Industry dynamics and limit pricing under uncertainty

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Abstract

This paper studies a model of entry deterrence with limit pricing in continuous time and in the presence of market uncertainty. Strategic considerations are richer than in the standard two-period model. Entry deterring limit pricing is only possible within lower and upper bounds of the market size and if the incumbent is stronger than the potential entrant. Our model also predicts that higher market uncertainty induces higher incidence of entry deterrence. Additionally, the model provides a new framework in which other continuous-time signaling games may be analyzed.

1 Introduction

In this paper we study a model of a monopoly firm using limit pricing to deter entry into the market. Information regarding the incumbent's cost is asymmetric like in Milgrom and Roberts (1982). When threatened by entry, the weak incumbent firm may, by setting low prices, pretend to be a strong one and thus prevent the entrant firm from entering. Unlike in the existing literature, in our model time is continuous and the market evolves stochastically.

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Remarkably, a large body of the non-game-theoretic literature on limit pricing preceding Milgrom and Roberts (1982) focused on dynamics and uncertainty (see, for instance, Kamien and Schwartz (1971), Gaskins (1971) and Flaherty (1980)). The theory was followed by empirical research testing industry dynamics predictions of limit pricing (Masson and Shaanan (1982)). The game theoretic revolution, which to a large extent invalidated those previous explanations of limit pricing, left us with a rational and convincing limit pricing story, but essentially a two-period and deterministic one. The aim of this paper is to extend limit pricing to a fully dynamic and stochastic setting and to study how limit pricing stemming from asymmetric information shapes industry dynamics.

The contributions of this paper can be seen from two angles. Firstly, the paper extends the literature on limit pricing by showing some results unavailable in two-period deterministic models. Secondly, the model provides a novel framework to study signaling games in continuous time and a stochastic environment.

The dynamic and stochastic setting makes the strategic considerations richer than in the two-period models. The decisions of the entrant to enter and of the incumbent to disengage from entry-deterring limit pricing resemble option exercise decisions. Moreover, both the decisions involve strategic incentives. An outcome of these strategic timing decisions are the bounds of the market size within which entry deterrence occurs. At the lower bound the incumbent ceases entry-deterring limit pricing practices and at the upper bound the entrant enters.

We identify two plausible equilibrium paths. Each of them arises for different parameter values. Remarkably, entry deterrence by limit pricing may occur only if the potential entrant is weaker than the incumbent. This result is at odds with what is usually inferred from the two-period deterministic models. Moreover, on the equilibrium path with entry deterrence, market structure exhibits history dependence, in the sense that timing of entry depends on the past evolution of the market demand.

Another interesting prediction of our model is that increased uncertainty induces higher incidence of entry deterrence. This is caused by the embedded options created by market uncertainty, which become more valuable in a more uncertain environment. At this point, the model contradicts the well-know result of Maskin (1999) that uncertainty reduces incidence of entry deterrence. Admittedly, the model of entry deterrence in Maskin (1999) differs from ours, nevertheless the contrasting result remains remarkable.

From a methodological point of view, our paper contributes to the literature on games in continuous time first developed by Fudenberg and Tirole (1985) and Simon and Stinchcombe (1989). In particular the model belongs to an important class of games with an underlying Brownian motion. The contract theory in continuous time (Holmstrom and Milgrom (1987), Sannikov (2006) and others) is an example of a growing literature which benefits from richness and tractability of the continuous time framework with Brownian stochastics. Another related literature concerns strategic investment decisions and strategic technology adoption in a real options framework (Grenadier (1996), Grenadier (2002), Lambrecht and Perraudin (2003), Murto (2004) and others). Despite some similarities, neither of these previous models have analyzed a signaling game.

The signaling model in continuous time is naturally linked to repeated signaling games in discrete time. A signaling model closest to our analysis is Kaya (2005). She studies separating equilibria in a (infinitely) repeated discrete signaling game. Some of the notions studied there in her general model reappear in our analysis, however our continuous and stochastic environment is richer and brings different consideration.

Timing of decisions to enter and to deviate adds another dimension to the signaling model. The timing game is methodologically related to the literature on stopping time games and option exercise games. The strategies and equilibrium notions are similar to Dutta and Rustichini (1995). The combination of strategic incentives and asymmetric information resembles that of Lambrecht and Perraudin (2003) and Morellec and Zhdanov (2005). In the present paper, however, by the nature of the limit pricing problem, the structure of asymmetric information is different.

