A Generalized Sharecropping Model of Retail Lease Contracting and Licensing Agreements

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December 2005

Abstract

A model of bilateral trade between an upstream supplier and a downstream producer is constructed, in which the upstream supplier confers long-term property usage rights to the downstream supplier in return for a base rental fee plus a percentage of verifiable sales production. Our model allows for the possibility that downstream sales production complements other activities of the upstream supplier to increase its total revenues. An optimal contract is designed that balances ex ante investment incentives of the downstream producer with ongoing reinvestment incentives of the upstream supplier. A number of important stylized facts associated with retail lease contracting are addressed, including why: i) retail leases contain base rents and often (but not always) contain an overage rental feature, ii) stores that generate greater externalities pay lower base rents and have lower overage rent percentages than stores that generate fewer externalities, iii) the overage rent option is typically well out-of-the-money at contract execution, and iv) stand-alone retail operations often sign leases that contain an overage rental feature.
I. Introduction and Motivation

Incentives to execute formal contracts derive from the division of labor and exchange, where division of labor implies delegation of responsibility. Comparative advantage underlies division of labor, but delegation introduces costs when there are conflicting objectives between principal and agent. The first contracts probably appeared in agriculture as bilateral agreements between landlords and their tenants.\(^1\) \textit{Sharecropping} arrangements were perhaps the most common such contract, in which the tenant agreed to share revenues from crop production with the landlord.

The structure of these contracts has puzzled economists for a long time. The reason is that a sharing arrangement appears to reduce incentives for the tenant to exert effort to maximize production, to the detriment of the landlord. A fixed payment rental contract with incentive payments made back to the tenant, where these payments correlate positively with production, would seem to Pareto-dominate the standard sharecropping contract.

It wasn’t until the development of agency theory that convincing arguments were offered to rationalize observed contracting practices. For example, Stiglitz (1974) emphasized risk-sharing with uncontrollable (weather-related) production. The sharecropping arrangement shifts risks of stochastically variable production from the risk-averse tenant to the better endowed, risk-neutral landlord. Tenant effort level, chosen prior to random production effects, is shown to approach the first-best level with an appropriately structured sharecropping contract. Others have

\(^1\) See Chapter 1 of Laffont and Martimort (2002).
since extended Stiglitz’s basic argument to explicitly account for landlord bargaining power, tenant financial constraints, and certain other supply effects (Newbery and Stiglitz (1979), Braverman and Stiglitz (1982), Eswaran and Kotwal (1985)).

Retail lease contracting offers similar puzzling features, in which retail tenants in a shopping center configuration typically pay base rents plus a percentage of sales when sales exceed an average threshold value. Interestingly, empirical analysis has shown that base rents and overage rent percentages vary systematically depending on the size of the retail tenant, where larger tenants pay lower base rents and overage rent percentages (see, e.g., Benjamin et al. (1992), Wheaton (2000), Gould et al. (2002)).

The combination of multiple tenants in a shopping center setting together with systematic variation in contract terms as a function of tenant size has sparked considerable interest among researchers. Brueckner (1993) was the first to consider externality as an explanation for both tenant agglomeration and observed contracting practices, in which larger tenants (anchor stores) generate positive externalities to the benefit of smaller tenants. The landlord in this setting operates as a discriminating monopolist. In the model, prior to consideration of tenant effort, an optimal allocation of space can be achieved with base rents only, with externality-generating tenants paying lower base rents than externality-consuming tenants. When tenant effort is considered, Brueckner shows that incentive payments made by the landlord back to the tenant is an optimal contract—an outcome that is exactly the opposite of what is observed in practice. Incentive payment percentages do increase with externality, however, so comparative static relations generally match up against empirically observed outcomes.

Others have offered risk-sharing arguments, but those arguments fail to explain observed retail contracting practices (see, e.g., Miceli and Sirmans (1995)). For example, although it may
be true that anchor tenants, which are firms with a national presence and sizable scale, are less risk-averse than smaller “mom-and-pop” retail tenants, it is doubtful that nationally recognized brand-name tenants that lease smaller spaces than anchor tenants (e.g., shoe or clothing specialty stores) are more risk-averse than the larger anchor tenants. These smaller specialty stores typically pay base and overage rental percentages that exceed anchor stores, however, which accords more closely to explanations that emphasize external as opposed to risk-sharing effects (see Gould et al. (2002) for further discussion of this issue).

Wheaton (2000) offers a compelling rationale for overage rental features that relies on providing incentive compatible contract terms for the landlord. In a shopping center setting, tenants sign long-term leases. An optimal mix of tenants is easily obtainable at the outset, but tenants disappear over time for idiosyncratic reasons. The landlord controls the releasing decision, which non-contractable. At the time of releasing, the landlord may have an incentive to sign a tenant that pays the most rent, regardless of its effect on other tenants. This high-rent-paying tenant will typically consume rather than generate externalities. Incumbent tenants may instead prefer an externality-generating tenant in order to maximize total shopping center sales. The overage rental contract can achieve this objective, and Wheaton shows it is an optimal contract when landlord hold-up problems exist.

Although rich and insightful, previous literature nonetheless fails to explain several important empirical facts that relate to retail lease contracting. First, as noted by Edelman and Petzold (1996), Gould et al. (2002), and others, the overage rent percentage option is typically well out-of-the-money at contract execution. Indeed, the overage rent threshold value is often two times or more the amount of initial sales, resulting in only a small percentage of total shopping center rents being attributable to overage rents. Anchor stores often do not have
overage rental clauses at all in their lease contract, and pay only minimal base rents. This suggests that overage rents are a low-powered rather than high-power incentive mechanism—contrary to the incentive mechanisms required in the models of Brueckner (1993), Wheaton (2000), and others.

Second, models in both the sharecropping and retail lease contracting literature generally assume complete bargaining power on behalf of the landlord, where complete bargaining power for the landlord biases contract terms toward the overage rental component. While complete bargaining power for the landlord may be appropriate in a sharecropping setting, we would not expect this to be true in general in a retail setting. When bargaining power tilts towards the tenant, a fixed contract emerges due to the tenant’s preference for such an outcome. This provides an alternative explanation as to why smaller, more localized tenants pay higher overage percentages than larger national chain stores.

Third, and perhaps most important, it is well known that stand-alone retail establishments (e.g., “big box” retailers) often execute lease contracts with overage rent features. These retail establishments neither generate nor receive externalities that are attributable to an optimal tenant mix (the landlord owns the single parcel of land occupied by the stand-alone, but typically does not own the surrounding properties). The tenants are also often national chain stores, which eliminates risk-sharing as a convincing explanation for contracting practices.

The purpose of this paper is to construct a model that addresses the important stylized empirical facts associated with retail lease contracting. To summarize, these facts are: i) retail leases contain base rents and often (but not always) contain an overage rental feature; ii) stores that generate more externalities pay lower base rents and have lower overage rent percentages than stores that generate less externalities; iii) the overage percentage rent option is typically well
out-of-the-money at contract execution, iv) the tenant often has significant bargaining power in setting contract terms, and v) stand-alone retail operations often sign leases that contain an overage rental feature.

II. Summary of Model and the Main Results

In an effort to explain the data, we offer a model of bilateral contracting that resembles the standard sharecropping model, but that also incorporates features that are central to a retail operating environment. These features include long lease terms, stochastic sales, costly effort (e.g., advertising) by the tenant that affects sales production, externalities generated by the tenant to the benefit of the landlord, and subsequent reinvestment by the landlord that increases productivity. Endogenously determined variables are the optimal contracting terms (base rent, overage rent threshold value, overage rent percentage), initial investment by the tenant, and the landlord’s reinvestment threshold and quantity.

In a setting with non-contractable specific investment, the basic tension is between providing incentives in the lease contract for initial investment by the tenant and incentives for subsequent investment by the landlord. High-powered overage rental features are unpopular with the tenant, and have the effect of decreasing initial investment to decrease total sales. Lower total sales subsequently decrease incentives for reinvestment by the landlord. Overage rents increase incentives for value-added reinvestment, which is preferred by the landlord. The optimal contract thus balances this tension, in which base rents trade off with overage rents in the optimal contract.

