Dynamic R&D Projects Selection*

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Comments Welcome

Abstract

Selecting between investing on R&D in incremental innovations and radical innovations is particularly challenging. In this paper, we focus on the problem of project selection under technical uncertainty and market uncertainty. After motivating the challenges and decisions facing firms using a real-life application from GM, we formulate a mathematical model of a firm that must develop its products in the presence of uncertainty. Specifically, the firm faces two options: (i) an incremental innovation project that is known to be relatively easy to develop and (ii) a radical innovation project that offers superior performance but whose development is much more difficult. We examine how characteristics of R&D projects such as projects' relative efficiencies and future benefits affect R&D investment policy, valuation and risk premia. Our analysis helps understand the appropriateness of the different development approaches. We illustrate our model with a Hybrid Electric Cars vs Hydrogen Fuel Cell Vehicles example as pursued by GM and note the managerial implications of our analysis.

Keywords: Real Options, R&D Projects, Stochastic Differential Equation, Managerial Flexibility, Project Management

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1. Introduction

The increasing emphasis on market leadership and shareholder value creation motivates many firms to focus on new product development as a source of growth, renewal, and competitive advantage. Project selection has been shown to be critical to the success of a firm's new products. While the crucial role of project selection in developing new products has been well recognized, the question of how to select projects amidst market and technical uncertainty largely remains open.

The potential uncertainty in the long and investment-intensive R&D process makes the management of innovations particularly challenging. In this paper, we address the problem of R&D investment under technical and market uncertainty. One of the key decision-making in R&D investment is the selection of alternative technologies (product standards). Frequently, the R&D team has to choose between more than one technology projects. The team may consider an incremental innovation type project or a radical innovation type project. The project selection problem in R&D investment has attracted much research attention. (Ali, Kalwani, and Kovenock 1993, Childs, Ott, and Triantis 1998, Krishnan and Bhattacharya 2002). In ex ante literature, investment decisions have been analyzed as a one-shot process, with a one time investment cost structure. However, real R&D investments are not necessarily one-shot decision problems. They usually involve ongoing efforts to commit resources. In addition, previous papers consider either technical uncertainty or market uncertainty separately, but R&D investments usually face both technical and market uncertainty simultaneously. Optimal project selection involves a trade-off between projects' expected returns and the risk associated with project development and commercialization. Risks come from multiple sources: technical uncertainty associated with the likelihood of success, time and costs of the R&D process, and market uncertainty related to potential future benefits. The costs, benefits, and the decisions faced by a R&D team are illustrated by the following real-life experience at General Motors (GM).

1.1. Project Selection at GM

In 1988, GM embarked on research and development of hybrids electric cars. GM also restarted its fuel cell program in the 1990s, after a long hiatus from the first fuel cell testing dating back to the 1960s. The pressure on the research team was that alternative technology might replace the existing Internal Combustion Engine which consumes gasoline. In 1998, GM decided to reconsider its project selection of R&D on alternative technologies. GM faced a dilemma in deciding how much investment to channel to the long-term, radical innovation, fuel cell vehicle project versus the short-term, incremental innovation, hybrid gasoline-electric car project. [1]

1.2. Research Questions Addressed in this Paper

The above GM example illustrates some of the challenges facing corporate executives in managing R&D projects amid technical and market uncertainty. In this context, this paper seeks to address the following questions:

1. When do technology projects involving incremental innovation or technology projects involving radical innovation deserve serious consideration?

2. What are the implications of parallel development of incremental and radical technologies, or inaction on both (i.e., waiting)?

3. How do features of R&D projects, such as their relative efficiencies and rewards and the state and variance of market conditions, affect R&D investment policy, and valuation of projects?

We begin answering these questions by developing a continuous-time option-pricing model to obtain qualitative and quantitative insights into the project selection problem, taking into account various sources of uncertainty and interactions. Technical uncertainty is modelled ex-

[1] See (Maccormack 2003) for a detailed description of this case.

plicitly, which allows an objective assessment of the exogenous risk associated with different types of projects and weighing of this risk against the expected returns. Market uncertainties about unknown future payoffs from successful R&D are modelled as stochastic processes.

In the model, R&D projects are dichotomized as " short-term/incremental " or " long-term/radical " innovation projects. Radical innovation projects are typically designed to find specific uses or markets for a promising technology or for a potential new product, and are expected to take a longer time to develop than incremental innovation projects. But once developed, these radical innovations may provide handsome returns. Incremental innovation projects aim to satisfy a perceived market need, usually take a shorter time to develop, and may result in lower returns on investment. The purpose of this paper is to determine whether a firm should devote resources to pursue a long-term radical innovation or a short-term incremental one. Our formulation is quite general. It seeks to ascertain which portfolio of projects a company should choose to fund, given differences in the technical and market uncertainties associated with each project.

This paper is related to a rich body of literature on project selection.[2] Our work fills the gap between market financial payoff variability, as addressed by the real option pricing literature (Childs, Ott, and Triantis 1998),Lint and Pennings (forthcoming) and technical uncertainty faced at the level of R&D management. (Ali, Kalwani, and Kovenock 1993, Krishnan and Bhattacharya 2002). [3] Our model also contributes to the real option literature and extends a single shot decision problem to a recurrent ongoing decision problem. The ongoing decision making is a more realistic setting for managerial flexibility and provides meaningful implications. Moreover, our focus is not only optimal investment policy, but also the valuation and risk premia of projects during the project selection process. [4]

[2]Kamien and Schwartz (1982) and Reinganum (1989) provide comprehensive reviews of economic research on project selection problems. Deshmukh and Chikte (1980), Weinberg (1990) and Krishnan and Ulrich (2001) give reviews of management research on project selection and resource allocation problems. For reviews of incremental innovation versus radical innovations, see Ali (1994) and Lynn, Morone, and Paulson (1996).

[3]For similar an approach see Tsekrekos (2001), Huchzermeier and Loch (2001).

[4]Berk, Green, and Naik (2004) analyze risk premia for one project. The discussions of risk premia for two projects shall be provided in Yao (2005).

From a technical point of view, this paper relates to recent papers on option pricing with regime shifts (Guo, Miao, and Morellec 2005) in the absence of fixed adjustment cost. Unlike previous research analyzing a recurrent investment decision with repeated states, we consider a partially repeated decision with absorbing states, since in the R&D context, the investment decision process ends with R&D success. Another difference is that we consider endogenous investment decision region changes, while they assume exogenous shifts of underlying diffusion process.

Structure of the paper The model is presented (§2), followed by the valuation (§3), optimal investment scenarios (§4) and results (§5). Next, the discussions and managerial implications of the model's results are provided (§6). Finally, we summarize findings and outline suggestions for future research (§7).

2. Model

Our proposed option pricing model is aimed at obtaining insights into the project selection problem that firms face in choosing between alternative new product development projects which differ in risk and reward level, under different states and variance of market conditions.

We provide a detailed mathematical formulation of the model, and a discussion of the modeling assumptions. The parameters are listed in Table 1.

We are given a standard Brownian motion B in $\mathbb{R}$ on a probability space $(\Omega, \mathcal{F}, P)$. We fix the standard filtration $\mathbb{F} = \{\mathcal{F}_t : t \geq 0\}$ of B and begin with the time horizon $[0, \infty)$.

Description	Parameter	Value Space
Market state	x	$\mathbb{R}$
Project	j	$\{1,2\}$
R&D state for project j	n_j	$\{0,1\}$
R&D state for system	n	$[n_1,n_2]$
Investment control for project j	u_j	$\{0,1\}$
Investment control for system	u	(u_1,u_2)
Success rate for project j	π_j	$\mathbb{R}$
Cash flow multiple for project j	θ_j	$\mathbb{R}$
Market growth drift	μ	$\mathbb{R}$
Market volatility	σ	$\mathbb{R}$

Table 1
Model parameters

The model under consideration here consists of one underlying process: $\{X_t, t \geq 0\}$ valued in the state space $\mathbb{X} \subset \mathbb{R}$. We will assume that the process follows a geometric Brownian motion, i.e., X_t satisfies the stochastic differential equation

$$dX_t = \alpha X_t dt + \sigma X_t dB_t, \tag{1}$$

where α is the drift parameter, measuring the expected growth rate of x, σ is the instantaneous standard deviation, and dB_t is the increment of a standard Brownian Motion.

Now we consider a firm that has opportunities to invest in two projects types j ($j = \{1,2\}$).

Suppose the firm's R&D state of project j is $n_j(t) \in \{0,1\}$, for all $j = \{1,2\}$, where state 0 represents incomplete R&D state, and state 1 refers to complete R&D state. We denote the system state $n_t = [n_1(t), n_2(t)]$.

We model the success of an active R&D process on project j as a Poisson process with parameter π_j, i.e.,

$$dN_t^{(j)} = \begin{cases} 1 & \text{with probability } \pi_j dt \\ 0 & \text{with probability } 1-\pi_j dt \end{cases}$$

The firm's decision at each point of time, given that it has not yet completed the R&D process, is whether to invest in R&D, i.e. to choose a control variable u_j from its set of feasible controls $\mathbb{U}^j : [0,\infty) \to \mathbb{A}$, where the actions set $\mathbb{A} = \{0,1\}$.

Project j's cost process is defined as $u_j I_j$, where I_j is the intensity level of the R&D investment.

Let $\theta_j \cdot X_t$ denote cash flow for project j, given that it is successfully completed at time t, where θ_j is a parameter of the reward multiple of project j. We assume that the first complete project will become the technology standard and grab all the potential benefits.

The utility functionals

$$\begin{gathered} L = L_1 + L_2; \\ L_j(t, x_t, n_1(t), n_2(t), u_1(t), u_2(t)) = \\ \int_t^\infty e^{-r(s-t)}(\zeta_j(s)\theta_j X_s - \kappa(n_s)u_j(s)I_j)ds; \\ j \in \{1,2\} \end{gathered} \tag{2}$$

where r is the discount factor, $\kappa(0,0) = 1; \kappa(1,0) = \kappa(0,1) = 0$, and

$$\zeta_j(t) = \begin{cases} 1 & \text{if } n_j(t) = 1 \\ 0 & \text{otherwise} \end{cases}$$

is the complete characteristic function for project j.

The information structure with feedback control is defined by the function $\eta(t) = \{x_t, n_t\}, t \in [0,\infty)$. The information space, N_η, is induced by its information η.

A policy in $\mathbb{P}_\eta(x_0, 0)$ is a mapping $\phi : [0,\infty) \times N_\eta \to \mathbb{U}$. Formally $u(t) = (u_1(t), u_2(t)), u_j(t) = \phi_j(t, x_t, n_t)$, for $j \in \{1,2\}$ and all $t \in [0,\infty)$.

3. Valuation

To analyze the value and risk premia of the projects, we assume the existence of a pricing kernel in the economy. The pricing kernel is given by the process:

$$d\nu_t = -r\nu_t dt - \varphi\nu_t dB_t, \tag{3}$$

We denote the market price of risk by:

$$\lambda = \sigma\varphi\rho, \tag{4}$$

where $\rho = corr(dX_t, d\nu_t)$.

