time $t+1$ on the condition that can be transited from $\theta (t)$ at time t. The transition probability from $\theta (t)$ to $\theta' \in \Theta (t + 1, \theta (t))$ is denoted by $q (\theta (t), \theta')$. Customers can choose to buy another product whose utility function is given by

$$U_0^n (t) = y^n (t) + v_0 (t) + \varepsilon_0^n (t), \tag{6}$$

where $\varepsilon_0^n (t)$ is another iid double exponential random variable. $v_0 (t)$ represents a utility of buying neither of products, namely, a larger $v_0 (t)$ implies an existence of strong competitors other than the two firms focused in the model.

Now consider the customer's product choices at each time. Suppose the level of each technology and the price for each firm I and S are given, respectively, by $d_k^j; k = I, S, j = new, old$ and $p_k (t); k = I, S$ and $\theta (t)$ is also given. The choice probability denoted by P_k[3] for customer n to choose firm $k's$ product can be derived by

$$\begin{aligned} P_k &= \Pr \left[U_k^n > U_{-k}^n, U_k^n > U_0^n \right] \\ &= \frac{\exp\left(\frac{a_k - p_k}{\mu}\right)}{\exp\left(\frac{aI - p_I}{\mu}\right) + \exp\left(\frac{a_S - p_S}{\mu}\right) + \exp\left(\frac{v_0}{\mu}\right)}. \end{aligned} \tag{7}$$

Note $-k = S$ in case of $k = I$ and vice versa. The choice probability of buying neither product is given by

$$P_0 = \frac{\exp\left(\frac{v_0}{\mu}\right)}{\exp\left(\frac{a_{I1} - p_I}{\mu}\right) + \exp\left(\frac{a_S - p_S}{\mu}\right) + \exp\left(\frac{v_0}{\mu}\right)}. \tag{8}$$

Since there are potentially N customers the expected demand of the firm $k\prime s$ products is given by $D_k = NP_k$. Thus, the expected revenue for each firm at time t, denoted by $R_k (t)$, is as follows.

$$\begin{aligned} R_k (t) &= D_k (t) \{p_k (t) - c_k (t)\} \\ &= NP_k (t) \{p_k (t) - c_k (t)\}. \end{aligned} \tag{9}$$

The above discussion is on the condition where the prices $p_k (t)$ are given. In our model both firms can choose the prices of their products so that their revenues are maximized under the price competition. Therefore, we derive the set of prices in equilibrium under Bertrand competition. Since

$$\frac{\partial P_k (t)}{\partial p_k (t)} = \frac{P_k (t) (P_k (t) - 1)}{\mu}, k = I, S,$$

[3]For the derivation of the choice probability see Anderson, Palma and Thisse[1992].

thus

$$\begin{aligned}\frac{\partial R_k(t)}{\partial p_k(t)} &= NP_k(t) + N\{p_k(t) - c_k(t)\}\frac{\partial P_k(t)}{\partial p_k(t)} \\ &= NP_k(t)\left\{1 + \{p_k(t) - c_k(t)\}\frac{(P_k(t) - 1)}{\mu}\right\}. \end{aligned} \tag{10}$$

By applying the first order condition

$$\frac{\partial R_k(t)}{\partial p_k(t)} = 0 \Leftrightarrow 1 + \{p_k(t) - c_k(t)\}\frac{(P_k(t) - 1)}{\mu} = 0 \tag{11}$$

$$\Leftrightarrow p_k(t) - c_k(t) = \frac{\mu}{1 - P_k(t)}, k = I, S. \tag{12}$$

The equilibrium prices which are denoted by $p_k^*(t), k = I, S$ can be derived by solving the above equations. Note due to the definition of the utility function $y^n(t) \geqq p_k(t)$ must hold[4].

