The fuzzy value of patent litigation

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Abstract

The vague notion of "probabilistic patents" (Lemley and Shapiro, 2005) is formalized through a model which combines real option theory and a fuzzy methodology. The imprecise ideas the patent holder possesses about her future profits, the validity and scope of the patent, the litigation costs, the court's decision. etc. under a regime of imperfect enforcement of property rights are specified using a more appropriate and promising concept of uncertainty through the theory of fuzzy sets. Such methodology is embedded within a real option approach, where the value of a patent includes the option value of litigation. We study how the value of a patent is affected by the timing and incidence of litigation. The main results are compared with the empirical findings of previous results.

1 Introduction

A patent is usually defined as a right to make exclusive use of an innovation at a predetermined cost for a predetermined period of time, i.e. the life of the patent. The patent holder may commercialize some products or licence her technology or use it for further developments. As such it can be interpreted as a real option. The interpretation of patents as real options presupposes an enforceable property right. Yet, an increased number of patents have registered a high frequency of disputes and litigation involving patent holders and alleged infringers, so that the risk that a patent will be declared invalid is substantial. There is a wide variation across patents in their exposure to risk: as Lanjouw and Schankerman (2001) have shown through detailed empirical evidence, for high-value patents and specific types of patentees the litigation risk can be quite high, in some cases almost offsetting what would otherwise be the R&D incentive provided by patent ownership. "Roughly half of all litigated patents are found to be invalid, including some of great commercial significance" (Lemley and Shapiro, 2005, page 76). Thus, because of uncertainty in the enforcement of property rights, it has been stated that " a patent does not confer upon its owner the right to exclude but rather a right *to try* to exclude by asserting the patent in court" (Lemley and Shapiro, 2005, page 75). Accordingly, the clarification of the norms about intellectual property right has been indicated as the main challenge for lawyers and politicians in the next twenty-five years (Greenspan, 2007).

Because of imperfect enforcement of property rights, most patents represent highly uncertain or probabilistic property rights. Lemley and Shapiro (2005) use the term *probabilistic patents.*

In this paper we translate the vague notion of probabilistic patents into a mathematical model, where the valuation of patents can be performed by a combination of real options and a fuzzy methodology. In order to capture the notion of vagueness about the validity and scope of patents under a regime of imperfect enforcement of property rights, we introduce a more appropriate and promising concept of uncertainty, through the theory of fuzzy sets. In this way, we are able to capture the vague and imprecise ideas the patent holder possesses about her future profits, the validity of the patent, the litigation costs, the court's decision. etc. Moreover, we embed such methodology within a real option approach, where the value of a patent includes the option value of litigation.

There are various papers applying the theory of real options to the valuation of patents although very few of them introduce the patent enforcement process explicitly. Pakes (1986) first estimated the distribution of the returns earned from holding patents as options which are renewed at alternative ages and require renewal fees. Bloom and van Reenen (2002) build on Pakes (1986) and derive empirical predictions on the relationship between patents and market uncertainty. Schwartz (2004) implements a simulation model to value patents as complex options, taking into account uncertainty in the cost-to-completion of the project and the possibility of abandoning the project. Takalo and Kanniainen (2000) model sequential real options, analysing the patenting decision and its effects in research, development and commercialization. Weeds (2002) also investigates the patenting decision under technological and market uncertainty with two competing firms. None of the abovementioned papers introduce the risk of litigation. To the best of our knowledge, the only analyses of the option value of litigation are Marco (2005) and Baecker (2007). However, Marco (2005) is mainly focused on the empirical estimates of patent litigation. Baecker (2007) develops at length both the theory and the numerical implementation of some jump-diffusion models, where the risk of litigation is exogenously given and negatively affects the value of the patent in the form of discontinuities or jumps in the value process. He also addresses some issues of endogenous patent risk through a model where the patent holder possesses full knowledge about the probability distribution of the litigation risk.

