Radiata Pine Optimal Stochastic Harvesting Model

EDUARDO NAVARRETE SUAREZ

Universidad de la Frontera, Departamento de Ingeniería de Sistemas

TEMUCO CHILE   enava@ufro.cl

PROSPER LAMOTHE FERNANDEZ

Universidad Autónoma de Madrid, Departamento de Financiación e Investigación Comercial

MADRID ESPAÑA prosper.lamothe@uam.es

Abstract

A new stochastic harvesting model for pine even age stand was developed under wood stock geometrical logistic and price geometric Brown diffusions, with risky decision agents, for a single rotation period.

The application of the model to a Chilean forest company stands increased the actual deterministic optimal cut volume in 70% average. It also showed the dominant role played by wood stock diffusion in relation to price diffusion, which had a non significant effect.

Two new convergent methods for estimating the parameter of the geometric logistic diffusion from unevenly spaced and highly concentrated series under stationary and non stationary state are presented.

Key words: Optimal tree cutting, logistical diffusion, real options

---

1: INTRODUCTION TO THE EXPLOITATION OF PINE RADIATA STANDS

Radiata pine exploitation in Chile

Radiata pine harvest in Chile is an important economical activity, contributes with 3% of its GNP and 13% of its exports, and generates more than 200.000 jobs.

Radiata pine forest is an artificial plantation of homogenous even age stands. Each stand is harvested simultaneously. Its yield depends on the local condition of soil and climate and on the silviculture intervention on the stands.

Optimal tree cutting models

The firsts optimal tree cutting time models were deterministic (see Faustman, 1845; Fisher, 1907 and Samuelson, 1976), and did not consider the irreversibility of forest investment, its price and its growth uncertainty. This investment type has significant operational flexibility, cutting time, soil abandonment and alternative soil use, which are the natural conditions for using Real Option models, (see Mascareñas et al, 2004).

The majority of Real Options early papers (see Clark & Reed, 1989; Thomson, 1992; Platinga et al, 1998; Insley, 2002; Insley & Rollins, 2005) considered price stochastic diffusion. Very few papers, such as Morck et al (1989) and Alvarez & Koskela (2006), considered also the wood stock diffusion. This last paper is the only one that separates both types of diffusions, price and stock, using an impulsive control model, in an agent neutral risk environment, which can be considered as a stochastic extension of Fisher model.

Research objective

Most of the revised research papers refer only to stochastic price diffusion; the only two papers which also considered wood stock stochastic diffusion did not applied their methodology to a real case. Given the dominant character that wood stock stochastic diffusion plays in the deterministic optimal cutting evaluations (see, Samuelson 1976; Faustaman, 1845), it is important to compare the impact in the optimal cutting policy of price and wood stock diffusions volatilities.

The objective of this paper is to evaluate whether the wood stock sigmoid diffusion has a dominant impact on the stochastic optimal cutting policy under risky decision agents.

2: OPTIMAL STOCHASTING HARVESTING MODEL

Linear diffusion equations

The model considers a geometric ITO diffusion for the wood stock and a geometric Brown diffusion for the wood price, respectively given by equations [1] and [2].

$$dV_t = \mu(V_t)\, dt + \sigma(V_t)\, dW \qquad [1]$$

$$dP_t = \alpha P_t dt + \beta P_t dW \qquad [2]$$

with:

V_t = Wood stock at time t

$\mu(V_t)$ = Wood stock diffusion drift rate

$\sigma(V_t)$ = Tree growth volatility

P_t = Wood stumpage spot price at time t

α = Wood price diffusion drift rate

β = Wood price volatility

Objective functional value

Under the assumption of a weak solution (V_t, t) of the diffusion equations [1, 2] and initial conditions ($V_0 \geq 0$, $P_0 \geq 0$), the Vicksellian single harvest functional objective is given by [3] (see Johnson, 2006; Alvarez & Koskela 2006),

$$F(V_0, P_0) = \frac{\sup}{\forall(t \geq t_o)} E^P(e^{-rt} P_t V_t) \qquad t_0 = \inf(t \geq 0 : V_t \leq 0) \qquad [3]$$

Reformulation of the Vicksellian problem

The stochastic model [1, 2, and 3] is difficult to solve. The following theorem 1 reduces it to a more amendable one dimensional stopping problem.

***Theorem 1*:** A probabilistic measure Q exists and is equivalent to the actual metric P, such that

$$F(V_0, P_0) = \frac{\sup}{\forall(t \geq t_o)} E^P(e^{-rt} P_t V_t) = P_o \sup E^Q [e^{-(r-\alpha)t} V_t] \qquad [4]$$

Furthermore, under the metric Q, the process V_t follows the diffusion

$$dV_t = \{\mu(V_t) + \beta\sigma(V_t\}_t dt + \sigma V_t d\overline{W} \qquad [5]$$

Proof. Replacing $P_t = P_0 e^{\alpha t} M_t$ in [3], since $M_t = \exp\{\beta W_t - 1/2\beta^2 t\}$ is a martingale, a new metric Q can be defined via the Radon-Nikodym derivative as $dQ/dP = M_t$, and considering that in this case β is positive, a straightforward application of Girsanov´s theorems I and II (Oksendal, 2000, pages 155-157), yields [4] and [5].