Next, we consider the dynamic strategy under the customers' preferences uncertainty. More concretely, we derive the equilibrium investment strategy at each time, which will affect the following the level of both technologies since both firms can choose the technology they invest. It is easy to prove that the investment decision is a Markov decision process that could depend on the current level of the technologies for both firms $\mathbf{d}(t) = \left(d_I^{old}(t), d_I^{new}(t), d_S^{old}(t), d_S^{new}(t)\right)$ as well as the customers' relative preference $\theta(t)$. To derive the optimal decision process we introduce the value function $V_k(t, \mathbf{d}(t), \theta(t)), k = I, S$ as follows. At each time t each firm has the following two options:Invest either in the old technology or in the new technology. The investment will be successful with probability q_k and the level of the technology is increased by Δd. Consequently, four set of investment decisions could be realized.

Suppose both the incumbent firm and the startup firm invest in the old technology. Let $\varphi_k^{old,old}(t, \mathbf{d}(t), \theta(t)), k = I, S$ denote the as follows.

$$\begin{aligned}\varphi_k^{old,old}(t, \mathbf{d}(t), \theta(t)) = R_k^*(t) - K_k^{old} \\ + \rho \sum_{\theta' \in \Theta(t+1, \theta(t))} q(\theta(t), \theta') E_k^{old,old}(t+1, \theta'),\end{aligned} \tag{13}$$

$$\begin{aligned}&E_k^{old,old}(t+1, \theta') \\ &= \left\{q_I q_S V_k\left(t+1, \left(d_I^{old}(t) + \Delta d, d_I^{new}(t), d_S^{old}(t) + \Delta d, d_S^{new}(t)\right), \theta'\right)\right\} \\ &+ q_I(1 - q_S) V_k\left(t+1, \left(d_I^{old}(t) + \Delta d, d_I^{new}(t), d_S^{old}(t), d_S^{new}(t)\right), \theta'\right) \\ &+ (1 - q_I) q_S V_k\left(t+1, \left(d_I^{old}(t), d_I^{new}(t), d_S^{old}(t) + \Delta d, d_S^{new}(t)\right), \theta'\right) \\ &+ (1 - q_I)(1 - q_S) V_k\left(t+1, \left(d_I^{old}(t), d_I^{new}(t), d_S^{old}(t), d_S^{new}(t)\right), \theta'\right).\end{aligned}$$

where $R_k^*(t)$ is the revenue for firm k at time t on the condition that both firms choose the optimal prices under the Bertrand competition. $\sum_{\theta' \in \Theta(t+1, \theta(t))} q(\theta(t), \theta') E_k^{old,old}(t+1, \theta')$

[4]In case $y^n(t) < p_k(t)$ is satisfied equation (11) is replaced by the equation $y^n(t) = p_k(t)$.

represents the expectation of the value function when $\theta(t)$ follows the Markov process where ρ represents a discount factor. $E_k^{old,old}(t+1,\theta')$ represents an expected value function when both firms invest in the old technology(superscript old, old) at time t.When both firms invest in the new technology $\varphi_k^{new,new}(t,\mathbf{d}(t),\theta(t))$ and $E_k^{new,new}(t+1,\theta')$ can be derived as follows.

$$\begin{aligned}\varphi_k^{new,new}(t,\mathbf{d}(t),\theta(t)) &= R_k^*(t) - K_k^{new} \\ &+ \rho \sum_{\theta'\in\Theta(t+1,\theta(t))} q(\theta(t),\theta') E_k^{new,new}(t+1,\theta'),\end{aligned} \tag{14}$$

$$\begin{aligned}&E_k^{new,new}(t+1,\theta') \\ &= \left\{q_I q_S V_k\left(t+1,\left(d_I^{old}(t), d_I^{new}(t)+\Delta d, d_S^{old}(t), d_S^{new}(t)+\Delta d\right),\theta'\right)\right\} \\ &+ q_I(1-q_S)V_k\left(t+1,\left(d_I^{old}(t), d_I^{new}(t)+\Delta d, d_S^{old}(t), d_S^{new}(t)\right),\theta'\right) \\ &+ (1-q_I)q_S V_k\left(t+1,\left(d_I^{old}(t), d_I^{new}(t), d_S^{old}(t), d_S^{new}(t)+\Delta d\right),\theta'\right) \\ &+ (1-q_I)(1-q_S)V_k\left(t+1,\left(d_I^{old}(t), d_I^{new}(t), d_S^{old}(t), d_S^{new}(t)\right),\theta'\right)\end{aligned}$$

In exactly the same manner, we can derive the rest of the four possible cases.

