# Price Cap Regulation
# Preliminary version

Nicos Savva and Fabien A. Roques[*][†]
Judge Business School, University of Cambridge[‡]

February 24, 2006

**Abstract**

We study the effect of price cap regulation on investment in new capacity in an oligopolistic (Cournot) industry. We use a continuous time model with stochastic demand. The contribution of this paper is both theoretical and practical. On the theoretical side, we show that there exists an optimal price cap that maximizes investment incentives. Just as in the case of deterministic demand, the optimal price cap is independent of market concentration. However, unlike the deterministic case, we show that this price cap does not restore the competitive equilibrium; there is still under-investment and companies are still enjoying positive rents. On the practical side, we perform sensitivity analysis, comparative statics and monte carlo simulations to examine the effect of price cap regulation at different levels of demand volatility, market concentration and lead times. The findings demonstrate that price cap regulation is ineffective in increasing investment in volatile markets, with high concentration and significant lead times. This casts doubts to whether price cap regulation can be effective in mitigating market power in liberalized electricity markets.

**Keywords:** Real options, stochastic games, price cap regulation, electricity markets

**JEL code:** C73, D92, L51, L94.

---

[*]The authors are grateful to David Newbery, William Nuttall, Stefan Sholtes, Karsten Neuhoff and two anonymous referees for their comments on earlier drafts of this paper.

[†]Research support from the Cambridge-MIT Institute under the project 045/P 'Promoting Innovation and Productivity in Electricity Markets' is gratefully acknowledged.

[‡]Contact details: Nicos Savva, Fabien Roques, Judge Business School, Cambridge, CB2 1AG, UK. Email: n.savva@jbs.cam.ac.uk, fabien.roques@gmail.com

# 1 Introduction

The wave of electricity industry liberalisation initiated during the 1980s and 1990s in many countries has dramatically changed the structure of the industry. While transmission and distribution remain regulated monopolies, generation has become a competitive industry in which prices are set in a wholesale market. In the early years of liberalisation, the focus of academic research and regulatory scrutiny concentrated mainly on short-term market efficiency and competitiveness. As the first territories to liberalise – among which are included England and Wales and some U.S. states – have now reached the end of their first investment cycle, much attention is being paid to assessing the long-term dynamic performance of the liberalised electricity industry.

One of the key issues is whether a liberalised electricity industry can deliver adequate investment to maintain security of supply, both in the regulated transmission and distribution sectors, and in the competitive generation business. This entails at least three subquestions which are the focus of this paper. Will electricity markets deliver the right equilibrium level of investment? Will investments be timely to prevent any delay in construction that might lead to temporary periods of capacity scarcity and high prices? And what is the effect of market power mitigation procedures that cap generators' bids on investment?

Many industry experts and academics defend the view that the idiosyncrasies of the electricity industry (such as concentrated markets, investments that are capital intensive and subject to a long construction time lag, and remaining non-market mechanisms or regulatory interventions such as price caps) are likely to result in delayed or under-investment (e.g. de Vries, 2004, Stoft, 2002). Joskow (2003) concludes from his study of the New England electricity market:

> "I think that there are good reasons to believe that spot market prices for energy and operating reserves alone, [...] are unlikely to provide adequate incentives to achieve generating capacity levels that match consumer's preferences for reliability. A variety of market and institutional imperfections contribute to this problem."

The model presented in this paper concentrates on the level and timing of investment in electricity markets, although it is applicable to any industry characterised by oligopolistic competition, uncertain demand and irreversible investment. We present a continuous time model of irreversible investment in an oligopolistic industry with stochastic demand, and introduce two critical characteristics of investments in electricity markets, a price cap and a construction lag.

The contribution of this paper is twofold.

First, we solve the Nash-Cournot symmetric industry equilibrium and demonstrate that the investment price trigger is an increasing function of market concentration, demand uncertainty, and of the length of the construction lag (and therefore the industry installed capacity is a decreasing function of these parameters). We demonstrate that under demand uncertainty, perfectly-rational, well-informed, risk-neutral investors will delay the construction of new capacity in an industry characterised by oligopolistic (Cournot) competition and with a construction time lag. This result gives some weight to the claim that electricity markets are likely to see delayed or under-investment. While models in the existing literature explain delayed or under-investment by advocating the risk aversion or bounded rationality of investors, in our model under-investment stems out of the irreversibility of investment and the uncertainty of demand.

Second, the model offers some new insights about the intertemporal effects of price cap regulation on investment under uncertainty. Our results underline the importance of taking into account the option value effect arising out of uncertainty in demand. As in the static models, we demonstrate that the optimal price cap level corresponds to the perfect competition entry price, but setting the price cap at the competitive level does not realise the competitive investment outcome and leads to under-investment. Contrary to perfect competition models, the investment price trigger is a non monotonic function of the level of the price cap, as the price cap has two effects on investment which work in opposite directions. On the one hand, the price cap has a negative impact on the 'option value effect' associated with demand uncertainty, as it caps potential upside profits while leaving unchanged potential downside losses, thereby providing a disincentive to investment. On the other hand, when the price cap is binding, increasing capacity in a Nash Cournot game does not lead to a reduction in price, hence providing an incentive to increase investment. We find that for a price cap lower than the competitive entry price, the impact of the price cap on the 'option value effect' dominates, such that the investment price trigger is a decreasing function of the price cap. Conversely, the 'market power mitigation' effect dominates for a price cap higher than the competitive entry price, such that the investment price trigger is an increasing function of the price cap.

Moreover, we show that the optimal price cap is an increasing function of the volatility of demand and of the length of the construction time lag. Sensitivity analyses and simulations suggest that not recognising the option value effects arising out of uncertainty in demand when determining the optimal level of a price cap can have a significant negative impact on investment. We show for instance that a price cap set at a conventionally optimal level without taking into account demand uncertainty can actually be counterproductive in an industry characterised by relatively highly volatile demand, as it may reduce investment and increase prices. Similarly, a conventionally optimal price cap set without taking into account the impact of the construction time lag on investment might reduce investment as compared to the oligopolistic case without price cap, and distort technology choices in favour of the technologies with the shortest construction lead time.

The rest of the paper is organised as follows. Section two reviews the relevant literature. Section three presents the continuous time model of irreversible investment in an oligopolistic industry with stochastic demand and studies the impact of market concentration on the Cournot Nash equilibrium investment strategies of the firms. Section four and five expand the model by introducing successively a price cap and a construction time lag, and studying their impact on investment. Section six presents some simulations to provide an idea of the quantitative magnitude of the effects identified by the model in the case of the electricity industry.

## 2 Literature review

There is an extensive literature modelling investment under demand uncertainty in electricity markets. Most models explain investment delays by assuming either that investors have bounded rationality or that they are risk averse. Murphy and Smeers (2003) build a two stage capacity-expansion game and show that the two-stage, closed-loop formulation leads to greater capacity than an open-loop, single-period formulation. Ford (1999, 2001) and Olsina et al. (2005) use a systems dynamics approach to model investments in an electric system with time to build and price caps. They assume that investors have limited rationality and information and their simulations show investment delays and under-investment.

The second advocated reason for under or delayed investment under demand uncertainty is investors' risk aversion. Neuhoff and de Vries (2005) show that when demand is uncertain and investors are risk adverse, generating companies invest in less generation capacity than is optimal when they cannot sign long-term contracts with retailers to hedge their risks. Earle and Schmedders (2001) introduce demand uncertainty and agent's risk aversion in a Cournot model and show that the introduction of a price cap may lead to higher average prices and lower production quantities.

The model presented in this paper introduces a new approach to study the impact of some of the idiosyncrasies of electricity markets (such as oligopolistic competition, a *wholesale* price cap, and a construction time lag) on investment level and timing in the electricity industry. We build on the theory of irreversible investment under uncertainty (often referred to as Real Options theory) initiated by McDonald and Siegel (1986) and developped extensively in Dixit and Pindyck (1994).[1] We model investment in power generation as a two-stage continuous time game, in which generators optimise the capacity utilisation and decide on their capacity investment.[2] This model relates therefore also indirectly to the literature looking at the impact of demand uncertainty on capacity choices and bidding strategies of generators in the spot market. However, the focus of this literature strand is on strategic use (i.e. withholding) of capacity to increase prices in the spot market, rather than on investment equilibrium and timing (see e.g. Crampes and Creti, 2003 and Le Coq, 2002).

Turning to the issue of price caps in electricity markets, Joskow and Tirole (2004) explore the impact of a wholesale price cap below the competitive price level and find that it creates a shortage of peaking capacity in the long

[1] There are many papers using the theory of irreversible investment under uncertainty to study investment issues in electricity markets, such as the impact of input price risk (Murto and Nese, 2002), of construction cost risk (Pindyck 1993), or of construction modularity (Gollier et al., 2005) on technological choices.

[2] However, we will prove later that our choice of demand function makes the first stage trivial as investors always produce at full capacity.

run when there is market power in the supply of peaking capacity. Grobman and Carey (2001) run simulations of investment in an electricity market with a price cap for either a social welfare maximizing or monopolistic agent. Their results show that the long run effects of a price cap on investment and spot prices differ significantly based upon the market structure.

In a broader perspective, the extensive literature focussing on price-cap regulation is also relevant to this paper (see e.g. Beesley and Littlechild,1989, Laffont and Tirole, 1993, and Laffont and Tirole, 2001). Our model concentrates on the intertemporal impact of price cap regulation on investment under uncertainty. Other models looking at this issue include Dobbs (2004) which investigates the monopoly case, and Dixit (1991) and Pindyck and Dixit (1994), which deal with the perfect competition case. The model presented in this paper can be viewed as an intertemporal model of price cap regulation with stochastic demand which generalises Dobbs (2004) and Dixit (1991)'s monopolistic and perfect competition models to the case of an oligopolistic (Cournot) industry.

## 3 The model

In this section we introduce the basic framework of the model, based on Baldursson (1998) and Grenadier (2000 and 2002). The model presented has its roots in the theory of irreversible investment under uncertainty initiated by McDonald and Siegel (1986) and developed in Dixit and Pindyck (1994). More precisely, this paper belongs to the growing literature on "strategic real options", which has developed recently by combining the theory of real options with game theory.[3] The model characterises optimal irreversible investment decisions of firms in an industry characterised by stochastic demand and oligopolistic (Cournot) competition. Although we present the model by referring to the electricity industry, it can be applied to any industry characterised by stochastic peak demand, oligopolistic (Cournot) competition, and irreversible investment.

[3] See Smit and Trigeorgis (2004) for an extensive review of this literature.

## 3.1 Model assumptions

### 3.1.1 Demand

In the case of the electricity industry, it is important to distinguish demand variability from demand uncertainty. The former corresponds to usual daily and seasonal demand fluctuations which are easy to forecast, while demand uncertainty is due to unexpected events (e.g. plant or transmission line breakdowns) and to weather unpredictability. In this perspective, the model stochastic demand function should be interpreted as the "residual demand" or "net demand". That is, the demand that generators which are not engaged in forward contracts face in the wholesale market. Therefore, the uncertainty in demand encompasses both the effects of aggregate demand shocks (unforecast increase in demand at short notice due to changes in weather, e.g. wind or temperature) and supply shocks (sudden breakdown of a "must run" base load plant that had its output contracted forward).

**Definition 1** *The price for electricity is determined endogenously by the aggregate inverse demand function which takes the isoelastic form:*

$$P(t) = X(t)Q(t)^{-\frac{1}{\gamma}} \tag{1}$$

*were $X(t)$ is a stochastic process, $Q(t)$ is the aggregate demand, and $\gamma$ is the elasticity parameter.*

The use of such a constant elasticity demand function is common in Real Options models (e.g. Dixit 1994, Grenadier 2002, and Dobbs 2004), as it simplifies the searcch of closed form solutions. Furthermore such demand specification has one degree of freedom (the elasticity constant $\gamma$) which allows for some sensitivity analysis to this critical parameter in electricity markets.

**Definition 2** *The stochastic shock $X(t)$ is assumed to follow a Geometric Brownian Motion (GBM) given by the equation:*[4]

$$dX = X\mu dt + X\sigma dz \tag{2}$$

*where $\mu$ is the drift of the demand process and $\sigma$ is the instantaneous standard deviation of the process. $dz$ is a Wiener process.*

[4] This can be generalised to any Ito process.

This specification implies that $X(t)$ is lognormally distributed. The GBM is a fairly general process whose use is widespread in the Real Options literature even though concerns are frequently voiced about its shortcomings (for example its tails are too flat). However, the focus of this paper is on long-term investment, and Metcalf and Gilbert (1995) show that using a GBM instead of a mean reverting process has little impact on cumulative investment in a similar real options application.

### 3.1.2 Supply

There are $N$ power producers in the industry, which produce the same homogenous good, electricity, using the same technology of production. We restrict the analysis to $N$ symmetric firms and assume that there is no entry. The model therefore represents a market in which firms have been successful at raising barriers to entry, through for instance vertical integration into electricity supply.[5]

The $N$ plants sell their production in a wholesale market at a single node. We abstract from transmission constraints and network effects which would render the model intractable and are not the focus of this paper (most other generation investment models make a similar assumption, see e.g. Murphy and Smeers, 2005).

Moreover, we do not model the impact of forward contracts between generators and retailers and assume that generators' output is entirely sold in a wholesale market. The issue of how forward contracts impact on generators's bidding behaviour and profit in spot markets is a problem which has attracted lots of research, but it is not the focus of this paper (see e.g. Allaz and Vila, 1993, Newbery, 1998, Green, 1999, and Murphy and Smeers, 2005).[6]

All plants have the same capacity $k(t)$, so that the total production capacity in the industry at time $t$ is $K(t)$, with the following relation:

[5] In the light of the recent European electricity industry concentration movement, it seems credible that the industry will eventually be dominated by an oligopoly of 5 to 7 major vertically integrated companies.

[6] This assumption is justified to keep the model tractable, Murphy and Smeers (2002) for instance make the same assumption, as well as all other investment models papers mentioned in the literature review section.

$$K(t) = \sum_{i=1}^{N} k(t) = Nk(t) \tag{3}$$

The $N$ identical producers produce each an amount $q(t)$ at time $t$, such that the total quantity produced at time $t$ is:

$$Q(t) = \sum_{i}^{N} q(t) = Nq(t) \tag{4}$$

## 3.2 The game

The timing of the game is as follows: each producer observes the evolution of demand and decides first how much to produce, $q(t)$, up to its capacity constraint $k(t)$. Then each producer decides whether or not to increase production capacity and by how much. This two stage game is repeated in continuous time.

