Optimal incentives to early exercise of public-private partnership investments under constrained growth*

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Abstract

This paper analyzes how certain incentives given to private concessionaires, in public-private partnerships, should be optimally determined to promote immediate investment, in a real options framework. Our model extends previous real options models, by considering that investment cost is lower than the project value only up to a certain demand level. This constrained growth model, while having the same trigger value, that induces investment, when it occurs before the maximum growth level, shows that investment may only be optimal after demand is above that level.

We show how investment subsidies, revenue subsidies and a minimum demand guaranty should be optimally arranged to make the option to defer worthless. These three types of incentives produce significantly different results when we compare the value of the project after the incentive is established and the moment when the related cash flows occur.

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1 Introduction

Large scale infrastructure investments have been increasingly promoted via Public-Private Partnerships (PPP) under a variety of arrangements. Those arrangements define the risk and return transferred from the public to the private sector. A correct valuation of the contractual arrangements is crucial for the bidding and negotiation of the PPP. Frequently, these projects are "out-the-money" and need investment incentives to be implemented. Some of these incentives are granted in the form of "contingent claims" or real options.

This paper studies the incentives which may be needed in an airport investment when the government seeks immediate investment. Under uncertainty of future cash flows, there is an incentive to delay investment (McDonald and Siegel 1986). The optimal threshold for investment occurs later than the traditional Net Present Value (NPV) rule suggests. Even when the NPV is positive, delaying investment may be optimal. The incentives given by the government, that grants the PPP concession, cannot ignore the effect of the option to defer, otherwise an insufficient incentive could delay investment, even after the concession is granted.

PPP, and their incentives, with real options features have been studied previously in the literature. PPP arrangements in infrastructure projects, and their risks, are discussed by Grimsey and Lewis (2002). In the present paper we focus on the revenue risk, but other sources of risk can be considered at the cost of a more complex model.

Alonso-Conde, Brown and Rojo-Suarez (2007) study the Melbourne CityLink Project PPP conditions, treated as real options, and how these options affect the incentive to invest. The value transferred from the public to the private sector, through government guarantees, is analyzed. The options valued are the private concessionaire option to defer the payments and the State option to cancel the concession. They show that, although the guarantees provided an investment incentive, the State has transferred considerable value to the private sector.

Different subsidies, guarantees and other incentives in PPP infrastructure projects have been previously studied by Cheah and Liu (2006) and Chiara, Garvin and Vecer (2007). Their focus is on the demand guarantee, which enhance project value. Mason and Baldwin (1988) discuss the case of a loan guarantee, and how the operating options influences both the value of the project, as well as the value guarantee.

Debt guarantee, provided by the government, reduces the cost of capital and raises the project value. This type of incentive is valued by Ho and Liu (2002), who model a PPP with value and investment costs behaving stochastically, accounting for the bankrupcy risk.

Moel and Tufano (2000) study the bidding terms of a copper mine privatization, where the probability of investment was preferred to the cash proceeds from the privatization. They suggest that, reducing the committed investment (exercise price), while reducing the option premium, induces more investment.

The real options embedded in airport projects have been studied by Smit (2003) combining real options and game theory to value airport expansion investments. Pereira, Rodrigues and Armada (2007) model an airport investment when the revenues and the number of passengers behave stochastically and negative or positive jumps occur randomly. Gil (2007) present a description of a wide range of real options embedded in airport investments.

In this paper, we focus on the incentives that can be given by the government in order to induce earlier investment on large scale infrastructures, i.e. , to induce investment in a moment when it is not yet optimal for a private concessionaire to exercise its option to invest.

In fact, under the government public welfare perspective, it can be optimal to start immediately the construction of an infrastructure. However, this may not be in accordance with the private value maximization perspective.

In such a context, a PPP can arise and the government can give the private concessionaire some incentives, in order to make the immediately investment an optimal decision. Technically, the incentive must compensate the private company for losing the option the postpone the project implementation (the so-called *option to defer*), becoming optimal the decision to invest now. We show how investment subsidies, revenue subsidies a a minimum number of passengers guaranty can be optimally established, and how they have different impacts on the project value.

A significant number of previous real options models (e.g.: McDonald and Siegel (1986)) have assumed that there is a “return shortfall” perpetually. On the other hand, most of them have assumed that the firm has an option to “buy” a project for a constant cost, lower than the its value. We propose a different approach, that we believe is more realistic. We assume that the constant investment cost can only produce a positive NPV until a certain “capacity” or demand level, after which the investment needed is, at least, equal to the value of the project. This is also the case when there are constrains to the project growth, such as the limits imposed to airport expansion. Note that this is equivalent to assume that after that level, for a firm holding a perpetual option to invest, the expected growth of the project value equals, or is greater than, the its expected equilibrium rate of return.

In such a constrained growth model, we show that, although the trigger value that induces investment is the same, when it occurs before the maximum capacity/growth level, the value of the investment opportunity is lower. We also show that investment may be only optimal after demand is above that level and how it is still optimal to invest, even installing a lower capacity than current demand.

This paper unfolds as follows. Section 2 derives the value of the project, the value option to invest and its optimal timing, without incentives under constrained growth. In Section 3 several types of incentives are analyzed, and we present a comparison of the immediate and future cash flows of the different incentives. Section 4 concludes the paper.

2 The value of a project with constrained growth

Let P be the number of passengers demanding a destination under the following stochastic process:

$$dP = \alpha P dt + \sigma P dz \tag{1}$$

where α is the (expected) growth rate of the number of passengers, σ the standard deviation, dz an increment of a Wiener process.

Each passenger produces a net revenue R, that is assumed to be constant.

Building an airport can take several years, and the decision of choosing the appropriate capacity is an important aspect in this type of projects. We assume that investing in scale will only add value up to a certain level (C) – the maximum infrastructure capacity – after which any additional investment will have a zero NPV[1]. Furthermore, we assume that the present value of the investment costs is:

$$I = K + kC \tag{2}$$

where K and k are, respectively, the fixed investment cost and the variable investment cost per passenger, and C the maximum capacity level. We assume that I is totally sunk once spent.

