Stage Financing, VC Short-Termism and Managerial Replacement as a Real Option*

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Abstract

This paper studies how the information available from potential investors determines an entrepreneur's choice of financing. In our two-period model, which allows stage financing, the entrepreneur chooses financing for his own new project from pool of potential investors. The pool includes business angels, venture capitalists and traditional atomistic investors. The entrepreneur's choice of financing depends on the additional value to the project brought by the investors' abilities to resolve over time the uncertainties about the project and by the actions they can take, such as replacing the manager or cutting the investment.

We obtain explicit analytic solutions for the choice of investor and for the amount of investment at each stage. Our results show that the entrepreneur chooses angel or venture capital financing when the intertemporal resolution of uncertainty creates value which exceeds the associated cost. The venture capitalist emerges as the preferred investor when potential manager replacement is an ex-ante valuable option. These results are consistent with observed market practice.

1. Introduction

When an entrepreneur is seeking financing for his project and considers several options, should he always choose the financier who demands the lowest share in the project? The answer is clear "No", if the choice of financier affects the payoffs of the project. The objective of this paper is to study how the information available from potential investors may determine an entrepreneur's choice of financing. The entrepreneur wants to maximize the value of his own stake in the project and so chooses investors whose participation maximizes the expected net present value of the project, thus maximizing the payoff to the entrepreneur. The main idea is that when all players are risk-neutral and there are no information asymmetries, the entrepreneur's choice of financing depends upon the investors' abilities to resolve the uncertainties that the project faces.

We consider three different random components that affect the project's outcome: 1) idiosyncratic (project specific, technology driven) uncertainty about the project payoff; 2) uncertainty about the manager's quality; and 3) market (non-company-specific) uncertainty which depends on more general factors.

We make the natural assumption that an entrepreneur with better managerial skills needs a smaller amount of financial investment to implement the project. Initially, the entrepreneur's capability as a manager is not known, even to the entrepreneur himself. This creates uncertainty about the required investment, while the realization of future payoff depends both on the project's specific uncertainty and on general market conditions. The project specific uncertainty is binary — the project is either viable or not — while the gross payoff of the viable project depends on the general market conditions.

The pool of potential investors includes an independent corporate entity, a business angel, a venture capitalist (VC) and traditional atomistic investors. As we discuss in Section 2, there are several important differences between these categories of investors. Since we consider entrepreneurial projects with a high degree of uncertainty about the required investment and about the payoff, we focus mainly on business angels and VCs as the most suitable investors for these projects. We call them "active investors," because of their ongoing involvement in the project. Both types have access to information production technologies and have the experience necessary to screen the projects. However, there exist important differences between

VCs and business angels, which play a key role in the entrepreneur's choice of investor. While business angels invest their own money, VCs act as managers of other people's money and require a fee for their professional advice. Due to cash constraints, angels typically invest in a single project, while VCs invest into a portfolio of several projects within the same industry. Another crucial difference is that only the VCs have the contractual right to replace the manager.

Different types of uncertainty can be efficiently resolved over different periods of time, thus determining the importance of specific information production technologies over the project's lifetime. For example, the technological uncertainty may be a very important factor at the beginning and very valuable information about the project's technological feasibility can be obtained from expert analysis of the project (project "screening"), whereas the market uncertainty becomes more important at the later stages. *An investor who has a superior information production technology about a source of uncertainty, crucial at the current phase of project development, should be the leading investor at this stage.* This leads to the observed fact that a project's sources of finance evolve and change over the lifetime of the project

An important characteristic of innovative projects is stage financing, whereas investments are made gradually, in stages. Our model has two stages of financing. By the beginning of the second stage, both business angels and VCs observe signals about the project's payoff and managerial capabilities of the entrepreneur and thus make their next stage investment contingent on the observed signal. While the signal about the project feasibility is observed before the first stage investment is disbursed, the precisions of the subsequent signals about the managerial capability of the entrepreneur and about the general market conditions, depends directly on the level of investment made at the first stage.

Using a two-stage financing scheme, we determine the optimal investment levels at the first and second stages for different investors as the solution to an NPV-maximization problem. Active investors make their second stage investment decision contingent on the signals about the project's payoff and the entrepreneur's managerial skills. Moreover, if necessary, the venture capitalist can replace (or sideline) the manager-entrepreneur. If such a development takes place, then a new manager, who is known to be good, is brought in to help the entrepreneur to manage the project. Hiring the new manager saves investors money on the remaining part of investment, but the new manager requires an extra fee to do the job.

Note the similarity and the difference between simple acquisition of a costly signal and learning from investment. In both cases, new information is produced at a cost. However, the cost of learning from investment is not equal to the money invested in the project at the first stage. The first-stage investment is a sunk cost, meaning that some bad projects will be subsequently continued in order to minimize the losses and some bad manager-entrepreneurs will not be replaced, it being better to terminate the project than to bring a replacement manager just for the remaining period. Therefore, the cost of learning from investment is a random variable with expected value equal to the expected value of wasted investment including the money already spent on a bad project, the money that will be invested in the bad project at the second stage, overinvestment into a good project and the lost opportunity of giving this project to the new good manager from the very beginning.

The second effect of learning from investment is more interesting, because it is quite different from traditional signal acquisition. In the presence of network externalities, learning can lead to an improvement in the distribution of the NPV. Thus, the investment already made either saves future investment costs or shifts the future payoffs upwards. For example, if investments made into several projects in the same industry lead to nonlinear industry growth, then at the second stage the set of accepted projects will be much bigger than it would be without learning from investment.

The main results of the paper are the following. First, we show that stage financing is *always* at least as good as one-time investment. We find the sets of parameters for which they are equivalent and for which stage financing has a strictly greater expected NPV. In the former case the entrepreneur is indifferent between the atomistic investors and the business angels and in the latter case financing by the business angels and the VCs is his preferable choice.

Second, taking into account the differences between the angels and the VCs described above, we obtain conditions for when the entrepreneur prefers business angels' financing. This occurs when replacement of the manager-entrepreneur is not optimal at any stage.

1.1. Literature review

We draw upon three separate streams of literature. The first one studies empirically the specifics of venture capital firms, their involvement in the projects they support and interaction between venture capitalists, angel investors and entrepreneurs. This empirical and descriptive literature enables us to better understand the venture capital investment process and to classify investors by their abilities to obtain information about different sources of uncertainty and to participate in the management of a project. The investment process is very well described in Sahlman (1990). Gompers (1995) describes in detail the stage financing process that allows venture capitalists to gather information and monitor the progress of firms whilst maintaining the abandonment option. The paper tests empirically the agency and monitoring cost predictions on a sample of 794 venture capital-backed companies. Venture capitalists concentrate investments in early stage companies and high-tech industries, where information asymmetries are significant and monitoring is valuable. Gompers (1995) shows that an increase in asset tangibility (thus, lower uncertainty) increases financing duration (timing between different investment stages) and reduces monitoring intensity. Time between investment stages declines as the potential for future investment opportunities increases (higher market-to-book ratios) and in industries with higher R&D. Several papers study the involvement of venture capitalists in the strategic decisions of their portfolio companies. Gorman and Sahlman (1989) report that venture capital firms in their sample have an average portfolio of nine entrepreneurial companies. The venture capitalists spend half of their time monitoring the portfolio companies and typical respondents said that during their affiliation with a venture capital firm, they had replaced three CEOs in portfolio companies. From the venture capitalist's point of view, weak management is the dominant cause of start-up failure. This fact motivates our model assumption that manager quality is an important variable and that the potential to change the manager can be important.

The second class of papers related to our study develops models of information production in stage financing. For example, Berk et al. (2003) develop a quite sophisticated and comprehensive model of a multi-stage investment project. Their work is similar to the current paper in so far as the project's profitability is revealed to the firm over its lifetime because technological uncertainty about required R&D effort can be resolved only through additional investment, whereas other uncertainties remain unresolved until the last period. Continuation decisions are made conditional on the resolution of systematic as well as unsystematic uncertainty.

Cabral (2000) employs a model in which different parties can engage themselves into a potentially rewarding joint venture. However, the free-rider problem hinders innovation — if the project is successful, the discovery (technological innovation) becomes a public good, and so the parties have an incentive to deviate at the beginning in order to free-ride. Bhattacharya and Chiesa (1995) have a similar model with applications closer to the current paper. They study the interaction between financial decisions and the disclosure of interim research results to competing firms. Technological knowledge revealed to a firm's financier(s) need not also flow to its R&D and product market competitors. The authors show that the choice of financing source can serve as a precommitment device for pursuing ex-ante efficient strategies in knowledge-intensive environments. Maug (2001) shows that the optimal ownership structure of a company changes during different stages of its life cycle, depending on the information available. The author states that "in those stages where firm-specific information is critical (start-up, restructuring) it is optimal for the company to be privately held." On the other hand, if industry-relevant information is more important, then the blockholders bring the company to an IPO. The model examines the timing of the IPO decision and it uses a monitoring cost approach. It shows that private blockholders (venture capitalists) decide to take the company public whenever this reduces their monitoring costs sufficiently to justify the loss from underpricing. This leads to going public inefficiently late. A venture capitalist with a lower cost of information has lower discounts in the IPO, that is these IPOs are less underpriced. Discounts increase with precision of the signal observed by informed outside investors. If investors learn about a new industry, going public creates an externality enabling other firms to bring their portfolio companies to IPO.

Finally, we consider papers which examine the optimal contract design. Admati and Pfleiderer (1994) derive a role for inside investors (venture capitalists) in resolving agency problems in a multi-stage financial contracting problem. They show that there exists a unique contract (fixed-fraction contract) that is robust with respect to optimal continuation. In this contract, the venture capitalist receives a fraction of the total payoffs and finances the same fraction of any future investment (strip financing). Repullo and Suarez

(2004) look into the complementarity between the financing and advisory roles of venture capitalists and find that the optimal contract between them and an entrepreneur has the characteristics of convertible preferred stock. This outcome results from a double-sided moral hazard problem that arises at the second stage of investment, when both the entrepreneur's and the advisor's (VC) efforts are crucial for increasing the proportion of potential non-zero payoffs. Hellmann, in a series of papers (Hellmann (1994), Hellmann (1998a) and Hellmann (1998b)), studies why and under what circumstances entrepreneurs would voluntarily relinquish control to venture capitalists. He also develops a theoretical model that examines the importance of a strategic motivation for making venture capital investment.

The rest of the paper is organized as follows. In Section 2 we discuss the main characteristics of different investors. Section 3 describes the model. In section 4 we consider improvements in the project's NPV from observing signals about payoffs. Section 5 derives the optimal choice of investor and the level of first-stage investment as solutions of the NPV-maximization problem with resolution of uncertainty about the manager's type. Section 6 concludes.

2. Different Types of Investors — What Distinguishes Them?

The focus of our paper is on the role of information in the choice of financing for a new project. Staying with his own parent company, i.e., his employer, might seem to be an obvious choice as a source of financing for an employed manager-entrepreneur with a novel idea. However, corporate industrial investment accounted for only \$ 0.5 bln (in a total of \$45.6 bln) in committed capital into new ventures in 1997 in the US. In Europe in 1996 corporate investors provided only \$ 413 mln out of a total of \$ 10.1 bln. These figures are quoted in Hellmann (1998b) who shows that the entrepreneur may prefer to go to an independent venture capitalist, because the parent company may be unable to credibly commit ex ante not to shirk with support, not to exercise self-interested control or not to invest in a rival internal venture. The outcomes crucially depend on the extent of complementarity / supplementarity (cannibalization) of the proposed new project with the profits of the corporate investor's core business, because the new venture generates an externality to it. In contrast to Hellmann (1998b), we consider how information production and management retention/replacement decisions influence the financing choice.

In a risk-neutral, frictionless world the major factors that distinguish investors are of the information and/or control rights nature. These factors include:

1. Ex-ante information that an investor already has;

2. Available information production technology and its cost;

3. Degree of control that an investor can exercise over the execution of the project;

We can divide investors with abilities to produce information about the company into the following groups

1. *Existing parent company*

2. *Traditional dispersed investors*

3. *Angels*

4. *Venture Capitalists*

We can characterize these groups according to the factors just described as following:

Existing parent company (internal investment) Information asymmetry between the parent company and the manager is quite low, especially concerning existing projects and the manager's abilities. There is little danger that the asymmetry will increase in the future. The future information production costs depend on the nature of the new project and can be quite high if the new project lies outside the company's core competence. The company can exercise complete control over the project — shut it down, replace the manager, inject additional investment. If the manager-entrepreneur sells his project to the company, then his compensation becomes part of the project's cost and is determined by the original contract. In this case the parent company gets all the benefits from the project and assumes all the costs. As we have already mentioned, the interests of the parent company and the entrepreneur do not necessary coincide.

Angels Business angels play an extremely important role in financing new start-ups. The lack of a reliable data source makes it difficult to give precise figures, but according to some estimates reported by Hellmann (1998b) angel capital provides from two to seven times the funding provided by "organized" venture capital. However, a typical individual angel's investment is smaller than that of a VC and varies in the range from \$ 50,000 to \$ 1 million.

In real life business angels are heterogeneous investors, some are very sophisticated while others are quite naive. However, in this paper we consider the angels who are experts and professionals, thus called "professional angels". Information asymmetry between them and the manager-entrepreneur is quite low, because usually angels invest into the projects in their area of expertise and give money to people whom they know (Prowse (1998)). There is little danger that the asymmetry will increase over time.

Although active angels are usually very involved in the company's operations, they rarely use employment contracts that penalize poor performance. Their role is more advisory. Therefore, we can assume that they do not make strategic decisions themselves. For example, they cannot fire a manager-entrepreneur.

Venture capitalists Sahlman (1990) describes the role and structure of venture capital. Venture capital organizations raise money for investment in early stage businesses with high potential and high risks. Their forms of organization vary from corporations and captive subsidiaries of banks and corporations to small business investment corporations (SBIC) and limited partnerships, with the latter being the most popular form of venture capital organization. By VCs we mean the managing partners of these partnerships. VCs are usually contribute about 1% of total partnership funds raised and receive compensation — the management fee — equal to 2.5 % of funds raised and a portion (about 20 %) of carried interest. Partnerships have a finite life (10 years, extendable for another 3). Most VCs specialize in a particular stage of development and/or in a particular industry. Experience is a key resource that the venture capitalist brings to the table in helping to nurture a developing firm. Agency conflicts are usually remedied by having entrepreneurs being paid in the form of equity interest which is proportionally larger for more successful performance ("earn-out"). Staged capital financing lessens potential asymmetric information moral hazard problems by creating an abandonment option and thus increasing the value of the project.

VCs and angels are involved in similar types of activities, namely, funding, monitoring, advising, formulating business strategy. Ehrlich et al. (1994) find that in comparison with business angels, VCs are more involved in the management of portfolio companies. They set higher performance standards, but give better feedback in return. The more formalized approach of VCs is often better for entrepreneurs with technical or scientific background and who have limited managerial experience. According to Gorman and Sahlman (1989), VCs spend roughly half of their time monitoring portfolio companies. From the VC's point of view, weak management is the dominant cause of start-up failure and decisions to replace CEOs in portfolio companies are not rare — Hannan et al. (1996) give the following attrition rates for companies' CEOs: in the first 20 months of a company's life, the likelihood that a non-founder is appointed as a CEO is 10%, after 40 months it is 40% and after 80 months it is 80%. Lerner (1995) finds that venture capitalists' involvement is more intense when the need for oversight is greater — venture capitalists' representation on the board increases around the time of CEO turnover, while the number of other outsiders remains constant.

Even if, at the initial stage, VCs may not possess all the information that is available to the entrepreneur, the high degree of monitoring of the company's progress and involvement in its activities make them well informed at later stages. Moreover, as we have mentioned, both business angels and venture capitalists bring to the company their own past experience and knowledge, thus producing information important to the company's success.