- Case where the incumbent firm invests in the old technology while the startup firm invest in the new technology: $\varphi_I^{old,new}(t,\mathbf{d}(t),\theta(t)), \varphi_S^{old,new}(t,\mathbf{d}(t),\theta(t))$
- Case where the incumbent firm invests in the new technology while the startup firm invest in the old technology:$\varphi_I^{new,old}(t,\mathbf{d}(t),\theta(t)), \varphi_S^{new,old}(t,\mathbf{d}(t),\theta(t))$

Based on these four cases the simultaneous game played by the two firms at time t can be defined. This game can be illustrated as the following table. In this paper we derive the Nash equilibrium. Let $V_k(t,\mathbf{d}(t),\theta(t))$ be the value function that corresponds to $\varphi_S^{j_1,j_2}(t,\mathbf{d}(t),\theta(t)); j_1, j_2 = old, new$ in equilibrium.

	old	new
old	$\left(\varphi_I^{old,old}, \varphi_S^{old,old}\right)$	$\left(\varphi_I^{old,new}, \varphi_S^{old,new}\right)$
new	$\left(\varphi_I^{new,old}, \varphi_S^{new,old}\right)$	$\left(\varphi_I^{new,new}, \varphi_S^{new,new}\right)$

In analyzing the dynamic equilibrium strategies we often encounter equilibrium selection problems. For simplicity in this paper, we first adopt (1) payoff dominance rule and (2)Incumbent firm advantage rule for specifying the unique equilibrium strategy.

3 Numerical Analysis

The model developed in this paper is difficult to analyze without numerical approach. First, to derive a solution of nonlinear equations (11) a numerical approach is required. Second, an exogenous uncertainty $\theta(t)$ is assumed as a discrete Markov process. Finally, we assume a project with a finite horizon that leads to a nonstationary strategy in general. In this section, therefore, we take the numerical approach to derive the dynamic equilibrium strategies of the firms with respect to the price decision as well as the investment decision. We also derive each firm's project value and analyze their strategies in the presence of the price and technology competition.

3.1 Price competition derived at each time point

We analyze the price competition when the customers' relative preference is given at some fixed time point. The analysis reveals the effects of the technology level of both firms on each firm's market share and revenue. The parameter values are set as follows.

$$\mu = 0, v_0 = 0, \theta = 1, N = 100, c_k = 1, k = I, S.$$

Note $\theta = 1$ implies that the customers equally evaluate the both functions embedded in the products on average. The first three figures show sensitivities of equilibrium prices(*Figure 1*), the market shares(*Figure 2*) and the revenues with respect to the level of the new technology for the startup firm $d_S^{new} \in [0, 15]$ when other parameters values are fixed as $d_I^{old} = 5, d_I^{new} = 0, d_S^{old} = 0$ and the reservation prices for all customers are equal,i.e., $y^n = 5, n = 1, \ldots, N$.

In *Figure 1* the equilibrium price for the incumbent firm tends to decrease and converges to 2 as the d_S^{new} increases while that for the startup firm reaches a ceiling of 5 due to the reservation price constraint. As a result shown in *Figure 2* and *Figure 3*, although the market share rises up to one hundred percent its revenue becomes in s-shape. Namely, the marginal revenue tends to decrease once the level of the function reaches some satisfactory level for the customers, which is clearly pointed out by Christensen[2000] as a cause of the innovator's dilemma. We can endogenously derive this phenomenon.

