Determining the Volatility and the Delay Option of a Petrochemical Project in Brazil

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ABSTRACT

The Real Option Theory (OR) offers a modern methodology for the valuation of an investment project because it considers the value of managerial flexibility facing project uncertainties. The present work seeks to study the deferral option value for a polypropylene petrochemical plant investment project. Perhaps the most critical step of OR is the estimation of the project volatility. This work estimates the project volatility for different cases, considering different possibilities for the uncertain variables modeling. The main uncertain variables are the price of the raw material and the price of the product. Three possibilities for price modeling were considered: Brownian Geometric Movement (BGM), Mean Reversion (MR), and Mean Reversion with Jumps (MRJ). A base case was selected for the volatility project and then the value of the deferral option was calculated through numerical approximations of the Black - Scholes partial differential equation.

1. INTRODUCTION

The Real Option Theory (OR) offers a modern methodology for the valuation of an investment project. It considers the value of managerial flexibility facing project uncertainties that may have a positive value. In other words, sometimes managers can attain the positive value of uncertainty. The value of such flexibility can not be estimated by the traditional methodology, thought deterministic cash flows and its net present value estimation. In contrast, OR manages to estimate the flexibility value through the representation of stochastic variables. Waiting to invest is one of the first real options managers can have. It is called the delay or deferral real option.

At least four new polypropylene (PP) units are being planned and announced for the next six years in Brazil and Latin America. If all these investments come true, it is not difficult to forecast a PP surplus in the region, negatively affecting the profitability of this industry for two or three year. These investments can be valued by the real option theory, in order to investigate if there is a value in waiting to invest.

The main uncertainties which the variation can positively affect the value of a petrochemical project are:

- The investment cost
- Raw-material prices
- Products prices
- The demand of products

In the real option calculation, the investment uncertainties can be neglected considering that this cost is undertaken in the beginning of the period.

Petrochemical prices have cyclical behavior. The petrochemical industry competes with the energy industry for its raw materials. But the petrochemical industry is much smaller than the energy industry in a manner that the petrochemical prices are highly correlated to energy and crude oil prices, that exhibit cyclical behavior.

The profitability of the petrochemical industry is mainly determined by the margin between products and its raw materials prices. The petrochemical prices margins also have a cyclical behavior and are related to supply-demand product balance. The demand generally grows in a steady fashion, as a function of the economic growth cycles. As the demand grows, margins recover, the industry profitability becomes attractive, all players will be likely to invest at the same time. In addition, new production units are capital intensive, as producers try to reduce relative fixed costs building large scale units. The result of all players investing in big units at the same time will be an excess of supply and after this, the prices will fall, the margins will fall and a new cycle will begin. The beginning of the cycle nobody invests because the margins are bad. So, as a result of big scale units and bad timing, the supply grows in large lumps, causing shocks in the supply-demand balance. These shocks also cause the cyclical behavior of the industry. Excess of capacity causes excess in supply, driving the prices and margins down. Butler et al. (1998) recommends the study of the cycles of the petrochemical products in order to plan the optimal investment timing for new projects.

Considering that the actual PP market is balanced, the future additions of capacity will manage to seize future forecasted demand growth. In a first approach, the demand growth can be seen as a function of the GDP growth.

There are two major categories of real option valuation models: the continuous time models and the discrete time models. The continuous time models solve the partial differential equation (PDE) of the option and its boundary conditions. In order to write the partial differential equation, the Ito´s Lemma is used in association with a solution method, like the contingent claims or the dynamic programming method. The stochastic modeling of processes can be modeled inside the option equation. It is possible to model one, two or three stochastic processes inside the equation, but the calculation becomes harder as the number of processes increase.

The discrete time modeling does not have to solve differential equation. One example of this methodology is the Copeland & Antikarov (2002) methodology that does real option valuation in four steps. The first step is to calculate the present value of the base case without flexibility using discounted cash flow valuation model. The second step is to model the uncertainty using event trees. The third step is to identify and incorporate managerial flexibilities creating a decision tree. And the final step is to calculate real option value.

The Copeland and Antikarov (2002) approach uses Monte Carlo simulation to provide distributions for the value of the underlying project. The Monte Carlo simulation can aggregate different stochastic processes for different variables such as prices, market and market size into a single volatility for the project. Then the project volatility can be used in a binomial tree to calculate the real option value with the same parameters as Cox, Ross & Rubinstein (1979).

The advantage of this methodology is the use of a single aggregate volatility value for the project, simplifying the calculation of the real option. Different stochastic processes can be used in the calculation and the number of processes is not limited.

Copeland and Antikarov (2002) methodology has two basic assumptions:

- The *Marketed Asset Disclaimer* (MAD) hypothesis;
- The value of the project follow a geometric Brownian motion.

MAD hypothesis says that the simulation of the value of the project can be used as a proxy of the risk return relation of the project. So it is possible to avoid the need of finding a proxy of the project in the market, what is sometimes difficult from the practical point of view. Sometimes is difficult to find a company in the market with exactly the same risk that the project.

The other Copeland and Antikarov (2002) assumption says that the value of the project follow a geometric Brownian motion (GBM) or, that the return of the project is normally distributed, based on Samuelson theorem (1965b).

This theorem says that even if there are different stochastic variables within a company, following different stochastic processes, the aggregate return value of the stock prices for this company will follow an arithmetic Brownian motion (with normal distribution). In other words, the value of the Project will follow a geometric Brownian motion (with log-normal distribution). If this assumption is true, we can use a binomial tree to value the option, because it assumes that the value of the underlying asset fallows a random walk, and this would be the simplest way to determine the real option value.