Joskow (1987) notes that, “Buyers and sellers make larger ex ante commitments to the terms of future trade, and rely less on repeated negotiations over time, when relationship-specific investments are more important.”
When positive externalities accrue to the landlord as a proportion of total sales, low initial investment by the tenant depresses landlord equity value. This causes the landlord to substitute base rents for percentage rents in order to increase initial investment. Greater external flows also allow the landlord to decrease base rents, which further increases initial investment. Thus, base rents and overage rental percentages move inversely with externality, which is consistent with observed practice.

This result does not depend explicitly on inter-store externalities, where externality-consuming tenants “subsidize” externality-generating tenants by paying higher rents. Rather, in our model, external flows that accrue to the landlord substitute for base and overage rents. This causes an increase in initial investment, to the benefit of both landlord and tenant, while also not depressing incentives for landlord reinvestment.

This substitution effect explains overage rent features with stand-alone retail operations. Overage rents are required to compensate for the absence of external flows in order to provide incentives for landlord reinvestment. Since stand-alone retail operations typically contain only one tenant, reinvestment can be especially important and easy to coordinate.

When the landlord has complete or nearly complete bargaining power, as is typically presumed in the literature, a contract with both base and overage rents emerges (see Edelman and Petzold (1996) for interview evidence). As the tenant assumes greater bargaining power, however, both base rents and overage rental percentages decrease. Indeed, given a moderate to large degree of tenant bargaining power, the overage rental feature disappears altogether to result in a base rental contract only. This result occurs even when external flows to the benefit of the landlord are relatively low. Variation in bargaining power can thus explain differences in retail lease contracts, independent of external effects. For example, variation in tenant bargaining
power can explain why stand-alone retail operations sometimes have overage features in their rental contracts and sometimes do not.

The optimal contract with dual agency delivers cumulative equity value of the landlord and tenant that is close to first-best. The differences are especially small when bargaining power is split equally between the two parties. This suggests that our presumption of a piecewise linear contract, which is what is observed in practice, causes little loss of efficiency relative to a more general (and complex) contract specifications.

We find that for most parameter values constellations, the overage rental percentage option is well out-of-the-money at contract execution. When external flows are an important component of landlord equity value, overage rents are not required to provide incentives for reinvestment. As external flows decrease, the landlord increases base rents in order to provide immediate compensation for the loss of external flow-based income. The tenant prefers paying higher base rents to sharing a significant portion of its income, and, while value added through reinvestment is positive, it has less value than that attributable to current cash flow in the base rent. Consequently, the optimal contract is such that base rent and overage rent percentages are relatively high when external flows are low, but the overage rent option remains well out-of-the-money.

Comparative statics for other parameter values are also considered. For example, when holding externality constant, a higher profit margin for a retailer causes base rents to increase and overage rental percentages to decrease. This result demonstrates that retail tenants prefer base rents to overage rents, with the two contract features being substitutes. It differs from our externality results, in which base rents and overage rent percentages move together as external flows substitute for both types rents. This result also suggests that one must be careful to
distinguish between differences in retail categories (which is probably best measured by externality) and differences in retailers within a retail category (which can be measured by profit margin).3

Our model is capable of explaining contracting practices for operations other than shopping centers; indeed, it is correct to assert that our model is not unique to a retail shopping center setting. Consequently, one can ask the question of why we don’t observe sharing contracts with certain other types of commercial operations, such as office or apartment property.

We contend that two factors are necessary for these sharing contracts to work in practice. First, a market structure must exist where an upstream supplier (landlord) provides an input (land) that crucially impacts downstream supplier productivity (crop production or retail sales). This is clearly the case with agricultural production or retail, but is much less clear in the case of, say office or apartment property, where numerous other factors are probably more important than location in the success or failure of the “operation.” Second, output directly attributable to the provision of upstream supply must be easily measurable and verifiable. This is not the case with many types of operations, in which the relevant information is difficult to measure or obtain.

While we obviously do not believe that inter-store externalities are not part of the reason for overage rent contracts with shopping centers, we believe there is more to the story. It is the absence of externality that makes these contracts especially attractive to landlords, as seen by use of similar contracts with stand-alone stores. Bargaining power also plays a role, where anchor stores and national chains have more of it than mom-and-pop retailers. In addition to retail, our model explains the existence of sharecropping contracts, in which landlord bargaining power and the absence of externality are primary reasons for the sharing feature. We would expect to observe the use of similar sharing contracts in other settings that satisfy the productivity and

3 See Edelman and Petzold (1996), who find exactly this result.
measurability criteria discussed above. Franchise agreements and the licensing of certain types of intellectual property are relevant examples.

III. Model

To fix ideas, consider a representative retail operation. Total current operating revenue, or sales, is $qs$, where $q$ is the quantity or quality of retail services and $s$ is the sales per unit of retail services. Retail services, $q$, summarizes such factors as store amenities and current condition. Over time, $q$ depreciates at a constant rate of $\delta$, $\delta>0$. Unit sales, $s$, are determined by the number and characteristics of consumers in households surrounding the operation. Unit sales evolve stochastically according to a geometric Wiener process, with drift parameter, $\mu$, and volatility parameter, $\sigma$, $\mu, \sigma>0$.

Retail operators have a comparative advantage in advertising and selling consumer goods, and real estate developers have a comparative advantage in the provision and maintenance of location and exterior space from which retail goods are sold. Thus, based on this division of labor, the developer (landlord) owns the land and shell that encloses the retail operation, and leases the interior space to the retailer (tenant). A long-term lease contract is offered by the landlord to the retailer once the exterior shell is in place, where terms of the contract are determined endogenously. A long-term lease is executed to address relationship-specific investments as well as other transactions costs (Joskow (1987), Williamson (1996)). Contract terms depend on the respective contributions of the tenant and landlord, where each can observe the other’s contributions to the overall success of the project, but courts cannot. Both parties are risk neutral, with a discount rate of $\iota$. 
Initially, the landlord either provides brand new retail space or redevelops existing space for the known retailer. After the lease has been signed, the retail tenant finishes the interior, advertises, and opens for business. Because the landlord’s initial investments are generic (non-specific to the investments made by the retailer once the lease has been executed) and completed prior to lease negotiation, the landlord’s initial investment is first-best conditional on the tenant’s anticipated actions. Consequently, the landlord’s initial investment can be taken as given, with the initial level of retail services normalized to 1.

Conditional on the terms of the signed lease contract, the retailer decides on its initial investment, $q_0$. This investment can be characterized as site specific physical investment as well as other activities that affect total sales, such as advertising. For simplicity, we package both effects into the level of retail services, $q$, rather than make a distinction between per unit sales and services. The retailer’s initial cost function is Cobb-Douglas, with cost equal to $\beta^\alpha q_0^{\alpha}$, $\alpha>0$, $\beta>1$. Investment and resulting production occur simultaneously at the initial time, $t=0$.

Later, after the commencement of operations, the landlord can repeatedly reinvest in the property. Reinvestment is non-contractable and in the landlord’s own interests. With each reinvestment, the landlord increases the level of services available for the retailer to $q$. The cost of the instantaneous reinvestment as of time $t$ is $q^\gamma e^{\rho t}$, where $\gamma>1$ and $\rho\geq0$. The term $e^{\rho t}$ adjusts for factors such as the cost of reinvestment. To ensure well-behaved solutions, the adjustment factor, $\rho$, is bounded from above by $\iota+\delta-\mu>0$.

The lease contract has three components: base rent, overage rent percentage (as a percentage of total sales), and the overage threshold value (above which overage rents are paid).

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4 One could get the same effect by separately adjusting initial sales, $s$, as long as the parameter values governing sales dynamics do not change.
At any point in time, $t$, both the base rent and the overage threshold value are proportional to current replacement cost of retail services, $q^\gamma e^{\rho t}$. In other words, the base rent and the overage threshold value have the respective forms: $aq^\gamma e^{\rho t}$ and $bq^\gamma e^{\rho t}$, $a,b>0$. Overage rent is paid whenever total sales, $qs$, exceed the overage threshold value. The actual paid overage rent is scaled by $p$, $0 \leq p \leq 1$. The constants $a$, $b$, and $p$ completely identify the lease contract. Terms of the contract are affected by the relative negotiating strength of the two parties, as well as by the anticipated initial investment of the tenant and subsequent reinvestment by the landlord.