Our paper is the first that combines a real option to litigate with a fuzzy valuation. The need for a fuzzy valuation comes from the common observation that the outcomes of a trial are difficult to forecast, legal costs are not easily predictable, it may be years before litigation is concluded, there may be divergence in parties' expectations about the court decision, future cash flows from commercialization are imprecise. Although the existing literature has identified the main determinants of litigation, it has not investigated how the value of a patent is affected by the timing and incidence of litigation under an appropriate framework of uncertainty. Section 2 presents the model of a patent under imperfect enforcement of property rights, where the relevant parameters are fuzzy. The model is solved analytically for infinitely lived patents and the main results are compared with the empirical findings of previous studies.

2 The fuzzy model of patent valuation

In what follows let us introduce the basic fuzzy-stochastic elements that are useful for our application (see also Zadeh, 1965; Dubois and Prade, 1980; 2000). A fuzzy number is a fuzzy set (depicted with tilde) of the real line R, which is commonly defined by a normal, upper-semicontinuous, fuzzy convex membership function $\mu : R \rightarrow [0,1]$ of compact support. The γ-cut of a fuzzy number is given by:

$\widetilde{\mu}_\gamma = \{x\epsilon R \mid \widetilde{\mu}(x) \geqq \gamma\}, \gamma\epsilon(0,1),$

and $\widetilde{\mu}_0 = cl\,\{x\epsilon R \mid \widetilde{\mu}(x) \geqq 0\}$, where cl denotes the closure of an interval. Let us write the closed intervals as $\widetilde{\mu}_\gamma = \left[\widetilde{\mu}_\gamma^-, \widetilde{\mu}_\gamma^+\right]$ for $\gamma\epsilon(0,1)$. Given two fuzzy numbers, $\widetilde{\mu}$ and $\widetilde{\eta}$, the partial order $\succsim$ on fuzzy numbers can be defined such that $\widetilde{\mu} \succsim \widetilde{\eta}$ means that $\widetilde{\mu}_\gamma^- \geq \widetilde{\eta}_\gamma^-$ and $\widetilde{\mu}_\gamma^+ \geq \widetilde{\eta}_\gamma^+$ for all $\gamma\epsilon(0,1)$. The arithmetic operations on two fuzzy numbers can be defined in the standard way, in terms of the γ-cuts for $\gamma\epsilon(0,1)$. In particular, for fuzzy numbers $\widetilde{\mu}$ and $\widetilde{\eta}$ the addition and subtraction $\widetilde{\mu} \pm \widetilde{\eta}$ and the scalar multiplication $a\mu$, where $a \geq 0$, are fuzzy numbers as follows:

$(\widetilde{\mu} + \widetilde{\eta})_\gamma = \left[\widetilde{\mu}_\gamma^- + \widetilde{\eta}_\gamma^-, \widetilde{\mu}_\gamma^+ + \widetilde{\eta}_\gamma^+\right],$

$(\widetilde{\mu} - \widetilde{\eta})_\gamma = \left[\widetilde{\mu}_\gamma^- - \widetilde{\eta}_\gamma^+, \widetilde{\mu}_\gamma^+ - \widetilde{\eta}_\gamma^-\right],$

$(a\widetilde{\mu})_\gamma = \left[a\widetilde{\mu}_\gamma^-, a\widetilde{\mu}_\gamma^+\right].$

Moreover, multiplication between two fuzzy numbers $\widetilde{\mu}$ and $\widetilde{\eta}$ is given by: $(\widetilde{\mu}\widetilde{\eta})_\gamma = \left[(\widetilde{\mu}\widetilde{\eta})_\gamma^-, (\widetilde{\mu}\widetilde{\eta})_\gamma^+\right],$

where $(\widetilde{\mu}\widetilde{\eta})_\gamma^- = \min\left[\widetilde{\mu}_\gamma^-\widetilde{\eta}_\gamma^-, \widetilde{\mu}_\gamma^-\widetilde{\eta}_\gamma^+, \widetilde{\mu}_\gamma^+\widetilde{\eta}_\gamma^-, \widetilde{\mu}_\gamma^+\widetilde{\eta}_\gamma^+\right]$

and $(\widetilde{\mu}\widetilde{\eta})_\gamma^+ = \max\left[\widetilde{\mu}_\gamma^-\widetilde{\eta}_\gamma^-, \widetilde{\mu}_\gamma^-\widetilde{\eta}_\gamma^+, \widetilde{\mu}_\gamma^+\widetilde{\eta}_\gamma^-, \widetilde{\mu}_\gamma^+\widetilde{\eta}_\gamma^+\right].$