Then we have

$$\begin{aligned} dX_t &= (\alpha - \lambda)X_t dt + \sigma X_t dB_t^Q, \\ &= \mu X_t dt + \sigma X_t dB_t^Q, \end{aligned} \tag{5}$$

where $\mu \equiv \alpha - \lambda$, B_t^Q is a standard Brownian Motion under a risk-neutral Q-martingale. [5]

[5] Let $\xi_t = exp(\int_0^t r_s ds)\frac{\nu_t}{\nu_0}, t \in [0,T]$. Then provided $var(\xi_T)$ is finite, there is an equivalent martingale measure Q with density process $\xi_t = E_t(\frac{dQ}{dP})$. Girsanov's theorem states that a standard Brownian Motion B^Q is defined by $dB_t^Q = dB_t + \varphi dt$, and therefore $dX_t = (\alpha - \sigma \cdot \varphi)X_t dt + \sigma X_t dB_t^Q$, where $\sigma \cdot \varphi = \sigma\varphi\rho = \lambda$.

The expected utility functional is defined as

$$V(t,x_t,n_t,\phi_1,\phi_2) = E_t^Q\left[L(t,x_t,n_t,u_1,u_2)|u_j(t) = \phi_j(t,x_t,n_t), j \in \{1,2\}\right] \tag{6}$$

From Ito's formulae with jumps,

$$dV_t = \mu_V(t)dt + \sigma_V(t)dB_t^Q + \beta_S^{(1)}(t)dZ^{(1)}(t) + \beta_S^{(2)}(t)dZ^{(2)}(t) \tag{7}$$

where $\mu_V, \sigma_V, \beta_S^{(1)}, \beta_S^{(2)}$ are adapted processes such that the integrals exist, with $\beta_S^{(1)}, \beta_S^{(2)}$ left-continuous.

$Z^{(j)}(t) = n_j(t)$,

$$dZ^{(j)}(t) = \kappa(Z(t-))u_j(t,X_t)dN_t^{(j)}. \tag{8}$$

where

$$u_j(t,X_t,n_t) = \kappa(Z(t-))u_j(t,X_t)$$

$$\kappa(0,0) = 1; \kappa(1,0) = \kappa(0,1) = 0$$

The process $Z^{(j)}(t) = n_j(t)$ has two possible states, say 0 and 1. When in state 0, given the investment decision $u_j(t,X_t) = 1$, the process $Z^{(j)}$ moves to state 1 after a time whose probability distribution is exponential with parameter π_j. State 1 is an absorbing state where $Z^{(j)}(t)$ will stay there forever.

Let $V_t = f(Z_t^{(1)}, Z_t^{(2)}, X_t, t)$, then

$$dV_t = \mathbb{D}f dt + f_S\sigma_S(t)dB_t^Q + f(Z_t^{(1)}, Z_t^{(2)}, X_t, t) - f(Z_{t-}^{(1)}, Z_{t-}^{(2)}, X_{t-}, t), \tag{9}$$

where $\mathbb{D}f = \frac{1}{2}\sigma_S^2 f_{SS} + \mu_S f_S + f_t$, the subscripts on f refer to the partial derivative.

Moreover, $dV_t = \mu_V(t)dt + dY_t$, for Y a local martingale and

$$\mu_V(t) = \mathbb{D}f + \pi_1(t)G_1(t) + \pi_2(t)G_2(t),$$

where

$$G_1(t) = f(Z_{t-}^{(1)} + \kappa(Z(t-))u_1(t,X_t), Z_t^{(2)}, X_t, t) - f(Z_{t-}^{(1)}, Z_t^{(2)}, X_t, t)$$

$$G_2(t) = f(Z_t^{(1)}, Z_{t-}^{(2)} + \kappa(Z(t-))u_2(t,X_t), X_t, t) - f(Z_t^{(1)}, Z_{t-}^{(2)}, X_t, t)$$

Proposition 1. *For a one player stochastic differential equation of prescribed fixed duration $[0,\infty)$, described by (1), and the objective functional (2), the admissible control $u \in \mathbb{U}$ and the information structure η, a feedback policy $\{\phi^* \in S_\eta\}$ provides an optimal control if there exists a suitably smooth function $J : [0,\infty) \times N_\eta \to \mathbb{R}$, satisfying the Hamiltonian-Jacobian-Bellman equation:*

$$\begin{aligned}\mathbb{D}J(t,x,n_t) + \zeta_t\theta X_t + \sup\nolimits_{u_t \in \mathbb{U}}\{u_t^1[\pi_1(n_t)(J(t,x,n_t^1+1,n_t^2) - J(t,x,n_t)) - I_1]\} + \\ \{u_t^2[\pi_2(n_t)(J(t,x,n_t^1,n_t^2+1) - J(t,x,n_t)) - I_2]\} = 0 \qquad (10)\end{aligned}$$

where

$$\mathbb{D}J(t,x,n_t) = \frac{1}{2}\sigma^2 x^2 J_{xx} + \mu x J_x + J_t \qquad (11)$$

where the subscripts on J refer to the partial derivative.

This proposition provides a sufficient condition for the objective function. The result follows from Fleming (1969) or Fleming and Rishel (1975) with application of Ito's formula with jumps. For a treatment of jumps see Duffie (2001, Appendix F).

We solve the system explicitly backward from $n = [n_1 = 1, n_2 = 0]$, or $[n_1 = 0, n_2 = 1]$ and towards the beginning. We let project 1 and 2 represent the incremental project and radical project, respectively. Without loss of generality, we assume $\pi_1 > \pi_2$, $I_1 < I_2$.

Value of the firm after completion of one project

Proposition 2. *If project 1 or project 2 is completed first, the value is*

$$V(x, n_1 = 1, n_2 = 0) \quad = \quad \theta_1 \frac{x}{r-\mu}, \tag{12}$$

or

$$V(x, n_1 = 0, n_2 = 1) \quad = \quad \theta_2 \frac{x}{r-\mu} \tag{13}$$

respectively. We assume $r > \mu$ for the project value to be finite.

Value of the firm before projects are completed At the state $[n_1, n_2] = [0, 0]$, the HJB equation can be rewritten as

$$\mathbb{D}V(t,x,n_t) + \sup_{u_t \in \mathbb{U}} \{u_t^1 [\pi_1 (\tfrac{\theta_1}{(r-\mu)} x - V(t,x,n_t)) - I_1]\} + \\ \{u_t^2 [\pi_2 (\tfrac{\theta_2}{(r-\mu)} - V(t,x,n_t)) - I_2]\} = 0 \tag{14}$$

First, consider the value of the firm in the *inaction region*, i.e. $u = (0,0)$. The solution for the value of the firm is given by

$$V(x,0,0) = c_1 x^{-\gamma_{1,r}} + c_2 x^{-\gamma_{2,r}}$$

where $\gamma_{1,y}$, $\gamma_{2,y}$ solve $\frac{1}{2}\sigma^2(-\gamma)(-\gamma-1) + \mu(-\gamma) - y = 0$, for $y > 0$,

$$\gamma_{1,y} = \frac{\mu - \sigma^2/2 - \sqrt{(\mu - \sigma^2/2)^2 + 2y\sigma^2}}{\sigma^2} < 0,$$
$$\gamma_{2,y} = \frac{\mu - \sigma^2/2 + \sqrt{(\mu - \sigma^2/2)^2 + 2y\sigma^2}}{\sigma^2} > 0,$$

Second, consider the value of the firm in the *incremental region*, i.e. $u = (1,0)$. The solution for the value of the firm is given by

$$V(x,0,0) = c_1 x^{-\gamma_{1,r+\pi_1}} + c_2 x^{-\gamma_{2,r+\pi_1}} + c_3 x + c_4$$

$$c3 = \frac{\pi_1 \theta_1}{(r-\mu+\pi_1)(r-\mu)},\ c4 = -\frac{I_1}{r+\pi_1}$$

Similarly, consider the value of the firm in the *radical region*, i.e. $u = (0,1)$. The solution for the value of the firm is given by

$$V(x,0,0) = c_1 x^{-\gamma_{1,r+\pi_2}} + c_2 x^{-\gamma_{2,r+\pi_2}} + c_3 x + c_4$$

$$c3 = \frac{\pi_2 \theta_2}{(r-\mu+\pi_2)(r-\mu)},\ c4 = -\frac{I_2}{r+\pi_2}$$

Finally, consider the value of the firm in the *parallel region*, i.e. $u = (1,1)$. The solution for the value of the firm is given by

$$V(x,0,0) = c_1 x^{-\gamma_{1,r+\pi_1+\pi_2}} + c_2 x^{-\gamma_{2,r+\pi_1+\pi_2}} + c_3 x + c_4$$

$$c3 = \frac{\pi_1 \theta_1 + \pi_2 \theta_2}{(r-\mu+\pi_1+\pi_2)(r-\mu)},\ c4 = -\frac{I_1+I_2}{r+\pi_1+\pi_2}$$

The boundary between different investment regions is given by a critical value, such that optimal investment region changes while x crosses it.

Proposition 3. *Suppose the state* $[n_1, n_2] = [0,0]$. *Let* $V^{(m)}(t, x_t, n_t), 1 \leq m \leq M$, *be functionals solved by HJB equations (10) with optimal investment regions u.*

$$V(x,0,0) = \begin{cases} c_1^{(0)} x^{-\gamma_1^{(0)}} & x < x^{(1)*}. \\ \dots & \\ c_1^{(m)} x^{-\gamma_1^{(m)}} + c_2^{(m)} x^{-\gamma_2^{(m)}} + c_3^{(m)} x + c_4^{(m)} & x^{(m)*} \le x < x^{(m+1)*}. \\ \dots & \\ c_2^{(M)} x^{-\gamma_2^{(M)}} + c_3^{(M)} x + c_4^{(M)} & x^{(M)*} \le x. \end{cases} \tag{15}$$

with

$$V^{(0)}(t,0,n_t) = 0 \tag{16}$$

$$\lim_{x \to \infty} V^{(M)}(t,x,n_t) \propto x \tag{17}$$

when $x = x^{(m)*}$,

$$V^{(m-1)}(t,x,n_t) = V^{(m)}(t,x,n_t) \tag{18}$$

$$\frac{d}{dx} V^{(m-1)}(t,x,n_t) = \frac{d}{dx} V^{(m)}(t,x,n_t) \tag{19}$$

$$\textit{either } \pi_1(V(t,x,n^1+1,n^2) - V(t,x,n^1,n^2)) - I_1 = 0 \tag{20}$$

$$\textit{or } \pi_2(V(t,x,n^1,n^2+1) - V(t,x,n^1,n^2)) - I_2 = 0 \tag{21}$$

Equation (16 to 17) are standard boundary conditions. The value matching conditions (18), smooth pasting conditions (19), and transitional boundary conditions (20, 21), are sufficient to solve for the parameters. For a heuristic argument of the value matching conditions and smooth pasting conditions see Dixit (1993, Section 3.8); a rigorous proof is in Karatzas and Shreve (1991, Theorem 4.4.9). The transitional boundary conditions follow from HJB equations (10).

As there is no closed form solution for general cases, numerical approximations are applied. [6]

4. Optimal Investment Scenarios

Proposition 3 characterizes the investment policy that maximizes firm value. This investment policy takes the form of a threshold policy and there exist total M thresholds for two projects. We will show that $M \leq 3$ and there are 7 possible scenarios of investment policy. We will also examine how these scenarios will emerge associated with various parameter values.

4.1. Some geometry and intuition

Define $f_j(x) \equiv \frac{\theta_j}{r-\mu}x - \frac{I_j}{\pi_j}$, $W_j(x) \equiv f_j(x) - V(x)$. Then we have $u_j = 1$ if $W_j(x) \geq 0$; $u_j = 0$ if $W_j(x) < 0$.

Since the first two terms in V are the addition to the expected present value made possible by the ability to control the u process, the constants $c_1^{(m)}, c_2^{(m)}$ are non-negative. So $V(x)$ is a convex ($V'(x)$ increasing) function, thus $W(x)$ is a concave ($W'(x)$ decreasing) function. Then there are at most two zero points for $W_j(x)$.

