Credible Capacity Preemption in a Duopoly Market under Uncertainty

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Abstract

This paper explores firms' incentives to engage in capacity preemption using a continuous-time real options game. Two *ex ante* identical firms can choose capacity and investment timing regarding the entry into a new industry whose demand grows until an unknown maturity date, after which it declines until it disappears. Previous literature usually predicts that the Stackelberg leader, whether endogenously or exogenously determined, is better off by building a larger capacity than its rival. In contrast, this paper proves that, under certain conditions about the demand function and the market growth rate, in equilibrium the first mover enters with a smaller capacity. If it had chosen the larger capacity, its competitor could, and in fact would use a smaller plant to force it out of the market. The result is driven by two facts: first, the large capacity firm lacks the incentive to preempt its competitor, because of its higher option value, which tends to delay its investment; second, the large firm also lacks commitment to fight for the market if its leadership is challenged by a smaller firm, because the smaller firm can credibly commit to stay in the market.

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1 Introduction

Theoretical industrial organization models have long recognized the value of first mover advantage regarding the timing of entry, which can be traced back as early as von Stackelberg (1934). By investing in a larger capacity ahead of its competitor, a firm can achieve higher profit than its follower[1]. Further, if firms are competing for the leadership in a market with uncertainty, making an earlier capacity commitment enables a firm to obtain Stackelberg leadership. In this case, the follower enjoys the value of flexibility, but its profit is generally lower than the leader. (Sadanand and Sadanand (1996), and Maggi (1996))

In real business world, however, we rarely observe leading firms engaging in large capacity preemption. In contrast, there are many cases in which either firms failed or deliberately gave up the opportunity to preempt their competitors with a large capacity. In the titanium dioxide industry, Du Pont chose not to expand its capacity to preempt its competitor, Ker McGee, even it seemed fairly reasonable[2]; In the digital video recorder (DVR) industry, Motorola decided not to produce DVR integrated set-top box until two years after the entry of its smaller competitor, Scientific Atlanta[3]; Finally in hot spot industry, Cometa Networks, a joint venture initiated by Intel, IBM, and AT&T as an acclaimed would-be industry leader in providing nation-wide wireless hot spot service, not only failed to preempt its competitor, T-mobile, but also failed to contest the first mover's leadership with a more ambitious network[4].

The difference between the theoretical prediction and business practice raises the question of why first entrants refrain from engaging in large capacity preemption while the advantages seem so compelling in theory. One probable reason is that the literature has focused on the cases in which uncertainty either does not exist or will be resolved immediately after the entry of the leader. Little is known in the case where the leader has to enter a developing market with evolving uncertainty. By introducing a new continuous-time real option game of timing, this article proves that under fairly general conditions about market demand, the leader would rather enter the market with a smaller capacity than its follower's. The reason is that to secure its market leadership, the leader not only needs to move ahead of its

[1] See Gal-Or (1985). Tirole (1988) also has nice discussion of Stackelberg leadership.

[2] See Ghematwat (1984). This has become a classical case to teach preemption to MBA students in many US business schools and the irony is that it turned out Du Pont chose not to preempt.

[3] See "Mototrola gets a digtal-video black eye." Wall Street Journal, April 15, 2004.

[4] See "Chill hits Hot Spots." Wall Street Journal, March 18, 2004.

competitor but also needs a credible commitment to stay in the market. Sinking a large amount of investment alone does not commit the leader to fight for the market. In fact, when a war of attrition broke out in a premature market, the large leader would have more incentive to abandon the market than its smaller follower. As a result, the leader in fact would enter the market with a smaller capacity.

The underlying driving force of this result lies in the evolving market uncertainty, which is modeled as an unknown market switching date. The market is originally growing but will switch to decline after a random date. The risk of sudden downturn creates an option value of delaying investment, as predicted by literature on investment uncertainty[5]. The difference, however, is that in the competition for market leadership, this option value confers a strategic advantage to the smaller firm: it enables the smaller firm to win the war of attrition in the growing market. The intuitions are as follows: first, as shown in Ghemawat and Nalebuff (1985), in the declining market a larger firm will exit earlier and leaving the smaller firm to monopolize the market. As a result, the smaller firm has more to lose if it exits the market earlier when the market is growing, which in turn strengthens the smaller firm's incentive to fight for the market. Consequently, large capacity preemption is not credible because rather than delaying the entry of the follower, it in effect would invite the smaller follower to jump into the market earlier to challenge the first mover's leadership and eventually force the larger leader out of the market by starting a war of attrition.

This paper is directly related to the literature on endogenous Stackelberg leadership. Most previous papers, however, understates the effect of market uncertainty on firms' strategic incentive to preempt by limiting the negative effect of adverse market outcomes. For example, Maggi (1996) assumes that the lower bound of the demand be large enough that a Cournot duopoly will never abandon the market in the second stage. In contrast, additional insights are provided by this paper, which focuses on modeling this situation by considering a market with unknown switching date. Unlike previous studies, the premature market switching to downturn will result in a loss on the incumbents if the market changes to decline prematurely. With this framework, the "bad news principle of irreversible investment" is applied to a game theoretic setup: the strategic effect of option value completely changes the predictions on a firm's equilibrium capacity and entry timing choices as stated in the previous papers.[6]

[5] See Dixit and Pindyck (1994) for an excellent survey.

[6] "Bad news principle of irreversible investment" was proposed by Bernanke (1983), which implies that only bad future market outcome matters in whether one should make irreversible investment today. See Ruiz-Aliseda and Wu (2004) for detail discussions.

It is also worth noting that most existing papers employ a two-stage model to explain the endogenous asynchronous entry of two firms into a new market, based on extended game with observable delay proposed by Hamilton and Slutzsky (1990)[7]. Maggi (1996) is the first paper to allow firms to choose capacity and entry timing simultaneously by dividing the game into a pre-commitment and a production stage. Because no production is allowed in the pre-commitment stage, in equilibrium only one firm becomes the *de facto* leader by making capacity commitment in the first stage, leaving the other firm to enter at the beginning of the second stage. As a result, the leadership race is reduced to a lottery on who gets to move in the first stage. By considering a continuous-time model, this paper allows firms to *compete* for the market leadership in a more realistic sense. Further, the paper shows that the first mover's advantage will vanish if firms are competing for the leadership in an emerging market where immediate large capacity investment is not feasible.

This paper is divided into seven sections. We start laying out the model in Section 2 and discuss a benchmark monopoly model in section 3. Section 4 and 5 solve the duopoly game of entry and exit. Section 6 explores possible relaxations of the assumptions and section 7 concludes the paper. Most proofs are presented in the Appendix.

2 Model

Consider a continuous-time game of entry and exit over the lifetime of a new industry with two *ex ante* identical firms.[8] As analyzed by Simon and Sinchcombe(1986), the game of continuous time is modeled as the limit of discrete time as period length goes to zero. Each firm can select its own capacity level and entry timing. The investment cost is I per unit of capacity. Once a firm sinks its investment cost, it can operate immediately and the operating cost is c per unit of capacity. Firms are free to exit the market at any time. Moreover, after exiting the market, a firm is allowed to restart its plant later by incuring the full investment cost again.[9] Specifically, let $K = \{0, q_1, q_2, ..., q_n\}$ with

[7] The main limitation of this model is that it separates firms capacity and entry timing decisions by requiring firms to announce their entry timing before making capacity choices. See, for example, Sadanand and Sadanand (1996), and van Damme and Hurkens (1999) and Hirokawa and Sasaki (2001)

[8] In this paper, we restrict the analysis to two potential entrants for several reasons: first, duopoly analysis is the first step to understand the strategic interaction in an oligopoly market. second, it is still a difficult problem to analyze firms' strategic investment in a real options framework, even if there are only two firms. Third, Fudenberg and Tirole (1985) shows that preemption games with three or more players usually involve more complexities since the rent equalization principle may not apply in all subgames.

[9] Restarting some technologies, such as Aluminum oxide, is not easy and will incur significant cost, as shown in Wells(1985). One key assumption for modeling continuous-time game as the limit of discrete time game is not to al-

$0 < q_1 < q_2 < ... < q_n$ denote a firm's possible plant sizes. At each date $t \geq 0$, firm i $(i = 1, 2)$ can choose a capacity level κ_i from its action set K_i, where K_i depends on whether the firm has invested before t. By choosing $\kappa_i = 0$, it will continue to stay out of the market, otherwise it can choose from n discrete capacity levels. If it has already been producing in the market or has entered the market before with capacity $\kappa \left(\in \{q_1, q_2, ..., q_n\}\right)$, its action set is limited to $K_i = \{0, \kappa\}$ .Action 0 means to exit the market immediately at t and κ means to stay in the market. In this paper, I assume that a firm cannot change its chosen capacity levels.[10]

Let $Q = \kappa_1 + \kappa_2$ denote the total capacity active in the market. As in Ghemawat and Nalebuff (1985) and Londregan (1990) , I assume that the firms produce at full capacity and sell all their output when active in the market.[11] This assumption also enables the model to focus on a firm's entry and exit decisions by avoiding the complicated discussion of capacity constrained competition. As a result, the short term market supply is inelastic, and the equilibrium price will vary only with demand shocks. Following Dixit and Pindyck (1994), the market price is determined by the product of two factors:

$$P(Y, Q) = Y D(Q) . \tag{1}$$

$D(\cdot)$ is a decreasing function of Q, which implies that the larger the total capacity of the active firms the lower the price. The lumpiness of capacity investment and disinvestment implies that adding or withdrawing capacity will result in a downward or upward jump of market price. The market price is also affected by a random demand shock represented by Y. To be concrete, Y represents the potential market demand, which is meant to capture the effect that the larger the size of the potential market, the higher the price given the fixed supply. The initial market size for this new product is $Y = y_0$, which grows exponentially over time at the rate of α_1. To introduce uncertainty, I assume that the market growth stops at an unknown date $\tilde{\tau}$ that is exponentially distributed with the parameter λ and density function $f(\tau) = \lambda e^{-\lambda\tau}$. Once the market stops growing, Y switches to decline at the rate of $\alpha_2 < 0$ from

low firms to exit and reentry infinite times. Costly reentry combined boundedness of a firm's financial resources are sufficient. See Simon and Stinchcombe(1986) for more details.

[10] This is for the simplicity of the model. It has two implications: first, firms are not allowed to expand or shrink its capacity during operation, otherwise the model becomes very complicated. Further, a more stringent implication is that it also implies that a firm cannot change its capacity during its reentry. Excluding this possibily allow us to focus on firms' initial entry capacity, which is also the central issue on capacity preemption and competition for Stackelberg leadership.

[11] In reality many firms, for example automakers, would rather reduce the selling price to keep the plant operating at full capacity (Mackintosh 2003). Also, due to the uncertainty of the market, firms might get stuck with their labor and raw material supply contract, which makes capacity shrinkage extremely difficult.

$Y(\tau)$, the level reached at the end of growing stage, until it disappears. In other words, the process of Y is evolving continuously except that there is a change of direction from growth to decline once τ is resolved. This uncertainty setup is different from that in previous papers on endogenous leadership, which usually assume that the demand realization could be high or low at a specific date known to both firms. In contrast, I assume that no firm knows how long the market growth will last. The switching of market demand from growth to decline is bad news to firms in the market. The uncertainty of this market switching creates option value and one main purpose of this paper is to study the effect of this "bad news" on firms' strategic investment behavior.[12]

Since firms are *ex ante* identical, either firm could become the leader. Hence I will only discuss the equilibrium in terms of leader's and follower's strategy without direct reference to their identity. However, one common issue of the continuous timing game is the coordination regarding players' making simultaneous move. I follow Dutta, Lach and Rustichini (1995), Grenadier (1996) and Weeds(2002) to make the following assumption to rule out *ex post* simultaneous entry:[13]

Assumption 1 *If both firms attempt to enter the market at the same date* t, *then only one of two firms will succeed and in this case, the probability of firm* 1 *entering first is* $p = \frac{1}{2}$.