Thus, to summarize, at each instant in time the retailer pays the landlord the total rent:

$$R = R(q,s,t) = aq^\gamma e^{\rho t} + p\text{Max}\{0, qs - bq^\gamma e^{\rho t}\}$$ (1)

Total sales at the site generate both profits for the retailer and possibly externalities that benefit the landlord. Retailer profits are proportional to total sales, where the profit margin, $\pi$, $0<\pi<1$, can depend on the type of product sold (the category of sales). Externalities that accrue to the landlord are also proportional to total sales, where the externality parameter is denoted as $\lambda$, $\lambda \geq 0$. At each instant in time the landlord thus realizes total benefits of $R + \lambda qs$. Concurrently, the retail tenant realizes net revenue of $\pi qs - R$.

In order to arrive at the optimal investment policies and lease contract, we can value the landlord and tenant’s revenue streams conditional on their actions. The equilibrium is determined in three stages, which we state in reverse order as a dynamic programming problem. In the first stage, conditional on the lease terms $\{a,b,p\}$ and initial investment by the tenant of $q_0$, the landlord determines the optimal level and timing of reinvestment in the property as a function of
states $q$ and $s$. The landlord’s resulting equity value has the expected present value, $V^L(q,s,t|a,b,p,q_0)$. Between reinvestment dates, this present value satisfies the pde:

$$0 = \frac{1}{2} \sigma^2 s^2 V_{ss}^L + \mu s V_s^L - \delta q V_q^L + V_t^L - t V^L + R + \lambda q s$$

(2)

The level of retail services at which reinvestment optimally occurs is $q^*$. At this point, the landlord reinvests to increase the level of retail services to $q^*$. Both the new level of retail services and the point at which reinvestment occurs maximize the landlord’s net investment gain:

$$0 = \max_{q^*} \left\{ V^L(q^*,s,t) - q^* e^{\rho t} - V^L(q,s,t) \right\}$$

(3)

The landlord’s minimum equity value occurs if per unit sales become zero. In this case, the tenant pays the base rent in perpetuity, which equals $aq^* e^{\rho t}$, and the landlord has no incentive to reinvest in the property.\(^5\) The resulting lower bound on landlord equity value is:

$$V^L(q,0,t) = \frac{aq^* e^{\rho t}}{\gamma \delta + t - \rho}$$

(4)

The tenant’s equity value is similarly valued. The tenant has expected present value, $V^T(q,s,t|a,b,p,q_0)$. Between reinvestment points this present value satisfies the pde:

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\(^5\) For simplicity we assume a “credit” tenant, in the sense that the tenant has other resources to fund the rental payments should sales become zero. Alternatively, payoffs between zero and the perpetual base rent could be considered as a result of zero sales, a change that would not alter the basic problem structure.
At the landlord’s optimal reinvestment point, the tenant’s present value must be continuous:

\[ V^T(\bar{q}^*, s, t) = V^T(q^*, s, t) \]  

(6)

Finally, if sales reach zero, the tenant is obligated to make the fixed lease payment. This obligation mirrors the landlord’s equity in (4):

\[ V^T(q, 0, t) = \frac{-aq^\gamma e^{\rho t}}{\gamma\delta + t - \rho} \]  

(7)

The preceding fully specifies the landlord’s optimal reinvestment problem. The second stage of the problem involves determination of the tenant’s optimal initial investment. Conditional on the lease terms \( \{a, b, p\} \), at time \( t=0 \) the tenant chooses the initial quantity, \( q_0^* \), to maximize its present value:

\[ W^T(a, b, p) = \max_{q_0} \left\{ V^T(q, 1, 0) - \alpha q^\beta \right\} \]  

(8)
where initial sales are normalized to 1 and an equity participation constraint of $W^T(a,b,p) \geq 0$ is imposed. The landlord’s equity has the resulting value, $W^L(a,b,p) = V^L(q_0^*, 1, 0)$.

The third stage of the model is a determination of the optimal lease contract. The optimal contract, $\{a^*, b^*, p^*\}$, maximizes a weighted average of the landlord’s and tenant’s initial equity value:

$$
max_{a,b,p} \left\{ \omega W^L(a,b,p) + (1-\omega) W^T(a,b,p) \right\}
$$

with $0 \leq \omega \leq 1$. The weight $\omega$ reflects the bargaining power of the landlord relative to the tenant. We would expect dominant landlords and those who hold space in highly desirable locations to have more bargaining power relative to tenants. Larger tenants and those with nationally recognized brands will typically have more bargaining power relative to landlords.

The first-best solution is one that maximizes the joint equity value of the landlord and tenant, $\hat{V} = V^L + V^T$. In between investment, joint equity value satisfies:

$$
0 = \frac{1}{2} \sigma^2 s^2 \dot{V}_s^2 + \mu s \dot{V}_s - \delta q \dot{V}_q + \dot{V}_i - t \dot{V} + q s(\lambda + \pi)
$$

If sales become zero, so does the joint equity value. The first-best reinvestment is triggered by the retail service level, $q$. At that point, the landlord reinvests to increase the level of retail services to $\bar{q}$. The first-best reinvestment policy is thus the pair $\{q^*, \bar{q}^*\}$ that solves the problem:
\[
0 = \max_{q,s} \left\{ \dot{V}(q,s,t) - \frac{\dot{s}}{s} e^{\nu t} - \tilde{V}(q,s,t) \right\}
\]  

(11)

Initial investment by the tenant maximizes joint equity value at \( t=0 \). This initial investment, \( q_{00}^* \), solves the problem:

\[
\max_{q_{00}} \left\{ \dot{V}(q,1,0) - \alpha q^\beta \right\}
\]  

(12)

There is no one unique lease contract in the first-best case, since lease payments are a zero-sum wealth transfer between agents. Rather, the tenant’s profit margin parameter, \( \pi \), together with the landlord’s externality parameter, \( \lambda \), determine joint equity value as a function of total sales. Total sales in turn depend on the optimal initial investment, \( q_{00}^* \), and the subsequent optimal reinvestment levels, \( \{q^*, \tilde{q}^*\} \), which fall out of the respective second- and first-stage optimization problems.

IV. Transformation and Solutions

The equity value equations (2) and (5) are partial differential equations that do not offer obvious general solutions. The objective of this section is to transform the partial differential equations into ordinary differential equations and then to solve these equations in order to characterize the optimal investment and contracting policies.

Define the transform variable as \( y = q^{1-s} e^{\nu t} \) and let \( F^i(y) = q^s e^{\nu t} V^i(q,s,t) \), where \( i=0 \) indicates the landlord and \( i=1 \) indicates the tenant. The transformed total value is the sum,
\[ F(y) = F^0(y) + F^i(y) \]. Value dynamics depend on whether an overage rent is being paid or not. For \( 0 \leq y \leq b \), no overage rent is required. We indicate this region with \( j = 0 \). Otherwise, for \( y > b \), an overage rent is being paid in addition to the base rent. This region is indicated by \( j = 1 \). Lastly, construct the three constants: \( \theta = \frac{y}{y-1} \); \( \psi = \delta + t - \mu \); \( \phi = \delta + t - \rho \), all of which are positive.

With this transformation, and using the indicated notation, equity value equations (2) and (5) simplify to the following ode:

\[
0 = \frac{1}{2} \sigma^2 y^2 F_{yy}^i + (\phi - \psi) y F_y^i - \phi F^i + \psi \xi_{i,j} y + \phi \xi_{i,j}
\]

where \( \xi_{i,j} = \left[ \frac{(1-i)\lambda + i\pi + p(1-2i)}{\psi} \right] \) and \( \xi_{i,j} = \frac{(1-2i)(a - pbj)}{\varphi} \) for \( i, j = 0,1 \).

The zero sales boundary conditions expressed in equations (4) and (7) become:

\[
F^i(0) = \xi_{i,0}
\]

for \( i = 0,1 \).

