Investment in Hightech Industries

An example from the LCD industry*

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Abstract

In this paper we study investments of firms in hightech industries. Typical examples are producers of consumer electronic products such as dvd players, LCD television sets, digital photo cameras, and mobile phones. There are two important characteristics in these markets. The first is (sharply) decreasing sales prices and the second is decreasing production costs. The introduction of new technologies shortens the life cycle of products. Furthermore, innovations in the production process reduce production costs and prices. The latter effect is strengthened by competition.

First we develop a standard real options investment model in which the sales price and the unit production costs per unit both follow geometric Brownian motions. However, using real life data from the LCD industry, we show that the sales price can not be described by a geometric Brownian motion. Consequently, a new real options model in discrete time is developed that fits to the real life data. Finally, the two resulting investment strategies of the models are compared.

*This is work in progress. The most recent version can be downloaded from: http://center.uvt.nl/staff/huisman/hightech.pdf

1 Introduction

In this paper we analyze the investment decision of firms in hightech industries. Typical examples of hightech industries are industries for electronic (consumer) products such as dvd players, LCD television sets, personal computers, MP3 players, photo cameras, mobile phones, and personal digital assistants. Anyone who has bought a personal computer in the past years knows that prices for personal computers have dropped very fast. After one year you could have bought the same or even a better personal computer for less money. The same holds for other products such as digital photo cameras, dvd players, and LCD television sets. Another feature of the past century is that hightech products become obsolete more quickly, i.e. the economic life of theses products becomes shorter and shorter. As an example think of the quick increase in the number of megapixels in a digital photo camera. Every new generation photo camera has more megapixels.and makes the previous generations on the one hand less attractive for the highend consumers, but on the other reachable for the more moderate consumers due to the lower prices. From a production side it is known that there is an enormous learning effect in the production of the products which leads to the fact that the production costs are decreasing over time. In general firms who produce hightech products are acting in an uncertain environment as they face sharp decreasing prices, rapid product changes, and decreasing production costs.

A lot of hightech industries face a phenomenon that is called the crystal cycle in the LCD industry (see also Mathews (2005)). In the LCD industry the capacity is limited in the sense that at this moment in time it is not possible to stop selling conventional TVs and only sell LCD TVs. When prices fall sharply a lot a consumers are attracted to the product, which rises the demand sharply. After a while the demand would exceed the available capacity which makes that firms increase their prices. Demand drops as a result of the higher prices and becomes smaller than the capacity. The firms have to lower their prices again and the next cycle starts. Mathews (2005) shows in his paper that in the period 1990 to 2003 there have been 5 cycles.

Real options theory is the appropriate tooling to analyze investment decisions under uncertainty. For an excellent introduction to the real options literature we refer to Dixit and Pindyck (1994). In most real options models uncertainty is incorporated via a geometric Brownian motion. Using real life data we test this assumption and find that price development in the LCD market does not follow a discretized geometric Brownian but an autoregressive process. From the literature of actuarial sciences we know that it is not possible to derive analytical solutions for dynamic programming models that control VAR models. Therefore, we use simulation to show the impact of the different stochastic process for the prices on the investment decision.

This paper is organized as follows. Besides this introduction there are four sections. In the second section the investment problem is analyzed with a real options model In Section 3 the model assumptions of the real options model are varified with real life data. Section 4 presents the adjusted model. The last section gives the conclusions.

2 Model

Our basic model is almost similar to the model in Dixit and Pindyck (1994, Section 6.5). Consider a firm that can undertake an irreversible investment by paying a sunk cost $I\,(>0)$. After the investment is made the firm can produce Q units of the product per time period. The price for the product at time t equals $P(t)$. Let $P(t)$ follow a geometric Brownian motion

$$dP(t) = \alpha_P P(t)\,dt + \sigma_P P(t)\,d\omega_P(t), \tag{1}$$

$$P(0) = P_0, \tag{2}$$

where $d\omega_P(t)$ is normally distributed with mean 0 and variance dt. The firm is assumed to be risk neutral and is assumed to discount with rate $r\,(>0)$.