Dynkins lemma can be use to developed an economic metric to optimize the stands growth.

Lemma: Dynkins formula, (see Oksendal, 2000)

Given a function g: $[0,\infty] \rightarrow R$ is C^1, with continuous first derivatives for any weak solution V_t of [5]; the function g (V_t) satisfies de Dynkin formula [6]

$$E^Q[\, e^{-(r-\alpha)\tau} g(V_t)\,] = P_o V_o + E^Q \int_0^{\tau} e^{-(r-\alpha)T} \Delta g(V_t) dt \qquad [6]$$

with $\Delta g(V_t) = ½\, \sigma^2 g''(V_t) + [\mu(V_t) + \beta\sigma V_t]\, g'(V_t) - (r\text{-}\alpha)\, V_t$

Corollary. Sience there is no correlation between price and volume diffusion, the expected net present value of the harvested stock at any future date T is given by

$$E^P(V_0,P_0)[\, e^{-rT} P_T V_T] = P_0\, V_0 + P_0 E^Q \int_0^{T} e^{-(r-\alpha)T} \pi(V_s) ds \qquad [7]$$

with the expected net economic wood stock metric growth $\pi(V_t)$ given by

$\pi\, (V_t) = \mu(V_t) + \beta\sigma\, V_t - (r\text{-}\alpha)\, V_t$

Proof. Applying the Dynkin formula to equations [4] and [5], and taking $g(V_t) = V_t$

$\Delta V_t = \pi\, (V_t) = \mu\, (V_t) + \beta\sigma\, V_t - (r\text{-}\alpha)\, V_t$

and

$$E^Q[e^{-(r-\alpha)\tau}\,] = V_0 + E^Q \int_0^{T} e^{-(r-\alpha)T} \pi(V_s) ds$$

which completes the proof.

Obviously, if the metric π (V) is positive for all values of V, there is no admissible optimal cut strategy. If it is negative for all V, the optimal strategy is to cut the stands immediately. If it is concave, it will achieve its maximum at the following Alvarez threshold point.

$$\frac{\partial \mu(V)}{\partial V} = (r - \alpha) - \beta\sigma \quad [8]$$

In this late case the optimal solution of the optimal stopping problem [4] subjected to the linear diffusion [5] is given by the following Hamilton-Jacobi-Bellman [HJB] equation (see Johnson, 2006).

$$\mathrm{Max}[\ \tfrac{1}{2}\sigma^2 V^2 F''(V) + [\mu(V) - \beta\,\sigma(V)]\ F'(V) - (r-\alpha)\ F(V)\ ,\ V - F(V)\ ] = 0 \qquad V \geq 0 \qquad [9]$$

Under the assumption of the existence of a frontier V* that divides the zone in two, a continuation (no-cutting), and stopping (immediate-cutting), the solution to the equation HJB is finally given by:

If V< V*, continuation region

$$\tfrac{1}{2}\sigma^2 V^2 F''(v) + [\mu(V) - \beta\,\sigma(V)]\ F'(V) - (r-\alpha)\ F(V) = 0 \qquad [10]$$

If V≥V*, stopping region

$$V - F(V) = 0 \qquad [11]$$

A solution of [10] is given by [12] (see Johnson T.C, 2006).

$$F(V) = \begin{cases} A\Psi(V) + B\Phi(V) & V < V^* \\ V & V \geq V^* \end{cases} \qquad [12]$$

Where Ψ (resp., Φ) is strictly increasing (resp., decreasing), functions since the payoff function are bounded, and small and V are positive and should remain bounded and positive as $V \to 0$, necessarily then $B \to 0$. The solution must also fulfill the so-called "smooth–pasting" condition at the free boundary point V*. So that

$$A\ \Psi(V^*) = V^* \quad \text{and} \quad A\,\Psi'(v^*) = 1 \qquad [13]$$

$A = v^*/\ \Psi(V^*) = 1/\Psi'(V^*)$ so V* must fulfill the following equation:

$\Psi (V^*) = V^* \; \psi'(V^*)$ [14]

$$W(V_0,P_0) = \begin{cases} PoV & V \leq V^* \\ Po\dfrac{\psi(V)}{\psi'(V^*)} & V \geq V^* \end{cases} \quad [15]$$

This equation is similar to theorem 2.8 developed by Alvarez and Koskela (2006), using the characterization of the excessive functions for linear diffusion.

3: LINEAR DIFFUSION MODELS

Diffusion characteristics

Stands stochastic diffusion growing models are not common in the bibliography and its basic references are in the optimal cutting models. The geometric Brown diffusion and the geometric Ornestein-Ulhembeck are the most common models. The basic requirement of the stands growing diffusion is its sigmoid pattern (see Garcia, 2003).

Geometric logistical diffusion

The logistic diffusion is an especial case of the geometric Ornestein-Ulhembeck model given by the expression [16].