First consider the case that $\theta_1 \leq \theta_2$. Since $\lim_{x\to\infty} W_2(x)/x = \frac{\theta_2}{r-\mu} - c_3^{(M)} > 0$, and $W_2(0) < 0$, then $W_2(x)$ has one zero point, denoted as x_2^*.

When $\theta_1 \leq \theta_2\pi_2/(r-\mu+\pi_2)$, we have $\lim_{x\to\infty} W_1(x)/x = \frac{\theta_1}{r-\mu} - c_3^{(M)} < 0$, then $W_1(x)$ has either zero or two zero points.

[6] See the optimization toolbox of MATLAB.

When $W_1(x)$ has no zero point, the investment region $u = (u_1, u_2)$ is denoted as $(0,0) \leftrightarrow (0,1)$, which means

$$u(x) = \begin{cases} (0,0) & x < x^{(1)*}. \\ (0,1) & x^{(1)*} \leq x. \end{cases} \tag{22}$$

where $x^{(1)*} = x_2^*$. This investment region scenario is denoted as scenario 1.

When $W_1(x)$ has two zero points, denoted as x_1^{L*}, x_1^{R*}, where $x_1^{L*} < x_1^{R*}$, the investment region $u = (u_1, u_2)$ could be the following three cases.

If $x_2^* \leq x_1^{L*} \leq x_1^{R*}$, denote the investment region as $(0,0) \leftrightarrow (0,1) \leftrightarrow (1,1) \leftrightarrow (0,1)$, which means

$$u(x) = \begin{cases} (0,0) & x < x^{(1)*}. \\ (0,1) & x^{(1)*} \leq x \leq x^{(2)*}. \\ (1,1) & x^{(2)*} \leq x \leq x^{(3)*}. \\ (0,1) & x^{(3)*} \leq x. \end{cases} \tag{23}$$

where $x^{(1)*} = x_2^*$, $x^{(2)*} = x_1^{L*}$, $x^{(3)*} = x_1^{R*}$. This kind of investment region scenario is denoted as scenario 2.

If $x_1^{L*} \leq x_1^{R*} \leq x_2^*$, denote the investment region as $(0,0) \leftrightarrow (1,0) \leftrightarrow (0,0) \leftrightarrow (0,1)$, which means

$$u(x) = \begin{cases} (0,0) & x < x^{(1)*}. \\ (1,0) & x^{(1)*} \leq x \leq x^{(2)*}. \\ (0,0) & x^{(2)*} \leq x \leq x^{(3)*}. \\ (0,1) & x^{(3)*} \leq x. \end{cases} \tag{24}$$

where $x^{(1)*} = x_1^{L*}$, $x^{(2)*} = x_1^{R*}$, $x^{(3)*} = x_2^*$. This kind of investment region scenario is denoted as scenario 3.

If $x_1^{L*} \leq x_2^* < x_1^{R*}$, denote the investment region as $(0,0) \leftrightarrow (1,0) \leftrightarrow (1,1) \leftrightarrow (0,1)$, which means

$$u(x) = \begin{cases} (0,0) & x < x^{(1)*}. \\ (1,0) & x^{(1)*} \leq x \leq x^{(2)*}. \\ (1,1) & x^{(2)*} \leq x \leq x^{(3)*}. \\ (0,1) & x^{(3)*} \leq x. \end{cases} \tag{25}$$

where $x^{(1)*} = x_1^{L*}$, $x^{(2)*} = x_2^*$, $x^{(3)*} = x_1^{R*}$. This kind of investment region scenario is denoted as scenario 4.

When $\theta_2 > \theta_1 > \theta_2\pi_2/(r-\mu+\pi_2)$, we have $\lim_{x\to\infty} W_1(x)/x = \frac{\theta_1}{r-\mu} - c_3^{(M)} > 0$, then $W_1(x)$ has one zero point x_1^*.

When $W_1(x)$ has one zero point x_1^*, the investment region $u = (u_1, u_2)$ could be $(0,0) \leftrightarrow (0,1) \leftrightarrow (1,1)$ if $x_1^* > x_2^*$, or $(0,0) \leftrightarrow (1,0) \leftrightarrow (1,1)$ if $x_1^* \leq x_2^*$. The two investment regions are illustrated as followed:

$$u(x) = \begin{cases} (0,0) & x < x^{(1)*}. \\ (0,1) & x^{(1)*} \leq x \leq x^{(2)*}. \\ (1,1) & x^{(1)*} \leq x. \end{cases} \tag{26}$$

where $x^{(1)*} = x_2^*$, $x^{(2)*} = x_1^*$.

$$u(x) = \begin{cases} (0,0) & x < x^{(1)*}. \\ (1,0) & x^{(1)*} \leq x \leq x^{(2)*}. \\ (1,1) & x^{(1)*} \leq x. \end{cases} \tag{27}$$

where $x^{(1)*} = x_1^*$, $x^{(2)*} = x_2^*$;

These two scenarios are denoted scenarios 5 and 6.

scenario	investment region
1	$(0,0) \leftrightarrow (0,1)$
2	$(0,0) \leftrightarrow (0,1) \leftrightarrow (1,1) \leftrightarrow (0,1)$
3	$(0,0) \leftrightarrow (1,0) \leftrightarrow (0,0) \leftrightarrow (0,1)$
4	$(0,0) \leftrightarrow (1,0) \leftrightarrow (1,1) \leftrightarrow (0,1)$
5	$(0,0) \leftrightarrow (0,1) \leftrightarrow (1,1)$
6	$(0,0) \leftrightarrow (1,0) \leftrightarrow (1,1)$
7	$(0,0) \leftrightarrow (1,0)$

Table 2
Scenarios

Similarly, consider the case where $\theta_2 < \theta_1$. Since $\lim_{x\to\infty} W_1(x)/x = \frac{\theta_1}{r-\mu} - c_3^{(M)} > 0$, and $W_1(0) < 0$, then $W_1(x)$ has one zero point, denoted as x_1^*.

When $W_2(x)$ has one zero point x_2^*, the investment region $u = (u_1, u_2)$ would be $(0,0) \leftrightarrow (1,0) \leftrightarrow (1,1)$, which is scenario 6, since $x_1^* < x_2^*$.

When $W_2(x)$ has no zero point, then the investment region $u = (u_1, u_2)$ is defined as $(0,0) \leftrightarrow (1,0)$,

$$u(x) \quad = \quad \begin{cases} (0,0) & x < x^{(1)*}. \\ (0,1) & x^{(1)*} \leq x. \end{cases} \tag{28}$$

where $x^{(1)*} = x_1^*$. This kind of investment region scenario is denoted as scenario 7.

We will show that $W_2(x)$ has at most one zero point given $\theta_1 > \theta_2$. Suppose that $W_2(x)$ has two zero points, we must have $\lim_{x\to\infty} W_2(x)/x = \frac{\theta_2}{r-\mu} - c_3^{(M)} < 0$, which implies that $\theta_2 \leq \theta_1\pi_1/(r-\mu+\pi_1)$. We can then find a dominated value function $V_1(x)$ which corresponds to investment region $(0,0) \leftrightarrow (0,1)$, such that $W_2(x) = f_2(x) - V(x) \leq f_2(x) - V_1(x) < 0$ for all $x \geq 0$.

In sum, the seven scenarios are illustrated in Table 2, and Figure 1 to Figure 7.

π_1 region	Scenarios						
	1	2	3	4	5	6	7
$\pi_1 \leq \pi_1^{(1)*}$	1				5	6	7
$\pi_1^{(1)*} < \pi_1 \leq \pi_1^{(2)*}$	1	2			5	6	7
$\pi_1^{(2)*} < \pi_1 \leq \pi_1^{(3)*}$	1	2		4		6	7
$\pi_1^{(3)*} < \pi_1$	1		3	4		6	7

Table 3
π_1 region, scenarios

π_1 region	Scenario Threshold θ_1^*			
	$\theta_1^{(l.1)*}$	$\theta_1^{(l.2)*}$	$\theta_1^{(l.3)*}$	$\theta_1^{(l.4)*}$
$\pi_1 \leq \pi_1^{(1)*}$	$\frac{\theta_2\pi_2}{r-\mu+\pi_2}$	θ_1^{56*}	$\frac{\theta_2(r-\mu+\pi_1)}{\pi_1}$	
$\pi_1^{(1)*} < \pi_1 \leq \pi_1^{(2)*}$	θ_1^{12*}	$\frac{\theta_2\pi_2}{r-\mu+\pi_2}$	θ_1^{56*}	$\frac{\theta_2(r-\mu+\pi_1)}{\pi_1}$
$\pi_1^{(2)*} < \pi_1 \leq \pi_1^{(3)*}$	θ_1^{12*}	θ_1^{24*}	$\frac{\theta_2\pi_2}{r-\mu+\pi_2}$	$\frac{\theta_2(r-\mu+\pi_1)}{\pi_1}$
$\pi_1^{(3)*} < \pi_1$	θ_1^{13*}	θ_1^{34*}	$\frac{\theta_2(r-\mu+\pi_1)}{\pi_1}$	$\frac{\theta_2(r-\mu+\pi_1)}{\pi_1}$

Table 4
θ_1 threshold

4.2. Scenario Analysis

The remaining analysis shows the specific investment region scenario under various parameter values. We assume that π_2 and θ_2 are given. First, we separate π_1 to four regions. For each region of π_1, the optimal investment scenario will change as θ_1 increases. The analysis of π_1 region and investment scenario is illustrated in Table 3. After identifying the π_1 region l by comparing π_1 with π_1 thresholds $\pi_1^{(i)*}, i \leq 3$, we will find the proper investment scenario by comparing θ_1 with θ_1 thresholds $\theta_1^{(l.k)*}, k \leq 4$. The scenarios' thresholds of θ_1 are displayed in Table 4. After that, from Table 3, we find the scenario number, which corresponds to a scenario in Table 2. Finally, we use Proposition 3 to solve the value function, x thresholds $x^{(m)*}$, with the corresponding investment scenario.

The derivations of the π_1 thresholds and θ_1 thresholds are provided in the appendix.

5. Results

We have characterized the investment policy that maximizes the value under various parameter values. Specifically, we first identify the π_1 region l by comparing the π_1 with three π_1 thresholds $\pi_1^{(i)*}, i \leq 3$. Then from Table 4, we identify the θ_1 region by comparing the θ_1 with the θ_1 thresholds $\theta_1^{(l.k)*}, k \leq 4$. After that, from Table 3, we find the scenario number, which corresponds to a scenario in Table 2. Finally, we use Proposition 3 to solve the value function, x thresholds $x^{(m)*}$, with the corresponding investment scenario.

The valuation formulas derived can thus be used to develop economic intuition regarding optimal investment policy, and the factors driving the choice between the two projects.

5.1. Investment policy

The investment region $u = \{(0,0),(0,1),(1,0),(1,1)\}$, corresponding to inaction, radical, incremental and parallel investment policies. We will discuss when each policy should be applied.

5.1.1. Inaction

Normally, the inaction investment region $u = (0,0)$ emerges when market condition x is weak, like in all scenarios 1 to 7. However, it could also happen when x is good, as in scenario 3, $(1,0) \rightarrow (0,0)$. The intuition is that a firm may drop an incremental innovation project when market conditions improve, anticipating that a radical innovation project coming. In the meantime, the firm may not invest in such a radical innovation project, as market conditions are not good enough.