Traditional investors Traditional investors, like debt and equity holders, have more limited capabilities to monitor and influence the start-up company's activities than do a parent company, business angels and VCs.

To summarize, we characterize these groups according to the factors just described as following:

Investors	Precision of information	Cost of information	Level of Control
Parent company	same as the manager	same as the manager	high
Angels	medium/high	low/medium	medium
Venture Capitalists	medium/high	low/medium	high
Other Investors	usually low	medium/high	low/medium

3. The Model

We consider a two-period model with three dates, $t = 0$, 1 and 2. There are three types of agents: the incumbent manager-entrepreneur, E, endowed with an individual project, potential investors and a potential replacement manager, M. All agents are assumed to be risk neutral and the riskless interest rate is normalized to zero.

3.1. The project

The entrepreneur has an investment project. The project requires initial investment at $t = 0$, and generates some random payoff at $t = 2$. Additional investment can be made at $t = 1$.

The required investment and future payoff are unknown and depend on three random variables:

1. The project type
2. The general market conditions
3. The manager's type

There are two types of projects: "star", S, and "failure", F. At $t = 0$ the type of the project is unknown and the ex-ante probability that the project is of type S is p_S.

If the project is of type S, then, assuming that it has received sufficient investment, it generates the payoff $\tilde{V}$, which is a random number with expected value $\mathrm{E}(\tilde{V}) = \bar{V} > 0$. If the project is of the F type, then its payoff is certainly zero.

Project S generates payoff $\tilde{V}$ only if the sufficient investment, $\tilde{I}$, has been made. The distribution of $\tilde{I}$ depends on the manager's type. The incumbent entrepreneur can be a good manager, G, or a bad manager, B. M is definitely good and, if hired, requires a fee proportional to the investment managed by him. At $t = 0$ nobody knows the managerial quality of the entrepreneur. The ex-ante probability of the entrepreneur being a good manager is q_G and the probability of him being a bad manager is $q_B = 1 - q_G$. If the manager is good and the entire project is managed by him, then $\tilde{I} = \tilde{I}_G$ and has a cumulative distribution Φ_G on some support $\left[0, \bar{I}\right]$. If the manager is bad and the entire project is managed by him, then $\tilde{I} = \tilde{I}_B$ and has a distribution Φ_B on the same support $\left[0, \bar{I}\right]$.

Manager	Project S		Project F	
	Sufficient Investment	Expected Payoff	Sufficient Investment	Expected Payoff
Good	$\tilde{I}_G \sim \Phi_G$	$p_S V$	$\tilde{I}_G \sim \Phi_G$	0
Bad	$\tilde{I}_B \sim \Phi_B$	$p_S V$	$\tilde{I}_B \sim \Phi_B$	0

Assumption 1. *The manager's type, the project's type and the star project's gross payoff (subsequent to "sufficient" investment) are independent*

Notice that because $\tilde{I}$ is a random variable, if we made an assumption that Φ_G stochastically dominates Φ_B, then it would be possible that the bad manager needed less investment than the good one. To avoid this complication, we consider the case when $\tilde{I}_G = \alpha \tilde{I}_B$, where $0 < \alpha < 1$.

Assumption 2. *We assume that* $\tilde{I}_G = \alpha \tilde{I}_B$

This assumption means that a good manager always needs to spend less than a bad manager in order to succeed with a star project. It implies that $\Phi_G(K) = \Phi_B\left(\frac{K}{\alpha}\right), K \geq 0$.

Assumption 3. *We assume that* $\Phi_G(K)$ *is smooth enough, that is* $\Phi_G'(K) = \phi_G(K)$ *exists everywhere on* $[0; \bar{I}]$

3.2. Investors

The entrepreneur has no personal wealth and seeks financing from investors. The pool of investors includes the business angel, A, the venture capitalist, VC, and atomistic investors. Atomistic investors and A require a share in the project's expected payoff equal to their investment and VC requires a fixed fee, $\delta > 0$, in addition to that. We call A and VC "active investors", because of their ongoing involvement in the project.

The entrepreneur aims to choose an investor, who would maximize the entrepreneur's own expected payoff. This may be achieved by requiring a low return on invested capital and also by providing additional information about the project's type and market conditions affecting the project's payoff and thus maximizing the project's net present value, which is equal to the entrepreneur's expected payoff.

3.3. Manager M

VC has access to a potential replacement manager, M. Thanks to VC's knowledge, contacts and experience in the industry manager M is always good, but to hire him would be costly. This additional cost is represented by the replacement cost, $c = c(K_M)$, to replace the incumbent manager-entrepreneur, where K_M is the remaining investment expenditure to be overseen by manager M.[1]

3.4. Information structure

Investors differ in their abilities to observe signals about the project's type, about the market conditions affecting the star project's payoff V and about the quality of E as a manager.

The signal about the project's type is a binary signal, T. The possible values of T are high (H) or low (L) and is observed at $t = 0$, before any actual investment is made. For simplicity we assume that an L signal indicating an F type, is perfect. That is, if the signal is low, then $\Pr(F|L) = 1$ and a person who observes the signal knows for sure that the project's type is F.

We distinguish between players' screening capabilities by differentiating the inferences each can make having observed an H signal. If the entrepreneur, or arms length atomistic investors, observe "high" (H_E), then this signal is "correct" with probability $p_S^E = \Pr(S|H_E)$. If the Angel or VC observe "high" (H_A), then this signal is "correct" with probability $p_S^A = \Pr(S|H_A)$. Furthermore, the signals are nested, $\Pr(H_E|H_A) = 1$, meaning that an active investor receiving H_A signal knows that the entrepreneur received an H_E signal.

Superior screening capability is characterized by $p_S^A > p_S^E$ so that their signal has higher precision than that of the entrepreneur.

If capital K_0 has been invested in the project at $t = 0$, then, by $t = 1$, E and active investors observe

[1] As we have discussed in Section 2, VC has the contractual right to replace the management of a poorly performing company. However, replacement does not necessary mean that the incumbent manager-entrepreneur has been fired. He may stay within the company and be moved to another position.

some signal, Ψ, about the potential payoff level V of project S. There is no direct cost of observing the signal, but it can be observed only if financing started at $t = 0$.

Active investors can learn the true type of E only at $t = 1$. If at $t = 0$ the investor does not replace E and invests K_0, then at the end of the first period, at $t = 1$, he and E himself learn E's type with probability $f(K_0)$.

3.5. The timeline

The decision tree is shown on Fig.(**??**) in Appendix I and the timeline is summarized in the table below.

$t = 0$	$t = 1$	$t = 2$
Signal T is observed	If $K_0 > 0$ then active investor	Required investment level, I,
K_0 is invested	observes Ψ.	is realized. If $I < K_0 + K_1$
	With probability $f(K_0)$	then the project's payoff
	manager's type is revealed	is realized.
	Retention/Replacement decision is made	
	K_1 is invested	

At $t = 0$, the entrepreneur approaches a potential investor with his project. An investor screens the project, receives signal T about the project's type and decides whether to invest in the project or not and how much to invest.

If positive investment has been made, then at $t = 1$ the active investor observes signal Ψ about project S's payoff and with probability $f(K_0)$ he learns the manager's type. If VC is the investor, then at that time he can make both the replacement decision and the further investment decision, and invest K_1. A makes only the further investment decision.

The project type and the true required investment level remain unknown until $t = 2$, the end of the second period. At $t = 2$, the required investment level is realized. If it is greater than the total investment made, then all the money is lost and the payoff is zero.[2] Otherwise, the project's type is realized, if it is S,

[2]Impossibility of making additional investment at time $t = 2$ can be interpreted as meaning that the cost of refurbishing an already finished project would be prohibitively high.

then the project generates payoff V and if it is F then the payoff is zero.[3]

Notice that K_0 represents capital initially invested into the project and not just the money spent to learn a signal about the project S's payoff and whether the manager is good or bad. Such a situation we interpret as "learning by doing", rather than as signal acquisition before undertaking the project or as costly monitoring over the course of the project. However, for this learning we can derive an analogue to the signal costs.

4. Information about the Project's Payoff

4.1. The Project's Type

In this section we consider information about the project's type only. When the entrepreneur observes signal H_E and seeks investment K, the project's expected net payoff is

$$\mathrm{E}[\mathrm{NPV}|H_E] = p_S^E \left(q_G \Phi_G(K) + q_B \Phi_B(K) \right) \mu - K.$$

where $\mu = \mathrm{E}(V)$. This is the value of the project if financed by atomistic investors. As we show in the following proposition, investment by active investors, A, creates additional value.

Proposition 1. *If the entrepreneur observes H_E, then by asking type A investor to provide financing, the entrepreneur creates additional value*

$$\begin{aligned} \Delta NPV &= \frac{p_S^A - p_S^E}{p_S^A} K \geq 0 \\ 0 &\leq \Delta NPV \leq (1 - p_S) K \end{aligned}$$

Proof. *: in Appendix IIIC .*

[3]Because of the players' risk neutrality, we can assume simply that the payoff is $p_s V$ if the investment is sufficient. The probabilistic character of the payoff is relevant in the next section.

The net expected payoff increases because of decrease in the expected changes in investment and not because of increase in the expected project payoff. All other things being equal, the higher the precision of the information that the investor possesses, the bigger the increase in the expected NPV is.

Example 2. *Let $K = 5$ and $q_G = 1$, $\Phi_G(5) = 1$, $\mu = 8$, $p_S^E = .5$ and $p_S^A = 1$. Even if atomistic investors observe $T = H$, they will refuse to finance the project ($.5 \times 8 - 5 = -1 < 0$). Simply by asking a type A investor to finance the project the entrepreneur expects to have NPV*

$$\mathrm{E}(NPV) = \Pr\left(H_A|H_E\right)(8-5) + \Pr\left(L_A|H_E\right)\cdot 0 = .5\cdot 3 + 0 = 1.5 > 0$$

That is, he expects NPV to increase by 2.5. Note, that this happens ***before*** *the investor actually screens the project! After the screening, the expected NPV is either 3, following approval, or* 0*,following rejection by A.*

It is important to mention that $K \neq \mathrm{E}(I)$, that is $\mathrm{E}(I)$ is not the expected investment by the investor. It is the expected required investment level, which is necessary for project S to succeed.

4.2. The Star Project's Payoff

Here we look at the mechanism of learning the star project's future payoff V. Payoff can depend on the macroeconomic conditions or some industry specifics. The first-best decision rule remains the same — all projects with positive expected NPV should get financing. However, as in the previous part, stage financing and observing signal Ψ at the first stage increase the project's ex-ante NPV. As in the previous case, this increase comes from the investment side — the active investor expects to make

As we have described in the previous section, before stage 1 the experienced investor(s) and/or entrepreneur has screened the projects. If investor had observed signal H indicating high possibility of a project being the star type S then the financing started.

In this part we consider stage 1 itself, when after making initial investment the players can get some signal Ψ about the overall payoff level V. There is no direct cost of observing the signal, but it can be observed only if capital K_0 has been invested in the project at stage 1. We assume that the signals about the project type and the project S payoff are independent, therefore we can consider them separately. Without loss of generality we can assume that the signals are observed sequentially. Without possibility to observe the signal the project should be undertaken iff

$$p_S^H \Pr\left(\tilde{I} \leq K\right) \mu - K \geq 0. \tag{1}$$

where

$$\Pr\left(\tilde{I} \leq K\right) = \left(q_G \Phi_G(K) + q_B \Phi_B(K)\right)$$

and p_S^H is either p_S^E or p_S^A depending on the type of investor who finances the project.

With the possibility to observe signal Ψ after K_0 was invested at stage 1, but before making the investment decision for stage 2, the expected NPV at the beginning of stage 1 ($t = 0$) becomes

$$\begin{aligned} \mathrm{E}\left[\mathrm{NPV}\right] \quad = \quad & \mathrm{E}_\Psi \left[\Pr\left\{\mathrm{E}\left(V|\Psi\right) > V_C\left(K_0\right)\right\} \left[p_S^H \Pr\left(\tilde{I} \leq K\right) \mathrm{E}\left(V\left|\mathrm{E}\left(V|\Psi\right) > V_C\left(K_0\right)\right.\right) - \left(K - K_0\right)\right]\right. \\ & \left. + \Pr\left\{\mathrm{E}\left(V|\Psi\right) < V_C\left(K_0\right)\right\} \left[p_S^H \Pr\left(\tilde{I} \leq K_0\right) \mathrm{E}\left(V\left|\mathrm{E}\left(V|\Psi\right) < V_C\left(K_0\right)\right.\right)\right]\right] - K_0 \end{aligned} \tag{2}$$

where

$$V_C\left(K_0\right) = \frac{K - K_0}{p_S^H \Pr\left(\tilde{I} \leq K\right)}.$$

Figure (**??**) shows the decision tree and payoffs for stage financing. Expression (2) takes into account the fact that after observing the signal Ψ, financing continues for all projects with NPV positive at $t = 1$, that is with respect to the remaining investment $K - K_0$. This means that for some projects continued at stage 2, the following inequality might hold

$$K - K_0 < p_S^H \Pr\left(\tilde{I} \leq K\right) \mathrm{E}\left(V|\Psi\right) < K.$$

Clearly, if the signal Ψ were observed before stage 1 this project would not have been undertaken, but since investment K_0 has already been made, the financing of this project will continue at stage 2 in order to minimize the losses. This peculiarity makes less obvious the fact that all projects undertaken with one-stage financing, would be undertaken with the two-stage financing as well, although it is quite intuitive that stage financing should increase the project's NPV (absent agency problems, the value of an option to stop financing can never be negative). We formally state this in the following proposition.

Proposition 3. *If after investing $K_0 < \mathrm{E}[I]$ into the project a signal Ψ about future payoff V can be observed, then the ex-ante expected NPV of such a two-staged project is never lower than the expected net present value of the same project financed in one stage. The expected NPV from the two-stage financing can be expressed as*

$$\begin{aligned} \mathrm{E}[NPV] &= \underbrace{p_S^H \Pr\left(\tilde{I} \leq K\right)\mu - K}_{\text{one-stage NPV}} + \mathrm{E}_\Psi \left\{ \Pr\left\{\mathrm{E}\left(V|\Psi\right) < V_C\left(K_0\right)\right\} \times \right. \\ &\left. \times \left[K - K_0 - p_S^H \Pr\left(\tilde{I} \leq K\right) \left(1 - \frac{\Pr\left(\tilde{I} \leq K_0\right)}{\Pr\left(\tilde{I} \leq K\right)}\right) \mathrm{E}\left(V\left|\mathrm{E}\left(V|\Psi\right) < V_C\left(K_0\right)\right.\right)\right]\right\} \qquad (3) \end{aligned}$$

where

$$V_C\left(K_0\right) = \frac{K - K_0}{p_S^H \Pr\left(\tilde{I} \leq K\right)}$$

Proof. *: in Appendix III.C*

In expression (3) the second term represents the value of abandonment option, which is non-negative.

As we have mentioned, K_0 is not equivalent to the cost of signal Ψ. To find the expected cost of signal Ψ, imagine a one-shot project with signal Ψ, costing $c(\Psi)$, being observed at $t = 0$. If the NPV of this imaginary project equals to the NPV of the project financed in two stages, then the expected cost of learning (the cost of correcting ignorance) in the two-stage project is equal to $c(\Psi)$.

$$\mathrm{E}_\Psi\left\{\Pr\left\{\mathrm{E}\left(V|\Psi\right)<V_C\left(K_0\right)\right\}\left[K-K_0-p_S^H\Pr\left(\tilde{I}\leq K\right)\left(1-\frac{\Pr\left(\tilde{I}\leq K_0\right)}{\Pr\left(\tilde{I}\leq K\right)}\right)\mathrm{E}\left(V\left|\mathrm{E}\left(V|\Psi\right)<V_C\left(K_0\right)\right.\right)\right]\right\} \quad (4)$$

$$= \mathrm{E}_\Psi\left\{\Pr\left\{\mathrm{E}\left(V|\Psi\right)<V_C\left(0\right)\right\}\left[K-p_S^H\Pr\left(\tilde{I}\leq K\right)\mathrm{E}\left(V\left|\mathrm{E}\left(V|\Psi\right)<V_C\left(0\right)\right.\right)\right]\right\} \quad (5)$$

Formally, we state it in the following corollary.

