A methodology to evaluate an option to defer an oilfield development

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Abstract

The purpose of this paper is the valuation of an option to defer an oilfield development. A methodology is implemented in order to choose the appropriate continuous-time stochastic processes for these risk factors: the crude oil price, the convenience yield and the risk-free interest rate. The analysis reveals that the convenience yield follows a mean-reverting process, the oil price is better fitted by the Geometric Brownian Motion with jumps and the risk-free interest rate can be considered constant. The valuation of the option to defer is based on the Monte-Carlo simulation adapting the Least-Squares simulation method for valuing American type options. Results indicate that using multi-factor pricing models leads to reject the project unlike the one-factor pricing model which leads to later investing at the option maturity.

Keywords: oilfield development valuation; uncertainty factors; stochastic diffusion processes; estimation; simulation

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The purpose of this paper is the valuation of an option to defer an oilfield development. A methodology is implemented in order to choose the appropriate continuous-time stochastic processes for these risk factors: the crude oil price, the convenience yield and the risk-free interest rate. The analysis reveals that the convenience yield follows a mean-reverting process, the oil price is better fitted by the Geometric Brownian Motion with jumps and the risk-free interest rate can be considered constant. The valuation of the option to defer is based on the Monte-Carlo simulation adapting the Least-Squares simulation method for valuing American type options. Results indicate that using multi-factor pricing models leads to reject the project unlike the one-factor pricing model which leads to later investing at the option maturity.

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1. Introduction

In this paper, we present a methodology to evaluate an option to defer an oilfield development. The value of this option depends on the stochastic evolutions of the crude oil price, the convenience yield and the risk-free interest rate. The choice of the continuous-time stochastic models corresponding to these uncertainty sources affects the value of the option to defer and consequently the development decision. Indeed, an investment decision based on a mean-reverting process could turn out to be quite different from the one based on a random-walk process (Dixit and Pindyck, 1994; Baker et al. 1998; Dias and Rocha, 1999 and Pelet, 2003).

In order to choose the appropriate continuous-time stochastic processes for these uncertainty sources, the analysis of their time series properties is required. In this study, our starting point is to examine the mean reversion of the crude oil price, the convenience yield and the risk-free interest rate. We extend the works of Gibson and Schwartz (1989) who examined the oil price and the convenience yield behaviors by testing the significance of the coefficient which reflects the mean-reverting pattern and those of Pindyck (1999) who ran Augmented Dickey-fuller and variance ratio tests to identify if the time series of the crude oil price is stationary. Indeed, we examine first the mean reversion without jumps of the crude oil price, the convenience yield and the risk-free interest rate by using unit root tests without breaks (Augmented Dickey-Fuller 1997; Phillips and Perron, 1988; Elliot et al. 1996 and Kwiaytkowski et al. 1992 tests, denoted respectively ADF, PP, ERS and KPSS tests). Perron (1989) showed that these tests will incorrectly fail to reject the nonstationarity hypothesis if the series presents at least a slope and/or a level shift. So, if we accept the unit root hypothesis, then we examine the mean reversion with jumps of these series. Unit root test allowing for a one-time structural change is carried out for this reason (Perron, 1993 test). If this test accepts the nonstationarity hypothesis, the next stage consists in testing the presence of multiple breaks (Bai and Perron, 1998-2003 tests). If the tests presented above do not allow selecting the appropriate stochastic processes, the final stage of the testing procedure is to simulate different continuous-time stochastic processes and the mean error between the simulated prices and the market ones.

After choosing the suitable continuous-time stochastic processes, we evaluate an option to defer a Tunisian oilfield development by means of one-factor and multi-factor pricing models. Since the option valuation problem is complex, it cannot be easily solved directly from the partial differential equation. In this paper, the problem is solved by the Monte-Carlo simulation adapting the Least-Squares simulation method developed by Longstaff and Schwartz (2001) for valuing American type options. The application of the real option theory requires the determination of the critical early-exercise values at the times when the option can be exercised. The linear interpolation developed by Grant et al. (1993) and Muβhoff et al. (2002) is used to calculate these critical values. To improve the efficiency of the Monte-Carlo method, the antithetic variable technique is considered to reduce the standard deviation of the estimate. The paper ends with a summary of the main conclusions.

2. Data description and oilfield development opportunity

The data used to analyze the time series properties of the risk factors consist of the spot crude oil price, the risk-free interest rate (Monetary Market Rate) and the convenience yield series. The spot oil prices on a monthly basis for the period from January 1960 to May 2004 (figure 1) were obtained from the International Financial Statistics Browser. The monthly observations of Monetary Market Rate (henceforth, TMM) during the period from 1990:1 through 2004:11 (figure 2) are available in the Tunisian Central Bank. The convenience yield (henceforth, convy) (figures 3 and 4) series are calculated by means of pairs of adjacent maturity futures contracts according to the Gibson and Schwartz (1989) formula. These series are computed by using daily prices of the futures crude oil contracts and the annualized three-month Treasury Bills over the period starting June 23, 1988 ending Mars 31, 2004 obtained respectively from the International Petroleum Exchange and the Board of Governors of the Federal Reserve System.