Once the market switches to decline, the demand will decrease continuously and ultimately cause the market price to fall below operating cost. Ghemawat and Nalebuff (1985) shows that there is a continuum of equilibria when both firms are the same size. The following assumption is introduced as a tie breaking rule[14]:

Assumption 2 *After the market switches, firm* i'*s capacity is randomly reduced by* δ_i *where* δ_i *is uniformly distributed on* $\left[0, \bar{\delta}\right]$, *where* $\bar{\delta}$ *is small. The realization of* δ_i'*s are common knowledge to both firms.*

[12] As mentioned by Bernanke (1983), firms' benefit from delaying investment comes from the avoidance of possible loss due to the occurrence of an unfavorable event, which he calls the "Bad News Principle of Irreversible Investment". My paper studies the linkage of this principle with firm's strategic capacity choice.

[13] This assumption might be the result of some institutional features; for example, to make the investment, the firms need to obtain a license, and the business license office randomly picks one of two firms to award the license at a time. This assumption can also be the result of random delay in the execution of investment decisions. For example, it can be implemented by the following mechanism: if two firms select the same entry timing t, their actual entry timing is actually $t + z$, where $z \in [0, \varepsilon]$ with ε small. Therefore, *ex post,* one firm always sees its competitor entering first. In this case, after updating its information immediately, it can choose to enter anyway to contest the first entrant's market leadership or to wait for its optimal entry date as a follower.

[14] Without this tie-breaking rule, I have to introduce mixed strategy equilibrium. It can be shown that as long as firms employ symmetric mixed strategy, the result of the paper still hold. So this assumption is for the simplicity of the paper. See Wu(2005) for more details.

This assumption is used to purify the equilibria when firms are of the same sizes so that we can clearly characterize their equilibrium payoff in the declining stage.

Finally, I also assume $\int_0^\infty f(\tau)(y_0 e^{\alpha_1 s} D(\kappa) - c)\kappa e^{-rs} ds < \infty$, for all $\kappa = q_1, q_2, ..., q_n$, which is used to guarantee the integrability of firm's expected payoff.[15] A sufficient condition for this is $r > \alpha_1 - \lambda$.

In this paper, I employ the solution concept of (pure strategy) Markov perfect equilibrium, which restrict the analysis to those strategies as a function of its payoff relevant state. The payoff relevant state is defined by an array of variables: $h_t = \{\kappa_t^1, \kappa_t^2, \omega\}$, where t is the current date, κ_t^i denotes firm $i's$ capacity at date t, and ω denotes whether the market has switched or not.

Numerically it is not difficult to derive the equilibrium of the above general model. In fact, a computed example is presented in Section 6. However, it is a daunting task to give an analytic solution to the model without further assumptions. Therefore, in the following discussion, I will introduce some simplifying assumptions and relax those assumption in Section 6. First, since the main question of this paper is whether a firm will be able to preempt its competitor credibly by choosing a larger capacity, it is sufficient to consider the case of two choices of plant sizes:

Assumption 3 *Firms can choose two sizes of plants: small (S) or large (L), with $S < L$.*

As above-mentioned, market switching from growth to decline means "bad news" for firms, because a firm's expected profit will decrease even if the demand is evolving continuously. Ruiz-Aliseda and Wu (2004) proves that even we allow demand Y to evolve continuously, there is still a value jumping down at the date when market switches. The following assumption further emphasize the risk of market switching by introducing a possible market crash at the switching date:

Assumption 4 *$L \leq 2S$ and the declining stage starts with demand level $Y = \bar{y}$, where $\bar{y} = \frac{c}{D(2S)}$.*

This assumption implies that once the declining stage starts, there is no room for two firms since the declining stage will start with a demand level $\bar{y}$ such that $P(\bar{y}, 2S) - c = 0$. Figure 1 shows a sample path of market evolution given this assumption, which implies that the continuation payoff associated to a declining industry is not contingent on the realization of a market switching date. However, the date $\tilde{\tau}$ at which this continuation payoff is realized is still uncertain. This assumption facilitates the tractability of the model, which allow us to provide an analytical solution of the model.[16]

[15] Note that this integral is the expected payoff in the growing stage for a monopoly firm that invests at date 0.

[16] This assumption simply divides the process of market evolution into two independent processes. As a result, there is

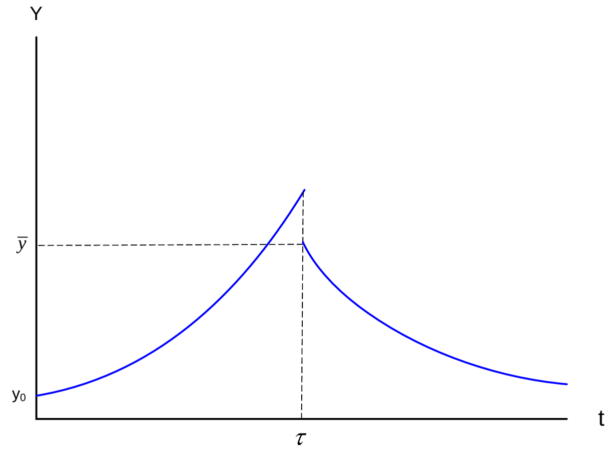

Figure 1: A sample path of market evolution

Before solving the model, let me define some notations. First, $L(t, \mu|\kappa)$ denotes the leader's expected payoff when it chooses capacity μ to enter at date t and the follower selects capacity κ. $F(t, \kappa|\mu)$ is defined similarly as the follower's payoff with capacity κ while the leader chooses μ. Also, let $M(t, \kappa)$ represent the expected payoff for a monopoly firm with capacity κ entering at t[17]. On the other hand, I use $\bar{t}$ and $\underline{t}$ to denote the leader's and follower's entry dates, respectively. Similarly, $\bar{t}_{\mu|\kappa}$ denotes the entry time for a *leader* with capacity μ when its *follower* chooses capacity κ, and $\underline{t}_{\kappa|\mu}$ represents the entry date of a *follower* with capacity κ when the *leader* picks capacity μ. Similarly, the monopoly entry date for a firm with capacity κ is denoted as t_{κ}^{m}. Finally, proofs are presented in the appendix if not in the main text.

3 Monopoly Entry and Exit Decisions

I first present some results regarding a monopoly firm's entry and exit decisions in this market, which provides a benchmark case that is useful for later discussions. This case also introduces the concept of marginal option value, which is a critical concept that affects a firm's investing behavior in this paper.

Consider a firm owning a monopoly right to invest in this new market. Since the market ultimately will switch to decline, it is natural to start with its exit decisions in the declining stage. Suppose that

also a possibility that the process might jump up if Y switches before it reaches $\bar{y}$. Actually for large enough investment cost, firms will not invest before Y reaches $\bar{y}$. Although there is a loss of generality with this assumption, it helps us to present some important results of market structure, which is not possible unless we resort to numerical methods.

[17] Note that $M(t, \kappa) = L(t, \kappa|0) = F(t, \kappa|0)$, because monopoly can be consider as a special case when your competitor choose not to enter the market.

when the declining stage begins, it has already entered the market with capacity κ. As a result, it picks t_κ^x as its exit date counting from the market switching date τ. Because there is no salvage value and no exit cost, it will not exit the market until the price falls below the production cost c. In this case, its continuation payoff in the declining stage is

$$W_\kappa^m = \int_0^{t_\kappa^x} \left(P\left(Y\left(s\right), \kappa\right) - c\right) \kappa e^{-rs} ds.$$

Note that $P\left(Y\left(s\right), \kappa\right) = \bar{y} e^{\alpha_2 s} D\left(\kappa\right)$, so the firm's monopoly exit date, t_κ^x, is the solution of the following equation

$$\bar{y} e^{\alpha_2 t_\kappa^x} D\left(\kappa\right) - c = 0 \tag{2}$$

The following lemma summarizes this monopoly firm's exit decision:

Lemma 1 *(i)If the market switching date occurs at τ and there is only one firm with capacity κ in the market, then it will exit the market at $t^x\left(\tau, \kappa\right) = \tau + \frac{1}{\alpha_2} \ln \frac{D(2S)}{D(\kappa)}$, $\kappa = S, L$. In particular, $t^x\left(\tau, S\right) > t^x\left(\tau, L\right)$.*

(ii) $\frac{W_S^m}{S} > \frac{W_L^m}{L}$, i.e., a smaller monopoly firm has a higher continuation payoff per unit of capacity invested.

Lemma 1 establishes that a smaller monopoly firm will be able to operate longer than a larger monopoly firm when the industry is declining, because for the same demand shock Y, the market price is higher for a firm of smaller capacity. As a result, when the market matures, it takes longer for the price to drop below the production cost if the firm is of smaller capacity than if it has a large capacity. Furthermore, the continuation payoff per unit of capacity for a monopoly firm with smaller capacity in the declining stage is greater than that for a firm with larger capacity.

It is clear that if $\frac{W_S^m}{S} > I$, the smaller firm is guarnteed positive payoff in this market, which allows the firm choosing small size S to enter the market immediately at date 0. To prevent this corner solution, in the remaining discussion I exclude the entry in the declining stage with following assumption:

Assumption 5 $\frac{W_S^m}{S} < I$.

Finally, the following lemma characterizes a monopoly firm's entry decision:

Lemma 2 *(i) A firm with capacity κ will choose to enter the market at t_κ^m, where t_κ^m satisfies*

$$P\left(Y\left(t_\kappa^m\right),\kappa\right)-c=rI+\lambda\left(-\frac{W_\kappa^m-I\kappa}{\kappa}\right) \tag{3}$$

(ii) $t_S^m<t_L^m$.

The monopoly firm is able to pick its entry date according to a rule of marginal cost equal to marginal revenue. The left hand side of Equation (3) is the marginal cost of waiting to invest, which is the instantaneous profit per unit of capacity lost by staying out. The right hand side represents the marginal value of waiting, which consists of two parts: rI is the per unit investment cost saved by delaying entry into the market and $\lambda\left(-\frac{W_\kappa^m-I\kappa}{\kappa}\right)$ is the marginal option value of waiting. To see why, observe that for each instant, there is probability λ that the market switches to decline, the occurrence of which will result in a continuation payoff of W_κ^m for the monopoly firm with capacity κ. By our assumption, $W_\kappa^m-I\kappa$ is negative, which implies that the firm will incur a loss when the market switches. However, if the firm delays investment by one instant, it will be able to avoid this loss by staying out of the market because once the market switches it will stay out of the market forever with a payoff of zero. As a result, this potential loss avoided multiplied by the probability of occurrence λ is exactly the marginal option value of waiting. Given this entry timing rule, a monopoly firm will pick the capacity level that generates highest expected payoff and enter the market at the optimal entry date specified by Lemma 2.

4 Duopoly exit in the declining industry

In the previous section, I discussed a benchmark case with only one entrant, in which this firm chooses an optimal entry date that equals the marginal cost to marginal value of investment. In particular, the marginal value of investment consists an option value component, which tends to delay entry. In the next three sections, I will extend this framework to the duopoly case, with the purpose to study the effect of this marginal option value on firms' strategic entry behavior.

Depending on whether the market has matured or not, the model is naturally divided into two phases, the growing industry and the declining industry. As a result, I solve the game backwards and start with the declining industry.

Suppose the market has switched from growth to decline. There are two cases to consider: monopoly and duopoly. The monopoly case is the one when the second firm did not enter the market before market switches. In this case, because there is only one firm in the market, it will exit the market at its monopoly exit date as derived in Lemma 1 since the other firm will not enter the declining market. If both firms are in operation before the decline begins, each firm will have to choose its exit date because the industry will ultimately become unprofitable. Depending on whether the firms are the same size or not, there are two cases to consider. The following proposition characterizes the perfect equilibrium of this subgame based on the sizes of the two firms.

Proposition 1 *Suppose there are two firms in operation when the market switches at date τ,*

(i) if the firms are of different sizes, the larger firm will exit immediately at date τ while the smaller firm will exit at its monopoly exit date $t^x(\tau, S)$.

(ii)if both firms are of the same size, as $\bar{\delta} \to 0$, one firm will exit the market immediately and the other firm will stop operation at its monopoly exit date.