Division between two fuzzy numbers $\widetilde{\mu}$ and $\widetilde{\eta}$, if allowed, is given by:

$\left(\frac{\widetilde{\mu}}{\widetilde{\eta}}\right)_\gamma = \left[(\frac{\widetilde{\mu}}{\widetilde{\eta}})_\gamma^-, (\frac{\widetilde{\mu}}{\widetilde{\eta}})_\gamma^+\right],$

where $(\frac{\widetilde{\mu}}{\widetilde{\eta}})_\gamma^- = \min\left[\frac{\widetilde{\mu}_\gamma^-}{\widetilde{\eta}_\gamma^-}, \frac{\widetilde{\mu}_\gamma^-}{\widetilde{\eta}_\gamma^+}, \frac{\widetilde{\mu}_\gamma^+}{\widetilde{\eta}_\gamma^-}, \frac{\widetilde{\mu}_\gamma^+}{\widetilde{\eta}_\gamma^+}\right]$ and $(\frac{\widetilde{\mu}}{\widetilde{\eta}})_\gamma^+ = \max\left[\frac{\widetilde{\mu}_\gamma^-}{\widetilde{\eta}_\gamma^-}, \frac{\widetilde{\mu}_\gamma^-}{\widetilde{\eta}_\gamma^+}, \frac{\widetilde{\mu}_\gamma^+}{\widetilde{\eta}_\gamma^-}, \frac{\widetilde{\mu}_\gamma^+}{\widetilde{\eta}_\gamma^+}\right].$

A fuzzy-number-valued map $\widetilde{X}$ is called a fuzzy random variable if $\left\{(\omega, x)\ \epsilon\ \Omega \times R \mid \widetilde{X}(\omega)(x) \geq \gamma\right\}$ for all $\gamma\epsilon(0,1)$. It is called integrably bounded if both $\omega \rightarrow \widetilde{X_\gamma}^-(\omega)$ and $\omega \rightarrow \widetilde{X_\gamma}^+(\omega)$ are integrable for all

$\gamma\epsilon(0,1)$. The expectation $E(\widetilde{X})$ of the integrably bounded fuzzy random variable $\widetilde{X}$ is also defined by a fuzzy number

$E(\widetilde{X})(x) = \sup_{\gamma\epsilon(0,1)} \min\left\{\gamma, \mathbf{1}_{E(\widetilde{X})_\gamma}(x)\right\}, x\epsilon R,$

where $E(\widetilde{X})_\gamma = \left[\int_\Omega \widetilde{X_\gamma}^-(\omega)dP(\omega), \int_\Omega \widetilde{X_\gamma}^+(\omega)dP(\omega)\right]$, $\gamma\epsilon(0,1)$.

Let us now introduce the valuation method based on fuzzy variables. Suppose an innovator owns a patent allowing her to generate additional cash flows from commercializing some product. For simplicity, let us assume that the protection period is infinite, which facilitates the derivation of a closed-form solution. Commercialization is related with some expected income which fluctuates randomly. This income can be either in the form of royalties or in the form of increased revenues from the ability to exclude others from the market. Let Π denote the net cash flow resulting from the patent, which is described by the following stochastic dynamics:

$$d\Pi_t = \Pi_t(\mu dt + \sigma dW_t)$$

where $\mu < r$ is the appreciation rate, r is the risk-free interest rate and σ is the volatility ($\mu\epsilon R$, $\sigma > 0$) and W_t is a standard Wiener process. Let $\left\{\widetilde{\Pi}_t\right\}_{t\geq 0}$ be a fuzzy stochastic process, which is specified as follows:

$\widetilde{\Pi}_t(\omega)(x) = \max\left\{1 - \mid \frac{x-\Pi_t(\omega)}{\alpha_t(\omega)} \mid, 0\right\},$

that is, the fuzzy random variable $\widetilde{\Pi}_t$ is of the triangular type, with centre $\Pi_t(\omega)$, and left-width and right-width $\alpha_t(\omega)$. Observe that the fuzziness in the process increases as $\alpha_t(\omega)$ becomes bigger. The choice of a triangle-type shape is not restrictive at all and is mainly adopted to facilitate computation.The γ-cuts of $\widetilde{\Pi}_t(\omega)(x)$ are $\widetilde{\Pi}^{\pm}_{t,\gamma}(\omega) = \left[\widetilde{\Pi}^-_{t,\gamma}(\omega), \widetilde{\Pi}^+_{t,\gamma}(\omega)\right] =$ $[\Pi_t(\omega) - (1-\gamma)\alpha_t(\omega), \Pi_t(\omega) + (1-\gamma)\alpha_t(\omega)]$.