$dV = \mu V (1- \gamma V) dt + \sigma V dw$ [16]

This expression grants the sigmoid form of the tree growing pattern, and the determination of its parameter can be easily done using the following ITO transformation:

If V = Ln(V), then by ITO $dv/ dV = 1/V$ and $d^2v/ d^2V = - 1/V^2$

$d\, Ln(V) = [\, \mu V (1- \gamma V)(1/V) - ½ \sigma^2 V^2/V^2] dt +(1/V) \sigma V dw$ [17]

which simplifies the differential equation [18].

$$dLn(V) = [\mu - \tfrac{1}{2}\ \sigma^2 - \mu\gamma V]\, dt + \sigma\, dw \qquad [18]$$

For a non homogenous time interval of the sample series Δt, equation [18] can be linearized by making dt = Δt and $\partial w = \sqrt{\Delta t}\,\delta_t$, with $\delta_t = N(0,1)$, and dividing equation [21] by Δt we finally arrive at equation [19]

$$dLn(V)/\Delta t = [\mu - \tfrac{1}{2}\ \sigma^2] - \mu\gamma V_{t-1} + (\sigma/\sqrt{\Delta t})\, \delta_t \qquad [19]$$

which can be fitted using linear regression [20]

$$R_t = a + b V_{t-1} + \varepsilon_t \qquad [20]$$

with $R_t = d\, Ln(V)/\Delta t$,

$a = \mu - \tfrac{1}{2}\sigma 2$ and $b = -\mu\gamma$

Volatility estimation by Quadratic variation of the stochastic processes

The property of quadratic variation for the stochastic processes, can be expressed by equation [21], see Ewald C. and Yang Z. (2007)

$$d[V_t, V_t] = \sigma^2 V_t^2\, dt \qquad [21]$$

Applying it to the discrete interval Δt and isolating the volatility we arrive at

$$\sigma_t^2 = \frac{[V_{t+1} - V_t]^2}{V_t^{\,2}\Delta_t} \qquad t = 1,2 \quad ,n \qquad [22]$$

Taking the mean value for the entire interval, we arrive at the following estimator of the volatility

$$\overline{\sigma^2} = \frac{1}{n-1}\sum_{1}^{n}\frac{[V_{t+1} - V_t]^2}{V_t{}^2\Delta_t} \qquad [23]$$

Stationary properties of the logistic diffusions

Merton (1975) showed that in the case of the geometric mean reversion, (logistical diffusion) the stationary Gamma equilibrium distribution is achieved when time tends to infinity, such as the age of the stands exploitation window.

Ewald & Zhang (2007) showed that the stationary moments for the Gamma distribution can be evaluated by the following recursive equation as function of the diffusion parameters:

$$E(V^{n+1}) = E(V^n)[\ 1/\gamma + (n-1)\sigma^2/(2\mu\gamma)] \qquad [24]$$

with $$E(V^n) = \overline{V^n} = \frac{\sum_{1}^{n} V_i^{\,n}}{n} \qquad [25]$$

Since only the two first moments are independent, see equations [26,27], the stationary parameter can be evaluate by equations [28,29] and the third parameter, the volatility "σ", can be evaluated by the quadratic variation method, as shown in equation [23].

$E[V^1] = 1/\gamma - \sigma^2/(2\mu\gamma)$ and [26]

$E[V^2] = 1/\gamma[1/\gamma - \sigma^2/(2\mu\gamma)]$ with [27]

$\gamma = E(V^1)/E(V^2)$ and [28]

$$\mu = \frac{\sigma^2}{2(1 - \gamma\overline{V^1})} \qquad [29]$$

4: EXPERIMENTAL DATA AND PARAMETER FITTING

Experimental data

The experimental data have been delivered by a Chilean forest company. These data pertain to 128 harvest stock of its stands between 1999 and 2005 and come from different sample plots, which belong to places with site index between 30 and 35 meters and which represent sites with forest aptitude one. These data are concentrated in the narrow time window between 20 and 26 years and with its biggest concentration between 20 and 22 years. This window makes very difficult to form significant representative temporal series of the growing processes. The age of the stand was calculated in fraction of years of non homogenous time intervals. In order to form an increasing age series, similar age stands were randomly eliminated forming a 107 temporal series single plot (see appendix 1). The total volume of commercial wood per hectare, in m^3/ha (VOLT/HECT), was calculated as the sum of pulp, industrial wood and pruning wood. The series for the complete site index range 30/35 is ploted in figures 4.1.