The equivalent risk-adjusted process of equation 1 is:

$$dP = (r - \delta) P dt + \sigma P dz \tag{3}$$

where $\delta = \mu - \alpha$ and μ is the equilibrium rate of return. Although the stochastic variable is not a traded asset, a general equilibrium model (e.g.: CAPM) can be used to compute the risk premium (λ) as if it was traded, provided that a stochastic variable time-series (number of passengers) is available. The equilibrium rate of return is:

[1] In other words, we are assuming that there is no option to expand the project beyond its maximum capacity level.

$$\mu = r + \lambda\sigma \tag{4}$$

with $\lambda = \rho_{PM}\dfrac{r_M - r}{\sigma_M}$. ρ_{PM} is the correlation between the variations of the number of passengers and the market and $\dfrac{r_M - r}{\sigma_M}$ the market price of risk.

Using the standard procedures, we have the non-homogeneous ordinary differential equation that must be followed by the project value, $V(P)$, immediately after investing:

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 V}{\partial P^2} + (r-\delta)P\frac{\partial V}{\partial P} - rV + \pi(P) = 0 \tag{5}$$

where $\pi(P) = R\min(P, C)$.

Given $\pi(P)$, we have two possible solutions for equation 15. In the region where $P < C$ (and so: $\pi(P) = RP$), the general solution takes the form:

$$V(P) = A_1 P^{\beta_1} + A_2 P^{\beta_2} + \frac{RPe^{-\delta n}}{\delta} \tag{6}$$

where A_1 and A_2 are constants to be determined, the third right-hand side term corresponds to a particular solution for the differential equation, and n represents the number of years for the construction. Additionally, β_1 and β_2 are as follows:

$$\beta_1 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{7}$$

$$\beta_2 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} - \sqrt{\left(-\frac{1}{2} + \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \tag{8}$$

Since $V(P)$ must tend to zero as P goes to zero (the airport has no value if there are no passengers), and given that P^{β_2} will tend to infinity as P approaches zero, the constant A_2 must be equal to zero.

In the region where $P \geqslant C$ (and so $\pi(P) = RC$), the general solution is:

$$V(P) = B_1 P^{\beta_1} + B_2 P^{\beta_2} + \frac{RCe^{-rn}}{r} \tag{9}$$

where the constants B_1 and B_2 remain to be determined, the last term corresponds to a particular solution for the differential equation, and β_1 and β_2 are as previously presented.

Due to the maximum capacity C, $V(P)$ must remain equal to the perpetuity $\dfrac{RCe^{-rn}}{r}$, even for large values of P. Noting that P^{β_1} tends to infinity as P goes to infinity, B_1 must be equal to zero.

This leaves the following solution for $V(P)$:

$$V(P)=\begin{cases} A_1P^{\beta_1}+\dfrac{RPe^{-\delta n}}{\delta} & \text{for } P<C \\ \\ B_2P^{\beta_2}+\dfrac{RCe^{-rn}}{r} & \text{for } P\geqslant C \end{cases} \tag{10}$$

The two remaining constants (A_1 and B_2) are found using the value matching and smooth pasting conditions at $P=C$[2]:

$$A_1C^{\beta_1}+\frac{RCe^{-\delta n}}{\delta}=B_2C^{\beta_2}+\frac{RCe^{-rn}}{r} \tag{11}$$

$$\beta_1A_1C^{\beta_1-1}+\frac{Re^{-\delta n}}{\delta}=\beta_2B_2C^{\beta_2-1} \tag{12}$$

The solution to these two linear equations for the two unknowns is:

$$A_1=\frac{C^{1-\beta_1}}{\beta_1-\beta_2}R\left(\frac{(\beta_2-1)\,e^{-\delta n}}{\delta}-\frac{\beta_2e^{-rn}}{r}\right) \tag{13}$$

$$B_2=\frac{C^{1-\beta_2}}{\beta_1-\beta_2}R\left(\frac{(\beta_1-1)\,e^{-\delta n}}{\delta}-\frac{\beta_1e^{-rn}}{r}\right) \tag{14}$$

After determining the value of the project, we want to find the value of the option to invest in this project. Its value-function, $F(P)$, has to satisfy the following differential equation:

$$\frac{1}{2}\sigma^2P^2\frac{\partial^2V}{\partial P^2}+(r-\delta)P\frac{\partial V}{\partial P}-rV=0 \tag{15}$$

The general solution for the equation takes the form:

$$F(P)=D_1P^{\beta_1}+D_2P^{\beta_2} \tag{16}$$

The following boundary conditions are used to find the two unknowns, as well as the optimal trigger value (P^*), which corresponds to the value P at which it is optimal to invest, starting the construction:

$$F(0) = 0 \tag{17}$$

$$F(P^*) = V(P^*)-kC-K \tag{18}$$

$$F'(P^*) = V'(P^*) \tag{19}$$

The first condition implies $D_2=0$; the other two conditions allow us to find D_1 and

[2] At $P=C$ the two functions must have the same value, and they must tangentially meet.

P^*. Note that, depending on the parameters, the trigger value can be in the $P < C$ or in the $P \geqslant C$ regions.

For the first case ($P^* < C$), the solution is found with the following value-matching and smooth-pasting conditions:

$$D_1 P^{*\beta_1} = A_1 P^{*\beta_1} + \frac{RP^* e^{-\delta n}}{\delta} - kC - K \tag{20}$$

$$\beta_1 D_1 P^{*\beta_1 - 1} = \beta_1 A_1 P^{*\beta_1 - 1} + \frac{Re^{-\delta n}}{\delta} \tag{21}$$

These equations yields the following solution for the trigger value of P:

$$P^* = \frac{\beta_1}{\beta_1 - 1} \frac{\delta (kC + K)}{Re^{-\delta n}} \tag{22}$$

The value-function for the option to invest is, in turn, as follows:

$$F(P) = \begin{cases} A_1 P^{\beta_1} + \dfrac{1}{\beta_1 - 1} (kC + K) \left(\dfrac{P}{P^*} \right)^{\beta_1} & \text{for } P < P^* \\ A_1 P^{\beta_1} + \dfrac{RPe^{-\delta n}}{\delta} - kC - K & \text{for } P^* \leqslant P < C \\ B_2 P^{\beta_2} + \dfrac{RCe^{-rn}}{r} - kC - K & \text{for } P \geqslant P^* \wedge P \geqslant C \end{cases} \tag{23}$$

For the second case ($P^* > C$), the solution is found with the following value-matching and smooth-pasting conditions:

$$D_1 P^{*\beta_1} = B_2 P^{*\beta_2} + \frac{RCe^{-rn}}{r} - kC - K \tag{24}$$

$$\beta_1 D_1 P^{*\beta_1 - 1} = \beta_2 B_2 P^{*\beta_2 - 1} \tag{25}$$

P^*, the trigger value of P, when it is greater than C, is:

$$P^* = \left[\frac{\beta_1}{B_2 (\beta_2 - \beta_1)} \left(\frac{RCe^{-rn}}{r} - kC - K \right) \right]^{\frac{1}{\beta_2}} \tag{26}$$