5.1.2. Incremental

Generally, an incremental region may come from an inaction region when market condition x rises, like $(0,0) \rightarrow (1,0)$ in scenario 3, 4, 6, and 7 , or from a parallel region when x worsens, like $(1,0) \leftarrow (1,1)$ in scenario 4 and 6. However, it is interesting to notice that an incremental region may also happen from an inaction region even when market condition x drops, as in scenario 3, $(1,0) \leftarrow (0,0)$, when a radical innovation is less likely to appear with lower market condition.

The incremental policy could be a dominant policy under certain conditions. Specifically, when $\theta_1 > \theta_1^{(I.M)} \equiv \theta_2(r - \mu + \pi_1)/\pi_1$, the investment policy follows scenario 7, i.e., $(0,0) \rightarrow (1,0)$. Thus the incremental innovation project dominates the radical innovation project, since a firm can consider when and whether to invest the incremental project only.

5.1.3. Radical

Similarly to the above mentioned incremental region, a radical region may come from an inaction region when x rises,like $(0,0) \rightarrow (0,1)$ in scenario 1, 2, 3, and 5, or from a parallel region when x worsens, like $(0,1) \leftarrow (1,1)$ in scenario 2 and 5. However, it is interesting to notice that a radical region may also happen from a parallel region even when market condition x increases, as $(1,1) \rightarrow (0,1)$ in scenario 2 and 4. The intuition is that as market condition rises, the future reward difference between the incremental and the radical innovation projects increases, then a firm may drop the incremental project, with the concern that the success of the easier incremental innovation may block the success of the more difficult but more rewardable radical innovation, i.e., the opportunity cost for the incremental innovation is higher than its potential benefits.

The radical policy could be a dominant policy under certain conditions. Specifically, when $\theta_1 < \theta_1^{(l.1)}$, the radical innovation project dominates the incremental innovation project.

5.1.4. Parallel

Generally, we find that a parallel region emerges when market conditions improve, like scenario 2, 4, 5, and 6. The only exception occurs with a radical region when market conditions worsen, like $(1,1) \leftarrow (0,1)$ in scenario 2 and 4. Similarly as above, the intuition is that when market condition drops, the future reward difference between the incremental and the radical innovation projects decreases, thus it is optimal to invest parallel.

5.2. Impact of market condition x on Investment policy

Project Start policy Usually, as market conditions improve from the inaction investment region, either incremental or radical projects could be started. For example, see $(0,0) \rightarrow (0,1)$, in scenario 1, 2 and 5; $(0,0) \rightarrow (1,0)$, in scenario 3, 4, 6, and 7; $(0,1) \rightarrow (1,1)$ in scenario

2, and 5; $(1,0) \rightarrow (1,1)$ in scenario 4 and 6. This framework shows whether incremental or radical projects should be developed under rising market conditions.

Exceptions occur when market conditions decline from either a radical region, i.e., $(1,1) \leftarrow (0,1)$ as in scenario 2 or 4, or an inaction region, i.e., $(1,0) \leftarrow (0,0)$ as in scenario 3, the incremental projects may be restarted. It is noteworthy to point out that $(0,1) \leftarrow (0,0)$ is impossible.

Project Stop policy Usually, as market conditions worsen from the parallel investment region, either incremental or radical projects could be stopped. For example, see $(0,0) \leftarrow (0,1)$, in scenario 1, 2 and 5; $(0,0) \leftarrow (1,0)$, in scenario 3, 4, 6, and 7; $(0,1) \leftarrow (1,1)$ in scenario 2 and 5; $(1,0) \leftarrow (1,1)$ in scenario 4 and 6. This framework illustrates whether incremental or radical projects should be stopped under downward market conditions.

One exception is that when market conditions improve from either a parallel region, i.e., $(1,1) \rightarrow (0,1)$ in scenario 2 or 4, or incremental region, i.e., $(1,0) \rightarrow (0,0)$ in scenario 3, the incremental project may be stopped. This finding shows that a firm should not always invest in an incremental project even when market conditions are rising.

Normally, a firm will begin to develop a project when market conditions improve and drop it when market conditions worsen. Contrary to this common intuition, we find that an incremental project may not be selected for investment even when market conditions rise, in anticipation of a radical project emerging.

5.3. Impact of π_1 on Investment policy

As π_1 rises, the π_1 region increases in the order of $1,2,3,4$.

Consider the following two interesting results.

1. As $\pi_1 < \pi_1^{(1)*}$, the investment scenario can only be 1, 5, 6 or 7. This π_1 region can be applied to the cooperative game for two symmetric two firms, with investment region $(0,0) \leftrightarrow (1,1)$. [7]

2. As $\pi_1^{(3)*} < \pi_1$, the investment region has no pattern of $(0,1) \leftrightarrow (1,1)$. This result has important implication in the GM case discussed later.

For simplicity, from now on, we will focus on the discussion of region $\pi_1^{(3)*} < \pi_1$, which provides rich implications for our examples, unless other π_1 regions are specifically mentioned. A general discussion of other π_1 regions could be provided in future.

5.4. Impact of θ_1 on Investment policy

As θ_1 rises, corresponding scenarios are selections increasingly from $\{1,2,3,4,5,6,7\}$, as shown in Table 3.

As θ_1 increases, the region to invest in incremental projects broadens since x_1^{L*} drops and $x_1^{(F*)}$ rises; the region to invest in radical projects is uplifted as x_2^* rises. This phenomenon is illustrated in Table 5.

The intuition is that a more profitable incremental project prefers an incremental and a parallel policy, rather than an inaction and a radical policy.

5.5. Impact of σ on Investment policy

As σ rises, the π_1 thresholds $\pi_1^{(2)*}$, $\pi_1^{(3)*}$, and the θ_1 thresholds $\theta_1^{(13)*}$, $\theta_1^{(34)*}$ also increase. As a result, for a given θ_1, the region for scenario 1 broadens, and the region for scenario 3 and 4 narrows.

[7] See (Weyant and Yao 2004).

θ_1	σ	$\pi_1^{(1)}$	$\pi_1^{(2)}$	$\pi_1^{(3)}$	$\theta_1^{(1)}$	$\theta_1^{(2)}$	$\theta_1^{(3)}$	$\theta_1^{(4)}$	x_1^{L*}	x_1^{R*}	$x_2^{(*)}$
16	.2	.0367	.044	.0714	18.0885	21.2271	40	100	∞		2.3124
24	.2	.0367	.044	.0714	18.0885	21.2271	40	100	.7843	2.8452	2.4038
35	.2	.0367	.044	.0714	18.0885	21.2271	40	100	.53	9.7843	2.7745
50	.2	.0367	.044	.0714	18.0885	21.2271	40	100	.3708	∞	3.6062
120	.2	.0367	.044	.0714	18.0885	21.2271	40	100	.1544	∞	∞
35	.3	.0367	.0501	.0849	22.7248	25.268	40	100	.6196	8.72	2.8108
35	.4	.0367	.0577	.1001	26.7241	28.3121	40	100	.7364	7.7697	2.8539

Table 5
Sensitivity analysis

Base Parameters: $\theta_2 = 80$, $\pi_1 = .2$, $\pi_2 = .05$, $I_1 = 50$, $I_2 = 150$, $\mu = .01$, $r = .06$.

As σ increases, the region to invest incremental project narrows since x_1^{L*} rises and x_1^{R*} drops, the region to invest radical project is uplifted as x_2^* rises.

These phenomena are illustrated in Table 5.

The intuition is that a volatile market prefers an inaction and a radical policy, rather than an incremental and a parallel policy.

6. Discussion: Model Implications and Managerial Insights

While the optimal investment policy derived in this paper takes the form of a threshold policy as in traditional real option models, two major differences arise within the present model. First, this optimal investment policy provides thresholds not for a one-time decision problem, but for a (partially) repeated decision problem. Real world R&D managers realize that active project management requires an ongoing decision making process. [8] We observe that firms may make recurrent start or stop investment decisions when market conditions change. This

[8] Cooper, Edgett, and Kleinschmidt (1998) have an intensive study of portfolio management as currently practiced in industry and define the decision making process on individual projects on an ongoing basis. The Real Option Group has applied an option-based strategic planning and control framework of Trigeogis (1996) to active management of investment projects over time.

observation can be applied to explain real world phenomena as well as to provide optimal investment policy for active management.

Second, the investment decision $u(x)$ may not be a monotone function of market state x. The insight is that R&D project managers need to consider the interaction between two projects, rather than focus on a single project alone. An incremental project may be stopped under super market conditions when the opportunity cost of losing a radical project outweighs the incremental project's own gain.

Furthermore, managers may want to select the incremental policy when the incremental project has good expected return, or when market conditions turn bad. On the contrary, they may need to choose the radical policy when the incremental project has unsatisfactory expected return, or when market conditions become pretty good.

6.1. Illustration: Project Selection Under Technology and Market uncertainty at GM

We now revisit the GM alternative technology example described above. In 1998, GM observed that the market was not as good as expected. The R&D team decided to stop the hybrid project, after choosing to start it in 1988. In the meantime, GM restarted its fuel cell program in the 1990s, assigning Byron McCormick to run it in 1997. As a result, GM focuses on the hydrogen car project. It seems that these investment patterns may be explained by scenario 2.

However, the booming hybrid car market established by Toyota and Honda in recent years made GM reconsider its project selection policy. In retrospect, the decision may not have served them well. Based on data collected from industry sources, we examine the profit from the various flexible approaches to see which scenario would have been more appropriate in the GM context.

Description	Parameter	Value
Success rate for incremental project 1	π_1	.2
Success rate for radical project 2	π_2	.05
Yearly investment cost for project 1	I_1	50
Yearly investment cost for project 2	I_2	150
Cash flow multiple for project 1	θ_1	
Cash flow multiple for project 2	θ_2	80
Market growth drift	μ	.01
Discount rate	r	.06
Market volatility	σ	.3

Table 6
GM parameters

The parameters' values are representative of GM and related industry, from Maccormack (2003) and experts in this area. In our model, the success rate for Gasoline Hybrid Electric Car (GHEC) and Hydrogen Fuel Cell Vehicle (HFCV) are .2 and .05, which make the expected success time 5 years and 20 years under conditions of consistent investments. The yearly cost for GHEC and HFCV under GM is estimated at \$50 million and \$150 million. [9] The parameters' values are listed in Table 6.

Without loss of generality, we assume that θ_2 is given. As shown in Table 5, $\pi_1^{(3)*} = .0849 < \pi_1$. Thus from Table 3, the scenario for a specific θ_1 can only be 1, 3, 4, 6 or 7. It follows that $(0,1) \leftarrow (1,1)$ is impossible, which means that the radical policy isn't optimal when market conditions turn bad from the parallel region. This observation tells us that from a starting state with the parallel policy, the R&D team may choose either the inaction policy, or the incremental policy, i.e., to develop the incremental technology alone, when market conditions worsen. The radical policy GM chose was not optimal with a downward market. This result also shows that the radical policy appears in scenario 1, 3, and 4. So the radical policy will appear either when the incremental project has low expected return (low θ_1, like

[9] It is noted that the cost of HFCV may be even higher in the whole industry, since GM may benefit from the collaboration of Hydrogen research with government. It can be shown that in such a higher radical project cost case, the main result and insights remain the same.

in scenario 1), or when market conditions turn good, rather than when market conditions turn bad, as observed by GM at 1998.