However, as argued by Lemley and Shapiro (2005), the value of a patent depends not only on the uncertainty about the commercial significance of the innovation being patented, but also on the uncertainty about the validity and scope of the legal right being granted. The latter introduces the notion of *probabilistic patents.* A patent does not confer an absolute right to exclude others from infringement; on the other hand, the actual scope and validity of a patent right and even whether the patent right will withstand litigation at all are uncertain and contingent issues. Therefore, following Marco (2005), a patent can be described as a portfolio consisting of two assets: an asset paying a stochastic cash flow Π and an option to litigate. Note however, that Marco (2005) considers the case of a patent infringement, while our analysis applies to challenge suits too. Since the option to litigate/sue can be exercised anytime prior to patent expiration, the decision to litigate can be modeled within a real option analysis.

Let us formalize the notion of a probabilistic patent. Based on the alleged infringement, a challenger may decide to litigate at any time $\tau\epsilon(t,\infty)$ and if successful receives a fraction θ of future net cash flows, which is determined by court and not known in advance by the two parties. Litigation may end up being successful or not: let p denote the probability of unsuccessful litigation, as from the beliefs of the patent-holder. In what follows, we assume that both θ and p are fuzzy numbers. Specifically, $\widetilde{\theta}(x) = \max\left\{1- \mid \frac{x-\theta}{\Theta} \mid, 0\right\}$, that is, it has a symmetric triangle-type shape, with centre θ and width Θ, where $0 < \Theta \leq \theta \leq 1 - \Theta$, and $\widetilde{p}(x) = \max\left\{1- \mid \frac{x-p}{\zeta} \mid, 0\right\}$, that is, it has a symmetric triangle-type shape, with centre p and width ζ, where $0 < \zeta \leq p \leq 1 - \zeta$. Finally, let $\widetilde{L}_i$, $i = 1,2$ denote the litigation costs incurred by the patent-holder ($i = 1$) and the challenger ($i = 2$). Both are fuzzy numbers, so that $\widetilde{L}_i = \max\left\{1- \mid \frac{x-L_i}{\lambda_i} \mid, 0\right\}$ that is, it has a symmetric triangle-type shape, with centre L_i and width $\lambda_i \geq 0$.

Analogously, we can specify the γ-cuts, that is,

$$\widetilde{\theta}^{\pm}_{\gamma} = \left[\widetilde{\theta}^{-}_{\gamma}, \widetilde{\theta}^{+}_{\gamma}\right] = [\theta - (1-\gamma)\Theta, \theta + (1-\gamma)\Theta];$$

$$\widetilde{p}^{\pm}_{\gamma} = \left[\widetilde{p}^{-}_{\gamma}, \widetilde{p}^{+}_{\gamma}\right] = [p - (1-\gamma)\zeta, p + (1-\gamma)\zeta];$$

$$\widetilde{L}^{\pm}_{i,\gamma} = \left[\widetilde{L}^{-}_{i,\gamma}, \widetilde{L}^{+}_{i,\gamma}\right] = [L_i - (1-\gamma)\lambda_i, L_i + (1-\gamma)\lambda_i].$$