The value of the option to invest is, for $P^* \geqslant C$:

$$F(P) = \begin{cases} \dfrac{\beta_2}{\beta_2 - \beta_1} \left(\dfrac{RCe^{-rn}}{r} - kC - K \right) \left(\dfrac{P}{P^*} \right)^{\beta_1} & \text{for } P < P^* \\ B_2 P^{\beta_2} + \dfrac{RCe^{-rn}}{r} - kC - K & \text{for } P \geqslant P^* \geqslant C \end{cases} \tag{27}$$

Parameter	Description	Value
P	Current number of passengers per year	15 million
α	Expected growth rate of P	0.02
σ	Standard deviation of P	0.08
R	Current mean net revenue per passenger	5
r	Risk-free interest rate	0.03
λ	Risk premium	0.3
n	Years of construction of the airport	7
K	Airport fixed investment cost	800 million
k	Airport variable investment cost	40

Table 1: Base-case parameters

It is well known that for real options without value growth constraints, like the one we have imposed trough a limited ability to generate positive NPV after C, δ must be positive, i.e. $\alpha < \mu$, otherwise investment will be delayed until the last available moment. For perpetual options, as above, investment would never be optimal.

However, in our model, a different picture arises. The solution above is valid for $\delta > 0$. When δ is negative, i.e. , the expected drift of the number of passengers is greater than its required rate of return, the project is allways delayed for $P < C$. The value of the project, then becomes:

$$V(P) = \begin{cases} G_1 P^{\beta_1} & \text{for } P < C \\ \\ H_2 P^{\beta_2} + \dfrac{RCe^{-rn}}{r} & \text{for } P \geqslant C \end{cases} \tag{28}$$

With these two equations, the two unknowns are:

$$G_1 = \frac{Re^{-nr}\beta_2 C^{1-\beta_1}}{r(\beta_2 - \beta_1)} \tag{29}$$

$$H_2 = \frac{Re^{-nr}\beta_1 C^{1-\beta_2}}{r(\beta_2 - \beta_1)} \tag{30}$$

The value of the option to invest ($F(P)$) and the trigger value (P^*) are found substituting B_2 for H_2 in equations 27 and 26, respectively. Therefore, the option to invest is given by:

$$F(P) = \begin{cases} \dfrac{\beta_2}{\beta_2 - \beta_1}\left(\dfrac{RCe^{-rn}}{r} - kC - K\right)\left(\dfrac{P}{P^*}\right)^{\beta_1} & \text{for } P < P^* \\ \\ H_2 P^{\beta_2} + \dfrac{RCe^{-rn}}{r} - kC - K & \text{for } P \geqslant P^* \geqslant C \end{cases} \tag{31}$$

3 Incentives to investment

Unless $P > P^*$, investment will be delayed. If immediate investment is intended, several incentives can be given. All of them must make the option to delay worthless ($F(P) = V(P)$), which implies, allways, a cost of $F(P) - V(P)$.

We proceed now to quantify the amount of the incentive and when it is due.

3.1 Fixed investment subsidy

A common incentive is to subsidize investment. Let S be the subsidy needed to make immediate investment optimal, i.e. to make P^* equal to P.

The subsidy amount, as a function of P, is:

$$S(P) = \begin{cases} (kC + K) - P\dfrac{\beta_1 - 1}{\beta_1}\dfrac{Re^{-\delta n}}{\delta} & \text{for } P < C \wedge \delta \geqslant 0 \\ \\ B_2 P^{\beta_2}\dfrac{\beta_2 - \beta_1}{\beta_1} + kC + K - \dfrac{RCe^{-rn}}{r} & \text{for } P \geqslant C \wedge \delta \geqslant 0 \\ \\ H_2 P^{\beta_2}\dfrac{\beta_2 - \beta_1}{\beta_1} + kC + K - \dfrac{RCe^{-rn}}{r} & \text{for } \delta < 0 \end{cases} \tag{32}$$

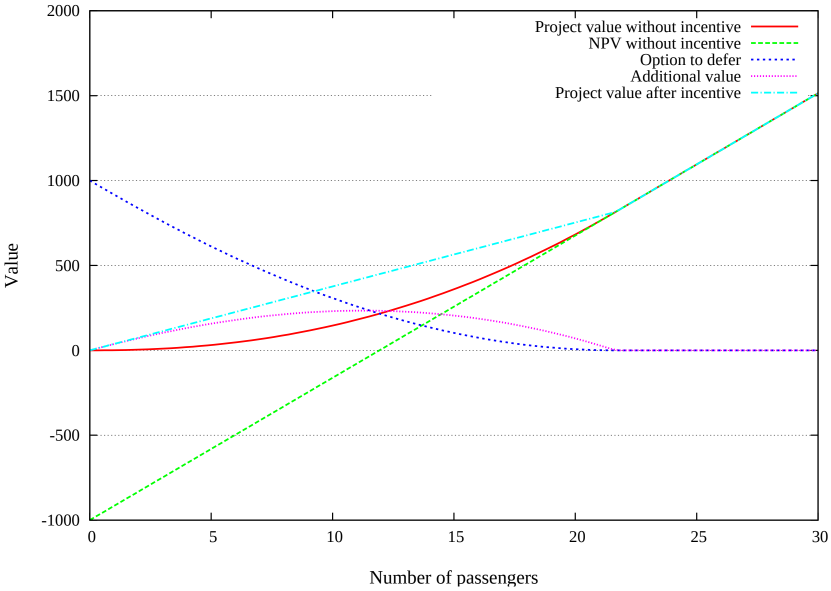
Lowering P^* to P, demands $S(P)$ immediately, but also increases the value of the project to:

$$F_1(P) = \begin{cases} A_1 P^{\beta_1} + \dfrac{1}{\beta_1 - 1}\left(kC + K - S(P)\right) & \text{for } P < C \wedge \delta \geqslant 0 \\ \\ \dfrac{\beta_2}{\beta_2 - \beta_1}\left(\dfrac{RCe^{-rn}}{r} - kC - K + S(P)\right) & \text{for } P \geqslant C \vee \delta < 0 \end{cases} \tag{33}$$