### 3.2.1 Firms' output decision

**Definition 3** *Normalizing variable and fixed costs to zero, the profit $\pi(t)$ that each producer makes per unit of production at time t is defined by:*

$$\pi(q_{0,}Q_{-i0,}, X_0, q(t), Q_{-i}(t), X(t)) = X(t)Q(t)^{-1/\gamma}q(t) \tag{5}$$

*where $(q_{0,}Q_{-i0,}, X_0)$ are the initial values of the state variables, and where $Q_{-i}(t)$ is the aggregate production of all firms but firm i at time t .*

We assume that all producers are profit maximizers, and that the market is concentrated enough (high $N$) or the demand is elastic enough (high $\gamma$) such that $N\gamma > 1$.

**Lemma 4** *Profit maximising producers produce at full capacity provided that $N\gamma > 1$.*

**Proof.** See Appendix 1

■

This Lemma makes the first stage of the game trivial: in the first stage agents will produce at full capacity and in the second they will decide if they want to invest in new capacity. In the rest of the paper, $q(t)$ refers equally to the firm's production output and installed capacity at time $t$ in order to simplify notation.

#### 3.2.2 Firms' irreversible investment problem

Investment in new capacity is assumed to be completely irreversible.[7] We assume that there is no technological progress nor physical depreciation of the assets, and the risk free discount rate is denoted by $\rho$. Capacity is infinitely divisible and becomes available instantaneously. We will relax this later assumption in the fifth section.

The firms continuously maximise their profit by expanding capacity incrementally whenever such strategy is profitable. When they increase capacity, they pay a sunk cost of $C$ per unit. The maximisation problem of firm $i$ at time $t$ consists of determining the firm's investment strategy, so that it maximises the expectation of its profit minus the cost of investing in more capacity. Each firm therefore faces a sequence of investment opportunities and must determine an exercise strategy for its path of investment. Agents have complete information, are rational, and risk neutral.

The optimal investment strategy of one firm has to take into account the other firms' investment decisions. It is determined as part of a Nash equilibrium, where firms compete à la Cournot. At each time $t$, firm $i$ will maximise the expected operating profit from existing as well as new capacity minus the cost of investing in more capacity, taking into account the strategies of the rest of the agents. We consider a closed loop equilibrium.

**Definition 5** *The capacity expansion problem of firm $i$ at time $t$ is an optimal control problem that can be formulated by the following objective functional $J$, where $\rho$ is the constant risk free discount rate, and the expectation*

[7] Investment is irreversible in the sense that firms cannot sell some of their capacity if demand drops. This assumption appears appropriate in electricity markets, in which investment is characterised by large sunk costs. In mathematical terms, the irreversibility assumption means that $k(t)$ is a non-decreasing function.

*operator E is conditional on the current state:*

$$J(q_0, Q_{-i0}, X_0, q(t), Q_{-i}(t), X(t)) =$$
$$\max_{q(t)\in[0,\infty)} \mathbf{E}[\int_{t=0}^{\infty} \pi(q_0, Q_{-i0}, X_0, q(t), Q_{-i}(t), X(t)) \exp(-\rho t) dt$$
$$- \int_{t=0}^{\infty} C \exp(-\rho t) dq(t)] \qquad (6)$$

Alternatively we can see each firm as owning a sequence of call options on the stochastic price of the output. The strike price is the investment costs. However, each firm fully recognises that the exercise of such options by its competitors will impact its own profits by reducing the market clearing price.

### 3.3 Nash equilibrium investment strategies

We restrict the analysis to symmetric Nash Cournot equilibrium, so that at time $t$, $q(t) = Q(t)/N$, for all firms. We follow Baldursson's (1998) and Grenadier's (2002) simplified approach to derive symmetric Nash-Cournot equilibrium strategies. Baldursson (1998) and Grenadier (2002) build on the seminal paper of Leahy (1993) and demonstrate that the investment Nash equilibrium investment strategy of an oligopolistic firm can be determined as if the firm was pursuing a form of myopic exercise strategy, while facing a modified aggregate demand curve.[8]

Assuming for a short while that the firms are price takers, the problem reduces to a standard application of the theory of investment under uncertainty developed in Pindyck and Dixit (1994), which can be solved by using either dynamic programming or contingent claims methods.

**Definition 6** *Let $V(q, Q_{-i}, X)$ be the optimal value of the objective functional $J$ defined by equation (6), which represents the value of the investment option for firm $i$ at time $t$.*[9]

As usual in the theory of irreversible investment under uncertainty, the decision rule will take the form of a critical price trigger $P^*$ such that it is

[8] Baldursson (1998) and Grenadier (2002) show that besides the monopoly and perfectly industry cases, it is also possible to solve the oligopoly case as a single agent optimisation problem. The procedure is just to pretend that the industry is perfectly competitive, maximising a "fictitious" objective function, using an "artificial" demand function.

[9] In rest of the paper we omit the time index $t$ to simplify notations.

optimal to invest once $P \geq P^*$. This price trigger $P^*$ for new investment constitutes a natural ceiling, above which the market price will never rise.

**Proposition 7** *The value $V$ of the investment option verifies the following differential equation when firms behave as price takers:*

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 V}{\partial^2 X} + \mu X \frac{\partial V}{\partial X} - \rho V + \pi = 0 \tag{7}$$

***Proof.*** *This is a standard result of Real Option theory obtained by applying Ito's lemma (see e.g. Dixit and Pindyck 1994).* ■

Following the transformation detailed in Grenadier (2002), let us change variables in order to work with $P$ as a stochastic variable, and define the marginal value of the firm $m$ as follows.

**Definition 8** *The marginal value of the firm $m(P(Q), Q_{-i}, q)$ is defined by $m = \frac{\partial V}{\partial q}(P(Q), Q_{-i}, q)$.*

We can now use the results obtained in the case of price taking firms to find the strategic Nash Cournot equilibrium investment strategies of the firms, using Grenadier's (2002) transformation.

**Proposition 9** *The symmetric Nash Cournot equilibrium investment strategies of the firms are defined by the following differential equation*

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 m}{\partial^2 P} + \mu P \frac{\partial m}{\partial P} - \rho m + \frac{\partial \pi}{\partial q} = 0 \tag{8}$$

*with*

$$\frac{\partial \pi}{\partial q} = P \frac{N\gamma - 1}{N\gamma} \tag{9}$$

*and the two free boundary conditions:*[10]

- *Value matching condition at the investment trigger $P^*$*

$$m(P^*(Q), Q_{-i}, q) = \frac{\partial V}{\partial q}(Q, Q_{-i}, q)\|_{V^*(Q,Q_{-i},q)} = C \tag{10}$$

[10] The smooth pasting conditions in this continuous time model are akin the first order necessary conditions for value maximisation in a static optimisation model.

- *Smooth pasting condition at the investment trigger $P^*$*

$$\frac{\partial m}{\partial P}(P^*(Q), Q_{-i}, q) = \frac{\partial^2 V}{\partial q \partial X}(Q, Q_{-i}, q)\|_{V^*(Q,Q_{-i},q)} = 0 \qquad (11)$$

***Proof.*** *See Grenadier (2002).* ■

The interpretation of this result is detailed in Grenadier (2002). The symmetric Nash Cournot equilibrium investment strategy of a firm corresponds to the equilibrium investment strategies of a myopic firm facing a modified aggregate demand curve, which is represented by the additional term $\frac{\partial \pi}{\partial q}$ in the differential equation (8).

### 3.4 Solution of the Nash equilibrium investment strategies

Let $\beta_1$ and $\beta_2$ be the roots of the second degree characteristic equation corresponding to equation (8) (see Appendix 2):

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}} > 1 \qquad (12)$$

and

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}} < 0 \qquad (13)$$

**Proposition 10** *The marginal value of the firm $m$ takes the following form:*

$$m(P(Q), Q_{-i}, q) = H_0(Q)P^{\beta_1} + A_0 P \qquad (14)$$

*with*

$$H_0(Q) = \frac{N\gamma - 1}{N\gamma(\mu - \rho)} \frac{1}{\beta_1}[C \frac{N\gamma(\mu - \rho)\beta_1}{(N\gamma - 1)(1 - \beta_1)}]^{1-\beta_1} \qquad (15)$$

*and*

$$A_0 = \frac{N\gamma - 1}{N\gamma(\rho - \mu)} \qquad (16)$$

***Proof.*** *See Appendix 2.* ■

**Proposition 11** *The investment price trigger $P^*$ is given by:*

$$P^* = C\,(\rho - \mu)\,\frac{\beta_1}{(\beta_1 - 1)}\frac{N\gamma}{(N\gamma - 1)} \qquad (17)$$

*with the condition $\gamma > \frac{1}{N}$. The investment trigger is a decreasing function of the number of firms $N$ in the industry.*

***Proof.*** *See Appendix 2.* ■

The first two terms are familiar. $P = C(\rho - \mu)$ represents the investment price trigger in a competitive industry without uncertainty, (see e.g. Laffont and Tirole, 2000, p.151). $\frac{\beta_1}{(\beta_1 - 1)} > 1$ is often termed an option value multiplier, as $P^*_{N=\infty} = C(\rho - \mu)\frac{\beta_1}{(\beta_1 - 1)}$ is the investment price trigger in a competitive industry under uncertainty (see e.g. Dixit and Pindyck, 1994).

### 3.5 Impact of market concentration on investment

The investment price trigger $P^*$ in the oligopolistic industry can be expressed as a function of the investment price trigger in a perfectly competitive industry:

$$P^* = \alpha P^*_{N=\infty} \tag{18}$$

where

$$\alpha = \frac{N\gamma}{(N\gamma - 1)} > 1 \tag{19}$$

and

$$P^*_{N=\infty} = C(\rho - \mu)\frac{\beta_1}{(\beta_1 - 1)} \tag{20}$$

$\alpha = \frac{N\gamma}{(N\gamma - 1)} > 1$ can be interpreted as a demand elasticity mark-up. That is, the investment price trigger at which a Cournot oligopoly adds to capacity $P^*$ is equal to the competitive investment entry price trigger $P^*_{N=\infty}$ multiplied by the oligopoly mark-up $\alpha$. Figure 4.1 illustrates the impact of market concentration on the investment trigger in an oligopolistic industry and a perfectly competitive industry with and without price cap. The investment price trigger is a decreasing function of the number of firms, and since the oligopoly mark-up $\alpha > 1$, the firms in the oligopolistic industry only add to capacity when price reaches a higher value than would be the case under competition. Prices are therefore at all times higher under oligopolistic competition, while, concomitantly, installed capacity is less.

Moreover, we retrieve in our model the expression of the investment price trigger $P^*$ in the two particular cases of monopoly and perfect competition. In the limit of a perfectly competitive industry (as $N$ goes to infinity), $\alpha$ tends towards one and equation (17) tends towards equation (20), which corresponds to the investment price trigger in a perfectly competitive industry (see Dixit and Pindyck (1994)). Moreover, when $N = 1$, $\alpha = \frac{\gamma}{\gamma - 1}$ which corresponds to the monopoly mark-up identified in Dobbs (2004).

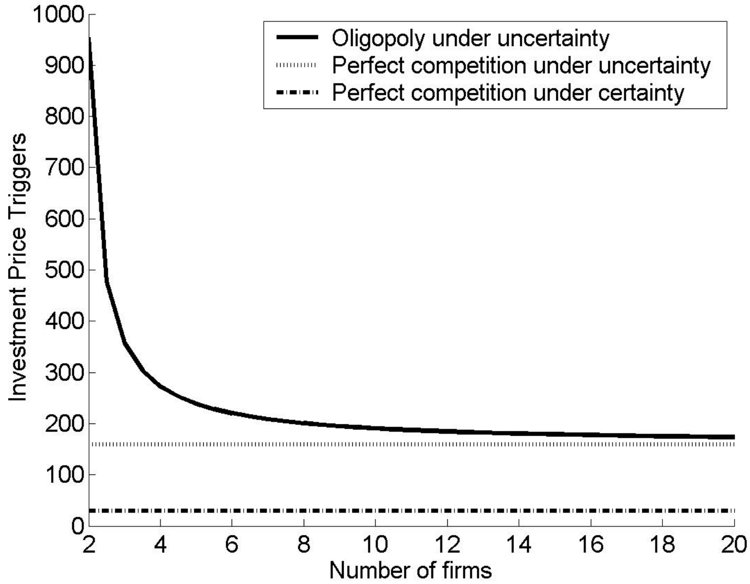


Figure 4.1 - Price Triggers vs. Market Concentration
$C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08$

In the next two sections we introduce successively two critical features of electricity markets into the Cournot Nash equilibrium irreversible investment model described in this section. We study successively the impact of a wholesale price cap and of a construction time lag on the level and timing of investment.

# 4 Impact of a price cap

Let us now assume that prices in the *wholesale* market are capped at a predetermined level by the regulator, $\bar{P}$. Two types of price caps need to be distinguished in electricity markets, namely *retail* and *wholesale* market price caps. This stylised model concentrates on the impact of *wholesale* price caps (see Joskow and Tirole (2004) and Stoft (2002) for a technical discussion of the impact of retail and wholesale price caps in electricity markets). Wholesale price caps constitute a widespread tool used by regulators to mitigate market power exercise in electricity markets, but are believed to have important side effects on investment.[11] Fraser (2003) highlights for instance

[11] In PJM and most of the US East Coast wholesale markets, there is for instance a 1000$/MWh bidding price cap.

the detrimental impact of price caps on investment that caused temporary capacity shortages and price increases in the late 1990s in Australia and in Canada (Alberta and Ontario). The price cap in this model can also be interpreted more broadly as any kind of regulatory rule that prevents prices from moving freely up when the market is tight. Such regulatory rules and engineering constraints on wholesale markets are detailed for instance in Stoft (2002), Joskow (2003), and Joskow and Tirole (2004).

