The value of operational flexibility by adding thermal to hydropower – a real option approach

by
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Abstract:

This paper presents a valuation study of operational flexibility for a hydropower operator restricted by contracts to deliver a steady flow of electricity to the contract counterpart. The hydropower operator has the flexibility to deliver from own production of hydro-electric generation, or deliver by buying option contracts of electricity from thermal electricity producers. The option may be in the form of a call option, or may be an implicit option created by having a separate thermal electricity plant that can be switched on and off. Long term industry contracts can make some operators obligated to always generate at a certain minimum level. Such operators cannot save the water in the reservoirs for peak price periods if this action compromises their ability to deliver the contracted minimum. If thermal generation is added and controlled, flexibility is enhanced and hence more generation can be allowed in peak price periods.

To assess this value of operational flexibility the switching option model of Kulatilaka (1988) is applied. The numerical calculations, introducing nuclear, coal fired or gas fired generation, show an option value for a hydro operator also controlling thermal generation of NOK 65 / NOK 45 / NOK 13, respectively, per MWh yearly generation capacity.

Key words: Operational flexibility, Real options, Electricity generation

1 Introduction

The focus of the paper is to assess the value of operational flexibility of a hydro based operator who has the possibility to add thermal power to his production. If a hydropower operator is restricted in minimum generation due to for example long term industry contracts, there is an added operational flexibility when thermal generation could alternatively be used at a cost lower than the current spot price of electricity. The added flexibility would intuitively represent value, if the option is optimally exploited. A key point in understanding the Norwegian (and Nordic) electricity market is the seasonal pattern of prices. Electricity demand is connected to heating requirements (31 % in 2001) (The Ministry of Petroleum and Energy, 2006a), which for obvious climatic reasons is much higher in the winter period compared to other seasons. The integration of thermal generation would therefore provide some obvious benefits for an operator restricted in scheduling planning by industry contracts. By using thermal power instead of hydropower in some parts of the year, in order to produce relatively more in peak price periods, one should yield an extra value, a premium, which must be taken into consideration when buying or renting thermal generation capacity. The research question for this paper is:

- What is the value of operational flexibility in generation when controlling thermal generation in addition to hydro?

The purpose of this paper is to calculate the value of operational flexibility by using the switching option model developed by Kulatilaka (1988). The aim is to calculate the impact on value of being able to switch between alternative sources of generating technologies in order to take advantage of higher electricity prices when national reservoir levels are low. The estimated value of this option is useful in several settings. This value must be taken into account when the rent or investment cost for thermal generation is assessed. The value can also be used to justify governmental subsidies at system level for initiating investments in thermal generation to avoid random fluctuations in supply due to variation in precipitation.

1.1 Background and motivation

The Norwegian power system is almost entirely dominated by hydro power. According to NVE (the regulator), hydro power provides more than 98% of electricity generation, whereas the remaining 2% is produced by wind or thermal sources[1]. This makes the Norwegian power system quite unique compared to other countries[2]. Hydropower is renewable, does not emit CO_2 and is in Norway a relatively cheap source of energy.

The generating capacity can be considerably increased by small scale hydro power plants[3]. Projects are also emerging based on alternative technologies, especially wind and thermal (gas fired)[4]. The power generation under construction will lead to a slight decrease in the hydro dependence from 98 % to possibly 94 % by 2010 (The Ministry of Petroleum and Energy, 2006a). Thermal power plants are currently, however, a controversial political issue. One gas fired thermal power plant is recently implemented[5] and others are commissioned. In addition, the possible introduction of coal-fired and even nuclear thermal power plants is debated, but none has reached the planning stage. There has also been an increased international interaction along with increased transmission capacity.

Hydropower generation represents a source of flexibility. The water can be “stored” in reservoirs thus creating an operational flexibility through which operators can adapt to demand and price signals[6]. This is a continuous optimalization problem faced by the generators in their scheduling planning, as studied by several (Fosso, Gjelsvik, Haugstad, Mo & Wangensteen, 1999; Näsäkkälä & Keppo, 2005). According to a recent valuation report on Statkraft SF[7] (Lehman Brothers, 2006) it would be reasonable to assume that this company could achieve a 10 % premium compared to the annual system average price (spot price) due to its ability to generate on demand when prices are high.

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[1] In 2007 the yearly middle production of hydro was 121.8 TWh, wind generation was 0.9 TWh and thermal generation was 1.5 TWh (www.nve.no).

[2] Norway is the 6th largest hydro power generator in the world (NVE, 2003).

[3] In a report from NVE (Norwegian Water Resources and Energy Directorate) (2004), the total estimated potential of small scale hydro power plants is in total 25 TWh with an investment cost below 3 NOK/kWh. Furthermore the estimated potential with investment cost between 3 NOK/kWh and 5 NOK/kWh is about 7 TWH, making a total of 32 TWh with the highest cost limitation. There is also a potential for improvements and expansion of existing hydro power plants. Due to the development of more advanced generating technology there is a potential for enhancing the effect of present plants by almost 12 TWh according to NVE (2006). Correspondingly the latest statistics from SSB (Statistics Norway) have raised the total potential of hydropower capacity in Norway from 186 TWh in 2003 to 205 TWH in 2004.

[4] The Government aims to have 3 TWh wind power generation within 2010 (The Ministry of Petroleum and Energy, 2006a). This corresponds to 1000 MW installed capacity and according to NVE (2007) this should be an achievable ambition. There are under construction gas fired thermal power plants that will provide 5 TWh before 2012 (NVE, 2007).

[5] The first plant at Kårstø started up in November 2007.

[6] This flexibility concerns operators with reservoirs and does not refer to those operating river plants.

[7] Statkraft SF is the state-owned generating company with an average generation of 42 TWh (almost 35 % of total national generation capacity) (Statkraft, 2007)

There are operators that only possess little operational flexibility. Some generators are restricted of long term industry contracts, and thereby obligated to always generate at a certain level[8]. Such operators have limited opportunities for saving water in the reservoirs for peak price periods. In such situations there is a genuine possibility of enhancing flexibility and hence postpone more generation to peak price periods if thermal generation is added and controlled. The following decision alternatives exist for the operator: 1) Use solely own generation restricted by the contracts, reservoir capacity and turbine capacity. 2) Save some of the water in the reservoirs and buy spot in the market in order to meet contract obligations. 3) Save some of the water in the reservoirs and instead use thermal generation, either from own plants or bought from an external plant to an agreed price (V^{Th}), in order to make more benefits of the heavy price fluctuation in the market. The focus in this paper is the value pr kWh thermal generation yearly capacity under the described circumstances of alternative 3.

The hydro dominant Nordic system has some special properties. Much because of the variability and uncertainty in rainfall, short time prices (spot and short forward) tend to be very volatile (see Figure 1). Reservoir levels, recent rainfall and weather forecasts have a great impact on short term prices. Therefore, short term electricity prices are often termed as "weather derivatives". The focus of the paper though, is to estimate the value of enhanced flexibility when a hydro based operator also controls thermal generated supply of gas-fired, coal-fired or nuclear. The results will also briefly be discussed in relation to system level analysis.

The paper is organized as follows: Section 2 examines the relationship between the spot price (system price) and three of the relatively short forward contracts traded at Nord Pool. This enables a thorough analysis of the forward-spot spread as the relevant alternative cost for hydro operation. This is linked to the data of reservoir levels, changes in reservoir levels and deviation from median reservoir level through a regression analysis. The findings enable the explanation of the forward-spot spread and hence the relevant alternative cost.

The results are utilized in Section 3 in a decision model based on the switching option model of Kulatilaka (1988) which implies an option value of a flexible situation per kWh yearly

[8] This is e.g. the case for several plants in Western and Northern Norway close to energy intensive factories.

thermal capacity. The pervasive uncertainty in the model lies in the national water reservoir levels, as representing the level for an average producer, and hence the alternative cost of hydro generation. The results will be discussed with the purpose of capturing the impact on value for an operator as well as benefits at system level. Section 4 draws the conclusions and implications.