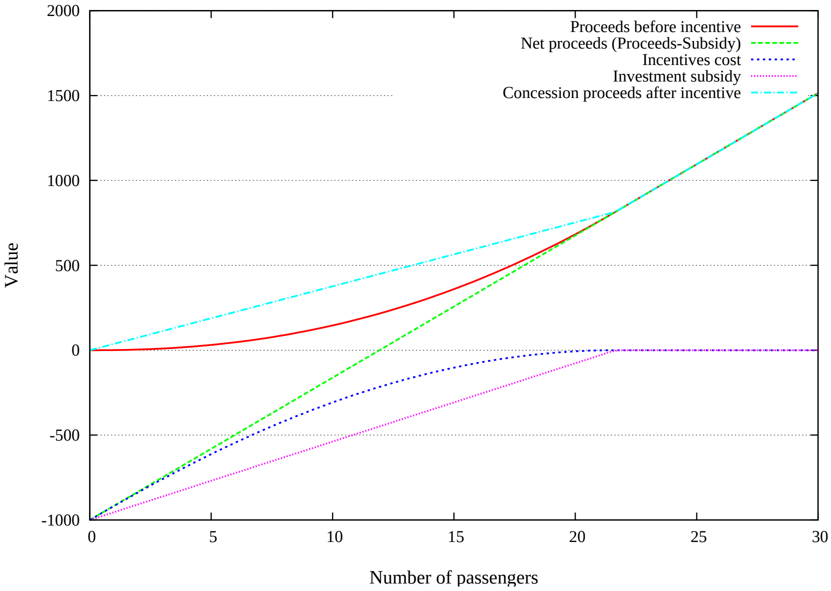
The subsidy makes the investment opportunity worthier by an amount equal to $A(P) = F_1(P) - F(P)$.

If the government pursues immediate investment, a subsidy of $S(P)$ must be given to the concessionaire who, in turn, is willing to pay $A(P)$, additionally to $F(P)$ and immediately, if that is intended. The net cost of this type of incentive is, therefore, $S(P) - A(P)$.

Figure 1 shows the results of a sensitivity analysis of the project value and the amount of incentive as a function of the number of passengers, using the parameters from Table 1. A higher number of passengers makes the project more valuable and reduces the value of the option to defer, thus reducing the incentive needed to build immediately the airport. From the government perspective, the maximum value that is expected to be received from the concessionaire, net of the incentives cost, is exactly the NPV of the project. The



(a) Concessionaire



(b) Government

Figure 1: Fixed investment subsidy - Number of passengers

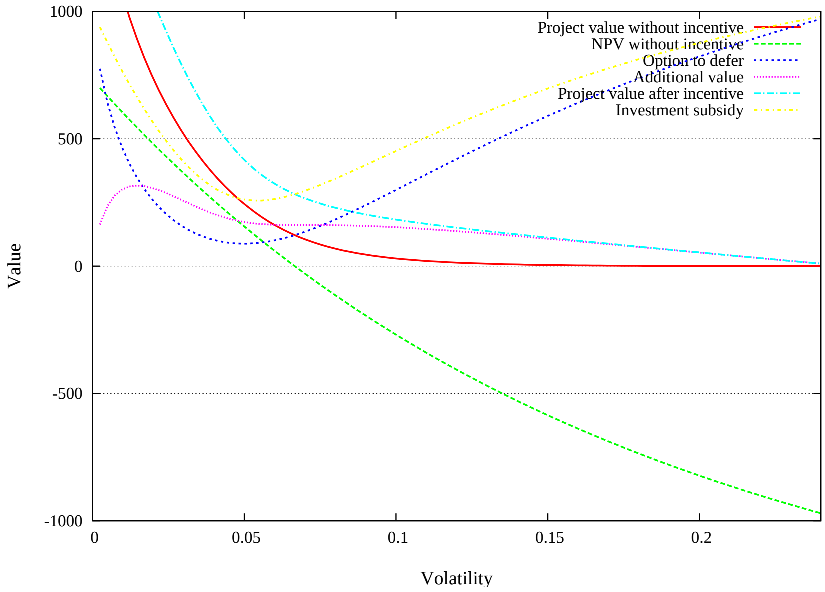


Figure 2: Fixed investment subsidy - Volatility

investment subsidy has, however, to be greater than the negative NPV to make investment optimal. Part of the subsidy is recovered through a higher value of the project.

A similar analysis for the volatility is shown in Figure 2. As we are doing a static comparative analysis, the risk premium (λ) and the expected growth rate (α) remain constant, with the required rate of return (μ) adjusting to volatility. This produces a negative relationship between uncertainty and both the NPV and the project value, while the option to defer, that is the same as the incentives cost, is increasing with volatility. The same occurs for the investment subsidy.

3.2 Revenue subsidy

Another incentive could be given in the form of a variable subsidy per passenger, increasing the revenue from R to $R + s(P)$. $s(P)$ must be enough to make immediate investment optimal, i.e. $P^* = P$:

$$s(P)=\begin{cases}\dfrac{\beta_1}{\beta_1-1}\dfrac{\delta}{e^{-\delta n}}\dfrac{kC+K}{P}-R & \text{for } P<C\wedge\delta\geqslant 0\\[2ex] \dfrac{\left(B_2P^{\beta_2}\dfrac{\beta_2-\beta_1}{\beta_1}+kC+K\right)-\dfrac{RCe^{-rn}}{r}}{\dfrac{Ce^{-rn}}{r}-\dfrac{B_2}{R}P^{\beta_2}\dfrac{\beta_2-\beta_1}{\beta_1}} & \text{for } P\geqslant C\wedge\delta\geqslant 0\\[2ex] \dfrac{\left(H_2P^{\beta_2}\dfrac{\beta_2-\beta_1}{\beta_1}+kC+K\right)-\dfrac{RCe^{-rn}}{r}}{\dfrac{Ce^{-rn}}{r}-\dfrac{H_2}{R}P^{\beta_2}\dfrac{\beta_2-\beta_1}{\beta_1}} & \text{for } \delta<0\end{cases} \tag{34}$$

The present value of the subsidy is obtained replacing R by $s(P)$ in equation 10 when δ is positive or in equation 28 when δ is negative.

With the revenue subsidy, the value of the project increases to:

$$F_1\left(P\right)=\begin{cases}A_1'P^{\beta_1}+\dfrac{1}{\beta_1-1}\left(kC+K\right) & \text{for } P<C\wedge\delta\geqslant 0\\[2ex] \dfrac{\beta_2}{\beta_2-\beta_1}\left(\dfrac{RCe^{-rn}}{r}-kC-K\right) & \text{for } P\geqslant C\vee\delta<0\end{cases} \tag{35}$$

where A_1' is A_1 in equation 13 with R replaced by $R+s(P)$.