The remainder of the paper is organized as follows. Section 2 introduces the model and presents the full information benchmark case. Section 3 describes strategies, possible equilibria and criteria to select among these equilibria. Section 4 derives best timing responses of both player in the pooling (entry deterring) equilibrium. In Section 5 we describe the equilibrium paths and discuss their implications for price and market dynamics. Impact of model parameters on the incidence of entry deterrence is further analyzed in Section 6. Section 7 contains some brief concluding remarks.

2 Model setup

2.1 Setting

The incumbent firm, denoted by index 1, operates already in the market. Its profits depend on four factors. Firstly, the profitability of the whole market evolves with a stochastic state variable Y following a geometric Brownian motion. Secondly, denoting the incumbent's cost type by $\theta \in \Theta = \{L, H\}$, the incumbent's technology may be of low marginal cost C_1^L or high cost C_1^H per unit of time[1], $C_1^L < C_1^H$. Thirdly, the incumbent may choose other than its monopoly price or quantity to imitate the behavior of another

[1] We concentrate our analysis on the two cost type case like in Bagwell and Ramey (1988) and Bagwell (2005). The main results are not limited to this case. An alternative assumption of, for instance, continuum of types would prolifterate considerations unessential for the main points of the paper.

cost type (a signaling rationale for such a behavior is explained later). And lastly, profits depend on the presence of the entrant firm (firm 2). The entrant's marginal cost C_2 is known with certainty. We assume that upon entry the two firms compete in quantities in the Cournot fashion. Below we show that these requirements on the profit flow function can be effectively captured by choosing an appropriate multiplicative constant for a stochastic state variable.

We assume a relatively general family of stochastic constant-elasticity demand functions, of which the inverse demand function at total output Q at time t is given by

$$P_t(Q) = Y_t Q^{-\frac{1}{\gamma}}. \tag{1}$$

$Y = \{Y_t : t \geq 0\}$ is a stochastic state variable following a geometric Brownian motion with drift μ_Y, volatility σ_Y and a standard Brownian motion W. Y is defined on a complete probability space $(\Omega, \mathcal{F}, P)$ and adopted to $\mathcal{F}_t$. γ is the demand elasticity and we assume that $\gamma > 1$ for the marginal monopoly profits to be increasing in Y.

The monopolist's optimal quantity choice q_M^θ at each state Y given its constant marginal cost C_1^θ is

$$q_M^\theta(Y_t) = \left(\frac{\gamma - 1}{\gamma}\frac{Y_t}{C_1^\theta}\right)^\gamma.$$

So its monopoly profit is

$$\pi_M^\theta(Y_t) = (Y_t)^\gamma \frac{C_1^\theta}{\gamma - 1}\left(\frac{\gamma C_1^\theta}{\gamma - 1}\right)^{-\gamma}. \tag{2}$$

Define now a new variable $X_t = f(Y_t) = (Y_t)^\gamma$. By Itô's lemma

$$dX_t = f' dY_t + \frac{1}{2} f'' dY_t^2 = \mu_X X_t dt + \sigma_X X_t dW_t,$$

where

$$\begin{aligned} \mu_X &= \gamma \mu_Y + \frac{1}{2}\gamma(\gamma - 1)\sigma_Y^2, \\ \sigma_X &= \gamma \sigma_Y. \end{aligned}$$

are constants and f' and f'' denote the first and second order derivatives. Therefore, X is also a geometric Brownian motion adopted to $\mathcal{F}_t$ with appropriately adjusted drift and volatility parameters. For notational convenience, in the following we suppress the subscripts on drift μ_X and volatility σ_X of X.

Note that $\frac{C_1^\theta}{\gamma-1}\left(\frac{\gamma C_1^\theta}{\gamma-1}\right)^{-\gamma}$ in (2) is constant over time, thus we conclude that with properly chosen parameters the profit flow in (2) may be expressed as a constant times a geometric Brownian motion. A similar equivalence can be shown for profit flows under duopolistic competition and monopolist's choice of another cost type's quantity. Hence, depending on the incumbent's true cost type, the cost type it imitates and the presence of the entrant, the incumbent's profit flow π_1 at time t is (to keep the notation parsimonious we do not explicitly write down these arguments in the profit function)

$$\pi_1(X_t) = \Pi_1 X_t. \tag{3}$$

Π_1 is a constant equal to either of

$$M^\theta = \frac{C_1^\theta}{\gamma - 1}\left(\frac{\gamma C_1^\theta}{\gamma - 1}\right)^{-\gamma}, \tag{4}$$