2 Operational Flexibility, the Alternative Cost of Hydro Generation and the Operational Cost of Thermal Generation

Operational flexibility is often treated as one of the most paramount real options, termed switching options. The main idea consists of the right to be able to switch between two different modes. This switching option enhances value if the value created by being flexible compared to rigid systems exceeds the extra cost. Switching options is mostly studied in relation to the energy industry, but is also applied to other industries such as shipping (Koekebakker, Ådland, & Sødal, 2006) and manufacturing (He & Pindyck, 1992; Kulatilaka & Trigeorgis, 1994).

A number of studies have focused on the applications of switching options with regard to valuation within the energy industry. This has particularly applied at plant level (Antikarov & Copeland, 2003; Bergendahl & Olsson, 2006; Fleten, Flåøyen, & Kviljo, 2007; Fleten & Näsäkkälä, 2005; Kulatilaka, 1993; Trigeorgis, 1996). Other studies have also been carried out concerning the utilization of the complementary characteristics of hydropower and other energy sources at system level (Bélanger & Gagnon, 2002; de Moraes Marreco & Tapia Carpio, 2006; de Neufville, 2001; Vogstad, 2000). Vogstad (2000) considers hydro versus wind energy in a Nordic context and concludes by estimating an additional value of up to 9 % through incorporating wind power in a hydro based system[9]. Application to firm level, though, where an operator controls more than one plant, is virtually non-existing.

The switching option value in the setting of this paper would concern the value of minimizing cost, quite analogue to the option of switching fuels (Kulatilaka, 1993). The idea is that the different cost structure in the different generation technologies can lead to financial benefits in a flexible system. Since the focus is on *operational* flexibility, the investment costs and fixed costs can be ignored. The relevant costs in thermo power generation consist then of operational cost and fuel cost, whereas this is by no means so obvious for hydro power

[9] The estimates vary according to different assumptions. The premium for a wind mill project ranges from 3.7 up to 9 %. The approach is though founded on simulation techniques and not option theory.

generation. The operational cost for hydro power is close to zero when maintenance is ignored (NVE, 2002). Hence, as corresponding cost, it seems more appropriate to relate to the alternative cost of hydro generation; this is the cost for present generation, thereby sacrificing later generation in peak price periods. This forward-spot spread follows a seasonal pattern and is very volatile and will be further discussed later on this study. The presence of flexibility thus brings advantages with regard to adapting to the uncertainty of the level of the alternative cost of hydro generation.

To meet contract obligations, an operator may trade in the market. However, if thermal generation is available to a lower price than the current spot price, this becomes a better source of generation in order to save water for peak price periods.

There are some assumptions to make before making the calculations. It is hard to neglect that an introduction of thermal generation would influence the electricity price pattern. Nevertheless, the Norwegian (and Nordic) system will remain hydro dominant. Investments in several thermal power plants of e.g. 10 TWh in total would still give a hydro dominance of approximately 93 %. In addition, bearing in mind that there is an increased construction and implementation of small scale hydro power plants as well, this percentage should grow even more. Hydro dominance would seem to continue, and there are arguments for relying on the validity of the presented model of the alternative cost of hydro generation. Therefore, despite being aware of this aspect, it is ignored in the calculations.

Another assumption is regarding the realism of the operator's situation. The approach assumes that there always will be generation due to lock-up in industry contracts, even if the alternative cost is high. At the same time there is thermal generation available. All investments are though undertaken, hence there are short term switching opportunities that are analyzed (Dixit, 1992). These assumptions may be viewed strong. Nevertheless, they are not out of range and do make the calculations viable.

2.1 Reservoir level, short term forward prices and the alternative cost of hydro generation

Previous studies of the relationship between the national reservoir level and the spot-forward spread (convenience yield) have been undertaken by Gjølberg & Johnsen (2001) and Botterud, Bhattacharya, & Ilic (2002). However, they stress that Nord Pool represented a young and possibly immature and inefficient market place at that time, and thus futures and forward prices were occasionally outside theoretical arbitrage reasoning. This having possibly been the case in 2001, one can argue that it is of interest to investigate these relationships now - at a time when the Nordic electricity market has matured and become more experienced.

This study intends to link analytical results from more comprehensive data (up to 2006) to the effect on value of a hydro-based power operator adding thermal generation to supply the load. The underlying hypothesis can be stated here: it is value enhancing to possess and control alternative generating technologies so that relatively more power is generated when prices are higher (and aggregate water reservoir levels are low). The aim of this study is to analyze the impact on value of being able to switch between alternative sources of generating technologies in order to take advantage of higher prices when national reservoir levels are running low.

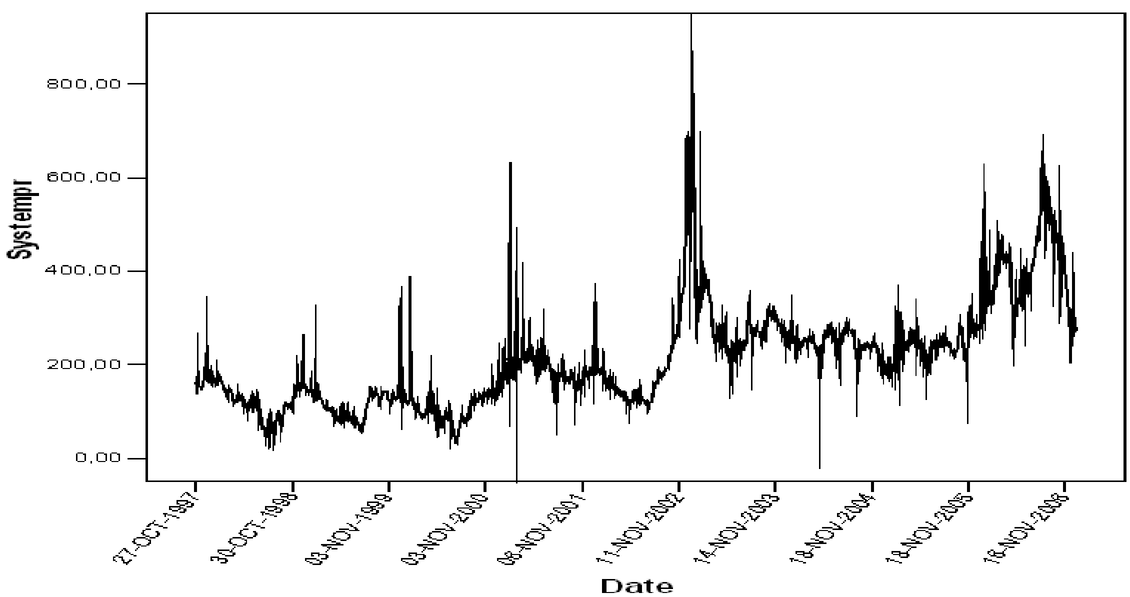

Figure 1: System price (spot price) development 27th October 1997 – 29th December 2006 (NOK pr MWh).

A prominent feature of both the Nordic and Norwegian electricity markets is the relatively low correlation between short and long term forward prices[10] (Koekebakker & Ollmar, 2005). Pilipovic (1998) claims that electricity prices exhibit "split personalities" because of the lack

[10] The structure of the forward contracts at Nord Pool is based on calendar month, quarter (three calendar months) and year contracts. Short term forward prices relate to the contracts within one year and long term prices to the contracts maturing more than one year ahead in time.

of consistency between long term and short term prices. Long term price driving factors have little impact on short term price changes and vice versa. Both Koekebakker (2002), Koekebakker & Ollmar (2005) and Lucia & Schwartz (2002) stress the seasonal pattern of electricity prices (see Figure 1).

Even if it is the long term prices that are usually of most interest in valuation issues, it is the relationships between reservoir level, spot price and *short* forward contracts that provide the focal point of this part of the study. This is explained by the aim of studying the forward-spot spread representing an alternative cost for hydro electric generation and hence having an impact on value. The relationship between spot and forward prices in general terms has been discussed on several occasions (Brennan, 1991). The classic equation states:

$$F_t(T) = S_t(1+r)^{T-t} + W - CY \qquad \textbf{(1)}$$

where $F_t(T)$ is the forward (or futures) price observed at time t for a contract that has maturity at time T, r is the risk free interest rate, W is the storage cost and CY denotes the convenience yield. In the hydro based electricity generation industry it would not seem a controversial assumption to neglect the storage cost and hence set $W = 0$.