From Table 2, we know that scenario 2 appears only in two cases: (i) $\pi_1^{(1)*} < \pi_1 \le \pi_1^{(2)*}$ and $\theta_1^{12*} < \theta_1 < \frac{\theta_2\pi_2}{r-\mu+\pi_2}$; (ii) $\pi_1^{(2)*} < \pi_1 \le \pi_1^{(3)*}$ and $\theta_1^{12*} < \theta_1 < \theta_1^{12*}$. So GM's strategy will make sense only when the hybrid technology become more difficult to develop and the potential profits are low. The intuition is that GM may have undervalued the potential profits of the hybrids project or have underestimated the possibility that incremental hybrid technology could become the standard. Instead of radical policy, they may either stay on parallel policy, or focus on incremental technology like Toyota.

6.2. Illustration: Project Selection Under Technology and Market uncertainty at Philips, DVD vs ORT

The intuition derived from the GM case can be generalized to other areas. Lets visit the case of Philips: DVD vs Optical Recording Tape (ORT) [10]. It is reported that Philips' R&D team focused on the radical ORT project. Finally, the ORT project, facing uncertain technology barriers, resulted in a high deliver cost and was abandoned, since DVD successfully surpassed ORT. In retrospect, as the success likelihood of ORT (π_2) is small, (0,1) may need to be replaced by (1,0). The intuition here is similar to the previous GM case.

6.3. Illustration: Project Selection Under Technology and Market uncertainty at Philips, Analog SD versus Digital SD

Lint and Pennings (forthcoming) provided a scenario of investment decisions as {Develop no standard, Develop digital standard, Develop both standards}, like scenario 5. We extend the scenario in at least three ways. The first is that there may be a situation to develop analog stan-

[10] More about this case see Lint and Pennings (1998)

dard instead of digital standard, like scenario 6. Another one is like scenario 4, {Develop no standard, Develop analog standard, Develop both standards, Develop digital standard}. When market conditions are super, it may be better to only develop the radical standard, which may result in higher profits. The third extension is like scenario 3, {Develop no standard, Develop analog standard, Develop no standard, Develop digital standard}. There may be an inaction region when market conditions are even better than an incremental region. The intuition is that a firm may stop the incremental project, anticipating a radical project is emerging.

7. Conclusion

This paper develops a real option model to examine the optimal investment policy for multiple projects that can be researched and developed separately or in parallel. Different from a discrete model with multiple stage decision points, we provide a continuous decision model where firms may make decisions on whether to invest or suspend R&D in each period before projects are completed. A numerical approximation to the solution of the problem is required. Our model provides intuition for various likely scenarios of parallel development and separate development.

Several important factors drive the decision of whether to develop the incremental innovation project, the radical innovation project, or both in parallel: the state and volatility of market conditions, the likelihood of success, the expected present value of future cash flows, and the costs of research and development. Generally, we find that parallel R&D is superior in cases when market conditions are good. The only exception is that an easy and low return incremental innovation project may be stopped under super strong market conditions and a radical innovation project is pursued alone. Normally, a firm will begin to develop a project when market conditions improve and drop it when market conditions fall. Contrary to this intuition, we find that an incremental project may be stopped even when market conditions improve, because a successful radical project is anticipated.

We illustrate our model with the GM alternative technology selection between hybrid electric cars and hydrogen fuel cell vehicles and note the managerial implications of our analysis.

There are some extensions of our model that could be interesting to pursue. The firm being modeled was essentially a monopolist; the effects of strategic interactions such as competition and collusion should be considered in future.

APPENDIX

Proof of Proposition 2 The result follows from Proposition 1.

Proof of Proposition 3 The result follows from Proposition 1.

A. Derivation of π_1 and θ_1 thresholds

We have three π_1 thresholds for four π_1 regions. The three π_1 thresholds are listed here:

$\pi_1^{(1)*} = I_1/I_2(r+\pi_2);$

$\pi_1^{(2)*} = \{\pi_1 : f_1(x^*;\theta_1,\pi_1) = f_2(x^*;\theta_2) = V(x^*)\}$, where $\theta_1 = \theta_2\pi_2/(r-\mu+\pi_2)$, value $V(.)$, threshold x^* are for investment scenario $(0,0) \leftrightarrow (1,1)$;

$\pi_1^{(3)*} = \{\pi_1 : f_1(x^*;\theta_1,\pi_1) = V(x^*), f_1'(x^*) = V'(x^*)\}$,[11] where value $V(.)$, threshold x^* are for investment scenario $(0,0) \leftrightarrow (0,1)$. The analytical solution

$$\pi_1^{(3)*} = \frac{\pi_2 I_1}{I_2(-1-\gamma_{1,r})(\frac{(r-\mu+\pi_2)\gamma_{1,r}\gamma_{2,r+\pi_2}}{(r+\pi_2)(1+\gamma_{1,r})(1+\gamma_{2,r+\pi_2})}-1)}$$

θ_1 thresholds are derived for each of the four π_1 regions.

[11] The notation $'$ represents the derivative with respect to x.

π_1 region 1: $\pi_1 \leq \pi_1^{(1)*}$

There are three θ_1 thresholds for four θ_1 regions.

$\theta_1^{(1.1)*} = \theta_2\pi_2/(r-\mu+\pi_2)$.

$\theta_1^{(1.2)*} = \theta_1^{56*}$, the threshold between scenarios 5 and 6, which can be solved by numerical approximation. [12]

$\theta_1^{(1.3)*} = \theta_2(r-\mu+\pi_1)/\pi_1$.

If $\theta_1 \leq \theta_1^{(1.1)*}$, we have investment region scenario 1.

If $\theta_1^{(1.1)*} \leq \theta_1 \leq \theta_1^{(1.2)*}$, the investment region scenario is 5.

If $\theta_1^{(1.2)*} \leq \theta_1 \leq \theta_1^{(1.3)*}$ the investment region scenario is 6.

If $\theta_1^{(1.3)*} \leq \theta_1$, the investment region scenario is 7.

π_1 region 2: $\pi_1^{(1)*} < \pi_1 \leq \pi_1^{(2)*}$.

There are four θ_1 thresholds for five θ_1 regions.

$\theta_1^{(2.1)*} = \theta_1^{12*}$,the threshold between scenario 1 and 2, which can be solved by numerical approximation. [13]

$\theta_1^{(2.2)*} = \theta_2\pi_2/(r-\mu+\pi_2)$.

$\theta_1^{(2.3)*} = \theta_1^{56*}$,the threshold between scenarios 5 and 6, which has been defined before.

$\theta_1^{(2.4)*} = \theta_2(r-\mu+\pi_1)/\pi_1$.

If $\theta_1 \leq \theta_1^{(2.1)*}$, we have investment region scenario 1.

If $\theta_1^{(2.1)*} \leq \theta_1 \leq \theta_1^{(2.2)*}$, the investment region scenario is 2.

[12] $\theta_1^{56*} = \{\theta_1 : f_1(x^*;\theta_1) = f_2(x^*;\theta_2) = V(x^*)\}$, where value $V(.)$, threshold x^* are for investment scenario $(0,0) \leftrightarrow (1,1)$.

[13] $\theta_1^{12*} = \{\theta_1 : f_1(x_1^*;\theta_1) = V(x_1^*), f_1'(x_1^*) = V'(x_1^*), x_1^* \geq x^*\}$, where value $V(.)$, threshold x^* are for investment scenario $(0,0) \leftrightarrow (0,1)$.

If $\theta_1^{(2.2)*} \leq \theta_1 \leq \theta_1^{(2.3)*}$ the investment region scenario is 5.

If $\theta_1^{(2.3)*} \leq \theta_1 \leq \theta_1^{(2.4)*}$ the investment region scenario is 6.

If $\theta_1^{(2.4)*} \leq \theta_1$, the investment region scenario is 7.

π_1 region 3: $\pi_1^{(2)*} < \pi_1 \leq \pi_1^{(3)*}$.

There are four θ_1 thresholds for five θ_1 regions.

$\theta_1^{(3.1)*} = \theta_1^{12*}$.

$\theta_1^{(3.2)*} = \theta_1^{24*}$,the threshold between scenario 2 and 4, which can be solved by numerical approximation. [14]

$\theta_1^{(3.3)*} = \theta_2\pi_2/(r-\mu+\pi_2)$.

$\theta_1^{(3.4)*} = \theta_2(r-\mu+\pi_1)/\pi_1$.

If $\theta_1 \leq \theta_1^{(3.1)*}$, we have investment region scenario 1.

If $\theta_1^{(3.1)*} \leq \theta_1 \leq \theta_1^{(3.2)*}$, the investment region scenario is 2.

If $\theta_1^{(3.2)*} \leq \theta_1 \leq \theta_1^{(3.3)*}$ the investment region scenario is 5.

If $\theta_1^{(3.3)*} \leq \theta_1$, the investment region scenario is 7.

π_1 region 4: $\pi_1^{(3)*} < \pi_1$

There are four θ_1 thresholds for five θ_1 regions.

$\theta_1^{(4.1)*} = \theta_1^{13*}$,the threshold between scenario 1 and 3, which can be solved by numerical approximation. [15]

[14] $\theta_1^{24*} = \{\theta_1 : f_1(x^{(1)*};\theta_1) = f_2(x^{(1)*};\theta_2) = V(x^{(1)*})\}$, where value $V(.)$, thresholds $x^{(1)*}$, $x^{(2)*}$ are for investment scenario $(0,0) \leftrightarrow (1,1) \leftrightarrow (0,1)$.

[15] $\theta_1^{13*} = \{\theta_1 : f_1(x_1^*;\theta_1) = V(x_1^*), f_1'(x_1^*) = V'(x_1^*), x_1^* \leq x^*\}$, where value $V(.)$, threshold x^* are for investment scenario $(0,0) \leftrightarrow (0,1)$.

$\theta_1^{(4.2)*} = \theta_1^{34*}$,the threshold between scenario 3 and 4, which can be solved by numerical approximation. [16]

$\theta_1^{(4.3)*} = \theta_2\pi_2/(r-\mu+\pi_2)$.

$\theta_1^{(4.4)*} = \theta_2(r-\mu+\pi_1)/\pi_1$.

If $\theta_1 \leq \theta_1^{(4.1)*}$, we have investment region scenario 1.

If $\theta_1^{(4.1)*} \leq \theta_1 \leq \theta_1^{(4.2)*}$, the investment region scenario is 3.

If $\theta_1^{(4.2)*} \leq \theta_1 \leq \theta_1^{(4.3)*}$ the investment region scenario is 6.

If $\theta_1^{(4.3)*} \leq \theta_1$, the investment region scenario is 7.

We have three π_1 thresholds for four π_1 regions. The three π_1 thresholds are listed here:

$\pi_1^{(1)*} = I_1/I_2(r+\pi_2)$;

$\pi_1^{(2)*} = \{\pi_1 : f_1(x^*;\theta_1,\pi_1) = f_2(x^*;\theta_2) = V(x^*)\}$, where $\theta_1 = \theta_2\pi_2/(r-\mu+\pi_2)$, value $V(.)$, threshold x^* are for investment scenario $(0,0) \leftrightarrow (1,1)$;

$\pi_1^{(3)*} = \{\pi_1 : f_1(x^*;\theta_1,\pi_1) = V(x^*), f_1'(x^*) = V'(x^*)\}$,[17] where value $V(.)$, threshold x^* are for investment scenario $(0,0) \leftrightarrow (0,1)$. The analytical solution

$$\pi_1^{(3)*} = \frac{\pi_2 I_1}{I_2(-1-\gamma_{1,r})(\frac{(r-\mu+\pi_2)\gamma_{1,r}\gamma_{2,r+\pi_2}}{(r+\pi_2)(1+\gamma_{1,r})(1+\gamma_{2,r+\pi_2})} - 1)}$$

θ_1 thresholds are derived for each of the four π_1 regions.