Corollary 4. *The expected cost of signal Ψ is equal to*

$$\begin{aligned} c(\Psi) &= \Pr\left(p_S^H\mathrm{E}\left(V|\Psi\right)>K\right)\left\{p_S^H\Pr\left(\tilde{I}\leq K\right)\mathrm{E}\left[V\left|p_S^H\Pr\left(\tilde{I}\leq K\right)\mathrm{E}\left(V|\Psi\right)>K\right.\right]-K\right\}- \\ &-\Pr\left\{\mathrm{E}\left(V|\Psi\right)>\frac{K-K_0}{p_S^H\Pr\left(\tilde{I}\leq K\right)}\right\}\left[p_S^H\Pr\left(\tilde{I}\leq K\right)\mathrm{E}\left(V\left|\mathrm{E}\left(V|\Psi\right)>\frac{K-K_0}{p_S^H\Pr\left(\tilde{I}\leq K\right)}\right.\right)-(K-K_0)\right]+K_0 \end{aligned}$$

Notice that even if the atomistic investors could acquire signal Ψ at cost $c(\Psi)$ before making the one-shot investment,they would not have an incentive to do that before they have actual stakes in the project. If they acquire the signal, then the signal cost becomes sunk at the time of the actual investment and, therefore, the investors cannot compensate this cost in the world of symmetric information.

Let us illustrate on two examples the value of information learned during the first stage and the expected cost of learning.

Example 5. *Consider the project which requires the total investment, $K=8$. Ex-ante, the payoffs distribution is the following:*

$$V_1=\begin{cases} 7 & \textit{with probability 0.25} \\ 17 & \textit{with probability 0.25} \\ 0 & \textit{with probability 0.5} \end{cases}$$

By straightforward calculation, the single stage financed project has $\mathrm{E}(NPV)=-2$. The project will not get financing. However, if after investing $K_0=2$ the active investor learns the exact payoff of the project, then $\mathrm{E}(NPV)=0.25\cdot(7-6)+.25\cdot(17-6)-2=1$. Notice, that if $V_2=7$ then the project will be continued, *because $K_0 = 2$ will have become a sunk cost. A requires a 33.3 percent stake in the project for providing financing at the first stage, and if ,alternatively, VC provides the initial capital and charges a fee, $\delta = 0.025$, then the VC's required stake is 33.75 percent. The equivalent information cost is $c(\Psi) = 1.75$.*

Example 6. *Suppose that V is normally distributed with $V \sim N(\mu, \sigma); r = \sigma^{-2}$ and that the players can observe a signal $\Psi = V + \varepsilon$, where ε is normally distributed $\varepsilon \sim N(0, \sigma_\varepsilon); r_\varepsilon = \sigma_\varepsilon^{-2}$ random noise independent of V. Then Ψ is normally distributed $\Psi \sim N(\mu, \sigma_\Psi); \sigma_\Psi^2 = \frac{R_\varepsilon}{r r_\varepsilon}, R_\varepsilon = r + r_\varepsilon$. Then we have the following corollary to Proposition 3:*

Corollary 7. *With stage financing and normally distributed payoff V and signal Ψ the expected NPV of the project becomes*

$$\begin{aligned}
\mathrm{E}[NPV] &= \underbrace{p_S^H \Pr\left(\tilde{I} \le K\right)\mu - K}_{\text{one-stage } NPV} \\
&+ p_S^H \Pr\left(\tilde{I} \le K\right)\sqrt{\frac{r_\varepsilon}{2\pi r R_\varepsilon}}\exp\left\{-\frac{rR_\varepsilon}{2r_\varepsilon\left(p_S^H \Pr\left(\tilde{I} \le K\right)\right)^2}\left(\mathrm{E}(I) - K_0 - p_S^H \Pr\left(\tilde{I} \le K\right)\mu\right)^2\right\} \\
&+ \Phi_Z\left(\sqrt{\frac{rR_\varepsilon}{r_\varepsilon\left(p_S^H \Pr\left(\tilde{I} \le K\right)\right)^2}}\left(K - K_0 - p_S^H \Pr\left(\tilde{I} \le K\right)\mu\right)\right)\left((K - K_0) - p_S^H \Pr\left(\tilde{I} \le K\right)\mu\right)
\end{aligned} \quad (6)$$

where $\Phi_Z(\cdot)$ is the cumulative density function of standard normal distribution.

The project should be undertaken iff (6) is positive.

Proof. *: in Appendix III.*

5. Learning the Manager's Type

In this section a further source of uncertainty is the manager's type. The total required investment into the project depends on this type. Initial investment into the project can potentially reveal the manager's type before the final investment is made. We show that a combination of parameters related to the manager's quality and his replacement cost on the one hand and to the investor's ability to learn the manager's quality on the other, determine the type of financing chosen for a particular project.

5.1. The benchmark case — one-shot investment scheme

First, as a benchmark case we consider a one-shot investment scheme — all capital is raised and invested at $t = 0$, $K_1 = 0$

No replacement

If no replacement is made and so the manager E manages the total investment K, then the optimal investment K_0^* is the solution to the optimization problem

$$\max_{K\in[0;\tilde{I}]} \mathrm{E}\left[\mathrm{NPV}\,|K\,\right] = \max_{K\in[0;\tilde{I}]} \left\{ p_S \left[q_G \Phi_B \left(\frac{K}{\alpha}\right) + q_B \Phi_B \left(K\right) \right] \mu - K \right\}, \tag{1}$$

subject to $\mathrm{E}\left[\mathrm{NPV}\,|K\,\right] \geq 0$. From the first- and second-order conditions we have that K_0^* is a solution to the equation

$$p_S \left[\frac{q_G}{\alpha} \phi_B \left(\frac{K}{\alpha}\right) + q_B \phi_B \left(K\right) \right] \mu - 1 = 0,$$

given that inequality

$$\frac{q_G}{\alpha^2} \phi_B' \left(\frac{K}{\alpha}\right) + q_B \phi_B' \left(K\right) < 0$$

holds at K_0^*. Otherwise K_0^* is the corner solution

$$K_0^* = \begin{cases} 0 & \text{if } p_S V \left[\frac{q_G}{\alpha} \phi_B\left(0\right) + q_B \phi_B\left(0\right)\right] < 1 \\ \alpha \overline{I} & \text{if } p_S V \left[q_G \frac{1}{\alpha} \phi_B\left(0\right) + q_B \phi_B\left(0\right)\right] > 1; p_S V q_B \phi_B\left(\alpha \overline{I}\right) < 1 \\ \overline{I} & \text{if } p_S V q_B \phi_B\left(\alpha \overline{I}\right) > 1 \end{cases} \tag{2}$$

giving

$$\mathrm{E}\left[\mathrm{NPV}\,|K_0^*\right] = \begin{cases} 0 & \text{if } K_0^* = 0 \\ p_S V\left[q_G + q_B \Phi_B\left(\alpha \overline{I}\right)\right] - \alpha \overline{I} & \text{if } K_0^* = \alpha \overline{I} \\ p_S V - \overline{I} & \text{if } K_0^* = \overline{I} \end{cases} \tag{3}$$

Replacement of the incumbent manager

If the investor decides to replace the incumbent manager, E, with the good manager, M, at $t = 0$, the replacement will be costly. We denote the replacement cost as c. We assume c is a function, $c(K)$, of how much money the manager M has to manage. The new optimal K_0^{**} is the solution to the optimization problem

$$\max_{K \in [0;\alpha \tilde{I}]} \mathrm{E}\left[\mathrm{NPV}\,|K\right] = \max_{K \in [0;\alpha \tilde{I}]} \left\{ p_S \Phi_B\left(\frac{K}{\alpha}\right) \mu - K - c(K) \right\}, \tag{4}$$

subject to $\mathrm{E}\left[\mathrm{NPV}\,|K_0^{**}\right] \geq 0$. From the first- and second-order conditions we have that K_0^{**} is either a solution to the equation

$$p_S \frac{\phi_B\left(\frac{K}{\alpha}\right)}{\alpha} \mu = 1 + c'(K),$$

given that the inequality

$$\frac{p_S \mu}{\alpha^2} \phi_B'\left(\frac{K}{\alpha}\right) - c''(K) < 0$$

holds at K_0^{**}. Otherwise, K_0^{**} is the corner solution

$$K_0^{**} = \begin{cases} 0 & \text{if } p_S \frac{\mu}{\alpha} \phi_B(0) < 1 \\ \alpha \overline{I} & \text{if } p_S \frac{\mu}{\alpha} \phi_B(0) > 1 \end{cases} \tag{5}$$

giving us

$$\mathrm{E}\left[\mathrm{NPV}\,|K_0^{**}\right] = \begin{cases} 0 & K_0^{**} = 0 & \text{if } p_S \frac{\mu}{\alpha} \phi_B(0) < 1 \\ p_S \mu - \alpha \overline{I} - c(\alpha \overline{I}) & K_0^{**} = \alpha \overline{I} & \text{if } p_S \frac{\mu}{\alpha} \phi_B(0) > 1 \end{cases} \tag{6}$$

5.2. Two-stage investment

Now we consider the case when investment is made in two stages. Assume that, at $t = 0$, the investor does not replace E and invests K_0. At $t = 1$ he and the manager E learn with probability $f(K_0)$ the manager's type. At that time the replacement decision and the further investment decision are made, and K_1 is invested and is managed by either the incumbent manager, E, or the new manager, M.

It costs $c(K_1)$ to replace the manager E, where K_1 is the investment managed by the manager M. At this point we put no specific constraints on $c(K_1)$. If E turns out to be bad, then his replacement can potentially save more than the cost of replacement. We assume that M can save money only on the part of the project that he manages and that the money already invested cannot be recovered. It is shown in Appendix II that under these assumptions the sum of the two investments K_0 and K_1 is sufficient for the star project with probability

$$\Pr\left\{\tilde{I} \leq K_{0,B} + K_{1,G|B}\right\} = \Phi_B\left(K_{0,B} + \frac{1}{\alpha}K_{1,G|B}\right) \tag{7}$$

where $K_{n,j}$ denotes that the manager of type j managed the money invested at time t_n, and $G|B$ means that the good manager has replaced the bad manager at the second stage.

In this case, the savings from replacing the bad manager by manager M are probably less than they would have been if the replacement had been made at $t = 0$, because manager M will run only the remaining part of the project.[4] The smaller K_0, the greater the gain from replacement given that E is revealed to be bad. On the other hand, $f(K_0)$, is an increasing function of K_0 and the more is invested into the project at $t = 0$, the higher is the probability of learning the incumbent manager's type.

We already know that if the manager's replacement is made at $t = 0$ the optimal value K_0^{**} is determined as a solution to maximization program (4). K_0^* maximizes NPV for the one-shot investment run by the incumbent manger E and is determined as a solution to maximization program (1).

Later we will show that both one-shot strategies are ex-ante weakly dominated by the optimal two-stage investment strategy.

[4] If there are some regions of K_0, where the investor saves more on the replacement costs than he loses on the unefficient management, then he will be interested in replacing the bad manager later.

At $t = 1$, the choice of decision for each of three possible realized manager's types (here we introduce type U as a dummy manager's type, if the manager's quality is not revealed) is predetermined by K_0 and $c(K_0)$. This means that, for each given K_0, we can determine what action the investor will make after the realization of the manager's type at $t = 1$.

However, in order to find K_0^{***}, the optimal initial investment under the two-stage investment scheme, we have to consider all five possible actions instead of three. To find the K_0^{***} that maximizes expected NPV at $t = 0$ under the two-stage investment scheme we have to solve the dynamic programming problem backwards. At $t = 1$ we have the following possible scenarios (see Fig. (**??**) in Appendix I) and Appendix II:

1. The manager E is bad. Then replacement/continuation decision depends on K_0^{***} and $c(K_1)$

   1.1. If the manager M replaces him at $t = 1$, then $K_{1,G|B}^{*} > 0$ is invested in the project at $t = 1$. The cost of replacement is $c(K_{1,G|B}^{*})$. $K_{1,G|B}^{*}$ is a solution to the equation

$$p_S \frac{\phi_B\left(K_0 + \frac{K_1}{\alpha}\right)}{\alpha} \mu_\Psi = 1 + c'(K_1),$$

   if the inequality

$$\frac{p_S \mu_\Psi}{\alpha^2} \phi_B' \left(K_0 + \frac{K_1}{\alpha}\right) - c''(K_1) < 0$$

   holds at $K_{1,G|B}^{*}$. Otherwise, $K_{1,G|B}^{*}$ is the corner solution

$$K_{1,G|B}^{*} = \alpha\left(\overline{I} - K_0\right) \quad \text{if } p_S \tfrac{\mu_\Psi}{\alpha} \phi_B\left(K_0\right) > 1$$

   giving us

$$\mathrm{E}\left[\mathrm{NPV}\,\middle|\,K_0, K_{1,G|B}^{*}\right] = p_S \mu_\Psi - K_0 - \alpha\left(\overline{I} - K_0\right) - c(\alpha\left(\overline{I} - K_0\right)) \quad K_0^{**} = \alpha\left(\overline{I} - K_0\right) \quad \text{if } p_S \tfrac{\mu_\Psi}{\alpha} \phi_B\left(K_0\right) > 1 \;\; ;$$

   1.2. If the manager E continues managing the project, then $K_{1,B}^{*}$ is a solution to the equation

$$p_S \phi_B\left(K_0 + K_1\right) \mu_\Psi = 1,$$

if the inequality

$$p_S \mu_\Psi \phi'_B (K_0 + K_1) < 0$$

holds at $K^*_{1,B}$. Otherwise, $K^*_{1,B}$ is the corner solution

$$K^*_{1,B} = \begin{cases} 0 & \text{if } p_S \mu_\Psi \phi_B (K_0) < 1 \\ \tilde{I} - K_0 & \text{if } p_S \mu_\Psi \phi_B (K_0) > 1 \end{cases}$$

giving us

$$\mathrm{E}\left[\mathrm{NPV} \,\middle|\, K_0, K^*_{1,B}\right] = \begin{cases} p_S \mu_\Psi - K_0 & 0 & \text{if } p_S \mu_\Psi \phi_B (K_0) < 1 \\ p_S \mu_\Psi - \overline{I} & K^*_{1,B} = \overline{I} - K_0 & \text{if } p_S \mu_\Psi \phi_B (K_0) > 1 \end{cases};$$

2. The manager E is good and continues managing the project. $K^*_{1,G} \geq 0$ is invested in the project at $t = 1$. $K^*_{1,G}$ is the solution to the equation

$$\frac{p_S \mu_\Psi}{\alpha} \phi_B \left(\frac{K_0 + K_1}{\alpha} \right) = 1,$$

if the inequality

$$\frac{p_S \mu_\Psi}{\alpha^2} \phi'_B \left(\frac{K_0 + K_1}{\alpha} \right) < 0$$

holds at $K^*_{1,G}$. Otherwise, $K^*_{1,G}$ is the corner solution

$$K^*_{1,G} = \begin{cases} 0 & \text{if } \frac{p_S \mu_\Psi}{\alpha} \phi_B \left(\frac{K_0}{\alpha} \right) < 1 \\ \alpha \tilde{I} - K_0 & \text{if } \frac{p_S \mu_\Psi}{\alpha} \phi_B \left(\frac{K_0}{\alpha} \right) > 1 \end{cases}$$

giving us

$$\mathrm{E}\left[\mathrm{NPV} \,\middle|\, K_0, K^*_{1,B}\right] = \begin{cases} p_S \mu_\Psi - K_0 & 0 & \text{if } \frac{p_S \mu_\Psi}{\alpha} \phi_B \left(\frac{K_0}{\alpha} \right) < 1 \\ p_S \mu_\Psi - \alpha \overline{I} & K^*_{1,B} = \alpha \overline{I} - K_0 & \text{if } \frac{p_S \mu_\Psi}{\alpha} \phi_B \left(\frac{K_0}{\alpha} \right) > 1 \end{cases};$$