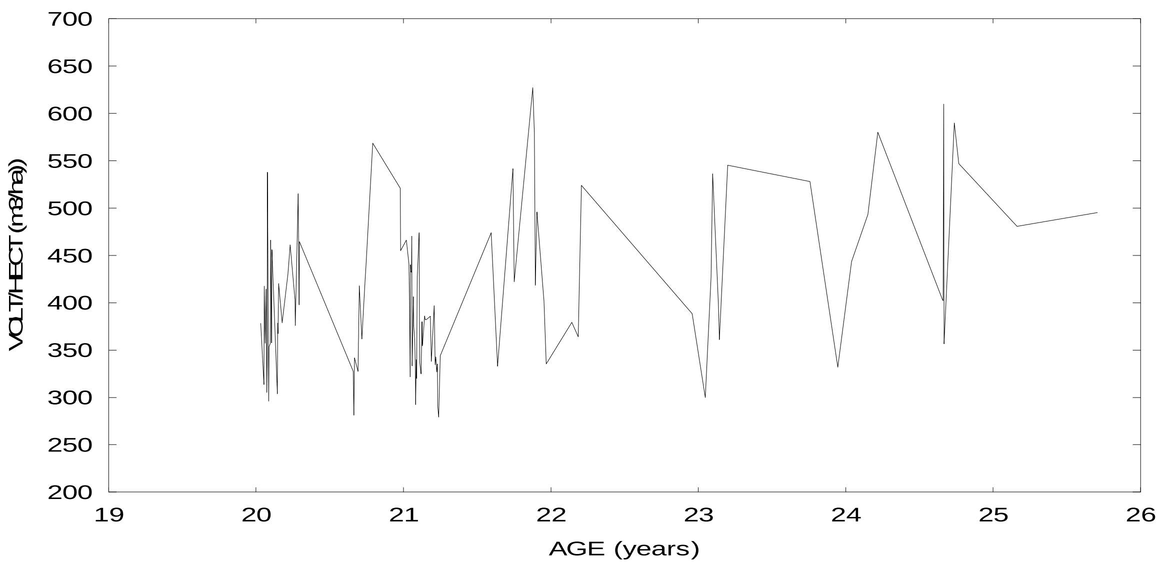

Figure 4.1 VOLT/HECT (m^3/tree) versus years, single plot

The series was also disaggregated by site index 30-31; 31 -32; 32-35 meters, in order to evaluate the impact of the site index in the stand volatility, see figure 4.2.

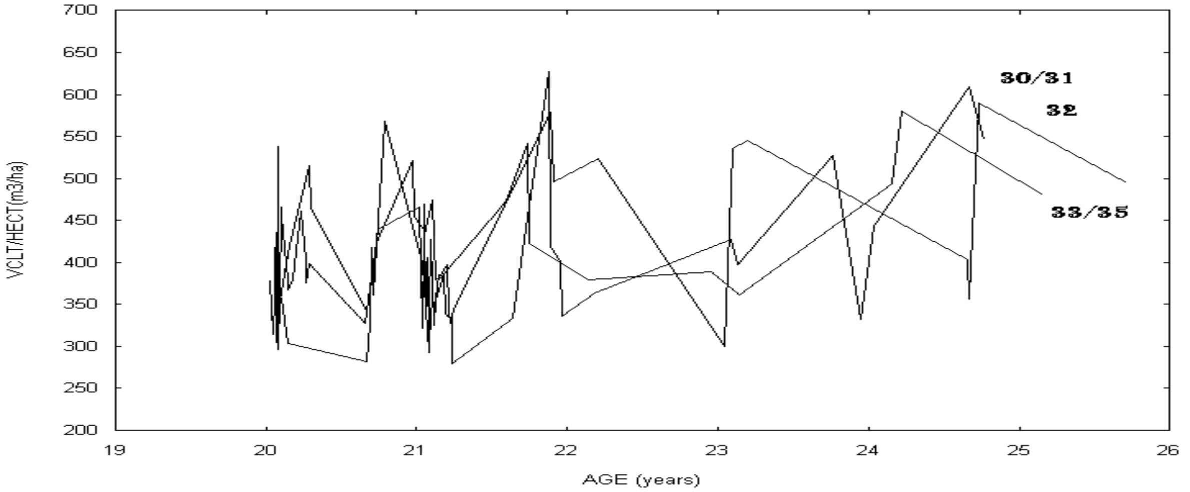

Figure 4.2. VOLT/HECT (m3/tree) for different stands (sites index: 30-31; 31-32; and 32-35 meters)

Logistic diffusion fitting

The Logistic diffusion model was fitted using the modified regression model, and the volatility was calculated using the quadratic variation method. The results are summarized in table 4.1 for the annual single sample plots, and for the three disaggregate site-index stands.

Table 4.1 Logistic diffusions, non stationary parameter estimation.

Site Index	Data	$\bar{a}$	$\bar{b}$	$\bar{\sigma}$	μ	γ
30/35	107	107.209	-0.263	2.856	111.186	0.00233
30/31	32	19.429	-0.053	1.451	20.481	0.00259
32	50	73.072	-0.185	2.422	76.006	0.00243
33/35	25	57.020	-0.134	1.417	58.024	0’.00231

Observing table 1, it is possible to conclude that the site index has only a moderate capacity to predict the stock volatility. Since the most representative and reliable site index 32 meters series only reduces the stock volatility of the global site index series 30/35 in 15,17 %.

Stationary parameter estimation

The stands exploitation window must occur near or in the stationary or asymptotic state of the logistic diffusion. Under this assumption the logistic parameter was evaluated using equations [28, 29], the results are shown in table 4.2.

Table 4.2. Logistic diffusions, stationary parameter estimation

Site Index	Data	$\overline{V^1}$	$\overline{V^2}$	$\overline{\sigma}$	μ	γ
30/35	107	406.85	171,877.5	2.856	114.61	0.00237
30/31	32	390.07	160,624.4	1.451	20.184	0.00243
32	50	409,52	173,260.2	2.422	87.468	0.00236
33/35	25	422.98	183,515.7	1.417	36.983	0.00230

The results of table 4.2 show an important convergence between the stationary and the non stationary parameters estimation, especially for the more reliable series 30/35 and 32 meters. This convergence justifies the assumption of the stationary behavior for the exploitation window.

Stands diffusions conclusions

Two observations can be concluded from the fitting results of tables 4.1 and 4.2. The first one is based on the amount of data and the aggregation level. The site index series 30/35 and 32 are the more representative and reliable. In the second place, given the stationary state assumption of the logistic diffusion in the harvest window, it is better to use the stationary parameter estimation to avoid the concentration gap.