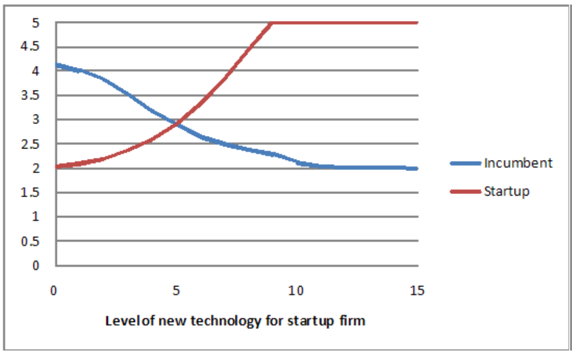

Figure1:A sensitivity of equilibrium prices

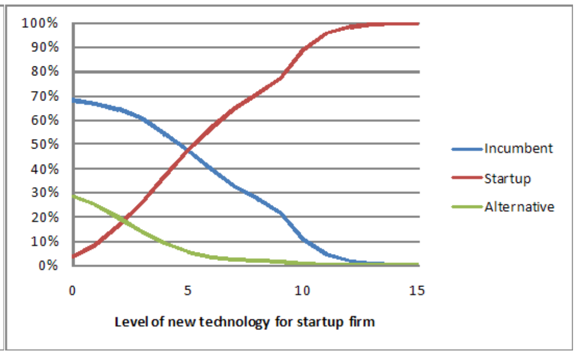

Figure2:A sensitivity of expected market shares

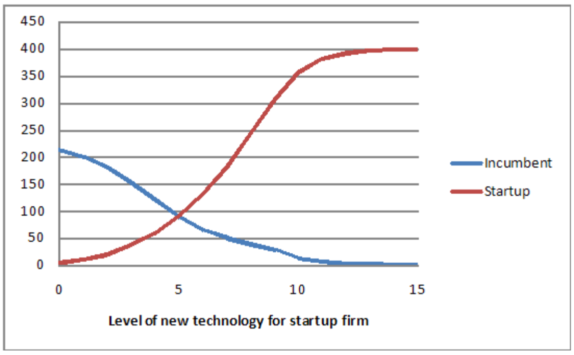

Figure 3:A sensitivity of expected revenues

3.2 The project values for the incumbent firm and the startup firm

The model developed in this paper deals with two sources of uncertainty. In the numerical example we focus on the uncertainty with respect to customers' relative preference $\theta(t)$. The other uncertainty about success of the investment is analyzed in Schivardi and Schneider(2008). Thus, we set the probabilities of the success in the technology investment are set as $q_I = q_S = 1$, which implies that the performance level of the function is always increased if the firm invest in either technology.

We suppose the finite lifetime of the project $T = 100$. We consider the binomial model as a discrete Markov process $\theta(t)$ with an initial value equal to one. We adopt CRR model so that we could interpret our Markov process as a discrete approximation of the geometric Brownian motion with a drift of $\mu_\theta = 20\%$ and a volatility of $\sigma_\theta = 40\%$[5].Thus, the rates of return and its probability in one period are given as follows

$$u_\theta = \exp\left(\sigma_\theta\sqrt{\Delta t}\right), d_\theta = 1/u_\theta,$$
$$q_{up} = \frac{r\_u_\theta}{u_\theta - d_\theta}, q_{down} = 1 - q_{up},$$

where $r = \exp\{\mu_\theta \Delta t\}$. At each time $t = 1, 2, \ldots, T$ both firms can invest either in the old technology or in the new technology to improve the performance level of the corresponding function. Thus, the equation

$$d_k^{old}(t) + d_k^{new}(t) = d_k^{old}(0) + d_k^{new}(0) + t, k = I, S.$$

is satisfied[6]. Suppose the initial technology level for each firm is given by $\mathbf{d}(0) = (5, 0, 0, 0)$, namely, only the level of the incumbent's old technology is in high-level at the beginning of the investment game, which implies a relative advantage of the incumbent firm against the startup firm. The marginal improvement of the function is $\Delta d = 0.1$.Other values are as follows:

$$v_0(t) \equiv 0, c_k = 0.1, \mu = 1, \rho = 1/1.2.$$

Case 1:Cost structures for both firms are same

We first analyze a case where the cost structures for both firms are same, i.e., $K_k^j = 10, k = I, S, j = old, new$. Although it does not reflect the situation on which we focus it is a building block for further analysis below. Note that the incumbent firm and the startup firm are symmetric except the initial level of the technology,i.e., $\mathbf{d}(0) = (5, 0, 0, 0)$.