### 4.1 Solution of the Nash-Cournot investment equilibrium strategies with a price cap

In this section, $P$ represents the hypothetical market-clearing or "shadow" price, which is monotonically related to the pressure of demand through equation (1). The investment price trigger above which it is optimal for firms to invest in more capacity is denoted $\bar{P}^*$ when a price cap $\bar{P}$ is implemented.

An imposed price cap higher than the natural investment trigger is simply irrelevant, as voluntary investment decisions will always generate enough capacity to keep the price below the price cap in the model stylised setting (next section will make the model more realistic by introducing a construction time lag). In other words, necessarily $\bar{P} \leq \bar{P}^*$ because $\bar{P}^*$ is a reflecting boundary for the price process. When the price is at the ceiling $\bar{P}$, excess demand is rationed in such a way that generators do not capture any of the scarcity rent.

**Definition 12** *The profit $\pi$ that each producer makes per unit of production at time $t$ is given by:*

$$\pi(q_{0,}Q_{-i0,}, X_0, q, Q_{-i}, X) = \min\left\{XQ^{-1/\gamma}q, \bar{P}q\right\} \tag{21}$$

*where $(q_{0,}Q_{-i0,}, X_0)$ are the initial values of the state variables, and where $Q_{-i}$ is the aggregate production of all firms but firm $i$ at time $t$.*

Solving now equation (8) using (21) requires us to distinguish two different cases depending on the level of the price cap $\bar{P}$:

1. Non-binding Price cap

   If $P \leq \bar{P}$, the price cap is not binding and we can use the results of the previous section. The marginal value of the firm $m$ takes the following

form:
$$m(P(Q), Q_{-i}, q) = H_1(Q)P^{\beta_1} + \frac{N\gamma - 1}{N\gamma(\rho - \mu)}P \qquad (22)$$

2. Binding Price cap

   If $P \geq \bar{P}$, new investment is made only when the pressure of demand rises to a critical level $\bar{P}^*$ at which the shadow price exceeds the imposed price cap $\bar{P}$ at which the actual price is stuck. Equation (9) becomes: $\frac{\partial \pi}{\partial q} = \bar{P}$

**Lemma 13** *When prices are capped at $\bar{P}$, the marginal value of the firm $m$ takes the following form (with $\beta_1 > 1$ and $\beta_2 < 0$ being the roots of the second degree characteristic equation corresponding to equation (8), see Appendix 2 for an analytic expression):*

$$m(P(Q), Q_{-i}, q) = H_2(Q)P^{\beta_1} + H_3(Q)P^{\beta_2} + \frac{\bar{P}}{\rho} \qquad (23)$$

*and verifies the two free boundary conditions:*

- *Value matching condition at the investment trigger $\bar{P}^*$*

$$m(\bar{P}^*(Q), Q_{-i}, q) = C \qquad (24)$$

- *Smooth pasting condition at the investment trigger $\bar{P}^*$*

$$\frac{\partial m}{\partial P}(\bar{P}^*(Q), Q_{-i}, q) = 0 \qquad (25)$$

***Proof.*** *Straightforward by following the same calculation steps as in Appendix 2.* ■

Bringing now the two cases in which the price cap is binding and in which it is not together, the value matching and continuity conditions at the price cap $\bar{P}$ give two additional boundary conditions, where the notation $\bar{P}^{(-)}$ and $\bar{P}^{(+)}$ refer respectively to the limit of the function considered below and above the price cap $\bar{P}$ :

$$m\left(\bar{P}^{(-)}, Q\right) = m\left(\bar{P}^{(+)}, Q\right) \qquad (26)$$

$$\frac{\partial m}{\partial P}\left(\bar{P}^{(-)}, Q\right) = \frac{\partial m}{\partial P}\left(\bar{P}^{(+)}, Q\right) \tag{27}$$

The system of four equations (24), (25), (26), and (27) with four unknowns $(H_1, H_2, H_3, \bar{P}^*)$ defines the symmetric Nash Cournot equilibrium investment strategies of a firm when prices are capped at $\bar{P}$. See Appendix 3 for an analytical expression.

## 4.2 Impact of a price cap on the oligopolistic investment price trigger

**Proposition 14** *When the regulator caps prices at $\bar{P}$, the investment price trigger $\bar{P}^*$ is given by:*

$$\bar{P}^* = [\lambda(\frac{\bar{P}}{\rho} - C)\bar{P}^{(\beta_2-1)}]^{1/\beta_2} \tag{28}$$

*with $\lambda$ defined as:*

$$\lambda = \frac{\beta_1}{\frac{(1-\beta_1)}{\alpha(\rho-\mu)} - \frac{\beta_1}{\rho}} \tag{29}$$

*where $\alpha$ is the elasticity mark-up defined previously:*

$$\alpha = \frac{N\gamma}{(N\gamma - 1)} > 1 \tag{30}$$

***Proof.*** *See Appendix 3.* ■

**Proposition 15** *There exists an interval $[\bar{P}_{\min}, \bar{P}_{\max}]$ over which the introduction of a price cap lowers the investment price trigger as compared to the oligopolistic industry investment trigger without price cap (i.e. $\bar{P}^* < P^*$). $\bar{P}_{\max}$ corresponds to the investment price trigger $P^*$ without price cap, over which any price cap is irrelevant.*

*Let $\bar{P}_{opt}$ denote the price cap level which minimises the investment price trigger $\bar{P}^*$. For $\bar{P} < \bar{P}_{opt}$, the investment price trigger $\bar{P}^*$ is a decreasing function of the level of the price cap $\bar{P}$, and tends towards infinity as $\bar{P}$ is lowered to zero.*

*Over the interval $[\bar{P}_{opt}, \bar{P}_{\max}]$, the investment price trigger $\bar{P}^*$ is an increasing function of the level of the price cap $\bar{P}$.*

***Proof.*** *See Appendix 4 and 5. No analytical solution for $\bar{P}_{\min}$ can be obtained in the general case, but an expression of the non-linear polynomial* *inequality is given in Appendix 4. Moreover, in the particular case in which* $\beta_2 = -1$, *an analytical solution of* $\bar{P}_{\min}$ *is given in appendix 4.* ■

Figure 4.2 shows the evolution of the investment price trigger $\bar{P}^*$ as a function of the level of the price cap $\bar{P}$. A first interesting result is that the relationship between the investment price trigger $\bar{P}^*$ and the level of the price cap $\bar{P}$ is not monotonic. This result contrasts with Dixit (1991) and Dixit and Pindyck (1994)'s findings that under perfect competition, the investment price trigger is a decreasing function of the level of the price cap.

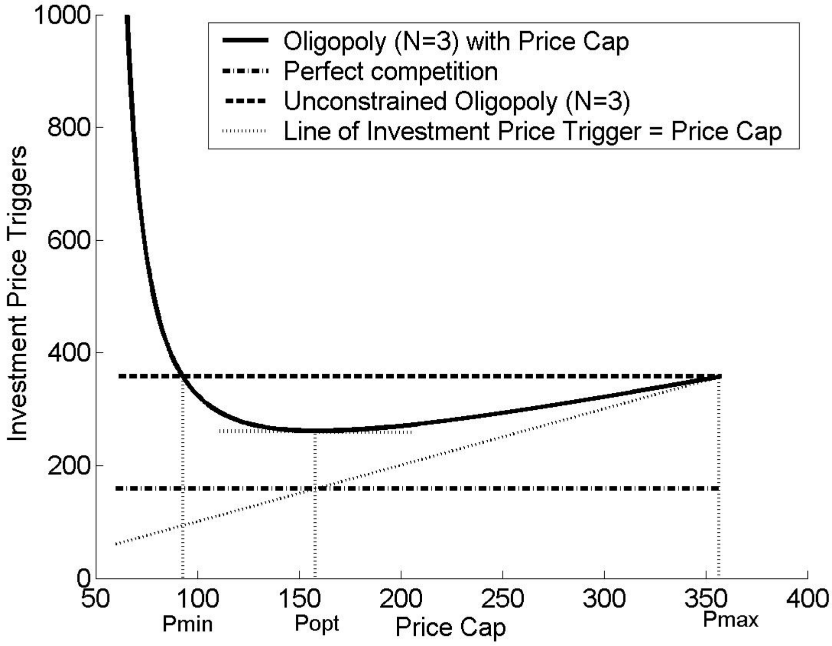


Figure 4.2 - Investment Price Trigger vs. Price Cap,
$C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08, N = 3.$

The intuition for this result is that under oligopolistic competition, the price cap has an impact not only on the *option value* characterising irreversible investments under uncertainty, but also has a *market power mitigation effect.* These two effects work in opposite directions as regards to investment incentives.

- Impact of a price cap on the *option value*

On the one hand, a price cap has a negative impact on the option value effect associated with demand uncertainty, as it caps potential upside profits while leaving unchanged potential downside losses, thereby providing a disincentive to investment. A price cap reduces the likelihood of future high profits, so that investors need to be confident that the pressure of demand will stay high for longer (and hence the price will be equal to the price cap) than when there is no price cap in order to commit to new investment. Hence, a tighter price cap implies that a greater current pressure of demand is needed to bring about investment. As Dixit (1991) explains in his model of perfect competition,

> "As the imposed ceiling is lowered toward the long-run average cost, the critical shadow price that induces new investment goes to infinity. In other words, if the imposed price cap is so low that at this point the return on capital is only just above the normal rate, then investors want to be assured that this state of affairs will last almost forever before they will commit irreversible capital."

- Impact of a price cap on *market power mitigation*

As seen before, under symmetric oligopolistic (Cournot) competition without price cap, the value maximising strategy of a firm is to reduce capacity as compared to the perfect competition case, in order to increase prices. But when there is a price cap, in the case where it is binding, increasing capacity in a Nash Cournot game does not lead to a reduction in price, hence providing an incentive to increase investment. In other words, a price cap reduces the ability of firms to leverage their market power by reducing capacity. Indeed, when the price cap is binding, firms have an incentive to invest as the increased capacity will not reduce price. A tighter price cap should therefore reduce the investment price trigger $\bar{P}^*$ and thereby induce greater investment and lower prices.

A price cap has therefore a *dual impact* on the investment price trigger in an oligopolitic industry, which explains the two regimes observed on Figure 2.

Over the interval $[\bar{P}_{opt}, \bar{P}_{\max}]$, the positive *market power mitigation impact* of the price cap dominates the negative impact of the price cap on the *option value effect*, so that the investment price trigger $\bar{P}^*$ is an increasing function of the level of the price cap $\bar{P}$.

On the contrary, for a price cap lower than the competitive entry price $\bar{P}_{opt}$, the impact of the price cap on the *option value effect* dominates, such that the investment price trigger $\bar{P}^{*}$ is a decreasing function of the level of the price cap $\bar{P}$ .

Moreover, the implementation of a price cap at a level lower than $\bar{P}_{\min}$ would be counterproductive, as it raises the investment price trigger above the oligopolistic price trigger without a price cap. As the price cap is lowered to zero, the investment price trigger tends towards infinity, as in the case of perfect competition.[12]

### 4.3 The optimal price cap

Let us now search for the optimal level of the price cap $\bar{P}_{opt}$, that is the level at which a regulator aiming to minimise the investment price trigger (and hence to maximise installed capacity and minimise prices) should set the price cap.

**Proposition 16** *The optimal level of the price cap* $\bar{P}_{opt}$ *is given by the following expression:*

$$\bar{P}_{opt} = \frac{\rho C(\beta_2 - 1)}{\beta_2} \tag{31}$$

*which is equal to (20),the investment price trigger in a competitive industry, and does not depend on* $N$ :

$$\bar{P}_{opt} = P^{*}_{N=\infty} = C\,(\rho - \mu)\,\frac{\beta_1}{(\beta_1 - 1)}$$

***Proof.*** *See appendix 5.* ■

In other words, under uncertainty, similarly to the certainty case, the optimal level of the price cap is equal to the competitive industry investment trigger price. Equation (31) indicates that the optimal price cap in this intertemporal model does not depend on the market concentration, but does depend on the volatility of demand and on the discount rate.

[12] These results are moreover consistent with Dobbs (2004), who studies the impact of price cap on a monopoly in a similar setting.

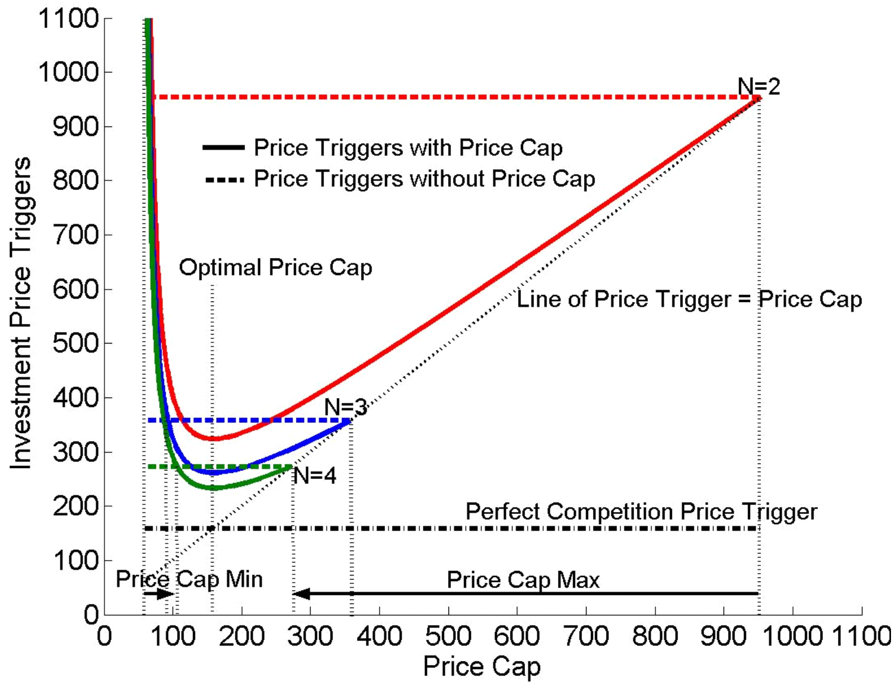


Figure 4.3 - Investment Price Triggers vs. Price Cap for different degrees of Market Concentration,
$C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08, .N = 2, 3, 4.$

Figure 4.3 illustrates the point that the optimal price cap does not depend on the market concentration, but that the bounds of the interval $[\bar{P}_{\min}, \bar{P}_{\max}]$ do. The higher the industry concentration (the lower $N$), the lower $\bar{P}_{\min}$, and the larger $\bar{P}_{\max}$. This is a fairly intuitive result: the higher the industry concentration, the larger the interval over which the introduction of a price cap is benefitial and lowers the investment price trigger as compared to the oligopolistic industry investment trigger without price cap. This generalises Dobbs' (2004) similar results obtained in the case of a monopoly to the case of a Cournot oligopoly.