3. The manager's quality is not revealed at $t = 1$.

   3.1. If the manager M replaces him at $t = 1$, then $K^*_{1,G|U} > 0$ is invested in the project at $t = 1$. The cost of replacement is $c(K^*_{1,G|U})$;

   3.2. If the manager E continues managing the project, then $K^*_{1,U} \geq 0$ is invested in the project at $t = 1$;

Note that $K_1 = 0$ means that no further investment is done at the second stage. This does not mean that the project is abandoned, because its potential payoff can still be greater than zero, if sufficient investment has been made. The optimal investment K_0^{***} is determined as an argument of the maximization program

$$
\begin{aligned}
&\max_{K_0 \in [0;\bar{I}]} \left\{ f(K_0) \left[q_G \left(p_S V \Phi_B \left(\frac{K_0 + K_{1,G}}{\alpha} \right) - K_0 - K^*_{1,G} \right) + \right. \right. \\
&+ q_B \max \left\{ p_S V \Phi_B \left(K_0 + K^*_{1,B} \right) - K_0 - K^*_{1,B}; \right. \\
&\left. \left. p_S V \Phi_B \left(K_0 + \frac{1}{\alpha} K^*_{1,G|B} \right) - K_0 - K^*_{1,G|B} - c\left(K^*_{1,G|B} \right) \right\} \right] + \\
&(1 - f(K_0)) \max \left\{ p_S \mu \left[q_G \Phi_B \left(\frac{K_0 + K^*_{1,U}}{\alpha} \right) + q_B \Phi_B \left(K_0 + K^*_{1,U} \right) \right] - K_0 - K^*_{1,U}; \right. \\
&\left. \left. p_S \mu \left[q_G \Phi_B \left(\frac{K_0 + K^*_{1,G|U}}{\alpha} \right) + q_B \Phi_B \left(K_0 + \frac{1}{\alpha} K^*_{1,G|U} \right) \right] - K_0 - K^*_{1,G|U} - c\left(K^*_{1,G|U} \right) \right\} \right\}
\end{aligned}
$$

We denote the optimal investment at $t = 1$ as $K^*_{1,j}$. At $t = 1$ $K^*_{1,j}$ can be written as $K^*_{1,j} = K^*_1(K'_0; j)$ given that K'_0 was invested at $t = 0$ and the manager's type at $t = 1$ is j, where $j = \{G, G|B, G|U, B, U\}$. Again, U means the unknown type and $G|U$ means that the manager M replaced the manager E whose type remained unknown.

Proposition 1. *The expected net payoff from an optimal one-shot investment strategy is never greater than the ex-ante expected net payoff from the optimal two-stage investment strategy with the possibility learning the manager's true type and an option to replace the manager at the second stage.*

Proof. : Consider $K_0' \leq K_0^*$, where K_0^* is the one-shot optimal investment without replacement. Then we have

$$\begin{aligned}
\mathrm{E}\left[\mathrm{NPV}|K_0^{***};K_1^*\right] &\geq \mathrm{E}\left[\mathrm{E}\left[\mathrm{NPV}|K_0';K_1^*(K_0';j)\right]\right] \\
&= f(K_0')\left[q_G\left(p_S V \Phi_B\left(\frac{K_0'+K_{1,G}^*}{\alpha}\right) - K_0' - K_{1,G}^*\right) + \right. \\
&\quad +q_B \max\left\{p_S V \Phi_B\left(K_0'+K_{1,B}^*\right) - K_0' - K_{1,B}^*;\right. \\
&\quad \left.\left. p_S V \Phi_B\left(K_0' + \frac{1}{\alpha}K_{1,G|B}^*\right) - K_0' - K_{1,G|B}^* - c\left(K_{1,G|B}^*\right)\right\}\right] + \\
&\quad + (1-f(K_0'))\,\mathrm{E}\left[\mathrm{NPV}|K_0^*\right] \\
&\geq f(K_0')\left[q_G\left(p_S V \Phi_B\left(\frac{K_0'+K_{1,G}^*}{\alpha}\right) - K_0' - K_{1,G}^*\right) + \right. \\
&\quad +q_B\left(p_S V \Phi_B\left(K_0'+K_{1,B}^*\right) - K_0' - K_{1,B}^*\right) + (1-f(K_0'))\,\mathrm{E}\left[\mathrm{NPV}|K_0^*\right] \\
&\geq f(K_0')\,\mathrm{E}\left[\mathrm{NPV}|K_0^*\right] + (1-f(K_0'))\,\mathrm{E}\left[\mathrm{NPV}|K_0^*\right] = \mathrm{E}\left[\mathrm{NPV}|K_0^*\right]
\end{aligned}$$

To prove that the one-shot scheme with replacement is at least weakly dominated by the two-stage investment scheme it suffices to notice that $\mathrm{E}\left[\mathrm{NPV}|K_0^{**}\right] = \mathrm{E}\left[\mathrm{NPV}|0;K_{1,G|U}^*\right] \leq \mathrm{E}\left[\mathrm{NPV}|K_0^{***};K_1^*(K_0^{***})\right]$ ■.

The intuition behind this proposition is the same as before — two-stage investment gives us an option to replace the manager at the second stage and the option cannot have negative value.

5.3. Example — a point-mass distribution

Consider an example when

$$\Phi_B(K) = \begin{cases} 0 & \text{if} \quad 0 \leq K < \overline{I} \\ 1 & \text{if} \quad \overline{I} \leq K \end{cases}$$

$$\Phi_G(K) = \Phi_B(\tfrac{K}{\alpha}) = \begin{cases} 0 & \text{if} \quad 0 \leq K < \alpha\overline{I} \\ 1 & \text{if} \quad \alpha\overline{I} \leq K \end{cases}$$

The replacement cost is a linear function of the remaining investment K^R managed by the manager M: $c(K^R) = \beta K^R$. The probability of learning the manager's type after investing K_0 is a concave function

$f(K_0)$

$$f\left(K_0\right)=\begin{cases}\gamma K_0 & \text{if} \quad 0\leq K_0\leq \frac{1}{\gamma}\\ 1 & \text{if} \quad \frac{1}{\gamma}<K_0\end{cases} \tag{8}$$

The ex ante probability of sufficient investment for the one-shot scheme is

$$\Pr\left[\tilde{I}\leq K\right]=\begin{cases}0 & \text{if} \quad 0\leq K<\alpha\bar{I}\\ q_G & \text{if} \quad \alpha\bar{I}\leq K<\bar{I}\\ 1 & \text{if} \quad \bar{I}\leq K\end{cases}$$

Since the second order condition of the maximization program (1) is never satisfied, we have a corner solution here, in which Eq. (3) becomes

$$\mathrm{E}\left\{\mathrm{NPV}|K_0^*\right\}=\begin{cases}0 & K^*=0 & \text{if} \quad \alpha\geq\frac{p_SVq_G}{\bar{I}};p_S\mu\leq\bar{I}\\ p_S\mu q_G-\alpha\bar{I} & K^*=\alpha\bar{I} & \text{if} \quad \alpha<\min\left(\frac{p_S\mu q_G}{\bar{I}};1-\frac{p_S\mu q_B}{\bar{I}}\right)\\ p_S\mu-\bar{I} & K^*=\bar{I} & \text{if} \quad p_S\mu>\bar{I};\alpha>1-\frac{p_S\mu q_B}{\bar{I}}\end{cases}$$

With immediate replacement of manager E at $t=0$ we have

$$\mathrm{E}\left[\mathrm{NPV}|K_0=\alpha\bar{I}\right]=p_S\mu-\alpha\bar{I}\left(1+\beta\right) \quad \text{if} \quad \alpha\left(1+\beta\right)<\frac{p_S\mu}{\bar{I}}$$

and the expected gain from the replacement is the difference between the two expressions

$$\mathrm{E}\left[\Delta\mathrm{NPV}|K_0^{**}\right]=\begin{cases}p_S\mu-\alpha\bar{I}\left(1+\beta\right) & \text{if} \quad \begin{cases}\alpha\geq\frac{p_S\mu q_G}{\bar{I}};\\ p_S\mu\leq\bar{I}\end{cases}\\ p_SVq_B-\alpha\bar{I}\beta & \text{if} \quad \alpha<\min\left(\frac{p_S\mu q_G}{\bar{I}};1-\frac{p_S\mu q_B}{\bar{I}}\right)\\ \bar{I}\left(1-\alpha\left(1+\beta\right)\right) & \text{if} \quad \begin{cases}p_S\mu>\bar{I};\\ \alpha>1-\frac{p_S\mu q_B}{\bar{I}}\end{cases}\end{cases}$$

Of course replacement is optimal whenever $\mathrm{E}\left[\Delta\mathrm{NPV}|K_0^{**}\right]>0$. The entrepreneur will be replaced either if his expected personal gains from voluntarily stepping down exceed the costs, namely the lost benefits of control and share of costs of hiring a new manager, or if the investors have enough voting power to enforce replacement decision even without entrepreneur's consent.

Remark 1. *If $\alpha(1+\beta) < \frac{p_S\mu}{\bar{I}}$ holds, then the possibility of manager replacement allows the firm to undertake some projects that would not be undertaken without the replacement. However, the cost of replacement is borne even when the replacement is not actually necessary, that is when the incumbent manager is of good quality, but that was not revealed. The possibility to always learn the manager's type and make replacement only when it is really necessary, could further increase the net payoff from the project.*

Two-stage investment

For two-stage financing, the first best investment scheme is a solution to the maximization program

$$\max_{K_0\in[0;\bar{I}]} \Big\{ f(K_0)\Big[q_G p_S\mu - \min\left(K_0;\alpha\bar{I}\right) + q_B \max\Big\{\mathrm{E}\left[\mathrm{NPV}|K_0;K^*_{1,B}\right];\mathrm{E}\left[\mathrm{NPV}|K_0;K^*_{1,G|B}\right]\Big\}\Big]$$
$$(1-f(K_0))\max\Big\{\mathrm{E}\left[\mathrm{NPV}|K_0;K^*_{1,U}\right];\mathrm{E}\left[\mathrm{NPV}|K_0;K^*_{1,G|U}\right]\Big\}\Big\} \tag{9}$$

We consider only the case when $\alpha\bar{I} < p_S\mu < \bar{I}$, that is when the expected net present value of the project is positive only if it is run by the good manager. That is we consider only the case when

$$\alpha < \frac{p_S\mu}{\bar{I}} < 1$$

1. If at $t=1$ the manager E turns out to be good, the investor continues investment into the project

$$\mathrm{E}\left\{\mathrm{NPV}|K_0;K^*_{1,G}\right\} = \begin{cases} p_S V q_G - \alpha\bar{I} & K_0 + K^*_{1,G} = \alpha\bar{I} \quad \text{if} \quad K_0 \le \alpha\bar{I} \\ p_S\mu - K_0 & K^*_{1,G} = 0 \quad \text{if} \quad K_0 > \alpha\bar{I} \end{cases}$$

2. If at $t=1$ the manager E turns out to be bad, then keeping the incumbent gives the investor

$$\mathrm{E}\left\{\mathrm{NPV}|K_0;K^*_{1,B}\right\} = \begin{cases} -K_0 & K^*_{1,B} = 0 \quad \text{if} \quad K_0 \le \bar{I} - p_S\mu \\ p_S\mu - \bar{I} & K_0 + K^*_{1,B} = \bar{I} \quad \text{if} \quad K_0 > \bar{I} - p_S\mu \end{cases} \tag{10}$$

Replacing him with manager M makes sense only if $K^*_{G|B} > 0$. This happens if $K_0 > \bar{I} - \frac{p_S\mu}{\alpha(1+\beta)}$

$$\begin{aligned} \mathrm{E}\left\{\mathrm{NPV}|K_0; K^*_{1,G|B}\right\} &= p_S\mu - K_0 - \alpha\,(1+\beta)\left(\bar{I} - K_0\right) \\ &= p_S\mu - \alpha\,(1+\beta)\,\bar{I} - K_0\,(1 - \alpha\,(1+\beta)) \end{aligned} \tag{11}$$

where $K^*_{1,G|B} = \alpha\left(\bar{I} - K_0\right)$

Comparing expressions (10) and (11) we obtain the following result that is graphically illustrated on Figure **??**:

Proposition 2. *If after the first stage when K_0 has been invested in the project, the incumbent manager's type is revealed as the bad type, the replacement of manager E by manager M at $t = 1$ is optimal for the following sets of parameters*

1. *For $p_S\mu_\Psi - \bar{I} > 0$, if $\alpha\,(1+\beta) < 1$ the replacement of the bad manager is always optimal at the second stage*

2. *For $p_S\mu_\Psi - \bar{I} < 0$, the total NPV of the project is negative, but replacement at the second stage and further investment can minimize total losses. Two possible outcomes are possible:*

   2.-i. if $\alpha\,(1+\beta) < \frac{p_S\mu}{\bar{I}} < 1$, the replacement of the bad manager is always optimal at the second stage

   2.-ii. if $\frac{p_S\mu}{\bar{I}} < \alpha\,(1+\beta) < 1$ the replacement is optimal only if $K_0 > \bar{I} - \frac{p_S\mu}{\alpha(1+\beta)}$. .

Corollary 3. *if $\alpha\,(1+\beta) > 1$, then the replacement is never optimal. The incumbent should stay. The investment should either continue to minimize the losses or the project should be abandoned. This decision depends on K_0.*

Remark 2. *Notice that the replacement of a bad manager is always optimal when β is less than the profitability index of a project managed by the good manager*

$$\beta < \frac{p_S\mu - \alpha\bar{I}}{\alpha\bar{I}}$$

3. If at $t = 1$ the manager's type is not revealed, the investor can either continue investment into the project run by manager E or replace the manager. Let us denote as U the case when the project is managed by the incumbent of the unknown type. If the investor does not replace the manager

$$\mathrm{E}\left\{\mathrm{NPV}|K_0; K_{1,U}\right\} = \begin{cases} -K_0 - K_{1,U} & \text{if} \quad K_0 + K_{1,U} \leq \alpha \bar{I} \\ q_G p_S \mu - K_0 - K_{1,U} & \text{if} \quad \alpha \bar{I} < K_0 + K_{1,U} \leq \bar{I} \\ p_S \mu - K_0 - K_{1,U} & \text{if} \quad \bar{I} < K_0 + K_{1,U} \end{cases}$$

The optimal investment at $t = 1$ when the manager's type has not been revealed, $K_{1,U}$, depends upon the value of parameter α. Figure **??** shows three possible cases for different values of α.