Wood price diffusion

Forest owners value their standing tree plantation in relation to stumpage price, discounting the harvest and transportation costs according to local market prices of the saw or pulp wood.

Under the assumption that the stumpage prices are highly correlated to the commercial prices of the logs, we use the historical prices of Chilean markets between the years 1985 and 2005 in order to fit the Brown diffusion for the commercial and pulp stumpage prices. The parameter of the price diffusion is shown in table 4.3. These results were corroborated by the stumpage commercial pulp prices (of) for a longer series of New Zealand, dating from 1955 to 2002.

The historical volatility for the Brown geometric diffusion processes can be easily calculated from the following modified price diffusion, by the ITO expression,

$$\partial \ln(P) = (\alpha - 1/2\sigma^2)\partial t + \sigma.\partial w \qquad [30]$$

Linearicing this expression, by using $\partial t = \Delta t$, $\partial \ln(P) = \ln(P_{t/}P_{t-1})$ = r_t and $\partial w = \sqrt{\Delta t}\Phi$, we obtain the following equations [34] which were used to calculate those parameters.

$$\bar{r} = \sum_{t=1}^{n} \frac{r_t}{n} = (\alpha\text{-}1/2\sigma^2) \qquad \sigma^2 = \frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2 \qquad [31]$$

The standard deviation of this series is the estimator of the historical volatility and the mean is equal to the ITO modified drift expression. The summary of Brown diffusion parameters of the pulp commercial and stumpage prices is given in Table 4.4

Table 4.3. Nominal Radiata Pine logs exportation prices

Years	Saw logs US$FOB/mts^3	Pulp log US$FOB/mts^3	Years	Saw logs US$FOB/mts^3	Pulp logs US$FOB/mts^3
1985	32	27	1996	65	52
1986	34	28	1997	62	55
1987	39	27	1998	52	54
1988	45	27	1999	49	53
1989	43	27	2000	46	42
1990	49	32	2001	48	34
1991	51	40	2002	46	41.6
1992	47	40	2003	45.9	37.4
1993	85	38	2004	48.6	33.0
1994	63	46	2005	57.0	33.5
1995	67	43			

Source: CONAF-INFOR

Table 4.4 Brown geometric processes**,** price diffusion parameters

WOOD	ANUAL DRIFT (%)	PRICE INCREASE DUE LOG THICKENING[1]	DETERMINISTIC DRIFT %	ANUAL VOLATILITY	%
Saw Wood	2.89	0.86%	1.3	17.35%	80.67%
Pulp wood	1.08	1.3%	1.9	13.17%	19.33%
Stumpage wood	2.54	0.9%	1.42	16.7%	
TOTAL	3.44		2.32		

[1] See E. Navarrete, Doctoral thesis UAM, 2004

Capital cost determination

The capital cost was estimated by CAPM model of the sector (see Brealey, Myers & Allen 2006):

$K_e = R_f + \beta (E(R_m) - R_f) = 5.3 + 0.67(12.7 – 5.3) = 10.3$ [32]

With

K_e	: Chile equity capital cost	
R_f	: Chile free risk rate	5.3%
$E(R_m)$	: Expected return of Chilean the market	12.7%
β	: Beta of Forest commercial company	0.67

5: RADIATA PINE STOCHASTIC VICKSELLIAN HARVESTING MODEL

Deterministic solution

Single rotation deterministic optimal cut

In the case of the single rotation the deterministic optimal cutting must fulfill the following optimal condition:

$$\frac{\partial(P_t V_t)}{\partial t} = (r - \lambda) P_t V_t \qquad [33]$$

Given the following deterministic drift for price and wood stock

$$\frac{\partial P_t}{\partial t} = \lambda P_t \; \frac{\partial P_t}{\partial t} = \lambda P_t \quad \text{and} \quad \frac{\partial V_t}{\partial t} = \varepsilon V_t (1 - \eta V_t) \qquad [34]$$

With the initial conditions (V_0, P_0), these equations integrate to

$$P_t = P_0 e^{\lambda t} \quad \text{and} \quad V_t = \frac{1}{(\eta + e^{-(\varepsilon t + c)})} \quad \text{with } c = \ln \frac{V_0}{1-\eta V_0} \qquad [35]$$

Replacing these equations in condition [33], the optimal Vicksellian deterministic cutting is given by

$$V^D = \frac{\varepsilon + \lambda - r}{\varepsilon \eta} \qquad [36]$$

The deterministic parameters for the wood stock can be easily obtained linearizing equation [34] by using the method developed for the logistic drift without the ITO correction, and are equal to the estimations of "a" and "b" of table 4.1. Taking λ = 2.32 and r= 0.103 and replacing this values in equation [36], the deterministic optimal cut is given in the table 5.1.