$$M^\theta(\tilde{C}) = \frac{\gamma\tilde{C} - (\gamma - 1)C_1^\theta}{\gamma - 1}\left(\frac{\gamma\tilde{C}}{\gamma - 1}\right)^{-\gamma}, \tag{5}$$

$$D_1^\theta = \begin{cases} \frac{\left[\gamma C_2 - (\gamma-1)C_1^\theta\right]^2}{(2\gamma-1)\left(C_1^\theta + C_2\right)}\left[\frac{\gamma\left(C_1^\theta + C_2\right)}{2\gamma-1}\right]^{-\gamma} & \text{if } C_1^\theta < \frac{\gamma}{\gamma-1}C_2, \\ 0 & \text{otherwise,} \end{cases} \tag{6}$$

depending on whether the incumbent is a monopolist choosing its monopoly strategy (M^θ), or if the incumbent is a monopolist imitating the monopoly strategy of a firm with marginal cost $\tilde{C}$ ($M^\theta(\tilde{C})$),[2] or if the incumbent firm operates in a duopoly (D_1^θ). It follows that $M^\theta > D_1^\theta \geq 0$, $M^L > M^H$, $M^\theta > M^\theta(\tilde{C})$ and $D_1^L > D_1^H$ for all θ and all $\tilde{C} \neq C^\theta$. From (5) it follows that a θ-type incumbent makes negative profit if it imitates the behavior of an incumbent with marginal cost lower than $\frac{\gamma-1}{\gamma}C_1^\theta$. Equation (6) says that θ-type incumbent is out of the market after entry if the entrant's cost is less than $\frac{\gamma-1}{\gamma}C_1^\theta$.

The incumbent's type is known to the incumbent firm itself but at the initial point of time the potential entrant does not know it. The prior probability that $\theta = H$ is η_0 and is known to the entrant. Upon entry firm 2 pays the entry cost of I and learns the cost type of the incumbent (the assumption that the cost uncertainty is resolved is made for simplicity; the entry cost may include a necessary market research cost). When the entrant enters the market its profits are affected by the costs level of the incumbent. Given that firm 1 is of θ type, firms 2's profit flow after entry is

$$\pi_2^\theta(X_t) = D_2^\theta X_t, \tag{7}$$

where

$$D_2^\theta = \begin{cases} \frac{\left[\gamma C_1^\theta - (\gamma-1)C_2\right]^2}{(2\gamma-1)\left(C_1^\theta + C_2\right)}\left[\frac{\gamma\left(C_1^\theta + C_2\right)}{2\gamma-1}\right]^{-\gamma} & \text{if } C_2 < \frac{\gamma}{\gamma-1}C_1^\theta, \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

[2] Precisely, $M^\theta(\tilde{C})$ is not necessary constant over time as $\tilde{C}$ may vary, but it is fixed for a given $\tilde{C}$.

The lower is the incumbent's cost the less profitable is the entry, that is $D_2^H > D_2^L$. If the entrant knows the incumbent's type to be θ and $C_2 \geq \frac{\gamma}{\gamma-1} C_1^{\theta}$, then the entrant cannot make positive profits and never enters.

2.2 Full-information benchmark

If the cost type of the incumbent is known then the incumbent cannot effectively use pricing to deter entry. Irrespective of its pricing policy before entry, the incumbent behaves optimally after entry, so that low prices before entry do not have commitment value necessary to discourage entrants.

In this situation, the entry decision is like in the real options models of entry or irreversible investment. It is standard to derive, see the appendix, that the entrant enters the market with the incumbent of type θ as soon as X_t is at or above

$$X_E^{\theta} = \frac{\beta_1}{\beta_1 - 1} \frac{r - \mu}{D_2^{\theta}} I, \tag{9}$$

where

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1,$$

is the positive root of the characteristic equation $\frac{1}{2}\sigma^2 \beta (\beta - 1) + \mu\beta - r = 0$. The formula implies that $X_E^L > X_E^H$. Denote the first passage times to these levels of the process at state X_0 by T_E^L and T_E^H, respectively.

3 Strategies and equilibria

3.1 Strategies

After entry, firms compete under symmetric information and their profit flows under optimal behavior are derived in (6) and (8). These duopolistic profits can be considered as termination payoffs the game before entry. From this point of view, the entrant makes only a single decision when to enter. The incumbent can use its pricing policy to signal its type. For example, similarly to the one-shot limit pricing model, the high cost incumbent may imitate the low cost type to deter entry or, alternatively, the low cost type may want to credibly signal its strong position by setting low prices. Moreover, the incumbent may (under unfavorable circumstances) deviate from a particular signaling strategy. In particular, the high cost type may cease its entry deterring strategy if the market becomes too small.