For small α, that is for

$$\alpha \leq \frac{q_G p_S \mu}{\bar{I}}$$

and the gains from having a good manager are high, expression (10) becomes

$$\mathrm{E}\left\{\mathrm{NPV}|K_0; K_{1,U}^*\right\} = \begin{cases} q_G p_S \mu - \alpha \bar{I} & K_{1,U}^* = \alpha \bar{I} - K_0 & \text{if} \quad K_0 \leq \alpha \bar{I} \\ q_G p_S \mu - K_0 & K_{1,U}^* = 0 & \text{if} \quad \alpha \bar{I} < K_0 \leq \bar{I} - q_B p_S \mu \\ p_S \mu - \bar{I} & K_{1,U}^* = \bar{I} - K_0 & \text{if} \quad \bar{I} - q_B p_S \mu < K_0 \leq \bar{I} \end{cases} \tag{12}$$

For medium α defined by the inequality

$$\frac{q_G p_S \mu}{\bar{I}} < \alpha \leq 1 - \frac{q_B p_S \mu}{\bar{I}}$$

expression (10) becomes

$$\mathrm{E}\left\{\mathrm{NPV}|K_0; K_{1,U}^*\right\} = \begin{cases} -K_0 & K_{1,U}^* = 0 & \text{if} \quad K_0 \leq \alpha \bar{I} - q_G p_S \mu \\ q_G p_S \mu - \alpha \bar{I} & K_{1,U}^* = \alpha \bar{I} - K_0 & \text{if} \quad \alpha \bar{I} - q_G p_S \mu < K_0 \leq \alpha \bar{I} \\ q_G p_S \mu - K_0 & K_{1,U}^* = 0 & \text{if} \quad \alpha \bar{I} < K_0 \leq \bar{I} - q_B p_S \mu \\ p_S \mu - \bar{I} & K_{1,U}^* = \bar{I} - K_0 & \text{if} \quad \bar{I} - q_B p_S \mu < K_0 \leq \bar{I} \end{cases} \tag{13}$$

For high α defined by the inequality

$$1 - \frac{q_B p_S \mu}{\bar{I}} < \alpha \leq 1$$

expression (10) becomes

$$\mathrm{E}\left\{\mathrm{NPV}|K_0; K^*_{1,U}\right\} = \begin{cases} -K_0 & K^*_{1,U} = 0 \quad \text{if} \quad K_0 \leq \bar{I} - p_S\mu \\ p_S\mu - \bar{I} & K^*_{1,U} = \bar{I} - K_0 \quad \text{if} \quad \bar{I} - q_B p_S \mu < K_0 \leq \bar{I} \end{cases} \tag{14}$$

Another alternative at stage 2 would be to replace the incumbent by manager M. Such replacement has cost βK_1. If $K_1 = 0$ then this is simply equivalent to leaving the project with the incumbent manager. Notice also that if $K_0 + K_1 = \alpha\bar{I}$ is the optimal level of investment, then replacement is suboptimal because of incurred cost βK_1 and no increase in the expected payoff. Then replacement potentially can be optimal only if $K^*_{1,G|U} = \alpha\left(\bar{I} - K_0\right)$

$$\begin{aligned} \mathrm{E}\left\{\mathrm{NPV}|K_0; K^*_{1,G|U}\right\} &= p_S\mu - K_0 - \alpha(1+\beta)\left(\bar{I} - K_0\right) \\ &= p_S\mu - \alpha(1+\beta)\bar{I} - K_0\left(1 - \alpha(1+\beta)\right) \end{aligned} \tag{15}$$

To find when replacement is optimal, one has to compare $\mathrm{E}\left\{\mathrm{NPV}|K_0;\ K^*_{1,U}\right\}$ and $\mathrm{E}\left\{\mathrm{NPV}|K_0;\ K^*_{1,G|U}\right\}$ for different combinations of parameters α and β. Replacement is optimal for such ranges of values of K_0 which correspond to the segments on Figure **??**, where a line for corresponding α (one of three piece-wise lines) lies below the "replacement" line (straight line with circle markers). We have the following proposition

Proposition 4. *If, after investment K_0 has been made in the project at $t = 0$, the incumbent manager's type remains unknown, then the replacement of manager E by manager M is optimal for the following sets of parameters:*

1. *for $\alpha \leq \frac{q_G p_S \mu}{\bar{I}}$ (α is small or q_G is high)*

   1.-i. *if $\alpha(1+\beta)(1-\alpha) < \frac{q_B p_S \mu}{\bar{I}}$ the replacement is always optimal*

1.-ii. *if* $\alpha(1+\beta)(1-\alpha) > \frac{q_B p_S \mu}{\bar{I}}; \alpha\beta < \frac{q_B p_S \mu}{\bar{I}}; \alpha(1+\beta) < 1$ *the replacement is optimal if the following is true*

$$0 < K_0 < \tfrac{q_B p_S \mu - \alpha\beta\bar{I}}{1-\alpha(1+\beta)} < \alpha\bar{I} \quad \text{or} \quad \alpha\bar{I} < \bar{I} - \tfrac{q_B p_S \mu}{\alpha(1+\beta)} < K_0 < \bar{I}$$

1.-iii. *if* $\alpha(1+\beta)(1-\alpha) > \frac{q_B p_S \mu}{\bar{I}}; \alpha\beta > \frac{q_B p_S \mu}{\bar{I}}; \alpha(1+\beta) < 1$ *the replacement is optimal if the following is true*

$$\alpha\bar{I} < \bar{I} - \frac{q_B p_S \mu}{\alpha(1+\beta)} < K_0 < \bar{I}$$

2. *for* $\frac{q_G p_S \mu}{\bar{I}} < \alpha \leq 1 - \frac{q_B p_S \mu}{\bar{I}}$ *(α is in the intermediate range)*

2.-i. *if* $\alpha(1+\beta)(1-\alpha) < \frac{q_B p_S \mu}{\bar{I}}$ *then replacement is always optimal*

2.-ii. *if* $\alpha(1+\beta)(1-\alpha) >; \alpha(1+\beta) < \frac{p_S \mu}{\bar{I}} < 1$ *then replacement is optimal if the following is true*

$$0 < K_0 < \tfrac{q_B p_S \mu - \alpha\beta\bar{I}}{1-\alpha(1+\beta)} < \alpha\bar{I} \quad \text{or} \quad \alpha\bar{I} < \bar{I} - \tfrac{q_B p_S \mu}{\alpha(1+\beta)} < K_0 < \bar{I}$$

2.-iii. *if* $\frac{p_S \mu}{\bar{I}} < \alpha(1+\beta) < \frac{p_S \mu}{q_B p_S \mu + (1-\alpha)\bar{I}} < 1$ *then replacement is optimal if the following is true*

$$0 < \bar{I} - \tfrac{p_S \mu}{\alpha(1+\beta)} < K_0 < \tfrac{q_B p_S \mu - \alpha\beta\bar{I}}{1-\alpha(1+\beta)} < \alpha\bar{I} \quad \text{or} \quad \alpha\bar{I} < \bar{I} - \tfrac{q_B p_S \mu}{\alpha(1+\beta)} < K_0 < \bar{I}$$

2.-iv. *if* $\frac{p_S \mu}{q_B p_S \mu + (1-\alpha)\bar{I}} < \alpha(1+\beta) < 1; \alpha(1+\beta) > \frac{p_S \mu}{\bar{I}}$ *the replacement is optimal if the following is true*

$$\alpha\bar{I} < \bar{I} - \frac{q_B p_S \mu}{\alpha(1+\beta)} < K_0 < \bar{I}$$

3. *for* $1 - \frac{q_B p_S \mu}{\bar{I}} < \alpha \leq 1$ *(α is large)*

3.-i. *if* $\alpha(1+\beta) < \frac{p_S \mu}{\bar{I}} < 1$ *then replacement is always optimal*

3.-ii. *if* $\frac{p_S \mu}{\bar{I}} < \alpha(1+\beta) < 1$ *then replacement is optimal for*

$$\bar{I} - \frac{p_S \mu}{\alpha(1+\beta)} < K_0 < \bar{I}$$

where $\bar{I} - \frac{p_S\mu}{\alpha(1+\beta)} < \alpha\bar{I}$

Proof. : The proof is straightforward. It follows from comparing (12), (13) and (14) with (15). Graphically it can be found from Figure **??**.

Corollary 5. *If both inequalities* $\alpha(1+\beta) < \frac{p_S\mu}{\bar{I}}$ *and* $\alpha(1+\beta)(1-\alpha) < \frac{q_B p_S\mu}{\bar{I}}$ *hold, then it is always optimal at stage 2 to replace the bad manager and the manager of unknown type*

Corollary 6. *For* $\alpha(1+\beta) > 1$ *it is never optimal to replace neither the bad manager nor the manager of an unknown type*

For other values of α and β different combinations of replacement/no replacement decisions are possible depending on the investment K_0 made at the first stage. For the sake of brevity we will show how to find optimal K_0^* for the cases when replacement is never optimal or when it is always optimal for managers who are not good.

Original investment K_0 when replacement is always optimal

Corollary 5 gives the conditions when the replacement of the incumbent manager is always optimal at the second stage if his type has not been revealed as good during the first stage. In this case optimal investment at the second stage is $K_1 = \alpha(\bar{I} - K_0)$. Different optimal K_0^{***} can emerge depending on the combinations of parameters $\alpha, \beta, \gamma, q_G$ and $\bar{I}$. It is possible that optimal K_0^{***} is greater than $\alpha\overline{I}$ when the gain from learning the manager's type is greater than the cost of overinvestment.

The exact value of K_0^{***} depends on the combination of parameters α, β and γ.

Proposition 7. *For* $\alpha(1+\beta) < \frac{p_S\mu}{\bar{I}}$ *and* $\alpha(1+\beta)(1-\alpha) < \frac{q_B p_S\mu}{\bar{I}}$ *and* $f(K_0)$ *defined by (8) the optimal investment* K_0^{***} *is determined as the solution to the maximization program*

$$p_S\mu - \alpha\,(1+\beta)\,\overline{I} + \max_{K_0}\left\{f(K_0)q_G\alpha\beta\overline{I} - (1 - q_G f(K_0))\,K_0\,(1-\alpha\,(1+\beta)) - q_G f(K_0)\left[K_0 - \alpha\overline{I}\right]^+\right\}$$

Possible expected K_0^{***} *and NPV are:*

1. $K_0^{***} = 0$ *and*

$$\mathrm{E}\left[NPV|K_0^{***}=0\right] = p_S\mu - \alpha(1+\beta)\overline{I}$$

1.-i. *for* $q_B > \frac{\beta}{(1-\alpha)(1+\beta)}$ and $\frac{2}{\overline{I}} + \frac{1-\alpha(1+\beta)}{q_G\alpha\overline{I}(1+\beta)} < \gamma < \frac{q_B(1-\alpha(1+\beta))}{q_G\alpha\beta\overline{I}}$

1.-ii. *for* $\alpha < \frac{1}{2}$; $q_G < \frac{1+(1-\alpha)(1+\beta)}{\alpha(1-\alpha)(1+\beta)}$; $q_G < \frac{1-\alpha(1+\beta)}{\alpha(1-2\alpha)(1+\beta)}$ *and*

$$\frac{1}{\overline{I}} < \gamma < \frac{1+(1-\alpha)(1+\beta)}{q_G\alpha\overline{I}(1-\alpha)(1+\beta)}$$

2. $K_0^{***} = \frac{1}{\gamma}$ *and*

$$\mathrm{E}\left[NPV|K_0^{***}=\frac{1}{\gamma}\right] = \underbrace{p_S\mu - \alpha(1+\beta)\overline{I}}_{NPV\ for\ K_0=0} + \alpha\beta\overline{I} - q_B\left(\alpha\beta\overline{I} + \gamma\left(1-\alpha\left(1+\beta\right)\right)\right)$$

2.-i. *for* $q_B < \frac{\beta}{(1-\alpha)(1+\alpha)}$ and $\gamma > \frac{1}{\alpha\overline{I}}$

2.-ii. *for* $q_B > \frac{\beta}{(1-\alpha)(1+\alpha)}$ and $\gamma > \frac{q_B(1-\alpha(1+\beta))}{q_G\alpha\beta\overline{I}}$

3. $K_0^{***} = \alpha\overline{I}$ *and*

$$\mathrm{E}\left[NPV|K_0^{***}=\alpha\overline{I}\right] = \underbrace{p_S\mu - \alpha(1+\beta)\overline{I}}_{NPV\ for\ K_0=0} + \alpha\beta\overline{I} - q_B\alpha\overline{I}(1-\alpha)(1+\beta)$$

3.-i. *for* $\alpha < \frac{1}{2}$; $(1+\beta) < \frac{\alpha}{(1-\alpha)^2}$; $q_G < \frac{1-\alpha(1+\beta)}{\alpha(1-2\alpha)(1+\beta)}$

$$\frac{1}{\overline{I}} < \frac{1+(1-\alpha)(1+\beta)}{q_G\alpha\overline{I}(1-\alpha)(1+\beta)} < \gamma < \frac{1-\alpha(1+\beta)}{q_G\alpha\overline{I}(1-2\alpha)(1+\beta)} < \frac{1}{\alpha\overline{I}}$$

4. $K_0^{***} = \frac{q_G\alpha(1+\beta)\gamma\overline{I}-(1-\alpha(1+\beta))}{2q_G\alpha(1+\beta)\gamma}$ *and*

$$\mathrm{E}\left[NPV|K_0^{***}=K_0^{FOC}\right] = \underbrace{p_S\mu - \alpha(1+\beta)\overline{I}}_{NPV\ for\ K_0=0} + \frac{\left(q_G\alpha(1+\beta)\gamma\overline{I}-(1-\alpha(1+\beta))\right)^2}{4q_G\alpha(1+\beta)\gamma}$$

4.-i. *for* $\alpha < \frac{1}{2}$; $q_G > \frac{1-\alpha(1+\beta)}{\alpha(1-2\alpha)(1+\beta)}$

$$\frac{1}{\overline{\overline{I}}} < \frac{1-\alpha\left(1+\beta\right)}{q_G\alpha\overline{I}\left(1-2\alpha\right)\left(1+\beta\right)} < \gamma < \frac{2}{\overline{\overline{I}}} + \frac{1-\alpha\left(1+\beta\right)}{q_G\alpha\overline{I}\left(1+\beta\right)} < \frac{1}{\alpha\overline{\overline{I}}}$$

5. $K_0^{***} = \frac{1}{\gamma}$ *and*

$$\mathrm{E}\left[NPV|K_0^{***}=\frac{1}{\gamma}\right] = \underbrace{p_S\mu - \alpha(1+\beta)\overline{I}}_{NPV\ for\ K_0=0} + q_G\left(1+\beta\right)\alpha\overline{I} - \frac{1}{\gamma}q_B\left(1-\alpha(1+\beta)+q_G\alpha\left(1+\beta\right)\right)$$

5.-i. *for* $q_B < \frac{\beta}{(1-\alpha)(1+\beta)}$

$$\frac{1}{\overline{\overline{I}}} < \frac{1-q_B\alpha\left(1+\beta\right)}{q_G\alpha\overline{I}\left(1+\beta\right)} < \gamma < \frac{1}{\alpha\overline{\overline{I}}}$$

Proof. : *The proof follows from finding optima as* γ *changes from* 0 *to values greater than* $\frac{1}{\alpha\overline{\overline{I}}}$.

When replacement is always optimal and $K_0^{***} = 0$, then E should be replaced immediately, before any investment has been made. Since E is the entrepreneur, it may be unlikely that he will choose such investment option.

Our model may explain why managers leave parent companies, obtain initial financing from venture capitalists and then perhaps come back (the projects are bought back) to the parent at the later stages. This happens in situations where the venture capitalists provide better screening (higher γ) or their replacement alternative is less costly (lower α). When the true manager type turns out to be good, the need for replacement is no longer an issue and this option can be relinquished. The manager-entrepreneur can return to the parent. This can also have an empirical implication — in exits through reacquisition we should see less managerial turnover (before and after reacquisition) than in exits through the IPOs.

We can interpret $K_0 = 0$ and replacement being optimal as the special case when it is optimal for the entrepreneur to sell the project at the very beginning to an independent company.

Original investment K_0 when replacement is never optimal

Replacement is never optimal in the situations when the cost of replacement, β, is "too high". This means that the VC's ability to replace the incumbent manager does not bring additional value and the project should be financed by A or by atomistic investors.

Note that learning the incumbent manager's type still creates additional value even when replacement is never optimal, because of the option to abandon the project. However, without replacement it is never optimal to invest more than $K_0 = \alpha \bar{I}$ in the first stage.