Figure 1. Logarithm of the crude oil price

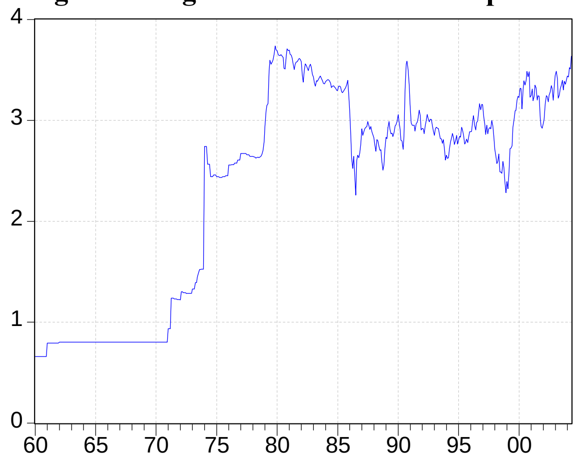


Figure 2. Monetary market rate in percentage

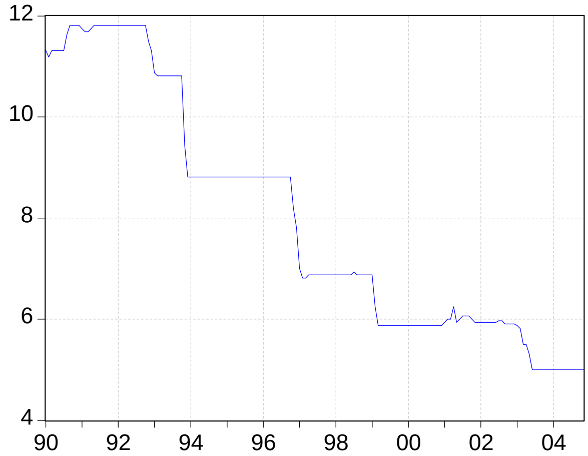


Figure 3. Convy derived from a pair of futures contracts expiring in months 1 and 2

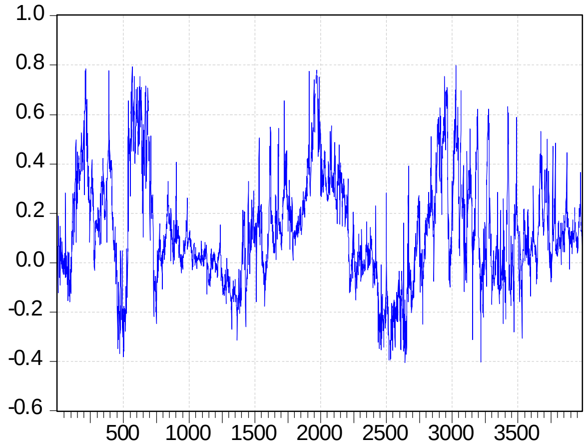
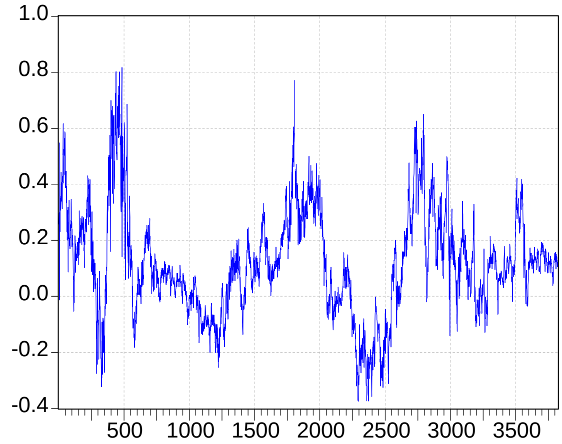


Figure 4. Convy derived from a pair of futures contracts expiring in months 2 and 3



The exploitation of the oilfield, which is the object of this study, began in 1974 after installing a first platform. In 1985, an additional oilfield was developed by installing a second platform. Encouraged by the important cash-flows generated by the news' oil wells, the company considered the option of spreading deposit to the East and the South-East in 1992 by installing a third platform. It had been confronted to choose between two alternatives: Installing a platform with six "legs" (henceforth, PF_6) or installing a platform with four "legs" (henceforth, PF_4).

The duration of the development phase is 3 years. The investment opportunity would be lost if the development phase didn't begin at one of those dates 1992, 1993 and 1994. So, the maturity option is 3 years, where the date 1994 is the final expiration date of the option. There is no possibility to shut down temporarily or to abandon the project. The project life is 17 years. The oil quantities in thousand barrels and the extraction cost and the depreciation per barrel are indicated respectively in figure 5 and table 1.

A royalty must be paid to the state in cash or in nature. For the considered oilfield, the company chooses nature royalty which is equal to 12.5 percent of the produced quantity. The risk-free interest rate r is 6%. The corporate tax rate denoted T is equal to 75 percent. 20 percent of the produced quantity must be sold on local market. The local price is equal to 90 percent of the exportation price.