Transformed optimal reinvestment by the landlord, as originally expressed in equation (3), is restated as:

\[
\theta = \max_{y, \bar{y}} \left\{ y^{-\theta} \left( F^0(y) - 1 \right) - \bar{y}^{-\theta} F^0(\bar{y}) \right\}
\]
where, because \( y \) moves inversely with \( q \), \( y^* \) is the new level for \( y \) immediately after reinvestment occurs and \( \bar{y}^* \) is the threshold at which reinvestment is triggered.

The tenant’s continuity condition at the point of reinvestment, as originally expressed in equation (6), becomes:

\[
(y^*)^\theta F^l(y^*) = (\bar{y}^*)^\theta F^l(\bar{y}^*)
\]  

Equation (8), the tenant’s optimal initial investment, is transformed to:

\[
G^l(a,b,p) = \max_{y_0} \left\{ y^{-\theta} F^l(y) - ay^{-\theta\beta/y} \right\}
\]  

in which \( y_0^* \) is the optimal value. Again, an equity participation constraint of \( G^l(a,b,p) \geq 0 \) is imposed.

Finally, if we let \( G^0(a,b,p) = (y_0^*)^{-\theta} F^0(y_0^*) \), from equation (9) the optimal contract, \( \{a^*, b^*, p^*\} \) is determined by:

\[
\max_{a,b,p} \left\{ \alpha G^0(a,b,p) + (1-\alpha) G^l(a,b,p) \right\}
\]  

A similar process yields the valuation equation and related optimizing relations for the first-best problem.

The general solutions to equation (13) are:
\[
F^i(y) = \begin{cases} 
F_{i,0}(y), & 0 \leq y \leq b \\
F_{i,1}(y), & y > b 
\end{cases}
\] (19)

with

\[
F_{i,j}(y) = A_{i,j}^1 y^{\eta_1} + A_{i,j}^2 y^{\eta_2} + \xi_{i,j} y + \xi_{i,j}
\] (20)

for \( i,j = 0,1 \). \( A_{i,j}^1 \) and \( A_{i,j}^2 \) are constants to be determined and

\[
\eta_1, \eta_2 = \left[ \frac{1}{2} - \frac{\phi - \psi}{\sigma^2} \right] \pm \sqrt{\left[ \frac{1}{2} - \frac{\phi - \psi}{\sigma^2} \right]^2 + \frac{2\phi}{\sigma^2}}. 
\]

It follows from the definitions of \( \phi \) and \( \psi \) that \( \eta_1 > 1 \) and \( \eta_2 < 0 \).

Particular solutions require solving for the constants \( A_{i,j}^1 \) and \( A_{i,j}^2 \), \( i,j = 0,1 \), as well as optimal reinvestment values, \( \underline{y}^* \) and \( \bar{y}^* \). We will consider the landlord \((i=0)\) problem first.

Because there is an absorbing lower bound, \( A_{0,0}^2 = 0 \). This leaves five unknowns, \( A_{0,0}^1 \), \( A_{0,1}^1 \), \( A_{0,1}^2 \), \( \underline{y}^* \), \( \bar{y}^* \), implying that five equations are required for identification.

At the overage threshold, \( b \), value matching and smooth pasting are required. This implies that:

\[
F_{00}(b) = F_{01}(b) \text{ and } F'_{00}(b) = F'_{01}(b) \] (21)
Additional restrictions are provided by equation (15) and the associated first-order conditions. To facilitate the analysis, we will assume that a base rental contract is in effect initially and right after reinvestment. That is, \( y_0 \leq b \) and \( \overline{y} \leq b \) is required. This is what is seen in practice (base rents only at execution of retail lease contract), and is a constraint that we don’t expect to bind for realistic parameter values. Then, because the overage rent region also includes base rent, we will require that \( p=0 \) when the reinvestment trigger point \( \overline{y} \leq b \); otherwise, for \( \overline{y} > b \), \( p \geq 0 \). This stipulation allows for the general possibility that reinvestment can be triggered in either the base rental or the overage rental regions, while also streamlining the analysis.

With this structure, equation (15) can be written as:

\[
(y^*)^\theta (F_{00}(y^*) - I) - (\overline{y}^*)^\theta F_{01}(\overline{y}^*) = 0
\]  

(22)

The associated first-order conditions are:

\[
\overline{y}^* F'_{00}(\overline{y}^*) - \theta (F_{00}(\overline{y}^*) - I) = 0
\]  

(23a)

\[
\overline{y}^* F'_{01}(\overline{y}^*) - \theta F_{01}(\overline{y}^*) = 0
\]  

(23b)

Finally, the solution must also satisfy the second-order conditions:

\[
(y^*)^2 F''_{00}(y^*) - 2 \theta y^* F'_{00}(y^*) + \theta(\theta + 1)(F_{00}(y^*) - I) < 0
\]  

(24a)

\[
(\overline{y}^*)^2 F''_{01}(\overline{y}^*) - 2 \overline{y}^* F'_{01}(\overline{y}^*) + \theta(\theta + 1)F_{01}(\overline{y}^*) > 0
\]  

(24b)
By taking the general solution in (20), solving for equation (23a), and subsequently applying the value-matching and smooth-pasting conditions stated in (21), we see that:

\[
A_{00}^{l}(y^{*}) = \frac{(y^{*})^{-\eta_{1}}}{\eta_{1} - \theta} \left[ (\theta - 1)\xi_{0,0} y^{*} + \theta(\xi_{0,0} - 1) \right]
\]  

(25a)

\[
A_{01}^{l}(y^{*}) = A_{00}^{l}(y^{*}) - A_{01}^{2}b^{\eta_{2} - \eta_{1}} - pb^{\eta_{2} - \eta_{1}} \left( \frac{1}{\psi} - \frac{1}{\phi} \right)
\]  

(25b)

\[
A_{01}^{2} = p \frac{b^{\eta_{2} - \eta_{1}}}{\eta_{1} - \eta_{2}} \left[ (1 - \eta_{1}) \frac{1}{\psi} + \eta_{1} \frac{1}{\phi} \right]
\]  

(25c)

The parameter space must be such that these values are positive. Note that \( A_{00}^{l} \) and \( A_{01}^{l} \) depend on the optimal reinvestment point, \( y^{*} \), which is yet to be determined. Also observe that \( A_{01}^{l} = A_{01}^{2} = 0 \) when \( p = 0 \). When this condition holds, optimal reinvestment occurs (or occurs as if) in the base rental region, as you would expect when percentage rents are not being paid.

By referencing the first-order conditions in (23), we obtain the following relations for \( y^{*} \) and \( \bar{y}^{*} \):

\[
y^{*} = \bar{y}^{*} \left[ \frac{F_{01}(\bar{y}^{*})}{F_{00}(\bar{y}^{*}) - 1} \right]^{-\frac{1}{\theta}}
\]  

(26a)

\[
\bar{y}^{*} = \theta \left[ \frac{F_{01}(\bar{y}^{*})}{F'_{01}(\bar{y}^{*})} \right]
\]  

(26b)

20
which must satisfy the second-order conditions stated in (24).

Equations (25) and (26) close the system, with resulting solutions providing the landlord’s equity value and optimal reinvestment policy.

The tenant’s equity value results from the relation, $F^t(y) = F(y) - F^0(y)$, where $y^*$ and $\bar{y}^*$ are exogenously determined by the landlord. The general solution for cumulative equity value, $F(y)$, is:

$$F(y) = A^1 y^{\eta_1} + (\xi_{0,0} + \xi_{1,0})y$$

(27)

Value matching at the point of reinvestment requires:

$$\left(y^*\right)^\theta \left(F(y^*) - 1\right) - \left(\bar{y}^*\right)^\theta F(\bar{y}^*) = 0$$

(28)

By substituting the general solution for $F(y)$ into (28) and solving for $A^1$, cumulative equity value can be determined. This relation can then be used in equation (17) to deliver the tenant’s optimal initial investment, $y_0^*$, where $y_0^*$ must satisfy second-order conditions as well as the participation constraint.

The last step is to use equation (18) to solve for the optimal contract, $\{a^*, b^*, p^*\}$, conditional on $y^*$, $\bar{y}^*$ and $y_0^*$. This can be done numerically with application of iterative techniques.
In the case of first-best, the general solution for equity value after initial investment and in between reinvestment is given in equation (27), where we now denote the value function as 
\( \hat{F}(\bar{y}) \) and the constant as \( A \).