The production costs per product are equal to $C(t)$. It is assumed that production costs also follow a geometric Brownian motion:

$$dC(t) = \alpha_C C(t)\,dt + \sigma_C C(t)\,d\omega_C(t), \tag{3}$$

$$C(0) = C_0. \tag{4}$$

We assume that $E[d\omega_P d\omega_C] = \rho dt$.

2.1 Stopping region

The profit flow of the firm after the investment is denoted by $\pi(P(t), C(t))$ and equal to

$$\pi(P(t), C(t)) = Q(P(t) - C(t)). \tag{5}$$

Denote the value of the firm in the stopping region by V, i.e. for $t \geq T$ it holds that

$$V(P(t), C(t)) = E\left[\int_{s=t}^{\infty} \pi(P(s), C(s)) \exp(-rs)\,ds\right]. \tag{6}$$

From now on we omit the time dependence of the variables as long as there is no confusion possible.

The Bellman equation that V must satisfy is given by

$$rV(P, C) = \pi(P, C) + \lim_{dt \downarrow 0} \frac{1}{dt} E[dV(P, C)]. \tag{7}$$

Expanding $E[dV(P, C)]$ with Ito's lemma gives

$$\begin{aligned} E[dV(P,C)] &= \alpha_C C \frac{\partial V(P,C)}{\partial C} dt + \alpha_P P \frac{\partial V(P,C)}{\partial P} dt \\ &+ \frac{1}{2}\sigma_C^2 C^2 \frac{\partial^2 V(P,C)}{\partial C^2} dt + \rho \sigma_C \sigma_P P C \frac{\partial^2 V(P,C)}{\partial P \partial C} dt + \frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 V(P,C)}{\partial P^2} dt. \end{aligned} \tag{8}$$

Substitution of (5) and (8) into (7) gives

$$\begin{aligned} rV(P,C) &= Q(P - C) + \alpha_C C \frac{\partial V(P,C)}{\partial C} + \alpha_P P \frac{\partial V(P,C)}{\partial P} \\ &+ \rho \sigma_C \sigma_P P C \frac{\partial^2 V(P,C)}{\partial P \partial C} + \frac{1}{2}\sigma_C^2 C^2 \frac{\partial^2 V(P,C)}{\partial C^2} + \frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 V(P,C)}{\partial P^2}. \end{aligned} \tag{9}$$

Note that doubling both the price P and the cost C will lead to a doubling of the value of the firm V. Therefore the optimal investment decision is only dependent on the ratio $\tau = \frac{P}{C}$ and the value of the firm should be homogeneous of degree 1 in (P, C), so that

$$V(P, C) = C\nu\left(\frac{P}{C}\right) = C\nu(\tau), \tag{10}$$

where $\nu(\tau)$ is now the function to be determined (see Dixit and Pindyck (1994, p. 210) for a similar argument). Differentiating (10) gives

$$\frac{\partial V(P,C)}{\partial C} = \nu(\tau) - \tau\frac{\partial \nu(\tau)}{\partial \tau}, \tag{11}$$

$$\frac{\partial V(P,C)}{\partial P} = \frac{\partial \nu(\tau)}{\partial \tau}, \tag{12}$$

$$\frac{\partial^2 V(P,C)}{\partial P \partial C} = -\frac{\tau}{C}\frac{\partial^2 \nu(\tau)}{\partial \tau^2}, \tag{13}$$

$$\frac{\partial^2 V(P,C)}{\partial C^2} = \frac{\tau^2}{C}\frac{\partial^2 \nu(\tau)}{\partial \tau^2}, \tag{14}$$

$$\frac{\partial^2 V(P,C)}{\partial P^2} = \frac{1}{C}\frac{\partial^2 \nu(\tau)}{\partial \tau^2}. \tag{15}$$