Figure 5. Petroleum quantities

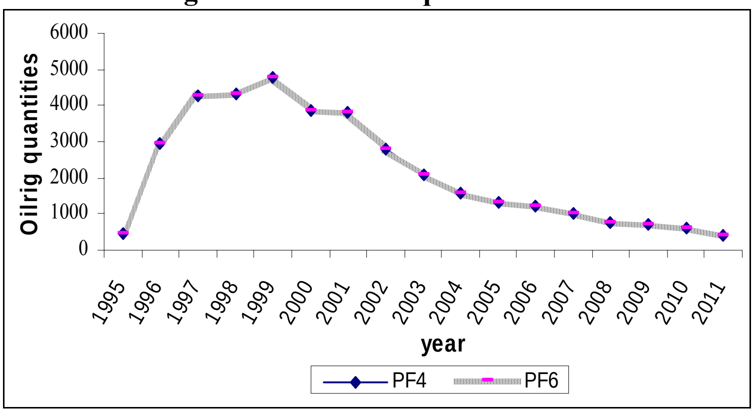


Table 1. Extraction cost and depreciation per barrel

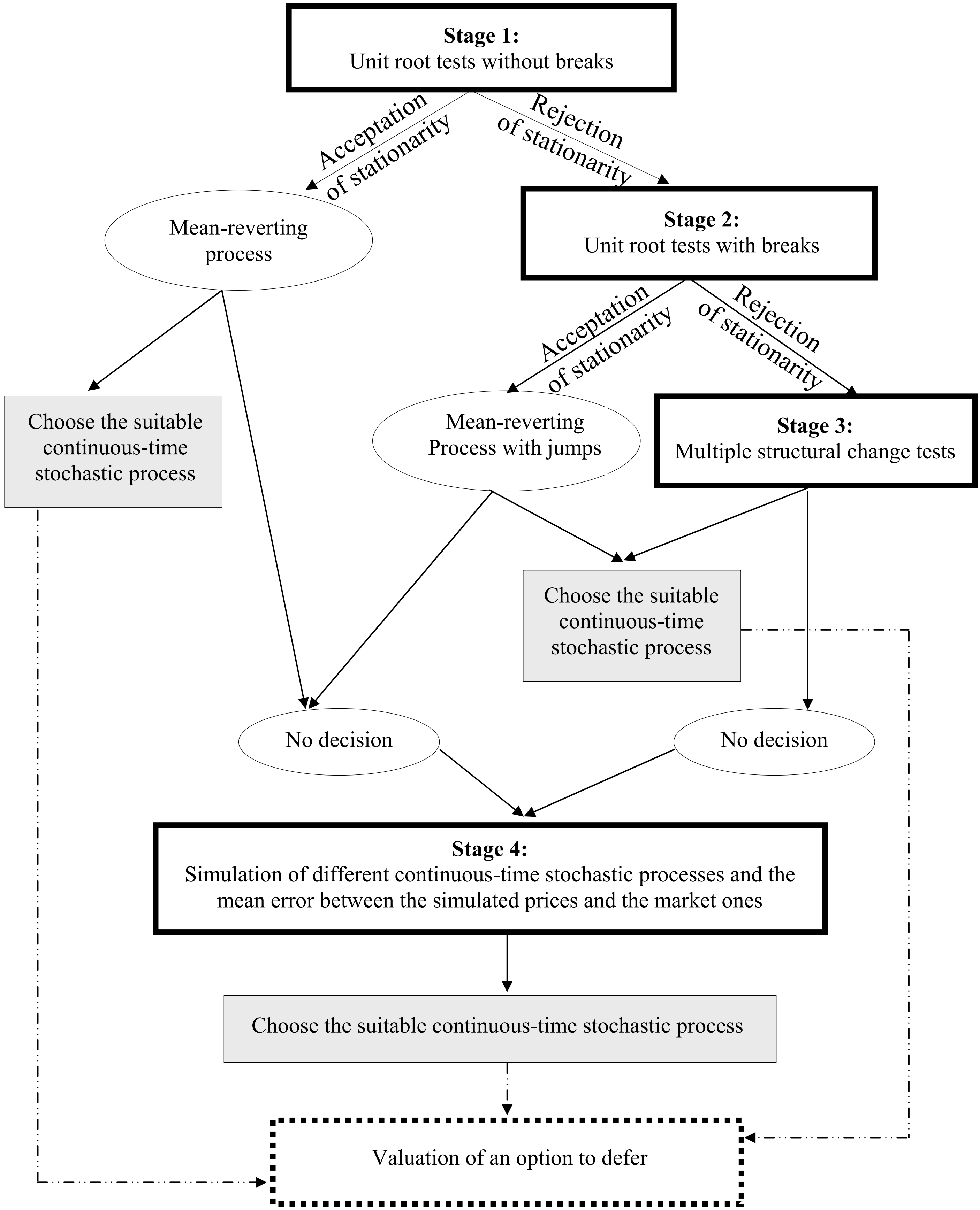
	Extraction cost	Depreciation
PF_4	1.81 $	5.52 $
PF_6	1.82 $	5.61 $

The data used to estimate the parameters of the multi-factor pricing models consist of weekly prices of futures crude oil contracts which cover the period from 2nd of January 1990 to 25th of August 2003. Four different maturities are used: the first, the third, the sixth and the ninth months of delivery.

3. Methodology

The methodology adopted in this paper to evaluate an option to defer is shown in Figure 6:

Figure 6. Methodology to evaluate an option to defer



We begin by testing whether the series of uncertainty sources present a unit root or not. (1) If we accept the stationarity hypothesis, we can conclude that the variable follows a mean-reverting process. In this case there are two possibilities: either we can choose the appropriate continuous-time stochastic process for this uncertainty source and then the testing procedure is stopped here or we carry on the testing procedure by simulating different continuous-time stochastic processes and the mean error between the simulated prices and the market ones. (2) If we reject the stationarity hypothesis, the failure to reject a unit root may be due to the presence of breaks in the time series. To that end, we apply unit root tests with breaks to examine the mean reversion with jumps of this series. (3) If we accept the stationarity hypothesis, the variable process can be described as a mean-reverting process with jumps. Likewise, there are two possibilities: either we can identify the suitable continuous-time stochastic process and then we can stop the testing procedure, or the choice will be based on the simulation of different stochastic processes. (4) If the stationarity hypothesis is rejected, then we run multiple structural change tests. If these tests aren't conclusive, we simulate different stochastic processes in order to decide on the best process for the uncertainty source. Since we choose the suitable continuous-time stochastic process, we evaluate the option to defer. In this paper we apply this methodology to evaluate an option to defer a Tunisian oilfield development.

Two categories of unit root test without breaks are used in stage 1: tests whose null hypotheses are nonstationarity (Augmented Dickey-Fuller, 1997; Phillips and Perron, 1988 and Elliot et al. 1996 tests) and test whose null hypothesis is stationarity (Kwiaytkowski et al. 1992 test). A unit root test with a structural break is applied in stage 2 (Perron, 1993 test). The multiple structural break tests used in stage 3 are developed by Bai and Perron (1998-2003)[1].

The simulation of continuous-time stochastic processes tackled in stage 4 necessitates the estimation of their parameters. For this reason, the Maximum Likelihood Estimation (henceforth, MLE) is implemented to estimate the parameters of the stochastic processes with and without jumps. MLE requires the density function which is unknown for the most part of the continuous-time stochastic processes. To approximate the density function[2], we use different methods: those of Aït-Sahalia (1999), Aït-Sahalia (2002) and Bakshi and Ju (2005)[3] applied to the stochastic processes without jumps and the approximation of Yu (2007) applied to jump-diffusion processes.

After choosing the appropriate continuous-time stochastic processes for the oil price, the risk-free interest rate and the convenience yield, the Least-Squares Monte-Carlo simulation approach is used to evaluate the option to defer. This method is developed by Longstaff and Schwartz (2001). It consists in: (1) Simulating many times the underlying assets price until the option maturity. (2) Determining the stopping rule matrix which indicates the time at which it is optimal to exercise the option along each path by working backwards from the option maturity. (3) Calculating the mean of the option discounted payoffs across all matrix entries.