Figure 4.4 shows two sensitivity analyses of the impact of demand volatility on the optimal price cap $\bar{P}_{opt}$, and of the impact of market concentration on the price range $[\bar{P}_{\min}, \bar{P}_{\max}]$ over which a price cap lowers the industry investment price trigger as compared to the oligopolistic industry investment trigger without a price cap.

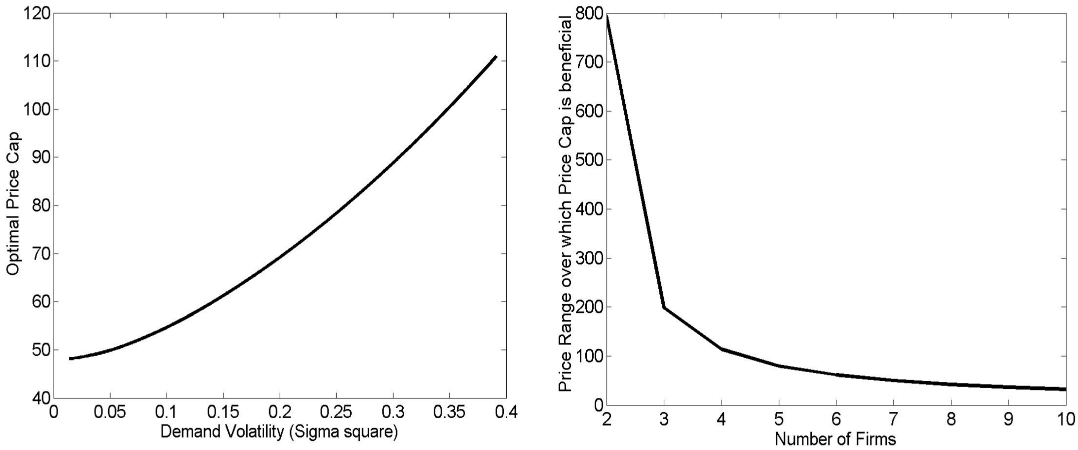


Figure 4.4 - Optimal Price Cap vs. Demand Volatility and Price Range over which a Price Cap lowers the Investment Trigger vs. Market Concentration, $C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08, N = 3$.

## 4.4 Efficiency of a Price Cap

The previous subsection revealed that, contrary to the results of models without demand uncertainty, setting the price cap at the competitive level $\bar{P}_{opt}$ does not realise the competitive outcome, as $\bar{P}^* (\bar{P}_{opt}) > P^*_{N=\infty}$, which is clearly illustrated on Figures 4.2 and 4.3. In other words, even with the optimal price cap, under uncertainty there will be under-investment and periods during which firms will impose quantity rationing as compared to the perfect competition case.

In this section, we present sensitivity analyses to investigate the efficiency of a price cap as a function of market concentration and demand volatility.

**Definition 17** *We define the efficiency e of a price cap $\bar{P}$ by the following formula*

$$e = 1 - \frac{\bar{P}^* - P^*_{N=\infty}}{P^* - P^*_{N=\infty}}$$

*where $P^*$ is the investment price trigger in the oligopolistic industry without price cap, $\bar{P}^*$ is the investment price trigger when the regulator caps prices at $\bar{P}$, and $P^*_{N=\infty} = C\left(\rho - \mu\right) \frac{\beta_1}{(\beta_1 - 1)} = \bar{P}_{opt} = \frac{\rho C(\beta_2 - 1)}{\beta_2}$ is the investment price trigger in a competitive industry under uncertainty without a price cap.*

The efficiency of a price cap can be understood as a measure of how close the price cap can take the market to the competitive outcome if set optimally.[13] It takes the value of 1 if it retrieves the competitive outcome and it is zero if, at best, it is irrelevant.

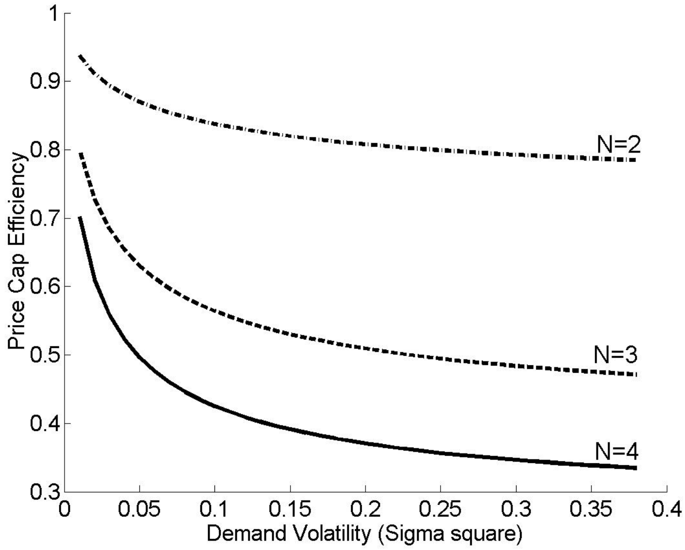


Figure 4.5 - Price Cap Efficiency vs. Demand Volatility, $C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08, N = 2, 3, 4.$

Figure 4.5 confirms the intuition, that the more concentrated the market, the more efficient a price cap is. Moreover, Figure 4.5 shows that the efficiency of a price cap depends critically on demand uncertainty, in particular when the market concentration is not too important.

## 4.5 Impact of neglecting demand volatility when setting the price cap

The previous results suggest that regulators should be careful to take into account the option value effects arising out of demand uncertainty when introducing a price cap. Figure 4.6 illustrates the importance of recognising this impact of demand uncertainty on the optimal price cap. It investigates the percentage error made when computing the optimal price cap and not

[13]An interesting extension left for further research would be to study the impact of a price cap on the market efficiency, i.e. on a measure of social welfare such as e.g.the sum of the firms' profits and of the consumer surplus. A proper evaluation of the latter would, however, require a more detailed study of quantity rationing: we assume in this model that quantity rationing is perfect, while in practice in electricity markets the lack of real time metering implies that load is unlikely to be curtailed according to consumers' value of lost load.

taking into account demand volatility. For instance, a price cap set at an optimal level when demand is constant represents a 39% under-estimation of the optimal price cap when the actual demand volatility is characterised by $\sigma^2 = 0.01$, and a 67% under-estimation when the actual demand volatility is characterised by $\sigma^2 = 0.1$. For relatively low actual volatilities of demand, such an under-estimated price cap would still lower the investment price trigger as compared to the non-regulated oligopoly case, and therefore increase investment. But for higher volatilities of demand, the price cap would be so low as compared to the optimal price cap that it is likely to be counter productive, in the sense that it would be so low as to to increase the investment price trigger as compared to the oligopolistic investment price trigger without a price cap, and therefore lead to under-investment.

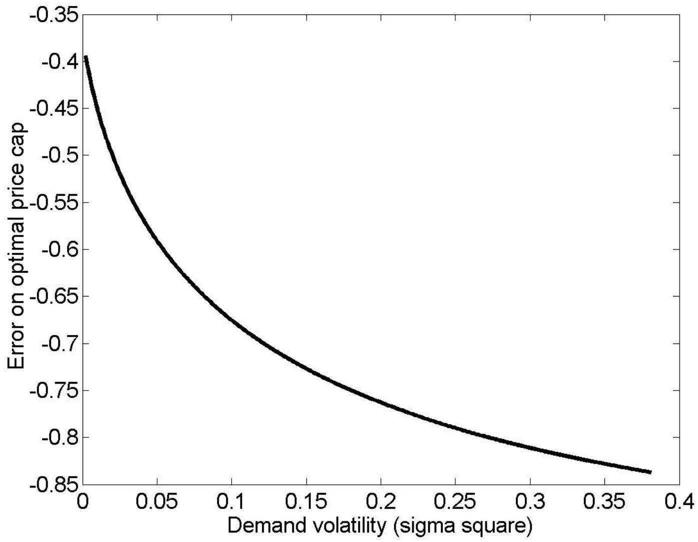


Figure 4.6 - Error on Optimal Pice Cap when not Taking into account Demand Volatility, $C = 600, \mu = 0.03, \gamma = 0.6, \rho = 0.08, N = 3$.

# 5 Impact of a construction time lag

In this section we introduce a construction time lag in the model, building on Bar-Ilan and Strange (1996), as well as on Grenadier's (2000 and 2002) models of real estate markets. The construction time lag is an important characteristic of investment projects in the electricity industry: for instance, the time lag between the investment decision to build a new power plant and the production start spans between three (gas turbine) and six years (nuclear plant) in the best case. This includes the time necessary to get regulatory and local approval, the actual construction period, and the final testing and commissioning stage.

This construction time lag is an important factor to be considered in the investment strategy of a firm, in particular when the determinants of the profitability of the investment are uncertain.[14] In our model, the profitability of a capacity increase will be determined by the electricity price when this capacity will become available. This electricity price will in turn depend on the demand when the capacity becomes available, as well as on the production capacity of the other producers at that time. This implies that the construction time lag should modify investors strategies as follows: investors must take into account both forecasts of future demand and the impact on the future price of production capacity which is being built at the time of the investment decision and which will become available in the future.

Let us call $\theta$ the time-lag between the time $t$ at which an investor takes the decision to increase capacity, and $t+\theta$ the time at which this capacity becomes operational.

**Lemma 18** *Since all the units under construction at time $t$ will be completed by time $t+\theta$, the profit flow during the period $[t, t+\theta]$ depends only on the capacity investments over the period $[t-\theta, t]$, while the profit flow after time $t+\theta$ depends only on the current level of committed capacity.*
***Proof.*** *See Grenadier (2000) for a formal proof.* ■

The marginal value of a firm $m(X(t), q(t-\theta), Q_{-i}(t-\theta))$ can therefore be expressed as follows:

$$m(t) = E\left[\begin{array}{c} \int_{t=\theta}^{\infty} \frac{\partial}{\partial q_i} \left(\pi_i \left(q_i(t-\theta), Q_{-i}(t-\theta), X(t)\right)\right) \exp(-\rho t) dt \\ -\frac{\partial}{\partial q_i} \int_{t=0}^{\infty} C \exp(-\rho t) dq_i(t) \end{array}\right] \tag{32}$$

where the expectation $E$ is taken over the stochastic parameter $X(t)$.

In the case where there is no price cap, the procedure to solve this integral is classic (see Grenadier 2000). A sketch of the proof is given in Appendix

[14] Gardner and Rogers (1999) examine a capacity mix model under demand uncertainty that explicitly accounts for differences in technology lead times, and show that lead time is a key design parameter that needs to be considered by investors alongside capital and operating costs.

6. When there is no price cap, the conditional expectation of demand with a construction lag of $\theta$ is:

$$\exp(-\rho\theta)E[(X(t+\theta)\lceil X(t)] = \exp((\mu-\rho)\theta)(X(t)) \tag{33}$$

When there is a price cap at $\bar{P}$, the conditional expectation of demand with a construction lag of $\theta$ is simply:

$$\exp(-\rho\theta)E[(X(t+\theta)\lceil X(t)] = \exp(-\rho\theta)(X(t)) \tag{34}$$

This implies that the construction time lag just introduces an extra multiplicative discounting factor of respectively $\exp(-\rho\theta)$ and $\exp((\mu-\rho)\theta)$ in the cases with and without price cap in the modified demand curve faced by an investor. It can be noticed that for $\theta = 0$, the discounting factor is equal to one.

## 5.1 Impact of a construction lag on the oligopolistic investment price trigger

We now turn to the impact of the construction time lag $\theta$ on the investment price triggers with and without price cap at $\bar{P}$ (denoted respectively now by $\bar{P}^*_\theta$ and $P^*_\theta$). When there is no price cap, the problem is standard and has been solved in Grenadier (2000).

In the case where there is a price cap at $\bar{P}$, deriving the investment trigger is more complex insofar as we need to distinguish two cases, as in section 3, depending on whether the price cap is binding or not. Following the same procedure as in section 3, we obtain a non linear system of four equations with four unknowns, which is detailed and solved in Appendix 7.

**Proposition 19** *The Nash Cournot equilibrium investment price trigger* $\bar{P}^*_\theta$ *with a construction lag of* $\theta$ *is given by the following formula:*

$$\bar{P}^*_\theta = [\frac{\beta_1(\exp(\rho\theta)C - \frac{\bar{P}}{\rho})}{\exp(\mu\theta)\frac{\beta_1-1}{\alpha(\rho-\mu)} - \frac{\beta_1}{\rho}}\bar{P}^{(\beta_2-1)}]^{1/\beta_2} \tag{35}$$

*where*

$$\alpha = \frac{N\gamma}{(N\gamma-1)} > 1 \tag{36}$$

**Proof.** See Appendix 7. ■

It is interesting to compare the investment price trigger $\bar{P}_\theta^*$ with a construction lag of $\theta$ with the investment price trigger $\bar{P}^*$ without construction lag:

$$\bar{P}^* = [\frac{\beta_1(C - \frac{\bar{P}}{\rho})}{\frac{\beta_1-1}{\alpha(\rho-\mu)} - \frac{\beta_1}{\rho}}\bar{P}^{(\beta_2-1)}]^{1/\beta_2} < \bar{P}_\theta^* \tag{37}$$

We can first notice that the two expressions are identical when $\theta = 0$. Moreover, we can interpret the two supplementary multiplicative factors in the expression of the investment price trigger $\bar{P}_\theta^*$ with a construction lag of $\theta$ years as follows.

The construction lag leads investors to modify their investment strategies in two ways. First, investors incorporate in the residual demand they expect to face the expected rate of demand growth over the coming $\theta$ years, and therefore multiply the fictional demand term at time $t$ by $\exp(\mu\theta)$, where $\mu$ represents the growth trend of the Geometric Brownian Motion characterising the evolution of demand.