Substitution of equations (11)-(15) into equation (9) gives

$$\begin{aligned} rC\nu(\tau) &= Q(P-C) + \alpha_C C\left(\nu(\tau) - \tau\frac{\partial \nu(\tau)}{\partial \tau}\right) + \alpha_P P\frac{\partial \nu(\tau)}{\partial \tau} \\ &\quad -\rho\sigma_C\sigma_P PC\frac{\tau}{C}\frac{\partial^2 \nu(\tau)}{\partial \tau^2} + \frac{1}{2}\sigma_C^2 C^2\frac{\tau^2}{C}\frac{\partial^2 \nu(\tau)}{\partial \tau^2} + \frac{1}{2}\sigma_P^2 P^2\frac{1}{C}\frac{\partial^2 \nu(\tau)}{\partial \tau^2}. \end{aligned} \tag{16}$$

Dividing by C and rewriting gives

$$(r-\alpha_C)\nu(\tau) = Q(\tau-1) + (\alpha_P-\alpha_C)\tau\frac{\partial \nu(\tau)}{\partial \tau} + \frac{1}{2}\left(\sigma_P^2 - 2\rho\sigma_C\sigma_P + \sigma_C^2\right)\tau^2\frac{\partial^2 \nu(\tau)}{\partial \tau^2}. \tag{17}$$

The general solution of (17) is given by

$$\nu(\tau) = A_1\tau^{\beta_1} + A_2\tau^{\beta_2} + Q\left(\frac{\tau}{r-\alpha_P} - \frac{1}{r-\alpha_C}\right), \tag{18}$$

where β_1 and β_2 are the positive and negative roots of the following quadratic equation

$$\frac{1}{2}\left(\sigma_P^2 - 2\rho\sigma_C\sigma_P + \sigma_C^2\right)\beta^2 + \left(\alpha_P - \alpha_C - \frac{1}{2}\left(\sigma_P^2 - \sigma_C^2\right)\right)\beta - (r-\alpha_C) = 0. \tag{19}$$

Since $\nu(0) = 0$ and $\lim_{\tau\to\infty}\nu(\tau) = Q\left(\frac{\tau}{r-\alpha_P} - \frac{1}{r-\alpha_C}\right)$ it must hold that $A_1 = 0$ and $A_2 = 0$. The analysis above implies the following equation for $\nu(\tau)$

$$\nu(\tau) = Q\left(\frac{\tau}{r-\alpha_P} - \frac{1}{r-\alpha_C}\right). \tag{20}$$

2.2 Continuation region

The value of the option to invest is denoted by $F(P, C)$ and must satisfy the following Bellman equation

$$rF(P,C) = \lim_{dt\downarrow 0}\frac{1}{dt}E\left[dF(P,C)\right]. \tag{21}$$

Applying Ito's lemma to $E\left[dF\left(P,C\right)\right]$ and substitution of the result in (21) gives the following differential equation

$$\begin{aligned} rF\left(P,C\right) &= \alpha_C C\frac{\partial V\left(P,C\right)}{\partial C}+\alpha_P P\frac{\partial V\left(P,C\right)}{\partial P} \\ &\quad +\rho\sigma_C\sigma_P PC\frac{\partial^2 V\left(P,C\right)}{\partial P\partial C}+\frac{1}{2}\sigma_C^2 C^2\frac{\partial^2 V\left(P,C\right)}{\partial C^2}+\frac{1}{2}\sigma_P^2 P^2\frac{\partial^2 V\left(P,C\right)}{\partial P^2}. \end{aligned} \tag{22}$$

We can use the same argument as we used to derive the value in the stopping region. Thus the value of the option to invest is only dependent on the ratio $\tau=\frac{P}{C}$ and the value of the option to invest should be homogeneous of degree 1 in (P,C), so that

$$F\left(P,C\right)=C\phi\left(\frac{P}{C}\right)=C\phi\left(\tau\right), \tag{23}$$

where $\phi\left(\tau\right)$ is the function to be determined. Differentiating (23) gives

$$\frac{\partial F\left(P,C\right)}{\partial C} = \phi\left(\tau\right)-\tau\frac{\partial\phi\left(\tau\right)}{\partial\tau}, \tag{24}$$

$$\frac{\partial F\left(P,C\right)}{\partial P} = \frac{\partial\phi\left(\tau\right)}{\partial\tau}, \tag{25}$$

$$\frac{\partial^2 F\left(P,C\right)}{\partial P\partial C} = -\frac{\tau}{C}\frac{\partial^2\phi\left(\tau\right)}{\partial\tau^2}, \tag{26}$$

$$\frac{\partial^2 F\left(P,C\right)}{\partial C^2} = \frac{\tau^2}{C}\frac{\partial^2\phi\left(\tau\right)}{\partial\tau^2}, \tag{27}$$

$$\frac{\partial^2 F\left(P,C\right)}{\partial P^2} = \frac{1}{C}\frac{\partial^2\phi\left(\tau\right)}{\partial\tau^2}. \tag{28}$$