The valuation is based on one-factor and multi-factor pricing models. The estimation of the multi-factor models' parameters is carried out by the simple and the extended Kalman filters.

The application of the real option theory to investment valuation needs the calculation of the critical exercise values V* at the times when the option can be exercised. At the option maturity, there is no temporal flexibility and the critical value is equal to the investment cost I. In order to determine the critical exercise values (before the option maturity), we use the linear interpolation developed by Grant et al. (1993) and Muβhoff et al. (2002)[4]. If the option can be exercised at the time t = T-1, the critical value at this time is the discounted cash-flows value generated by the project for which the investor is indifferent with regard to exercise immediately the option or to

[1] We wish to thank Dr Pierre Perron for the Gauss program available from his home page.

[2] The approximate density functions corresponding to the stochastic processes with and without jumps are calculated using mathematica. The estimation is carried out by Matlab.

[3] We wish to thank Dr Yacine Aït-Sahalia and Nengjiu Ju for their help.

[4] We wish to thank Dr Dwight Grant and Dr Oliver Muβhoff for their help.

defer the exercise time. In other words, it is the discounted cash-flows value V that equate the option intrinsic value (V-I) with the option continuation value. This value falls between two discounted cash-flows values which yield a change of sign of the difference of intrinsic value and continuation value.

The increase of the number of simulation trails allows reducing the estimate standard deviation given that this last is inversely proportional to the square root of the simulations number. Therefore, different numbers of simulations trials are carried out in this study. A ninety five per cent confidence limits given by $\pm\frac{1.96W}{\sqrt{N}}$ is computed, where M is the option payoffs mean, W is the option payoffs standard deviation and N is the simulations number.

In order to improve the efficiency of the Monte-Carlo method, the antithetic variable technique is considered. It consists in introducing a negative dependence between pairs of random variables. Let U uniformly distributed over the interval [0, 1]. If we generate a path using as input U_1, U_2….., U_N, we can simulate a second path using (1-U_1), (1- U_2)….. (1-U_N) without changing the law of the simulated process (Glasserman, 2003). When simulating continuous-time stochastic processes based on a standard normal random variable, the antithetic variable technique can be implemented by Z_1, Z_2….., Z_N sequence with -Z_1, -Z_2….., -Z_N sequence (Glasserman, 2003).

The results of the application of this methodology to the valuation of a Tunisian oilfield development are presented in the following section.

4. Results analysis

4.1. Stationarity of the uncertainty sources

The value of the option to defer an oilfield development depends on the uncertainty relating to the crude oil price, the convenience yield and the risk-free interest rate. To explore whether these series follow a random-walk or a mean-reverting process, we examine if these series are stationary. Table 2 displays the results of unit root tests without breaks.

Table 2. Results of ADF, PP, ERS and KPSS tests for the oil price, the Convy and the TMM series

Series	ADF-Statistic	PP-Statistic		
		l_4	l_{12}	l_{wn}
Crude oil price	0.6824	0.7549	0.8555	0.7296
Convenience yield	-6.1025***	-9.020***	-10.738***	-8.9265***
TMM	-1.0833	-0.7485	-0.7415	-0.7485
	DF-GLS-Statistic		P_T-Statistic	
	$z_t = \{1,t\}$	$z_t = \{1\}$	$z_t = \{1,t\}$	$z_t = \{1\}$
Crude oil price	-1.555	0.371	10.626	43.880
Convenience yield	-5.453***	-5.306***	1.373***	0.390***
TMM	-2.104	0.736	9.418	78.816

	KPSS-Statistic					
	η_τ			η_μ		
	l_4	l_{12}	l_{wn}	l_4	l_{12}	l_{wn}
Crude oil price	1.3494	0.5205	1.5677	5.5045	2.0903	6.4045
Convenience yield	0.5064	0.2044***	0.5530	0.7433	0.2996***	0.8117
TMM	0.5034	0.2178	0.5034	3.4611	1.2974	3.4611

*l_4 and l_{12} are selected bandwidths based on the Schwert (1989) method; l_{wn} is a selected bandwidth based on the Newey and West (1987) method; $z_t = \{1, t\}$ is a regression with constant and trend; $z_t = \{1\}$ is a regression with constant; η_τ is a regression with constant and trend; η_μ is a regression with constant and the symbol *** indicates the acceptation of the stationarity hypothesis at the 1% risk level.*

For the convy series (derived from a pair of futures contracts expiring in months 1 and 2), the ADF, PP, DF-GLS and P_T tests reject the null hypothesis at the 1% risk level[5]. Given that the nonstationarity tests provide results indicating that the convy series is stationary[6], we can conclude that the data generating process of this series is stationary and consequently it would be preferable to use a mean-reverting process to describe the stochastic evolution of the convy in continuous time. Figures 3 and 4 show that the convenience yield can be negative. So, in this study, the Ornstein-Uhlenbeck process is chosen to model the convenience yield when evaluating the option to defer using multi-factor pricing models and then the testing procedure is stopped at stage 1 for this uncertainty source.

The dynamic of the Ornstein-Uhlenbeck process is written as:

$$dC_t = k(\alpha - C_t)dt + \sigma_c dz_t \qquad (1)$$

Where C is the convenience yield, k is the speed reversion of the convenience yield, α is the equilibrium level to which the convenience yield tends to revert, σ_c is the convenience yield's standard deviation and dz_t is the Wiener increment.

Stationarity and nonstationarity tests applied to the TMM series accept the presence of unit root and so, indicate that this series is a random-walk. The rejection of the stationarity hypothesis may be due to the presence of at least one break in this time series. We carry on the testing procedure for this series. The unit root test with a structural break is applied to this series at stage 2. The test results are regrouped in table 3.