According to Pindyck (1990), the convenience yield is highly convex in inventories, becoming large as inventory level is low. This is clearly related to the expectations of availability in the contract period. Electricity does though possess some peculiar properties. Because of the lack of storage possibilities, some careful considerations should be made. Botterud, Bhattacharya, & Ilic (2002) points out that asymmetrical aspect do exist between the supply and demand side of a hydro power based electricity market. They argue that a certain degree of flexibility in generation supply certainly does exist, which can further be used for profit purposes during price peaks in the day ahead spot market. There is, however, no corresponding situation on the demand side, with limited opportunities to adjust demand according to the price level. Strong incentives do exist therefore for a risk averse demand side to lock in as much as possible of expected future demand in the forward/futures market. The consequence is a hypothesis of negative convenience yield and a negative risk premium in keeping with the contango hypothesis. Their empirical findings based on data from 1995 - 2001[11] support the hypothesis.

More flexibility in power generation than implied by Botterud et al. (2002) does, however, exist. In the Norwegian context the water reservoirs are capable of storing water with a low probability of overflow (NVE, 2006). This enables operators to act with a certain degree of flexibility and generate more when prices are high. But when water reservoirs are running low, this flexibility diminishes. In sum this should lead to a theoretical relationship between reservoir level and CY. From **(1)** one obtains (when W is ignored):

$$CY_t = S_t(1+r)^{T-t} - F_t(T) \qquad \textbf{(2)}$$

The definition of convenience yield is “the flow of services accruing to the owner of a physical inventory but not to the owner of contract for future delivery” (Brennan, 1991). Because of the peculiar properties of electricity, this parameter often has a negative value concerning short forward contracts (Kjærland, 2007). The absolute value of the CY does then refer to an alternative cost for hydro electric power generation. The relevant alternative cost, C^H, for generation operators is though consequently the forward-spot spread. Formally one gets:

$$C_t^H = \frac{F_t(T)}{(1+r)^{T-t}} - S_t \qquad \textbf{(3)}$$

Table 1: Descriptive statistics of spot and relevant forward prices (NOK/MWh).

	Number of observations	Min.	Max.	Mean	Standard deviation
System price	108	46.02	610.82	213.36	107.295
Forward one month	108	69.08	591.84	221.60	115.050
Forward two months	108	76.50	624.37	223.23	114.210
Forward three months	108	74.75	664.27	222.96	114.031

[11] They studied the risk premium for four types of futures contracts, with maturity 1, 4, 26 and 52 weeks ahead. The absolute value of the negative risk premium increased from 1.5 % to 18.3 %.

in which C^H denotes the alternative cost for hydro generation. This forward-spot spread is an important parameter when analyzing this industry. C^H can be viewed as an alternative cost for the operator because present generation can lead to lost future production in peak price periods. C^H captures the value per kWh of sacrificing generation some months ahead when prices may be higher. This relationship should therefore be examined carefully, to establish what available data reveals concerning this parameter.

Table 2: Estimation of the C^H (equation (3), NOK/MWh).

	Number of observations	Min.	Max.	Mean	Standard deviation
C^H one month forward contract	108	-59.72	95.28	7.44	25.86
C^H two month forward contract	108	-127.69	126.28	8.27	36.72
C^H three month forward contract	108	-132.30	182.90	7.22	44.84

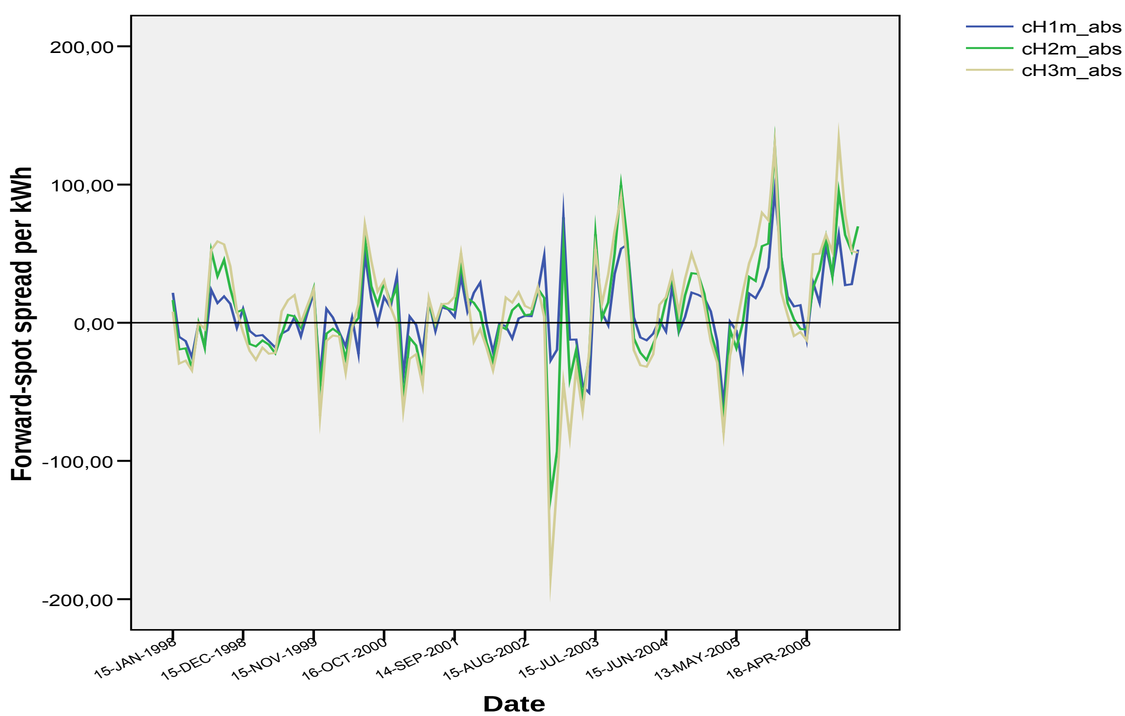

Figure 2: C^H (forward-spot spread) for one, two and three month forward contracts 1998 to 2006 (equation (3)).

This leads to an empirical estimation of C_t^H in equation **(3)**. The spot price is the so-called system price[12]. The system price development is shown in Figure 1.

In order to analyze the forward price one needs to choose some of the forward or futures contracts which capture the difference between spot price and near forward price. As pointed out by Lucia & Schwartz (2002) the issue of sufficient liquidity should be taken into consideration. The financial market at Nord Pool has developed and changed since the introduction of financial futures in 1994, since the design of financial instruments considers the needs of the different participants (Nord Pool, 2005). The forward contract structure from 2004 is based on calendar month, quarter (three calendar months) and year contracts. To capture the intended relationship all the three monthly forward contracts are chosen. These are observed from late 2003 to 2006[13]. During the period 1998 to 2002, the weekly block contacts are used. By using the weekly forward contracts one can estimate the corresponding forward prices to the monthly contracts from 2003. Furthermore, the 15th of each month is chosen and with three prices each month being observed. The sample consists then of 108 observations (9 years). Some summary statistics can be found in Table 1. As risk free interest rate is used the monthly average of the nominal NIBOR (Norwegian InterBank Offered Rate) rate of respectively one, two and three months, obtained from Norges Bank (2007).

The data is used to calculate the alternative cost, C^H. The average results are shown in Table 2 and the data is plotted in Figure 2[14]. Figure 3 shows the average C^H for the above-mentioned three contracts together with the national reservoir level development. The figures reveal heavy fluctuations and a seasonal pattern as previously commented on. This relationship is confirmed when making a more systematic approach in a correlation analysis as shown in Table 3.