In this paper we will use the same idea of simulation for calculating the aggregate volatility of the project, but with some modifications comparing to the Copeland and Antikarov (2002) methodology, as proposed by Dias (2006). Also proposed by Dias (2006), will be another modification to Copeland and Antikarov (2002) methodology. The real option value will be calculated using an approximation of the Black-Scholes-Merton differential partial equation, instead of using the binomial three.

With these two modifications, the methodology used in the present work tries put together the best features of the discrete time and continuous time modeling. It will gather together different stochastic processes in a single volatility for the project and will calculate the real option of delay value using the Black-Scholes-Merton PDE, resulting in a precise estimation.

The volatility calculation will be performed using different stochastic processes for the prices of raw material and product. Three different processes will be considered:

- Brownian Geometric Motion
- Mean Reverting motion
- Mean reverting with jumps

All the values used will be real with real discount rate and all the cash flows will be in constant currency. The cash flow will be for 300 thousand tons per year polypropylene unit.

First the parameters for different stochastic processes will be obtained. Then the aggregate volatility will be estimated considering different stochastic processes for the prices. Finally the option value will be calculated for base case of volatility value.

This paper is organized as follows: the second section discusses stochastic processes and shows the estimated parameters for the different uncertain variables involved. The third section discusses how to estimate the volatility of the project and its real option value. The forth section shows the results of simulation and discusses selection of the volatility base case for the real option calculation. The fifth section presents the real option calculation and final conclusions.

2. STOCHASTIC PROCESSES

The most important uncertain variables able to generate a positive value of waiting to invest in a polypropylene project are:

- Propylene prices (raw-material prices);
- Polypropylene prices (product prices);
- Brazilian demand growth for polypropylene (modeled as a GDP function).

These variables can be modeled in order to have a stochastic cash flow of the project and to determine the aggregate volatility of the project.

These variables can be modeled using three different stochastic processes:

- Geometric Brownian Motion (GBM);
- Mean Reversion Process (MRM);
- Combined process of Mean Reversion with jumps (MRMJ).

The stochastic process parameters for propylene and PP prices will be estimated based on a 17 years long monthly time series, from 1990 to 2006.

The GDP prices and PP demand are available in a yearly basis from 1963 to 2006.

2.1 The Geometric Brownian Motion (GBM)

A continuous-time GBM can be represented by the following differential equation:

$$\frac{dP}{P} = \alpha \cdot dt + \sigma \cdot dz \tag{1}$$

Where:

$dz = \varepsilon\sqrt{dt}$ = Wiener increment; (2)
$\varepsilon = N(0,1)$ =normal distribution with mean 0 and standard deviation 1; (3)
α is the drift;
σ is the volatility of P, or its standard deviation;

For simulations of the GBM it is necessary to "discretize" the differential equation (eq. 1). This can be done through Ito´s Lemma. The resulting equation is (Dias, 2006):

$$P_t = P_0\left\{\exp\left[\left(\alpha - \frac{\sigma^2}{2}\right)\cdot \Delta t + \sigma \cdot N(0,1)\cdot\sqrt{\Delta t}\right]\right\} \tag{4}$$

In the case of risk neutral simulation, the previous equation can be modified resulting the following equation:

$$P_t = P_0\left\{\exp\left[\left(\mu - \delta - \frac{\sigma^2}{2}\right)\cdot \Delta t + \sigma \cdot N(0,1)\cdot\sqrt{\Delta t}\right]\right\} \tag{5}$$

The α and σ parameters of equation (4) is estimated by the equations (Dias, 2006):

$$\sigma^2 = N \cdot Var\left[\ln\left(\frac{P_t}{P_{t-1}}\right)\right] \tag{6}$$

$$\alpha = N \cdot \left\{mean\left[\ln\left(\frac{P_t}{P_{t-1}}\right)\right] + 0{,}5\frac{\sigma^2}{N}\right\} \tag{7}$$

N is the number of periods per year, in the case series data is given in moths (N=12) or days (N=365) and the annual α and σ are needed.

The expected value of V_0 and its variances in the t instant are given by:

$$E[V(t)] = V_0 e^{\alpha t} \tag{8}$$

$$Var[V(t)] = V_0^2 e^{2\alpha t}\left(e^{\sigma^2 t} - 1\right) \tag{9}$$

The estimated GBM parameters for the propylene are shown below (table 1):

ln(Pt/Pt-1)	a. m.	a. a.
mean	0,26%	3,12%
variance	0,96%	11,51%
standard deviation	9,79%	33,93%
alpha	0,74%	8,87%

Table 1 - GBM parameters for propylene

And the GBM parameters for the PP prices are shown below (table 2):

$Ln(P_t/P_{t-1})$	a. m.	a. a.
mean	0,05%	0,62%
variance	0,46%	5,55%
standard deviation	6,80%	23,55%
alpha	0,28%	3,39%

Table 2 - GBM parameters for PP

For both price series unit root tests were performed and it was not possible to reject the random walk hypothesis.

2.2 The Mean Reverting Process (MR)

The simplest mean reverting process is called Arithmetic Ornstein-Uhlinbeck process (Dixit & Pindyck, 1994):

$$dx = \eta \cdot (\bar{x} - x) \cdot dt + \sigma \cdot dz \qquad (10)$$

Where $\bar{x}$ is the equilibrium level and η is the reversion speed.