Table 3. Results of the unit root test with a structural break for the TMM series

		Perron test: IO version (model C)			
Break dates	t	τ	Critical value (τ = 0.5)		
			1%	5%	10%
1996 :11	-3.6492	0.5	-4.90	-4.24	-3.96
		Perron test: AO version (model B)			
	t	τ	Critical Value (τ = 0.5)		
			1%	5%	10%
1999 :02	-3.5144	0.5	-4.49	-3.93	-3.65

Model B allows for a change in slope; Model C allows for a change in both intercept and slope; $\tau = \frac{T_b}{T}$ *;* T_b *is the break date; T is the sample size and t is the t-statistic.*

The nonstationarity hypothesis isn't rejected. The non-rejection of the null hypothesis may be due to the presence of multiple breaks. So, multiple structural change tests developed by Bai and Perron (1998-2003) are applied to this series at stage 3. Table 4 displays the results of multiple structural change[7] tests (pure model) corresponding to a level shift, a slope shift and a level and slope shift.

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[5] Table 2 shows that the KPSS test statistic is affected by the choice of the bandwidth. Indeed, in the cases of l_4 and l_{wn}, the convy series is not stationary unlike the case of l_{12}.

[6] The same results are obtained for the convy series derived from pairs of futures contracts expiring in months 2-3; 3-4; 4-5; 5-6; 6-7; 7-8; 8-9; 9-10; 10-11. The Results are available upon request.

[7] The Results of multiple structural change tests with a nonparametric correction and multiple structural change tests using partial model are available upon request.

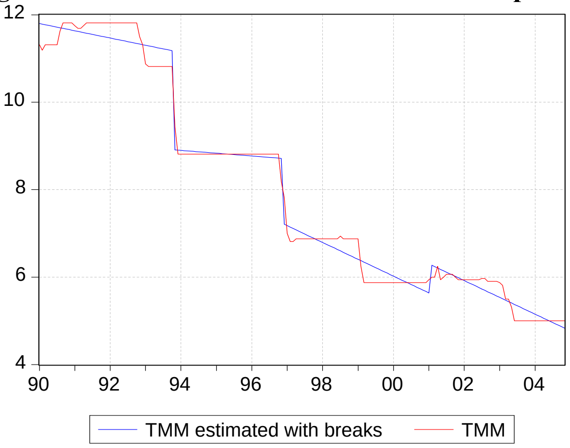
Table 4. Results of multiple structural change tests for the TMM series: pure model

	cor_u = 0	het_u = 0	het_z = 1	ε = 0.2	m = 3
		x_t= {Ø}, z_t= {1} and p= 0			
Tests	UDmax	WDmax	$supF_T(1)$	$supF_T(2)$	$supF_T(3)$
	2140.6768***	388 4.500***	7356.629***	1778.131***	2140.6768***
	$supF_T(2\|1)$	$supF_T(3\|2)$			
	1463.6033***	108.4104***			
	BIC	LWZ	Seq		
Number of breaks	3	3	3		
	T_1	T_2	T_3		
Break dates	1993 :10	1996 :11	1999 :10		
Confidence intervals	(1993 :9-1993 :11)	(1996:10- 1996:12)	(1999 :8 – 2000 :1)		
		x_t={Ø}, z_t={t} and p= 0			
Tests	UDmax	WDmax	$supF_T(1)$	$supF_T(2)$	$supF_T(3)$
	226.1908***	336.2265***	186.7576***	226.1908***	185.2906***
	$supF_T(2\|1)$	$supF_T(3\|2)$			
	382.3574***	198.2025***			
	BIC	LWZ	Seq		
Number of breaks	3	3	3		
	T_1	T_2	T_3		
Break dates	1993 :10	1996 :12	1999 :11		
Confidence intervals	(1993 :9 -1994 :8)	(1996:11 -1997:2)	(1999:10 - 2000:1)		
		x_t={Ø}, z_t= {1, t} and p= 0			
Tests	UDmax	WDmax	$supF_T(1)$	$supF_T(2)$	$supF_T(3)$
	433.9159***	609.8089***	229.1624***	433.9159***	365.8036***
	$supF_T(2\|1)$	$supF_T(3\|2)$			
	351.2022***	49.1154***			
	BIC	LWZ	Seq		
Number of breaks	3	3	3		
	T_1	T_2	T_3		
Break dates	1993 :10	1996 :11	2001 :1		
Confidence intervals	(1993:9 -1993 :11)	(1996:10-1996:12)	(2000 :12 -2001:3)		

Seq: Sequential procedure developed by Bai and Perron (1998-2003).
BIC and LWZ: information criteria.
*The symbol *** indicates that the tests are significant at the 1% risk level.*
cor_u = 0: errors are serially uncorrelated.
het_u = 0: errors are identically distributed.
het_z = 1: *data have different distributions across segments.*

Both UDmax and WDmax tests are highly significant, so we distinguish at least one break. $SupF_T(l)$, l={1;2 ;3}, and $supF_T(3| 2)$ are all significant at the 1% risk level. The three information criteria select three breaks. We conclude that the non-rejection of the null hypothesis under which the TMM series is not stationary is due to the presence of several switching regime. Figure 2 shows that there aren't remarkable fluctuations in the TMM historical series. The evolution of TMM series estimated with three breaks (figure 7) shows that after each break the value of TMM is nearly constant for a given period. So, for this study, the TMM is considered constant when evaluating the option to defer the oilfield development and the testing procedure is stopped at stage 3 for this series.

Figure 7. TMM estimated with 3 level and slope shifts



For the crude oil price series, the computed ADF test statistic is superior to the critical values at the 1%, 5% and 10% risk levels, so we accept the unit root null hypothesis. Whatever the bandwidth chosen, the PP test also accepts the nonstationarity hypothesis at the 1%, 5% and 10% risk levels. The DF-GLS and P_T tests are unable to reject the presence of unit root too. The computed KPSS test statistic is superior to the critical values at the 1%, 5% and 10% risk levels, and then we reject the null hypothesis. However, the non-rejection of the nonstationarity hypothesis may be due to the presence of at least one break in the crude oil price series. That is why we continue the testing procedure. The unit root test with a structural break is applied to this series at stage 2. Table 5 gives the results of this test.