The main differences between a revenue subsidy and an investment subsidy are the moment when they occur and the amounts involved. While the net cost is the same (equal to the value of the option to defer), an investment subsidy here is assumed to be paid immediately and the revenue subsidy is assumed to be paid in the future. On the other hand, the additional value that the concessionaire is willing to pay, is also different and higher for the revenue subsidy case (Equations 33 and 35).[3]

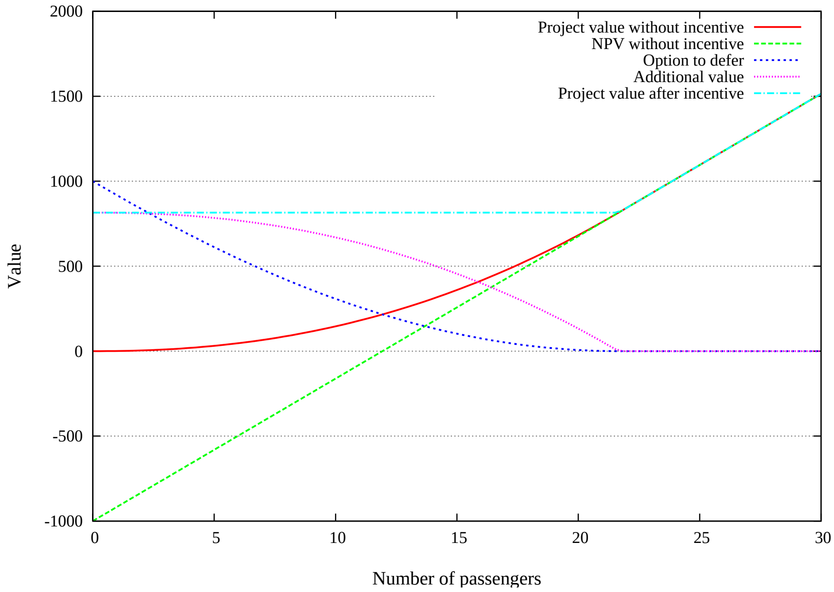
Figure 3 shows that the revenue subsidy, per passenger, needs to increase up to infinity as we move closer to zero passengers. Differently from the investment subsidy, the project value, before the trigger value of P, is decreasing.

3.3 Guaranteed number of passengers

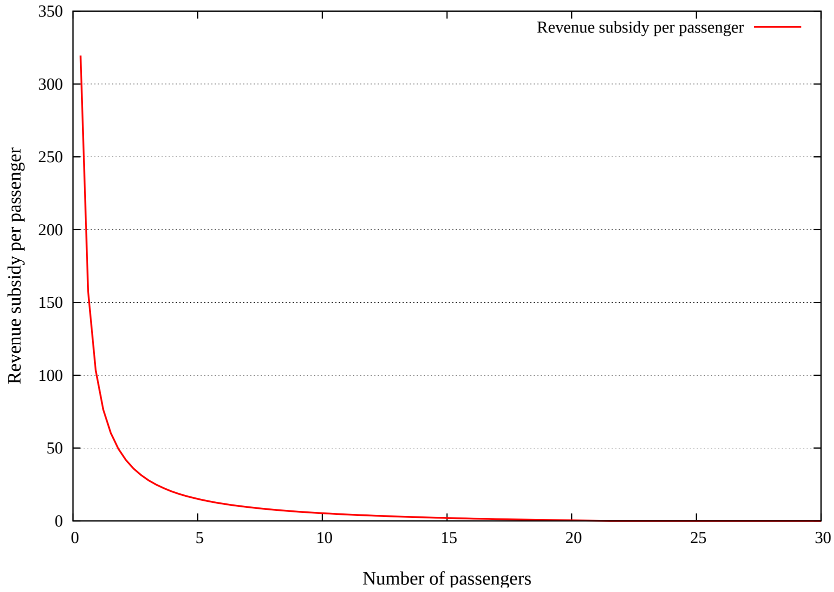
In public-private partnerships, the government guarantees, frequently, a minimum number of passengers ($\underline{P}$): the concessionaire receives the revenue of every passenger and a subsidy for the difference between the actual number of passengers, P, and $\underline{P}$, whenever $P<\underline{P}$.

Under these setting, the ordinary differential equation that $V(P)$ must satisfy is:

[3] Note that $A_1'>A_1$ and $\dfrac{\beta_2}{\beta_2-\beta_1}$ is allways positive.



(a)



(b)

Figure 3: Revenue subsidy

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 V}{\partial P^2} + (r-\delta)P\frac{\partial V}{\partial P} - rV + \pi(P) = 0 \tag{36}$$

where $\pi(P) = R\max(\underline{P}, \min(P, C))$.

We have now three possible solutions for equation 36, in the regions $P \leqslant \underline{P}$, $\underline{P} < P < C$ and $P \geqslant C$.

In the region where $P \leqslant \underline{P}$ (and, so: $\pi(P) = R\underline{P}$), the general solution takes the form:

$$V(P) = M_1 P^{\beta_1} + M_2 P^{\beta_2} + \frac{R\underline{P}e^{-rn}}{r} \tag{37}$$

Since $V(P)$ must tend to zero as P goes to zero, the constant M_2 must be equal to zero.

In the region where $P \leqslant \underline{P}$ (and so $\pi(P) = RP$), the general solution takes the form:

$$V(P) = N_1 P^{\beta_1} + N_2 P^{\beta_2} + \frac{RPe^{-\delta n}}{\delta} \tag{38}$$

In the region where $P \geqslant C$ (and so $\pi(P) = RC$), the general solution is:

$$V(P) = Q_1 P^{\beta_1} + Q_2 P^{\beta_2} + \frac{RCe^{-rn}}{r} \tag{39}$$

As with B_1 above, Q_1 must be set equal to zero.