The expected value of mean reverting and the variance of the mean reverting process x(T) are represented below:

$$E[x(T)] = x(0)e^{-\eta T} + \bar{x}(1 - e^{-\eta T}) \qquad (11)$$

$$Var[x(T)] = (1 - e^{-2\eta T}) \cdot \frac{\sigma^2}{2\eta} \qquad (12)$$

In order to use equation (10), it is necessary to “discretize” it. The discrete equation for real simulation is given below (Dias, 2004):

$$P(t) = \exp\left\{ x_{t-1}e^{-\eta \cdot \Delta t} + \bar{x} \cdot \left(1 - e^{-\eta \cdot \Delta t}\right) + \sigma \sqrt{1 - \left(\frac{e^{-2\eta \cdot \Delta t}}{2\eta}\right)} \cdot N(0,1) - (1 - e^{-2\eta \cdot t}) \cdot \frac{\sigma^2}{4\eta} \right\} \qquad (13)$$

In the case of risk neutral simulation, the previous equation can be modified resulting in the following equation:

$$P(t) = \exp\left\{ x_{t-1} e^{-\eta \cdot \Delta t} + \left[\bar{x} - \frac{(\mu - r)}{\eta} \right] \cdot \left(1 - e^{-\eta \cdot \Delta t}\right) + \sigma \sqrt{1 - \left(\frac{e^{-2\eta \cdot \Delta t}}{2\eta} \right)} \cdot N(0,1) - (1 - e^{-2\eta \cdot t}) \cdot \frac{\sigma^2}{4\eta} \right\}$$

According to Dias (2006), the parameters of this equation can be estimated from the following regression:

$$\ln(P_t) - \ln(P_{t-1}) = a + (b-1)\ln(P_{t-1}) + \varepsilon_t \qquad (14)$$

From the parameters a and b of the regression, it is possible to calculate the parameters of the mean reversion process using equation (14), (15) e (16) that follows (Dias, 2006).

$$\eta = -\ln(b) N \qquad (15)$$

$$\sigma = \sigma_\varepsilon \sqrt{N} \sqrt{\frac{2 \ln b}{b^2 - 1}} \qquad (16)$$

$$\bar{P} = \exp\left[\frac{\left(a + 0{,}5 \frac{\sigma^2}{N} \right)}{1 - b} \right] \qquad (17)$$

σ_ε is the Standard error of the regression (eq. 14).

The MR parameters for the prices are presented below:

	Propylene	**PP**
a	**0,1996**	**0,1940**
b	**0,9686**	**0,9714**
Regression standard error, σ_e	**9,77%**	**6,78%**
P(t = 0)	**1060**	**1400**
Volatility, σ (%a.a.)	**34,38%**	**23,81%**
Reversion speed η (a.a.)	**0,3831**	**0,3478**
Long term average price P_bar	**671,09**	**968,01**

Table 3 –MR parameters for prices

2.3 Combined process of Mean Reversion with Jumps (MRJ)

Dias & Rocha (1998) first modeled a Poisson process with mean reversion in real options literature. A stochastic process of Arithmetic Ornstein-Uhlenbeck mean reverting with discrete jumps can be represented by (Dias, 2004):

$$dx = \eta \cdot (\bar{x} - x) \cdot dt + \sigma \cdot dz + dq \tag{18}$$

This is a Wierner process and also a Poisson process. The jump process is represented by dq, and is independent of dz (Dias, 2004). Most of the time dq is equal to zero. But jumps can happen of an uncertain size ϕ occuring with frequency λ. According to Dias (2004), dq= 0 with probability 1 – λ dt and dq = ϕ with λ dt probability. The size of the jump ϕ is modeled as a probability distribution. According to Merton (1976) apud Dias (2004), the probability distribution ϕ is log-normal. Dias (2004) assumes a symmetric distribution to the jumps up and down in order that the average size becomes zero and so it is not necessary to compensate the Poisson process.

The expected value of x(T) is the same of the mean reverting process. The variance is given by the following equation (Dias, 2004):

$$Var[x(T)] = (1 - e^{-2\eta T}) \cdot \frac{(\sigma^2 + \lambda \cdot E[\phi^2])}{2\eta} \tag{19}$$

Where $E[\phi^2] = \int \phi^2 \cdot f(\phi) \cdot d\phi$, pois $E[\phi^2] \neq (E[\phi\ ])^2$ (Dias, 2004).

The "discretization" of eq. 18 results in the following equation (Dias, 2004):

$$P(t) = \exp\left\{ x_{t-1} e^{-\eta \cdot \Delta t} + \bar{x} \cdot \left(1 - e^{-\eta \cdot \Delta t}\right) + \sigma \sqrt{1 - \left(\frac{e^{-2\eta \cdot \Delta t}}{2\eta}\right)} \cdot N(0,1) + jumps - (1 - e^{-2\eta T}) \cdot \frac{(\sigma^2 + \lambda \cdot E[\phi^2])}{4\eta} \right\} \tag{20}$$

For risk neutral simulation eq. 20 is modified and result

$$P(t) = \exp\left\{ x_{t-1} e^{-\eta \cdot \Delta t} + \left[\bar{x} - \frac{(\mu - r)}{\eta}\right] \cdot \left(1 - e^{-\eta \cdot \Delta t}\right) + \sigma \sqrt{1 - \left(\frac{e^{-2\eta \cdot \Delta t}}{2\eta}\right)} \cdot N(0,1) + jumps - (1 - e^{-2\eta T}) \cdot \frac{(\sigma^2 + \lambda \cdot E[\phi^2])}{4\eta} \right\} \tag{21}$$

Symmetric distributions for jumps are imposed, so that the sum of jumps up and jumps down are zero. Also the probability of a jump up is the same that a jump down probability.