Table 5. Results of the unit root test with a structural break for the crude oil price series

		Perron test: IO version (model A)			
Break dates	t	τ	Critical Value (τ = 0.3)		
			1%	5%	10%
1972 :12	-3.6010*	0.3	-4.39	-3.76	-3.46
		Perron test: AO version (model A)			
	t	τ	Critical Value (τ = 0.5)		
			1%	5%	10%
1973 :12	-2.5927	0.5	-4.32	-3.76	-3.46

Mode A allows for a trend's level shift; the symbol indicates the acceptation of the stationarity hypothesis at the 10% risk level;* $\tau = \frac{T_b}{T}$ *;* T_b *is the break date; T is the sample size and t is the t-statistic.*

The null hypothesis is rejected at the 10% risk level which is generally considered as a high risk level. At this stage, we can't draw any definitive conclusion about the time series proprieties of the crude oil price. In order to choose the appropriate stochastic process for this uncertainty source, we move on to the stage 4 which consists in simulating different continuous-time stochastic processes[8] and the mean error between the simulated prices and the market ones.

[8] The continuous-time stochastic processes which are simulated in this study are: the Geometric Brownian Motion with and without jumps, the Ornstein-Uhlenbeck process with and without jumps, the Pearl-Verhulst process with and without jumps, the Schwartz (1997) process with and without jumps, the Inhomogeneous Geometric Brownian Motion with and without jumps and the Cox-Ingersoll-Ross process with and without jumps. The simulation is carried out using Excel. We wish to thank Dr Marco Dias for his help.

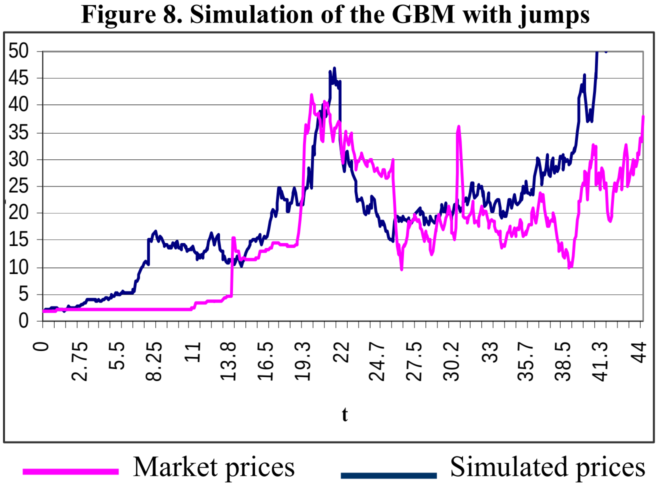
4.2. Estimation and simulation of the oil price process

The simulation of continuous-time stochastic processes requires the estimation of their parameters. In this study, the estimation is carried out using the Maximum Likelihood Estimation[9]. The estimated parameter values of the Geometric Brownian Motion (henceforth, GBM) without jumps obtained through the approximate density functions developed by Aït-Sahalia (1999), Aït-Sahalia (2002) and Bakshi and Ju (2005) are equal to those obtained by the exact density function. The estimated parameter values of the Ornstein-Uhlenbeck process without jumps obtained through the approximate density functions developed by Aït-Sahalia (1999) and Bakshi and Ju (2005) are equal to those obtained by the exact density function. However, the estimated parameter values of this process using the approach of Aït-Sahalia (2002) are different from those obtained by the exact density function. For this process, the likelihood maximized value of the Aït-Sahalia (2002) approximation is inferior to the ones obtained through the exact density function and the other approximate density functions. When we simulate the fitted processes without jumps, we use the values of parameters estimated by the approximation method that presents the higher value of the maximum likelihood. For the GBM and the Ornstein-Uhlenbeck process we use the estimated parameter values obtained by maximizing the exact density function[10].

The simulated paths, compared to the historical oil prices curve, and the frequencies of mean errors between the simulated prices and the market ones, show that[11]: (1) A process with jumps is better than a process without jumps to model the crude oil price with an increase in kurtosis. (2) The Cox-Ingersoll-Ross process and the Inhomogeneous Geometric Brownian Motion are unable to describe the oil price stochastic evolution in spite of the decreases in the mean error in comparison with the processes without jumps. (3) The GBM with jumps (figure 8) is the suitable stochastic process for the crude oil price. (4) A switching equilibrium level for the mean-reverting processes with jumps (Ornstein-Uhlenbeck process with jumps, Pearl-Verhulst process with jumps and Schwartz (1997) process with jumps) is better than a constant one[12].

So, in this study, the GBM with jumps is chosen to model the oil price when evaluating the option to defer the oilfield development.

Figure 8. Simulation of the GBM with jumps



The dynamic of the GBM with jumps is written as:

$$dS_t = (\mu - c) S_t \, dt + \sigma_S \, S_t \, dZ_t + J \, S_t \, dq_t \qquad (2)$$

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[9] The estimated parameter values using MLE are available upon request.

[10] The simulation of stochastic processes with jumps is carried out using the values of parameters estimated by the approximation of Yu (2007). Parameter estimates are available upon request.

[11] The sample paths of simulated processes and the statistics of the Monte-Carlo simulation of the mean error are available upon request.

[12] Pindyck (1999) suggested a switching equilibrium level for these processes to model the oil price too.

where S is the oil price, μ is the instantaneous expected return, c is the convenience yield, σ_s is the oil price's standard deviation, dz_t is the Wiener increment, J is the jumps size, dq_t is equal to 0 with probability 1 – ηdt, dq_t is equal to 1 with probability ηdt and η is the jumps intensity.

A calibration of a GBM with compensated stochastic jumps developed by Jeanblanc and Privault (2002) shows that the jumps considered as stochastic are better described by a mean-reverting process. Since we are interested in jumps up and jumps down, we choose the Ornstein-Uhlenbeck process to describe the evolution of jumps considered as stochastic when evaluating the option to defer the oilfield development using three-factor pricing model.