Substitution of equations (24)-(28) into equation (22), dividing by C and rewriting gives

$$\left(r-\alpha_C\right)\phi\left(\tau\right)=\left(\alpha_P-\alpha_C\right)\frac{\partial\phi\left(\tau\right)}{\partial\tau}+\frac{1}{2}\left(\sigma_P^2-2\rho\sigma_C\sigma_P+\sigma_C^2\right)\tau^2\frac{\partial^2\phi\left(\tau\right)}{\partial\tau^2}. \tag{29}$$

The general solution of equation (29) is equal to

$$\phi\left(\tau\right)=B_1\tau^{\beta_1}+B_2\tau^{\beta_2}, \tag{30}$$

where β_1 and β_2 are the positive and negative root of equation (19). The option to invest will be worthless if the price equals zero, i.e. $\phi(0)=0$, therefore it must hold that $B_2=0$.

2.3 Investment trigger

The value matching and smooth pasting conditions are given by

$$\phi\left(\tau^*\right) = \nu\left(\tau^*\right)-I, \tag{31}$$

$$\left.\frac{\partial\phi\left(\tau\right)}{\partial\tau}\right|_{\tau=\tau^*} = \left.\frac{\partial\nu\left(\tau\right)}{\partial\tau}\right|_{\tau=\tau^*}. \tag{32}$$

Substitution of (18) and (26) in (31) and (32) gives

$$B_1\tau^{\beta_1} = Q\left(\frac{\tau}{r-\alpha_P}-\frac{1}{r-\alpha_C}\right)-I, \tag{33}$$

$$\beta_1 B_1\tau^{\beta_1-1} = \frac{Q}{r-\alpha_P}. \tag{34}$$

Therefore τ^* is given by the following equation

$$\tau^* = \frac{\beta_1}{\beta_1 - 1}\left(\frac{r-\alpha_P}{r-\alpha_C} + \frac{(r-\alpha_P)\,I}{Q}\right). \tag{35}$$

We conclude that a firm should invest the first time that τ is larger than τ^*.

3 Example from the LCD Industry

We have a dataset from a firm that is active in the LCD industry. As we argued before in such an industry the typical features are decreasing production costs and even more strongly decreasing prices. Price and cost data are presented in index form. There are 24 observations in the dataset. In Figure 1 the data is plotted.

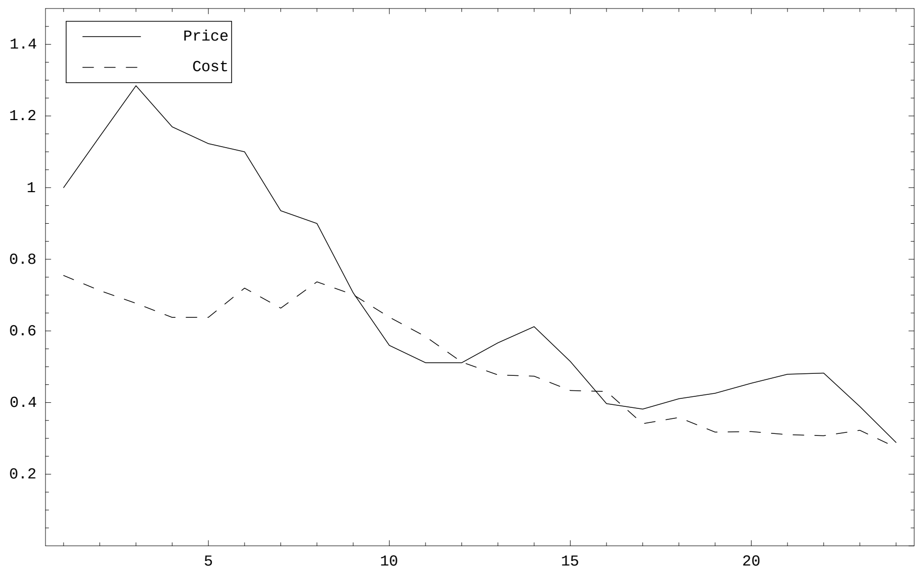


Figure 1: Price and cost data.