Doing so, there is not the need of compensations (Dias, 2004). The jump parameters are shown below:

Frequency of Jump up (λu)	0,12500
Frequency of Jump down (λd)	0,12500
Average jump up length	0,69315
Average jump down length	-0,69315
$E[\phi^2]$	0,48045

Table 4 – Jump parameters

The correlation of propylene and polypropylene jumps are considered 90%, close to the correlation of annual prices.

2.4 Choosing the Right Stochastic Process

Dixit & Pindyck (1994) p. 78 mention that the random walk hypothesis for crude oil prices, tested through unit root tests, is not rejected using 30 to 40 years long time series. In these cases, it is possible to use a random walk model like the geometric Brownian motion to model crude oil prices. Petrochemical prices are correlated to crude oil prices. As mentioned before, the random walk hypothesis also was not rejected for a 17 years long monthly time series for prices of propylene and PP.

On the other hand, according to Geman (2005), commodity prices are better represented by mean reverting processes as long as generally commodity prices do not present tendency over long periods of time. In other words, it is not expected that commodity prices increase over the time, or have a positive trend. However, if there are economic cycles affecting these prices, the prices may exhibit a trend (Geman, 2005).

The choice of the stochastic process suitable for petrochemical prices is a difficult decision. On one hand, petrochemicals are commodities, should have limited prices and exhibit some reversion like behavior. On the other hand, petrochemical also have cyclical behavior. A petrochemical price can be compounded into two parts: the first part is the cost of production. The main cost of production normally is the cost of petrochemical raw materials. These raw materials are highly correlated to the crude oil price, that has a cyclical behavior. The second part of petrochemical prices is the margin between its costs and its market price. The market price is a function of the supply-demand balance. In this balance, the supply is cyclical. Generally all players tend to invest when the prices are high and then nobody invest until the demand is balanced again. The demand grows in a steady fashion but is dependant on the economic general cycles. So, petrochemical prices are cyclical mainly due to three factors: crude oil prices cycles, investment cycles and the general economic cycles.

Commodity prices with cyclical behavior can also be modeled as a mean reverting process with shocks in the supply-demand balance (Geman, 2005). This more realistic model combines mean reverting processes with jumps (Dias, 2006). The jumps represent the shocks in the supply demand balance. This is the case of the combined process of mean reversion with jumps and it is suitable for modeling commodities prices, like the petrochemical prices, that are exposed to shocks in supply-demand balance.

The internal demand growth variable will be considered as a function of the GDP of Brazil. The function is determined and tested in a regression of the internal market and the GDP. The GDP will be considered as a random walk and modeled through a GBM.

3. ESTIMATING THE VOLATILITY AND THE REAL OPTION VALUE OF THE PROJECT

The aggregate volatility of the Project will be calculated in a different way from the proposed of Copeland & Antikarov (2002). In order to obtain the aggregate volatility of the project Copeland & Antikarov (2002), simulate the uncertain variables for all the periods and calculate the volatility for the return of the project. It is proposed that the simulation should be done in a single period, in the first year of operation of the plant. The remaining periods the expected values are calculated based on the first period value. It will be simulated the value of the project. According to Dias (2006), this is a way to avoid working with the logarithm of negative numbers and simulating problems and simulation problems resulting from it.

From the simulation it will be obtained the values of the mean and the variance of the project value. Considering that the project value follows a geometric Brownian motion, equations (7) e (8) can be used to obtain α and σ of the Project value. The resulting equation are below:

$$\alpha = \frac{\ln\left\{\frac{E[V(t)]}{V_0}\right\}}{t} \tag{22}$$

$$\sigma = \sqrt{\frac{\ln\left\{\frac{\mathrm{var}[V(t)]}{e^{2\alpha \cdot t} \cdot V_0} + 1\right\}}{t}} \tag{23}$$

Substituting (22) into (23), gives:

$$\sigma = \sqrt{\frac{\ln\left\{\frac{\mathrm{var}[V(t)]}{(E[V(t)])^2} + 1\right\}}{t}} \tag{24}$$

Note that σ does not depend on V_0.

The return of a firm can be represented as its growth rate a plus the dividend yield paid to the shareholders

$$\mu = \alpha + \delta \tag{25}$$

The α value can be determined by the eq (22) in the case of real simulation. The value of μ is the hurdle rate of the Project. Then, it is possible to determine δ from eq (25):

$$\alpha = \mu - \delta \tag{26}$$

In the case of risk neutral simulation, the starting point is equations (8) and (9), but drift will be modified using the risk neutral drift, α'.

$$\alpha' = r - \delta \tag{27}$$

Where r is the risk free rate.