First a sensitivity with respect to the initial value of the customers' relative preference $\theta(0)$ is examined. *Figure 4* illustrates values of $V_k(0, \mathbf{d}(0), \theta(0)), k = I, S$ with respect to $\theta(0)$.V_I and V_S represent initial values of the incumbent firm I and the startup firm S, respectively. The sensitivities for both firms values are similar that reflects the fact that the two firms are symmetric. They are constant when $\theta(0)$ is around less than one, which implies that when the initial value $\theta(0)$ is too small it is unrealistic for both firms to invest in the

[5] Cox, Ross and Rubinstein(1979)

[6] From a computational viewpoint this equation can be used to avoid the curse of dimensionality in deriving the dynamic equilibrium strategies.

new technology. In case of $\theta(0) > 1$ on the other hand, the firms prefer to invest in the new technology in response to the customers' preference, which increases the firms' values.

Next, a sensitivity with respect to the risk of customers preference uncertainty is examined. *Figure 5* shows values of $V_k(0, \mathbf{d}(0), \theta(0)), k = I, S$ with respect to σ_θ. Clearly the sensitivity is small, which is different from the case observed in the standard real option analysis. *Figure 6* shows a sensitivity of the firms values with respect to $d_I^{old}(0)$. The value of the firm are either increasing or decreasing monotonically.

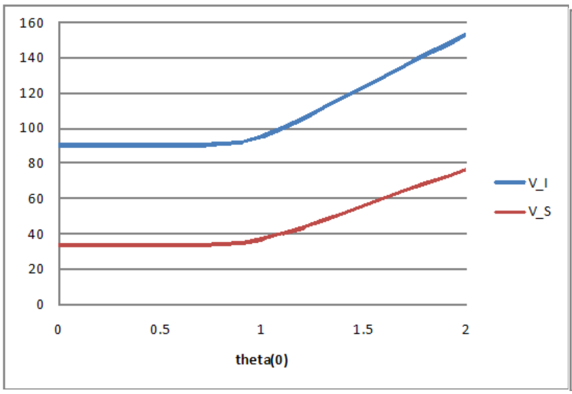

Figure 4:A sensitivity of firms values to $\theta(0)$

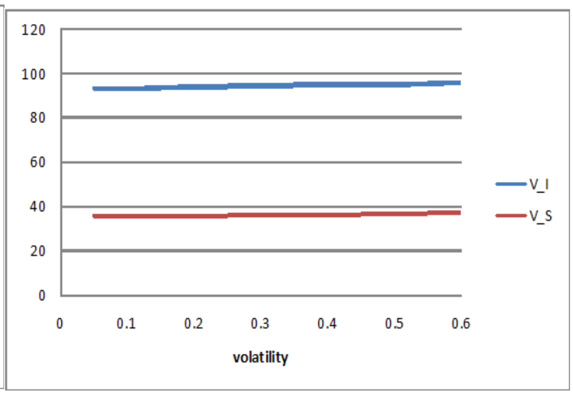

Figure 5:A sensitivity of firms values to σ_θ

Case 2:Cost structures for both firms are asymmetric

Next, we deal with the case where the cost structure of the incumbent firm is different from that of the startup firm, which reflect our motivation outlined in the introduction. The investment costs are given as follows.

$$K_I^{old} = 10, K_I^{new} = 12, K_S^{old} = 15, K_S^{new} = 10$$

It implies that the incumbent firm tends to invest in the old technology since it has many experiences with respect to the old technology while the startup firm tends to invest in the new technology. *Figure 7* shows a sensitivity of the firms values which respect to the initial value of the customers' relative preference $d_I^{old}(0)$, which corresponds to *Figure 4*. In this case the firms values are not monotonic. As for the startup firm it starts increasing around $\theta(0) \approx 0.2$. It can be interpreted that the inequality of $K_S^{old} > K_S^{new}$ forces the startup firm to switch to the new technology even when $\theta(0)$ is relatively small. In fact the thorough analysis in this numerical example reveals that the startup firm start investing in the new technology at the beginning of the project when $\theta(0) > 0.23$ while the incumbent firm still keep investing in the old technology. It is interesting to note that the change of the startup firm's investment strategy affects not only the startup firm's value but also the incumbent firm's value. In case where $\theta(0) \in (0.23, 0.3)$ it increases the incumbent firm's market share and revenue because the startup firm starts investing in the new technology which is not evaluated by the customers at that time. In case where $\theta(0) > 0.3$, on the other hand, there increases a possibility where the customers' will prefer the second function in the future. But the incumbent firm still tends to choose the old technology since investing in the new technology is more costly for the incumbent firm. Consequently, the incumbent firm's value decreases when $\theta(0) \in (0.3, 1.15)$. Finally, when $\theta(0) > 1.15$ both firms' values increases because they invest in the new technology from the beginning.