The literature of real options focus mostly on geometric Brownian motions. In Dixit and Pindyck (1994, p. 72) it is described that the geometric Brownian motion of equation (1) can be discretized as follows

$$\Delta P(t) = \alpha_P P(t) + \sigma_P P(t)\,\omega_P(t).$$

Rewriting gives

$$\frac{\Delta P(t)}{P(t)} = \alpha_P + \sigma_P \omega_P(t). \tag{36}$$

Similarly, we derive the discrete version for the production cost

$$\frac{\Delta C(t)}{C(t)} = \alpha_C + \sigma_C \omega_C(t). \tag{37}$$

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.039937	0.016385	-2.437475	0.0233
R-squared	0.000000	Mean dependent var	-0.039937	
Adjusted R-squared	0.000000	S.D. dependent var	0.078578	
S.E. of regression	0.078578	Akaike info criterion	-2.206948	
Sum squared resid	0.135839	Schwarz criterion	-2.157579	
Log likelihood	26.37990	Durbin-Watson stat	2.486781	

Table 1: Estimations for growth rate of production costs.

3.1 Estimation of parameters

First we estimate the parameters of the costs process that is described in equation (37) and after that we estimate the parameters of the price process of equation (36).

3.1.1 Costs

In Figure 2 the relatively cost changes are plotted.

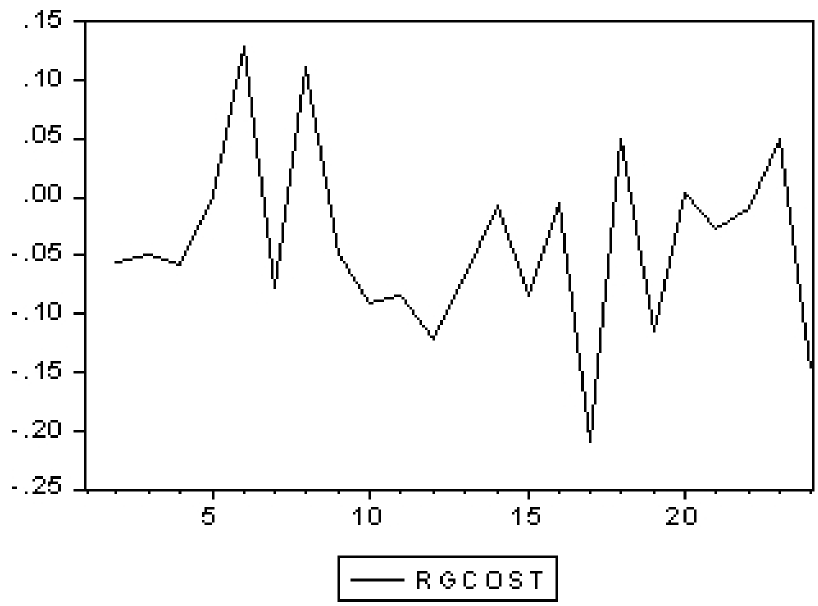
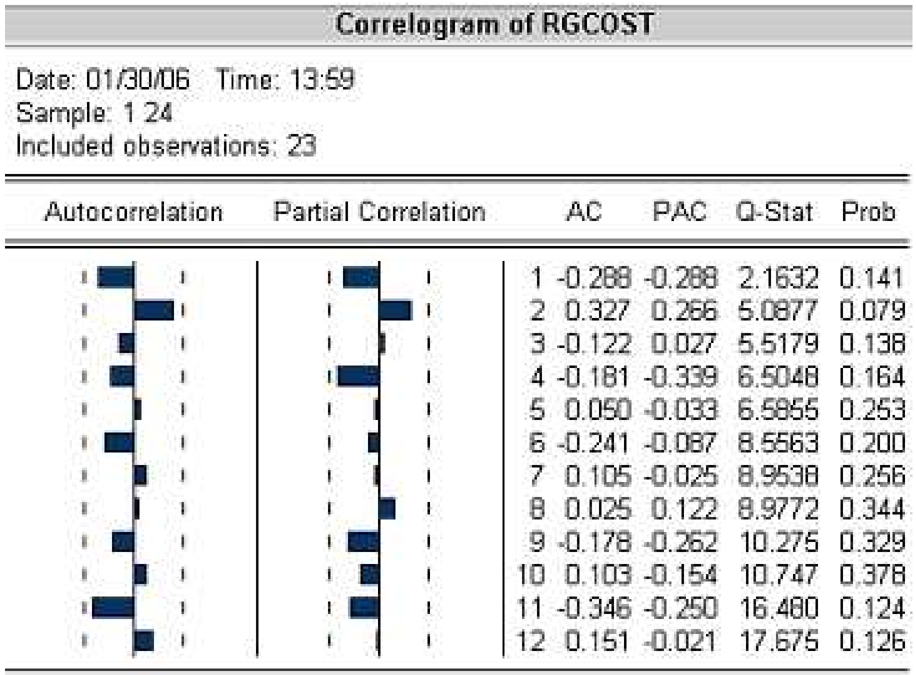