When firms are asymmetric, it is always the larger firm that exit market first, which is a well-known result first presented in Ghemawat and Nalebuff (1985). No firm is willing to operate at a loss once the declining industry starts, while at the same time, each firm wants its competitor to leave the market so that it can stay until its monopoly exit date. If the firms' sizes are different, the smaller firm has a credible threat to outlast the larger firm because as a monopoly, the smaller firm can operate longer than the larger firm by Lemma 1. Realizing this, in a subgame perfect equilibrium, the larger firm had better exit the market at the duopoly exit date to avoid any losses. By Assumption 4, once the declining industry starts, there is no room for the coexistence of two firms, hence the larger firm will exit immediately once the market matures.

However, if both firms have the same capacity, Assumption 2 introduces a perturbation of the firms' sizes by randomly shrinking each firm's capacity by a small amount. As discussed in Ghemawat and Nalebuff (1985) , this purturbation will purify continuum of mixed strategy equilibrium. As a result, *ex post* one firm always realizes that it will never be able to outlast the other firm in the war of attrition, and thus it is in its best interest to leave the market immediately. The following corollary characterize firms' continuation payoff in the declining stage:[18]

[18]Note that in the declining stage, the firms' sizes are slightly different from their entry sizes as a result of the random

Corollary 1 *If two firms with different sizes are in the market when the declining stage starts, let $W(\kappa|\mu)$ denote the continuation payoff of a firm with capacity κ if its competitor has capacity μ. Thus,*

$$\begin{aligned} W(L|S) &= 0 \\ W(S|L) &= \int_0^{t_S^x} \left(\bar{y}e^{\alpha_2 s} D(S) - c\right) S e^{-rs} ds = W_S^m \end{aligned}$$

If the declining stage starts with two firms of the same size,

$$\begin{aligned} W(L|L) &= \frac{1}{2} W_L^m, \textit{ if both firms are of size } L \\ W(S|S) &= \frac{1}{2} W_S^m, \textit{ if both firms are of size } S. \end{aligned}$$

Assumption 2 is actually a tie breaking rule. Without this assumption, there is a continuum of mixed strategy equilibria as shown by Ghemawat and Nalebuff (1985), one of which is the symmetric mixed strategy equilibrium in which two firms each choose a distribution of exit dates between market maturity date and monopoly exit date and the choice of this distribution will completely dissipates the rent of all future possible monopoly profits because firms must be indifferent between staying and exiting.[19]

5 Duopoly entry in the growing industry

Given firms' exit strategy in the declining stage, we can analyze firms' entry strategies in the growing industry. By Assumption 5, the expected profit from the declining stage alone is not enough to cover the investment cost, which implies entry only occur before market switches. Because two firms are *ex ante* identical, either firm could preempt the other. Once a firm observes the entry of its competitor before it makes any move, it has two choices: either accepts its position as the follower or jumps into the market to contest the first entrant's leadership by forcing it out of the market. In previous studies of endogenous leadership, leader never concerns the possibility of being forced out by the follower because

capacity reduction of $\delta_i' s$. However, I assume $\bar{\delta}$ to be small so that the payoff change due to this slight size change is ignored. Thus I abuse the notation here by ignoring the small changes of firms' continuation payoff due to the tiny random shrinkage of firms' sizes.

[19] See Anderson, Goeree and Holt (1998), Fudenberg and Tirole (1986). In fact, without Assumption 2, the results of this paper still hold if we require firms to choose this symmetric mixed strategy equilibrium in the decline phase. Further, there many asymmetric equilibria but in this paper, I will not consider those asymmetric equilibria because it will give one firm ex ante advantage in the declining stage.

competition for market leadership ends when one firm enters the market. By allowing the follower contest the first entrant's leadership, I divide the analysis of firms' entry behavior into three steps. First, I assume that the follower takes the entry of the leader as a *fait accompli*, which means that it does not try to contest the leader's position. With this assumption, I analyze the follower's entry decision given the leader's capacity and entry timing choice. Then I proceed to analyze of leader's best choice of size and entry date given the prediction of follower's best response, which will identify the potential equilibrium. Finally, I check whether the leader's strategy is indeed immune to the possible contest by a follower in this equilibrium.

5.1 Optimal entry date as a follower

Suppose the current date is t_0 and one firm has already entered the market with capacity μ. If the follower chooses capacity κ to enter the market at $t \geq t_0$, it will achieve the following expected payoff, which is discounted back to date 0 :

$$F(t, \kappa|\mu) = \int_t^{\infty} f(\tau) \begin{pmatrix} \int_t^{\tau} \left[P(Y(s), \mu+\kappa) - c\right] \kappa e^{-rs} ds \\ +e^{-r\tau} W(\kappa|\mu) - I\kappa e^{-rt} \end{pmatrix} d\tau. \tag{4}$$

Note that the follower will be able to enter the market if and only if the market switching date τ is later than the entry date t chosen by the follower. In particular, the follower's value function consists of three parts: $\int_t^{\tau} \left[P(Y, \mu+\kappa) - c\right] \kappa e^{-rs} ds$ is the expected discounted profit from the duopoly growing market until the maturity date. The second part, $e^{-r\tau} W(\kappa|\mu)$, is the discounted continuation payoff associated with the declining stage, and the third part $I\kappa e^{-rt}$ is the total investment cost properly discounted.

Given the leader's capacity choice, if the follower selects capacity κ, the following proposition gives its best entry timing.

Proposition 2 *Let t_0 be the current date and the leader has entered the market with capacity κ.If the follower chooses capacity κ, taking the leader's position as fait accompli, then it will enter the market at date is $\underline{t}_{\kappa|\mu} = \max\{t_0, t^f\}$, where t^f satisfies*

$$P\left[Y\left(t^f\right), \mu+\kappa\right] - c = rI + \lambda\left(-\frac{W(\kappa|\mu) - I\kappa}{\kappa}\right) \tag{5}$$

The follower's entry decision rule is similar to that of a monopoly firm discussed in Section 3 except that the firm is a duopoly now. The left hand side of Equation (5) is the marginal cost of waiting to invest, which is the instantaneous profit per unit of capacity. The right hand side represents the marginal value of waiting to invest, consisting of two parts: rI is the per-unit investment cost saved by delaying entry into the market, and the second part is the marginal option value of waiting. For each instant there is probability λ that the market switches to decline, which will render a payoff of $\frac{W(\kappa|\mu)}{\kappa}$ per unit of capacity and it is smaller than per unit cost of investment I. Hence $-\left(\frac{W(\kappa|\mu)}{\kappa} - I\right)$ is positive and represents the loss avoided in the event that the market suddenly begins to decline. As a result, this potential loss avoided multiplied by λ, the probability of market switching in the next instant, becomes the marginal option value of waiting.[20]

Define $\phi(\kappa|\mu) = -\lambda\left(\frac{W(\kappa|\mu)}{\kappa} - I\right)$, which is the marginal option value per unit of capacity for a firm choosing κ while its competitor selects capacity μ. The following corollary shows a relationship of follower's optimal entry date in two cases where the resulting duopoly market structure is the same.

Corollary 2 $\underline{t}_{S|L} \leq \underline{t}_{L|S}$, *i.e., if the leader chooses large capacity and the follower chooses small capacity, the follower delays its investment less than if the leader chooses a small capacity and the follower chooses a large capacity.*

Proof. Since $\phi(L|S) > \phi(S|L)$, we must have

$$\underline{t}_{S|L} = \frac{1}{\alpha_1} \ln \frac{rI + c + \phi(S|L)}{y_0 D(L+S)} < \frac{1}{\alpha_1} \ln \frac{rI + c + \phi(L|S)}{y_0 D(S+L)} = \underline{t}_{L|S}$$

■

When the resulting duopoly market structure is the same: one large firm and one small firm, the option value effect is the sole driving force in the difference in entry timing. In particular, when the follower chooses smaller capacity, it can enter the market earlier than if it chooses the larger capacity to enter the market. This implies that in this uncertain, large capacity preemption might achieve just the opposite: inviting the follower coming into the market earlier, even if the follower has no intension to contest the first entrant's leadership. The smaller option value of waiting for the smaller firm gives

[20] Note that this formula is actually the option value of waiting on the per unit capacity basis, which is obtained by dividing total loss avoided by the capacity level. Also, the monopoly entry date t_κ^m derived in Lemma 2 is actually a special case of $\underline{t}_{\kappa|\mu}$, as if the leader selects capacity $\mu = 0$.

it more incentive to invest earlier. This will have an effect on leader's entry capacity choice. Before moving on the analysis on the leader's entry strategy, I first summarize the follower's optimal strategy. Given the follower's optimal entry timing, we are ready to study its optimal capacity choice. It turns out that given the leader's capacity choice of μ, the follower's optimal capacity choice depends on a threshold $\alpha_\mu = \frac{(\lambda+r)\left[\ln D(\mu+S)-\ln D(\mu+L)-\ln \underline{\eta}^\mu\right]}{\ln L-\ln S-\ln \underline{\eta}^\mu}$, where $\underline{\eta}^\mu = \frac{rI+c+\lambda\phi(S|\mu)}{rI+c+\lambda\phi(L|\mu)} < 1$ and $\mu = S, L$. In fact, $\underline{\eta}^\mu$ can be viewed as an option value factor, because it compares the investment cost with the incorporated marginal option value being the sole difference. In particular, if the market growth rate is $\alpha_1 < \alpha_\mu$, the follower will choose small capacity S; otherwise it will select large capacity L if $\alpha_1 \geq \alpha_\mu$. The relationship between α_S and α_L is generally ambiguous without the further assumption about the demand function $D(Q)$. The following lemma gives a sufficient conditions for $\alpha_S < \alpha_L$ when the demand is linear[21]:

Lemma 3 *Let $D(Q) = a - bQ$. If $2L + S > \frac{a}{b} > 2S + L$, then $\alpha_S < \alpha_L$.*

In the following discussions, I will concentrate on the case of $\alpha_S < \alpha_L$, because in this case the threshold of choosing a large capacity is higher when the leader engage is larger, which is consistent with previous literature in the sense that large Stackelberg leader induce the follower to smaller.

Proposition 3 *Let the current date be $t_0 < t_S^m$ and suppose $\alpha_S < \alpha_L$. Given Assumptions 1-5, if $\alpha_1 \leq \alpha_S$, then the follower has a dominant strategy to choose small capacity. If $\alpha_L > \alpha_1 > \alpha_S$, the follower will choose a capacity level different from the leader's choice. If $\alpha_1 > \alpha_L$, the follower will always choose large capacity.*

Proposition 3 establishes a linkage between the follower's optimal capacity choice and the market growth rate. Generally speaking, given a particular market price, choosing a larger capacity might result in greater profit because the revenue is increasing in capacity. However, in this model, this is not always the case because if the follower chooses a large capacity, he will delay his entry into the market. As a result, its expected payoff from large capacity might be less than the payoff from the small capacity because of the additional waiting. Proposition 3 shows that if the market growth rate is slow, the follower would enter the market earlier by choosing a small capacity. If the growth rate is intermediate, the follower would choose its capacity depending on the leader's choice. In this case, although the follower might prefer to choose large capacity, it might be forced to choose small capacity

[21] The proof also provides a necessary and sufficient condition for $\alpha_S < \alpha_L$.

if the leader chose large size strategically. Finally, if the market is growing really fast, the follower will choose a large capacity no matter what capacity the leader chooses.

5.2 Leader's Entry Date and Capacity Choice

Once one firm invests, the other firm either accepts its follower position and adopt a best response as lay out in Propostition 3, or contests the first entrant's leadership by following the first entrant immediately in order to force the leader out of the market. This leadership contest is more likely to succeed when the the market is premature and the demand is too small to support two firms. In this case, the leadership contest results in a war of attrition. The first entrant might be induced to exit the market and (possibly) come back later to avoid the losses from the war of attrition. Nevertheless, if this were the case, the leader should not have entered the market in the first place. This immediately implies that the leadership contest will never happen along the equilibrium path. Therefore, the analysis of the leader's entry strategy is divided into the two steps: in the first step, we discuss the leader's capacity choice and entry timing assuming the follower not challenging the leader's position. Once we identify the potential equilibrium, we move on to discuss whether this equilibrium action can indeed survive a contest.