So, by modifying equation (8) and (9) with equation (26) will result

$$E[V(t)] = V_0 e^{(r-\partial)t} \tag{29}$$

$$Var[V(t)] = V_0^2 e^{2(r-\partial)t}\left(e^{\sigma^2 t} - 1\right) \tag{30}$$

From the simulation it will be obtained the values of the mean and the variance of the project value. Considering that the project value follows a geometric Brownian motion, From equations (29) and (30), will be obtained

$$r - \partial = \frac{\ln\left\{\frac{E[V(t)]}{V_0}\right\}}{t} \tag{31}$$

$$\sigma = \sqrt{\frac{\ln\left\{\frac{\mathrm{var}[V(t)]}{e^{2(r-\partial)\cdot t}\cdot V_0} + 1\right\}}{t}} \tag{32}$$

Equations (31) and (32) are similar to equations (22) and (23), but can be used in risk neutral simulations.

Substituting (31) into (32), also will result in equation (24). Once one have the risk adjusted hurdle rate, m and the risk free rate r, the value of δ for the process can be calculated using equation (27). Then the value of a can be calculated using equation (26).

3.1 Deferral Option Estimation

After determining the volatility for the project, the next step will be the estimation of the real option to defer the project.

Starting with the Black-Scholes Differential partial equation shown below, the option type, if European or American, will depend on the boundary conditions of the equation.

$$\frac{\partial F}{\partial t} + \frac{1}{2} \cdot \sigma^2 \cdot V^2 \cdot \frac{\partial^2 F}{\partial V^2} + (r - \partial) V \frac{\partial F}{\partial V} - rF = 0 \qquad (33)$$

In order to obtain the Black-Scholes Formula for European call option, an only boundary condition is needed (Wilmott, 2007):

F(V, T)=Max(V – X; 0)

In this case, equation (33) will have an analytical solution resulting in the Black-Scholes Formula.

In the case of an American call option, there are four boundary conditions (Dias, 2006):

For V=0, F(0;t)=0

For t=T, F(V, T)=Max(V-X; 0)

And the last two boundary conditions are in the threshold:

$F(V^*,t)= V^*$ - I (“smooth-pasting condition”)

$\frac{\partial F}{\partial V}(F^*, t) = 1$ (“high contact condition”)

Dias (2006) developed a *software* called Timing that solves the Black-Scholes PDE for an American call option through the numerical approximations of Bjerksund & Stensland (available at http://www.puc-rio.br/marco.ind/timing.html). This software graphs the F(V, t) values, and the threshold curve V*(t), and calculates the probability of exercise (in the case of waiting to invest) and the expected first hitting time. The Timing software will be used in order to determine the deferral option value instead of using a binomial tree and the binomial equation. Timing software is more precise than the binomial formula.

4. RESULTS

All discounted cash flows have the following premises:

- Constant currency (2006 American dollars);
- Real values for the cash flow (all prices are real values, and there is no inflation);

- Discounted values to Jan. 2008 (t=0);
- 10 year of economic life of the project;
- No perpetuity or terminal value is considered;
- Regarding taxes, only the PIS/ COFINS recovery is considered;
- Other taxes considered are: PIS/ COFINS 9,25%, ICMS 18%, income tax and CSLL, 34%. CPMF: 0,38%;
- Currency R$ 2,00/ US$;
- Total investment: 300 million dollars and depreciation in 10 years;
- Capital investment schedule: 10% in the first year; 60% in the second; 30% in the third year;
- For the depreciation/ amortization calculations a 5% a.a. inflation is considered.

The total capacity of the plant is 300 kt/a of propylene. It was considered a production curve of 80, 90 e 95% of the total production capacity.

The change in working capital will be the total cash equivalent to: 5 days of operational costs; 30 days of product inventory; 30 days of raw-material inventory; 45 days of product term sales; 60 days of raw material term supply; 15 days of other materials supply; 15 days of other payments; 15 days of wages; e 15 days of taxes. In the last year of investment, a provision equivalent of 80% of the working capital is made.

Fixed and variables cost of the plant were obtained from literature, from a typical propylene unit.

The risk free rate and the adjusted discount rate considered are respectively 4,32% and 9,02%.

4.1 Modeling PP Growth Demand

An attempt to model the PP demand growth as a function o the GDP growth stochastic process was made.

The Brazilian Gross Domestic Product (GDP) time series from 1963 to 2006 are shown below (currency: 2006 constant Reais)