Figure 2: Growth rate of production costs.

The results of estimating (37) are in Table 1. From which it is clear that the constant relatively cost decrease is equal to 3.99% with a standard error of 1.64%.

From Figure 3 it is clear that there is no (partial) autocorrelation left, so that the costs of the hightech product follow a type of a random walk or a discrete geometric Brownian motion.



Correlogram of RGCOST

Date: 01/30/06 Time: 13:59
Sample: 1 24
Included observations: 23

	AC	PAC	Q-Stat	Prob
1	-0.288	-0.288	2.1632	0.141
2	0.327	0.266	5.0877	0.079
3	-0.122	0.027	5.5179	0.138
4	-0.181	-0.339	6.5048	0.164
5	0.050	-0.033	6.5855	0.253
6	-0.241	-0.087	8.5563	0.200
7	0.105	-0.025	8.9538	0.256
8	0.025	0.122	8.9772	0.344
9	-0.178	-0.262	10.275	0.329
10	0.103	-0.154	10.747	0.378
11	-0.346	-0.250	16.480	0.124
12	0.151	-0.021	17.675	0.126

Figure 3: Correlogram of the growth rate of production costs.

3.1.2 Price

The relatively price changes are plotted in Figure 5.

Inspecting the correlogram of the inflation rates as displayed in Figure 6. We observe that there two significant partial autocorrelations with a demping sinusodual autocorrelation function. So that a second order autocorrelation model seems to be appropriate.

Estimating results of these AR(2) are presented in Table 2. From this table it is clear that both autoregressive terms are significant:

$$\frac{\widehat{\Delta P\,(t)}}{P\,(t)} = -\underset{(0.0270)}{0.057} + \underset{(0.214)}{0.642}\frac{\Delta P\,(t-1)}{P\,(t-1)} - \underset{(0.212)}{0.458}\frac{\Delta P\,(t-2)}{P\,(t-2)} \text{............R}^2\text{=0.355, DW=1.96.} \qquad (38)$$

From the standard errors within brackets under the estimated parameters in equartion (38) it is clear that all parameters are significant at the 5% level, while the Durbin Watson test statistic of 1.96 gives an indication of no overall autocorrelation of the remaining residuals. This observation is confirmed while inspecting the residuals in more detail in Figure 6. There is no significant residual (partial) autocorrelation left. Therefore equation (38) can be considered as the final characterization of the inflation rates of the hightech product. This implies a considerable deviation from the discrete geometric Brownian motion in (36).