Second, investors modify their investment strategies by taking into account the return on capital lost by spending money at time $t$ for an investment that will only start generating returns at time $t + \theta$ . Hence they simply increase the cost of investing today $C$ by the return on capital they could earn during the $\theta$ years it takes for the capacity to become operational, therefore replacing $C$ by $\exp(\rho\theta)C$, where $\rho$ represents the risk free discount rate.

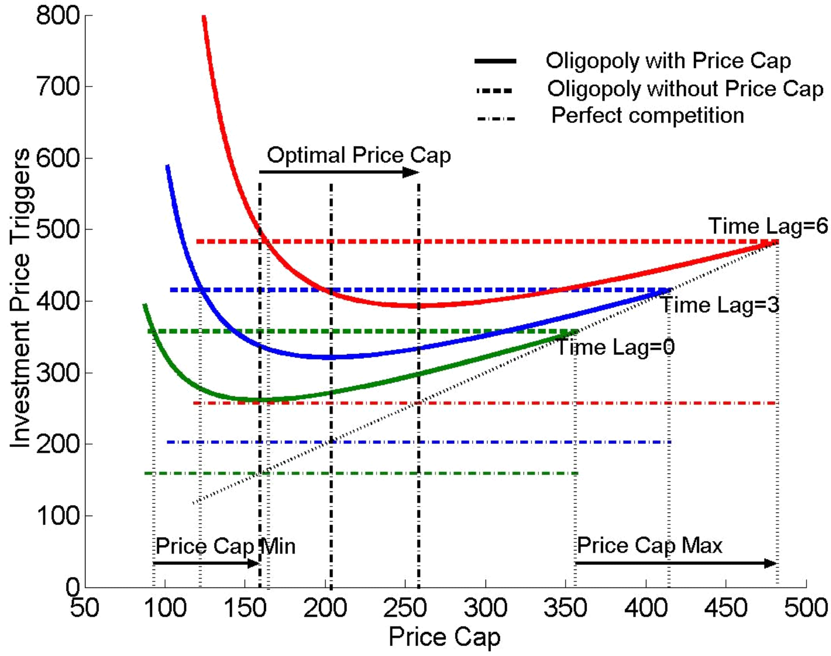


Figure 4.7 - Investment Price Triggers vs. Price Cap for Different Construction Time Lags (years),
$C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08, N = 3.$

Figure 4.7 shows the impact of the price cap on the investment price trigger for different construction time lags. It shows that a construction time lag increases firms' investment price triggers because of the increased cost of investing today when taking into account the risk free discount rate, as explained before. The next subsection will focus on the impact of a construction time lag on the optimal price cap and on the efficiency of price cap regulation.

## 5.2 Optimal choice of a price cap with a construction lag

As shown by equation (35), the Nash Cournot equilibrium investment price trigger $\bar{P}^*_\theta$ with a construction lag of $\theta$ is a function of the price cap level $\bar{P}$. We now derive the optimal intertemporal price cap $\bar{P}_{\theta opt}$ that a regulator can chose to maximise investment and lower prices.

**Proposition 20** *The optimal level of the price cap* $\bar{P}_{\theta opt}$ *with a construction* *time lag of $\theta$ years is given by:*

$$\bar{P}_{\theta opt} = \exp(\rho\theta)\frac{\rho C(\beta_2 - 1)}{\beta_2} \tag{38}$$

**Proof.** See Appendix 8. ■

This expression indicates that similarly the case without construction time lag studied in the previous section, the optimal price cap does not depend on market concentration, but does depend on the volatility of demand and on the discount rate. Figure 4.8 shows that the optimal price cap $\bar{P}_{\theta opt}$and the price range $[\bar{P}_{\theta \min}, \bar{P}_{\theta \max}]$ over which a price cap lowers the industry investment price trigger as compared to the oligopolistic industry investment trigger without price cap are increasing (exponential) functions of the construction time lag $\theta$ (years).

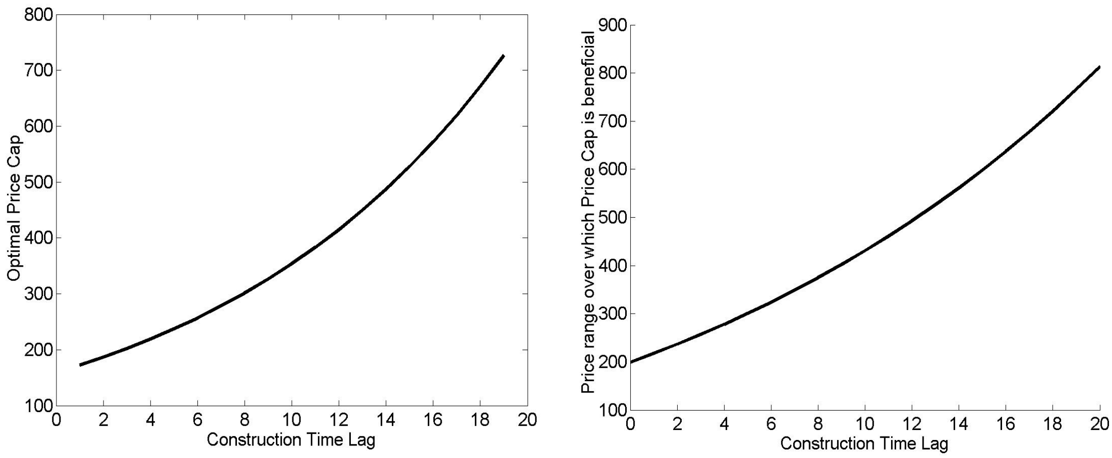


Figure 4.8 - Optimal Price Cap and Price Range over which a Price Cap lowers the Investment Trigger vs. Contruction Time Lag (years), $C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08, N = 3.$

It is interesting to compare the expression of the optimal level of the price cap with a construction lag of $\theta$ years $\bar{P}_{\theta opt}$with the expression (31) of the optimal level of the price cap without construction lag $\bar{P}_{opt}$. The two are linked by the following simple relation:

$$\bar{P}_{\theta opt} = \exp(\rho\theta)\bar{P}_{opt} \tag{39}$$

This can be interpreted as follows: when determining the optimal level of a price cap, the regulator should take into account the impact of the construction lag on investors' investment decisions by setting the price cap at a higher level than it would be without construction time lag. This increase of the price cap should reflect the return on capital that investors would earn on the money spent on their investment during the $\theta$ years it takes for the capacity to become available. The regulator should therefore multiply the optimal price cap without construction time lag by the factor $\exp(\rho\theta)$, where $\rho$ represents the risk free discount rate.

## 5.3 Impact of a construction time lag on the efficiency of a price cap

Figure 4.9 shows that the efficiency of a price cap (defined previously as a measure of how close the price cap can take the market to the competitive outcome if set optimally) is a concave decreasing function of the construction time lag $\theta$. This raises questions as to the efficiency of price cap regulation in industries which exhibit both long lead times and significant demand uncertainy, such as the electricity industry.

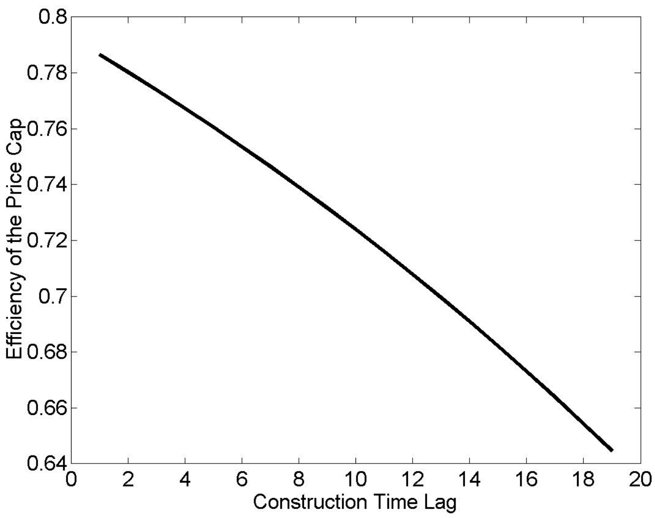


Figure 4.9 - Impact of a Construction Time Lag on the Efficiency of a Price Cap, $C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08, N = 3$.

## 5.4 Impact of not taking into account the construction time lag

The previous results suggest that regulators should be careful to take into account the construction time lag characterising the industry when introducing a price cap. Figure 4.10 illustrates the importance of recognising this impact of the construction time lag on the optimal price cap. It investigates the percentage error made when computing the optimal price cap and not taking into account the construction time lag. For instance, a price cap set at an optimal level if there was no construction time lag represents a 10% under-estimation of the optimal price cap when the actual construction time lag is 3 years, and a 50% under-estimation when the actual construction time lag is 6 years. For relatively short construction time lags, such an under-estimated price cap would still lower the investment price trigger as compared to the non-regulated oligopoly case, and therefore increase investment. But for longer construction time lags, the price cap would be so low as compared to the optimal price cap that it is likely to be counter productive, in the sense that it would be so low as to to increase the investment price trigger as compared to the oligopolistic investment price trigger without a price cap, and therefore lead to under-investment.

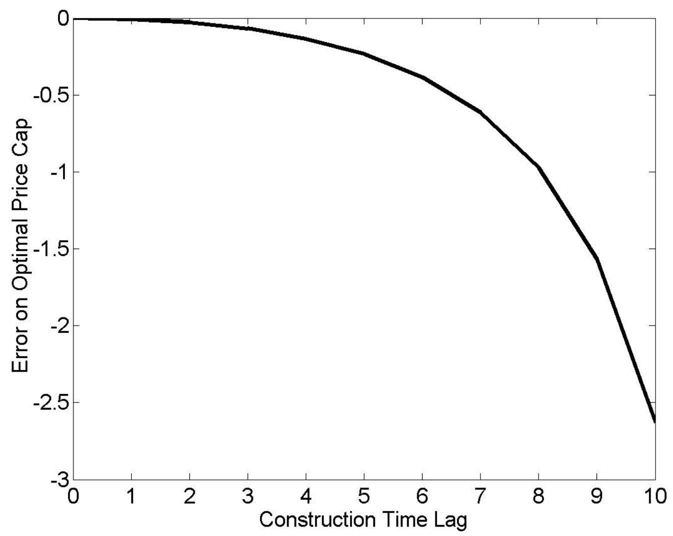


Figure 4.10 - Error on the Optimal Price Cap when not taking into account the Construction Time Lag ,
$C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08, N = 3.$

Moreover, these results raise questions as regard to the appropriate choice of price cap in an industry in which technologies with different construction lag times compete for the production of the same good, such as the electricity industry. If a regulator does not take into account the impact of the construction time lag on the optimal price cap under demand uncertainty, the regulator will not only induce under-investment, but will also introduce a bias against technologies which have a long construction lag time. This is particularly relevant to the electricity industry, in which a wholesale price cap is likely to reduce investment in technologies with a long construction time, such as nuclear power plants, as compared to technologies which are quicker to build, such as combined-cycle gas turbines (see the second part of this thesis for a discussion of the impact of the construction time lag on the economics of differnet genration technologies).

# 6 Simulations

In this section we aim to provide some insights on the magnitude of the investment delays caused either by the exercise of market power, by a construction time lag, or by a counter productive use of price caps by the regulator (i.e. a price cap set at a level such that it increases the investment price trigger as compared to the case without ptrice cap). We simulate a realisation of the stochastic demand, and observe the corresponding capacity expansion and price paths.[15]

## 6.1 Simulation parameters

The model is implemented numerically using usual parameter values in electricity markets, but does not pretend to offer a precise characterisation of investment in electricity markets, due to its stylised nature. Rather, the aim of this section is to provide some insights on the order of magnitude of the quantitative impact on the delays and under-investment identified in the previous sections. Table 4.11 summarizes the main model variables and the default simulation parameters. Initial capacity is normalised to one, as we are interested in comparing the relative level of investments. The annual average demand growth is set to 3%, the risk-free discount rate at 8%. The

[15] The numerical solutions and simulations were done using MATLAB software. The code is available from the author upon request.

price elasticity with respect to production is set at 0.6, and the construction time lag at 3 years. It should be noted that the results presented in this section are qualitatively robust over a wide range of parameters.

| Base case parameters | Value | Unit |
|---|---|---|
| Fixed investment costs[16] | $C = 600$ | US$/kW |
| Volatility | $\sigma^2 = 0.3$ | p. year |
| Trend of GBM ($\mu < \rho$) | $\mu = 0.03$ | % d. growth p. yr. |
| Price elasticity | $\gamma = 0.6$ | p. year |
| Risk free discount rate | $\rho = 0.08$ | % |
| Number of firms ($N > 1/\gamma$) | $N = 3$ | |
| Initial production quantity | $Q_0 = 1$ | Normalised |
| Optimal Price cap | $\bar{P} = 158.94$ | US$ |
| Initial demand | $X = 160$ | MW |
| Initial time | $t = 0$ | Year |
| Construction time lag | $\theta = 0$ | Year |

Table 4.11 - Base case simulation parameters

## 6.2 Efficiency of a price cap

Figure 4.12 shows a typical demand and price realisation together with the evolution of capacity in the three cases of perfect competition, oligopoly without price cap, and oligopoly with an optimal price cap.

[16] Recent estimates from the US Department of Energy suggest that the capital cost for a new plant range from US$ 400/KW for an open cycle gas turbine, US$ 600 for a combined-cycle gas turbine, US$ 1200/KW for a coal plant, to US$ 2000/KW for a nuclear plant (DOE 2004).