[12] The system price is the equilibrium price when net congestion is ignored. Due to congestion there are normally different equilibriums in different areas (Norway is divided into three zones), but the system price reflects the general spot price relevant for the analysis performed in this paper.
[13] From late 2005 to 2006 the observed price are in Euro and is changed into Norwegian currency (NOK) by using the actual exchange rate the trading date (obtained from Norges Bank (2007)).
[14] The mean of the convenience yield is negative according to theses data, consistent with the contango hypothesis - the reason being the peculiar properties of electricity as e.g. explained by Botterud et al. (2002) and Koekebakker (2002).

Reservoir statistics are provided from the database of NVE (the regulator). NVE collects and publishes reservoir levels on a weekly basis from 1998 – 2006. Figure 3 and Figure 4 show this for 2001 to 2006. One can recognize the heavy decrease in autumn 2002 causing the extremely high prices in late 2002 and early 2003. This also causes extremely low C^H – as can be seen in both Figure 2 and Figure 3. Figure 4 also reveals the low reservoir level in late summer/early autumn 2006, followed by an unusual increase during the rest of the year, which is due to an extremely mild and wet autumn. This explains the high prices during the late summer/early autumn, whereas there was a significant decrease in price levels for rest of the year. The observations of extremely high C^H in the early autumn of 2006 can be recognized in Figure 2 and Figure 3.

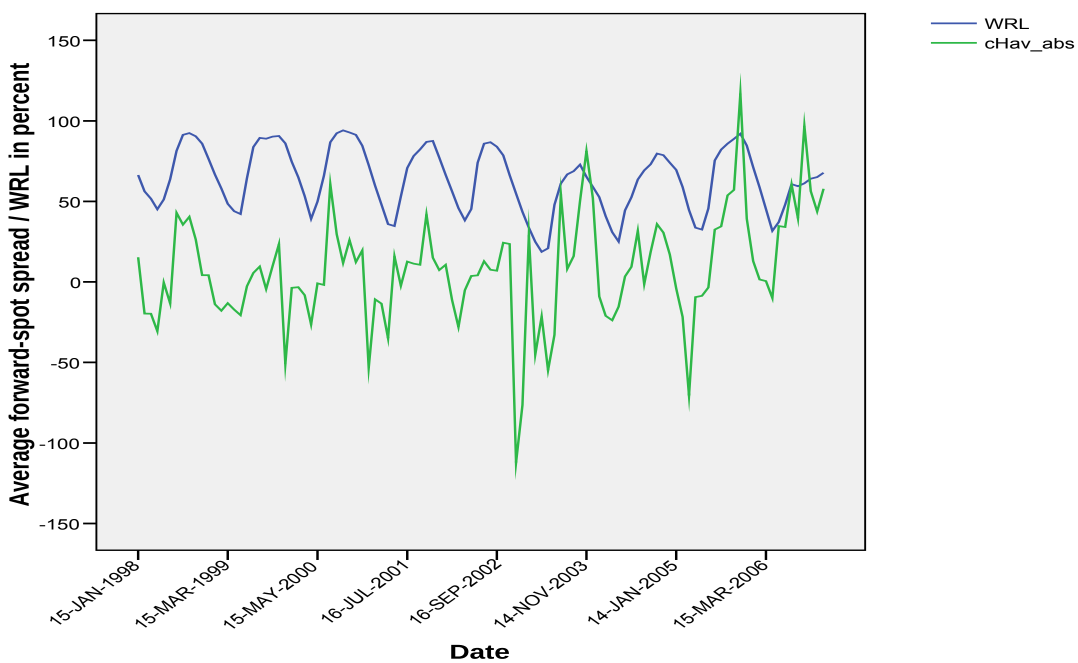

Figure 3: Average forward-spot spread (C^H) and national water reservoir level (*WRL*, in percent) 1998 – 2006. The positive correlation can be observed.

Table 3: Correlation (Pearson) for C^H and the water reservoir levels at national level (*WRL*).

	C^H 1 month	C^H 2 months	C^H 3 months	Average C^H
WRL	0.370	0.457	0.492	0.475

2.2 Explaining the alternative cost, C^H

The above analysis reveals that spot price, forward price and the forward-spot spread fluctuate greatly and this can be correlated to the reservoir level, published each week by the regulator (NVE). To further test the described relationship between reservoir level and the forward-spot spread, the following regression equation is estimated:

$$C_t^H = \beta_0 + \beta_1 WRL_t + \varepsilon_t \qquad \textbf{(4)}$$

in which C^H denotes the average forward-spot spread (as defined in equation **(3)**) at time t of one, two and three months forward contracts and WRL denotes the national reservoir level in percent of maximum capacity.

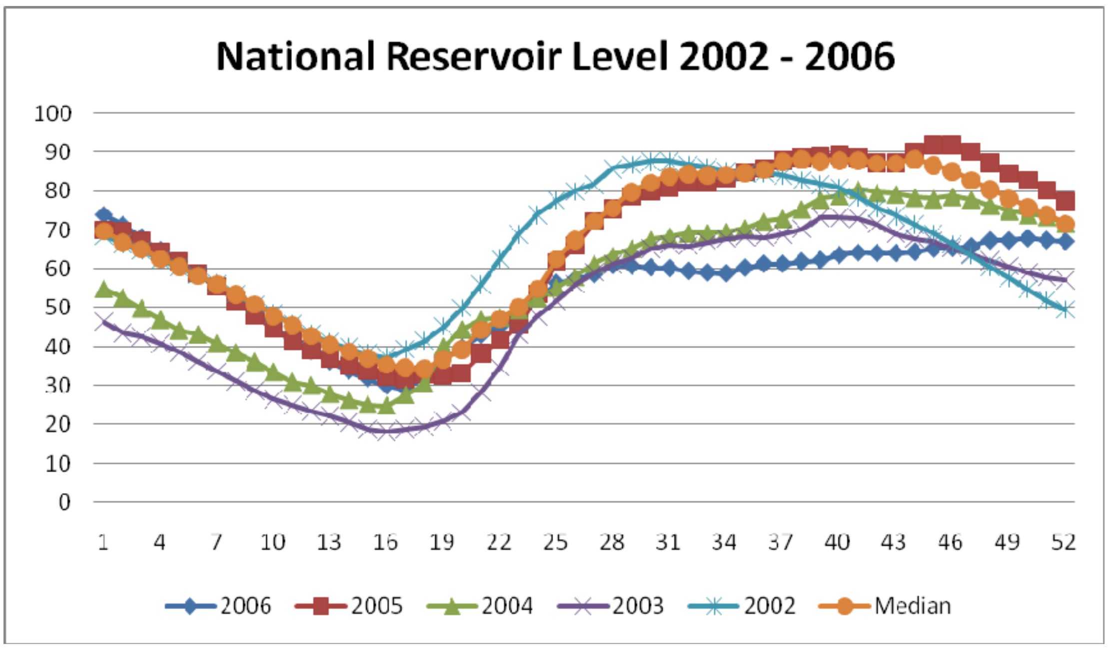

Figure 4: Reservoir inventory at national level 2001 – 2006, in per cent of maximal capacity. The X-axis consists of week no. "Median" is for each week the median level of national reservoir levels 1970 – 2006, as disclosed by NVE.

The industry is very much concerned with changes in reservoir levels, leading to include the last week reservoir change observation in the model (Gjølberg & Johnsen, 2001). To also try to capture the hydrological situation, one includes a variable that measures the deviation from the median reservoir level. This variable captures the situation if it is a "wet" or "dry" year. Hence an extension of the model becomes:

$$C_t^H = \beta_0 + \beta_1 WRL_t + \beta_2 \Delta WRL_t + \beta_3 \Delta MED_t + \varepsilon_t \qquad \textbf{(5)}$$

in which *ΔWRL* denotes the change in reservoir level during the last week in percentage points ($\Delta WRL_t = WRL_t - WRL_{t-1}$) and ΔMED_t ($= WRL_t - MED_t$) denotes the difference between median reservoir level the actual week and the reservoir level as disclosed by the NVE. Descriptive statistics and correlations (Pearson) of these independent variables is reported in Table 4.