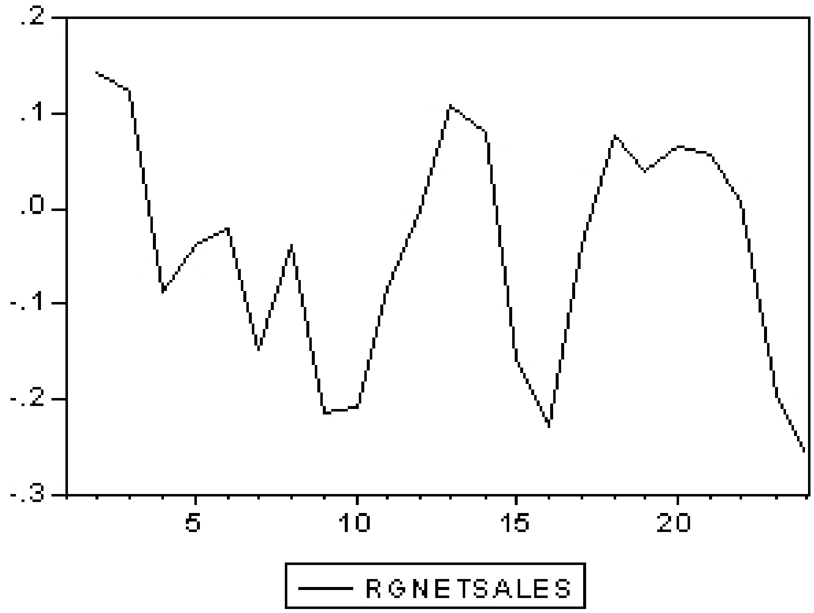
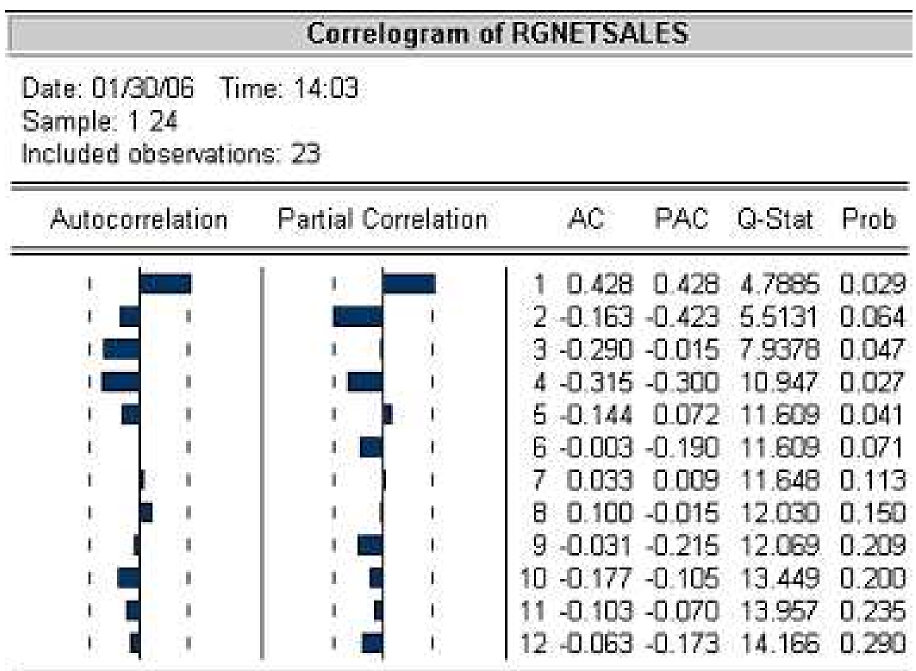


Figure 4: Growth rate of price.