π_1 region 1: $\pi_1 \leq \pi_1^{(1)*}$

There are three θ_1 thresholds for four θ_1 regions.

[16] $\theta_1^{34*} = \{\theta_1 : f_1(x^{(2)*};\theta_1) = f_2(x^{(2)*};\theta_2) = V(x^{(2)*})\}$, where value $V(.)$, thresholds $x^{(1)*}$, $x^{(2)*}$ are for investment scenario $(0,0) \leftrightarrow (1,0) \leftrightarrow (0,1)$.

[17] The notation $'$ represents the derivative with respect to x.

$\theta_1^{(1.1)*} = \theta_2\pi_2/(r-\mu+\pi_2)$.

$\theta_1^{(1.2)*} = \theta_1^{56*}$, the threshold between scenarios 5 and 6, which can be solved by numerical approximation. [18]

$\theta_1^{(1.3)*} = \theta_2(r-\mu+\pi_1)/\pi_1$.

If $\theta_1 \leq \theta_1^{(1.1)*}$, we have investment region scenario 1.

If $\theta_1^{(1.1)*} \leq \theta_1 \leq \theta_1^{(1.2)*}$, the investment region scenario is 5.

If $\theta_1^{(1.2)*} \leq \theta_1 \leq \theta_1^{(1.3)*}$ the investment region scenario is 6.

If $\theta_1^{(1.3)*} \leq \theta_1$, the investment region scenario is 7.

π_1 region 2: $\pi_1^{(1)*} < \pi_1 \leq \pi_1^{(2)*}$.

There are four θ_1 thresholds for five θ_1 regions.

$\theta_1^{(2.1)*} = \theta_1^{12*}$,the threshold between scenario 1 and 2, which can be solved by numerical approximation. [19]

$\theta_1^{(2.2)*} = \theta_2\pi_2/(r-\mu+\pi_2)$.

$\theta_1^{(2.3)*} = \theta_1^{56*}$,the threshold between scenarios 5 and 6, which has been defined before.

$\theta_1^{(2.4)*} = \theta_2(r-\mu+\pi_1)/\pi_1$.

If $\theta_1 \leq \theta_1^{(2.1)*}$, we have investment region scenario 1.

If $\theta_1^{(2.1)*} \leq \theta_1 \leq \theta_1^{(2.2)*}$, the investment region scenario is 2.

If $\theta_1^{(2.2)*} \leq \theta_1 \leq \theta_1^{(2.3)*}$ the investment region scenario is 5.

If $\theta_1^{(2.3)*} \leq \theta_1 \leq \theta_1^{(2.4)*}$ the investment region scenario is 6.

[18] $\theta_1^{56*} = \{\theta_1 : f_1(x^*;\theta_1) = f_2(x^*;\theta_2) = V(x^*)\}$, where value $V(.)$, threshold x^* are for investment scenario $(0,0) \leftrightarrow (1,1)$.

[19] $\theta_1^{12*} = \{\theta_1 : f_1(x_1^*;\theta_1) = V(x_1^*), f_1'(x_1^*) = V'(x_1^*), x_1^* \geq x^*\}$, where value $V(.)$, threshold x^* are for investment scenario $(0,0) \leftrightarrow (0,1)$.

If $\theta_1^{(2.4)*} \leq \theta_1$, the investment region scenario is 7.

π_1 region 3: $\pi_1^{(2)*} < \pi_1 \leq \pi_1^{(3)*}$.

There are four θ_1 thresholds for five θ_1 regions.

$\theta_1^{(3.1)*} = \theta_1^{12*}$.

$\theta_1^{(3.2)*} = \theta_1^{24*}$,the threshold between scenario 2 and 4, which can be solved by numerical approximation. [20]

$\theta_1^{(3.3)*} = \theta_2\pi_2/(r-\mu+\pi_2)$.

$\theta_1^{(3.4)*} = \theta_2(r-\mu+\pi_1)/\pi_1$.

If $\theta_1 \leq \theta_1^{(3.1)*}$, we have investment region scenario 1.

If $\theta_1^{(3.1)*} \leq \theta_1 \leq \theta_1^{(3.2)*}$, the investment region scenario is 2.

If $\theta_1^{(3.2)*} \leq \theta_1 \leq \theta_1^{(3.3)*}$ the investment region scenario is 5.

If $\theta_1^{(3.3)*} \leq \theta_1$, the investment region scenario is 7.

π_1 region 4: $\pi_1^{(3)*} < \pi_1$

There are four θ_1 thresholds for five θ_1 regions.

$\theta_1^{(4.1)*} = \theta_1^{13*}$,the threshold between scenario 1 and 3, which can be solved by numerical approximation. [21]

$\theta_1^{(4.2)*} = \theta_1^{34*}$,the threshold between scenario 3 and 4, which can be solved by numerical approximation. [22]

[20] $\theta_1^{24*} = \{\theta_1 : f_1(x^{(1)*};\theta_1) = f_2(x^{(1)*};\theta_2) = V(x^{(1)*})\}$, where value $V(.)$, thresholds $x^{(1)*}$, $x^{(2)*}$ are for investment scenario $(0,0) \leftrightarrow (1,1) \leftrightarrow (0,1)$.

[21] $\theta_1^{13*} = \{\theta_1 : f_1(x_1^*;\theta_1) = V(x_1^*), f_1'(x_1^*) = V'(x_1^*), x_1^* \leq x^*\}$, where value $V(.)$, threshold x^* are for investment scenario $(0,0) \leftrightarrow (0,1)$.

[22] $\theta_1^{34*} = \{\theta_1 : f_1(x^{(2)*};\theta_1) = f_2(x^{(2)*};\theta_2) = V(x^{(2)*})\}$, where value $V(.)$, thresholds $x^{(1)*}$, $x^{(2)*}$ are for investment scenario $(0,0) \leftrightarrow (1,0) \leftrightarrow (0,1)$.

$\theta_1^{(4.3)*} = \theta_2\pi_2/(r-\mu+\pi_2)$.

$\theta_1^{(4.4)*} = \theta_2(r-\mu+\pi_1)/\pi_1$.

If $\theta_1 \leq \theta_1^{(4.1)*}$, we have investment region scenario 1.

If $\theta_1^{(4.1)*} \leq \theta_1 \leq \theta_1^{(4.2)*}$, the investment region scenario is 3.

If $\theta_1^{(4.2)*} \leq \theta_1 \leq \theta_1^{(4.3)*}$ the investment region scenario is 6.

If $\theta_1^{(4.3)*} \leq \theta_1$, the investment region scenario is 7.

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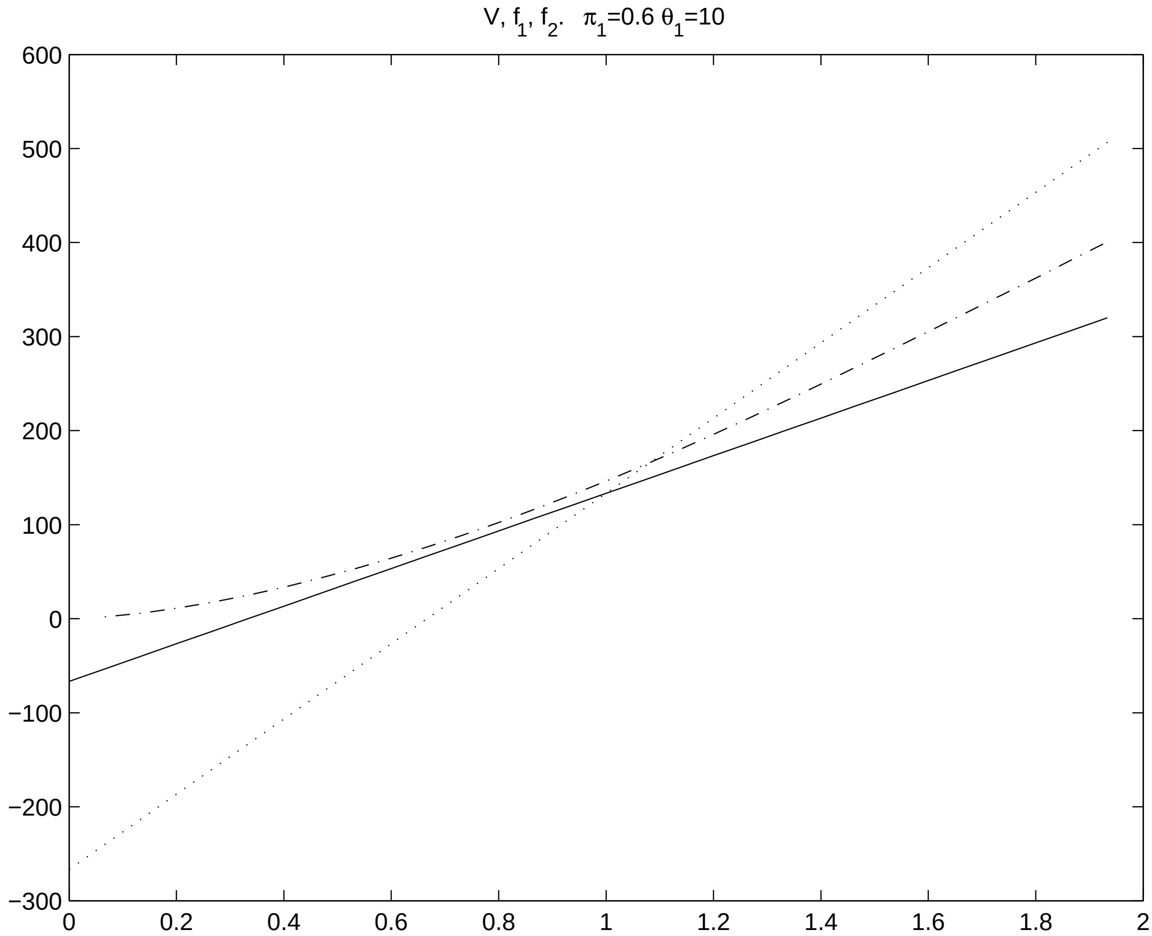

Figure 1. Scenario 1.
The dash-dotted line refers to V(x). The solid line refers to $f_1(x)$. The dotted line refers to $f_2(x)$. In this figure, $x_2^* < x_1^{L*} = x_1^{R*} = \infty$, where x_1^{L*} and x_1^{R*} are $W_1(x)$'s zero points, x_2^* is $W_2(x)$'s zero point, $W_j(x) = f_j(x) - V(x), j = \{1,2\}$.

