A Logit Model of Brand Choice and Purchase Incidence: A Real Options Approach

Hiroto Suzuki[a,*], Makoto Goto[b] and Takahiro Ohno[a]

[a] *School of Creative Science and Engineering, Waseda University*

[b] *Graduate School of Finance, Accounting and Law, Waseda University*

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Abstract

We develop a model of consumers' purchase behavior under uncertainty of price by using real options approach. Retailers, discount stores and super markets, frequently conduct price promotions, and the promotions to stockpile products are frequent, too. In that category, there are trends to purchase at lower price, and postpone the purchase at higher price. That is, they make decision to purchase or postpone by their memory of the past price and the actual price. In the other words, under price uncertainty, they make the decision considering the option value to postpone purchase. Therefore we develop a consumers' purchase incidence and brand choice model considering postpone option by using real options approach.

Keywords: Purchase incidence, Brand choice, Postpone options

JEL classification: M31, D81

1 Introduction

Retailers, discount stores and super markets, frequently conduct price promotions, and the promotions to stockpile products are frequent, too. The main purpose of this promotion is to attract customers to the shop. Therefore, revealing the consumers' purchase behavior, when and what they purchase, is very effective to retailers conducting price promotions,

*Corresponding author. Address: 3-4-1 Okubo, Shinjuku-ku, Tokyo 169-8555, Japan; Phone: +81-3-5286-3192; E-mail: hiroto-suzuki@aoni.waseda.jp

on fixing the products and the discounted price. In the category of stockpile products, consumers attempt to purchase at lower price, and postpone the purchase at higher price. That is, they make decision to purchase or postpone by comparing the past price of their memory and the actual price. In the other words, under price uncertainty, they make the decision considering the option value to postpone purchase.

Consumers' processes to purchase, there are processes of category purchase incidence and brand choice (Howard and Sheth, 1969), and both oh the processes are strong effected by the price (Guadagni and Little, 1998; Gupta, 1988, 1991). Therefore, to determine the effect of price promotion, modeling the above two processes and revealing the effect to the each processes are needed.

The purpose of our study is to develop consumers' category purchase incidence and brand choice decision model considering real options value to postpone purchase. Our model is based on nested logit model, the model is tractable on estimating the parameters and modeling.

The studies about consumers' purchase incidence are separated by the used data, aggregate and disaggregate. The studies on aggregate data, they focus the number of the purchasing occurrence during a fixed period (Ehrenberg, 1967; Morrison and Schmittlein, 1981; Zufryden, 1978). These models are easy to estimate and usually fit and predict the data well, but these are omitted the marketing variables. And in late years, by the spread of POS system, studies of developing the models used disaggregate data are caught on. The models used aggregate data are separated, modeling the interpurchase time and the purchase incidence (purchase or no purchase). As the studies of modeling the interpurchase time, there are two approaches, by a simple linear regression (Neslin et al., 1985), and hazard model (Cox, 1972; Jain and Vilcassim, 1991). However, Wheat and Morrison (1990) shows the models of the purchase incidence are superior to the model of interpurchase time. In the models about the purchase incidence, they model both the binary outcome of purchase during the interval of interest, and the brand choice, by using Nested Logit Model (Bucklin and Lattin, 1991; Bucklin and Gupta, 1992). These studies are developed with various directionality, integrated the model of the quantity of purchase (Gupta, 1988; Chiang, 1991; Chintagunta, 1993; Mela et al., 1998), the shopping basket analysis (Chintagunta and Haldar, 1998; Manchanda et al., 1999), elaborating the purchase incidence model (Bell and Bucklin, 1999; Sun et al., 2003; Siddhartha et al., 2004).

Bell and Bucklin (1999) develop a substantive understanding of the role that reference

points play in consumers' decisions about when to buy in a product class. (Siddhartha et al., 2004) develop a brand choice model in which the no-purchase outcome is carefully modeled and allowed to correlate with the unobservable that influence brand choice. However, the formers' model focuses the difference of marketing variables, in the other words, consumers compare the reference point at previous purchase time and purchase opportunity of the moment. And latter doesn't consider the influence the uncertainty of the products price to consumers' purchase behavior. On the other hand, (Sun et al., 2003) develop a dynamic structural model for analyzing the effect of promotion on brand switching in an uncertain promotion environment. Consumers' purchase behavior to stockpile product is decided by their stock and the price, and they attempt to maximize their utilities considering the option of forgoing or accelerating consumption. The model can appeal the above consumer's characteristics, but the method to estimate the parameters is complicacy. Therefore the method to estimate the parameters of Nested Logit Model is very simple and easy to treat.