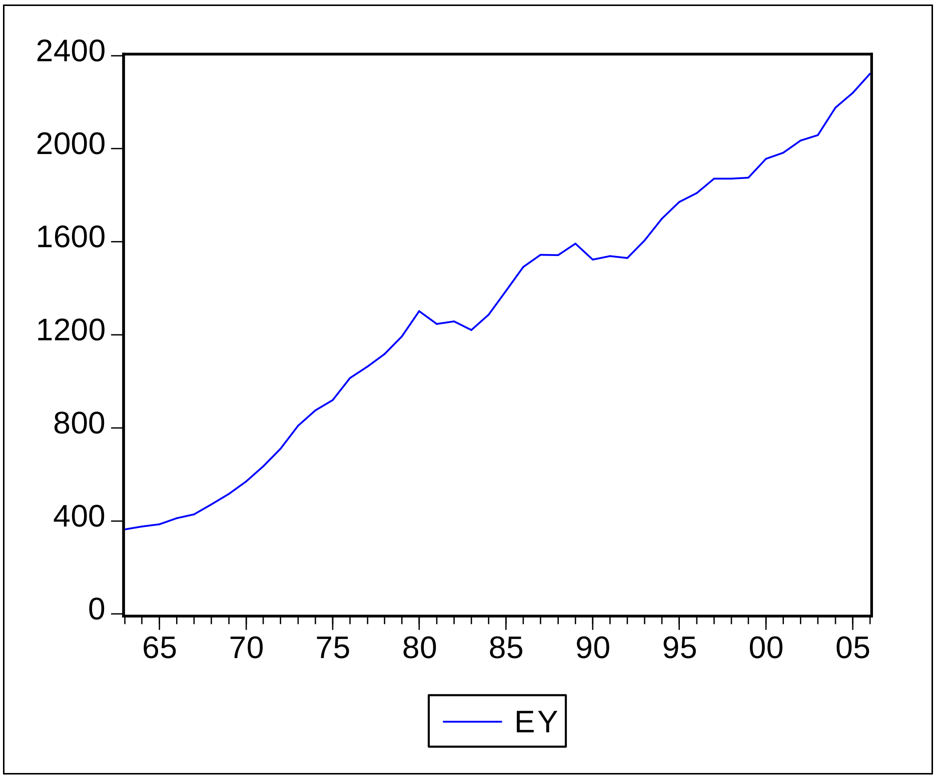

Figure 1 - GDP 1963 to 2006

It is clear that the Brazilian GDP has a drift. Additionally, unit root test does not reject the random walk for this series. So, there is no evidence against the use of a GBM to model the Brazilian GDP.

The GBM parameters for the Brazilian GDP (estimated from the series above) are shown below.

$\ln(P_t/P_{t-1})$	a. a.
mean	4,22%
variance	0,16%
standard deviation	4,03%
alpha	4,31%

Table 5 – GBM Parameters for GDP

The polypropylene market was estimated based on ABIQUIM 2006 data. In 2005, the total polypropylene production was 1212,2 kt/a. In the same period the total volume of imported polypropylene was 85,59 kt/a and the total volume exported was 227,76 kt/a. Then the Brazilian demand of polypropylene in 2005 was 1212,20-227,76+85,59 = 1070,03 kt/a.

If the total demand of PP is regressed against the total brazilian the Gross National Product (GDP) the following graph is obtained:

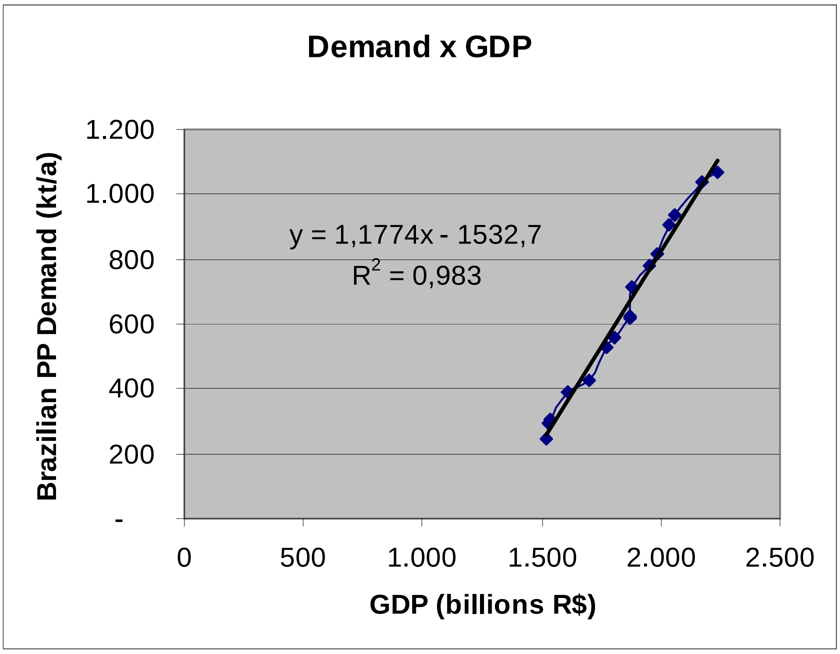

Figure 2 – Demand x GDP Regression

The PP demand is highly correlated to the GDP in Brazil. The equation shown is the regression equation and it will be used to link the GDP processes to the PP demand. So

Total demand (t) = 1,1774 x GDP (t) -1532,7 (34)

The polypropylene prices were different according to their destination. If the polypropylene is exported, its prices will be the reference price minus US$ 80 per ton. If the polypropylene is sold in the internal Brazilian market, its prices will be the reference price plus US$ 150.

As a first approximation, it will be considered the growth of Brazilian market does not affect the international prices, once the prices are internationally determined and Brazil is a relatively small market comparing to the Chinese market and the rest of world's market.

The internal sales of the Project will be the difference between the PP demand and supply in Brazil. The total demand will be given using equation (34). For each year the GDP value is simulated and then the total demand is calculated using equation (34).

The actual installed capacity of PP plants in Brazil is 1335 kt/a. Considering that the maximum production capacity is 95% of the total installed capacity, and that until 2011 will have two 300 kt/a polypropylene new units and an expansion of 100kta of an existing unit, the total PP supply in 2011 considered is 1968,25 kt/a.

A minimum import of 10% will be considered for all years, considering that is not possible to fully occupy the internal PP market.

4.2 Calculating the Project Volatility

The project aggregated volatility was calculated for different cases, considering different possibilities of stochastic processes for price modeling. The correlation between prices considered is 74% (monthly prices correlation). In each simulation, V_0 is the project value for N(0;1) = 0. This is also the value of project without flexibility for each case.

First set of simulations were made considering the different stochastic process for the propylene prices, PP prices and considering a GBM for the GDP process. The results are in table 6 below. For each case, both real and risk neutral simulations were made.