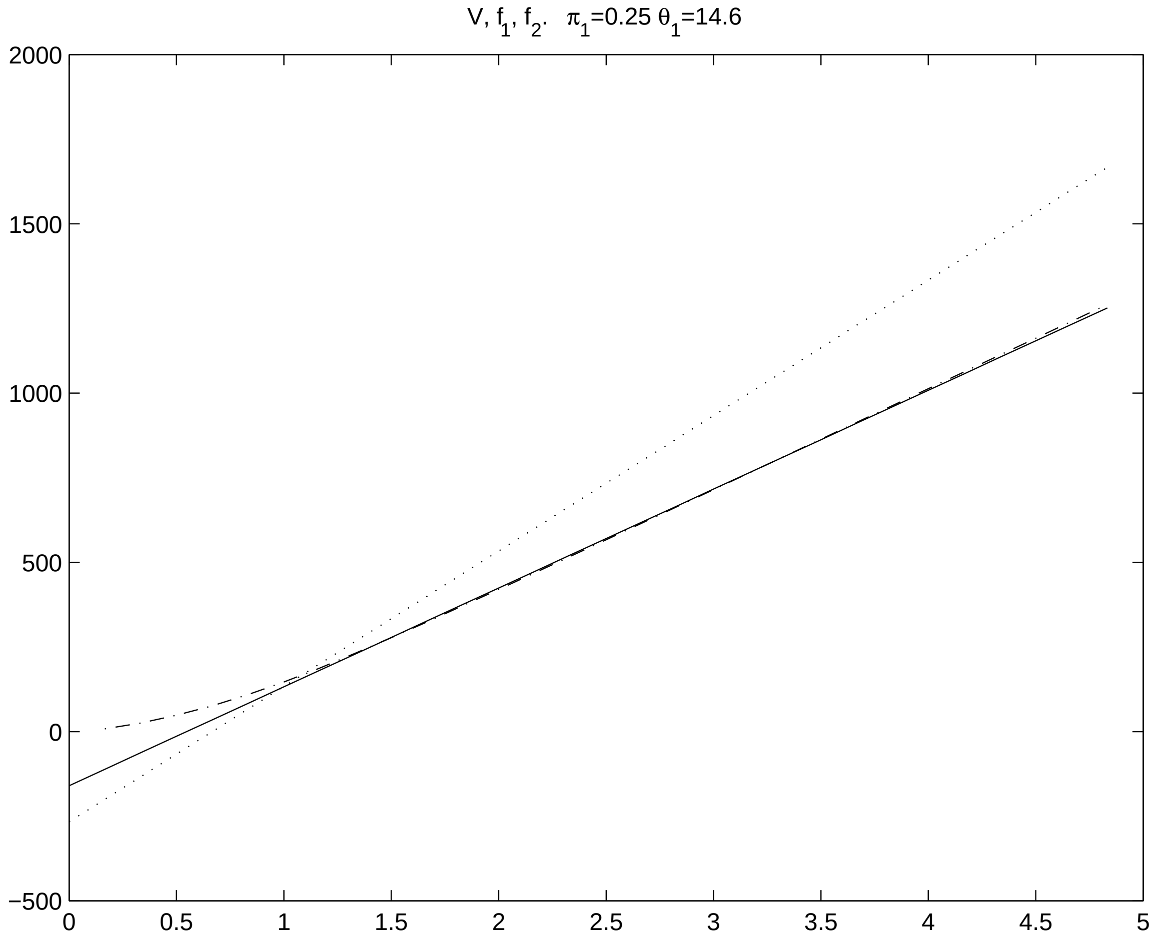

Figure 2. Scenario 2.
The dash-dotted line refers to V(x). The solid line refers to $f_1(x)$. The dotted line refers to $f_2(x)$. In this figure, $x_2^* < x_1^{L*} < x_1^{R*} < \infty$, where x_1^{L*} and x_1^{R*} are $W_1(x)$'s zero points, x_2^* is $W_2(x)$'s zero point, $W_j(x) = f_j(x) - V(x), j = \{1, 2\}$.

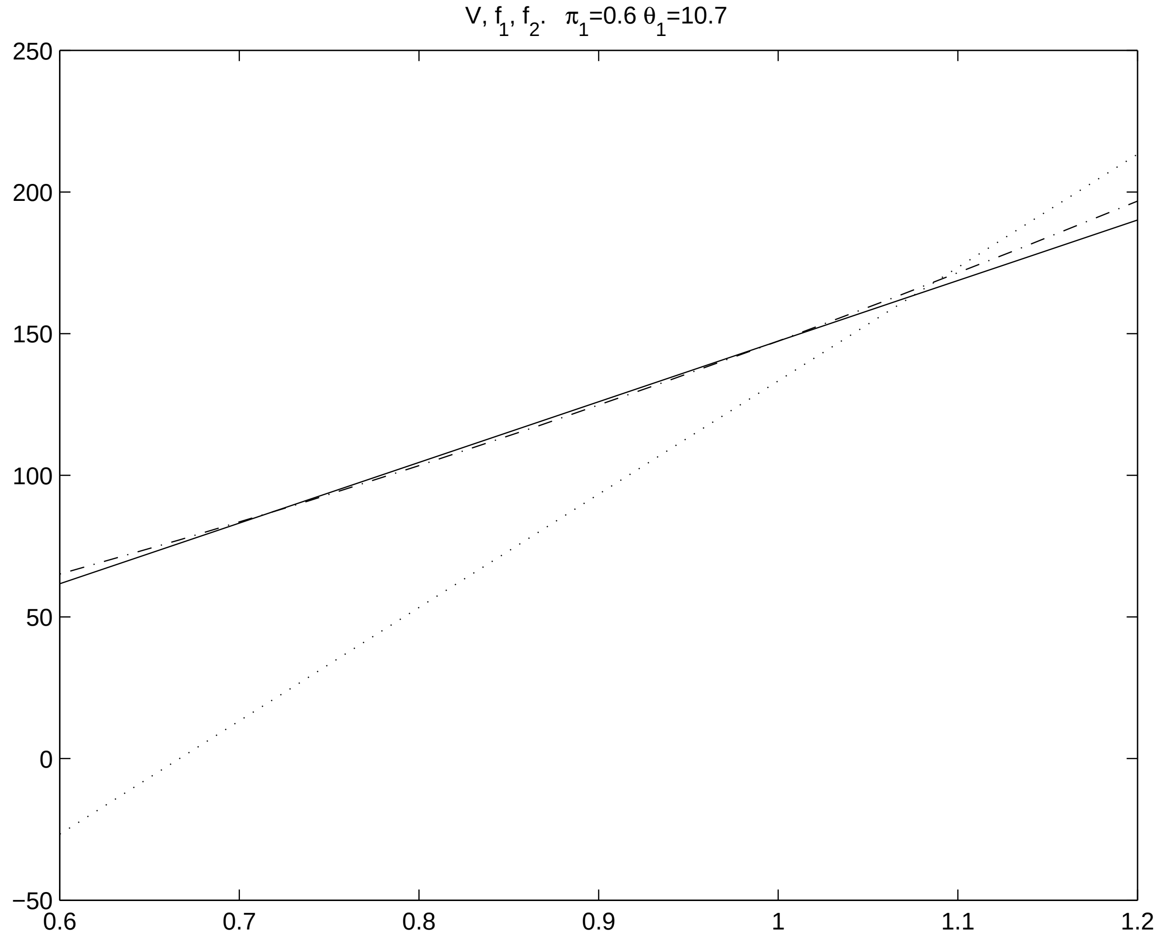

Figure 3. Scenario 3.
The dash-dotted line refers to V(x). The solid line refers to $f_1(x)$. The dotted line refers to $f_2(x)$. In this figure, $x_1^{L*} < x_1^{R*} < x_2^* < \infty$, where x_1^{L*} and x_1^{R*} are $W_1(x)$'s zero points, x_2^* is $W_2(x)$'s zero point, $W_j(x) = f_j(x) - V(x), j = \{1,2\}$.

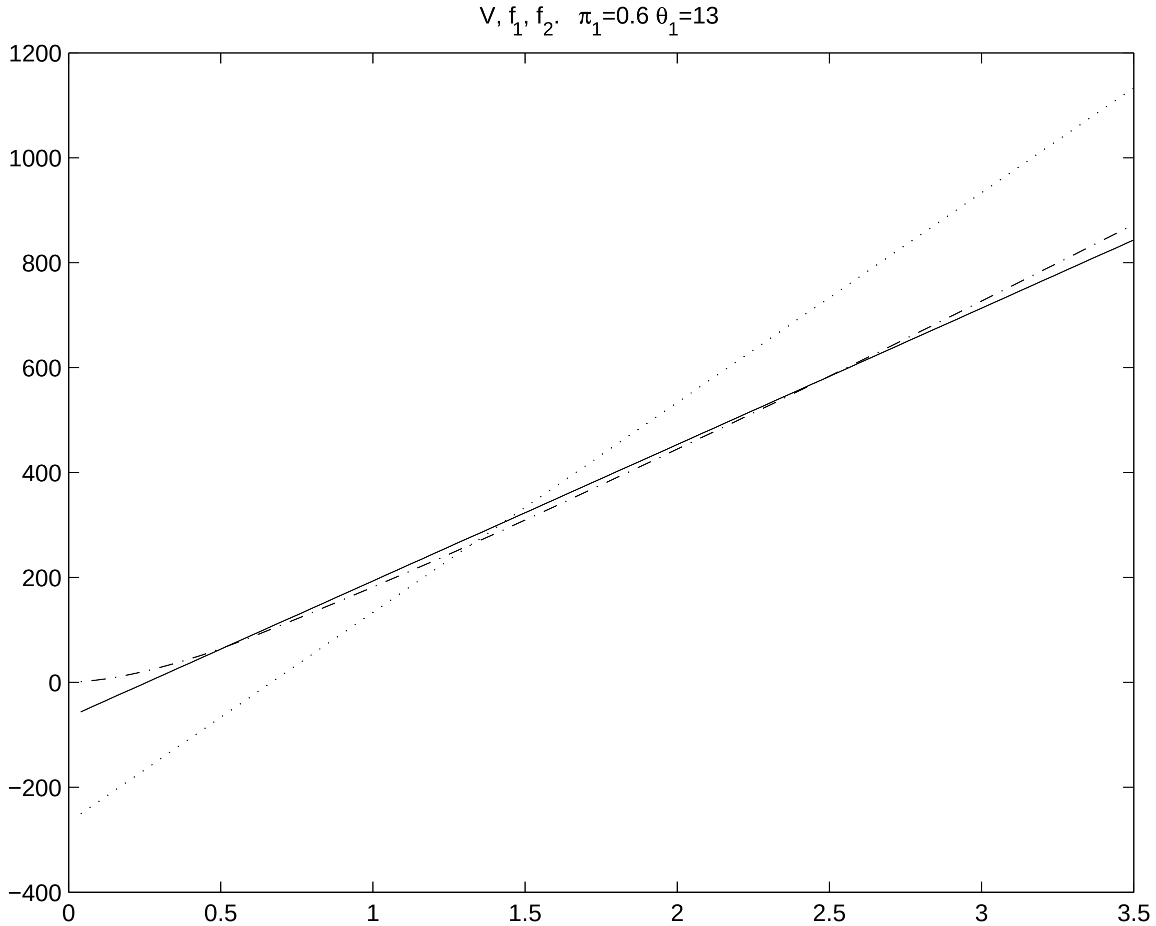

Figure 4. Scenario 4.
The dash-dotted line refers to V(x). The solid line refers to $f_1(x)$. The dotted line refers to $f_2(x)$. In this figure, $x_1^{L*} < x_2^* < x_1^{R*} < \infty$, where x_1^{L*} and x_1^{R*} are $W_1(x)$'s zero points, x_2^* is $W_2(x)$'s zero point, $W_j(x) = f_j(x) - V(x), j = \{1,2\}$.