Table 5.1: Deterministic optimal cut

Site Index	$\varepsilon = a$	$\eta = -a/b$	Optimum	Actual average
30/35	107.11	0.00246	414.92	406.85
32	73.02	0.00253	407.26	409.52

Stochastic solution

Alvarez and Koskela (2006) economic expected net growth of the harvested stock, π(V) is given by the expression [37] for the logistic wood stock diffusion:

$$\pi(V) = \mu V(1-\gamma V) + \beta\sigma V - (r-\alpha)V \qquad [37]$$

This is a concave function which achieves a maximum at the threshold $V^{A,}$ that makes its derivative cero.

$$V^A = \frac{\mu - (r-\alpha) + \beta\sigma}{2\gamma\mu} \qquad [38]$$

In this case the stochastic solution for the case of a logistic geometric diffusion, with μ (V) = μV (1- γV) and $\sigma_V = \sigma V$, is given by the following differential equation:

$$\tfrac{1}{2}\sigma^2 V^2 F''(v) + [\mu V(1-\gamma V) + \beta\sigma V] F'(V) - (r-\alpha) F(V) = 0 \qquad [39]$$

Dixit & Pindyck (1994) propose the following function to solve the ordinary differential equation [29]:

$$\Psi(V) = V^{\theta} f(V)$$

Replacing $W = A\Psi(V) = AV^{\theta}f(V)$, the ordinary differential equation of the continuation region takes the following form:

$$V^{\theta}f(V)[\tfrac{1}{2}\sigma^2\theta(\theta-1)+(\mu+\beta\sigma)\theta-(r-\alpha)]+V^{\theta+1}[\tfrac{1}{2}\sigma^2 Vf''(V)+(\sigma^2\theta+\mu\{1-\gamma V-\beta\sigma\}f'(V)-\mu\gamma\theta f(V)]=0$$

This will be true for all V>0 only if

$$\tfrac{1}{2}\sigma^2\theta(\theta-1)+(\mu+\beta\sigma)\theta-(r-\alpha)=0 \quad \text{and} \qquad [40]$$

$$\tfrac{1}{2}\sigma^2 Vf''(V)+(\sigma^2\theta+\mu\{1-\gamma V+\beta\sigma\} f'(V)-\mu\gamma\theta f(V)=0 \qquad [41]$$

Taking the positive root of equation [40]

$$\theta_\sigma = \frac{1}{2} - \frac{\mu}{\sigma^2} - \frac{\beta}{\sigma} + \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2} - \frac{\beta}{\sigma})^2 + \frac{2(r-\alpha)}{\sigma^2}} \quad . \qquad [42]$$

Equation [41] corresponds to the Kummer equation with the following hipergeometric solution series

$$M(x;a;b) = 1 + \frac{a}{b}\frac{x}{1} + \frac{a(a+1)}{b(b+1)}\frac{x^2}{2!} + \frac{a(a+1)(a+2)}{b(b+1)(b+2)}\frac{x^3}{3!} + + \qquad [43]$$

with parameters x= $\frac{2\mu\gamma V}{\sigma^2}$, a= θ b=2θ + $\frac{2(\mu+\beta\sigma)}{\sigma^2}$.

The solution of ψ for the continuation region is

$$\psi_\sigma = V^\theta \, M\{ \frac{2\mu\gamma V}{\sigma^2}, \theta , 2\theta + \frac{2(\mu+\beta\sigma)}{\sigma^2} \} \qquad [44]$$

The optimal solution V^{opt} is given by the smooth pasting condition [14]. It was programmed in Maple and the optimal values are shown in table 5.2 for annual VOLT/HECT 30/35 and 32 meters site index diffusion series.

Table 5.2 Vicksellian optimal stochastic cut

Site index (mts)	μ	γ	$\overline{\sigma}$	α	β	V^{opt} m^3/ha
30/35	114.61	0.00237	2.856	0.034	0.167	716.082
32	87.47	0.00236	2.422	0.034	0.167	692.349

Table 5.3 summarizes the optimal cut policy results for both site index models 30/35 and 32 meters, and compares them with the deterministic optimal value V^D and the actual cut volume $\overline{V}$ in value and in % increase. For the 30/35 and the 32 site index series, the stochastic optimum increases the deterministic optimal cut in 76,01% and 69,07% , and the actual average cut in a 75,58% and 70,00% respectively.

Table 5.3 Variable VOLT/HECT, optimal cutting results

Site index (mts)	V^{opt} m^3/ha	$\overline{V}$ m^3/ha	% de $\overline{V}$	V^D m^3/ha	% de V^D
30/35	716.082	406.85	76.01	414.918	72.58
32	692.349	409.50	69.07	407.258	70.00

A sensitivity analysis of the effect of the stock and price volatility over the optimal stochastic cut was done. The results for the stock volatility are shown in figure 5.1 and for price volatility in figure 5.2.

These figures clearly shows the dominant effect of the stock volatility. A 20% increase in the actual stock volatility of the site index 30/35 series produces a 7,35% increase in the optimal cut. The price volatility has no significant effect a similar 20% increase on the same series will only produce a 0, 00084 % increase of the optimal cut value. This result is crucial for the application of Real Option models which normally evaluate this type of problems, without considering the wood stock volatility effect.