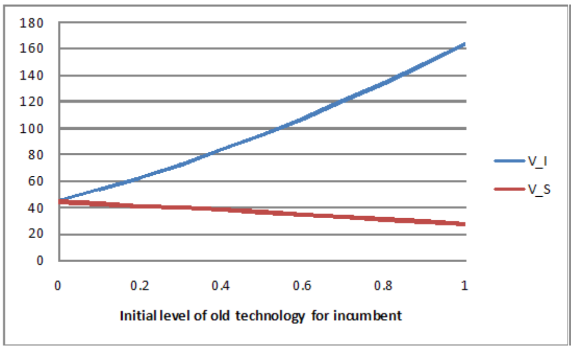

Figure 6:A sensitivity of firms values to $d_I^{old}(0)$

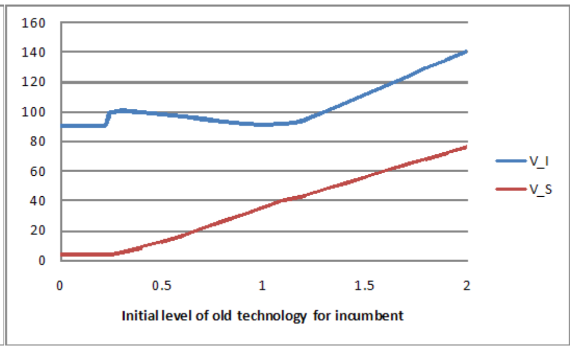

Figure 7:A sensitivity of firms values to $d_I^{old}(0)$

3.3 Analysis along with typical sample paths

Previous examples reveal that the value of the incumbent firm is usually larger than that of the startup firm. This fact, however, does not contradict Christensen[2000]'s innovator's dilemma where a leader firm sometimes loses its dominant position even when its management keeps making rational decisions. This is partly because the firms' values are evaluated ex ante while Christensen's observations are ex post facto. At the beginning of the investment game the incumbent firm can acquire the larger market share and thus, larger revenues due to the ability to produce better products for the customers. Those revenues are accumulated with relatively large discount factors. However, it should be noted that the incumbent firm could be defeated by the startup firm in the future if the customers' preference would be changed rapidly.

Motivating from the above discussion we examine the investment strategies for both firms along with particular time series of the customers' preference $\theta(t), t = 1, \ldots, T$. We use Monte Carlo method to generate samples with respect to the relative preference $\theta(t), t = 1, \ldots, T$, which are governed on the binomial tree. To generate one sample path T iid uniform variates $\mathbf{u} = (u_1, \ldots, u_T)$ are required,i.e., the relative preference increases at time t when $u_t < q_{up}$ and it decrease otherwise. Once we specify the particular path we can easily derive the equilibrium investment strategies for both firms along with the path. Hence, we can derive distributions with respect to the market share and revenue at a given time t.

Case 1:Cost structures for both firms are same

We first consider a case where $K_k^j = 10, k = I, S, j = old, new$ and examine histogram of ex post firms values, market share and their revenues at maturity out of 1000 samples. We confirm that the incumbent firm always dominates the startup firm in terms of the market share at the end of the project. This result is attributed to the fact that since both firms have the same cost structure they always invest in the same technology in response to the relative preference changes, which results in keeping the incumbent firm's relative advantage of the old technology. We have confirmed $d_S^{new}(t) = d_I^{new}(t)$ are always satisfied in the 1000 sample paths, which leads to hold the equation $a_S(t) < a_I(t)$. This result could justify the mitigating strategy for the firm to keep its dominant position.