The value matching condition stated in equation (28) can be used in this case to determine \( A \) as a function of the reinvestment values, \( y^* \) and \( \bar{y}^* \). In this case the value-matching condition becomes:

\[
\left( \frac{y^*}{y} \right)^{-\theta} \left( \hat{F}(\frac{y^*}{y}) - 1 \right) \left( \frac{\bar{y}^*}{y} \right)^{-\theta} \hat{F}(\frac{\bar{y}^*}{y}) = 0
\]

which produces:

\[
A\left( \frac{y^*}{y}, \frac{\bar{y}^*}{y} \right) = \frac{\left( \frac{\bar{y}^*}{y} \right)^{1-\theta} - \left( \frac{y^*}{y} \right)^{1-\theta}}{\left( \frac{y^*}{y} \right)^{1-\theta} - \left( \frac{\bar{y}^*}{y} \right)^{1-\theta}}
\]

The difference between the first-best problem and the cumulative value problem with agency is that the reinvestment values for \( y \) are endogenously determined in the first-best case.

The first-order (smooth-pasting) conditions that follow from (29) can be used to determine \( y^* \) and \( \bar{y}^* \). After some algebraic manipulation and simplification, we find that:
\[ y^* = -\gamma \left( \xi_{0,0} + \xi_{1,0} \left( \frac{y^*}{y} \right)^{\eta_1 - 1} \right) \]  

\[ \left( \frac{y}{y^*} \right) = \theta(\eta_1 - 1) \left[ (\theta - 1)\eta_1 \left( \frac{y}{y^*} \right)^{\eta_1 - 1} + \eta_1 - \theta \right]^{-1} \left( \frac{1}{1 - \theta} \right) \]  

This fully closes the system. From here the optimal initial investment, \( y_{00}^* \), can be determined, subject to satisfying the associated second-order condition and participation constraint.

V. Optimal Lease Contract and Investment Policy

For reference, the appendix summarizes the numerous parameters and variables contained in the model. The optimal investment and contracting problem is solved numerically as a dynamic program for each of the three stages (reinvestment, initial investment, contract), where a hill-climbing algorithm is implemented using a modified Newton-Raphson technique. A range of different starting values for the endogenous quantities were specified in order to help assure identification of a universal optimum.

V.A. Base Case

To assess the optimal contract and associated comparative statics, numerical solutions are obtained for a variety of eligible parameter value constellations. For purposes of specifying a base case, the following parameter values are employed: \( \gamma = 2.0; \ \delta = .03; \ \epsilon = .05; \ \rho = .03; \ \mu = .03; \)
\[\sigma = 0.10; \alpha = 1.0; \beta = 2.0; \lambda = 0.20; \pi = 0.30; \omega = 0.50.\] We believe these parameters values to be realistic and representative of those typically encountered in a retail operating environment.

This particular case corresponds to a tenant that is generating positive externalities that accrue to the landlord, and is a case that has sparked considerable interest in the literature. In a shopping center setting, the externality-generating tenant would likely be an anchor tenant or national chain store with a well-known brand name. This base case produces the following results, as summarized in Table 1.

**Table 1**

**Optimal Contracting and Re/Investment Policy (Base Case)**

<table>
<thead>
<tr>
<th>Contract Variables</th>
<th>Re/Investment Values</th>
<th>Equity Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a^* = 0.49)</td>
<td>(q_0^* = 3.906)</td>
<td>(W^L = 40.16)</td>
</tr>
<tr>
<td>(b^* = 0.953)</td>
<td>(q^* = 0.891se^{-0.03t})</td>
<td>(W^T = 22.67)</td>
</tr>
<tr>
<td>(p^* = 0.0)</td>
<td>(q^* = 4.219se^{-0.03t})</td>
<td></td>
</tr>
</tbody>
</table>

Base rent equals about 19 percent of total sales at contract execution. In this case the optimal contract consists of base rent only – there is no overage rent, since \(p=0.0\). This finding is consistent with contracts often observed in practice, in which externality-generating tenants pay no overage rents. Not surprisingly, tenants have a strong aversion to actually paying such rents.

Because overage rents are eliminated in this contract, initial investment by the tenant is encouraged and is in fact relatively high. At \(q_0^* = 3.906\), initial investment is only slightly below the \((t=0)\) steady-state reinvestment optimum of \(\bar{q}^* = 4.219\). This compares to the first-best level
of initial investment, $q^*_{00} = 3.861$, suggesting that agency-affected initial investment is very close to first-best. In fact, the tenant slightly overinvests relative to first-best due to relatively low base rents and the absence of overage rents.

The landlord benefits from such investment, as it increases the initial base rent as well as the value of external flows. External flows, which are a function of tenant gross sales, are thus seen to directly substitute for overage rents, which are also a function of gross sales. In other words, overage rents, which might ordinarily be required to provide proper incentives for reinvestment, become less critical as an optimal contracting mechanism in the presence of positive external effects.

Initial investment and the quality/quantity of services bounds for reinvestment are reported in their primitive variable quantities, $q^*_0$, $q^*$, $q^*$, respectively. Note that the reinvestment bounds are increasing in the level of sales, $s$, and decreasing in time, $t$. Because sales are expected to increase at a rate of 3 percent annually, the net expected change in these bounds over time is minimal.

Reinvestment with dual agency occurs less often than first-best; that is, the landlord dynamically underinvests. This is primarily the result of the agency-based reinvestment trigger value being lower the corresponding first-best value. Without overage rents, the landlord has less incentive to reinvest. The landlord avoids overage rents in the optimal contract to extract a higher initial investment from the tenant, which increases the rate of external flow and hence equity value. Thus, because external flows depend directly and immediately on the level of retail services, $q$, the agency-based optimal contract is such that the tenant overinvests and the landlord subsequently underreinvests.
Initial equity values for the landlord and tenant are 40.16 and 22.67, respectively. It is useful to decompose these values, as this exercise provides insight into optimal contract determination. Landlord equity value has three components: i) rental cash flow stream, ii) externality flow, and iii) reinvestment option value. Because the optimal contract includes base rents only in this base case, the rental component is easily valued using equation (4), and equals 9.35. The externality component is also easily valued by capitalizing the time $t=0$ flow, $\lambda q_0^*$, at the rate of $\psi$. The resulting value is 15.62. The reinvestment option value component can be obtained by taking the difference between total equity value and the rental and externality components. The resulting option value is 15.19.

The externality flow component of total landlord equity value exceeds the rental component, which highlights the landlord’s reduced incentives to require overage rents in the optimal contract. The tenant prefers base rents to overage rents, in the sense that overage rent payments reduce net profits while providing no offsetting benefits. Indeed, all else equal, in comparison to overage rents, the tenant prefers an increase in the base rent that results from reinvestment, since reinvestment increases $q$ which in turn increases profits, $\pi q_0^* s$. Lower base rent and elimination of overage rent feed back to encourage more initial investment by the tenant, which is also preferred by the landlord.

Reinvestment option value is a large component to total equity value, implying that convexity in the landlord’s payoff function creates significant value. This value is derived from three sources. First, there is the direct value effect of reinvestment that establishes a lower bound on $q$. Second, reinvestment increases value derived from the externality flow, since external effects are proportional to total sales, $qs$. Third, the ability to make repeated reinvestments adds value relative to a one-time option to reinvest (Williams (1997)).
The tenant’s total equity value can be similarly decomposed. These components are: i) capitalized profits from sales prior to rental expense; ii) capitalized rental expense; iii) option value associated with reinvestment by the landlord; and iv) the cost of initial investment. These four components are 23.44, –9.35, 23.84, and –15.26 respectively. Capitalized profits are easily obtained by dividing initial $\pi q^*_0 s$ by $\psi$. Capitalized rental cost mirrors that of the landlord. Option (convex payoff) value is a sizeable component of tenant equity value, and follows from the positive effects of reinvestment on total profits. This component is large due to value associated with the jump in profits when reinvestment occurs (with profit margin, $\pi=0.30$). Option value is significant even after accounting for base rent increases that partly offset the benefit, and follows because externalities figure into the landlord’s cost of reinvestment to decrease the base rental component, $a$. The final component is initial investment cost, which derives from a quadratic production function technology with initial investment, $q^*_0$.