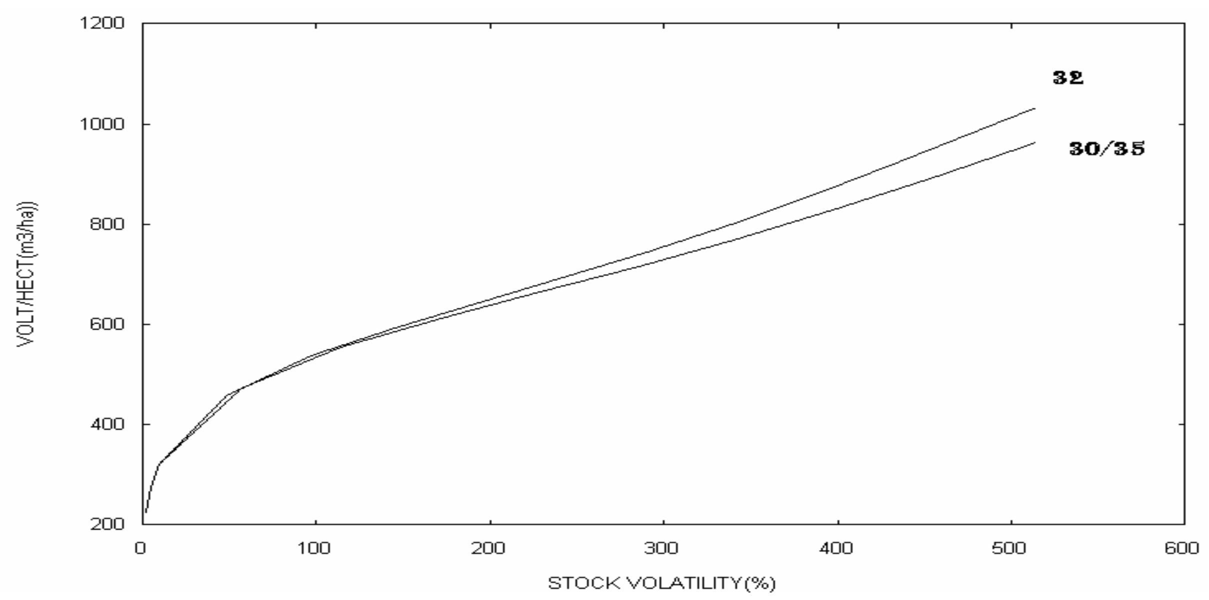

Figure 5.1. Optimal tree cut volume wood stock volatility sensitivity analysis

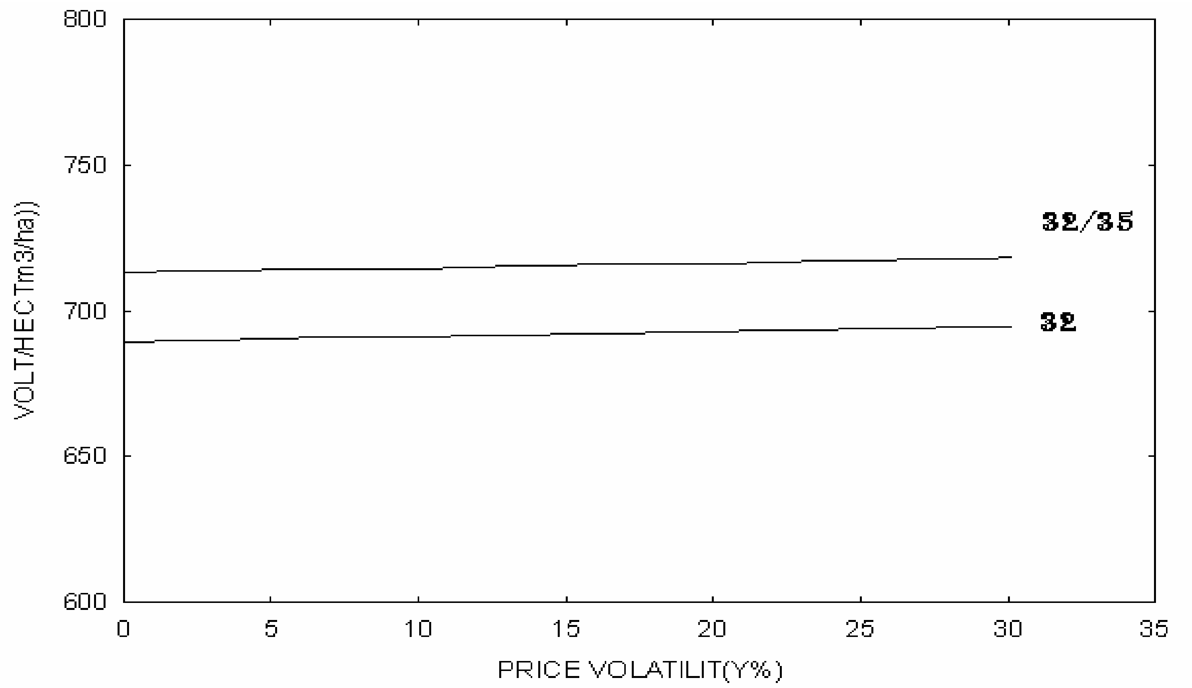

Figure 5.2 Optimal tree cutting volume price volatility sensitivity analysis

6: CONCLUSIONS AND RESULTS

1. The actual cutting policy is similar to the Vicksellian deterministic optimal cutting volume, showing that the company gives no consideration to stochastic effects.

2. The effect of diffusion stock and price volatility is significant, and from the theoretical point of view it increases the optimal stochastic cutting policy for the 30/35 stand in 72,58% from the deterministic optimal cutting.