Table 4: Descriptive statistics and correlations of the independent variables (*WRL*, *ΔWRL*, *ΔMED*) in the regression equation (5).

Variable	Number of observations	Min.	Max.	Mean	Standard deviation	Correlation (Pearson)	
						ΔWRL	*ΔMED*
WRL	108	18.7	94.1	63.85	19.49	0.103	0.331
ΔWRL	108	-3.5	7.5	-0.11	2.46		0.192
ΔMED	108	-26.4	19.1	-2.62	10.28		

The weekly disclosure of NVE does emphasize both the change in reservoir level and the observation in light of the median level the actual week. Observers and commentators in the industry do the same. Hence, there is a solid foundation for the choice of these independent variables.

Table 5: Results of regression analysis of the relation between average forward-spot spread (C^H) and national water reservoir levels 1998 – 2006 (T-values in brackets).

Equation	*n*	β_0	β_1	β_2	β_3	DW[15]	$\overline{R}^2$
(4)	108	-45.192	0.827			1.161	0.218
		(-4.543)	(5.551)				
(5)	108	-56.170	0.962	4.496	-1.104	1.371	0.378
		(-5.848)	(6.820)	(4.193)	(-4.072)		

The estimation results of the regression analyses are reported in Table 5 and Table 6. A plot of the results of equation **(5)** is shown in Figure 5. The results are consistent with the results of Gjølberg & Johnsen (2001). There is a significant positive relationship between water reservoir level and the forward-spot spread, which may surprise. Nevertheless, this is the

[15]Lower critical value of DW for 100 observation and 3 explanatory variables is 1.61.

empirical findings. At low reservoir levels there is a negative forward-spot spread, as indicated by the negative constant term. At a reservoir level of about 55 percent, the spread becomes positive (equation **(4)**). When reservoir levels are high, the spot price is low and hence, the forward-spot spread is high. When reservoir levels are low, the spot price is higher and consequently the spread becomes negative. The spot price seems to dominate the short forward prices. This explains the positive sign of the β_1-coefficient of equation **(4)** and **(5)**.

The Durbin Watson test shows that autocorrelation does exist in the models. This leads us to perform a robust test of the model (Gujarati, 2003; Wooldridge, 2003). The robust test shows slightly different T-values, but all coefficients remain significant at the 1 % level. No multicollinarity was detected ($VIF < 1.2$ for all three independent variables).

Table 6: Results of the regression analysis of equation (5) with respectively, one, two and three months C^H as the dependent variable (T-values in brackets).

Equation (5)	***n***	β_0	β_1	β_2	β_3	$\overline{R}^2$
1 month	108	-36.603	0.651	1.043	-0.991	0.252
		(-4.565)	(5.532)	(1.166)	(-4.386)	
2 months	108	-58.589	1.007	4.530	-1.173	0.346
		(-5.505)	(6.446)	(3.815)	(-3.909)	
3 months	108	-73.319	1.228	7.917	-1.146	0.439
		(-6.086)	(6.945)	(5.890)	(-3.374)	

The results show that, taking into consideration the last week change in reservoir levels and the deviation from the median value of the reservoir level, hypotheses that these independent variables have impact on the forward-spot spread are supported. This forward-spot spread on the studied contracts is sensitive to inventory information published from the regulator every week. The findings also confirm that the hydrological conditions, depending on the observations are done in a “wet” or “dry” year, also have influence. The observed C^H and predicted C^H (based on equation **(5)**) are plotted in Figure 5. It is observable that the extreme situations, as winter 2002/03 (very low *WRL* after a “dry” autumn and cold part of the early winter) and late autumn 2006 (“wet” and mild period), are not fully captured by the model.

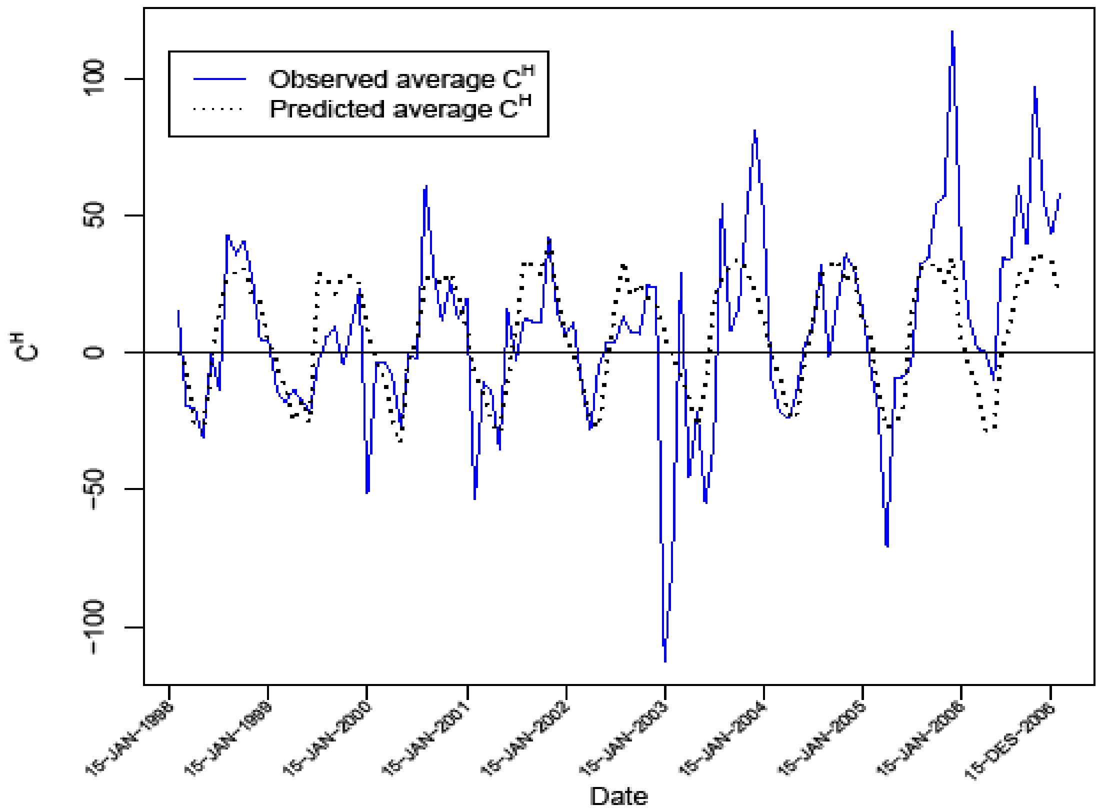

Figure 5: Observed average C^H (solid line) versus predicted average C^H (dotted line) (equation (5)).

This completes the analysis of the forward-spot spread. When reservoir levels are running low, there is a negative forward-spot spread, making the alternative cost negligible. Hence, there are no benefits involved in including alternatives. However, in times when the forward-spot spread is high, the alternative cost of generating is significant. Thus it becomes economically interesting to have the opportunity to switch to alternative generation in order to generate more in peak price periods, assumed that the cost of using such generation is lower that the spot price.

2.3 Operational cost of thermal generation

The relevant cost of thermal generation is operational costs and fuel costs, if the plant is owned by the hydro-operator, or the agreed price, V^{Th}, if there is an option agreement with a thermal operator.

The cost of operating a gas fired thermal power plant is complex, and depends particularly on exogenously determined gas prices. According to Bolland (2006), the operational cost for an average gas fired thermal power plant in the Norwegian context would be NOK 0.0243/kWh and the fuel cost NOK 0.2855/kWh - based on a gas price of NOK 1.73/Sm^3 [16]. Hence, the flexibility value involved in despatching gas fired thermal power to a hydro producer would yield a low switching option value. However, gas prices are highly volatile and have at present (2007) reached a high level compared to for instance 2004 prices which were much lower (average price in e.g. 2004 was NOK 0.97/Sm^3 (SSB (Statistics Norway), 2007).