Total equity value net of the initial investment is thus 62.83. This compares to the first-best total equity value of 64.38, implying that the optimal contract under dual agency delivers almost 98 percent of the total possible value. Such a high percentage occurs with equal bargaining power for the landlord and tenant. With unequal bargaining power, total net equity value declines relative to first-best (for example, total net equity value is 92 percent of first-best when $\omega=0.90$). This is not surprising, as unequal bargaining power at contract execution leverages one agent’s ability to extract rents from the other agent, which distorts investment policy to decrease total equity value.

Comparison to first-best suggests that the imposition of a piece-wise linear contract appears to result in little or no efficiency loss relative to the specification of a more general, and complex, contract form.
To summarize, this base case yields a relatively low base rent, no overage rent, and significant initial investment by the tenant. Externalities generated by the tenant to the benefit of the landlord play a central role in the results, where externalities substitute for overage rents in the optimal contract. This contract delivers total value that is close to first-best, where equal bargaining power among agents is a central reason for the outcome. This suggests simple linear contracts observed in practice are highly efficient relative to more complex contracting alternatives.

V.B. Comparative Statics

Much of the focus on optimal contracting in a retail setting involves the effects of externality (e.g., Brueckner (1993), Wheaton (2000)). Empirical studies suggest that this focus is warranted, as evidence indicates that base rents and overage rent percentages move inversely with the magnitude of the externality (as proxied by store size or category of sales). Moreover, while many retail contracts contain overage rent clauses, they are often far out-of-the-money at contract execution and are never expected to actually pay. Empirical study suggests a first-order relation between external effects and the “moneyness” of the overage rental option at the time of contract execution.

This literature generally examines externality in the context of an agglomerative mix of tenants, where externality-consuming tenants “subsidize” externality-generating tenants through higher base and overage rents. In Wheaton (2000), overage rents are uniquely employed as an incentive compatibility contracting mechanism. This and other related literature cannot explain the existence of overage rent contracts in stand-alone retail establishments, however. Our model,
with its focus on the individual tenant, which may or may not generate an externality to the benefit of the landlord, offers a structure to examine such an issue.

An additional important but unaddressed issue is the relative degree of bargaining power that exists between the landlord and the tenant. A likely reason for the lack of focus on bargaining power is that externality and bargaining power are often correlated in the data, where tenants that generate more externalities are thought to have relatively greater bargaining power. We would expect to see frequent exceptions to this rule of thumb, however. In certain markets or at certain desirable locations (in which the landlord is a spatial monopolist), landlords will enjoy market power relative to tenants—even with those tenants that might be highly desirable due to their brand name recognition. Conversely, market conditions or location might dictate that even small, non-externality-generating tenants enjoy considerable bargaining power relative to the landlord.

To highlight these effects, in this section we will initially focus on externality, \( \lambda \), and relative bargaining power, \( \omega \), as they impact optimal contracting and investment policies. Specifically, given base case parameters, we vary \( \lambda \) and \( \omega \) to generate comparative static results. We believe the following results to be robust, as comparative static relations are consistent across other eligible parameter value constellations. In terms of optimal contracting variables, \( \{a^*, b^*, p^*\} \), we obtain the following relations:

\[
\frac{\partial a^*}{\partial \lambda} < 0, \quad \frac{\partial a^*}{\partial \omega} > 0: \text{An increase in the rate of external flow causes a reduction in base rent, as external flows substitute for both base and overage rents. An increase in the bargaining power of the landlord causes an increase in base rent, as more weight is given to initial landlord equity value in the determination of optimal contract parameters, } \{a^*, b^*, p^*\}.
\]
These results demonstrate the importance of distinguishing between external and bargaining related effects, as an increase in bargaining power for the landlord increases base rent, which often causes a decrease in initial investment. In contrast, an increase in the external flow causes the landlord to decrease base rent in equilibrium, thereby facilitating initial investment by the tenant, due the importance of the increase in external flows on equity value.

\[
\frac{\partial p^*}{\partial \lambda} \leq 0, \quad \frac{\partial p^*}{\partial \omega} \geq 0: \text{An increase in the rate of external flow causes a decrease in the overage rent percentage when } p \text{ is initially positive. In the case of initial } p^* = 0, \text{ overage rent percentage remains equal to zero as } \lambda \text{ increases. An increase in the bargaining power of the landlord results in an increase in the overage rent percentage when } \omega \text{ is initially positive. In the case of initial } p^* = 0, \text{ overage rent percentage remains equal to zero as } \omega \text{ decreases.}
\]

Figure 1 shows how, with base case parameter values, overage rent percentage changes as a function of \( \lambda \) and \( \omega \). Given equal bargaining power, overage rent is zero for \( \lambda \) in excess of 0.16. Below this point it increases exponentially to peak at approximately 12.6 percent when \( \lambda = 0 \). Similarly, overage rents are optimally set to zero for moderate to low values of \( \omega \). Only when the landlord has significant bargaining power relative to the tenant do overage rents appear in the optimal contract.

These findings demonstrate that the absence of external effects is a crucial cause of overage rents in the optimal contract. Without significant external flows from the tenant, the landlord requires overage rents to enhance incentives for reinvestment in the retail space. In the sense of aligning the ex post incentives for the landlord, this is similar to Wheaton’s model of optimal contracting with externalities. Where we part company is that Wheaton’s results rely on
multi-tenant inter-store external effects. Retail tenants agree to insert high-powered incentives \textit{vis-à-vis} the overage rent component to reduce \textit{ex post} opportunism by the landlord when reconfiguring the tenant mix.

**Figure 1**

*Overage Rent Percentage, \( p \), as a Function of \( \lambda \) and \( \omega \)*

**Panel A: Externality, \( \lambda \)**

**Panel B: Relative Bargaining Power, \( \omega \)**

Our model addresses a single tenant without explicit reference to other tenants. A tenant that generates no externalities can exist as a stand-alone operation, without any agglomerative effects. We find that overage rents are an optimal contracting mechanism in this case, as the tenant wishes to affect \textit{ex post} incentives of the landlord as they apply to their particular site
(they are unconcerned about off-site effects). Thus, we can uniquely explain the existence of overage rent contracts with stand-alone retail operations.

Interestingly, an analysis of data on stand-alone big-box retail operations reveals that overage rents only sometimes appear in the lease contract. Our model provides an explanation for this outcome: landlords with high relative bargaining power are able to secure such rents, whereas tenants with relatively high bargaining power are able to exclude the overage rental feature.

Our no-externality, sharing contract result can also explain sharecropping contracts, where, in our context, the landlord makes major periodic investments in the land to restore its productivity. In a typical sharecropping setting, we would expect the landlord to possess significant bargaining power, which further enhances the overage rent component of the optimal contract. Our model thus offers an alternative perspective on sharecropping arrangements, which typically rely on risk-sharing and capital constraints to justify observed contracting outcomes.

Given that the absence of externality provides a powerful rationale for overage rent contracts, why don’t we observe this contracting mechanism in other commercial leasing settings such as with office and apartment buildings? We believe it is largely due to two factors. First, sharing contracts require verifiable downstream production, which is easily satisfied in a retail setting due to required reporting of sales for sales tax purposes. In other settings, verifiability of tenant income or sales can be difficult. Second, a market structure exists where the upstream supplier (the landlord) provides an input (well-located land) that is crucial to the success or failure of the downstream supplier (the retail tenant). A strong first-order causative relation between location and total production of the tenant is much less apparent with other types of commercial property uses.
Similar contracts should therefore be observed in other settings that satisfy the necessary conditions just discussed. Franchise agreements and licensing arrangements involving certain types of intellectual property are two such settings. An initial reading of the literature suggests our model does a good job of explaining the data.

\[
\frac{\partial (q_0^{1-\gamma} / b^*)}{\partial \lambda} < 0, \quad \frac{\partial (q_0^{1-\gamma} / b^*)}{\partial \omega} < 0: \text{An increase in external flow causes an increase in initial investment. Given that } \gamma > 1, \text{ this implies an increase in the expected time to hitting the overage rental threshold } (q_0^{1-\gamma} \text{ decreases relative to } b^*) \text{ and thus a decrease in the relevance of the overage rent component of the optimal contract. The comparative static relation is indeterminate in the case of the relative bargaining power of the landlord.}
\]

The externality comparative static result is consistent with the empirical fact that externality-generating tenants pay less overage rent (Gould et al. (2002)). Further examination of total initial sales relative to the overage rent threshold value when overage rents are part of the optimal contract reveals that the overage rental option is well out-of-the-money at contract execution. Depending on parameter values, the overage rent threshold is found to be two to four times initial total sales. This outcome is also consistent with the data, which show that overage rent thresholds are often two or more times initial sales. A desire to provide the tenant incentives for initial investment appears to be the cause of this result, where the landlord compensates for lower-powered overage rents with higher base rents.