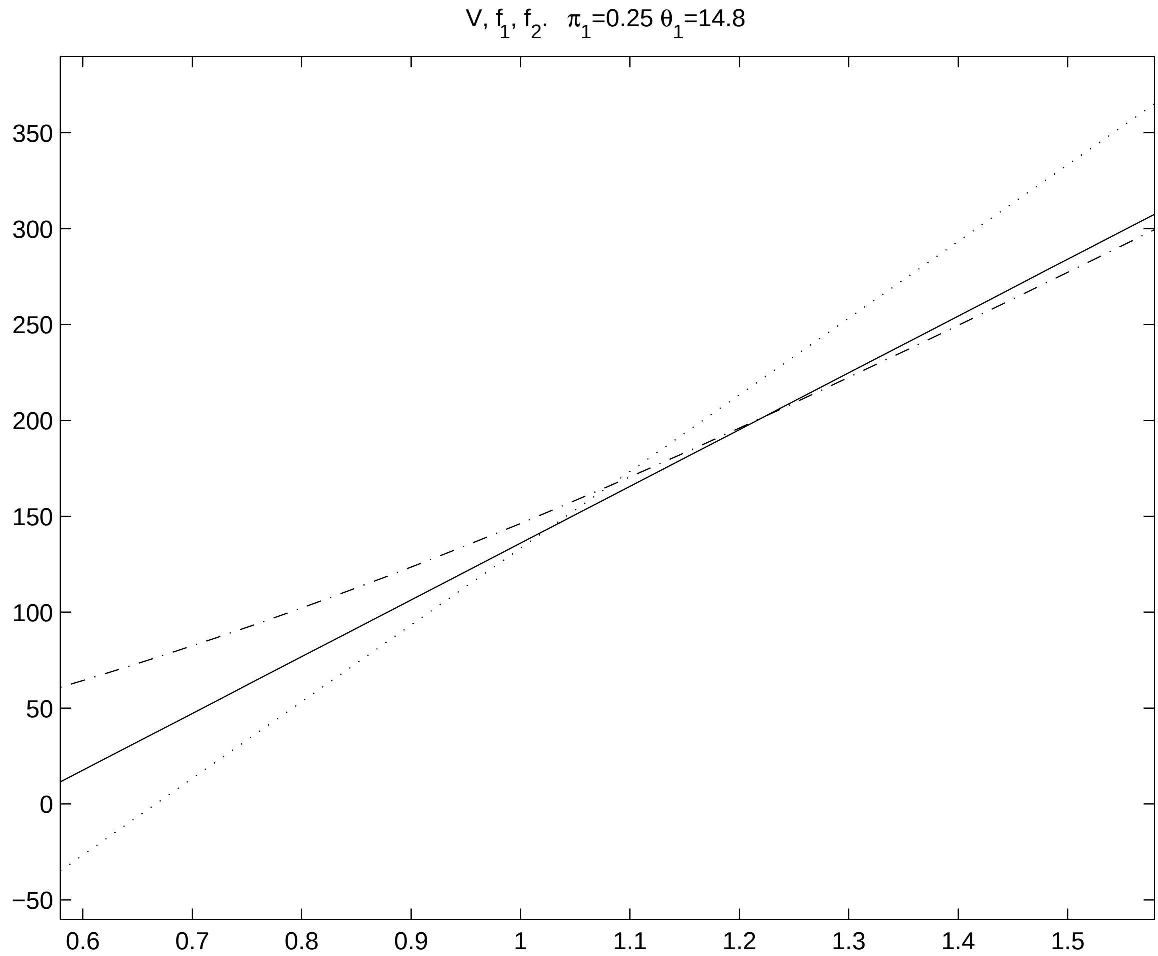

Figure 5. Scenario 5.
The dash-dotted line refers to V(x). The solid line refers to $f_1(x)$. The dotted line refers to $f_2(x)$. In this figure, $x_2^* < x_1^{L*} < x_1^{R*} = \infty$, where x_1^{L*} and x_1^{R*} are $W_1(x)$'s zero points, x_2^* is $W_2(x)$'s zero point, $W_j(x) = f_j(x) - V(x), j = \{1,2\}$.

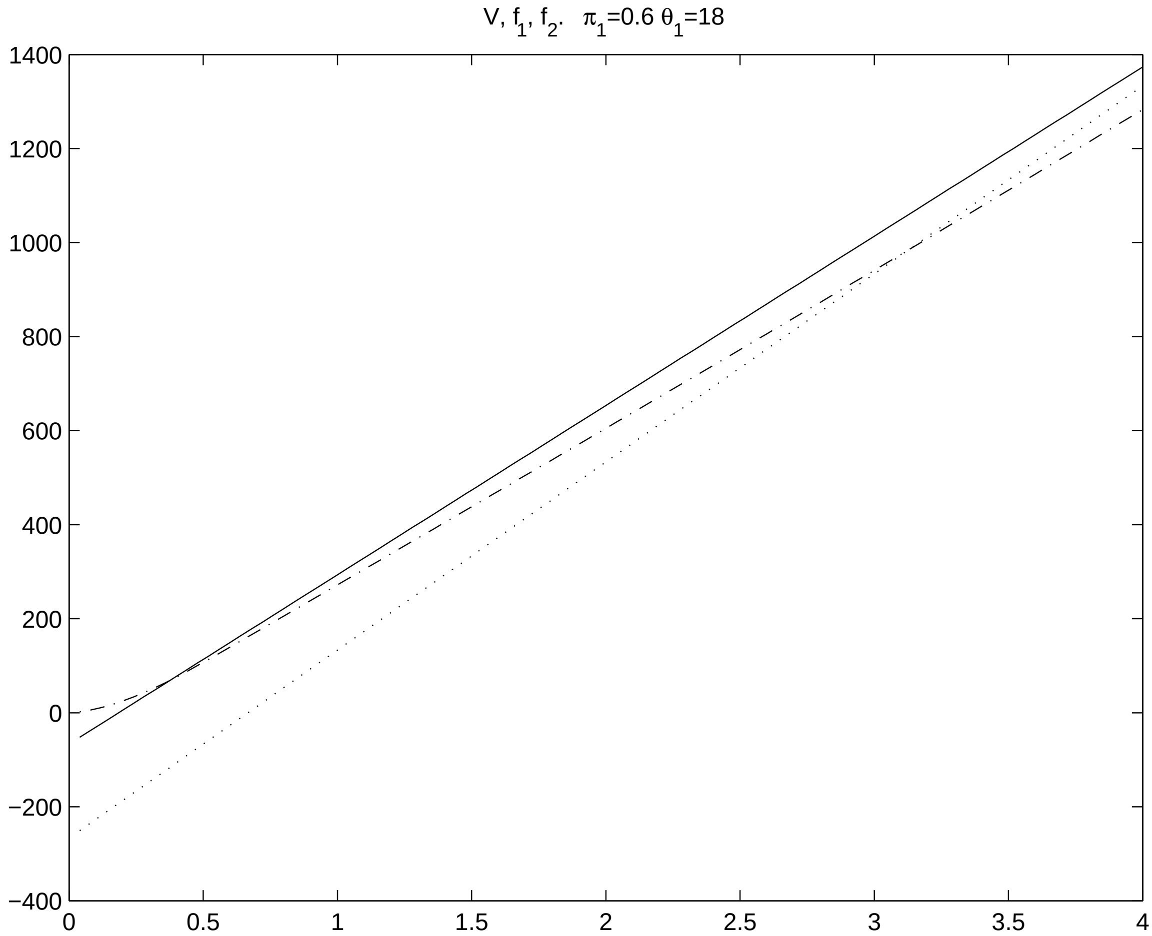

Figure 6. Scenario 6.
The dash-dotted line refers to V(x). The solid line refers to $f_1(x)$. The dotted line refers to $f_2(x)$. In this figure, $x_1^{L*} < x_2^* < x_1^{R*} = \infty$, where x_1^{L*} and x_1^{R*} are $W_1(x)$'s zero points, x_2^* is $W_2(x)$'s zero point, $W_j(x) = f_j(x) - V(x), j = \{1,2\}$.

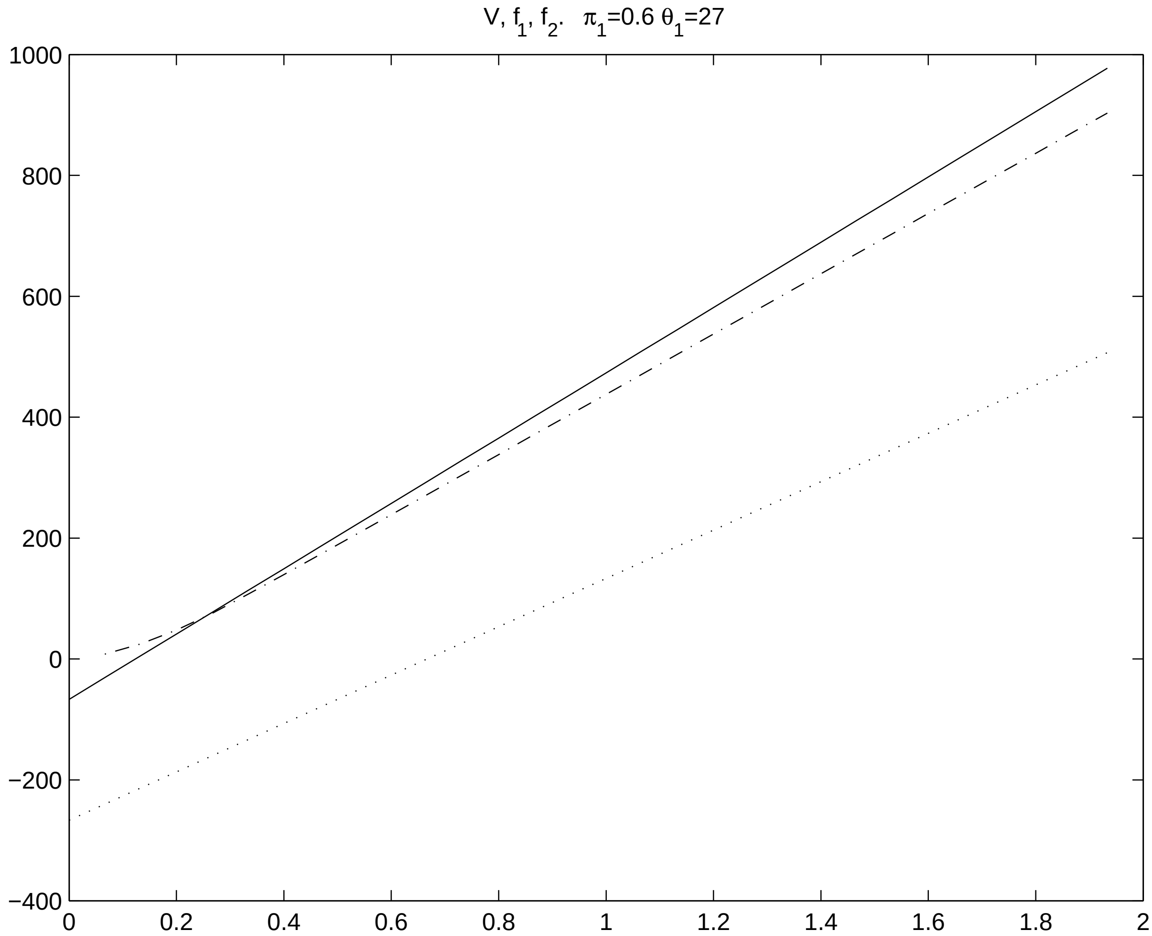

Figure 7. Scenario 7.
The dash-dotted line refers to V(x). The solid line refers to $f_1(x)$. The dotted line refers to $f_2(x)$. In this figure, $x_1^{L*} < x_1^{R*} < x_2^* = \infty$, where x_1^{L*} and x_1^{R*} are $W_1(x)$'s zero points, x_2^* is $W_2(x)$'s zero point, $W_j(x) = f_j(x) - V(x), j = \{1,2\}$.