All agents are assumed to follow a policy of value-maximization. The optimal litigation time $\tau*$ is chosen by the challenger in response to the resolution of uncertainty related to Π_t over time. The optimal stopping time $\tau*$ is defined as the first time Π_t exceeds a critical level which is sufficiently high to justify the cost of litigation. More specifically, we will find $\tau* = \inf\left[t : \widetilde{\Pi}*^{\pm}_{\gamma} < \Pi_t\right]$, where $\widetilde{\Pi}*^{\pm}_{\gamma}$ is obtained in Proposition 1. The value of the patent under the risk of litigation for the patent-holder can be specified in terms of the both ends of the γ-cuts as follows:

$$\widetilde{V}^{\pm}_{\gamma}(\Pi_t, t) =$$

$$\sup_{\tau\epsilon(t,\infty)} E\left\{\int_t^{\infty} e^{-r(s-t)}\widetilde{\Pi}^{\pm}_{s,\gamma} ds - \int_{\tau}^{\infty} e^{-r(s-t)}(\widetilde{p}\widetilde{\theta}\widetilde{\Pi}_s)^{\pm}_{\gamma} ds - e^{-r(\tau-t)}\widetilde{L}^{\pm}_{1,\gamma}\right\}.$$

which is equivalent to the expected present value of cash flows from commercialization minus the option to litigate gained by the challenger, minus the present value of additional litigation costs.

Following Yoshida (2002, 2003) we introduce a reasonable assumption A.1, which allows us to simplify the formulas although it is not necessary for the argument.

Assumption A.1. The stochastic process $\alpha_t(\omega)$ is specified by $\alpha_t(\omega) = c\Pi_t(\omega)$, where $0 < c < 1$. The values Θ, ζ, λ_i are specified by $\Theta = b\theta$ and $\zeta = dp$, where $0 < b, d$ and $b + d - bd < 1$; $\lambda_i = f_i L_i$, where $0 < f_i < 1$.

Assumption A.1. is reasonable since $\alpha_t(\omega)$ is related to the value $\Pi_t(\omega)$, so that $\widetilde{\Pi}_{t,\gamma}{}^{\pm}(\omega) = (1 \pm (1-\gamma)c)\Pi_t(\omega)$.

Analogously, $\widetilde{\theta}_{\gamma}^{\pm} = (1 \pm (1-\gamma)b)\theta$; $\widetilde{p}_{\gamma}^{\pm} = (1 \pm (1-\gamma)d)p$; $\widetilde{L_{i,\gamma}}^{\pm} = (1 \pm (1-\gamma)f_i)L_i$.

We can prove Proposition 1:

Proposition 1 *The payoff from commercializing under imperfect patent protection due to the risk of litigation is given by:*

$\widetilde{V}_{\gamma}^{\pm}(\Pi_t, t) =$

$\left[\frac{\widetilde{\Pi}_{t,\gamma}^{-}}{r-\mu} - \widetilde{L_{1,\gamma}}^{+} - \frac{(\widetilde{p}\widetilde{\theta}\widetilde{\Pi}_t)_{\gamma}^{+}}{r-\mu}, \frac{\widetilde{\Pi}_{t,\gamma}^{+}}{r-\mu} - \widetilde{L_{1,\gamma}}^{-} - \frac{(\widetilde{p}\widetilde{\theta}\widetilde{\Pi}_t)_{\gamma}^{-}}{r-\mu}\right]$, *if* $\Pi_t > \widetilde{\Pi}*_{\gamma}^{\pm}$,

$\left[\frac{\widetilde{\Pi}_{t,\gamma}^{-}}{r-\mu} - ((\widetilde{L}_1 + \frac{\varepsilon}{\varepsilon-1}\widetilde{L_2})(\frac{\Pi_t}{\widetilde{\Pi}*})^{\varepsilon})_{\gamma}^{+}, \frac{\widetilde{\Pi}_{t,\gamma}^{+}}{r-\mu} - ((\widetilde{L}_1 + \frac{\varepsilon}{\varepsilon-1}\widetilde{L_2})(\frac{\Pi_t}{\widetilde{\Pi}*})^{\varepsilon})_{\gamma}^{-}\right]$,

if $\Pi_t < \widetilde{\Pi}*_{\gamma}^{\pm}$,

where $\widetilde{\Pi}*_{\gamma}^{\pm} = \frac{\epsilon(r-\mu)}{\epsilon-1}\left[\frac{(1-(1-\gamma)f_2)L_2}{\theta p(1+(1-\gamma)(b+d+bd(1-\gamma))}, \frac{(1+(1-\gamma)f_2)L_2}{\theta p(1-(1-\gamma)(b+d-bd(1-\gamma))}\right]$

and $\epsilon = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2r}{\sigma^2}} > 1.$