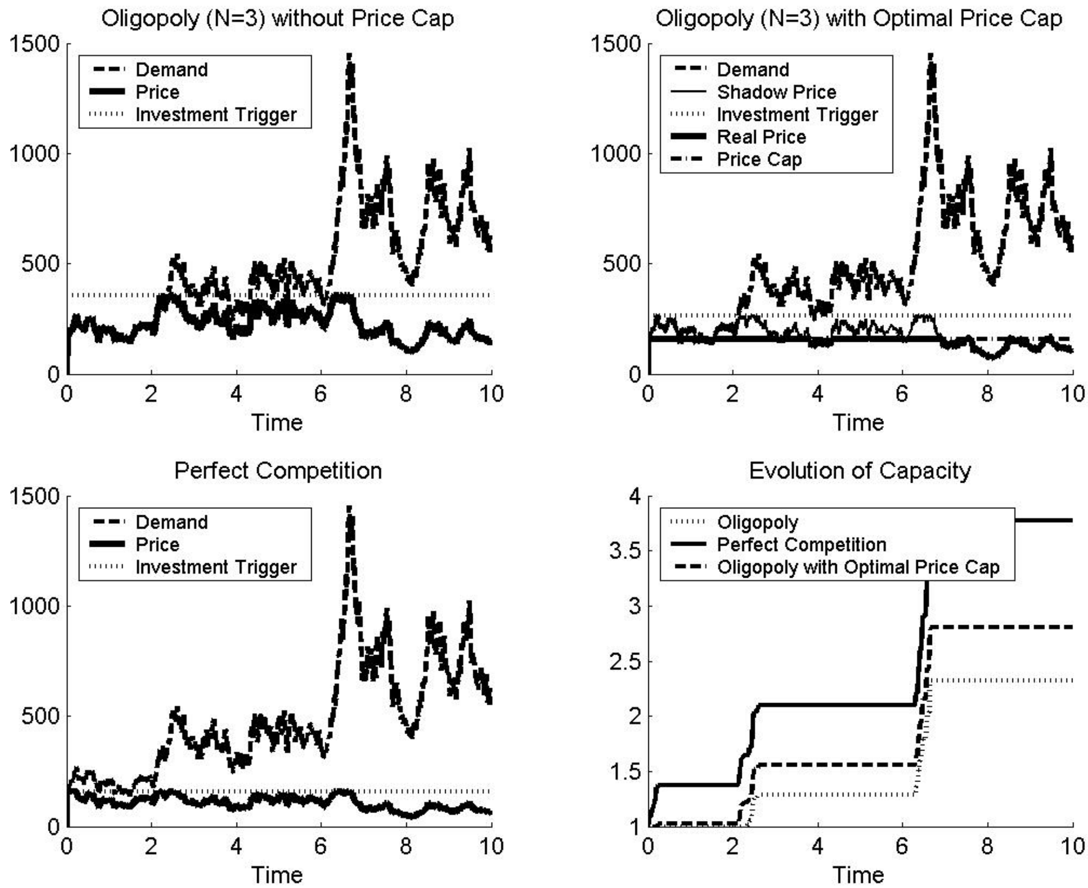


Figure 4.12 - Price and Capacity Evolutions for one typical Demand Realisation,
$C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08, N = 3, X_0 = 160.$

It can first be observed by comparing the upper and lower left hand side figures that the Nash Cournot equilibrium investment trigger price is much higher in the oligopolistic case than in the perfect competition case. Firms strategically restrict investment and impose quantity rationing on consumers in order to raise prices. Because of the higher level of the price trigger in the oligopolistic case, this investment trigger is hit less often such that less capacity is being built over time, which can be seen on the bottom right hand side figure.

When a price cap is introduced at the optimal level (top right hand side figure), i.e. at the competitive investment price, the investment price trigger is lower than for a non regulated oligopoly. However, as already pointed out in section four, setting the price cap at a competitive level does not realise the competitive investment outcome. The bottom right hand side figure shows indeed that if the regulated oligopoly industry has more capacity installed at any time than the oligopolistic industry without price cap, it still does not see as much capacity being installed as in the competitive industry.

### 6.3 Impact of a construction time lag

We now turn to the issue of under-investment related to the construction time lag. The price cap is set at the optimal level when there is no construction time lag. The left-hand side of Figure 4.13 shows a typical demand realisation and the associated endogenous price evolution in both cases with and without construction time lag. If it takes time to build capacity, when the without construction time lag investment trigger is reached, no investment takes place. The price needs to reach the higher investment trigger with a construction lag to motivate new build, and even when it does so, it takes time before capacity becomes operational. This is shown by the time lag between the time at which prices reach the investment trigger on the left hand side of Figure 4.13, and the time at which capacity actually increases on the right hand side. During this time lag, price can increase dramatically if demand keeps growing as capacity cannot be adjusted immediately.

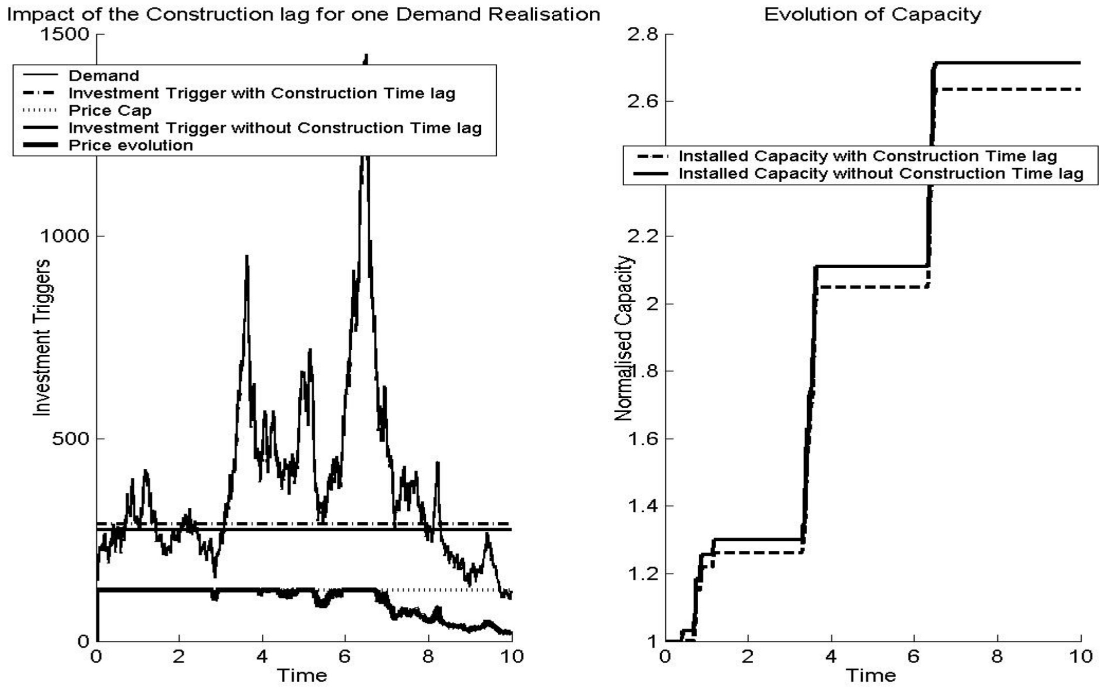


Figure 4.13 - Price and Capacity Evolutions with a Construction Time Lag for one typical Demand Realisation,
$C = 600, \sigma^2 = 0.3, \mu = 0.03, \gamma = 0.6, \rho = 0.08, N = 3, X_0 = 160, \theta = 3.$

## 6.4 Sensitivity Analysis

Figure 4.12 and Figure 4.13 showed the evolution of prices, and installed capacity for a typical demand realisation. In this subsection, we use a Monte Carlo simulation to get some insights on the average underinvestment in both a non-regulated oligopolistic industry and a regulated oligopolistic industry with an optimal price cap as compared to the perfectly competitive industry. We compute the average installed capacity after 10 years over 10,000 demand realisations in the three cases and show the following ratios:

- $\frac{Q_{Olig}}{Q_{comp}}$ represents the ratio of the average installed capacities after 10 years in a non-regulated oligopolistic industry and in the perfectly competitive industry.
- $\frac{Q_{OPC}}{Q_{comp}}$ represents the ratio of the average installed capacities after 10 years in a regulated oligopolistic industry with an optimal price cap

and in the perfectly competitive industry.

Table 4.14 shows the extend of under-investment for varying values of the base-case parameters introduced in Table 4.11. The results of the simulation suggest that the impact of imperfect competition on installed capacity can be quantitatively quite significant, insofar as the non-regulated oligopolistic industry and the regulated industry have on average respectively only 69% and 78% of the perfectly competitive industry installed capacity after 10 years with the base-case parameters. Table 4.14 shows also that variations of the investment cost, volatility of demand, load growth, price elasticity, market concentration, and construction time lag significantly impact both the extend of underinvestment, and the efficiency of price cap regulation.

| Investment cost | | | Volatility | | | Load growth | | | Price elasticity | | | Discount rate | | | Number of firms | | | Construction time lag | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| K | $\frac{Q_{Olig}}{Q_{comp}}$ | $\frac{Q_{OPC}}{Q_{comp}}$ | $\sigma^2$ | $\frac{Q_{Olig}}{Q_{comp}}$ | $\frac{Q_{OPC}}{Q_{comp}}$ | m | $\frac{Q_{Olig}}{Q_{comp}}$ | $\frac{Q_{OPC}}{Q_{comp}}$ | γ | $\frac{Q_{Olig}}{Q_{comp}}$ | $\frac{Q_{OPC}}{Q_{comp}}$ | ρ | $\frac{Q_{Olig}}{Q_{comp}}$ | $\frac{Q_{OPC}}{Q_{comp}}$ | N | $\frac{Q_{Olig}}{Q_{comp}}$ | $\frac{Q_{OPC}}{Q_{comp}}$ | θ | $\frac{Q_{Olig}}{Q_{comp}}$ | $\frac{Q_{OPC}}{Q_{comp}}$ |
| **300** | 0,62 | 0,74 | **0,001** | 0,61 | 0,97 | **-0,03** | 0,76 | 0,82 | **0,35** | 0,71 | 0,82 | **0,04** | 0,65 | 0,74 | **2** | 0,56 | 0,71 | **0** | 0,69 | 0,78 |
| **400** | 0,64 | 0,75 | **0,01** | 0,61 | 0,87 | **0,00** | 0,72 | 0,80 | **0,4** | 0,70 | 0,81 | **0,06** | 0,67 | 0,76 | **3** | 0,69 | 0,78 | **1** | 0,70 | 0,78 |
| **500** | 0,66 | 0,76 | **0,1** | 0,63 | 0,77 | **0,01** | 0,71 | 0,79 | **0,5** | 0,69 | 0,79 | **0,07** | 0,68 | 0,77 | **4** | 0,76 | 0,82 | **2** | 0,71 | 0,78 |
| **600** | 0,69 | 0,78 | **0,3** | 0,69 | 0,78 | **0,03** | 0,69 | 0,78 | **0,6** | 0,69 | 0,78 | **0,08** | 0,69 | 0,78 | **5** | 0,81 | 0,85 | **3** | 0,71 | 0,78 |
| **700** | 0,72 | 0,80 | **0,5** | 0,74 | 0,81 | **0,04** | 0,68 | 0,77 | **1** | 0,69 | 0,75 | **0,1** | 0,71 | 0,79 | **7** | 0,86 | 0,88 | **5** | 0,73 | 0,78 |
| **800** | 0,74 | 0,81 | **0,7** | 0,76 | 0,83 | **0,05** | 0,67 | 0,77 | **1,5** | 0,69 | 0,73 | **0,15** | 0,75 | 0,83 | **10** | 0,91 | 0,92 | **10** | 0,77 | 0,80 |
| **900** | 0,76 | 0,83 | **1** | 0,80 | 0,85 | **0,07** | 0,66 | 0,76 | **2** | 0,69 | 0,73 | **0,2** | 0,78 | 0,85 | **20** | 0,95 | 0,96 | **20** | 0,85 | 0,86 |

Figure 4.14 - Average underinvestment after 10 years for 10,000 demand realisations with and without price cap.

Similarly, we run a Monte-Carlo simulation to compute the average price markup after 10 years over 10,000 demand realisations in both a non-regulated oligopolistic industry and a regulated oligopolistic industry with an optimal price cap as compared to the competitive price, and compute the following ratios:

- $\frac{P_{Olig}}{P_{comp}}$ represents the ratio of the average prices after 10 years in a non-regulated oligopolistic industry and in the perfectly competitive industry.

- $\frac{P_{OPC}}{P_{comp}}$ represents the ratio of the average prices after 10 years in a regulated oligopolistic industry with an optimal price cap and in the perfectly competitive industry.

Table 4.15 shows the markup over the competitive price for varying values of the base-case parameters introduced in Table 4.11. The average price in the non-regulated oligopolistic industry represents on average 193% of the average competitive price after 10 years with the base-case parameters, and the average price in the regulated industry represents on average 139% of the competitive price. Table 4.15 shows also that variations of the investment cost, volatility of demand, load growth, price elasticity, market concentration, and construction time lag significantly impact the markup in both the non-regulated and the regulated oligopolistic industries.

| Investment cost | | | Volatility | | | Load growth | | | Price elasticity | | | Discount rate | | | Number of firms | | | Construction time lag | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| K | $\frac{P_{Olig}}{P_{comp}}$ | $\frac{P_{OPC}}{P_{comp}}$ | $\sigma^2$ | $\frac{P_{Olig}}{P_{comp}}$ | $\frac{P_{OPC}}{P_{comp}}$ | m | $\frac{P_{Olig}}{P_{comp}}$ | $\frac{P_{OPC}}{P_{comp}}$ | γ | $\frac{P_{Olig}}{P_{comp}}$ | $\frac{P_{OPC}}{P_{comp}}$ | ρ | $\frac{P_{Olig}}{P_{comp}}$ | $\frac{P_{OPC}}{P_{comp}}$ | N | $\frac{P_{Olig}}{P_{comp}}$ | $\frac{P_{OPC}}{P_{comp}}$ | θ | $\frac{P_{Olig}}{P_{comp}}$ | $\frac{P_{OPC}}{P_{comp}}$ |
| **300** | 2,22 | 1,49 | **0,001** | 2,25 | 1,01 | **-0,03** | 1,73 | 1,35 | **0,35** | 4,91 | 1,55 | **0,04** | 2,08 | 1,47 | **2** | 3,30 | 1,53 | **0** | 1,93 | 1,38 |
| **400** | 2,13 | 1,46 | **0,01** | 2,25 | 1,11 | **0** | 1,83 | 1,38 | **0,4** | 3,31 | 1,52 | **0,06** | 2,00 | 1,43 | **3** | 1,93 | 1,39 | **1** | 1,90 | 1,40 |
| **500** | 2,02 | 1,43 | **0,1** | 2,18 | 1,37 | **0,01** | 1,88 | 1,38 | **0,5** | 2,31 | 1,45 | **0,07** | 1,96 | 1,41 | **4** | 1,59 | 1,31 | **2** | 1,88 | 1,42 |
| **600** | 1,93 | 1,39 | **0,3** | 1,93 | 1,39 | **0,03** | 1,93 | 1,39 | **0,6** | 1,93 | 1,39 | **0,08** | 1,93 | 1,39 | **5** | 1,43 | 1,26 | **3** | 1,86 | 1,44 |
| **700** | 1,85 | 1,35 | **0,5** | 1,82 | 1,36 | **0,04** | 1,96 | 1,39 | **1** | 1,43 | 1,26 | **0,1** | 1,87 | 1,36 | **7** | 1,28 | 1,19 | **5** | 1,80 | 1,44 |
| **800** | 1,79 | 1,32 | **0,7** | 1,81 | 1,34 | **0,05** | 1,98 | 1,41 | **1,5** | 1,26 | 1,18 | **0,15** | 1,77 | 1,30 | **10** | 1,18 | 1,14 | **10** | 1,71 | 1,49 |
| **900** | 1,75 | 1,31 | **1** | 1,80 | 1,34 | **0,07** | 2,03 | 1,39 | **2** | 1,18 | 1,14 | **0,2** | 1,69 | 1,27 | **20** | 1,08 | 1,07 | **20** | 1,55 | 1,50 |

Figure 4.15 - Average markup over the competitive price after 10 years for 10,000 demand realisations with and without price cap.