Suppose the leader chooses capacity κ and the follower selects capacity μ, where $\kappa, \mu \in \{S, L\}$. In previous subsection, I show that $\underline{t}_{\mu|\kappa}$ is the optimal entry date for a follower with capacity μ if the leader has chosen κ. If the leader enters later than $\underline{t}_{\mu|\kappa}$, the follower will invest right after the leader's entry, otherwise the leader will enjoy a period of monopoly before $\underline{t}_{\mu|\kappa}$. Let t denote the leader's entry date and the leader's payoff $L(t, \kappa|\mu)$ is

$$L(t,\kappa|\mu) = \begin{cases} \begin{array}{c} \int_t^{\underline{t}_{\mu|\kappa}} f(\tau) \left[\begin{array}{c} \int_t^{\tau} (P[Y(s),\kappa]-c)\,\kappa e^{-rs}ds \\ -e^{-rt}I\kappa + e^{-r\tau}W_\kappa^m \end{array} \right] d\tau \\ +\int_{\underline{t}_{\mu|\kappa}}^{\infty} f(\tau) \left[\begin{array}{c} \int_t^{\underline{t}_{\mu|\kappa}} (P[Y(s),\kappa]-c)\,\kappa e^{-rs}ds - e^{-rt}I\kappa \\ +\int_{\underline{t}_{\mu|\kappa}}^{\tau} (P[Y(s),\mu+\kappa]-c)\,\kappa e^{-rs}ds \\ +e^{-r\tau}W(\kappa|\mu) \end{array} \right] d\tau \end{array} & if\ t < \underline{t}_{\mu|\kappa} \\ \int_t^{\infty} f(\tau) \left[\begin{array}{c} \int_t^{\tau} (P[Y(s),\mu+\kappa]-c)\,\kappa e^{-rs}ds \\ -e^{-rt}I\kappa + e^{-r\tau}W(\kappa|\mu) \end{array} \right] d\tau & if\ t \geq \underline{t}_{\mu|\kappa} \end{cases} . \quad (6)$$

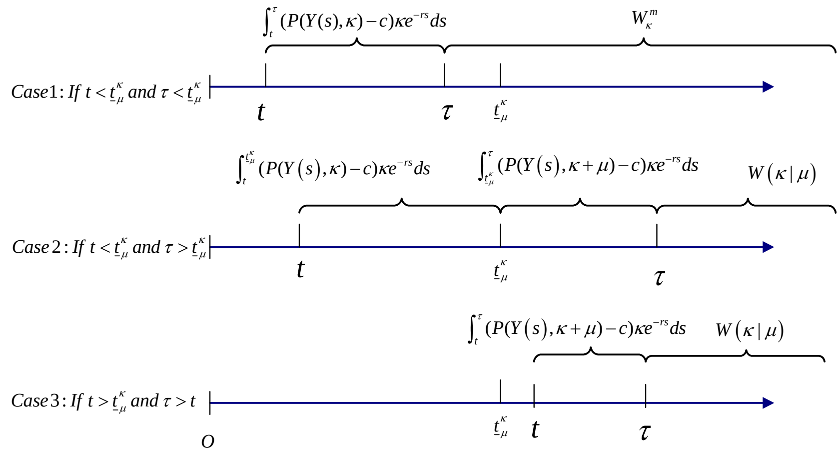

Figure 2: Leader's Payoff

Figure 2 illustrates the components of the leader's payoff defined in Equation (6). In the first case, the leader enters the market before the market maturity date τ is realized, and the market switches before the follower's optimal entry date, *i.e.*, $\tau < \underline{t}_{\mu|\kappa}$. In this case, the leader will monopolize the market until its exit and receive a payoff of two parts: a discounted profit stream, $\int_t^\tau (P[Y(s), \kappa] - c)\,\kappa e^{-rs}ds$, before the market switches, and a discounted monopoly continuation payoff W_κ^m in the declining stage[22]. In the second case, if the market matures later than $\underline{t}_{\mu|\kappa}$, the leader's monopoly position ends at $\underline{t}_{\mu|\kappa}$ due to the follower's entry. So its payoff after $\underline{t}_{\mu|\kappa}$ consists of two part: $\int_{\underline{t}_{\mu|\kappa}}^\tau (P[Y(s), \kappa+\mu] - c)\,\kappa e^{-rs}ds$, monopoly profit before the market switches and a duopoly continuation payoff of $W(\kappa|\mu)$. This case is characterized in the middle of Figure 2. Finally, case 3 considers the situation that the leader chooses $t \geq \underline{t}_{\mu|\kappa}$. If so, the follower will enter the market immediately with capacity μ as long as its payoff is nonnegative. In this case, the leader will be earning its duopoly profit immediately after its entry; *i.e.*, its payoff as a leader equals the profit it would have earned had it entered as a follower with capacity κ while its opponent enters earlier with capacity μ. Hence for $t > \underline{t}_{\mu|\kappa}$, $L(t, \kappa|\mu) = F(t, \kappa|\mu)$.

Proposition 3 implies that the follower's optimal entry date will depend on the demand growth rate α_1. With the assumption $\alpha_S < \alpha_L$, there are three cases to consider. If $\alpha_1 < \alpha_S$, the follower always chooses small capacity and if $\alpha_1 > \alpha_L$, the follower always builds a large plant regardless of the leader's capacity choice. The most interesting case is $\alpha_S < \alpha_1 < \alpha_L$, because if the leader chooses small capacity

[22] W_κ^m is derived in Section 3.

S, the follower will select L,and if the leader chooses large capacity L the follower will choose S. As a result, in the case of $\alpha_S < \alpha_1 < \alpha_L$, the leader might be able to preempt the follower with a large capacity and thus force the follower to become a small competitor, which is the result usually predicted in previous studies. However, this might not be the case if we account for the effect of option value on firm's strategic incentive. As a result, the discussion of firms' equilibrium strategies are divided into two subsections: we start with analyzing the equilibrium without leadership contest by presenting two lemmas, after which, we introduce the possibility of leadership contest and demonstrate the effect of option value on firms' strategic preemptive incentives.

Equilibrium without Leadership contest

Because $\alpha_S < \alpha_1 < \alpha_L$, the leader and follower always choose different capacities. From Equation (6), we know that the graph of the leader's payoff function $L(t,\kappa|\mu)$ contains two concave curves linked at $\underline{t}_{\mu|\kappa}$. For $t < \underline{t}_{\mu|\kappa}$, $L(t,\kappa|\mu)$ is first increasing in t until it reaches t^m_κ, the monopoly entry date for a firm with capacity κ[23]. If $t > \underline{t}_{\mu|\kappa}$, $L(t,\kappa|\mu)$ coincides with the payoff of a firm with capacity κ following a leader with capacity μ. Figure 3 shows the leader's and follower's payoffs as a function of the leader's entry date. If the leader chooses L and the follower picks S, curve $CEFGL$ maps the payoff function for the large leader, $L(t,L|S)$. Note that this curve contains two parts, the first part (CEF) is the payoff when the large leader enters before the follower's optimal entry date $\underline{t}_{S|L}$, which reaches its maximum at t^m_L, the monopoly entry date for a large firm. The second part (FGL) is the leader's payoff if it enters later than $\underline{t}_{S|L}$, which is less because its competitor will enter immediately after the leader's entry. In this case, the leader will not earn monopoly profit but will earn duopoly payoff after date $\underline{t}_{S|L}$, which is overlapped with the payoff of a follower with capacity L after a leader with capacity S. The second part of the leader L's payoff is maximized at $\underline{t}_{L|S}$, which is greater than $\underline{t}_{S|L}$ as shown in Corollary 2, so the leader L's payoff has a kink at F. Similarly, curve $BDJK$ is the payoff for a leader with capacity S when its follower selects capacity L.

[23] $L(t,\kappa|\mu)$ can be rewritten as $L(t,\kappa|\mu) = L(t,\kappa|0) - A(\kappa|\mu)$,where $L(t,\kappa|0) = \int_t^\infty f(\tau)\left(\int_t^\tau (y_0 e^{\alpha_1 s} D(\kappa) - c)\kappa e^{-rs} ds - e^{-rt} I\kappa + e^{-r\tau} W^m_\kappa\right) d\tau$ is the expected monopoly profit for a firm with capacity κ to enter at date t, which is concave over t and maximized at t^m_κ, and $A(\kappa|\mu) = \int_{\underline{t}_{\mu|\kappa}}^\infty f(\tau)\left(\begin{array}{c}\int_{\underline{t}_{\mu|\kappa}}^\tau y_0 e^{\alpha_1 s}(D(\kappa) - D(\mu+\kappa))\kappa e^{-rs} ds \\ +e^{-r\tau}(W^m_\kappa - W(\kappa|\mu))\end{array}\right) d\tau$ is independent of the leader's entry date t. The term $A(\kappa|\mu)$ can be viewed as the *negative impact* on the leader caused by the presence of a follower. This term is decreasing in $\underline{t}_{\mu|\kappa}$, *i.e.*, the earlier the follower enters the market, the smaller the leader's payoff will be. Therefore, when choosing its plant size, the leader has incentive to delay the entry of the follower as much as possible.

On the other hand, if the follower does not contest the leadership, it will not enter the market until its optimal entry date as a follower. In particular, a follower with capacity L when following a leader with capacity S will wait until $\underline{t}_{L|S}$, thus its payoff $F(t, L|S)$ as a function of leader's entry date t is a constant before $\underline{t}_{L|S}$, which is shown as the horizontal line YG in Figure 3. After date $\underline{t}_{L|S}$, the follower's payoff starts decreasing which is denoted by curve FL. So the curve YGL is the large follower′s payoff. Similarly, curve AHK depicts the small follower′s payoff.

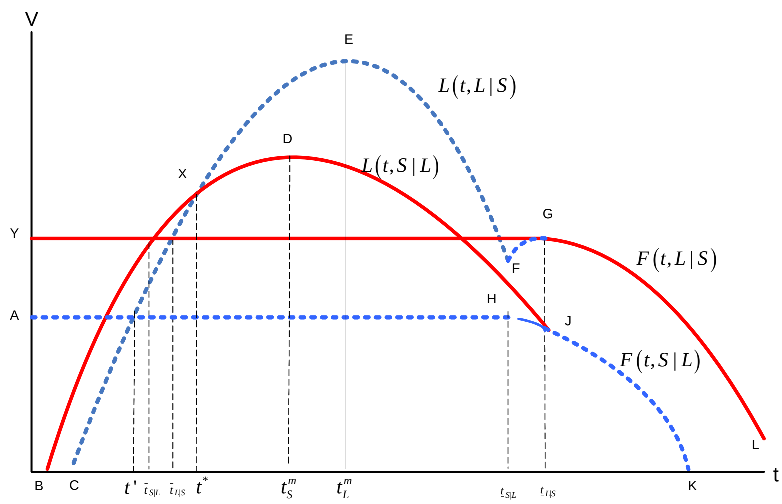

Figure 3: Leader and Follower's payoff functions

Figure 3 illustrates the leader and follower's payoff functions when they are of different capacities. The following lemma gives the sufficient conditions for a unique intersection between two leader's payoff function to the left of t_L^m, which is point X in Figure 3.