The dynamic of the Ornstein-Uhlenbeck process is written as:

$$dJ_t = k_1(\alpha_1 - J_t)dt + \sigma_J dz_t \qquad (3)$$

where J is the jumps size, k_1 is the speed reversion of jumps, α_1 is the equilibrium level to which jumps tend to revert, σ_J is the jump's standard deviation and dz_t is the Wiener increment

4.3. Value of the option to defer

After choosing the suitable continuous-time stochastic processes for the risk factors, we evaluate a petroleum reserve using a real option approach (option to defer) based on one-factor and mutlti-factor pricing models. For the single-factor model, the relevant state-variable is the crude oil price which follows the GBM with jumps. The second uncertainty source added in the two-factor model is the convenience yield which follows the Ornstein-Uhlenbeck process. The third factor used in the three-factor model is the jumps modelled by the Ornstein-Uhlenbeck process.

For the one-factor model, the oil price evolves according to:

$$dP_t = (r - c)P_t\, dP_t + \sigma_p\, P_t\, dZ_t + J\, P_t\, dq_t \qquad (4)$$

where r is the risk-free interest rate, c is the convenience yield[13], σ_p is the oil price's standard deviation, dz_t is the Wiener increment, J is the jumps size (We use log-normal distribution for jumps size like Merton, 1976), dq_t is equal to 0 with probability 1 – ηdt, dq_t is equal to 1 with probability ηdt and η is the jumps intensity.

To apply the Longstaff and Schwartz (2001) approach to evaluate the option to defer the oilfield development, the oil price is simulated many times[14] in order to determine the discounted cash-flows values V generated by the project at the dates t = 1994, t = 1993 and t = 1992 at which the option can be exercised. Since the simulation of these discounted cash-flows values is achieved, we deduce first the optimal early exercise decision matrix at the dates t = 1993 and t = 1992. Then, we determine the optimal stopping rule matrix at the dates t = 1994, t = 1993 and t = 1992. Finally, we calculate the mean of the option discounted payoffs which is the value of the option to defer F(V). In order to calculate the critical exercise values V*, the linear interpolation is applied at the dates t = 1993 and t = 1992. Table 6 regroups the values of the option to defer the oilfield development F(V) and the critical values V*.

Table 6. Option to defer values and critical values: one-factor model

	1992				
	F(V)		**1992**	**1993**	**1994**
PF_6	227 779.39	**V**	233 100.20	267 616. 06	282 491.23
	F(V)+I				
	399 203.34	**V***	344 844.08	302 631.05	192 611.95

[13] For this model c is assumed constant. For detailed calculations of c for these two platforms, see "Pricing an Option to Defer a Tunisian Oil Field Development", Kaffel, B., Abid F., Kaaniche L (2005).

[14] The simulations number chosen is 2000. The simulation is carried out using Excel.

	1992				
	F(V)		**1992**	**1993**	**1994**
PF_4	213 105.84	**V**	223 166.10	256 568.02	282 833.90
	F(V)+I				
	382 124.52	**V***	342 211.99	308 739.80	189 909.39

At the date t = 1992, we note that the project full cost [F(V) + I] and the critical value V* are superior to the discounted cash-flows value V for the two platforms. So, the petroleum company shouldn't begin the development phase at this date. At the date t = 1993, we also note that the critical value is superior to V. At the option maturity (t = 1994), the critical value is inferior to the discounted cash-flows value for the two platforms. Moreover, the value of the option to defer corresponding to PF_6 is superior to this of PF_4. So we can conclude that, when we use a one-factor pricing model for which the oil price is the relevant state-variable, the petroleum company should begin the development phase at the option maturity by installing the platform with six legs (PF_6).

The multi-factor pricing models fit the futures prices better than the one-factor pricing models (Schwartz, 1997). Indeed, the mean square error of the single-factor models is superior to the one of the multi-factor models. Besides, the volatilities implied by the single-factor models are different from the historical volatilities unlike the ones implied by the multi-factor models. Next, we evaluate the option to defer by means of multi-factor models and we examine whether the investment decision is affected when we use multi-factor models compared to the one-factor model. The valuation of the option to defer the oilfield development is under taken using two-factor and three-factor pricing models[15].

In a world where there are no arbitrage opportunities and under the risk-neutral probability, the joint stochastic process corresponding to the two-factor model is:

$$\begin{cases} dS = (r - C)S\,dt + \sigma_S S\,dz_S + J\,S\,dq_S \\ dC = [k(\alpha - C) - \lambda]dt + \sigma_C dz_C \end{cases} \quad (5)$$

with $E[dz_S \times dz_C] = \rho dt$

where S is the oil price, C is the convenience yield, σ_s is the oil price's standard deviation, dz_s is the Wiener increment associated with the oil price, k is the speed reversion of the convenience yield, α is the equilibrium level to which the convenience yield tends to revert, σ_c is the convenience yield's standard deviation, dz_c is the Wiener increment associated with the convenience yield, λ is the risk premium associated with the convenience yield and ρ is the coefficient correlation between the two Brownian motions associated with S and C.

The third factor added in the three-factor model is the jumps. The joint stochastic process corresponding to the three-factor model is:

$$\begin{cases} dS = (r - C)S\,dt + \sigma_S S\,dz_S + J\,S\,dq_S \\ dC = [k(\alpha - C) - \lambda]dt + \sigma_C dz_C \\ dJ = [k_1(\alpha_1 - J) - \lambda_1]dt + \sigma_J dz_J \end{cases} \quad (6)$$

with $E[dz_S \times dz_C] = \rho_1 dt$, $E[dz_S \times dz_J] = \rho_2 dt$ and $E[dz_J \times dz_C] = \rho_3 dt$

where k_1 is the speed reversion of jumps, α_1 is the equilibrium level to which jumps tend to revert, λ_1 is the risk premium associated with jumps, σ_J is the jump's standard deviation, dz_J is the Wiener

[15] Calculation details of the simple and the extended Kalman filters used respectively to estimates the parameters of two-factor and three-factor pricing models are available upon request.

increment associated with the jumps and ρ_1, ρ_2 and ρ_3 are the coefficients correlation between the three Brownian motions associated respectively with S and C, S and J and J and C.

For this last model, we proceed in this way: between two times of jumps t_k and t_{k+1}, we multiply S_t by (1 + J). So, $S_{t+1} = S_t*(1+J)$ (Swishchuk, 2004).