# 7 Conclusions

This paper presented a continuous time model of irreversible investment in an oligopolistic industry with stochastic demand, and introduced two critical characteristics of investments in electricity markets, a price cap and a construction time lag. The contribution of this paper is twofold.

First, we solved the Nash-Cournot symmetric industry equilibrium and demonstrated that the investment price trigger is an increasing function of market concentration, demand uncertainty, and of the length of the construction time lag (and therefore the industry installed capacity is a decreasing function of these parameters). We demonstrated that under demand uncertainty, perfectly-rational, well-informed, risk-neutral investors will delay and underinvest in new capacity in an industry characterised by oligopolistic (Cournot) competition and a construction time lag. Numerical simulations showed that the investment delay and under-investment due to the exercise of market power, a construction time lag, or a non optimal price cap can be significant. This gives some weight to the claim that electricity markets are likely to see delayed or under-investment, because of the industry concentration or the remaining non market mechanisms such as price caps that prevent prices from moving up freely and remunerate investors appropriately in times of capacity scarcity.

Second, the model provided some new insights about the intertemporal effects of price cap regulation on investment under uncertainty. Our results underline the importance of taking into account the option value effect arising out of uncertainty in demand. As in the static models, we demonstrated that the optimal price cap level corresponds to the perfect competition entry price, but setting the price cap at the competitive level does not realise the competitive investment outcome and leads to under-investment. Contrary to perfect competition models, the investment price trigger is a non-monotonic function of the level of the price cap, as the price cap has two effects on investment which work in opposite directions. On the one hand, the price cap has a negative impact on the *option value effect* associated with demand uncertainty, as it caps potential upside profits while leaving unchanged potential downside losses, thereby providing a disincentive to investment. On the other hand, when the price cap is binding, increasing capacity in a Nash Cournot game does not lead to a reduction in price, hence providing an incentive to increase investment. We found that for a price cap lower than the competitive entry price, the impact of the price cap on the option value effect dominates, such that the investment price trigger is a decreasing function of the price cap. Conversely, the *market power mitigation* effect dominates for a price cap higher than the competitive entry price, such that the investment price trigger is an increasing function of the price cap.

Moreover, we showed that the optimal price cap is an increasing function of the volatility of demand and of the length of the construction time lag. Sensitivity analyses and simulations suggest that not recognising the option value effects arising out of uncertainty in demand when determining the optimal level of a price cap can have a significant negative impact on investment. We showed for instance that a price cap set at a conventionally optimal level without taking into account demand uncertainty can actually be counter-productive in an industry characterised by relatively highly volatile demand, as it may reduce investment and increase prices. Similarly, a conventionally optimal price cap set without taking into account the impact of the construction time lag on investment might reduce investment as compared to the oligopolistic case without price cap, and distort technology choices in favour of teh technologies with the shortest construction lead time.

# 8 Appendices

## 8.1 Appendix 1

In this appendix we prove that producers produce always at full capacity.

The profit of firm $i$ is given by equation (5). The marginal profit resulting from a marginal increase in capacity is given by

$$\frac{\partial \pi}{\partial q} = XQ^{-1/\gamma} - 1/\gamma X Q^{-\frac{1}{\gamma}-1} q \tag{40}$$

Given that $q = \frac{Q}{N}$, a little algebra gives

$$\frac{\partial \pi}{\partial q} = XQ^{-1/\gamma} \frac{N\gamma - 1}{N\gamma} = P \frac{N\gamma - 1}{N\gamma} \tag{41}$$

As we assume that $\gamma > \frac{1}{N}$, $\frac{\partial \pi}{\partial q} > 0$ which demonstrates that producers produce always at full capacity.

## 8.2 Appendix 2

We name $\beta_1$and $\beta_1$the roots of the 2nd degree polynomial characteristic equation of the differential equation (8):

$$\frac{1}{2}\sigma^2 \beta(\beta - 1) + \mu\beta - \rho = 0 \tag{42}$$

The two roots are defined by the two following equations

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}} > 1 \tag{43}$$

and

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2\rho}{\sigma^2}} < 0 \tag{44}$$

The solution of the differential equation (8) takes the following form

$$m(P(Q), Q_{-i}, q) = H_0(Q)P^{\beta_1} + H_1(Q)P^{\beta_2} + A_0 P + B_0 \qquad (45)$$

where $A_0$ and $B_0$ are two constants.

$\beta_2 < 0$ implies that $H_1(Q) = 0$ (because $P^{\beta_2}$ tends to infinity as $P$ goes to zero.

$A_0 P + B_0$ is a particular solution of the differential equation, which gives after a little algebra the equation (16) for $A_0$ and $B_0 = 0$.

Using now the two boundary conditions (10) and (11) yields after a little algebra the analytical expression of the investment price trigger $P^*$ given in equation (17) and of the function $H_0(Q)$ given in equation (15).

Lastly,

$$\frac{\partial P^*}{\partial N} = \frac{\partial}{\partial N}\left(\frac{C\,(\rho-\mu)\,\beta_1}{(\beta_1-1)}\frac{N\gamma}{N\gamma-1}\right) \qquad (46)$$

which gives

$$\frac{\partial P^*}{\partial N} = \frac{C\,(\rho-\mu)\,\beta_1}{(\beta_1-1)}\frac{(-\gamma)}{(N\gamma-1)^2} < 0 \qquad (47)$$

because $\gamma > 0$ , $\rho > \mu$ , and $\frac{\beta_1}{(\beta_1-1)} > 0$.

### 8.3 Appendix 3

The system of four equations (24), (25), (26), and (27) with four unknowns $(H_1, H_2, H_3, \bar{P}^*)$ defines the symmetric Nash Cournot equilibrium investment strategies of a firm when prices are capped at $\bar{P}$. An analytical expression of these four equations is given below by equations (48), (49), (50) and (51):

$$H_2\bar{P}^{*\beta_1} + H_3\bar{P}^{*\beta_2} + \frac{\bar{P}}{\rho} = C \qquad (48)$$

$$H_2\beta_1\bar{P}^{*\beta_{1-1}} + H_3\beta_2\bar{P}^{*\beta_{2-1}} = 0 \qquad (49)$$

$$H_1\bar{P}^{\beta_1} + \frac{N\gamma-1}{N\gamma\,(\rho-\mu)}\bar{P} = H_2\bar{P}^{\beta_1} + H_3\bar{P}^{\beta_2} + \frac{\bar{P}}{\rho} \qquad (50)$$

$$H_1\beta_1 P^{\beta_1-1} + \frac{N\gamma-1}{N\gamma\,(\rho-\mu)} = H_2\beta_1\bar{P}^{\beta_1-1} + H_3\beta_2\bar{P}^{\beta_2-1} \qquad (51)$$

This system is non linear but possesses unusual features which make it possible to solve and find an analytical solution for $\bar{P}^*$.

We provide a sketch of the calculations and leave the intermediary steps to the interested reader.

Subtracting $\frac{\bar{P}^*}{\beta_1}$.(49) from (48) to eliminate $H_2$ yields

$$H_3 = \bar{P}^{*(-\beta_2)}(C - \frac{\bar{P}}{\rho})\frac{\beta_1}{\beta_1 - \beta_2} \tag{52}$$

Subtracting $\frac{\bar{P}}{\beta_1}$.(51) from (50) to eliminate $H_2$ yields

$$H_3 = \bar{P}^{(1-\beta_2)}\frac{[\frac{(\beta_1-1)}{\alpha(\rho-\mu)} - \frac{\beta_1}{\rho}]}{\beta_1 - \beta_2} \tag{53}$$

where we introduce $\alpha = \frac{N\gamma}{(N\gamma-1)}$ to simplify the notations.

Equating (52) and (53) to eliminate $H_3$ gives

$$\bar{P}^{*\beta_2} = \frac{\beta_1}{\frac{(\beta_1-1)}{\alpha(\rho-\mu)} - \frac{\beta_1}{\rho}}(C - \frac{\bar{P}}{\rho})\bar{P}^{(\beta_2-1)} \tag{54}$$

which upon rearrangement yields the following analytical solution for $\bar{P}^*$

$$\bar{P}^* = [\lambda(\frac{\bar{P}}{\rho} - C)\bar{P}^{(\beta_2-1)}]^{1/\beta_2} \tag{55}$$

with $\lambda$ defined as

$$\lambda = \frac{\beta_1}{\frac{(1-\beta_1)}{\alpha(\rho-\mu)} - \frac{\beta_1}{\rho}} \tag{56}$$

## 8.4 Appendix 4

In this appendix we prove that there exists an interval $[\bar{P}_{\min}, \bar{P}_{\max}]$ over which the introduction of a price cap lowers the investment price trigger as compared to the oligopolistic industry investment trigger without price cap (i.e. $\bar{P}^* \leq P^*$). $\bar{P}_{\max}$is equal to the investment price trigger $P^*$ without price cap above which the price cap is irrelevant.

Let us define $\Delta = \bar{P}^* - P^*$ the difference between the industry investment price trigger with and without price cap at $\bar{P}$. To demonstrate proposition (16), it is sufficient to prove that $\Delta \leq 0$ over the interval $[\bar{P}_{\min}, \bar{P}_{\max}]$ .

From (28) and (17) we have

$$\Delta = [\lambda(\frac{\bar{P}}{\rho} - C)\bar{P}^{(\beta_2-1)}]^{1/\beta_2} - \alpha C \frac{\beta_1}{(1-\beta_1)} \tag{57}$$

To simplify the calculations we define $\zeta = \alpha C \frac{\beta_1}{(1-\beta_1)}$, so that $\Delta \leq 0$ is equivalent successively to

$$[\lambda(\frac{\bar{P}}{\rho} - C)\bar{P}^{(\beta_2-1)}]^{1/\beta_2} \leq \zeta \tag{58}$$

and because $\beta_2 < 0$ to

$$\frac{\lambda}{\rho}\bar{P}^{\beta_2} - \lambda C \bar{P}^{(\beta_2-1)} - \zeta^{\beta_2} \geq 0 \tag{59}$$

Analytical solutions of this non-linear inequality cannot be found in the general case.

However, we can find an analytical solution in the particular case in which $\beta_2 = -1$. In this case (59) becomes

$$\frac{\lambda}{\rho}\bar{P}^{(-1)} - \lambda C \bar{P}^{2} - \zeta^{(-1)} \geq 0 \tag{60}$$

which is equivalent to

$$\bar{P}^{2} - \frac{\lambda\zeta}{\rho}\bar{P} + \lambda\zeta C \leq 0 \tag{61}$$

Noting $\bar{P}_{\min}$ and $\bar{P}_{\max}$the two roots of this second degree polynomial equation, defined as follows

$$\bar{P}_{\min} = \frac{\frac{\lambda\zeta}{\rho} - \sqrt{(\frac{\lambda\zeta}{\rho})^2 - 4\lambda\zeta C}}{2} \tag{62}$$

and

$$\bar{P}_{\max} = \frac{\frac{\lambda\zeta}{\rho} + \sqrt{(\frac{\lambda\zeta}{\rho})^2 - 4\lambda\zeta C}}{2} \tag{63}$$

we obtain that $\Delta = \bar{P}^{*} - P^{*} \leq 0$ over the interval $[\bar{P}_{\min}, \bar{P}_{\max}]$.

### 8.5 Appendix 5

From (28) we have

$$\bar{P}^{*\beta_2} = \lambda(\frac{\bar{P}}{\rho})^{\beta_2} - \lambda C\bar{P}^{(\beta_2-1)} \tag{64}$$

Differentiating this expression relatively to the price cap $\bar{P}$ we obtain

$$\frac{\partial(\bar{P}^{*\beta_2})}{\partial\bar{P}} = \beta_2\frac{\partial\bar{P}^*}{\partial\bar{P}}\bar{P}^{*(\beta_2-1)} = \frac{\lambda}{\rho}\beta_2\bar{P}^{(\beta_2-1)} - \lambda C(\beta_2-1)\bar{P}^{(\beta_2-1)} \tag{65}$$

$\frac{\partial\bar{P}^*}{\partial\bar{P}} = 0$ implies

$$\frac{\lambda}{\rho}\beta_2\bar{P}^{(\beta_2-1)} = \lambda C(\beta_2-1)\bar{P}^{(\beta_2-1)} \tag{66}$$

and the optimal level of the price cap $\bar{P}_{opt}$is therefore given by the following expression

$$\bar{P}_{opt} = \frac{\rho C(\beta_2-1)}{\beta_2} \tag{67}$$

In the rest of this appendix we demonstrate that this later expression is equal to (20), the investment price trigger in a competitive industry.