And the studies about price and consumers' purchase behavior, consumers evaluate the actual price by their reference price. The reference price is treated as weighted average (Lattin and Bucklin, 1989), the previous price (Bell and Bucklin, 1999), the simple average and geometric average of past N period (Rajendran and Tellis, 1994). However, by these approaches, consumers' forward looking behavior is not able to be treated.

Therefore we develop a consumers' purchase incidence and brand choice model by Nested Logit Model considering postpone option by using real options approach. By using real options approach, we can represent the value to postpone purchase, and develop more elaborated consumers' purchase behavior model than previous studies. In addition, our model's method to estimate the parameters is simple, and easy to treat.

2 The Model

We develop a consumers' purchase incidence and brand choice model considering the option value to postpone purchase under price uncertainty. We model the probabilities of brand choice and purchase incidence as a Nested Logit Model, in the model the probability of purchase incidence is a function of the expected maximum utility of the brand choice outcome. And we model the option value to postpone purchase as real options approach.

First, it is thought that consumers purchase stockpile products not to lack of stock, so to model these utilities, modeling the inventory of consumer is needed. Based on Bucklin

and Lattin (1991), the consumer h's inventory of the category at purchase timing[1] t, INV_τ^h is given by

$$INV_t^h = INV_{t-1}^h + Q_{t-1}^h - CR^h I_{t-1}^h, \tag{1}$$

where Q_{t-1}^h is quantity of the category purchased at $t-1$ by consumer h, CR^h is estimated rate of consumption, and I_{t-1}^h is interval of time from $t-1$ to t. Next, the time when the stock of consumer h gets exhausted is given by

$$T^h = \frac{INV_0^h}{CR^h}. \tag{2}$$

Therefore, we can estimate the number of times to come shopping before the stock of consumer h gets exhausted as

$$LS^h = \frac{T^h}{AT^h}, \tag{3}$$

where AT^h is average interval time of purchase. In our model, LS^h is discrete and non-negative, we treat it as cutting off a decimal. Then, the remaining time before the stock of consumer h gets exhausted is given by

$$\tau = LS^h - t. \tag{4}$$

From now on, we use 'time' for the remaing time.

The probability of consumer h purchases brand i at time τ is modeled as follows:

$$P_\tau^h(i) = P_\tau^h(inc) P_\tau^h(i|inc). \tag{5}$$

First of the right side is the probability of purchase incidence, and second of the right side is the probability of brand choice, conditional on purchase incidence.

The probability of brand choice, conditional on purchase incidence is modeled by the Multinomial Logit Model as follows:

$$P_\tau^h(i|inc) = \frac{\exp\left(U_{i,\tau}^h\right)}{\sum_k \exp\left(U_{k,\tau}^h\right)}. \tag{6}$$

Deterministic component of consumer h's utility for purchasing brand i at time τ is given as follows:

$$U_{i,\tau}^h = u_i + \boldsymbol{\beta} \boldsymbol{X}_{i,\tau}^h. \tag{7}$$

The parameter u_i is a parameter of brand-specific and $\boldsymbol{\beta}$ is a vector of response coefficients. The vector $\boldsymbol{X}_{i,\tau}^h$ means marketing variables and consumer-specific variables. Then, consumer h's utility is given by

$$\tilde{U}_{i,\tau}^h = U_{i,\tau}^h + \varepsilon_i^h, \tag{8}$$

[1]Note that 'purchase timing' is different from 'actual time.' Purchase timing t represents t-th purchase.

where ε_i^h is random component of the utility.

The probability of purchase incidence is modeled by Binomial Logit Model as follows:

$$P_\tau^h(inc) = \frac{\exp\left(W_{1,\tau}^h\right)}{\exp\left(W_{1,\tau}^h\right) + \exp\left(W_{0,\tau}^h\right)}. \tag{9}$$

$\tilde{W}_{1,\tau}^h$ is a utility including random component of consumer h's purchase incidence at time τ, and $\tilde{W}_{0,\tau}^h$ is a no-purchase of it. Consumers consider the value of the options to postpone purchase, so we model the value on these utilities:

$$\tilde{W}_{1,\tau}^h = W_{1,\tau}^h + \varepsilon_1^h, \tag{10}$$

$$W_{1,\tau}^h = \pi_{1,\tau} + \theta_{1,\tau} CV_\tau^h, \tag{11}$$

$$\tilde{W}_{0,\tau}^h = W_{0,\tau}^h + \varepsilon_0^h, \tag{12}$$

$$W_{0,\tau}^h = \pi_{0,\tau} + \theta_{0,\tau} OV_\tau^h, \tag{13}$$

where ε_1^h and ε_0^h are random components of the utility. CV_τ^h is category value modeled by Ben-Akiva (1985). Mathematically, they show that the expected maximum utility from the brand choice decision is given by log of the denominator of the brand choice probability:

$$CV_\tau^h = \ln\left(\sum_k \exp\left(U_{k,\tau}^h\right)\right). \tag{14}$$

Under price uncertainty consumers consider the value of the option to postpone purchase, whose value is modeled as OV_τ^h. This is a value consumer h postpone purchase at τ. To model the postpone value, we model the price of brand i at τ by binomial model:

$$x_{i,\tau} = \begin{cases} u x_{i,\tau-1} & \text{with probability } q_i, \\ d x_{i,\tau-1} & \text{with probability } 1 - q_i. \end{cases} \tag{15}$$