Proof. The result is obtained from the fuzzification of the following standard argument. The option value of litigation held by the challenger and denoted by $O(\Pi_t, t)$ satisfies the following partial differential equation:

$$\frac{1}{2}\sigma^2\Pi_t^2\partial_{\Pi_t}^2 O + \mu\Pi_t\partial_{\Pi_t}O - rO - p\theta\Pi_t = 0$$

with the final condition $-L_2$ if $\Pi_t > \Pi*$. Finally, $\widetilde{V}_{\gamma}^{\pm}(\Pi_t, t)$ can be easily obtained observing that

$\widetilde{V}_{\gamma}^{\pm}(\Pi_t, t) = \frac{\widetilde{\Pi}_{t,\gamma}^{\pm}}{r-\mu} - \widetilde{O}_{\gamma}^{\pm}(\Pi_t, t) - E\left[e^{-r(\tau*-t)}(\widetilde{L_{1,\gamma}}^{\pm} + \widetilde{L_{2,\gamma}}^{\pm})\right]$. ■

$\widetilde{\Pi}*_{\gamma}^{\pm}$represents the critical value between the stopping region where litigation occurs (for $\Pi_t > \widetilde{\Pi}*_{\gamma}^{\pm}$) and the continuation region (for $\Pi_t < \widetilde{\Pi}*_{\gamma}^{\pm}$). Observe that $\widetilde{\Pi}*_{\gamma}^{\pm}$is included in the set $\frac{\epsilon(r-\mu)}{\epsilon-1}\left[\frac{(1-f_2)L_2}{\theta p(1+b+d+bd)}, \frac{(1+f_2)L_2}{\theta p(1-b-d+bd)}\right]$ which is positive. Note that because of the fuzzy modelling litigation occurs if the cash flow resulting from the patent is larger than $\widetilde{\Pi}*_{\gamma}^{+}$,while if it is less that $\widetilde{\Pi}*_{\gamma}^{-}$then waiting becomes the optimal strategy. In the intermediate range we cannot conclude for any of the two occurrences, so that this area will be called the "indecision area". The membership function for $\widetilde{\Pi}*$ is plotted in Figures 1-6 for various parameter values. Observe that the shape of the critical value is asymmetric to the right, although the driving parameters have a symmetric membership, implying that litigation is postponed in the fuzzy model in comparison with a non-fuzzy model, where the critical value is the crisp value $\widetilde{\Pi}*_1^{\pm} = \frac{\epsilon(r-\mu)}{\epsilon-1}\frac{L_2}{\theta p}$.

3 Sensitivity analysis

In this section a numerical implementation is performed. It is useful to study the effects of the model parameters on the critical value $\widetilde{\Pi}*$. Figures 1-6 allow us to view the results graphically. It is shown how the fuzzy shape of the critical value changes as L_2 changes (Figure 1), as p changes (Figure 3) and as θ changes (Figure 5). The dashed curves represent the shape of the critical value for the highest values of the parameters, the thin solid curves are related to the intermediate value and the thick solid curves to the lowest value. For $\gamma = 1$ we obtain the crisp value, which is increasing in L_2 and decreasing in θ, p. Therefore, the challenger hastens litigation if his cost of litigation decreases, the probability of successful litigation increases, the fraction of future net cash-flows increases. Figures 2,4,6 display the impact of "fuzziness". In Figure 2 an increase in fuzziness is measured by an increase in f_2, in Figure 4 by an increase in d and in Figure 6 by an increase in b. The dashed curves represent the shape of the critical value for the highest values of the parameters, the thin solid curves for the intermediate value and the thick solid curves give the lowest value. Note that as fuzziness increases, the fuzzy shape of the critical value enlarges, the membership function becomes more asymmetric and shifts to the right, implying that litigation tends to be postponed. In other words, a fuzzier model,predicts a wider range of values for Π in the "indecision area".

Thus, the following testable implications can be found: (i) higher p, θ lead to more litigation (in keeping with Marco,2005); (ii) the greater the fuzziness over the patent "strength" - probability of patent validity (p) and patent scope (θ) - the more delayed becomes litigation; (iii) the greater the fuzziness over the profit flow obtained with the patent the less likely becomes litigation (in keeping with Lanjouw and Schankerman (2001)).