$\beta_1$ and $\beta_2$ are the two roots of the characteristic equation of (8) and therefore verify the two following relations:

$$\beta_1 + \beta_2 = 1 - \frac{2\mu}{\sigma^2} \tag{68}$$

and

$$\beta_1\beta_2 = -\frac{2\rho}{\sigma^2} \tag{69}$$

From (68) and (69) we obtain

$$\rho - \mu = -\frac{\sigma^2}{2}[\beta_1\beta_2 - (\beta_1+\beta_2) + 1] \tag{70}$$

Using again (69) to replace $-\frac{\sigma^2}{2}$ by $\frac{\rho}{\beta_1\beta_2}$ and rearranging inside the brackets yields

$$\rho - \mu = \frac{\rho}{\beta_1\beta_2}[(\beta_2-1)(\beta_1-1)] \tag{71}$$

and therefore the optimal level of the price cap is equal to the investment price trigger in the competitive industry without price cap:

$$\bar{P}_{opt} = \frac{\rho C(\beta_2 - 1)}{\beta_2} = C\frac{(\rho-\mu)\,\beta_1}{(\beta_1 - 1)} = P^*_{N=\infty} \tag{72}$$

### 8.6 Appendix 6

By changing variables with $t = t - \theta$ in equation (32), the marginal value of a firm becomes

$$m = \exp(-\rho\theta)E\left[\begin{array}{c} \int \frac{\partial}{\partial q_i}\left(\pi_i\left(q_i(t)\,, Q_{-i}(t)\,, X(t+\theta)\right)\right)\exp(-\rho t)dt \\ -\frac{\partial}{\partial q_i}\int C\exp(-\rho t)dq_i(t) \end{array}\right] \tag{73}$$

which, omitting the time index $t$ can also be written as

$$m = \exp(-\rho\theta)E\left[m(X(\theta), Q_{-i}, q_i)\lceil X(0) = X_0\right] \tag{74}$$

Following the procedure detailed in the appendix of Bar-Ilan and Strange (1996), we obtain

$$E\left[X(\theta), Q_{-i}, q_i)\lceil X(0) = X_0\right] = E\left[(X(t+\theta)\lceil X(t) = X(t)\exp(\mu\theta)\right] \tag{75}$$

which yields finally

$$\exp(-\rho\theta)E\left[(X(t+\theta)\lceil X(t) = X(t)\exp((\mu-\rho)\theta)\right] \tag{76}$$

with $\mu < \rho$.

### 8.7 Appendix 7

The system of four equations (24), (25), (26), and (27) with four unknowns $(H_1, H_2, H_3, \bar{P}^*)$ defines the symmetric Nash Cournot equilibrium investment strategies of a firm when prices are capped at $\bar{P}$. Applying the additional discount factor of respectively $\exp((\mu-\rho)\theta)$ and $\exp(-\rho\theta)$ in the cases without and with price cap in the modified demand curve faced by an investor yields the following analytical expression of the system:

$$H_2\bar{P}_\theta^{*\beta_1} + H_3\bar{P}_\theta^{*\beta_2} + \exp(-\rho\theta)\frac{\bar{P}}{\rho} = C \tag{77}$$

$$H_2\beta_1\bar{P}_\theta^{*\beta_{1-1}} + H_3\beta_2\bar{P}_\theta^{*\beta_{2-1}} = 0 \tag{78}$$

$$H_1\bar{P}^{\beta_1} + \exp((\mu-\rho)\theta)\frac{N\gamma - 1}{N\gamma\,(\rho-\mu)}\bar{P} = H_2\bar{P}^{\beta_1} + H_3\bar{P}^{\beta_2} + \exp(-\rho\theta)\frac{\bar{P}}{\rho} \tag{79}$$

$$H_1\beta_1 P^{\beta_1-1} + \exp((\mu-\rho)\theta)\frac{N\gamma - 1}{N\gamma\,(\rho-\mu)} = H_2\beta_1\bar{P}^{\beta_1-1} + H_3\beta_2\bar{P}^{\beta_2-1} \tag{80}$$

Similarly to the system solved in Appendix 3, this system is non linear but possesses unusual features which make it possible to solve and find an analytical solution for $\bar{P}_\theta^*$. We provide below a sketch of the calculations.

Subtracting $\frac{\bar{P}^*}{\beta_1}$.(78) from (77) to eliminate $H_2$ yields

$$H_3 = \bar{P}_\theta^{*(-\beta_2)}(C - \exp(-\rho\theta)\frac{\bar{P}}{\rho})\frac{\beta_1}{\beta_1-\beta_2} \tag{81}$$

Subtracting $\frac{\bar{P}}{\beta_1}$.(80) from (79) to eliminate $H_2$ yields

$$H_3 = \bar{P}^{(1-\beta_2)}\frac{\exp((\mu-\rho)\theta)\frac{(\beta_1-1)}{\alpha(\rho-\mu)} - \exp(-\mu\theta)\frac{\beta_1}{\rho}}{\beta_1-\beta_2} \tag{82}$$

where we introduce $\alpha = \frac{N\gamma}{(N\gamma-1)}$ to simplify the notations.

Equating the last tow equations to eliminate $H_3$ gives

$$\bar{P}_\theta^{*\beta_2} = \frac{\beta_1(\exp(\rho\theta)C - \frac{\bar{P}}{\rho})}{\exp(\mu\theta)\frac{\beta_1-1}{\alpha(\rho-\mu)} - \frac{\beta_1}{\rho}}\bar{P}^{(\beta_2-1)} \tag{83}$$

which eventually yields the following analytical solution for $\bar{P}_\theta^*$

$$\bar{P}_\theta^* = [\frac{\beta_1(\exp(\rho\theta)C - \frac{\bar{P}}{\rho})}{\exp(\mu\theta)\frac{\beta_1-1}{\alpha(\rho-\mu)} - \frac{\beta_1}{\rho}}\bar{P}^{(\beta_2-1)}]^{1/\beta_2} \tag{84}$$

$$\bar{P}_\theta^* = [\frac{\beta_1(e^{\rho\theta}C - \frac{\bar{P}}{\rho})}{e^{\mu\theta}\frac{\beta_1-1}{\alpha(\rho-\mu)} - \frac{\beta_1}{\rho}}\bar{P}^{(\beta_2-1)}]^{1/\beta_2}$$

### 8.8 Appendix 8

From (35) we have

$$\bar{P}_\theta^{*\beta_2} = \frac{\beta_1 \exp(\rho\theta) C}{\exp(\mu\theta)\frac{\beta_1 - 1}{\alpha(\rho-\mu)} - \frac{\beta_1}{\rho}} \bar{P}^{(\beta_2-1)} - \frac{\frac{\bar{\beta}_1}{\rho}}{\exp(\mu\theta)\frac{\beta_1 - 1}{\alpha(\rho-\mu)} - \frac{\beta_1}{\rho}} \bar{P}^{\beta_2} \tag{85}$$

Differentiating this expression relatively to the price cap $\bar{P}$ we obtain

$$\frac{\partial(\bar{P}_\theta^{*\beta_2})}{\partial \bar{P}} = \beta_2 \frac{\partial \bar{P}_\theta^*}{\partial \bar{P}} \bar{P}_\theta^{*(\beta_2-1)} \tag{86}$$

$$= \frac{\beta_1 \exp(\rho\theta) C}{\exp(\mu\theta)\frac{\beta_1 - 1}{\alpha(\rho-\mu)} - \frac{\beta_1}{\rho}} (\beta_2 - 1) \bar{P}^{(\beta_2-2)} \tag{87}$$

$$- \frac{\frac{\bar{\beta}_1}{\rho}}{\exp(\mu\theta)\frac{\beta_1 - 1}{\alpha(\rho-\mu)} - \frac{\beta_1}{\rho}} \beta_2 \bar{P}^{(\beta_2-1)} \tag{88}$$

$\frac{\partial \bar{P}_\theta^*}{\partial \bar{P}} = 0$ implies

$$\beta_2 \bar{P}^{(\beta_2-1)} = \exp(\rho\theta) C (\beta_2 - 1) \bar{P}^{(\beta_2-2)} \tag{89}$$

and the optimal level of the price cap $\bar{P}_{\theta opt}$is therefore given by the following expression

$$\bar{P}_{\theta opt} = \exp(\rho\theta) \frac{\rho C(\beta_2 - 1)}{\beta_2} = \exp(\rho\theta) \bar{P}_{opt} \tag{90}$$

# References


[1] Allaz, B. and J.-L Vila. (1993), "Cournot competition, forward markets and efficiency," Journal of Economic Theory, 59, 1–16.

[2] Baldursson, F. (1998), "Irreversible investment under uncertainty in oligopoly", Journal of Economic Dynamics and Control 22: 627-644.

[3] Bar-Ilan, A., and W. C. Strange (1996), "Investment lags", American Economic Review, 86: 611-622.

[4] Beesley M., and S. Littlechild (1989), "Regulation of privatized monopolies in the United Kingdom", Rand Journal of Economics 20 (3): 454-472.

[5] Crampes, C. and A. Creti (2003), "Capacity Competition in Electricity Markets," IDEI Working Paper, Mimeo.

[6] Dixit, A., (1991), "Irreversible Investment with Price Ceilings", Journal of Political Economy 99 (3).

[7] Dixit, A. and. R. Pindyck (1994), "Investment Under Uncertainty", Princeton University Press.

[8] Dobbs I. (2004), "Intertemporal price cap regulation under uncertainty," Economic Journal, 114: 421-440.

[9] DOE (2004), Annual Energy Outlook 2004 (DOE/EIA-0554), U.S. Dept. of Energy/Energy Information Administration, February 2004.

[10] Earle, R. and K. Schmedders (2001), "Demand uncertainty and risk-aversion : why price caps may lead to higher prices", October 2001, Mimeo.

[11] Ford, A. (1999), "Cycles in competitive electricity markets: a simulation study of the western United States," Energy Policy, 27: 637-658.

[12] Ford, A. (2001), "Waiting for the Boom: A Simulation Study of Power Plant Construction in California," Energy Policy, 29: 847-869.

[13] Fraser, P. (2003), "Power Generation Investment in Electricity Markets", OECD/IEA publication 2003.

[14] Gardner, D. and J Rogers (1999), "Planning Electric Power Systems Under Demand Uncertainty with Different Technology Lead Times," Management Science, 45(10): 1289-1306.

[15] Gollier C., D. Proult, F. Thais and G. Walgenwitz (2005), "Choice of nuclear power investments under price uncertainty: Valuing modularity," Energy Economics 27(4): 667-685.

[16] Green, R. (1999), "The electricity market in England and Wales," The Journal of Industrial Economics, 47: 107–124.

[17] Grenadier, S. (2002), "Option exercise games: an application to the equilibrium investment strategies of firms", The Review of Financial Studies 15 (3): 691-721.

[18] Grenadier, S. (2000), “Equilibrium with Time-to-build: A Real-options Approach”, in M. Brennan and L. Trigeorgis (eds.), Project flexibility, Agency, and Competition, Oxford University Press.

[19] Grobman, J. and J. Carey (2001), “Price caps and investment: long-run effects in the electric generation industry”, Energy Policy 29: 545-552.

[20] de Joode J., and M. Boots (2005), “Concepts Of Investment Risks And Strategies In Electricity Generation,”, ECN Working Paper ECN-C–05-061, June 2005.

[21] Joskow, P. (2003), “The difficult transition to competitive electricity markets in the U.S.,” Cambridge-MIT Institute Electricity Project Working Paper 28.

[22] Joskow P., and J. Tirole (2004), “Reliability and Competitive Electricity Markets,” CMI Electricity Project Working Paper 53.

[23] Laffont, J.-J. and J. Tirole (1993), “A theory of incentives in procurement and regulation”, MIT Press, Cambridge, Massachusetts, USA.

[24] Laffont, J.-J. and J Tirole (2001), “Competition in Telecommunications,” MIT Press, Cambridge, Massachusetts, USA.

[25] Leahy, J. (1993), “Investment in competitive equilibrium: The optimality of myopic behaviour”, The Quarterly Journal of Economics 108 (4): 1105-1133.

[26] Le Coq, C. (2002), “Strategic Use of Available Capacity in the Electricity Spot Market,” SSE/EFI Working Paper, Series in Economics and Finance No 496.

[27] McDonald, R. and D. Siegel (1986), "The Value of Waiting to Invest," The Quarterly Journal of Economics, MIT Press, 101(4): 707-27.

[28] Metcalf, G. an K. Hassett (1995), “Investment under alternative return assumptions: comparing random walks and mean reversion,” National Bureau of Economic Research Technical Working Paper N. 175, March 1995.

[29] Murphy, F. and Y. Smeers (2003), "Generation capacity expansion in imperfectly competitive restructured electricity markets," Operations Research, to appear.

[30] Murphy, F. and Y. Smeers (2005), "Forward markets may not decrease market power when capacities are endogenous," UCL Center for Operations Research and Econometrics Working Paper Number 2005/28.

[31] Murto, P. and G. Nese (2002), "Input price risk and optimal timing of energy investment: choice between fossil- and biofuels," Working Paper No. 25 / 02, Institute for Research in Economics and Business Administration, Bergen, Norway, MIMEO.

[32] Neuhoff, K. and L. de Vries (2004), "Insufficient Incentives for Investment in Electricity Generation," Utilities Policy, 4: 253-268.

[33] Newbery, D. (1998), "Competition, contracts, and entry in the electricity spot market," RAND Journal of Economics, 29(4): 726–749.

[34] Newbery (2005), "Electricity liberalisation in Britain and the evolution of market design," Energy Journal - chapter.

[35] Olsina, F., F. Garces, and H.-J. Haubrich (2005), "Modelling long-term dynamics of electricity markets," Energy Policy, in Press.

[36] Pindyck, R. (1993), "Investments of uncertain cost," Journal of Financial Economics 34: 53-76.

[37] Smit H., and L. Trigeorgis (2004), "Strategic Investment. Real Options and Games", Princeton University Press, 2004.

[38] Stoft, S. (2002), "Power Systems Economics - Designing Markets for Electricity", IEEE Press, John Wiley & Sons.

[39] de Vries (2004), "Securing the public Interest in electricity generation markets: the myths of the invisible hand and the copper plate", Ph.D. thesis.