Lemma 4 *If* $L(0, S|L) > L(0, L|S)$ *and* $L(t_S^m, S|L) < L(t_L^m, L|S)$, *there exists a unique* $t^* < t_L^m$ *such that* $L(t^*, S|L) = L(t^*, L|S)$ *.*

This paper asks the question whether the first entrant can credibly preempt its follower with a large capacity. $L(t_\kappa^m, \kappa|\mu)$ is the maximum payoff for a firm with capacity κ if it enter at its optimal entry date t_κ^m. $L(t_S^m, S|L) < L(t_L^m, L|S)$ gives the leader incentive to preempt the follower with large capacity, which is a necessary condition when comparing with previous papers on Stackelberg leadership, because in previous papers, the leader usually chooses a large capacity and receives more payoff than its follower. As shown in Figure 3, if the leader is indeed exogenously chosen, it will choose large capacity as long as

$L(t_S^m, S|L) < L(t_L^m, L|S)$. In order to compare our result to previous studies, I maintain the assumption that $L(t_S^m, S|L) < L(t_L^m, L|S)$, which implies that large capacity preemption is at least feasible and we can focus on its credibility.[24]In addition, I assume that $F\left(\underline{t}_{L|S}, L|S\right) > F\left(\underline{t}_{S|L}, S|L\right)$, which implies that the follower prefers to be preempted by a smaller firm rather than a larger firm because it can take advantage of a faster-growing market by choosing larger capacity without too much concern about the adverse option value effect. As a result, if the leader successfully installs a large capacity and commits to staying in the market, the follower will suffer as a Stackelberg follower with less payoff.

Let L_X denote the leader's payoff at X. Because X is the intersection of two leader's payoff functions at date t^*,we must have $L_X = L(t^*, S|L) = L(t^*, L|S)$. Since $F\left(\underline{t}_{L|S}, L|S\right) > F\left(\underline{t}_{S|L}, S|L\right)$,there are three cases to consider depending on the relationship between L_X and two optimal payoffs of followers. This is easily seen in Figure 3, horizontal line YG corresponds to payoff level $F\left(\underline{t}_{L|S}, L|S\right)$ and line AH corresponds to payoff level $F\left(\underline{t}_{S|L}, S|L\right)$. Those two lines divided the vertical axis into three segments and the intersection X may lie in any of the three segments. The following three lemmas studies those two cases individually and discuss the preemption equilibrium with the assumption that the follower takes the leader's position as *fait accompli.*

Lemma 5 *(Rent Equalization with a small leader)Suppose the follower takes the leader's position as fait accompli. If* $L_X \geq F\left(\underline{t}_{L|S}, L|S\right) > F\left(\underline{t}_{S|L}, S|L\right)$, *the leader will enter at* $\bar{t}_{S|L}$ *with capacity* S *and the follower will enters at* $\underline{t}_{L|S}$ *with capacity* L, *where* $\bar{t}_{S|L}$ *satisfies* $L\left(\bar{t}_{S|L}, S|L\right) = F\left(\underline{t}_{L|S}, L|S\right)$.

This Lemma characterized a rent equalization equilibrium when firms are allow to choose both capacity and entry timing. $L_X > F\left(\underline{t}_{L|S}, L|S\right)$ implies that there exist two dates, $\bar{t}_{S|L}$ and $\bar{t}_{L|S}$, such that $L\left(\bar{t}_{S|L}, S|L\right) = L\left(\bar{t}_{L|S}, L|S\right) = F\left(\underline{t}_{L|S}, L|S\right)$. In Figure 3, $\bar{t}_{S|L}$ is the date at which the horizontal line YG crosses the smaller leader's payoff function $BDJK$, which is the rent equalization date. That is, if the smaller leader enters at this date, it earns the same expected payoff as the follower. However, to reach the same payoff level, the leader with capacity L needs to wait until $\bar{t}_{L|S}$, which is later than $\bar{t}_{S|L}$. Observe that $\bar{t}_{S|L} < t^*$, that is, large capacity preemption is not feasible, so the first entrant will choose capacity S to enter the market at $\bar{t}_{S|L}$. This is due to two facts: first, the first entrant will not enter the market before $\bar{t}_{S|L}$, because it can achieve higher payoff as a follower. On the other hand,

[24]If this condition is violated, it means that the large capacity choice is actually an over capacity, which results in lower payoff even if the leadership is not contested. Therefore, even an exogenously determined leader would not choose large capacity. See Wu(2005) for details.

it could not be an equilibrium for the leader to later than $\bar{t}_{S|L}$. To see why, suppose firm 1 is the leader and enters at $t > \bar{t}_{S|L}$, firm 2 can always best respond by entering at $t - \varepsilon$ to preempt firm 1, which contradicts the fact that firm 1 is the leader. As a result, the leader has to enter with capacity S at $\bar{t}_{S|L}$. Since no leadership contest is allowed, the follower will wait until its optimal entry date $\underline{t}_{L|S}$ and enter the market with capacity L. Previous studies usually assume the first entrant can enter the market immediately. Suppose firms cannot invest until date t^*. The following lemma shows that there is a symmetric subgame perfect equilibrium with first entrant preempting the second entrant with large capacity and achieves higher expected payoff.

Lemma 6 *If $F\left(\underline{t}_{L|S}, L|S\right) > L_X > F\left(\underline{t}_{S|L}, S|L\right)$ and the market is not open until t^*, the leader enters immediately at t^* with capacity L and the follower will enter at $\underline{t}_{S|L}$ with capacity S, where t^* satisfies $L\left(t^*, S|L\right) = L\left(t^*, L|S\right) = L_X$.*

For $t < t^*$, large capacity preemption is not feasible because the leader is better off by choose a small capacity plant. Suppose the game starts at t^*, Lemma 6 proves that in equilibrium both firm will try to preempt its competitor with a large capacity if no market contest will happen. This result extends Maggi (1996) to a continuous-time setting: if immediate investment is possible, the leader will try to invest in larger capacity, which trades off flexibility with the advantage of earlier commitment.[25] This result relies on the assumption that the game begins at t^*. If the firm is allowed to move before t^*, there is no pure strategy subgame perfect equilibrium with the leader enters as a large firm. Observe that if firm 1's strategy is to wait until t^* to enter as a large firm. If firm 2 adopts the same strategy, each firm has equal probability of entering as a large leader. So the expected payoff for firm 2 is $\frac{1}{2}L\left(t^*, L|S\right) + \frac{1}{2}F(\underline{t}, S|L)$. However, firm 2 can improve its payoff by investing at $t - \varepsilon$ with a smaller capacity, which guarantees its winning of the leadership contest with a payoff slightly below $L\left(t^*, L|S\right)$ but strictly greater than $\frac{1}{2}L\left(t^*, L|S\right) + \frac{1}{2}F(\underline{t}, S|L)$. On the other hand, the leader enters as a small firm is not an symmetric pure strategy equilibrium, because the other firm will choose not to enter the market until $\underline{t}(L|S)$ so it can earn higher payoff as a large firm. The following lemma shows that there exists a subgame perfect $\varepsilon-$equilibrium with the leader enters as a smaller firm first. A subgame perfect $\varepsilon-$equilibrium is a strategy profile defined from a specific date t onwards when no player can deviate to

[25] Maggi (1996) considered a two stage setting with leader acts at the first stage but the market is not open until second stage.

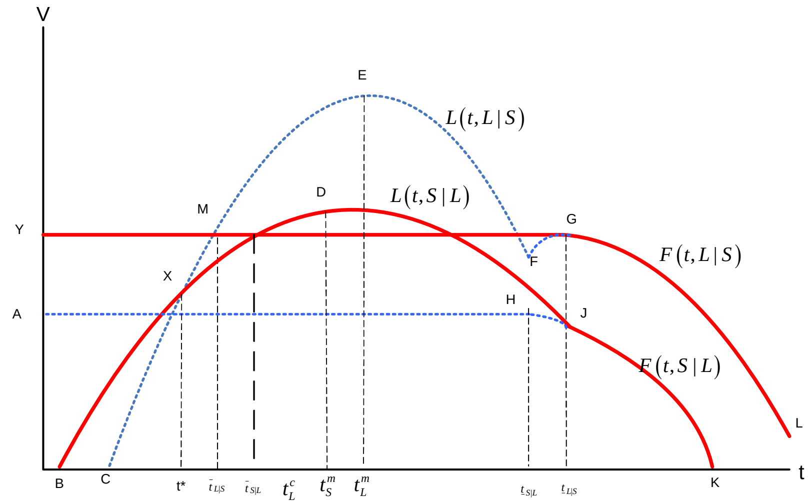

Figure 4: Small Leader Large Follower

any other strategy and gain more than ε. If $\varepsilon = 0$, then this is a subgame perfect equilibrium.[26]

Lemma 7 *If* $F\left(\underline{t}_{L|S}, L|S\right) > L_X > F\left(\underline{t}_{S|L}, S|L\right)$, *there is a subgame perfect* $\varepsilon-$*equilibrium with the leader enters with capacity* S *and the followers enters with capacity* L.

The case of $F\left(\underline{t}_{L|S}, L|S\right) > L_X > F\left(\underline{t}_{S|L}, S|L\right)$ is shown in Figure 4. The difference between Figure 4 and Figure 3 is that YG crosses CE at a date later than t^* because $L_X < F\left(\underline{t}_{L|S}, L|S\right)$. In this case, in an asymmetric equilibrium, the leader enters with capacity S at a date slightly earlier than t^* and the follower wait until $\underline{t}_{L|S}$. In fact, as shown in Figure 4, if the leader enters later than t^*, it is always better off by choosing large capacity rather than small capacity. However, competition for market leadership will force the first entrant to enter earlier than t^* with a smaller capacity.

Finally, if $F\left(\underline{t}_{L|S}, L|S\right) > F\left(\underline{t}_{S|L}, S|L\right) \geq L_X$, the following lemma shows another rent equalization equilibrium with the leader entering as a large firm and the follower being the smaller firm. In this case, both firms are earning the same payoff and the large capacity preemption is credible if the follower is not allowed to contest its leadership.

Lemma 8 *(Rent equalization with a large leader)Suppose the follower takes the leader's position as fait accompli. If* $F\left(\underline{t}_{L|S}, L|S\right) > F\left(\underline{t}_{S|L}, S|L\right) \geq L_X$, *the leader will enter at* $\bar{t}_{L|S}$ *with capacity* L *and the follower will enters at* $\underline{t}_{S|L}$ *with capacity* S, *where* $\bar{t}_{L|S}$ *satisfies* $L\left(\bar{t}_{L|S}, L|S\right) = F\left(\underline{t}_{S|L}, S|L\right)$.

[26] Please see Laraki et.al. (2003) for more discussions on this concept applied to continuous-time games of timing.

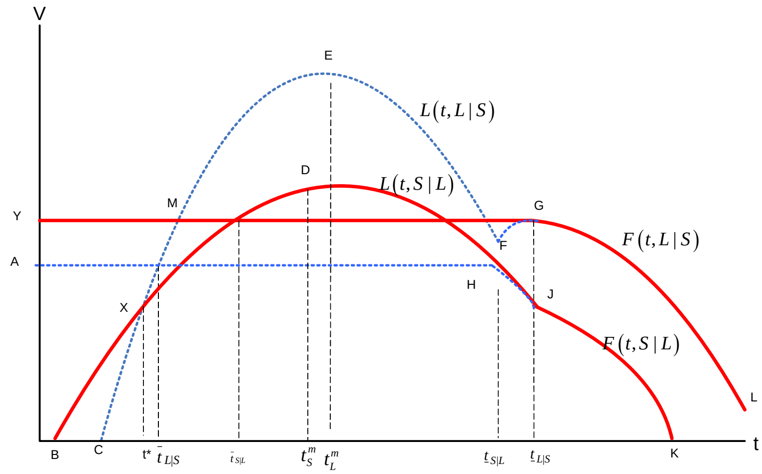

Figure 5: Rent equalisation with large leader

Contestability Condition

In previous subsection, I assume that the follower will not contest the first entrant's leadership. Removing this assumption opens the possibility for the follower to challenge the first entrant's leadership. In particular, the follower, after seeing its competitor entering the market, has two options: it can wait until its optimal entry date as a follower or it can enter earlier and trigger a war of attrition in the growing market. Of course, it would not start a war unless it could force the leader out of the market, otherwise it would have entered at a suboptimal date. An immediate implication is that choosing the same capacity to contest the leadership will not succeed, because both firms will earn the same payoff and the leader does not have a dominant strategy to exit the market. Consequently, a firm will contest the other firm's leadership if and only if, by choosing a different capacity, it can replace the leader as a monopoly by driving it out of the market.