This leaves the following solution for $V(P)$:

$$V(P) = \begin{cases} M_1 P^{\beta_1} + \dfrac{R\underline{P}e^{-rn}}{r} & \text{for } P \leqslant \underline{P} \\[2ex] N_1 P^{\beta_1} + N_2 P^{\beta_2} + \dfrac{RPe^{-\delta n}}{\delta} & \text{for } \underline{P} < P < C \\[2ex] Q_2 P^{\beta_2} + \dfrac{RCe^{-rn}}{r} & \text{for } P \geqslant C \end{cases} \tag{40}$$

The four unknown constants (M_1, N_1, N_2 and Q_2) are found using the value matching and smooth pasting conditions at $P = \underline{P}$ and $P = C$.

$$M_1 = \frac{C^{1-\beta_1} - \underline{P}^{1-\beta_1}}{\beta_1 - \beta_2} R \left(\frac{(\beta_2 - 1)e^{-\delta n}}{\delta} - \frac{\beta_2 e^{-rn}}{r} \right) \tag{41}$$

$$N_1 = \frac{C^{1-\beta_1}}{\beta_1 - \beta_2} R \left(\frac{(\beta_2 - 1)e^{-\delta n}}{\delta} - \frac{\beta_2 e^{-rn}}{r} \right) \tag{42}$$

$$N_2 = \frac{\underline{P}^{1-\beta_2}}{\beta_2 - \beta_1} R \left(\frac{(\beta_1 - 1)e^{-\delta n}}{\delta} - \frac{\beta_1 e^{-rn}}{r} \right) \tag{43}$$

$$Q_2 = \frac{C^{1-\beta_2} - \underline{P}^{1-\beta_2}}{\beta_1 - \beta_2} R \left(\frac{(\beta_1 - 1)e^{-\delta n}}{\delta} - \frac{\beta_1 e^{-rn}}{r} \right) \tag{44}$$

$F(P)$, has to satisfy the following differential equation:

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 V}{\partial P^2} + (r-\delta)P\frac{\partial V}{\partial P} - rV = 0 \tag{45}$$

The general solution, for this equation, takes the form:

$$F(P) = S_1 P^{\beta_1} + S_2 P^{\beta_2} \tag{46}$$

The following boundary conditions are used to find the two unknowns and trigger value (P^*):

$$\begin{aligned} F(0) &= 0 & (47)\\ F(P^*) &= V(P^*) - kC - K & (48)\\ F'(P^*) &= V'(P^*) & (49) \end{aligned}$$

The first condition implies $S_2 = 0$; the other two conditions allow us to find S_1 and P^*. Note that, depending on the parameters, the trigger value can be in either the three regions.

For the first region, $P \leqslant \underline{P}$, investment is never optimal. As the revenues of $\underline{P}$ passengers are guaranteed, the concessionaire, unless forced (which is the likely situation) to operate when the number of passengers is below that level, will delay investment until the number of passengers reaches $\underline{P}$.[4]

For the second region, where $\underline{P} < P < C$, and proceeding as before, the solution for the trigger value of P is obtained solving the following nonlinear equation:

$$(\beta_1 - \beta_2)\, N_2 P^{*\beta_2} + (\beta_1 - 1)\, P^* \frac{Re^{-\delta n}}{\delta} - \beta_1 (kC + K) = 0 \tag{50}$$

The value of the option to invest is given as follows:

$$F(P) = \begin{cases} S_1 P^{\beta_1} & \text{for } P < P^* \\ N_1 P^{\beta_1} + N_2 P^{\beta_2} + \dfrac{RPe^{-\delta n}}{\delta} - kC - K & \text{for } P^* \leqslant P < C \\ Q_2 P^{\beta_2} + \dfrac{RCe^{-rn}}{r} - kC - K & \text{for } P \geqslant P^* \wedge P \geqslant C \end{cases} \tag{51}$$

For the third case ($P^* > C$), the solution is very similar to the solution without $\underline{P}$. The trigger value of P is given by equation 26, replacing B_2 with Q_2, i.e. :

[4]It is straightforward to show that smooth pasting between $F(P)$ and $V(P)$ is impossible.

$$P^{*}=\left[\frac{\beta_1}{Q_2\left(\beta_2-\beta_1\right)}\left(\frac{RCe^{-rn}}{r}-kC-K\right)\right]^{\frac{1}{\beta_2}} \tag{52}$$

The value of the option to invest is, for $P^{*}\geqslant C$:

$$F(P)=\begin{cases}\dfrac{\beta_2}{\beta_2-\beta_1}\left(\dfrac{RCe^{-rn}}{r}-kC-K\right)\left(\dfrac{P}{P^{*}}\right)^{\beta_1} & \text{for } P<P^{*}\\[2ex] Q_2P^{\beta_2}+\dfrac{RCe^{-rn}}{r}-kC-K & \text{for } P\geqslant P^{*}\geqslant C\end{cases} \tag{53}$$

Solution when δ is negative

As before, when the expected growth rate of the number of passengers is greater than the required rate of return, the concessionaire is better off delaying operations until P reaches C, which includes the region before $\underline{P}$. However, when $P<\underline{P}$ it has at least a value corresponding the the number of passangers guaranteed, i.e. : $V(0)=\frac{R\underline{P}e^{-rn}}{r}$. The value of the project, immediately after investment, is then given by:

$$V(P)=\begin{cases}T_1P^{\beta_1}+\dfrac{R\underline{P}e^{-rn}}{r} & \text{for } P<C\\[2ex] U_2P^{\beta_2}+\dfrac{RCe^{-rn}}{r} & \text{for } P\geqslant C\end{cases} \tag{54}$$

At $P=C$ these to value-functions meet tangently, given the following solution for T_1 and U_2:

$$T_1 = \frac{Re^{-nr}\beta_2C^{-\beta_1}\left(C-\underline{P}\right)}{r\left(\beta_2-\beta_1\right)} \tag{55}$$

$$U_2 = \frac{Re^{-nr}\beta_1C^{-\beta_2}\left(C-\underline{P}\right)}{r\left(\beta_2-\beta_1\right)} \tag{56}$$

The value of the option to invest ($F(P)$) and the trigger value (P^{*}) are found substituting Q_2 for U_2 in equations 53 and 52, respectively.

The effect of changing $\underline{P}$

Figure 4 (a) shows how NPV and the option to invest change as $\underline{P}$ increases. A higher number of guaranteed passangers increases the NPV. That effect is, as expected, more pronounced for lower values of P. As we approach the level above which the NPV stops increasing (C), the probability of using the safeguard provided by the guaranty is lower and, after a certain level is negligible.