Correlogram of RGNETSALES

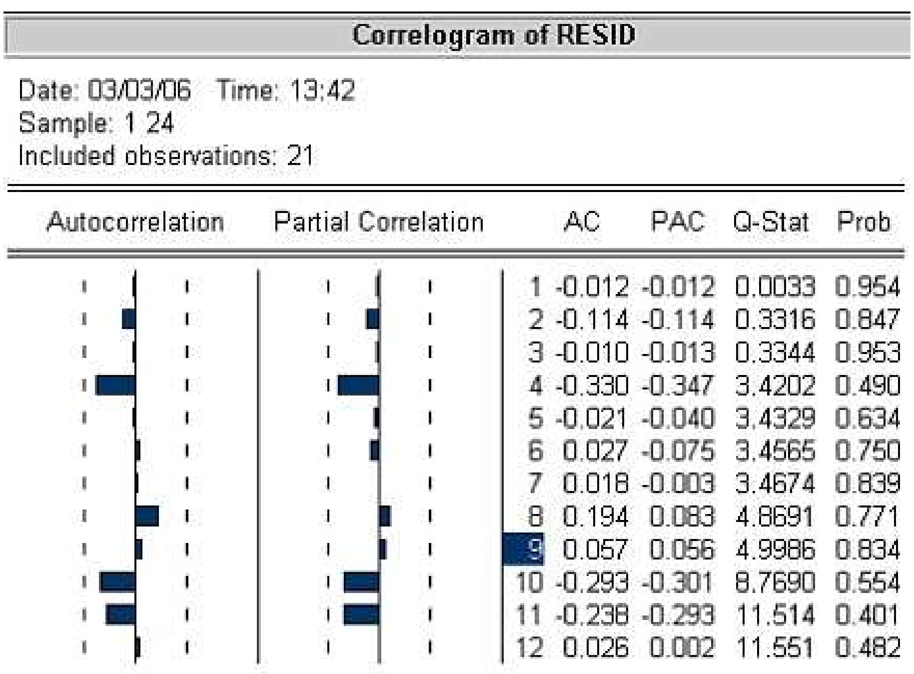
Date: 01/30/06 Time: 14:03
Sample: 1 24
Included observations: 23

	AC	PAC	Q-Stat	Prob
1	0.428	0.428	4.7885	0.029
2	-0.163	-0.423	5.5131	0.064
3	-0.290	-0.015	7.9378	0.047
4	-0.315	-0.300	10.947	0.027
5	-0.144	0.072	11.609	0.041
6	-0.003	-0.190	11.609	0.071
7	0.033	0.009	11.648	0.113
8	0.100	-0.015	12.030	0.150
9	-0.031	-0.215	12.069	0.209
10	-0.177	-0.105	13.449	0.200
11	-0.103	-0.070	13.957	0.235
12	-0.063	-0.173	14.166	0.290

Figure 5: Correlogram of the growth rate of price.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.056766	0.026998	-2.102569	0.0498
AR(1)	0.641629	0.214374	2.993035	0.0078
AR(2)	-0.458357	0.211572	-2.166436	0.0439
R-squared	0.354810	Mean dependent var		-0.061677
Adjusted R-squared	0.283123	S.D. dependent var		0.115440
S.E. of regression	0.097742	Akaike info criterion		-1.681411
Sum squared resid	0.171962	Schwarz criterion		-1.532194
Log likelihood	20.65482	F-statistic		4.949389
Durbin-Watson stat	1.963321	Prob(F-statistic)		0.019373
Inverted AR Roots	.32+.60i	.32 -.60i		

Table 2: Estimations for growth rate of price.



Correlogram of RESID

Date: 03/03/06 Time: 13:42
Sample: 1 24
Included observations: 21

	AC	PAC	Q-Stat	Prob
1	-0.012	-0.012	0.0033	0.954
2	-0.114	-0.114	0.3316	0.847
3	-0.010	-0.013	0.3344	0.953
4	-0.330	-0.347	3.4202	0.490
5	-0.021	-0.040	3.4329	0.634
6	0.027	-0.075	3.4565	0.750
7	0.018	-0.003	3.4674	0.839
8	0.194	0.083	4.8691	0.771
9	0.057	0.056	4.9986	0.834
10	-0.293	-0.301	8.7690	0.554
11	-0.238	-0.293	11.514	0.401
12	0.026	0.002	11.551	0.482

Figure 6: Correlogram of the residuals of the growth rate of price.

4 Adjusted Model

[Simulation]

5 Conclusion

Using real life data we have shown that the assumption that the price development follows a geometric Brownian motion is not correct. In fact it turns out that the price development follows a second order autoregressive process. Using simulations we have shown the impact of the different price process on the investment timing.

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Mathews, J. A. (2005). Corporate strategy - strategy and the crystal cycle. *California Management Review*, 47, 6–32.