The comparative static with respect to bargaining power is indeterminate. An increase in \( \omega \) has only small effects on both initial investment and the overage rent threshold, with the net effect being indeterminate. Closer examination of the component effects shows that the ratio
\[ q_0^{1-\gamma} / b^* \] decreases with extremes in relative bargaining power, and is generally at a maximum when landlord and tenant have roughly equal bargaining power.

In addition to optimal contracting relations, comparative statics for \( \lambda \) and \( \omega \) with respect to the landlord’s reinvestment policy also yield interesting results.

\[
\frac{\partial (q^* / q^*)}{\partial \lambda} < 0, \quad \frac{\partial (q^* / q^*)}{\partial \omega} > 0: \text{An increase in the rate of external flow causes the landlord to reinvest more frequently. The comparative static is indeterminate with respect to relative bargaining power.}
\]

External flows increase with total sales, thus providing the landlord with increased incentives to reinvest to offset depreciation-related declines in sales. It is interesting to note that both \( q^* \) and \( \overline{q}^* \) increase with an increase in \( \lambda \), indicating a higher trigger point for reinvestment as well as an increase in the total quantity/quality of retail services as a result of reinvestment. This happens because the landlord directly benefits from such a policy in its equity value, whereas the tenant does not have to bear the direct cost of this policy because external effects are exogenous.

In a number of cases, especially those with moderate positive rates of external flow and balanced bargaining power, we obtain rental contracts with the overage rent threshold value, \( b^* \), in excess of \( y^* \), and where \( p=0 \) constraint is binding. This implies a contract in which an overage rent percentage is specified, but where that rent is never expected to be paid (since, in the model, reinvestment occurs prior to hitting the overage rent threshold). This is an arrangement that is often observed in practice, where contracting parties incorporate an overage
Although outside of our model, this finding suggests that one might incorporate such a contracting feature as an incentive compatibility device, whereby the landlord is able to harvest overage rents should the tenant or some exogenous factor somehow limit the landlord’s ability to undertake value enhancing actions after the contract is signed (see Joskow (1987) for more on the relevance of specific investment and hold-up in the context of long-term contracting).

\[
\frac{\partial (q_0^*/q^*)}{\partial \lambda} < 0, \quad \frac{\partial (q_0^*/q^*)}{\partial \omega} < 0: \text{An increase in the rate of external flow causes a decrease in the distance between initial quantity/quality of services and the reinvestment trigger value, implying a shorter expected time to initial reinvestment. A similar relation holds for an increase in landlord bargaining power.}
\]

The underlying reasons for these similar comparative statics are very different. An increase in \(\lambda\) causes an increase in both initial investment and the reinvestment trigger, with a relatively larger increase in the reinvestment trigger. This causes reinvestment to occur sooner on average to offset depreciation-related declines in total sales, which is in the landlord’s interest, since reinvestment increases total sales to increase the rate of external flow.

In contrast, an increase in landlord bargaining power has only a small effect on initial investment by the tenant, but causes a significant increase in the reinvestment trigger value, \(q^*\), to decrease the expected time to initial reinvestment. The landlord prefers more frequent reinvestment because it increases equity value, especially when \(\lambda\) is positive.

---

6 Gould et al. (2002) find that only 18 percent of stores with overage rent thresholds specified in the contract actually reach the threshold.
In Table 2 we report comparative static relations for other selected parameters. Recall that the quality/quantity of retail services, \( q \), depreciates at a rate of \( \delta \). An increase in \( \delta \) thus has a negative impact on total sales, to the detriment of the retail tenant and the landlord that benefits from external flows. It also affects reinvestment policy of the landlord, who must adjust both the timing and quantity of reinvestment to compensate for lost productivity.

<table>
<thead>
<tr>
<th>Endogenous Quantity</th>
<th>( \delta )</th>
<th>( \mu )</th>
<th>( \sigma )</th>
<th>( \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a^* )</td>
<td>+</td>
<td>–</td>
<td>–</td>
<td>+</td>
</tr>
<tr>
<td>( p^* )</td>
<td>+</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>( q_0^{<em>1-\gamma} / b^</em> )</td>
<td>+</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>( \bar{q}^* / q^* )</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>( q_0^* / \bar{q}^* )</td>
<td>+/-</td>
<td>+</td>
<td>+/-</td>
<td>+/-</td>
</tr>
</tbody>
</table>

These effects cause the landlord to increase both the base rent and the overage rent percentage to compensate for the higher rate of depreciation. The overage rent option also becomes more valuable, as seen in \( q_0^{*1-\gamma} / b^* \). The steady-state reinvestment ratio, \( \bar{q}^* / q^* \), increases to compensate for the higher rate of depreciation. Both values, \( \bar{q}^* \) and \( q^* \), decrease relative to the base case to compensate for a higher rate of depreciation. Initial reinvestment

Table 2

Comparative Static Relations for Selected Parameters
timing, as reflected in the ratio, \( q_0^* / q^* \), is indeterminate, as both the numerator and denominator decrease as a result of an increase in \( \delta \).

All else equal, an increase in the drift rate of sales, \( \mu \), increases equity value for both the landlord and the tenant. This in turn causes the equilibrium base rent and overage rent percentage to decrease. The moneyness of the overage rent option also decreases. The steady-state reinvestment ratio, \( q^*/q^* \), increases to compensate for a higher drift rate in sales. A similar dynamic explains the increase in the initial reinvestment ratio, \( q_0^*/q^* \).

An increase in the volatility of sales is beneficial for the landlord, as it increases the value of the overage rent option. This causes a decrease in the equilibrium base and overage rent percentage. Increased volatility in sales increases the option moneyness ratio, \( q_0^{l-\gamma}/b^* \), however, since higher volatility decreases the expected time to hitting a barrier. The steady-state reinvestment ratio, \( q^*/q^* \), increases to compensate for the fact that an increase in volatility reduces the expected time to reinvestment. The initial reinvestment ratio, \( q_0^*/q^* \), is indeterminate, as both initial investment and the reinvestment trigger value decrease as a result of an increase in sales volatility.

Interestingly, as the tenant’s profit margin increases, base rent increases and overage rent percentage decreases. An increase in profit margin implicitly provides the tenant greater bargaining power, as an increase in \( \pi \) increases tenant equity value without directly affecting landlord equity value. From the tenant’s perspective, there is a tradeoff between base rent and overage rent percentage. The tenant prefers base rent, which explains the comparative static relations. Increased profits and the beneficial adjustment in contract terms causes an increase in
initial investment by the tenant, which in turn decreases the moneyness of the overage rent option. A lower overage rent percentage decreases reinvestment incentives of the landlord, to increase the steady-state reinvestment ratio, $q^* / q^*$. The initial reinvestment ratio comparative static is indeterminate.

We would caution against interpreting $\pi$ in our model as indicating a separate “category of retail stores”. For example, anchor stores generally have lower sales per square foot than smaller specialty stores, where anchor stores also generally have lower base rents and overage rent percentages in their leases. Sales per square foot in this case correlates closely with externality and relative bargaining power, which are the real causes of differentials in the optimal rental contract, and which generate comparative statics that are consistent with the data in terms of retail store categories. The appropriate way to interpret the comparative static with respect to $\pi$ is as a change in profitability of a store within a particular retail category (such as men’s apparel or a jewelry store).