When simulations for polypropylene and PP prices are performed together using a GBM model, the value of V_0 becomes negative in most of the iterations. The reason is that the raw material drift (propylene) is greater than the product drift (PP drift), so that the margin of PP and propylene became negative for most of the iterations. As a result, $E[V_0]$ is negative. The value of V_0 is also negative. This is not consistent to a GBM for the value of the project. So cases 5 and 6 can not be used.

In the case of risk neutral simulation, the new drift (equation 27) of the GDP becomes negative. This significantly affects the value of V, resulting in a greater variance of V, and a different volatility. So is not recommended to use simulations 2 and 4 for the real option calculation.

The real simulation considering MRJ´s for prices and a GBM for the GDP (case 3) resulted in a greater value for α than μ. In other words, the project profits will grow more in average than the discount rate. In this case, δ becomes negative. So this case was not considered also.

Case	1	2	3	4	5	6
Simulation	Real	Risk neut.	Real	Risk neut.	Real	Risk neut.
Propylene price	MR	MR	MRJ	MRJ	GBM	GBM
PP price	MR	MR	MRJ	MRJ	GBM	GBM
Demand	GBM	GBM	GBM	GBM	GBM	GBM
V(t = 0), MM USD	280,5	143,59	278,6	94,52	(52,0)	(249,83)
$E[V(t_{simul})]$	303,8	148,0	306,5	101,7	(112,4)	(317,9)
$Var[V(t_{simul})]$	4.879,8	5.280,5	17.908,8	21.530,0	199.512,8	213.732,8
σ_V	22,70%	46,48%	41,77%	106,08%	167,95%	106,60%
α_V	7,99%	7,71%	9,57%	12,04%	77,07%	28,79%
δ_V	1,03%	1,31%	-0,55%	-3,02%	-68,05%	-19,77%

Table 6 – Simulation results for stochastic process for prices and GDP

The next set of simulation results do not consider any stochastic process for the GDP and PP demand growth. It is shown in table 7 below. The problem with the GBM margin for prices is still present. So cases 11 and 12 can not be used. The real and risk neutral results considering MR and MRJ for prices become closer. The problem of obtaining a negative dividend yield happens again in cases 8, 9 and 10.

Comparing case 1 to case 7 it is possible to see that the demand growth variable does not result in a much higher variance of V_0 or a much higher volatility. The effect of the demand growth is not additive to the volatility of the project for two reasons: first, this variable affect the value of the project in its first years of operation, until demand growth develops enough and all the production can be sold in the internal market. Also the effect of chance in the demand result on more or less exports that are sold with discount in the international market.

Case	7	8	9	10	11	12
Simulation	Real	Risk neut.	Real	Risk neut.	Real	Risk neut.
Propylene price	MR	MR	MRJ	MRJ	GBM	GBM
PP price	MR	MR	MRJ	MRJ	GBM	GBM
Demand	CONST.	CONST.	CONST.	CONST.	CONST.	CONST.
V(t = 0), MM USD	281,1	361,96	279,13	310,28	(51,44)	(34,07)
$E[V(t_{simul})]$	304,1	384,0	306,8	326,1	(112,1)	(93,4)
$Var[V(t_{simul})]$	4.259,9	9.936,5	17.326,1	21.478,9	198.857,6	213.713,8
σ_V	21,22%	25,54%	41,11%	42,89%	167,99%	179,94%
α_V	7,88%	10,60%	9,45%	9,69%	77,94%	105,58%
δ_V	1,14%	-1,58%	-0,43%	-0,67%	-68,92%	-96,56%

Table 7 – Simulation results for stochastic process for prices

A last set of simulation are shown below (table 8) considering only stochastic processes for PP prices. This would be the case of an integrated company from crude oil production to PP production. In this case the raw material price can be regarded as a constant transferring price.

Case	13	14	15	16	17	18
Simulation	Real	Risk neut.	Real	Risk neut.	Real	Risk neut.
Propylene price	CONST.=671	CONST.=671	CONST.=671	CONST.=671	CONST.=671	CONST.=671
PP price	MR	MR	MRJ	MRJ	GBM	GBM
Demand	CONST.	CONST.	CONST.	CONST.	CONST.	CONST.
V(t = 0), MM USD	355,72	359,35	349,63	355,20	876,87	772,19
$E[V(t_{simul})]$	393,7	372,4	402,2	393,4	1.002,8	854,4
$Var[V(t_{simul})]$	8.043,5	5.234,0	31.108,8	38.317,1	167.065,8	181.761,0
σ_V	22,49%	19,25%	41,94%	47,04%	39,20%	47,15%
α_V	10,14%	8,27%	14,02%	14,90%	13,42%	14,81%
δ_V	-1,12%	0,75%	-5,00%	-5,88%	-4,40%	-5,79%

Table 8 – Simulation results for stochastic process for PP prices only

The mean reversion result in the smaller values for the volatilities. The volatility considering BGM or MRJ have similar values.

4.3 Deferral Option Estimation

Case number 1 was chosen for the real option calculations as a base case. The real option parameters are shown below:

V_0	280,48	σ	22,7%
X = I	285,82	α	7,99%
τ	10	δ	1,03%

Table 9 – Real Option Parameters

Using these parameters, the delay option was estimated through the Timing Program (available at http://www.puc-rio.br/marco.ind/timing.html). The outputs of Timing Program are shown below.