Even when replacement is not optimal, to find K_0^{***} we still have to consider different combinations of parameters α and β in order to find out whether it is optimal to continue financing the project run by a bad manager or the manager of unknown type at stage 2.

Proposition 8. *For $\alpha(1+\beta) > 1$ and $f(K_0)$ defined by (8) the optimal investment K_0^{***} is determined as the solution to the maximization program*

$$\max_{K_0} \left\{ f(K_0) \left[q_G \left(p_S \mu - \alpha \overline{I} \right) + q_B \max \left(p_S \mu - \overline{I}; -K_0 \right) \right] + (1 - f(K_0)) \max \left(-K_0; q_G p_S \mu - \alpha \overline{I} \right) \right\}$$

Expected NPV and K_0^{***} are:

1. *For* $\alpha < \min \left(\frac{q_G p_S \mu}{\overline{I}}; \frac{2\left(\overline{I} - p_S \mu\right)}{\overline{I}} \right); \gamma < \frac{2}{\alpha \overline{I}}$ *or* $\frac{q_G p_S \mu}{\overline{I}} < \alpha \leq 1 - \frac{q_B p_S \mu}{\overline{I}}; \frac{\alpha \overline{I} - q_G p_S \mu}{q_B \left(\frac{\alpha \overline{I}}{2} \right)^2} < \gamma < \frac{2}{\alpha \overline{I}}$

$$\mathrm{E}\left[NPV\right] = q_G p_S \mu - \alpha \overline{I} + \gamma q_B \left(\frac{\alpha \overline{I}}{2} \right)^2; K_0^{***} = \frac{\alpha \overline{I}}{2}$$

2. *For* $\frac{2(\overline{I}-p_S\mu)}{\overline{I}} \leq \alpha < \frac{q_G p_S \mu}{\overline{I}}; \gamma < \frac{1}{\alpha \overline{I}}$

$$\mathrm{E}\left[NPV\right] = q_G p_S \mu - \alpha\overline{I} + \gamma q_B \alpha \overline{I}\left(p_S\mu - (1-\alpha)\overline{I}\right); K_0^{***} = \alpha\overline{I}$$

3. *For* $\frac{2(\overline{I}-p_S\mu)}{\overline{I}} \leq \alpha < \frac{q_G p_S \mu}{\overline{I}}; \gamma > \frac{1}{\alpha \overline{I}}$

$$\mathrm{E}\left[NPV\right] = q_G p_S \mu - \alpha\overline{I} + q_B\left(p_S\mu - (1-\alpha)\overline{I}\right); K_0^{***} = \frac{1}{\gamma}$$

4. *For* $\frac{q_G p_S \mu}{\overline{I}} < \alpha \leq 1 - \frac{q_B p_S \mu}{\overline{I}}; \gamma < \frac{\alpha\overline{I} - q_G p_S \mu}{q_B\left(\frac{\alpha \overline{I}}{2}\right)^2}$

$$\mathrm{E}\left[NPV\right] = 0; K_0^{***} = 0$$

5. *For* $1 - \frac{q_B p_S \mu}{\overline{I}} < \alpha;$

$$\mathrm{E}\left[NPV\right] = 0; K_0^{***} = 0$$

Proof. *: in Appendix III.*

6. Conclusion

We have shown how a combination of stage financing and different information production technologies available to investors changes the expected net present value of an entrepreneurial project and, therefore, determines the choice of investor by the entrepreneur. Different sources of uncertainty can be critical for the project's payoff at different stages of its development. Therefore, at each stage the entrepreneur must attract investors with the highest level of expertise in the relevant area. For example, professional angels and venture capitalists obtain better information about a project's potential outcomes and emerge as providers of capital when this information is crucial. Venture capitalists and parent companies who are the industry's experts and who have access to a pool of professional managers, can furnish least costly managerial replacement and so should be the main source of capital at the stage when managerial quality is decisive.

This model explains the empirically observable fact that many entrepreneurial ventures are being bought back by parent companies. This happens once information asymmetry is low and uncertainty has been resolved at the previous stage financed by active investors.

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A. Appendix I

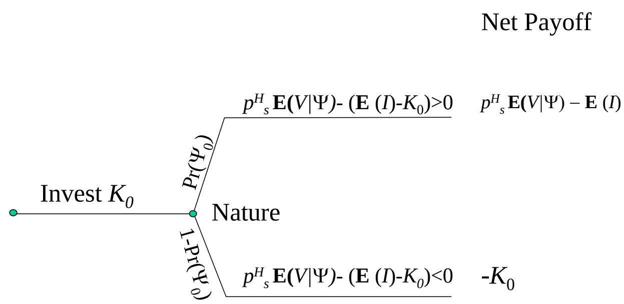

Figure 1. Decision tree and payoffs for stage financing when learning V

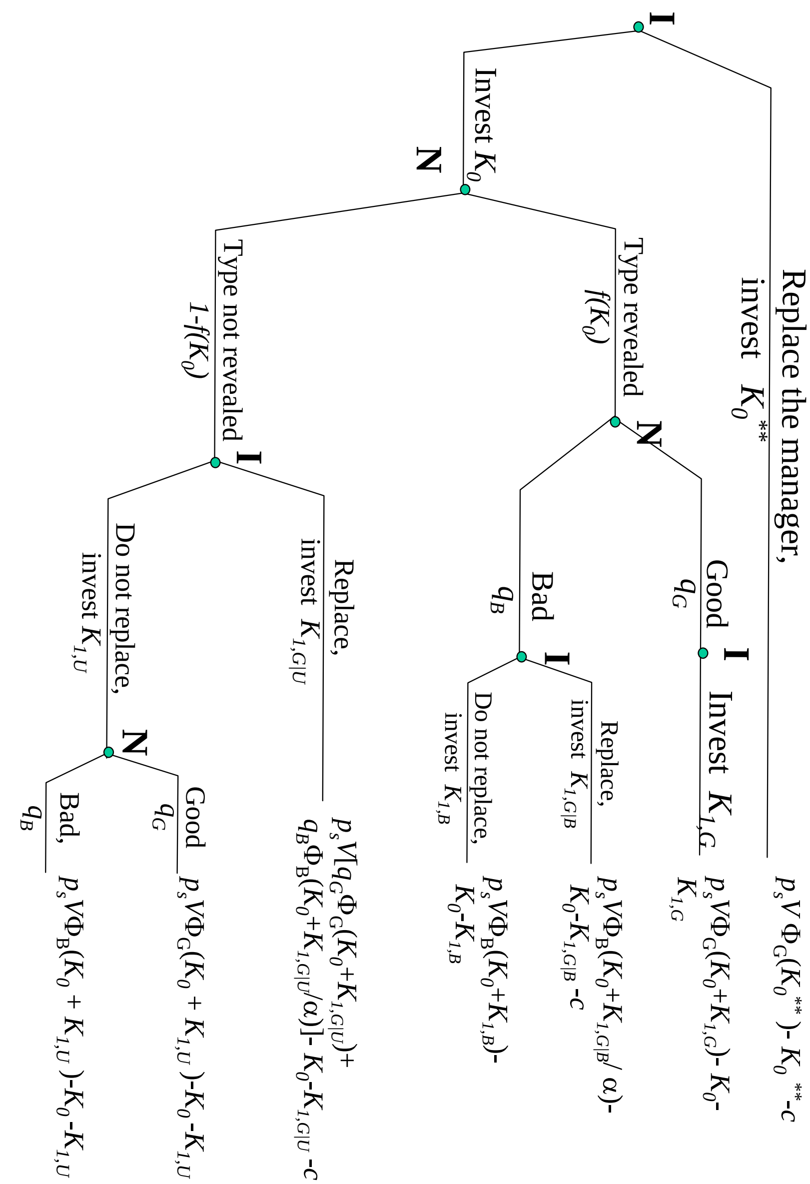

Figure 2. Decision tree for stage financing

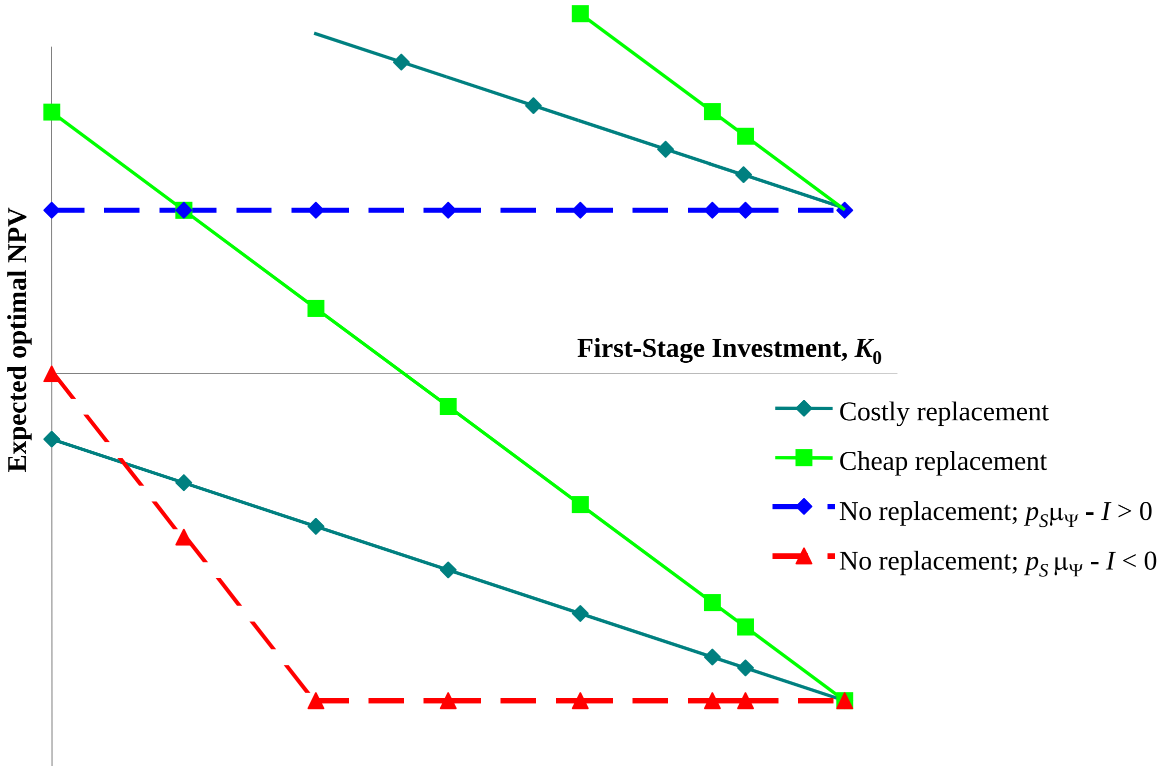

Figure 3. Net payoffs when the manager's type is revealed as B at $t = 1$. Solid lines show the net payoffs when the bad manager is replaced at $t = 1$. Dashed lines show the net payoffs without replacement. Replacement is optimal, when the solid line is above the dashed line.The upper dashed line corresponds to the positive net payoff (μ_Ψ is high), if no replacement is made. In this case the replacement is optimal whenever $\alpha(1+\beta) < 1$. The lower dashed line corresponds to the negative net payoff (μ_Ψ is low). Replacement is always optimal, if $\alpha(1+\beta) < \frac{p_S \mu_\Psi}{\overline{I} - K_0} < 1$. If $K_0 < \overline{I} - \frac{p_S \mu_\Psi}{\alpha(1+\beta)}$, then it is optiomal to abandon the project.

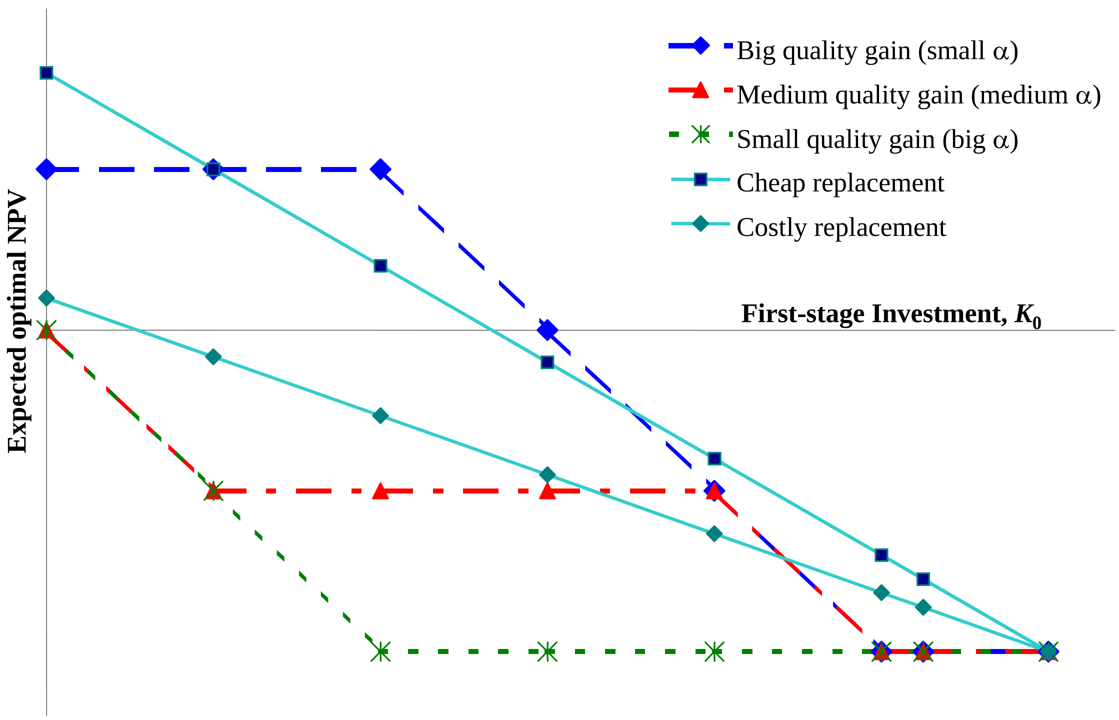

Figure 4. Net payoffs for different parameters α when the manager's type remains unknown at $t = 1$. Solid lines show net payoffs when the manager is replaced at $t = 1$. Replacement is optimal when the solid line is above the dashed line.