The operational cost of nuclear and coal fired thermal plants is lower. Concerning the operational cost of nuclear power, a number of country-specific factors do exist. Technological improvements have nevertheless lowered the cost considerably, making nuclear energy the cheapest alternative compared to other non-hydro generation technologies. According to WNA (World Nuclear Association, 2005), the operating cost, including fuel and maintenance, in Finland and Sweden is currently at a level of NOK 0.08/kWh. This then represents the relevant cost of an input parameter in the model of proposed in this paper.

The operational cost of coal fired thermal power plants is higher than that of nuclear plants, but lower than plants fuelled with gas. According to statistics from the Nuclear Energy Institute (2007), the average cost for U.S. plants is approximately NOK 0.14/kWh. This operating cost can serve as the base case input parameter, even if there are some factors that are complicating transference to a Norwegian setting.

Table 7: The operational cost and fuel cost used in the analysis of different types of thermal generation, along with an estimated external renting price. NOK/kWh.

C^{Th} Gas-fired	C^{Th} Coal-fired	C^{Th} nuclear	V^{Th}
0.31	0.14	0.08	0.30

However, if the situation is that thermal generation is rented from another operator, the relevant parameter is the agreed price, termed V^{Th}. The level of V^{Th} would obviously be independent from type of fuel, but be probably somewhere below the general long forward prices traded at Nord Pool. A careful estimate would be NOK 0.30/kWh.

[16] According to Statistics Norway this was the average gas price in 2006 (SSB (Statistics Norway), 2007).

This provides the basis for a further analysis in the next session, aimed at estimating the value of being able to switch between hydro and an alternative of thermal generation. The numbers used in the following analysis is shown in Table 7.

3 Model Description and Numerical Analysis

3.1 The decision model framework

This session describes the model for quantifying the value of the operational flexibility provided by controlling both hydropower and a type of thermal power (de Moraes Marreco & Tapia Carpio, 2006). The model assumes a situation where a generating company can switch and operate in either one of two different modes, *H* (hydro) or *Th* (thermal). If thermal is bought externally, there are two conditions for exercising such an option to become economically interesting; 1) $C^H > 0$ and 2) $V^{Th} < S$ (spot price).

The switching aspect relates though only to a portion of the hydropower generation. Because of the contract obligations the operator cannot produce under a certain level. But there is an option value for every kWh below this level that can be replaced by available thermal generation in times when C^H is high. The following description relates to this part of the production.

Associated with each mode is a cash flow depending on the uncertainty incorporated in the model. In each period, in this context one week, the operator can choose which mode to operate in. The nature of the situation described in this paper suggests focusing on the cost flows attached to each mode. Hence, in each mode one can compare the alternative cost for hydro energy generation with the operational and fuel cost of thermal generation (respectively gas, coal and nuclear) or V^{Th}. The objective is to minimize the operational cost flow in each period, which in the setting of this paper is each week.

In the model there is a focus on the cost flow generated in week *t* at either mode hydro (C^H) or mode thermal (C^{Th})[17]. Switching costs relating to interchanging between the two modes are assumed to be zero. This could be problematic if thermal generation was owned by the hydropower generator, since switch on/off costs are considerable. However, if the operator possesses an option of renting thermal generation capacity from another entity, this should not

[17] Alternatively V^{Th}, as described previously.

cause controversy. The model provides then the net present value of cost saving per kWh yearly available thermal power capacity.

The driving uncertainty in the model is the inflow in the water reservoirs, modelled by a stochastic process. We assume that change in reservoir level (*ΔWRL*) in each week is truncated normal distributed with expectation the average change in each week 1998-2006 and a standard deviation based on the same time series (see appendix 1). The focus on *ΔWRL* is justified due to obvious lack of independence between WRL_t and WRL_{t-1}. However, it seems more reasonable to assume independence between *ΔWRL* and WRL_{t-1}. Hence, one gets:

$$\begin{aligned} &WRL_t \equiv WRL_{t-1} + \Delta WRL_t \\ &E(WRL_t \mid WRL_{t-1}) = WRL_{t-1} + E(\Delta WRL_t \mid WRL_{t-1}) \approx WRL_{t-1} + E(\Delta WRL_t) \end{aligned} \quad \textbf{(6)}$$

This enables to incorporate the uncertainty in downpour, inflow and hence the reservoir level. Change in reservoir level is a variable with a seasonal pattern. But the reservoir level statistics make it possible to calculate for each week the average and standard deviation (see appendix 2). These figures serve as input parameters for simulating the alternative cost of hydro generation according to equation **(5)**, which is utilised later in this section. Hence, one incorporates in the model the stochastic and seasonal pattern of inflow and thereby the great differences in alternative cost throughout the fiscal year[18].

When the basis of simulating *ΔWRL* has been established, one can follow the model framework of Kulatilaka (1988). The purpose is to calculate the option value of possessing both hydro and thermal power when relating to the inflow and hence reservoir level as the stochastic, uncertain factor. This option value is calculated as the difference between the values of the flexible situation compared to the rigid situation without thermal generation. As previously described, the C^H represents an alternative cost for an operator, which can be high in some parts of the year. This means that the option to switch between hydro and thermal generation is worth calculating for those weeks of the year when C^H is at a high level. For the weeks when the negative outcome concerning inflow leads to a higher expected C^H than the operational cost of a thermal power plant, one obtains an option value, due to the opportunity of being able to switch from *H* to *Th*.

[18] The simulated values of *WRL* are programmed to be truncated by the max and min value for each week disclosed by NVE for the period 1970 – 2007 (see the *R* source code in appendix 2).

The actual model derived from Kulatilaka (1988) provides the following: the flexible situation is studied for a period of one year; $T = 52$. At time *T-1*, when one week remains of the total period, the water reservoir level will be WRL_{T-1} and the operational cost for the last week of the period will either be $C^{H}{}_{T-1}$ or $C^{Th}{}_{T-1}$ (V^{Th}) depending on which mode one is operating in. When only one period remains, the value can be calculated with certainty given by the minimum cost of the two possible modes, either C^{Th} (V^{Th}) or the estimated C^{H}. If one denotes the actual cost of the flexible situation C^{F} one gets:

$$C^{F}_{T-1} = \min\left[C^{H}(WRL_{T-1}), C^{Th}{}_{T-1}\right] \qquad \textbf{(7)}$$

At time *T-2*, the cost of the flexible system will be the cost of the next period (week) that minimizes this period's operational cost plus the expected value from the last period (*T-1*). This gives:

$$C^{F}_{T-2} = \min\left[C^{H}(WRL_{T-2}), C^{Th}{}_{T-2}\right] + \rho^{-1} E_{T-2} C^{F}_{T-1} \qquad \textbf{(8)}$$

where ρ is the risk free discount factor for the week (one period) equal to ($1+r_f$) (alternatively: $\rho = e^{r_f \Delta t}$).

In each period the operator must contemplate switching to the other node, comparing the expected alternative cost of hydro to the operational cost and fuel cost of thermal generation. To capture the switching option value one relates to a summarization of the cost saving. This is a simplified version of the conceptual model of Kulatilaka (1988), since no switching cost leads to avoiding that the value is depending on modes. The net present value of yearly saved cost in this setting becomes:

$$NPV_{costsavings} = C^{H} - C^{F} = \frac{1}{52}\sum_{t=1}^{52}\left[\rho^{-1} \cdot C^{H}(WRL_t) - \min\left[C^{H}(WRL_t), C^{F}_{t}\right]\right] \qquad \textbf{(9)}$$

where; $t = 1,....52$; and $\rho = e^{r_f \Delta t}$. The optimalization problem is each week to choose the mode minimizing the cost for that week. No switching costs simplify the calculations. The equation gives the net present value of the yearly cost saved per kWh through the accessibility of thermal generation in the flexible situation. This calculation makes it possible to estimate the net present value of saved cost by having a flexible situation compared to a rigid situation of purely hydropower.