Tables 7 and 8 report the parameter values of the two-factor and the three-factor pricing models estimated using respectively the simple and the extended Kalman filters:

Table 7. Parameter estimates: two-factor model

Parameter	Value
σ_s	0.2634
σ_c	0.0006
κ	0.1438
μ	0.2212
α	0.1331
ρ	0.3237
λ	0.0562
σ_3	$4.6548\ 10^{-5}$
σ_4	0.0134
σ_5	0.0023
σ_6	$4.508\ 10^{-5}$
σ_junps	0.1214
μ_jumps	0.0025
η	0.7539
Likelihood function	649.0214

μ is the oil price's immediate return.
σ_3, σ_4, σ_5, σ_6 standard deviations of error for measurement equation.

Table 8. Parameters estimates: three-factor model

Parameter	Value
σ_s	0.0891
σ_c	0.0963
σ_J	0.2114
κ	0.1232
κ_1	0.2445
μ	0.1521
α	0.1075
α_1	0.04270
ρ_1	0.0941
ρ_2	0.0013
ρ_3	0.0010
λ	0.0023
λ_1	0.0018
σ_4	0.7827
σ_5	0.4551
σ_6	0.7432
σ_7	0.5410
Likelihood function	657.1605

μ is the oil price's immediate return.
σ_4, σ_5, σ_6, σ_7 standard deviations of error for measurement equation.

We note a positive correlation between the crude oil price and the convenience yield processes' residuals which indicates a positive relationship between the unexpected changes in these two state-variables. Furthermore, a weak correlation between the jumps and the oil price and the jumps and the convenience yield is remarked for three-factor models.

The option pricing based on a multi-factor model has to take account of the correlation between the stochastic variables. The simulation of a normal distribution vector with parameters N(M, ∑) where M is a mean vector and ∑ is a covariance matrix is required. The simulation of a vector $Z = [Z_1\ Z_2\ \ldots..Z_d]$ reverts to simulate $W = A*Z$ where $W = [W_1\ W_2\ \ldots..W_d]$ with a covariance matrix $\sum_W$. So, the simulation of the matrix W reduces to finding a matrix A such that $AA^T = \sum_W$ (Glasserman, 2003). The Cholesky factorization allows calculating this matrix. The results corresponding to the option to defer values F(V) and the critical values V* using multi-factor models are summarized in the following tables (tables 9 and 10).

Table 9. Option to defer values and critical values: two-factor model

	1992				
	F(V)		**1992**	**1993**	**1994**
PF_6	134 524.88	**V**	174 434.92	175 347.11	185 782.10
	F(V)+I				
	305 948.83	**V***	232 226.74	231 949.99	192 611.95

	1992 F(V)		1992	1993	1994
PF_4	98 741.47	**V**	142 163.72	158 149.56	187 032.88
	F(V)+I 270 165.42	**V***	239 973.74	232 752.16	189 909.39

Table 10. Option to defer values and critical values: three-factor model

	1992 F(V)		1992	1993	1994
PF_6	10 185.63	**V**	157 219.68	171 353.09	185 955.97
	F(V)+I 181 609.57	**V***	282 950.31	193 733.81	192 611.95
	1992 F(V)		**1992**	**1993**	**1994**
PF_4	9 593.94	**V**	156 445.10	169 767.49	185 217.61
	F(V)+I 181 017.88	**V***	193 500.06	193 143.01	189 909.39

We note that the project full cost is superior to the discounted cash-flows value V at the date t = 1992 for the two platforms. In addition, at the dates t = 1993 and t = 1994, the critical values V* are superior to the discounted cash-flows values. So we can conclude that, when we use multi-factor models, the petroleum company shouldn't exercise this option and consequently shouldn't develop this oilfield.

When increasing the number of simulations from 2000 to 5000, the standard deviation of the estimate is reduced from 7 to 5.75 and the confidence limits from ± 14.30 to ± 11.28. We also note, when the simulations number increases from 20000 to 25000, the reduction of the confidence limits is smaller (from ± 6.50 to ± 6.01). The use of the antithetic variable technique reduces more the standard deviation of the estimate. Indeed, the range of the ninety five per cent confidence limits is reduced from ± 6.01 to ± 2.88.

Table 11 gives the value of the option to defer the oilfield development and the critical values using 25000 simulations together with the antithetic variable technique.

Table 11. Option to defer value and critical values (three-factor model): 25000 simulations together with antithetic variable technique

	1992 F(V)		1992	1993	1994
PF_6	10 008.72	**V**	156 714.76	170 687.95	185 983.42
	F(V)+I 181 432.67	**V***	196 560.56	194 438.87	192 611.95

We note that, when we use the antithetic variable technique, the investment decision isn't affected. Indeed, the critical values are superior to the discounted cash-flows values at the dates t = 1992, t = 1993 and t = 1994 and consequently the company shouldn't develop this oilfield.

5. Conclusion

The choice of the appropriate continuous-time stochastic processes for the underlying assets has important consequences for the valuation of oil deposits and for the development decision. This paper presents a methodology to evaluate an option to defer an oilfield development. The testing procedure to analyze the time series properties of the crude oil price, the convenience yield and the risk-free interest rate, with the purpose of identifying the stochastic processes that reflect as likely as possible their dynamics behaviors, consists in: (1) Running unit root tests with and without breaks. (2) Running multiple structural change tests. (3) Simulating different stochastic continuous-time processes and the mean error between the simulated prices and the market ones. After identifying the appropriate stochastic processes for these risk factors, the valuation of the option to defer is carried out by means of one-factor, two-factor and three-factor pricing models. The Least-Squares simulation method is applied for the option valuation. To improve the efficiency of the Monte-Carlo method, the antithetic variable technique is considered. The results analysis show that the Ornstein-Uhlenbeck process is the suitable process to model the convenience yield, the Geometric Brownian Motion with jumps is the appropriate process to describe the stochastic behavior of the crude oil price and the risk-free interest rate can be considered constant when pricing an option to defer an oilfield development. The case study illustrates that the investment decision depends on the number of stochastic variable kept for the valuation of a petroleum investment using real option theory. Indeed, according to the multi-factor pricing models, the company shouldn't develop the oilfield unlike the single-factor model.

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