B. Appendix II

B.1. Probabilities of sufficient investment

We have to prove that

$$\Pr\left\{\tilde{I} \leq K_{0,B} + K_{1,G|B}\right\} = \Phi_B\left(K_{0,B} + \frac{1}{\alpha}K_{1,G|B}\right) \tag{1}$$

and

$$\Pr\left\{\tilde{I} \leq K_{0,B} + K_{1,G|U}\right\} = q_B\Phi_B\left(K_{0,B} + \frac{1}{\alpha}K_{1,G|B}\right) + q_G\Phi_B\left(\frac{1}{\alpha}\left(K_{0,B} + K_{1,G|B}\right)\right) \tag{2}$$

Remember that $\tilde{I}_G = \alpha\tilde{I}_B$, $\alpha < 1$, where $\tilde{I}_G$ is the required investment level if the project is undertaken entirely by the good manager and $\tilde{I}_B$ is the required investment level if the project is undertaken entirely by the bad manager. We assume that the manager M can save money only on the part of the project that he manages and that the money already invested cannot be recovered. Then the probability that $K_{0,B}+K_{1,G|B}$ is greater than the required investment level is

$$\begin{aligned}
\Pr\left\{\tilde{I} \leq K_{0,B} + K_{1,G|B}\right\} &= \Pr\left\{\tilde{I}_B \leq K_{0,B} + K_{1,B}\right\} \\
&= \Pr\left\{\tilde{I}_B \leq K_{0,B}\right\} + \Pr\left\{K_{0,B} \leq \tilde{I}_B \leq K_{0,B} + K_{1,B}\right\} \\
&= \Pr\left\{\tilde{I}_B \leq K_{0,B}\right\} + \Pr\left\{0 \leq \tilde{I}_B - K_{0,B} \leq K_{1,B}\right\} \\
&= \Pr\left\{\tilde{I}_B \leq K_{0,B}\right\} + \Pr\left\{0 \leq \tilde{I}_B - K_{0,B} \leq K_{1,B}\right\} \\
&= \Pr\left\{\tilde{I}_B \leq K_{0,B}\right\} + \Pr\left\{0 \leq \alpha\left(\tilde{I}_B - K_{0,B}\right) \leq K_{1,G|B}\right\} \\
&= \Pr\left\{\tilde{I}_B \leq K_{0,B}\right\} + \Pr\left\{0 \leq \tilde{I}_B - K_{0,B} \leq \frac{1}{\alpha}K_{1,G|B}\right\} \\
&= \Pr\left\{\tilde{I}_B \leq K_{0,B}\right\} + \Pr\left\{K_{0,B} \leq \tilde{I}_B \leq \frac{1}{\alpha}K_{1,G|B} + K_{0,B}\right\} \\
&= \Phi_B\left(K_{0,B} + \frac{1}{\alpha}K_{1,G|B}\right).
\end{aligned}$$

To find $\Pr\left\{\tilde{I} \leq K_{0,B} + K_{1,G|U}\right\}$ it suffices to notice that

$$\Pr\left\{\tilde{I} \leq K_{0,B} + K_{1,G|U}\right\} = q_B \Pr\left\{\tilde{I} \leq K_{0,B} + K_{1,G|B}\right\} + q_G \Pr\left\{\tilde{I}_G \leq K_{0,G} + K_{1,G}\right\}$$

and then result (2) follows.□

B.2. Possible outcomes at $t = 1$

If the manager E is not replaced at the beginning and K_0^{***} is invested at $t = 0$, then depending on the realization of the manager's type, amount K_0^{***} invested at $t = 0$ and other parameters five different investor's decisions about replacing the manager are generally possible, namely: 1) the investor can replace the bad manager, 2) he can keep the bad manager, 3) he keeps the good manager, 4) he keeps the manager, whose type is not revealed, and 5) he replaces the manager of unknown type.

With probability $f(K_0^{***})$ the manager's type is revealed at $t = 1$ and then we have the following three scenarios:

1. The manager E is bad and has to be replaced by the manager M. After K_0^{***} was invested at $t = 0$, the cost of replacement is $c(K_{1,G|B}^*)$. Then the optimal investment $K_{1,G|B}^*$ is determined from

$$\max_{K_{1,G|B}\in\left[0;\alpha\left(\bar{I}-K_0^{***}\right)\right]} \mathrm{E}\left[\mathrm{NPV}\left|K_{1,G|B}\right.\right] = p_S\mu_\Psi\Phi_B\left(K_0^{***}+\frac{1}{\alpha}K_{1,G|B}\right) - K_0^{***} - K_{1,G|B} - c\left(K_{1,G|B}\right)$$

From the first- and second-order conditions we have that $K_{1,G|B}^*$ is either a solution to the equation

$$p_S\mu_\Psi\frac{1}{\alpha}\phi_B\left(K_0^{***}+\frac{1}{\alpha}K_{1,G|B}\right) - 1 - c'\left(K_{1,G|B}\right) = 0;$$

given that

$$p_S\mu_\Psi\phi_B'\left(K_0^{***}+\frac{1}{\alpha}K_{1,G|B}\right) - c''\left(K_{1,G|B}\right) < 0$$

at $K_{1,G|B}^*$. Otherwise $K_{1,G|B}^*$ is a corner solution[5]

$$K_{1,G|B}^* = \alpha\left(\overline{I}-K_0^{***}\right) \quad \text{if } p_S\mu_\Psi\left(1-\Phi_B\left(K_0^{***}\right)\right) > \alpha\left(\overline{I}-K_0^{***}\right) + c\left(\overline{I}-K_0^{***}\right) \tag{3}$$

[5] Corner solution $K_{1,G|B}^* = 0$ is not feasible in this case because it is dominated by $K_{1,B}^* = 0$

and then

$$\mathrm{E}\left[\mathrm{NPV}\left|K^{*}_{1,G|B}\right.\right] = p_S\mu_\Psi - \alpha\overline{I} - (1-\alpha)K_0^{***} - c\left(\alpha\left(\overline{I} - K_0^{***}\right)\right) \quad \text{at } K^{*}_{1,G|B} = \alpha\left(\overline{I} - K_0^{***}\right). \tag{4}$$

2. The manager E is bad, but he is not replaced. The optimal investment $K^{*}_{1,B}$ is determined from

$$\max_{K_{1,B}\in\left[0;\bar{I}-K_0^{***}\right]} \mathrm{E}\left[\mathrm{NPV}\left|K_{1,B}\right.\right] = p_S\mu_\Psi\Phi_B\left(K_0^{***} + K_{1,B}\right) - K_0^{***} - K_{1,B}$$

From the first- and second-order conditions we have that $K^{*}_{1,B}$ is a solution to the equation

$$p_S\mu_\Psi\phi_B\left(K_0^{***} + K_{1,B}\right) - 1 = 0;$$

$$K^{*}_{1,B} = \phi_B^{-1}\left(\frac{1}{p_S\mu_\Psi}\right) - K_0^{***};$$

if the following ineqaulity holds

$$\mu_\Psi\phi'_B\left(K_0^{***} + K_{1,B}\right) < 0$$

at $K^{*}_{1,B}$ and then

$$\mathrm{E}\left[\mathrm{NPV}\left|K^{*}_{1,B}\right.\right] = p_S\mu_\Psi\Phi_B\left(\phi_B^{-1}\left(\frac{1}{p_S\mu_\Psi}\right)\right) - \phi_B^{-1}\left(\frac{1}{p_S\mu_\Psi}\right).$$

Otherwise, $K^{*}_{1,B}$ is a corner solution

$$K^{*}_{1,B} = \begin{cases} 0 & \text{if } p_S\mu_\Psi\left(1 - \Phi_B\left(K_0^{***}\right)\right) < \overline{I} - K_0^{***}; \\ \overline{I} - K_0^{***} & \text{if } p_S\mu_\Psi\left(1 - \Phi_B\left(K_0^{***}\right)\right) > \overline{I} - K_0^{***}, \end{cases} \tag{5}$$

and then

$$\mathrm{E}\left[\mathrm{NPV}\left|K^{*}_{1,B}\right.\right] = \begin{cases} p_S V\Phi_B\left(K_0^{***}\right) - K_0^{***}; & \text{at } K^{*}_{1,B} = 0; \\ p_S\mu - \overline{I}; & \text{at } K^{*}_{1,B} = \overline{I} - K_0^{***}; \end{cases} \tag{6}$$

3. The manager E is good and continues to manage the company. The optimal investment $K^*_{1,G}$ is determined from

$$\max_{K_{1,G}\in\left[0;\alpha\bar{I}-K_0^{***}\right]} \mathrm{E}\left[\mathrm{NPV}\,|K_{1,G}\right] = p_S\mu_\Psi\Phi_B\left(\frac{K_0^{***}+K_{1,G}}{\alpha}\right) - K_0^{***} - K_{1,G}$$

From the first- and second-order conditions we have that $K^*_{1,G}$ is a solution to the equation

$$p_S\mu_\Psi\frac{1}{\alpha}\phi_B\left(\frac{K_0^{***}+K_{1,G}}{\alpha}\right) - 1 = 0,$$

$$K^*_{1,G} = \alpha\phi_B^{-1}\left(\frac{\alpha}{p_S\mu_\Psi}\right) - K_0^{***}$$

if the following inequality holds

$$\mu_\Psi\phi'_B\left(\frac{K_0^{***}+K_{1,G}}{\alpha}\right) < 0$$

at $K^*_{1,G}$. In that case the expected optimal NPV is

$$\mathrm{E}\left[\mathrm{NPV}\,\middle|K^*_{1,G}\right] = p_S\mu_\Psi\Phi_B\left(\phi_B^{-1}\left(\frac{\alpha}{p_S\mu_\Psi}\right)\right) - \alpha\phi_B^{-1}\left(\frac{\alpha}{p_S\mu_\Psi}\right)$$

Otherwise, the corner solution for $K^*_{1,G}$ is

$$K^*_{1,G} = \begin{cases} 0 & \text{if } p_S\mu_\Psi\left(1-\Phi_B\left(K_0^{***}\right)\right) < \alpha\overline{I} - K_0^{***} \\ \alpha\overline{I} - K_0^{***} & \text{if } p_S\mu_\Psi\left(1-\Phi_B\left(K_0^{***}\right)\right) > \alpha\overline{I} - K_0^{***} \end{cases} \tag{7}$$

$$\mathrm{E}\left[\mathrm{NPV}\,\middle|K^*_{1,G}\right] = \begin{cases} p_S\mu_\Psi\Phi_B\left(\frac{K_0^{***}}{\alpha}\right) - K_0^{***}, & \text{at } K^*_{1,G} = 0; \\ p_S\mu_\Psi - \alpha\overline{I}, & \text{at } K^*_{1,G} = \alpha\overline{I} - K_0^{***}. \end{cases} \tag{8}$$

With probability $1 - f(K_0^{***})$ the manager's type is not revealed at $t = 0$. Such realization has two possible outcomes:

4. The manager E is not replaced. The optimal investment $K^{*}_{1,U}$ is defined from

$$\max_{K_{1,U}\in\left[0;\bar{I}-K_0^{***}\right]} \mathrm{E}\left[\mathrm{NPV}\left|K_{1,U}\right.\right] = p_S\mu_\Psi\left[q_G\Phi_B\left(\frac{K_0^{***}+K_{1,U}}{\alpha}\right)+q_B\Phi_B\left(K_0^{***}+K_{1,U}\right)\right]-K_0^{***}-K_{1,U}$$

From the first- and second-order conditions we have that $K^{*}_{1,U}$ is either a solution to the equation

$$p_S\mu_\Psi\left[q_G\frac{1}{\alpha}\phi_B\left(\frac{K_0^{***}+K_{1,U}}{\alpha}\right)+q_B\phi_B\left(K_0^{***}+K_{1,U}\right)\right]-1=0,$$

given that

$$\mu_\Psi\left[q_G\frac{1}{\alpha^2}\phi'_B\left(\frac{K_0^{***}+K_{1,U}}{\alpha}\right)+q_B\phi'_B\left(K_0^{***}+K_{1,U}\right)\right]<0$$

at $K^{*}_{1,U}$ or it is a corner solution

$$K^{*}_{1,U}=\begin{cases} 0, & \text{if } \begin{cases} \overline{I}-K_0^{***}>p_S\mu_\Psi\left(1-\left[q_G\Phi_B\left(\frac{K_0^{***}}{\alpha}\right)+q_B\Phi_B\left(K_0^{***}\right)\right]\right), \\ \alpha\overline{I}-K_0^{***}>p_S\mu_\Psi\left[q_G\left(1-\Phi_B\left(\frac{K_0^{***}}{\alpha}\right)\right)+q_B\left(\Phi_B\left(\alpha\overline{I}\right)-\Phi_B\left(K_0^{***}\right)\right)\right]; \end{cases} \\ \alpha\overline{I}-K_0^{***}, & \text{if } \begin{cases} \overline{I}\left(1-\alpha\right)>p_S\mu_\Psi\left(1-\left[q_G+q_B\Phi_B\left(\alpha\overline{I}\right)\right]\right), \\ \alpha\overline{I}-K_0^{***}<p_S\mu_\Psi\left[q_G\left(1-\Phi_B\left(\frac{K_0^{***}}{\alpha}\right)\right)+q_B\left(\Phi_B\left(\alpha\overline{I}\right)-\Phi_B\left(K_0^{***}\right)\right)\right]; \end{cases} \\ \overline{I}-K_0^{***}, & \text{if } \begin{cases} \overline{I}-K_0^{***}<p_S\mu_\Psi\left(1-\left[q_G\Phi_B\left(\frac{K_0^{***}}{\alpha}\right)+q_B\Phi_B\left(K_0^{***}\right)\right]\right), \\ \alpha\overline{I}-K_0^{***}>p_S\mu_\Psi\left[q_G\left(1-\Phi_B\left(\frac{K_0^{***}}{\alpha}\right)\right)+q_B\left(\Phi_B\left(\alpha\overline{I}\right)-\Phi_B\left(K_0^{***}\right)\right)\right]; \end{cases} \end{cases}; \tag{9}$$

and then

$$\mathrm{E}\left[\mathrm{NPV}\left|K^{*}_{1,U}\right.\right]=\begin{cases} p_S\mu_\Psi\left[q_G\Phi_B\left(\frac{K_0^{***}}{\alpha}\right)+q_B\Phi_B\left(K_0^{***}\right)\right]-K_0^{***}, & \text{at } K^{*}_{1,U}=0; \\ p_S\mu_\Psi\left[q_G+q_B\Phi_B\left(\alpha\overline{I}\right)\right]-\alpha\overline{I}, & \text{at } K^{*}_{1,U}=\alpha\overline{I}-K_0^{***}; \\ p_S\mu_\Psi-\overline{I}, & \text{at } K^{*}_{1,U}=\overline{I}-K_0^{***}, \end{cases} \tag{10}$$

5. The manager E is replaced by the manager M. Then the optimal investment $K^{*}_{1,G|U}$ is defined as a solution to the maximization program

$$\max_{K_{1,G|U}\in\left[0;\alpha\left(\bar{I}-K_0^{***}\right)\right]} \mathrm{E}\left[\mathrm{NPV}\left|K_{1,G|U}\right.\right]$$

$$= p_S\mu_\Psi\left[q_G\Phi_B\left(\frac{K_0^{***}+K_{1,G|U}}{\alpha}\right)+q_B\Phi_B\left(K_0^{***}+\frac{1}{\alpha}K_{1,G|U}\right)\right]-K_0^{***}-K_{1,G|U}-c\left(K_{1,G|U}\right)$$

From the first- and second-order conditions follows that $K_{1,G|U}^*$ is either a solution to the equation

$$p_S\mu_\Psi\frac{1}{\alpha}\left[q_G\phi_B\left(\frac{K_0^{***}+K_{1,G|U}}{\alpha}\right)+q_B\phi_B\left(K_0^{***}+\frac{1}{\alpha}K_{1,G|U}\right)\right]-1-c'\left(K_{1,G|U}\right)=0,$$

given that

$$p_S\mu_\Psi\left(\frac{1}{\alpha}\right)^2\left[q_G\phi_B'\left(\frac{K_0^{***}+K_{1,G|U}}{\alpha}\right)+q_B\phi_B'\left(K_0^{***}+\frac{1}{\alpha}K_{1,G|U}\right)\right]-c''\left(K_{1,G|U}\right)<0$$

at $K_{1,G|U}^*$ or it is a corner solution

$$K_{1,G|U}^*=\begin{cases}\alpha\overline{I}-K_0^{***} & \text{if } (1-\alpha)\alpha\left(\overline{I}-K_0^{***}\right)+c\left(\alpha\left(\overline{I}-K_0^{***}\right)\right)-c\left(\alpha\overline{I}-K_0^{***}\right)>p_S\mu_\Psi\left(1-\left[q_G+q_B\Phi_B\left(\overline{I}-\frac{1-\alpha}{\alpha}K\right.\right.\right.\\ \alpha\left(\overline{I}-K_0^{***}\right) & \text{if } (1-\alpha)\alpha\left(\overline{I}-K_0^{***}\right)+c\left(\alpha\left(\overline{I}-K_0^{***}\right)\right)-c\left(\alpha\overline{I}-K_0^{***}\right)<p_S\mu_\Psi\left(1-\left[q_G+q_B\Phi_B\left(\overline{I}-\frac{1-\alpha}{\alpha}K\right.\right.\right.\end{cases} \tag{11}$$

and then

$$\mathrm{E}\left[\mathrm{NPV}\left|K_{1,G|U}^*\right.\right]=\begin{cases}p_S\mu_\Psi\left[q_G+q_B\Phi_B\left(\overline{I}-\frac{1-\alpha}{\alpha}K_0^{***}\right)\right]-\alpha\overline{I}-c\left(K_{1,G|U}^*\right), & \text{at } K_{1,G|U}^*=\alpha\overline{I}-K_0^{***};\\ p_S\mu_\Psi-\alpha\overline{I}-(1-\alpha)K_0^{***}-c\left(K_{1,G|U}^*\right). & \text{at } K_{1,G|U}^*=\alpha\left(\overline{I}-K_0^{***}\right).\end{cases} \tag{12}$$