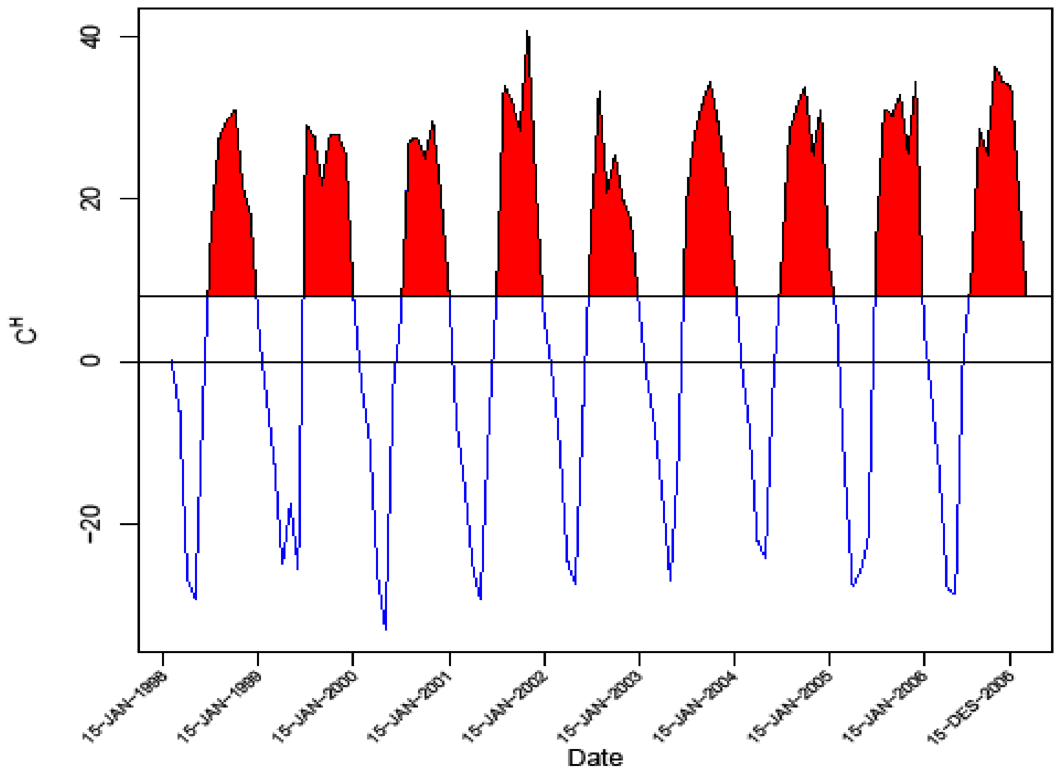

Figure 6: The shaded area represents the costsavings calculated in equation (6.9) per kWh yearly nuclear generation that is used instead of hydro when $C^H > C^{Th}$. The areas are limited of the line of estimated C^H (equation (6.5)) and the operational and fuel cost of nuclear of NOK 0.08/kWh.

6.3.2 Numerical analysis

The numerical calculations give the results shown in Table 8. The option values based on equation **(5)** and equation **(9)** can be interpreted as the flexible value of introducing thermal power generation for a hydro-based operator in order to generate 1 kWh in a year. The option values are highest for nuclear due to the low operational cost, and lowest for thermal power plants fuelled by gas. The value in the case of nuclear is illustrated as the shaded area in Figure 6.

This option value is the net present value of cost savings due to being able to switch to thermal generation in times when the alternative cost of hydro generation as a stochastic variable exceeds the operational cost of a thermal power plant or renting price [19]. The value is a result of high C^H during some parts of the year.

Table 8: The option value based on different types of thermal generation (per kWh yearly generation capacity) based on equation (9). The value represents a premium for a hydro based operator of being able to switch to thermal generation in times when the alternative cost for hydro is high.

Type of thermal generation	*Input parameters*	*Equation **(9)** Value[20], yearly generation 1kWh*	*Standard deviation (equation **(9)**)*
Gas fired	$C(C_t^m, m, t)/52$ $C^{Th} = NOK\,0.31/kWh$ ($r = 0.052$)	NOK 0.0128	0.0223
Coal fired	$C(C_t^m, m, t)/52$ $C^{Th} = NOK\,0.14/kWh$ ($r = 0.052$)	NOK 0.0453	0.0525
Nuclear	$C(C_t^m, m, t)/52$ $C^{Th} = NOK\,0.08/kWh$ ($r = 0.052$)	NOK 0.0652	0.0660
Externally bought	$C(C_t^m, m, t)/52$ $V^{Th} = NOK\,0.30/kWh$ ($r = 0.052$)	NOK 0.0219	0.0264

Following these results, one can comment on some implication for an operator implementing thermal power generation in addition to hydropower generation. The rent of some thermal generation in order to have the opportunity to switch from hydro to thermal in some parts of the year for some of the production give some benefits, if not V^{Th} is too high. If thermal generation is controlled by the operator, the value of flexibility becomes higher. If e.g. a producer controls 100 GWh yearly from a thermal nuclear producer (constant through the

[19] The risk free rate is set to 5.2 % p.a. which yields a weekly discount factor of 0.10 %. This is close to the current risk free rate in Norway (October 2007), however this parameter has little impact on the switching option value.

[20] The numbers are a result of 10000 simulations; see the *R* source code in appendix 2.

year) which all can be used for saving water to peak price periods, the value of the enhanced flexibility would be NOK 6.5 million.

The numbers calculated in this subsection give reasonable input regarding the flexibility value which is relevant in the negotiations of the rent in order to have access to thermal generation or in the assessment of an investment in a thermal power plant. The numbers show a possible significant value and should be considered in the described situation.

3.3 Discussion

Hydro operators face constantly the optimalization problem of use now or later of the water in their reservoirs. No obligation exists for constant output. However, a large part of the production for a significant number of generating companies is locked up in long term industry contracts, limiting the possibility of scheduling the production to peak price periods. By having an option to control thermal power in addition to hydro, there is realism in the calculations presented which should be considered in renting issues or investment decisions.

Another aspect to comment is the uncertainty of fuel prices. The development of the cost of nuclear power as fuel seems quite stable and not particularly volatile. The cost of coal as fuel depends on the location, but seems far less volatile than petroleum. Nevertheless, stochastic elements do exist in the cost of thermal generation that are ignored in this analysis, and hence this represents a shortcoming. However, the value of operational flexibility has intuitively represented a value and has been taken into account as a qualitative aspect in such assessments. But by using the approach presented in this paper, there is a solid foundation for measuring the impact the switching option aspects has for the value at both firm and system level.

This approach may also hold valid at system level. There would always be a demand to be met, and thereby the presented approach yields trustworthy results. The possibility of import could question this point. Nevertheless, the congestion in the net capacity can partly meet this argument. The calculations can hence be discussed in view of the governmental subsidies (The Ministry of Petroleum and Energy, 2006b). If the alternative cost for hydro generation can also be interpreted as a deficit cost in a macro perspective, the findings can justify and legitimate a part of possible subsidies, as done by de Moraes Marreco & Tapia Carpio (2006). Even if uncertain factors do exist in this approach, the results show that the switching option aspect represents a value that should not be ignored. This should definitely be incorporated in the valuation of the alternatives to hydropower.

The findings show that the complementary argument is valid and that the switching option aspect should be included in the economical assessments of adding alternative generation technologies. The values in Table 8 provide e.g. the willingness of paying for the option of renting thermal generation capacity.

4 Conclusions and Implications

This paper represents a real option approach to the value of operating flexibility in the Norwegian generating industry when adding thermal generation to hydropower. The key assumption is the operator's restriction in scheduling due to long term industry contracts. By applying the real option model framework of Kulatilaka (1988), one has been able to estimate the option value per kWh available thermal generation that can be used for saving water to peak price periods. Moreover, estimates have been presented of the net present value of minimizing costs between the alternative cost of hydro and operational cost and the fuel cost of different types of thermal generation in the described situation where large parts of the hydropower generation are locked up in industry contracts.

