Valuation of a Power Plant Under Production Constraints

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Real option models for valuating power plants are often criticised for not taking into account important features of these physical assets such as switching costs, minimum on-off times, ramp rates or non-constant heat rates. Incompleteness of electricity markets is also an important issue while valuating these kind of options.
We study the valuation problem of a power plant in a continuous time commodity market, in the presence of frictions, mainly production constraints and market incompleteness.

To this end, we consider a commodity market with a price process

$$\frac{dS_t^i}{S_t^i} \;=\; \mu(t, S_t^i)dt + \Sigma_i(t, S_t^i)dW_t \ , \ 1 \le i \le n \ , \ t \in [0, T] \ .$$

An agent, whose preferences are defined by the exponential utility function

$$U(x) \;:=\; -e^{-\eta x}, \ x \in \mathbb{R} \ ,$$

owns a power plant paying ψ_t^0 and ψ_t^1 at time t, respectively in shut-down mode and start-up mode. Fixed costs C_0 and C_1 are paid at each mode switch. In addition, the use of the power plant is restricted by minimal shut-down and start-up durations δ_0 and δ_1, so that a control of the power plant is a sequence $\theta = (\theta_n, n \ge 0)$ of stopping times with values in $[0, T]$, such that, for all $n \ge 0$

$$\theta_0 = 0, \ \theta_{2n+1+i} \ge (\theta_{2n+i} + \delta_i) \wedge T \ .$$

The gain from managing the power plant is thus given by

$$B_t^\theta \;:=\; \int_0^t \psi(u, \theta)\, du - \sum_{n \ge 1} \left(C_0 \mathbf{1}_{\{0 < \theta_{2n} \le t < T\}} + C_1 \mathbf{1}_{\{0 < \theta_{2n-1} \le t < T\}} \right) \ ,$$

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where

$$\psi(u,\theta) \;:=\; \sum_{n\geq 0}\left(\psi^0_u \mathbf{1}_{\{\theta_{2n}\leq u<\theta_{2n+1}\}} + \psi^1_u \mathbf{1}_{\{\theta_{2n+1}\leq u<\theta_{2n+2}\}}\right) .$$

The agent is also allowed to trade on the commodity market and we denote by π her portfolio and

$$X^{x,\pi}_t \;:=\; x + \int_0^t \sum_{i=1}^n \pi^i_u \frac{dS^i_u}{S^i_u}$$

the associated wealth process.

We define the value of the power plant by means of utility indifference. For this purpose we define the indirect utility function of the manager respectively in the presence and absence of the power plant:

$$V_0(x) \;:=\; \sup_{(\theta,\pi)\in\mathcal{T}_\infty\times\mathcal{A}_0} \mathbb{E}\left[U\left(X^{x,\pi}_T + B^\theta_T\right)\right] \tag{0.1}$$

$$v_0(x) \;:=\; \sup_{\pi\in\mathcal{A}_0} \mathbb{E}\left[U\left(X^{x,\pi}_T\right)\right] \tag{0.2}$$

The utility valuation of the power plant is then defined by

$$p_0(x) \;:=\; \inf\{p\in\mathbb{R} \;:\; v_0(x+p)\geq V_0(x)\}\,.$$

Our main result expresses the price p_0 as

$$p_0 \;=\; \overline{Y}_0 - y_0 \;,$$

where y_0 is the initial value of a backward stochastic differential equation and $\overline{Y}_0$ is the initial value of a coupled system of reflected backward stochastic differential equations. We also formulate the PDE and the variational inequalities associated to these equations.

We prove that, in the absence of friction, this utility-based value reduces to the classical no-arbitrage valuation. We also provide numerical applications via PDE and BSDE methods to compute the value of a coal power plant in a complete and incomplete market.