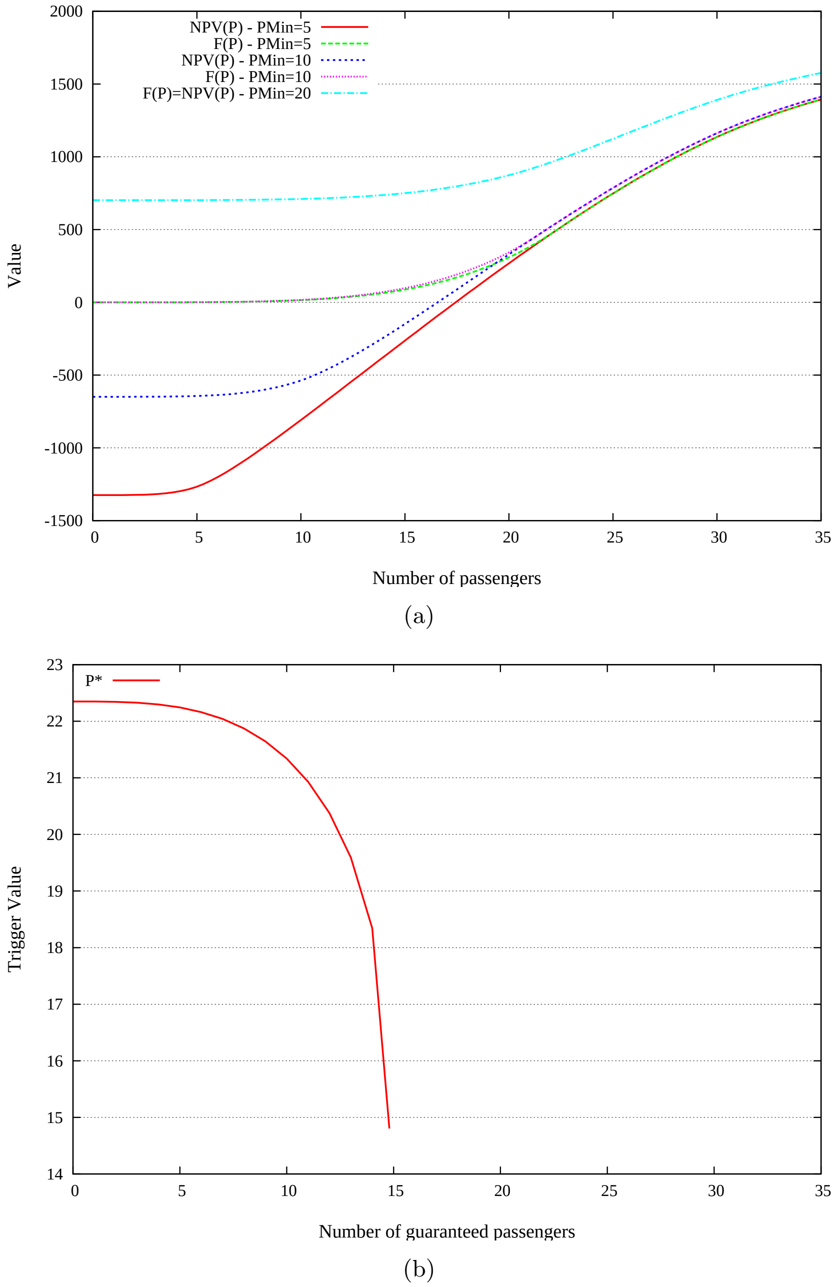


Figure 4: Changing the number of guaranteed passengers

As we increase the level of $\underline{P}$, investment is optimal for lower values of P, i.e. , P^* decreases with $\underline{P}$ (Figure 4 (b)). There is a threshold above which the option to invest equals the NPV, making immediate investment optimal, regardless P. That occurs when the NPV is allways positive, even for $P = 0$. The threshold is:

$$\underline{P}^* = \frac{r\,(kC + K)}{Re^{-rn}} \tag{57}$$

Optimal incentive

Figure 5 (a) shows that the additional value, induced by a guaranteed level of passengers, is lower than for the previous incentives, which means that incentives cost is closer to the value of the option to defer. Figure 5 (b) shows how the optimal level of the guaranty must increases from zero (for $P = P^*$) to the maximum level needed ($\underline{P}^*$) as the investment is less "in-the-money". A higher volatility increases the level of $\underline{P}$ needed.

3.4 Immediate vs future cash flows

We now compare the moment of the payment of the incentives and the net proceeds from the incentives. Figure 6 shows the government cash flows induced by the incentives. The investment subsidy is the only incentive, of those presented above, that is due immediately[5]. All the other types of incentives are due after concession is granted, with positive cash flows, related with the additional project value, received immediately. The revenue subsidy is the type of incentive that delays more the payments and anticipates more the receipts, while the guaranty of a number of passengers generates a lower positive cash flow immediately, which is compensated by a lower futures commitment. The degree of government's commitment to future generations of tax payers is likely to influence the choice of the types of incentives.

4 Concluding remarks and future research

Building a large scale infrastructure involves large sunk costs which, as is suggested by the real options literature, under uncertainty, produces an incentive to delay investment. These projects have been frequently developed by public-private partnerships and usually the government who grants the concession, seeks immediate investment. A correct valuation of these incentives is crucial to promote the desired outcome and to avoid an excessive value transfer to the private sector.

We quantify the optimal investment subsidy, revenue subsidy, and guaranteed number of passengers that prompts immediate investment. Furthermore, we show that these type

[5] Actually, any incentive can be deferred or anticipated at the risk-free rate.