Case 2:Cost structures for both firms are asymmetric

In this case the technology investment strategies for both firms could be different due to the asymmetric cost structure. *Figure 8* and *Figure 9* show the histogram of both firms' ex post values out of 1000 samples. When we compare the project values for both firms it is confirmed that the value of the incumbent firm is larger than that of the startup firm[7].

Figure 10 and *Figure 11* plot the market share and the amount of revenues for both firms at maturity, respectively. These figures clearly illustrate there could be cases where the incumbent firm would be defeated by the startup firm in the end. Namely, in the presence of uncertainty even the rational firm could lose its market share. To illustrate possible scenarios we pick up three different sample paths. *Figure 12* shows a sample path of $\theta(t), t = 1, \ldots, T$ and resulting revenues for both firms at each time t. In this case the market share and the revenue of the startup firm will go beyond those of the incumbent firm during the lifetime of the project. This is because in the first half of the project where $\theta(t)$ does not change rapidly the incumbent firm keep investing in the old technology while the startup firm starts investing in the new technology. In the second half the incumbent firm switches to invest in the new technology in response to an increase of $\theta(t)$ but never catch up the startup firm's function level because the startup firm keeps investing in the new function from the beginning.

In the next *Figure 13* $\theta(t)$ moves below one which means that the customers relative preference never changes. In this case the incumbent firm keeps investing in the old technology while the startup firm keeps in the new technology that results in the big difference of revenues between the two firms. Finally, in *Figure 14* since $\theta(t)$ starts increasing from the beginning both firms invests in the new technology. Consequently, the revenue of the startup firm never goes beyond that of the incumbent firm during the project lifetime.

These ex post analyses indicate that even though both firms make the same equilibrium strategies in response to the customers preference change, the realized results look quite differently. In the first example after the incident it looks as if the incumbent firm fails to respond correctly against the technology changes and loses its dominant position. The second case looks like the disruptive new technology turns out not to catch the customers' preference expected in advance. The third case looks that the incumbent firm sagaciously responds to the market change and invest in the new technology.

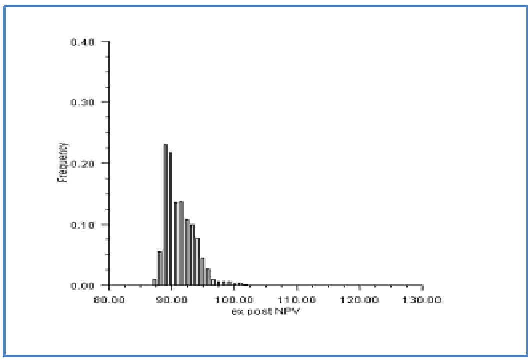

Figure 8 : Ex post distribution of the incumbent firm's project value

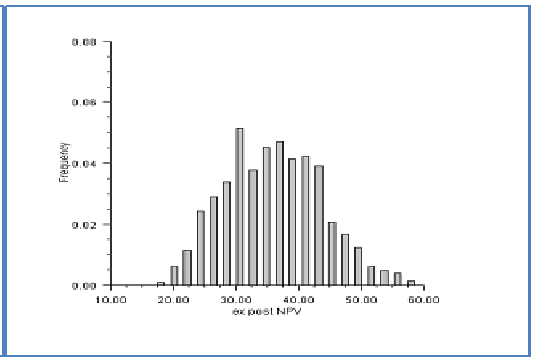

Figure 9 : Ex post distribution of the startup firm's project value

[7]Note the average of the ex post project values converge to the expectation of firm value due to the law of large numbers.