Therefore, we simply need to discuss the leadership contest between a small firm and a large firm. Once war of attrition is triggered, if neither firm is earning positive payoff, both firms might want to exit the market. As this is a war of attrition happens along the growing market, the winner depends on which firm has a shorter exit region. In particular, the following proposition shows that if the war of attrition is started at t_0, each firm has a continuous exit region in the form of $[t_0, t^c_\kappa]$ where $\kappa \in \{S, L\}$, and t^c_κ is the last date that a firm with capacity κ wants to exit the market as long as the market continues to grow. The following proposition identifies the winner by comparing the last exit dates t^c_κ.

Proposition 4 *If a war of attrition happens at* $t < t_L^c = \frac{1}{\alpha_1} \ln \frac{c - \lambda \frac{W(L|S)}{L}}{y_0 D(S+L)}$ *between a firm with capacity* S *and the other firm with capacity* L *when the market is growing, the firm with capacity* L *will exit the market immediately.*

Because the market demand is growing, for a firm with capacity $\kappa \{\in \{S, L\}\}$, t_κ^c is the last day that it will consider exiting the market. Therefore, by comparing t_S^c and t_L^c, we can identify the equilibrium in this war of attrition subgame. In the proof, I show that $t_\kappa^c = \frac{1}{\alpha_1} \ln \frac{c - \lambda \frac{W(\kappa|\mu)}{\kappa}}{y_0 D(S+L)}$, $\kappa \neq \mu$. A quick comparison of t_S^c and t_L^c reveals that the only difference is an item on the numerator, $\frac{W(\kappa|\mu)}{\kappa}$, which is the per unit capacity marginal option value for a firm to delay its exit from the market. To see why, suppose the market is growing at the moment. Observe that $W(\kappa|\mu)$ is also the continuation payoff for a firm with capacity κ after market swtiches. By choosing to stay in the market in the war of attrition, the firm can avoid losing $W(\kappa|\mu)$ when the market suddenly switches to decline, which, multiplied by the probability of market switching λ, is exactly the expected potential losses avoided by choosing not to exit the market, or in a word, it is the option value of delaying exit.

Corollary 1 shows that $\frac{W(S|L)}{S} > \frac{W(L|S)}{L}$, which implies $t_S^c < t_L^c$. This is, a smaller firm has higher option value and thus reduces its willingness to exit the market. As a result, its exit region is shorter than that of the large firm. In turn, this implies that if the leader chooses capacity L and enters at $t < t_L^c$, then the other firm can enter immediately with a smaller capacity and force the leader out of the market, because once the war of attrition is triggered, the exit region of the follower is $[t, t_S^c]$,which is shorter than $[t, t_L^c]$; hence in a subgame perfect equilibrium the firm with the larger capacity will not be able to win the war of attrition because for the subgame beginning at $t > t_S^c$, the smaller firm definitely will stay in the market while the large firm is still in its exit region.

Given this result, we can check the contestability condition in the two cases discussed in the previous subsection. In particular, if $\bar{t}_{L|S} < t^*$ as shown in Figure 3, Lemma 5 proves that the smaller firm can enter the market earlier if the follower takes the leader as *fait accompli*. If the follower is allowed to contest the first mover's lead, the smaller leader can still survive because its advantage in winning the war of attrition further strengthens its credibility to stay in the market.

However, this is not the case when $t^* < \bar{t}_{L|S}$. First, a firm choosing smaller capacity will not enter the market until $\bar{t}_{S|L}$ which guarantees it at least as much payoff as the follower. However, if the firm chooses to be large, its leader's payoff will reach this level at $\bar{t}_{L|S}$ which is earlier than $\bar{t}_{S|L}$. In fact, the leader is better off by choosing large capacity if it wants to enter later than t^*. The question is whether it can credibly choose large capacity to preempt the follower. The following proposition summarizes those two cases and gives a set of sufficient conditions under which the leader holds its entry until $\bar{t}_{S|L}$, at which time it starts operation in a small capacity.

Proposition 5 *Given Assumptions 1-5 and assuming $\alpha_S < \alpha_1 < \alpha_L$, and $L(t_L^m, L|S) > L(t_S^m, S|L) > F\left(\underline{t}_{L|S}, L|S\right) > F\left(\underline{t}_{S|L}, S|L\right)$, there is a unique subgame perfect equilibrium in which the leader enters at $\bar{t}_{S|L}$ with smaller capacity S while the follower enters at $\underline{t}_{L|S}$ with larger capacity, if the following conditions are satisfied:*

$$\frac{D(S)}{D(S+L)} \geq \frac{c + rI + \phi(S|L)}{c} \tag{7}$$

where $\phi(S|L) = -\lambda\left(\frac{W(S|L)}{S} - I\right)$.

The condition $\alpha_S < \alpha_1 < \alpha_L$ implies that the resulting duopoly market structure is asymmetric with one small firm and one large firm. Condition (7) is a sufficient condition that guarantees that a small firm will win the war of attrition in the growing stage, because $\frac{D(S)}{D(S+L)} \geq \frac{c+rI+\phi(S|L)}{c}$ implies $t_S^m \leq t_L^c$, which is a sufficient condition for $\bar{t}_{S|L} < t_L^c$ since $\bar{t}_{S|L} \leq t_S^m$. This condition implies that if the larger firm entered earlier than $\bar{t}_{S|L}$, it would be forced out by a smaller firm. Hence a large firm should not enter the market even with $\bar{t}_{L|S} < \bar{t}_{S|L}$. The proof of the proposition considers two cases. When $\bar{t}_{L|S} \leq t^*$, as shown in Figure 3, we have $\bar{t}_{S|L} < \bar{t}_{L|S}$, in which case it is intuitively evident that the smaller firm can enter earlier credibly. In contrast, $\bar{t}_{L|S} > t^*$, as shown in Figure 4, a larger firm is able to enter earlier by the rent equalization principle, but its action is not credible. Since $\bar{t}_{S|L} < t_L^c$, if the first firm selects larger capacity to preempt the second firm, the second firm can enter at $\bar{t}_{S|L}$ to force the first one out of the market. In this sense the larger capacity preemption is simply not credible.

Note that condition (7) is a sufficient condition. Actually as long as $\bar{t}_{S|L} < t_L^c$, the smaller firm can always wait until $\bar{t}_{S|L}$ to enter the market, because no larger firm will be able to preempt its competitor, otherwise it would be forced out by a smaller firm at $\bar{t}_{S|L}$.

Finally, we briefly discuss two other cases: one is when the market growth rate $\alpha_1 < \alpha_S$ and the other is $\alpha_1 > \alpha_L$. In both cases, the main economic forces that drive the result are those two analyzed in the previous section: first, a higher option value causes a firm with large capacity to delay its investment timing, which in turn reduces its incentive to preempt its competitor. Second, even if the first entrant is able to enter the market with a large capacity, it might not want to build a large plant because it may not survive the leadership contest if its follower jumps into the market with a small capacity. In particular, if $\alpha_1 < \alpha_S$, the follower always enters at a smaller capacity no matter what capacity the leader chooses. In this case, if the leader wants to choose large capacity, it must take into account the disadvantage it faces when the market switches. Under some similar conditions in equilibrium both the leader and follower will enter the market with small capacity.[27] This will be the case when firms are impatient given the uncertainty of the market, for example in the Wi-Fi market, some people claim that the most important question faced by the potential entrants is not how large its size is, but how fast it enters the market.[28]

If the market is growing very fast as $\alpha_1 > \alpha_L$, a noncontesting follower always chooses large capacity to enter the market. Whether the leader chooses large capacity will completely depends on whether the follower will be able to use small capacity to force the leader out of the market, because the follower will only choose small capacity if she is able to start a war of attrition and gets higher payoff. Therefore, we can apply a similar analysis and the discussion is omitted here since in equilibrium the leader will always choose a capacity to prevent a follower from contesting its leadership. As a result, it never chooses a capacity greater than its follower.

6 Discussions

6.1 Multiple Capacities

In the previous discussions, there are two essential assumptions: capacity choice is binary and the path of market evolution is discontinuous at the date when market switches. The latter is justified because in many cases the introduction of new product by a third party has a significant negative effect on the demand for the old product. In fact, the main intuition of the paper does not rely on either of these assumptions. As long as the number of capacity levels is finite, the numerical analysis will be reduced to a comparison of two capacity levels. I relax both assumption by computing a numerical example with three capacities choices and continuous market evolution. Table 1 is list the results of

[27] The proofs of those results are available upon request. I omitted the details here since it only provides limited additional insights.

[28] See "Cometa hot spots to get cold shoulder?", CNET news.com. July 21, 2003.

Table 1: Leader and follower's equilibrium payoffs and capacity choices

	Leader		Follower	
α_1	Capacity	Payoff	Capacity	Payoff
0.15	1.5	4.674	1.5	4.674
0.16	1.5	6.495	1.5	6.495
0.17	1.5	8.833	1.5	8.833
0.18	1.8	9.944	1.5	9.944
0.19	1.8	13.26	1.5	13.26
0.20^e	1.5	18.20	1.8	20.94
0.21^e	1.5	22.39	1.8	27.54
0.22	1.8	30.88	1.8	30.88
0.23	1.8	55.07	1.8	55.07
0.24	1.8	75.32	1.8	75.32

Source: Calculated by the author.

$Parameters: y_0 = 0.2; \lambda = 0.2; \alpha_2 = -0.3; r = 0.1; I = 50; c = 3.$

Firm Capacity Choices: $Q = \{1.5, 1.8, 2.1\}$

$e : \varepsilon - equilibrium;$

numerical solution of a model with three capacities. The algorithm is very simple. First, we derive the noncontesting follower's optimal capacity choice given the leader's different capacity choices. Second, we investigate which rent equalization equilibrium is most preferred by the leader and check whether it can sustain a potential contest from the follower. If not, we keep tracking other equilibria. Table 1 illustrates three types of equilibria: rent equalization with equal capacities, rent equalization with large leader and asymmetric $\varepsilon - equilibrium$ with smaller leader. A common pattern is that the leader is not better off than the follower and some times earning less payoff than the follower. In particular, if $\alpha_1 = 0.20$ or 0.21, if market opens right at the first date that the leader is indifferent from choosing large or small capacity, that is, date t^* in previous discussion, the leader will be able to choose large capacity and earns higher payoff, as proved in previous section. However, it is not an equilibrium for both firm to wait until this day to compete for the Stackelberg leadership with a large capacity.

6.2 Cost Advantage

In my model, firms have the same unit cost regardless of its plant size. Proposition 4 shows that the smaller firm always has an advantage in the war of attrition. Especially if the market is growing, the contestability advantage of the smaller firm forces the leader to choose smaller capacity to enter the market. However, in reality a large plant is usually associated with smaller average cost level because of the fixed cost effect. The result of this paper relies on the fact that the small firm has an advantage in the declining industry, which is the capability of winning the war of attrition as shown in Ghemawat and Nalebuff (1985). However, they also show that even if introducing differential cost due to economies of scale might complicate the analysis, there is still a nontrivial set of parameters that support the original result. Nevertheless, it is also worth noting that, there is an additional aspect in my model that might change the result, which is the fact that the large capacity preemption might indeed become credible if the larger firm has a cost advantage. As shown in Proposition 4, the comparison of two firm's last exit date relies on the fact that both firms have the same operating cost. However, if $c_S \neq c_L$, the relationship between two last exit date, t_S^c and t_L^c, are ambiguous, since $t_\kappa^c = \frac{1}{\alpha_1} \ln \frac{c_\kappa - \lambda \frac{W(\kappa|\mu)}{\kappa}}{y_0 D(S+L)}$. One immediate implication is that if $t_L^c < t_S^c$, then a firm can credibly preempt the other firm with a large capacity, it actually must invest earlier than in the case of no cost advantage. The larger firm can credibly preempt the other firm which reduces the follower's payoff. Consequently, it intensifies the competition for leadership and actually forces the leader to invest earlier. This result implies that a firm that chooses to be large and that exploits its cost advantage might actually be worse off because of the induced intensified competition for the leadership.