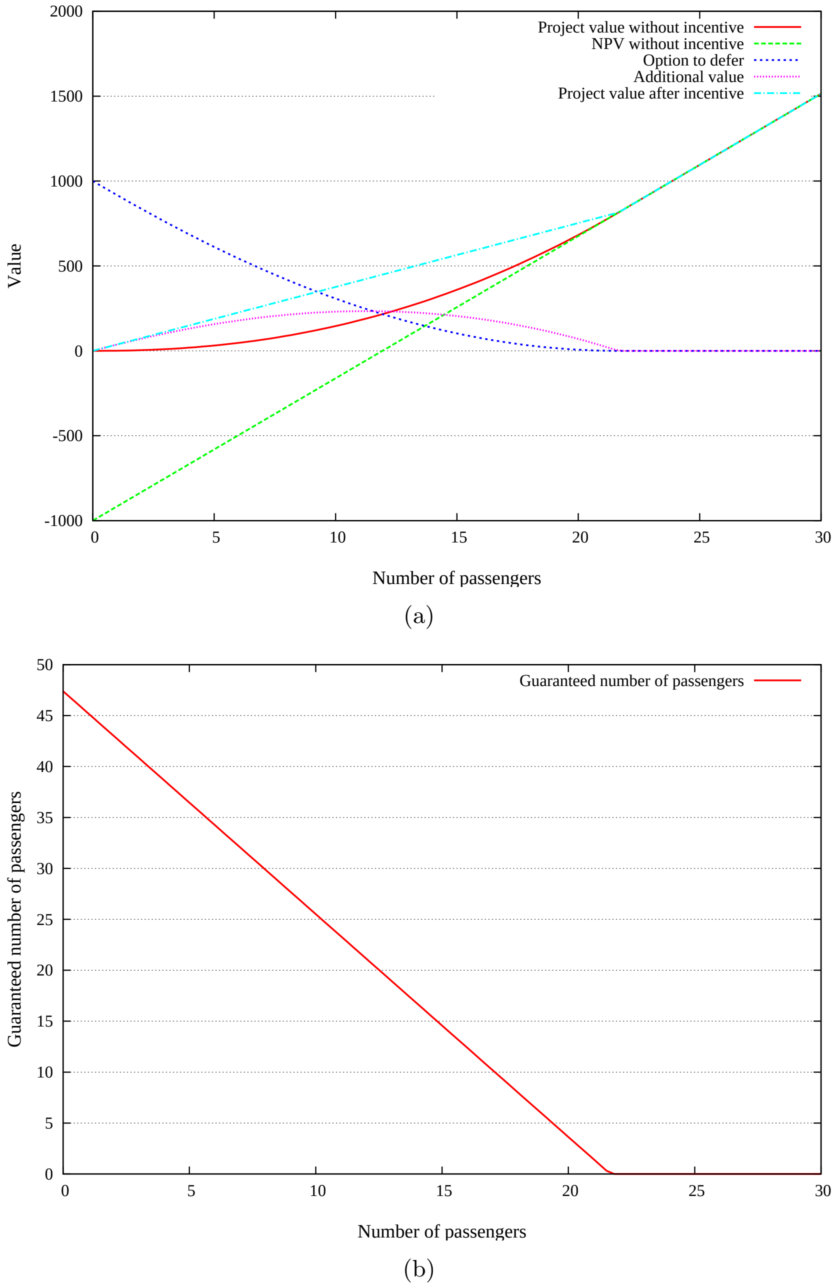
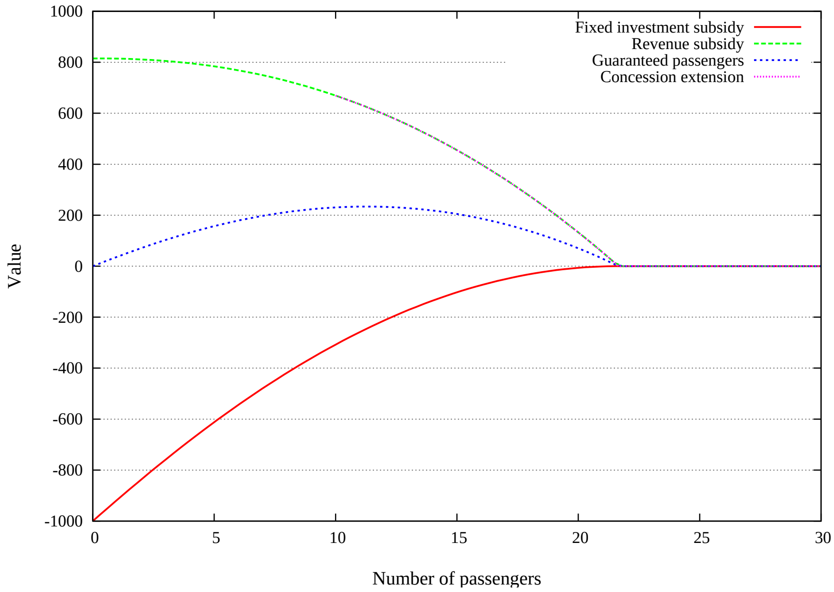
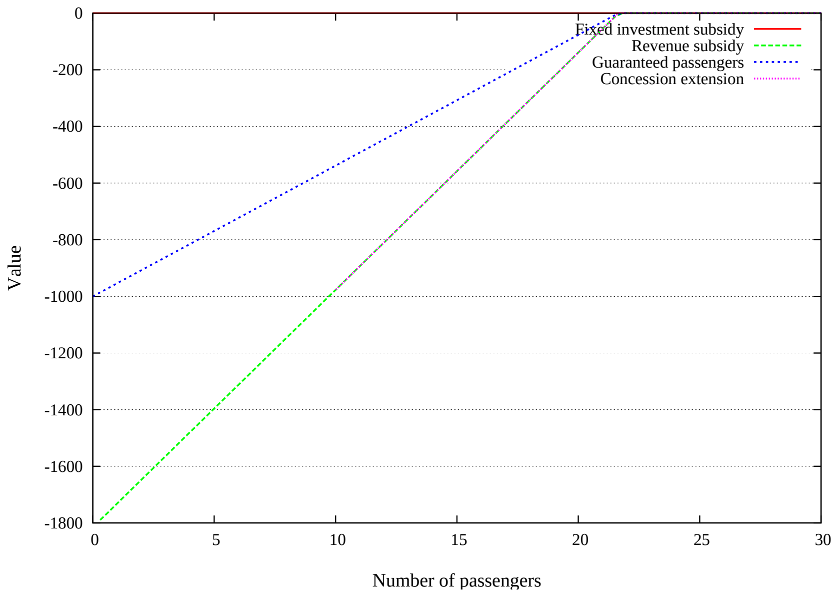


Figure 5: Guaranteed number of passengers



(a) Immediate



(b) Future

Figure 6: Immediate vs future cash flows

of incentives are due in different amounts and moments, with the revenue subsidy being more likely, when the government favors current to future tax payers.

This paper also extends previous real option models (e.g.: McDonald and Siegel (1986)) assuming that deviations from the equilibrium growth rate are only temporary. This is not only more realistic but also allows for an early exercise incentive, even when the project has a higher return than the equilibrium rate of return, in which case previous models have suggested that investment is never optimal for perpetual options.

Several extensions can be made to this paper. Other options, as the expansion or the bankruptcy options can be added to the model. Optimal capacity choice is another important issue in large scale projects. Other assumptions about the stochastic behavior of the two segments, namely mean-reverting processes, could be considered. Adding more stochastic variables is also another feasible extension.

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