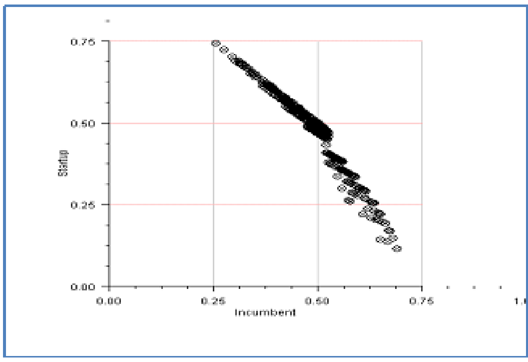

Figure 10: The market shares of both firms at the end of the project maturity

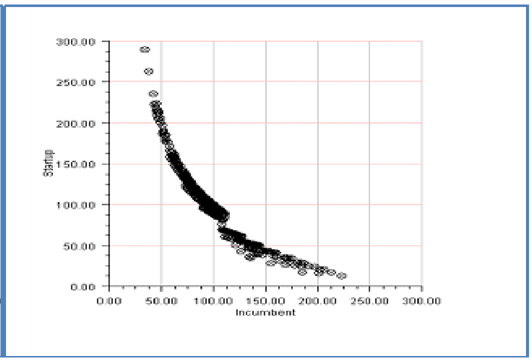

Figure 11: The revenues of both firms at maturity

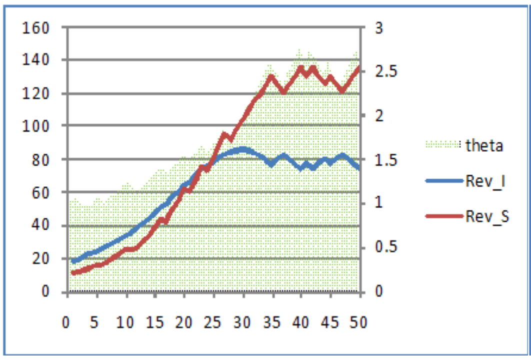

Figure 12: First sample

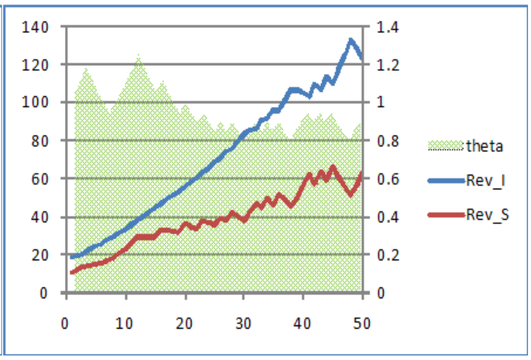

Figure 13: Second sample

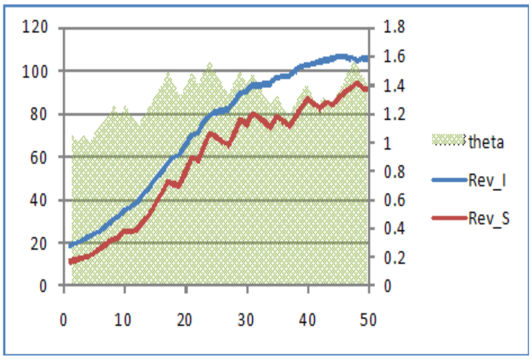

Figure 14: Third sample

4 Concluding Remarks

This paper investigates the equilibrium investment strategies of the incumbent firm and the startup firm under customers' preferences uncertainty. We assume a discrete Markov process as the customers' preference uncertainty. At each discrete time point both firms have options to invest either in the old technology or in the new technology that can improve the level of the corresponding function embedded in the merchandise, respectively.

The incumbent firm, which owns a superior old technology, produces merchandise that can satisfy current customers at the beginning of the investment game. The startup firm, which possesses an inferior old technology, does not capture the customers' satisfaction but it has a possibility to cultivate a new technology that can attract the customers in the future if the customers' preferences are changed. We consider two types of equilibria in our valuation model. The first one is a price equilibrium at each time point derived from the Bertrand competition. The other one is a Markov perfect equilibrium where each firm can invest either in the old technology or in the new technology depending on customers' preferences which are modeled as a Markov process.

Our numerical experiences reveal distribution of the project values, market shares, and revenues for both firms. Furthermore, with the Monte Carlo simulation we derive some possible results that could be interpreted differently by the ex post facto analysis. Especially we analyze the reason the leader firm could be defeated by the entrant firm even if they make rational investment decisions in the presence of uncertainty.

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