7 Conclusion

This paper explores firms' incentives to perform large capacity preemption in a market with evolving uncertainty. In contrast to previous literature, this paper demonstrates that making earlier capacity commitment is not sufficient for a firm to secure Stackelberg leadership. Indeed, the possibility of leadership contest by a smaller second mover prevents the first entrant to invest in large capacity, because it lacks the credibility to fight for the market. It is commonly believed that preemption probably destroys option value, however, this paper emphasizes that the differentiation in option value might in turn change a firm's incentive to preempt.

This research certainly raises more questions than the ones addressed in this paper. For example, if firms are allowed to accumulate capacities rather than making one shot investment, they might race to entry by starting small and keep the option to grow later. Although this does not change the result of this paper, it does warrant further studies. Most recent studies on industry dynamics have been focused on the asymmetries generated by firm specific idiosyncrasies as pioneered by Ericson and Pakes (1995), while literature has been pretty silent on the evolution of industry structure due to firms' strategic interactions under uncertainty, which is of course a topic for future research.

8 Appendix

Proof of Lemma 1

Proof. (i) By Assumption 4, $\bar{y} = \frac{c}{D(2S)}$. Substitute it into Equation (2) and obtains firm's monopoly exit date, $t^x_\kappa = \frac{1}{\alpha_2} \ln \frac{D(2S)}{D(\kappa)}$, which is counting from the market switching date τ. As a result, its exit date with respect to date 0 is $t^x(\tau, \kappa) = \tau + \frac{1}{\alpha_2} \ln \frac{D(2S)}{D(\kappa)}$. In particular, $D(S) > D(L)$ implies $t^x_S > t^x_L$ and thus $t^x(\tau, S) > t^x(\tau, L)$ for all τ.

(ii) Note that $\frac{W^m_\kappa}{\kappa} = \int_0^{t^x_\kappa} (\bar{y} e^{\alpha_2 s} D(\kappa) - c) e^{-rs} ds$. This result follows immediately from the facts that $D(S) > D(L)$ and $t^x_S > t^x_L$. ■

Proof of Lemma 2

Proof. Consider a firm with capacity κ, $\kappa \in \{S, L\}$, its expected payoff when entering at date t is:

$$M(t, \kappa) = \int_t^\infty f(\tau) \left(\int_t^\tau (y_0 e^{\alpha_1 s} D(\kappa) - c) \kappa e^{-rs} ds + e^{-r\tau} W^m_\kappa - I \kappa e^{-rt} \right) d\tau$$

Hence

$$\frac{\partial M(t, \kappa)}{\partial t} = e^{-(\lambda + r)t} \left(-y_0 e^{\alpha_1 t} D(\kappa) \kappa + c\kappa + (\lambda + r) I \kappa - \lambda W^0_\kappa \right) = 0$$

Therefore let t^m_κ denote the solution of the above first order conditions, we have

$$P(Y(t^m_\kappa), \kappa) - c = rI + \lambda \left(-\frac{W^m_\kappa - I\kappa}{\kappa} \right)$$

where $P(Y(t^m_\kappa), \kappa) = y_0 e^{\alpha_1 t^m_\kappa} D(\kappa)$, and it is routine to check that the second order condition is satisfied at t^m_κ :

$$t^m_\kappa = \frac{1}{\alpha_1} \ln \frac{c + (\lambda + r) I - \lambda \frac{W^m_\kappa}{\kappa}}{y_0 D(\kappa)}$$

Since $\frac{W_S^m}{S} > \frac{W_L^m}{L}$ and $D(S) > D(L)$, we must have

$$t_S^m < t_L^m.$$

■

Proof of Proposition 1

Proof. Let t be the current date, and suppose the market switches at τ. Define $t' = t - \tau$ to be the current date counting from the switching date τ. Once the decline stage begins, if two firms are of different sizes, then we know the market price is

$$p(\bar{y}, S+L) = \bar{y}D(S+L) < \bar{y}D(2S) = c$$

This implies that if both firms stay in the market, they will be earning negative profit. Also note that no firm will stay longer than its monopoly exit date t_κ^x, $\kappa = S, L$. Hence we only need to consider the date $t' < t_S^x$.

If $t_S^x > t' > t_L^x$, the larger firm will exit the market immediately because even if the smaller firm leaves the market in the next instant and the larger firm becomes a monopoly, it will still exit the market because it has already passed its monopoly exit date. Realizing that the dominant strategy for the larger firm is to leave, the smaller firm will stay in the market.

Consider the case of $t_L^x \geq t' \geq 0$. If $t' = t_L^x$, the larger firm will exit the market for sure and the smaller firm will stay. Consider $t' = t_L^x - \varepsilon$, by staying in the market, the smaller firm will incur a loss until date t_L^x when the larger firm leaves the market. Hence the smaller firm's profit is

$$\int_{t_L^x-\varepsilon}^{t_L^x} \left(\bar{y}e^{\alpha_2 s}D(S+L) - c\right)Se^{-rs}ds + \int_{t_L^x}^{t_S^x} \left(\bar{y}e^{\alpha_2 s}D(S) - c\right)Se^{-rs}ds > 0$$

for ε sufficiently small. Therefore, the smaller firm will stay in the market for sure. In this case, the larger firm will have to leave the market at $t' = t_L^x - \varepsilon$. By the same reasoning, the larger firm will leave the market at each date $t' \geq 0$. Hence we have proved part (i).

(ii) In this case two firms are of the same size. By Assumption 2, firm $i's$ size is decreased by $\delta_i, i = 1, 2$. With probability $\frac{1}{2}$, $\delta_1 > \delta_2$. In this case, *ex post* firm 1 will become smaller than the other firm. By the same reasoning of part (i), firm 2 will have to exit the market earlier than firm 1, while the latter can stay until its monopoly exit date. As a result, if $\bar{\delta} \to 0$, firm 2 will exit immediately after market switches. ■

Proof of Proposition 2

Proof. Differentiating (4) with respect to t, we will have

$$\begin{aligned}\frac{\partial F(t,\kappa|\mu)}{\partial t} &= -f(t)\left(e^{-rt}W(\kappa|\mu) - I\kappa e^{-rt}\right) \\ &\quad + \int_t^{\infty} f(\tau)\left(\left(-y_0 e^{\alpha_1 t} D(\mu+\kappa) - c\right) e^{-rt}\kappa + rI\kappa e^{-rt}\right) d\tau = 0\end{aligned}$$

which can be simplified as

$$y_0 e^{\alpha_1 t} D(\mu+\kappa) - c = rI + \lambda\left(-\frac{W(\kappa|\mu) - I\kappa}{\kappa}\right)$$

Thus[29]

$$t^f = \frac{1}{\alpha_1}\ln\frac{c + rI + \lambda\left(-\frac{W(\kappa|\mu)-I\kappa}{\kappa}\right)}{y_0 D(\mu+\kappa)}$$

In particular, $t_0 > t^f$, then the firm will enter immediately since its payoff is decreasing. ■

Proof of Lemma 3

Proof. Note that

$$\begin{aligned}&\alpha_L - \alpha_S \\ =\; &(\lambda+r)\frac{\left(\ln\frac{D(L+S)}{D(2L)} - \ln\underline{\eta}^L\right)\left(\ln\frac{L}{S} - \ln\underline{\eta}^S\right) - \left(\ln\frac{D(2S)}{D(S+L)} - \ln\underline{\eta}^S\right)\left(\ln\frac{L}{S} - \ln\underline{\eta}^L\right)}{\left(\ln\frac{L}{S} - \ln\underline{\eta}^L\right)\left(\ln\frac{L}{S} - \ln\underline{\eta}^S\right)} \\ =\; &\frac{(\lambda+r)}{\left(\ln\frac{L}{S} - \ln\underline{\eta}^L\right)\left(\ln\frac{L}{S} - \ln\underline{\eta}^S\right)}\left[\begin{array}{c}\ln\frac{L}{S}\left(\ln\frac{D(L+S)}{D(2L)} - \ln\frac{D(2S)}{D(S+L)}\right) \\ -\ln\underline{\eta}^S\left(\ln\frac{D(L+S)}{D(2L)} - \ln\frac{L}{S}\right) \\ -\ln\underline{\eta}^L\left(\ln\frac{L}{S} - \ln\frac{D(2S)}{D(S+L)}\right)\end{array}\right]\end{aligned}$$

[29]It is easy to check the second-order condition is satisfied.

So $\alpha_L > \alpha_S$ if and only if $\begin{array}{c} \ln\frac{L}{S}\left(\ln\frac{D(L+S)}{D(2L)} - \ln\frac{D(2S)}{D(S+L)}\right) \\ -\ln\underline{\eta}^S\left(\ln\frac{D(L+S)}{D(2L)} - \ln\frac{L}{S}\right) \\ -\ln\underline{\eta}^L\left(\ln\frac{L}{S} - \ln\frac{D(2S)}{D(S+L)}\right) \end{array} > 0$. In particular, if $D(Q) = a - bQ$, it is routine to check that $2L + S > \frac{a}{b} > L + 2S$ is sufficient. ■

Proof of Proposition 3

Proof. Note that

$$F(t, \kappa|\mu) = \int_t^\infty f(\tau)\left(\int_t^\tau \left(y_0 e^{\alpha_1 s} D(\mu+\kappa) - c\right)\kappa e^{-rs} ds + e^{-r\tau} W(\kappa|\mu) - I\kappa e^{-rt}\right) d\tau$$

From the first order condition, we know that $y_0 e^{\alpha_1 \underline{t}_{\kappa|\mu}} D(\mu+\kappa)\kappa = c\kappa + (\lambda + r) I\kappa - \lambda W(\kappa|\mu)$ .Hence we must have

$$F_\kappa^\mu\left(\underline{t}_{\kappa|\mu}\right) = e^{-(\lambda+r)\underline{t}_{\kappa|\mu}}\left(c\kappa + (\lambda+r) I\kappa - \lambda W(\kappa|\mu)\right)\left(\frac{\alpha_1}{(\lambda + r - \alpha_1)(\lambda + r)}\right)$$

Hence

$$\frac{F_S^\mu\left(\underline{t}_{S|\mu}\right)}{F_L^\mu\left(\underline{t}_{L|\mu}\right)} = e^{-(\lambda+r)\left(\underline{t}_{S|\mu} - \underline{t}_{L|\mu}\right)} \frac{c + (\lambda+r) I - \lambda\frac{W(S|\mu)}{S}}{c + (\lambda+r) I - \lambda\frac{W(L|\mu)}{L}} \frac{S}{L}$$

Define $\underline{\eta}^\mu = \frac{c+(\lambda+r)I-\lambda\frac{W(S|\mu)}{S}}{c+(\lambda+r)I-\lambda\frac{W(L|\mu)}{L}} \leq 1$. Note that $\underline{t}_{S|\mu} - \underline{t}_{L|\mu} = \frac{1}{\alpha_1}\ln\left[\frac{D(\mu+L)}{D(\mu+S)}\underline{\eta}^\mu\right]$ .Hence

$$\frac{F_S^\mu\left(\underline{t}_{S|\mu}\right)}{F_L^\mu\left(\underline{t}_{L|\mu}\right)} = \left[\frac{D(\mu+L)}{D(\mu+S)}\underline{\eta}^\mu\right]^{-\frac{\lambda+r}{\alpha_1}} \underline{\eta}^\mu \frac{S}{L}$$

Therefore, if $\alpha_1 < \alpha_\mu = \frac{(\lambda+r)\left[\ln D(\mu+S) - \ln D(\mu+L) - \ln\underline{\eta}^\mu\right]}{\ln L - \ln S - \ln\underline{\eta}^\mu}$, $F_S^\mu\left(\underline{t}_{S|\mu}\right) > F_L^\mu\left(\underline{t}_{L|\mu}\right)$. The results of the proposition then follow naturally. ■