The alternative cost for hydropower operators has been developed and modelled based on data from Nord Pool and the regulator (NVE). This result in two versions of a model explaining the forward-spot spread (C^H) based on water reservoir level and the hydrological situation. The adjusted R squared for the three-factor model reaches 0.44 at the highest (three month forward contracts, equation **(5)**).

The numerical calculations of the switching option value show that there are significant option values when thermal power plants are controlled by a hydro operator. However, if thermal capacity is rented externally, the option value depends on the agreed price. If this price is sufficiently low, an option value emerges. The calculations are useful in order to either 1) assessment of own thermal investments, or 2) in negotiations with thermal operators of option contracts. In both situations, the switching option aspect would provide relevant information in valuation assessments.

Another implication is that ignoring the option value aspect can lead to underinvestment in nuclear and coal fired thermal generation compared to gas fired plants. In other words, from the viewpoint of flexibility, the least profitable alternative is gas-fired thermal generation. Nevertheless, this is the only thermal generation actually implemented in the Norwegian power system.

The value of a flexible system can justify and legitimate governmental subsidies. This assumes that the alternative cost of hydro generation can be linked to a kind of deficit cost at system level. If the estimations of C^H are interpreted in this way, the calculations suggested in this paper partly provide a valid argument for subsidies of alternative power generation, which in turn depends on such support being profitable.

The stochastic nature of this industry makes it challenging to analyze valuation issues. The uncertainty of this paper is related to the uncertainty in reservoir levels throughout the year. The regression equations of C^H are also disputable since they only partially explain the forward-spot spread. Nevertheless, the estimations of switching option values are relevant and provide insight into the value of operating a situation with flexibility.

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Appendix 1: National Water Reservoir Level (WRL) Statistics

Week no. (t)	Average WRL (percent) (1998-2006)	Standard deviation (WRL)	Median (WRL, in percent)	Average ΔWRL (1998-2006)	Standard deviation ΔWRL	Max WRL (1970-2007)	Min WRL (1970-2007)
1	67.12	10.05	69.8	-2,81	0,82	46,4	76,8
2	64.86	10.18	66.8	-2,27	0,81	43,5	74,6
3	62.61	9.75	65.1	-2,24	0,86	42,5	71,7
4	59.96	9.51	62.6	-2,66	0,41	40,7	68,9
5	57.53	9.49	60.6	-2,42	0,67	38,5	66,9
6	55.09	9.16	58.3	-2,44	0,74	36,2	65,0
7	52.51	9.05	56.0	-2,58	0,64	33,7	62,0
8	50.02	9.09	53.5	-2,49	1,20	31,1	61,8
9	47.44	9.25	50.8	-2,58	1,14	28,6	60,1
10	44.76	9.16	48.0	-2,69	0,50	26,5	58,0
11	42.06	8.83	45.5	-2,70	0,52	24,9	57,6
12	39.78	8.35	42.8	-2,28	0,69	23,4	58,0
13	37.67	8.14	40.5	-2,11	0,66	22,1	56,8
14	36.20	8.40	38.8	-1,47	1,11	20,5	55,4
15	34.53	8.46	37.1	-1,67	0,60	18,7	53,8
16	33.29	8.17	35.4	-1,24	0,69	17,3	52,4
17	33.81	8.14	34.6	0,52	1,51	18,7	52,7
18	35.78	8.63	34.2	1,97	1,59	19,4	57,8
19	38.86	8.84	36.8	3,08	3,13	20,9	62,1
20	42.50	10.22	39.2	3,64	1,75	23,0	64,1
21	46.76	10.05	44.4	4,26	1,19	27,1	65,1
22	50.59	9.79	47.2	3,83	2,20	29,5	67,8
23	55.00	9.66	50.1	4,41	2,32	35,7	74,3
24	59.78	9.83	54.9	4,78	1,75	40,6	79,1
25	64.34	9.89	62.6	4,57	1,55	44,5	84,8
26	68.44	10.54	67.5	4,10	1,68	46,6	88,4
27	71.87	10.68	72.3	3,42	1,53	50,0	91,3
28	74.67	11.02	75.7	2,80	0,69	52,4	93,2
29	76.66	11.58	79.8	1,99	1,06	53,8	94,7
30	78.21	11.72	82.2	1,56	1,05	55,2	95,4
31	78.92	11.64	83.9	0,71	0,76	56,4	96,3
32	79.29	11.88	84.5	0,37	0,99	57,0	95,6
33	79.71	12.07	84.2	0,42	0,94	57,2	97,3
34	80.17	12.18	84.4	0,46	0,92	58,3	97,1
35	80.69	11.76	84.8	0,52	0,75	59,5	97,2
36	81.06	11.39	85.6	0,37	0,87	59,7	97,2
37	81.38	11.31	87.6	0,32	0,85	58,9	96,5

38	81.96	10.92	88.3	0,58	1,10	58,1	96,6
39	82.44	10.31	87.6	0,49	1,46	57,8	96,5
40	82.71	9.75	88.0	0,27	1,29	60,0	95,9
41	82.81	9.59	87.9	0,10	1,50	62,2	96,7
42	81.87	9.79	87.2	-0,94	1,09	63,1	97,1
43	81.41	10.38	87.1	-0,46	1,04	63,4	96,5
44	81.59	11.21	88.2	0,18	1,68	64,3	95,1
45	80.97	11.23	86.7	-0,62	1,31	65,1	93,0
46	79.74	11.19	85.2	-1,22	1,16	65,3	91,9
47	78.30	11.11	82.8	-1,44	1,03	63,5	90,2
48	76.78	10.87	80.5	-1,52	1,34	60,6	87,7
49	75.12	10.87	78.1	-1,66	0,93	57,7	86,1
50	73.33	10.70	75.8	-1,79	1,09	54,9	84,6
51	71.53	10.52	74.0	-1,80	1,02	52,1	81,7
52	69.49	10.10	71.6	-2,04	0,98	49,6	78,8

Appendix 2: The R source code

The R source code made for the simulations of equation **(6.9)**:

```
library(foreign)
library(msm)
pdf(file="CH_simulering-WRL%003d.pdf", onefile=FALSE)
N=10000
value=numeric(N)
SimWRL=numeric(52)
SimWRLchange1=numeric(52)
xx =read.spss('H:/PhD/Endringer WRL.sav',
          to.data.frame=TRUE)
WRLchange1.mean=xx[1:52,1]
# WRLchange1.mean
WRLchange1.sd=xx[1:52,2]
# WRLchange1.sd
MedianWRL=xx[1:52,3]
# MedianWRL
MinWRL=xx[1:52,4]
MaxWRL=xx[1:52,5]
WRLuke1.mean=67.1
WRLuke1.sd=10
cTH=8    #atom
# cTH=14    #kull
# cTH=31    #gass
plot(MinWRL,col="red",pch=20,ylim=c(15,100))
points(MaxWRL,col="blue",pch=20)
for (i in 1:N) {
   SimWRL[1]=rtnorm(1,mean=WRLuke1.mean,sd=WRLuke1.sd,lower=MinWRL[1],
      upper=MaxWRL[1])
   SimWRLchange1[1]=rnorm(1,mean=WRLchange1.mean,sd=WRLchange1.sd) #ikke
trunkert
   for (j in 2:52) {
      SimWRLchange1[j]=rtnorm(1,mean=WRLchange1.mean[j],sd=WRLchange1.sd[j],
```

```
            lower=MinWRL[j]-SimWRL[j-1],upper=MaxWRL[j]-SimWRL[j-1])
       SimWRL[j]=SimWRL[j-1]+SimWRLchange1[j]
    }
   points(SimWRL)

   EstCH=-56.170+0.962*SimWRL+4.496*SimWRLchange1-1.104*(MedianWRL-
SimWRL)
#   EstCH
   CF=pmin(EstCH,cTH)
#   CF
   Diff=EstCH-CF
#   Diff
   value[i]=0
   for (j in seq(52,1,-1)) value[i]=value[i]/1.001+Diff[j]
   value[i]=value[i]/52
#   value[i]
}
hist(value)
mean(value)
sd(value)
dev.off(which = dev.cur())
```