C. Appendix III

Proof. Proposition 1: Investor A will contribute money only after he has observed high signal. In this case the projects's expected NPV is

$$\mathrm{E}[\mathrm{NPV}|H_A] = p_S^A \left(q_G \Phi_G(K) + q_B \Phi_B(K)\right) \mu - K.$$

Before inviting investor A the entrepreneur does not know whether the signal will be H_A or L_A, but he knows the ex-ante probability of H_A

$$\Pr(H_A|H_E) = \frac{\Pr(H_E|H_A)\Pr(H_A)}{\Pr(H_E)} = \frac{p_S^E}{p_S^A}$$

Here we used

$$\begin{aligned}
\Pr(H_A|S) &= 1; \Pr(H_E|H_A) = 1; \\
\Pr(H_A) &= \frac{\Pr(H_A|S)\Pr(S)}{\Pr(S|H_A)} = \frac{p_S}{p_S^A} \\
\Pr(H_E) &= \frac{\Pr(H_E|S)\Pr(S)}{\Pr(S|H_E)} = \frac{p_S}{p_S^E}
\end{aligned}$$

Now, after investor A has screened the project, it will go ahead only if the signal is H_A. So, the mere fact of being screened by investor A makes the expected NPV equal to

$$\begin{aligned}
\mathrm{E}\left[\mathrm{NPV}|H_E;\ \text{A invited}\right] &= \Pr(H_A|H_E)\left(p_S^A \left(q_G \Phi_G(K) + q_B \Phi_B(K)\right) \mu - K\right) + \Pr(L_A|H_E) \cdot 0 \\
&= \frac{p_S^E}{p_S^A}\left(p_S^A \left(q_G \Phi_G(K) + q_B \Phi_B(K)\right) \mu - K\right) = p_S^E \left(q_G \Phi_G(K) + q_B \Phi_B(K)\right) \mu - \frac{p_S^E}{p_S^A} K
\end{aligned}$$

The screening by A increases the project's NPV by

$$\Delta\mathrm{NPV} = \frac{p_S^A - p_S^E}{p_S^A} K$$

Since $p_S^A > p_S^E$, $\Delta\text{NPV} > 0$.

$$\frac{p_S^A - p_S^E}{p_S^A} = 1 - \frac{p_S^E}{p_S^A} \leq 1 - p_S^E \leq 1 - p_S$$

Therefore, $\Delta\text{NPV} \leq (1 - p_S)\,\mathrm{E}(I)$ ■

Proof. Proposition 3: We have to show that the expected ex-ante NPV for one stage investment

$$\mathrm{E}\left[\text{NPV}|\text{one-shot}\right] = p_S^E \Pr\left(\tilde{I} \leq K\right)\mu - K$$

is never greater than the expected ex-ante NPV for the two stage investment. Let us denote $\mu_\Psi = \mathrm{E}\left(V|\Psi\right)$ and

$$V_C\left(K_0\right) = \frac{K - K_0}{p_S^H \Pr\left(\tilde{I} \leq K\right)}.$$

The expected NPV from the two-stage investment is

$$\begin{aligned} \mathrm{E}\left[\text{NPV}|\text{two-stage}\right] &= \mathrm{E}_\Psi \Big\{\Pr\left\{\mu_\Psi > V_C\left(K_0\right)\right\}\left[p_S^H \Pr\left(\tilde{I} \leq K\right)\mathrm{E}\left(V\left|\mu_\Psi > V_C\right.\right) - \left(K - K_0\right)\right] \\ &\quad + \Pr\left\{\mu_\Psi < V_C\left(K_0\right)\right\} p_S^H \Pr\left(\tilde{I} \leq K_0\right)\mathrm{E}\left(V\left|\mu_\Psi < V_C\right.\right)\Big\} - K_0. \end{aligned}$$

Here we used the fact that at the second stage investment $K - K_0$ will be made iff the expected payoff conditional on signal Ψ will exceed a critical value $V_C\left(K_0\right)$

$$\mu_\Psi > V_C\left(K_0\right) = \frac{K - K_0}{p_S^H \Pr\left(\tilde{I} \leq K\right)}$$

Let us denote Subtracting expression for the one-stage investment from the expression for the two-stage investment and using the law of iterated expectations for μ

$$\begin{aligned} \mu &= \mathrm{E}(V) = \mathrm{E}_\Psi\left\{\Pr\left\{\mu_\Psi > V_C\left(K_0\right)\right\}\mathrm{E}\left(V\left|\mu_\Psi > V_C\left(K_0\right)\right.\right)\right. \\ &\quad \left. + \Pr\left\{\mu_\Psi < V_C\left(K_0\right)\right\}\mathrm{E}\left(V\left|\mu_\Psi < V_C\left(K_0\right)\right.\right)\right\}, \end{aligned}$$

we get

$$\begin{aligned}
\Delta\,\mathrm{E(NPV)} &= \mathrm{E}\left\{\Pr\{\mu_\Psi > V_C(K_0)\}\left[p_S^H \Pr\left(\tilde{I}\le K\right)\mathrm{E}\left(V\,|\mu_\Psi > V_C(K_0)\right)-(K-K_0)\right]\right.\\
&\quad\left.+\Pr\{\mu_\Psi < V_C(K_0)\}\,p_S^H \Pr\left(\tilde{I}\le K_0\right)\mathrm{E}\left(V\,|\mu_\Psi < V_C(K_0)\right)\right\}-K_0+K-\\
&\quad-\mathrm{E}_\Psi\left\{p_S^H \Pr\left(\tilde{I}\le K\right)\left[\Pr\{\mu_\Psi > V_C(K_0)\}\mathrm{E}\left(V\,|\mu_\Psi > V_C(K_0)\right)+\Pr\{\mu_\Psi < V_C(K_0)\}\mathrm{E}\left(V\,|\mu_\Psi < V_C(K_0)\right)\right]\right.\\
&= \mathrm{E}_\Psi\left\{\Pr\{\mu_\Psi < V_C(K_0)\}\left[K-K_0-p_S^H\,\mathrm{E}\left(V\,|\mu_\Psi < V_C(K_0)\right)\left(\Pr\left(\tilde{I}\le K\right)-\Pr\left(\tilde{I}\le K_0\right)\right)\right]\right\}\\
&= \mathrm{E}_\Psi\left\{\Pr\{\mu_\Psi < V_C(K_0)\}\left[K-K_0-p_S^H\,\mathrm{E}\left(V\,|\mu_\Psi < V_C(K_0)\right)\Pr\left(\tilde{I}\le K\right)\left(1-\frac{\Pr\left(\tilde{I}\le K_0\right)}{\Pr\left(\tilde{I}\le K\right)}\right)\right]\right\}\\
&\ge \mathrm{E}_\Psi\left\{\Pr\{\mu_\Psi < V_C(K_0)\}\right\}\left[K-K_0-(K-K_0)\left(1-\frac{\Pr\left(\tilde{I}\le K_0\right)}{\Pr\left(\tilde{I}\le K\right)}\right)\right]=0.
\end{aligned}$$

The inequality follows from

$$\mathrm{E}\left[V|\,\mathrm{E}\left[V|\Psi\right]<V_C\right]\le V_C$$

Notice that equality (3) is the equality part in the expression above ■3

Proof. Corollary 7: Conditional expectation of V after observing signal Ψ, $\Psi = V + \varepsilon\ \Psi \sim N\left(\mu, \frac{R_\varepsilon}{rr_\varepsilon}\right)$ is equal to

$$\mu_\Psi = \mathrm{E}\left[V|\Psi\right] = \frac{\mu r + \Psi r_\varepsilon}{R_\varepsilon}.$$

Then the ex-ante probability of observing "continuation" signal becomes

$$\begin{aligned}
\Pr\left\{\mu_\Psi > \frac{K-K_0}{p_S^H \Pr\left(\tilde{I}\le K\right)}\right\} &= \Pr\left\{\Psi > \frac{(K-K_0)\,R_\varepsilon - p_S^H \Pr\left(\tilde{I}\le K\right)\mu r}{p_S^H \Pr\left(\tilde{I}\le K\right) r_\varepsilon}\right\}\\
&= \Pr\{\Psi > \Psi_0\} = 1-\Phi_Z\left(\frac{\Psi_0-\mu}{\sqrt{\frac{R_\varepsilon}{rr_\varepsilon}}}\right)\\
&= \Phi_Z\left(\frac{\mu-\Psi_0}{\sqrt{\frac{R_\varepsilon}{rr_\varepsilon}}}\right) = \Phi_Z\left(\frac{\sqrt{rR_\varepsilon}}{p_S^H\sqrt{r_\varepsilon}}\left(p_S^H \Pr\left(\tilde{I}\le K\right)\mu-(K-K_0)\right)\right)
\end{aligned}$$

where

$$\Psi_0 = \frac{(K - K_0) R_\varepsilon - p_S^H \Pr\left(\tilde{I} \leq K\right) \mu r}{p_S^H \Pr\left(\tilde{I} \leq K\right) r_\varepsilon}.$$

Expected value of V given that "continuation" signal is observed is

$$\begin{aligned}
\mathrm{E}\left[V | \Psi > \Psi_0\right] &= \frac{1}{\Pr\left\{\Psi > \Psi_0\right\}} \int_{\Psi_0}^{\infty} \frac{\mu r + x r_\varepsilon}{R_\varepsilon} p_\Psi(x) dx \\
&= \mu \frac{r}{R_\varepsilon} + \frac{r_\varepsilon}{\Pr\left\{\Psi > \Psi_0\right\} R_\varepsilon} \int_{\Psi_0}^{\infty} x p_\Psi(x) dx
\end{aligned}$$

where $p_\Psi(\cdot)$ is the probability density function of Ψ. The integral is equal to

$$\begin{aligned}
\int_{\Psi_0}^{\infty} x p_\Psi(x) dx &= \int_{\Psi_0}^{\infty} x \exp\left\{-\frac{(x-\mu)^2}{2\sigma_\Psi^2}\right\} dx \\
&= \frac{\sigma_\Psi}{\sqrt{2\pi}} \int_{\Psi_0}^{\infty} \exp\left\{-\frac{(x-\mu)^2}{2\sigma_\Psi^2}\right\} d\left(\frac{(x-\mu)^2}{2\sigma_\Psi^2}\right) + \mu \int_{\Psi_0}^{\infty} p_\Psi(x) dx \\
&= \mu \Pr\left\{\Psi > \Psi_0\right\} + \frac{\sigma_\Psi}{\sqrt{2\pi}} \exp\left\{-\frac{(\Psi_0 - \mu)^2}{2\sigma_\Psi^2}\right\}
\end{aligned}$$

where $\sigma_\Psi^2 = \frac{R_\varepsilon}{r r_\varepsilon}$. Then

$$\mathrm{E}\left[V | \Psi > \Psi_0\right] = \mu + \frac{r_\varepsilon}{\Pr\left\{\Psi > \Psi_0\right\} R_\varepsilon} \frac{\sigma_\Psi}{\sqrt{2\pi}} \exp\left\{-\frac{(\Psi_0 - \mu)^2}{2\sigma_\Psi^2}\right\}$$

Remember that

$$\Pr\left\{\mu_\Psi < \frac{K - K_0}{p_S^H \Pr\left(\tilde{I} \leq K\right)}\right\} = \Phi_Z\left(\frac{\Psi_0 - \mu}{\sqrt{\frac{R_\varepsilon}{r r_\varepsilon}}}\right) = \Phi_Z\left(\frac{\sqrt{r R_\varepsilon}}{p_S^H \sqrt{r_\varepsilon}}\left((K - K_0) - p_S^H \Pr\left(\tilde{I} \leq K\right) \mu\right)\right)$$

and from (3) we get

$$\begin{aligned}\mathrm{E}\left[\mathrm{NPV}\right] &= \underbrace{p_S^H \Pr\left(\tilde{I} \leq K\right)\mu - K}_{\text{one-stage NPV}} + \mathrm{E}_\Psi \left\{ \Phi_Z \left(\frac{\Psi_0 - \mu}{\sqrt{\frac{R_\varepsilon}{r r_\varepsilon}}} \right) \times \right. \\ &\quad \left. \times \left[K - K_0 - p_S^H \Pr\left(\tilde{I} \leq K\right) \left(1 - \frac{\Pr\left(\tilde{I} \leq K_0\right)}{\Pr\left(\tilde{I} \leq K\right)} \right) \mathrm{E}\left(V \left| \mathrm{E}\left(V|\Psi\right) < V_C\left(K_0\right)\right.\right) \right] \right\}\end{aligned}$$

$$\begin{aligned}\mathrm{E}\left[\mathrm{NPV}\right] &= p_S^H \Pr\left(\tilde{I} \leq K\right)\mu - K + p_S^H \Pr\left(\tilde{I} \leq K\right) \frac{r_\varepsilon}{R_\varepsilon} \frac{\sigma_\Psi}{\sqrt{2\pi}} \exp\left\{ -\frac{\left(\Psi_0 - \mu\right)^2}{2\sigma_\Psi^2} \right\} \\ &\quad + \Phi_Z \left(\frac{\left(\left(\mathrm{E}\left[I\right] - K_0\right) - p_S^H \Pr\left(\tilde{I} \leq K\right)\mu \right)}{\frac{r_\varepsilon}{R_\varepsilon}\sigma_\Psi p_S^H \Pr\left(\tilde{I} \leq K\right)} \right) \left\{ \left(\mathrm{E}(I) - K_0\right) - p_S^H \mu \right\}\end{aligned}$$

■

Proof. Proposition 8: When $\alpha(1+\beta) > 1$ and $\alpha < \min\left(\frac{q_G p_S \mu}{\overline{I}}; \frac{2\left(\overline{I} - p_S\mu\right)}{\overline{I}}\right); \gamma < \frac{2}{\alpha\overline{I}}$ the project should be abandoned unless manager E is good and expression (9) becomes

$$\max_{K_0} \left\{ f(K_0)\left[q_G\left(p_S\mu - \alpha\overline{I}\right) + q_B \max\left(p_S\mu - \overline{I}; K_0\right)\right] + (1 - f(K_0)) \max\left(-K_0; q_G p_S \mu - \alpha\overline{I}\right) \right\}$$

and

$$\begin{aligned}&\max_{K_0} \left\{ f(K_0)\left[q_G\left(p_S\mu - \alpha\overline{I}\right) - q_B K_0 \right] + (1 - f(K_0)) \left[q_G p_S \mu - \alpha\overline{I} \right] \right\} \\ &q_G p_S \mu - \alpha\overline{I} + \max_{K_0} \left\{ f(K_0) q_B (\alpha\overline{I} - K_0) \right\}\end{aligned}$$

subject to

$$q_G p_S \mu - \alpha\overline{I} + \max_{K_0} \left\{ f(K_0) q_B (\alpha\overline{I} - K_0) \right\} \geq 0$$

For $K_0 < \frac{1}{\gamma}$ it is equivalent to

$$\max_{K_0} \left\{ \alpha \overline{I} K_0 - K_0^2) \right\}$$

subject to

$$q_G p_S \mu - \alpha \overline{I} + \max_{K_0} \left\{ \gamma K_0 q_B (\alpha \overline{I} - K_0) \right\} \geq 0$$

Then the result follows from (12), (13) and (14). Proof for other parts of the proposition is similar.□