2.1) The value of the threshold (level of optimal immediate investment) V' *

V* = **1973,7** **$ millions**

V*/I = **6,906** **unitary threshold (see chart below)**

Options theory suggestion: WAIT AND SEE as the optimal action under uncertainty

2.2) The value of the option to invest (or the value of the investment opportunity) F

F = **100,750** **$ millions**

So, the option premium value (or the waiting value) = F - NPV = 99,67 $ millions

2.3a) Probability of Exercise Occurrence with Hybrid Quasi-Monte Carlo Simulation (HQMC)

Probab* = **36,4%**

2.4a) The expected first hitting time T* for V' to cross the threshold curve, given that the exercise occurr

T* = **9,99** **years**

(using HQR)

Table 10 – Timing Output

From table 10, the calculated real deferral option was US$ 100,75 million. Timing also draws two charts: the threshold cart and the real option value chart. These charts are shown below.

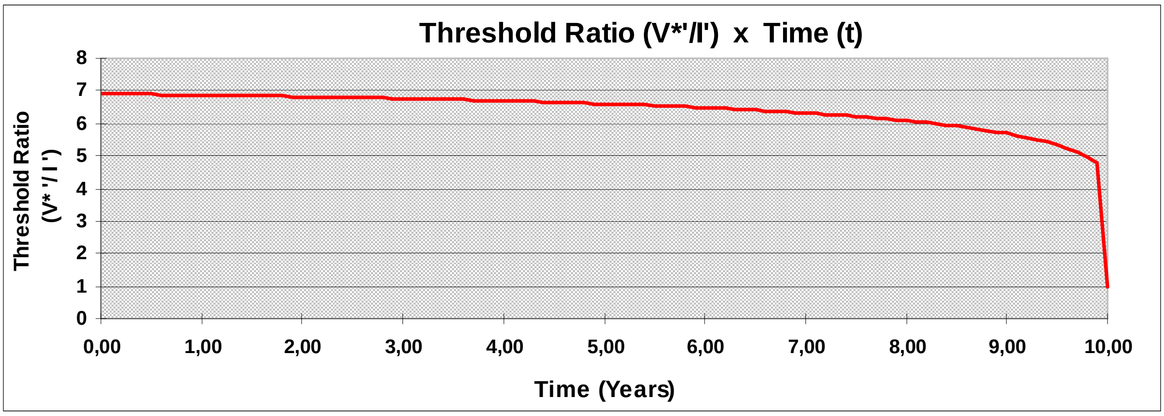

Figure 3 – Timing Program Threshold Chart

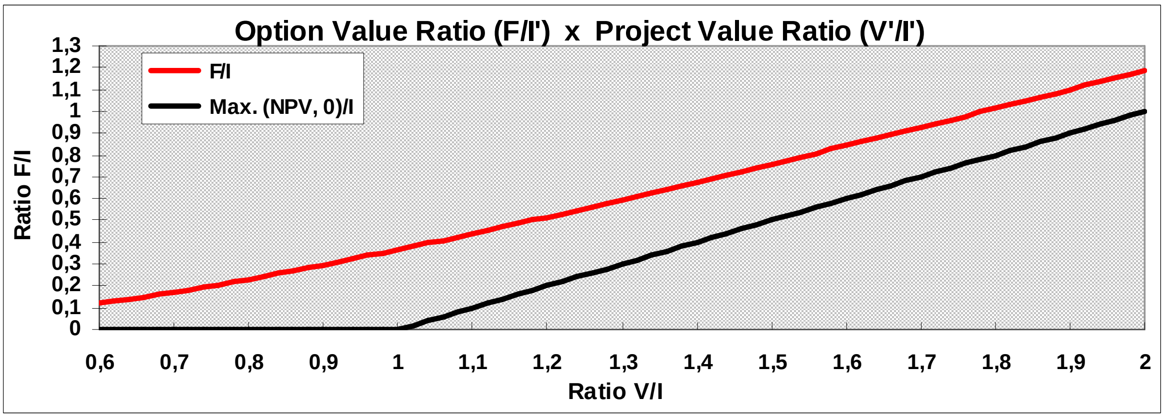

Figure 4 – Timing Program Real Option Value Chart

Alternatively, the real option value can be estimated for the case 7 that considers MR for prices only. The parameters of the real option calculation are show below. The estimated value for the option to delay in this case is US$ 93,02 million. The value of real option in base case and alternative case are close to each other, as the its volatilities values were close to each other too. Again the effect of the demand growth is not additive, as discussed before, and it could be neglected.

5. CONCLUSIONS

Considering the selection of stochastic models, BGM has a drawback. It has a trend that depends the data available for its estimation. The estimated trend can be a value that is not consistent in the long term. In the present work, BGM models for raw material and product could not be used together in simulations, once the raw material drift had a higher value than the product drift value. This is a situation that is unbearable in the long term, otherwise all PP producers would shut their doors. In this point MR process can be more realistic that BGM. However, modeling petrochemical prices with pure mean reverting process is not recommended, once petrochemical projects have cyclical behavior. In this case, MR process can underestimate the volatility of the project, and the real option value. The solution would be a combined process like MRJ. The MRJ volatility can be close to BGM volatility (see cases 15 and 17).

Comparing risk neutral simulation with real simulation for BGMs processes, sometimes the risk neutral simulation can result in negative values for the risk neutral drift, as in the case of the GDP process. These could be a draw back for the risk neutral simulation.

The uncertainties related to petrochemical prices generate a value to wait even in the most conservative cases.

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