3. The wood stock diffusion is the dominant stochastic process. An increase of 20% of its volatility will increase the optimal cutting in 7, 53% for the site index series 30/35. A similar increase in the price volatility, 20% does not produce a significant increase for the same series.

4. The site index 32 disaggregate stands diminished natural wood stock volatility of the aggregation 30/35 in more than 15,17% and actually is the only parameter that business uses to evaluate wood growth volatility.

5. Obviously, the Vicksellian models are subjected to Samuelson (1976) critics, since they do not considers the rent of the land used as forest resource. Correction of this objection, such as Faustman (see Samuelson, 1976) model, should give a lower optimal cutting.

6. The other important model restriction the narrow data windows, forces the logistical curves to achieve their stationary state. This gap justifies the selection of the stationary parameter.

7. Finally, several innovations are developed in the present article. A new model for stochastic optimal harvesting with price and stock stochastic diffusion for risky decision agents from the Vicksellian point of view is proposed. A solution based on the Hamilton-Jocobi-Bellman differential inequation is developed for the models. Two approximate methods were developed to estimate the parameter of the logistical diffusion from data unequally distributed in time and highly concentrated. The first method applies under the normal non stationary assumption state and the second applies under the stationary limiting state of the logistical diffusion. Both methods produced convergent parameter that validate the state stationary assumption of the wood diffusion and justify the selection of the second method to reduce the effect of the concentration gap of the harvest window of data.

REFERENCES

1. Alvarez L.H.R. Koskela E., (2006). “Optimal Harvesting under Resource Stock and Price Uncertainty,” *Journal of Economics Dynamics & Control*, doi: 10.1016 /j.jidc 2006, 08.003.
2. Brennan M.E., Schwartz E., (1985). “Evaluating natural resource investments”, *J. of Business* 58(2): 135-157.
3. Brealy, R.A., Myers, S.C., Allen, F., (2006). *Principles of Corporate Finance*, McGraw-Hill, New York (8 ed).
4. Clarke R., Reed W., (1989). “The Tree Cutting Problem in a Stochastic Environment,” *Journal of Economics Dynamics and Control* N°13. 569-595.
5. Dixit A., Pindick R., (1994). *Investment under uncertainty*, Princeton, University Press.
6. Ewald, C.;Yang Z.(2007) Geometric Mean Reversion: Formulas for the Equilibrium Density and Analytical Moment Matching.Preprint.
7. Faustaman M., (1849 translation 1995). “Calculation of the Value which Forest Land and Immature Stands posses for Forestry,” Republished, *Journal of Forest Economics* 1: 7-44.
8. Fisher F., (1907). *The Rate of Interest*, New York, Macmillan.
9. Fundación Chile, (2000). *Modelos de Simulación de Pino Radiata y Eucaliptus*.
10. Garcia O., (2005) *Unifying Sigmoid Univariate Growth Equations*, FBMIS.
11. Hanson T., Ryan D., (1998). “Optimal harvesting with population a price dynamics,” *Mathematical Bioscience*. 1-18.
12. Hull J., (2000). *Options Futures & Others Derivatives*, Prentice Hall.
13. Insley M., (2002). “A Real Option Approach to the Valuation of a Forestry on Investment,” *Journal of Environmental Economics and Management*. Vol. 44, 471-492
14. Insley M., Rollins K., (2005). “On solving the multi-rotational timber harvesting problem with stochastic prices: a linear complimentarily formulation.” *American Journal of Agriculture Economics* 87, 735-755.
15. Johnson T.C., (2006). *The optimal Timing of Investment Decisions*, PhD thesis, University of London.

16. Lungu F., Oksendal B., (1996). "Optimal Harvesting from a Population in a Stochastic Crowded Environment," *Mathematical Bioscience*, 145, 47-75.
17. Mascareñas J., Lamothe P., López F., Luna W., (2004). *Opciones Reales y Valoraciones de Activos*, Prentice.
18. Merton R. (1975), An asymptotic Theory of Growth under Uncertainty, Reviews of Economics Studies, 42,375-393.
19. MININCO (2006). Santibáñez P. Los Angeles.
20. Morck, R., E. Schwartz, D. Stangeland, 1989. "The valuation of forestry resources under stochastic prices and inventories", *J. Finance*. *Quant*. Anal. 24: 473-487.
21. Navarrete E., (2004). *Determinación del tiempo de corta óptima de una plantación de pino radiata mediante opciones reales*, PhD Thesis Universidad Autónoma de Madrid
22. Newman D. H., (1988). *A Discussion of the Concept of the Optimal Forest Rotation and a Review of the Recent Literature, USDA Forest Service*, Southeastern Forest Experimental Station General Technical Report SE-48.
23. Oksendal, B., (2000). *Stochastic Differential Equations*, (Faith Ed.) Springer Verlag.
24. Plantinga A.J., (1998). "The optimal timber rotation: An option approach," *Forest Science*, 44 1992-202.
25. Samuelson P, (1976). "Economics of Forestry in an evolving society," *Economic Inquiry* 14. 466-491.
26. Saphores J. (2003), "Harvesting a renewable resource under uncertainty." *Journal of Economics Dynamics and Control* 28, 509-529. .
27. Thompson T.A., (1992). "Optimal forest rotation when stumpage prices follow a diffusion processes," *Land Economics*, 68, 329.