Proof of Lemma 4

Proof. Since $L(0, S|L) > L(0, L|S)$, $L(t_S^m, S|L) < L(t_L^m, L|S)$ implies that there exists a t^* such that $L(t^*, S|L) = L(t^*, L|S)$. Also note that

$$\frac{\partial \frac{L(t,S|L)}{S}}{\partial t} - \frac{\partial \frac{L(t,L|S)}{L}}{\partial t} = e^{-(\lambda+r)t}\left(-\lambda\left(\frac{W_S^m}{S} - \frac{W_L^m}{L}\right) - y_0 e^{\alpha_1 t}\left(D(S) - D(L)\right)\right) < 0$$

Hence, we must have $\frac{\partial L(t,S|L)}{\partial t} = \frac{\partial \frac{L(t,S|L)}{S}}{\partial t} S < \frac{\partial \frac{L(t,S|L)}{S}}{\partial t} L < \frac{\partial \frac{L(t,L|S)}{L}}{\partial t} L = \frac{\partial L(t,L|S)}{\partial t}$. That is, for all $t_L^m > t > t^*$, $L(t, L|S) > L(t, S|L)$ and for all $t < t^*$, $L(t, L|S) < L(t, S|L)$, which proves the uniqueness of t^*. ■

Proof of Lemma 5

Proof. First, $L_X > F\left(\underline{t}_{L|S}, L|S\right)$ implies that there exists two dates $\bar{t}_{S|L} < t^*$ and $\bar{t}_{L|S} < t^*$ such that $L\left(\bar{t}_{S|L}, S|L\right) = L\left(\bar{t}_{L|S}, L|S\right) = F\left(\underline{t}_{L|S}, L|S\right)$. As shown in Figure 3, $\bar{t}_{S|L}$ and $\bar{t}_{L|S}$ are two dates at which YG crosses two payoff functions $L(t, S|L)$ and $L(t, L|S)$, respectively. We are considering a symmetric equilibrium. Without loss of generality, suppose firm 1 is the leader. As shown in Figure 3, firm 1 has no incentive to enter as a larger firm before date t^*. If firm 1 enters at $\bar{t}_{S|L}$ with capacity S, firm 2 will best respond by entering at $\underline{t}_{L|S}$ with capacity L and earning the same payoff. If firm 1 chooses to enter at $t > \bar{t}_{S|L}$, firm 2 will enter at $t - \varepsilon$ to secure its leadership,which contradicts the assumption that firm 1 is the leader. On the other hand, if firm 1 enters at $t < \bar{t}_{S|L}$, the follower has no incentive to compete for the leadership at t since by waiting until date $\underline{t}_{L|S}$ it can earn higher expected payoff by investing in larger capacity. But given this, firm 1 can actually delay its entry and earn higher payoff. As a result, the unique Markov perfect equilibrium is the leader enters at $\bar{t}_{S|L}$ with capacity S and the follower enters at $\underline{t}_{L|S}$ with capacity L. ■

8.1 Proof of Lemma 6

Proof. We first show that the strategy described in this lemma is indeed an equilibrium. Without loss of generality, assume that firm 1 is the leader. If firm 1 succeeds in entering at t^* with capacity L, firm 2's best response is to choose small capacity and enter at $\underline{t}_{S|L}$ as shown in Proposition 3. However, if firm 1 chooses to enter later than $t > t^*$, firm 2's best response is to enter earlier with capacity L at

$t-\varepsilon$, since it will guarantee its leadership and a higher expected payoff than if it tries to compete for leadership at t. ■

8.2 Proof of Lemma 7

Proof. Because $L(t, S|L)$ is continuous and strictly greater than $L(t, L|S)$ for $t < t^*$, there exists $\bar{t}$ such that for $t \in [\bar{t}, t^*)$, $L_X - L(t, S|L) < \varepsilon$. Define a strategy profile as follows: firm 1 enters at $\bar{t}$ with capacity S if no firm has entered before $\bar{t}$, otherwise enters as L at t^* if no firm has enters between $\bar{t}$ and t^*, otherwise enter at $\underline{t}(S|L)$ if firm 2 has built a plant of capacity L otherwise enter at $\underline{t}(L|S)$ if firm 2 has invested with capacity S. Firm 1 enters at t^* with capacity L if no firm has entered, otherwise, enter at $\underline{t}(S|L)$ if firm 1 has built a plant of capacity L otherwise enter at $\underline{t}(L|S)$ if firm 1 has invested with capacity S. I claim this is a subgame perfect $\varepsilon-$equilibrium. To see why, given firm 2's strategy, firm 1 had better enter as a small firm before t^* to guarantee itself a payoff slightly below L_X, in particular, firm 1 cannot deviate its entry time later than t^* because firm 2 would have entered with capacity L at t^*. On the other hand, firm 1 will not profit more than ε by deviating to any date between $\bar{t}$ and t^*. On the other hand, give firm 1's strategy, firm 2 has no incentive to deviate since its payoff is strictly higher. ■

8.3 Proof of Lemma 8

Proof. Define the firm 1's strategy as follows: enter at $\bar{t}_{L|S}$ with capacity L if no firm has enter the market, otherwise if firm 2 has built a plant with capacity S, entering at $\underline{t}_{L|S}$ with capacity L, or if firm 2 has built a plant with capacity L, enter at $\underline{t}_{S|L}$ with capacity S. Given firm 1's strategy, I claim that firm 2's best response is to adopt the same strategy. To see why, observe that if $F\left(\underline{t}_{L|S}, L|S\right) > F\left(\underline{t}_{S|L}, S|L\right) \geq L_X$, as shown in Figure (**??**), entering earlier than $\vec{t}_{L|S}$ is not optimal since it is better off even it was preempted by firm 1 with large capacity. Entering later than $\bar{t}_{L|S}$ will result being preempted by firm 1. As a result, the best response is to enter as a large firm at the $\bar{t}_{L|S}$. In this case, the leader and follower earns the same payoff. ■

Proof of Proposition 4

Proof. Suppose a firm chooses capacity κ and its competitor chooses μ where $\kappa \neq \mu$ and both κ and μ $\in \{S, L\}$. Firm κ needs to choose an exit date t_κ when the market is growing to maximize its expected value:

$$\begin{aligned} V_\kappa^x(t_\kappa) &= \max_{t_\kappa > t} \int_t^{t_\kappa} f(\tau) \left(\int_t^\tau (y_0 e^{\alpha s} D(S+L) - c) \kappa e^{-rs} ds + e^{-r\tau} W(\kappa|\mu) \right) d\tau \\ & \quad + \int_{t_\kappa}^\infty f(\tau) \left(\int_t^{t_\kappa} (y_0 e^{\alpha s} D(S+L) - c) \kappa e^{-rs} ds + \frac{V_\kappa^c}{\Pr(\tilde{\tau} > t_\kappa)} \right) d\tau \end{aligned} \tag{8}$$

The first part of (8) is the expected payoff when the market switches before the exit date chosen by firm κ for the growing industry while the second part is the expected payoff when firm κ exits the market at t_κ. Note that, once Firm κ exit the market, it will achieve a continuation payoff V_κ^c which is independent of its exit date. Let t_κ^c be the unique solution of the following equation,

$$\begin{aligned} \frac{\partial V_\kappa^x(t_\kappa)}{\partial t_\kappa} &= f(t_\kappa) \left(\int_t^{t_\kappa} (y_0 e^{\alpha_1 s} D(S+L) - c) \kappa e^{-rs} ds + e^{-rt_\kappa} W(\kappa|\mu) \right) \\ & \quad - f(t_\kappa) \left(\int_t^{t_\kappa} (y_0 e^{\alpha_1 s} D(S+L) - c) \kappa e^{-rs} ds \right) \\ & \quad + \int_{t_\kappa}^\infty f(t_\kappa) \left(\left(y_0 e^{\alpha_1 t_\kappa} D(S+L) - c \right) \kappa e^{-rt_\kappa} \right) \\ &= e^{-(\lambda + r) t_\kappa + \lambda t} \left(\lambda W(\kappa|\mu) + \left(y_0 e^{\alpha_1 t_\kappa} D(S+L) - c \right) \kappa \right) \\ &= 0 \end{aligned}$$

That is

$$t_\kappa^c = \frac{1}{\alpha_1} \ln \frac{c - \lambda \frac{W(\kappa|\mu)}{\kappa}}{y_0 D(S+L)}$$

Since $S < L$ and $W(S|L) > 0 = W(L|S)$,

$$c - \lambda \frac{W(S|L)}{S} < c - \lambda \frac{W(L|S)}{L}$$

we must have

$$t_S^c < t_L^c$$

which means that the smaller firm has a shorter exit region than the larger firm. Hence the smaller firm can win the war of attrition in the growing industry, because once the date reaches t_S^c, if the smaller firm stays in the market, then the larger firm will exit immediately. By working backward, we can prove the result. This proves the result that the larger firm can not contest the smaller firm's monopoly position. ■

Proof of Proposition 5

Proof. Since $t_L^c = \frac{1}{\alpha_1} \ln \frac{c}{y_0 D(S+L)}$ and $t_S^m = \frac{1}{\alpha_1} \ln \frac{c+rI-\lambda\left(\frac{W_S^m}{S}-I\right)}{y_0 D(S)}$, $\frac{D(S)}{D(S+L)} \geq \frac{c+(\lambda+r)I-\lambda\frac{W_S^m}{S}}{c}$ implies $t_L^c - t_S^m \geq 0$.

Also $\bar{t}_{L|S} < t^*$ implies $\bar{t}_{L|S} < \bar{t}_{S|L}$. Suppose $\bar{t}_{L|S} \geq \bar{t}_{S|L}$. then we have

$$F\left(\underline{t}_{L|S}, L|S\right) = L\left(\bar{t}_{L|S}, L|S\right) > L\left(\bar{t}_{S|L}, L|S\right) > L\left(\bar{t}_{S|L}, S|L\right) = F\left(\underline{t}_{L|S}, L|S\right),$$

which is a contradiction. Note that $\bar{t}_{S|L} < t_S^m$ implies $\bar{t}_{L|S} < t_L^c$. Therefore, the leader will not enter the market with a large capacity because it will be forced out by a follower with a smaller capacity.

Given that the follower enters at $\underline{t}_{L|S}$ with a payoff of $F\left(\underline{t}_{L|S}, L|S\right)$, the leader will not enter earlier than $\bar{t}_{S|L}$ since for $t < \bar{t}_{S|L}$, $L(t, L|S) < L(t, S|L) < F\left(\underline{t}_{L|S}, L|S\right)$. On the other hand, if the leader delays entry at $\bar{t}_{S|L}$, the other firm will enter the market immediately. However, if the leader enters at $\bar{t}_{S|L}$ with a smaller capacity, the other firm will not preempt the leader by entering earlier because its payoff is smaller by preempting the leader. It cannot force the leader out of the market by investing in larger capacity. Hence, the follower's best response to the leader's entry at $\bar{t}_{S|L}$ is to wait until $\underline{t}_{L|S}$ and enter with a large capacity. ■

Proof of Lemma ??

Proof. First we will show that simultaneous entry is not an equilibrium. Suppose the two firms choose to enter simultaneously with the same capacity μ. In this case, they will choose to enter at date $\underline{t}_{\mu|\mu}$. However, since $L\left(t_\mu^m, \mu|\mu\right) > F\left(\underline{t}_{\mu|\mu}, \mu|\mu\right)$, it is always better for one firm to deviate and enter earlier at t_μ^m and achieve better payoff. There is no equilibrium in simultaneous entry. Hence, in a pure strategy equilibrium, we must have one firm chooses to be a leader and the other to be a follower.

Suppose firm 1 chooses l and firm 2 chooses f in the precommitment stage. Since $L\left(t_L^m, L|S\right) > L\left(t_S^m, S|S\right)$ and given that firm 2 chooses S, the best response for firm 1 is to chooses large capacity L

to enter at date t_L^m. On the other hand, $\alpha_1 < \alpha_L$ implies $F\left(\underline{t}_{S|L}, S|L\right) > F\left(\underline{t}_{L|L}, L|L\right)$. Provided that firm 1 will commit to its investment and there is no exit allowed, firm 2's best response is to choose S and enter at $\underline{t}_{S|L}$. ■

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