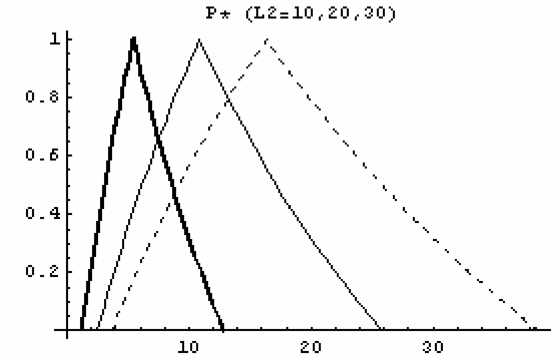

Figure 1

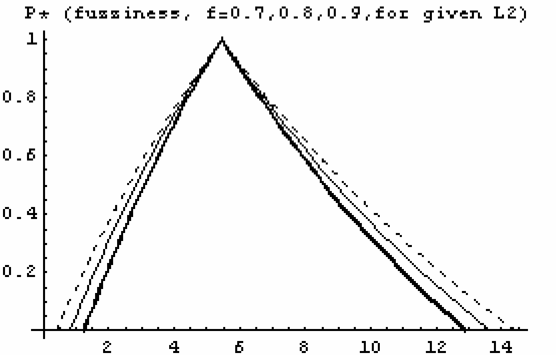

Figure 2

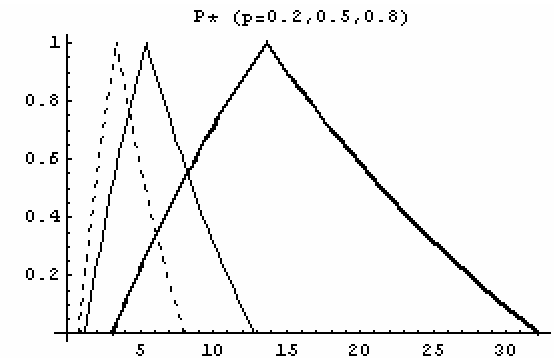

Figure 3

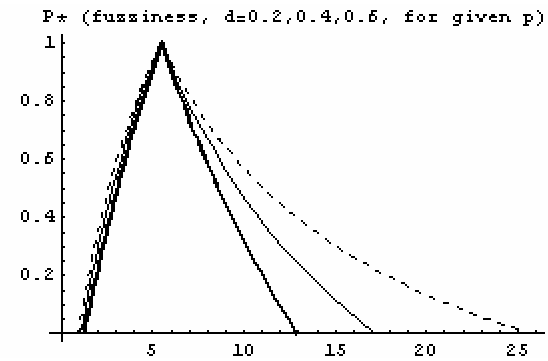

Figure 4

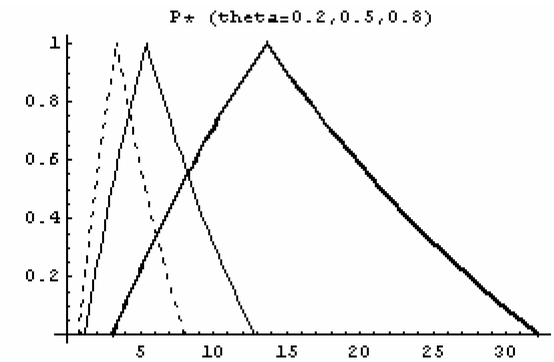

Figure 5

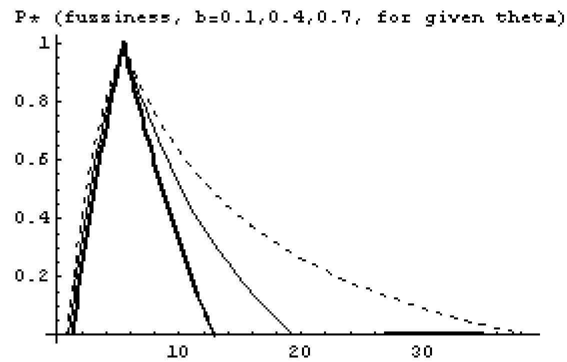

Figure 6

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