When price of brand j is only uncertain, consumer h's expected utility at any time τ is given by

$$V_\tau^h(x_{j,\tau}) = P_\tau^h(inc)\tilde{W}_{1,\tau}^h(x_{j,\tau}) + \left(1 - P_\tau^h(inc)\right)\tilde{W}_{0,\tau}^h(x_{j,\tau}). \tag{16}$$

Note that the price variable $x_{j,\tau}$ is described explicitly. Then, the value of postpone the purchase at τ is given by

$$OV_\tau^h(x_{j,\tau}) = q_j V_{\tau-1}^h(u x_{j,\tau}) + (1 - q_j) V_{\tau-1}^h(d x_{j,\tau}). \tag{17}$$

When the stock of consumer h gets exhausted,

$$OV_0^h(x_{j,0}) = 0. \tag{18}$$

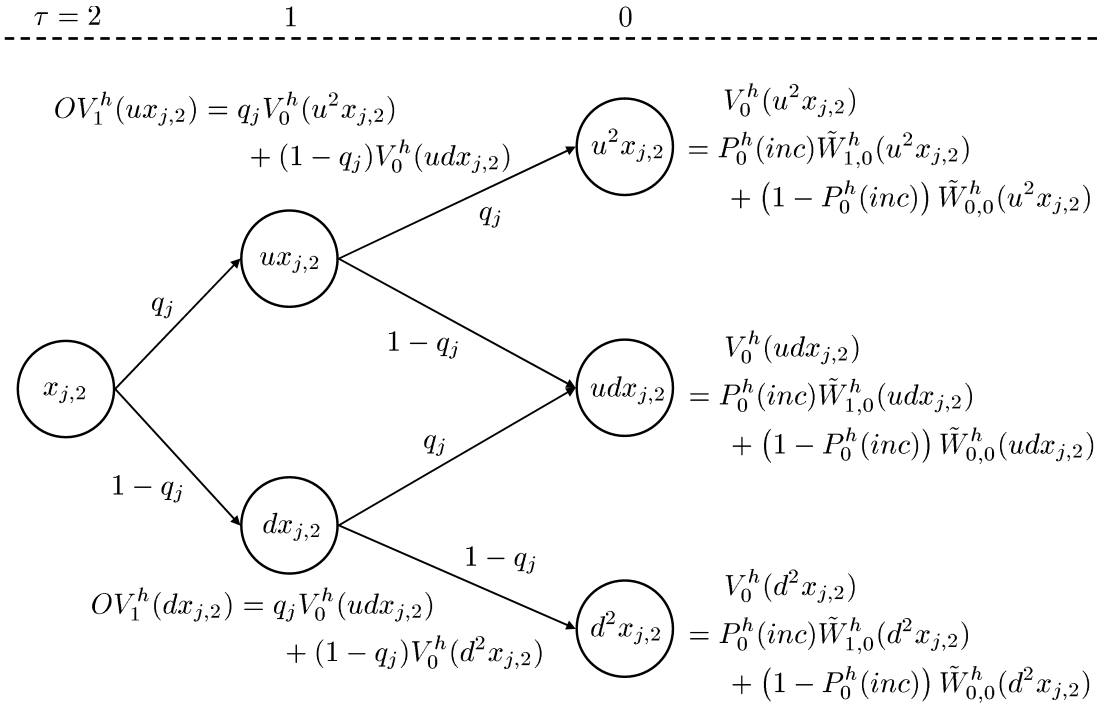

Figure 1: The image of caluculation by the binomial model ($LS^h = 2$).

Therefore, in this case, estimating the parameters is very simple as normal Nested Logit Model. Next, when $\tau = 1$, estimating the parameters is in the same way of $\tau = 0$, by using result of $\tau = 0$. Repeated the same process, we can estimate all of the parameters. Figure 1 displays the image of caluculation of equations (16)–(17) by the binomial model.

3 Conclusion

In this paper, we developed consumers' purchase incidence and brand choice model by Nested Logit Model considering postpone option by using real options approach. By using real options approach, we could represent the value to postpone purchase, and develop more elaborated consumers' purchase behavior model than previous studies.

For future subjects, we will extend the model to all the price of the products under uncertainty and verification.

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