VI. Conclusion

We have constructed a model of bilateral trade between an upstream supplier (landlord) that confers property usage rights to downstream producer (tenant). In return for usage rights, the downstream producer pays a base fee (rent) plus a percentage of verifiable sales production that exceeds an overage threshold value. Our model allows for the possibility that downstream production complements other activities of the upstream supplier to increase total revenues. It also recognizes different levels of bargaining power that may exist between agents. In designing an optimal contract, the upstream supplier wants to provide incentives to the downstream
producer to make high initial investments while also maintaining its own incentives to reinvest to enhance productivity.

Endogenously determined quantities in our model are optimal contracting terms (base rent, overage rent threshold, overage rent percentage), initial investment by the downstream producer, and the upstream supplier’s reinvestment threshold and quantity. In this paper we specifically consider a retail lease contracting environment. We find that when positive externalities accrue to the benefit of the landlord, they substitute for both base and overage rent in the optimal contract. Lower rents cause higher initial investment by the tenant, which enhances landlord as well as tenant equity value.

This result implies it is the absence of external effects that explains the existence of overage rents in settings where production is verifiable and highly dependent on upstream usage rights (e.g., location in the case of land). This allows us to explain the existence of overage rental contract features with stand-alone retail operations and one-off licensing agreements.

Sharecropping and related contracting literature generally presume complete bargaining power on behalf of the landlord. When bargaining power is allowed to vary, we find that strong landlord bargaining power causes an overage rental contract to emerge with higher overage rent percentages in addition to higher base rents. Bargaining power that is more balanced or that favors the tenant results lower overage rent percentages, and can even eliminate the overage rent feature altogether. Variation in bargaining power can thus explain cross-sectional differences in overage rental contract terms, independent of external effects.

Our model also explains other important empirical facts documented with retail lease contracting, including the fact that overage rents are typically well out-of-the-money at contract
execution. A piecewise linear contract specification is also found to be close to first-best, suggesting little loss in efficiency relative to specifying more complex contracts.

We conclude by observing that, while inter-store externalities are certainly relevant to contract design in a multi-agent retail setting, there are other factors such as relative bargaining power and landlord reinvestment incentives that are central to the contracting problem. Indeed, we can explain observed relations in retail contracting without explicit consideration of inter-store externality, and then go beyond existing models to explain overage rent contract features with stand-alone retail operations and intellectual property licensing agreements. Our setting suggests it is the importance of upstream supply to downstream sales production and the verifiability of these sales that are necessary for these sharing contracts to work in practice, as opposed to inter-store externalities per se.
References


Appendix

Summary of Model Parameters and Variables

Parameters and Variables in the Basic Model

$q$: Quantity or quality of retail services. At time $t=0$, a choice variable of the tenant (initial investment, $q_0$). After time $t=0$, a choice variable of the landlord (reinvestment). Each reinvestment produces a new quantity/quality of retail services, $\overline{q} = \overline{Q}(s,t)$. Reinvestment occurs when quantity/quality of retail services reaches the lower bound, $q = \underline{Q}(s,t)$.

$s$: Sales per unit of retail services, reflecting factors such as amenities and location. Sales evolve stochastically according to a geometric Wiener process with drift parameter $\mu$ and volatility $\sigma$. The initial value of unit sales is normalized to 1.

$\delta$: Constant rate of depreciation of the quantity/quality of retail services, $q$.

$\iota$: The constant riskless rate of interest. This quantity satisfies the following inequality: $\iota > \mu - \delta$.

$aq^\beta$: Cobb-Douglas production function for initial investment by the tenant, $\alpha > 0$ and $\beta > 1$.

$\overline{q}^\gamma e^{\alpha x}$: Cobb-Douglas production function for follow-on reinvestment by the landlord, $\gamma > 1$ and $0 \leq \rho < \delta - \mu$.

$aq^\gamma e^{\alpha x}$: Base rent paid by retail tenant, $a > 0$, where $a$ is a choice variable of the landlord as part of optimal contract determination.

$bq^\gamma e^{\alpha x}$: Overage rent threshold value, $b > 0$, where $b$ is a choice variable of the landlord as part of optimal contract determination.

$pqs$: Overage rent paid when total sales, $qs$, are greater than the overage threshold value, $bq^\gamma e^{\alpha x}$, $0 \leq p < 1$. Overage rental percentage, $p$, is a choice variable of the landlord as part of optimal contract determination.

$R$: Total rent paid by retail tenant to landlord at a particular point in time, where

$R = R(q,s,t) = aq^\gamma e^{\alpha x} + \max\{0, qs - bq^\gamma e^{\alpha x}\}$.

$\pi qs$: Profits to the retail tenant prior to payment of rent, with profit margin $\pi$, 0 < $\pi$ < 1, which can depend on the category of sales.
\( \lambda qs \): Externalities captured by the developer/landlord, \( \lambda > 0 \), which accrue in addition to the base rent, \( R \).

\( V^L(q,s,t) \): Landlord equity value for \( t > 0 \).

\( V^T(q,s,t) \): Tenant equity value for \( t > 0 \).

\( W^L(a,b,p) \): Landlord equity value at \( t=0 \) conditional on optimal initial investment by the tenant.

\( W^T(a,b,p) \): Tenant equity value at \( t=0 \) conditional on optimal initial investment and net of the cost of that investment.

\( \omega \): Bargaining power of the landlord relative to the tenant at the time of lease contract execution, \( 0 < \omega < 1 \).

\( \hat{V}(q,s,t) \): Aggregate equity value of the first-best problem, \( \hat{V}(q,s,t) = V^L(q,s,t) + V^T(q,s,t) \).

\( q, q, q_0 \): Optimal reinvestment threshold value, new quantity/quality of retail services from reinvestment, and optimal initial investment, respectively, that result from first-best solution.

**Transform Variables and Further Variable Definitions**

**Transform variable:** \( y = q^{1-\gamma} e^{-\rho t} \)

Transformed equity value function: \( F^i(y) = q^{1-\gamma} e^{-\rho t} V^i(q,s,t) \), \( i = 0 \) indicates the landlord, \( i = 1 \) indicates the tenant.

\( F(y) \): Transformed aggregate equity value, where \( F(y) = F^0(y) + F^1(y) \).

\( \theta = \frac{\gamma}{\gamma - 1} \).

\( \psi = \delta + t - \mu \).

\( \phi = \delta \gamma + t - \rho \).

\( \zeta_{ij} = [(1-i)\lambda + iP(1-2i)]/\psi \), \( i,j = 0,1 \).

\( \xi_{ij} = (1-2i)(a-pb)/\phi \), \( i,j = 0,1 \).

\( \bar{y} \): Transformed new level for \( y \) immediately after reinvestment occurs.

\( \bar{y} \): Transformed threshold at which reinvestment is triggered.
$y_0$: Transformed optimal level of initial investment.

$G^0(a,b,p)$: Transformed landlord equity value function at $t=0$, conditional on optimal initial investment by the tenant.

$G^i(a,b,p)$: Transformed tenant equity value function at $t=0$ conditional on optimal investment and net of the cost of initial investment.

$F^i(y) = \begin{cases} 
F_{i,0}(y), & 0 \leq y \leq b \\
F_{i,1}(y), & y > b 
\end{cases}$.

$\eta_1, \eta_2 = \frac{1}{2} - \frac{\phi - \psi}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\phi - \psi}{\sigma^2}\right)^2 + \frac{2\phi}{\sigma^2}}$.

$A^i_{j,i}, A^2_{j,i}, i, j = 0, 1$: Constants from particular equity value solutions.

$\hat{F}(y)$: Transformed aggregate equity value for the first-best problem.

$A$: Constant from particular solution for the first-best problem.

$y$: Transformed new level for $y$ immediately after reinvestment occurs in the first-best problem.

$\bar{y}$: Transformed threshold at which reinvestment is triggered in the first-best problem.

$y_{00}$: Transformed optimal level of initial investment in the first-best problem.

$\hat{G}(y)$: Transformed aggregate equity value for the first-best problem at $t=0$, net of cost of initial investment.