To facilitate a more formal analysis of firms' strategies, we introduce some further restrictions and notions. First of all, for most of the paper we concentrate on pure strategies, that is we do not allow randomization over stopping times and cost types. Mixed strategies considerably complicate the analysis but may be of importance to the model and we discuss them in Section 5.2. Secondly, we would like to consider Markov strategies, i.e. strategies which depend only on the current state. But the entrant's strategy at t will depend on the current X_t and its belief w.r.t. the incumbent's type η_t, which in turn depends on past X and past incumbent's behavior. Thus, to make the strategies Markovian, the model's state space is augmented by the Markov process $\{\eta_t \in [0,1] : t \geq 0\}$, which is the entrant's belief accordingly updated using Bayes rule whenever possible.

First, we consider the entrant's strategic decision. As known from the literature on control of Brownian motion, the entry decision will take a form of an entry trigger in X. Formally, the strategy of firm 2 is $X_E : [0,1] \to \mathbb{R}_+$, such that for each belief about the incumbent's type it is optimal to enter if the market variable has a value in $[X_E(\eta), \infty)$.[3] Note that in the current notation the full information triggers are $X_E^L \equiv X_E(0)$ and $X_E^H \equiv X_E(1)$.

The incumbent's decision at each instant in the game before entry is to choose a cost type $\tilde{C}_t$. The incumbent's chosen cost level process $\tilde{C} = \{\tilde{C}_t \in \mathbb{R}_+ : t \geq 0\}$ is adapted to $\mathcal{F}_t$. $\tilde{C}$ can be the incumbent's own true cost type, and then $\tilde{C} = C_1^\theta$ is constant. At each t, $\tilde{C}_t$ can be also another fictitious cost type. Given $\tilde{C}$, the incumbent selects prices corresponding to the optimal decision of a monopolist with marginal cost $\tilde{C}_t$. The entrant observes market prices and X_t and can infer $\tilde{C}_t$. The strategy $\tilde{C}$ serves as a signaling devise in a manner similar to the one-shot limit pricing models.

As explained in detail below in section 3.4, the high cost incumbent may want to take a 'deviating action' at low levels of X, which terminates the signaling game. This timing decision makes the second dimension of the incumbent's strategy. The termination strategy denoted $X_D \in \mathbb{R}_+$ is such that the incumbent takes a discrete action in $\tilde{C}$ that terminates the signaling game if X_t is in $[0, X_D]$.

We conclude this discussion with a formal definition of the strategy space.

Definition 1 *In the state space $\{X, \eta\}$, the Markov strategy profile of both firms is defined as $\{X_E, X_D, \tilde{C}\}$.*

3.2 Equilibria types and equilibrium selection

3.2.1 Signaling game

Under asymmetric information, when the entrant does not know the incumbent's cost, there exist two types of potential pure-strategy equilibria. In a separating equilibrium

[3] See also Section 3.2.2 for a discussion on the validity of trigger strategies of both the entrant and the incumbent.

types choose different prices which credibly convey information about the incumbent type. The true types are revealed to the entrant. In a pooling equilibrium both the incumbent types set the same price and no cost information is revealed.

Already the simple two-period signaling games suffer from a great multiplicity of equilibria. In the standard two-period limit pricing model corresponding to our setting, typically there is a continuum of separating equilibria and, if existent at all, a continuum of pooling equilibria. Some of the equilibria that are subgame perfect are still not plausible. A number of approaches have been taken in the literature to refine the set of equilibria i.e. to select the most plausible ones. The intuitive criterion of Cho and Kreps (1987) requires that, for any out-of-equilibrium price, the entrant puts zero weight on the incumbent type that is sure to lose by this choice compared to the equilibrium price, even when followed by the most favorable response of the entrant. This criterion leads to a unique separating equilibrium at the minimum level of signaling. Such a separation equilibrium is also called the Riley outcome after Riley (1979).

In dynamic games it is typically difficult to apply the intuitive criterion or other refinements that restrict in some way off-equilibrium-path beliefs. Especially in a continuous-time game the set of possible out-of-equilibrium paths is extremely large and it may be a daunting task to analyze reasonability of beliefs associated to these paths. Recently, Kaya (2005) proposed to use the least cost criterion for selecting plausible separating equilibria. While not as formal as the usual restrictions on beliefs, the least cost separation is in its spirit similar to the intuitive criterion. The strong player selects the cheapest sequence of actions that would not be selected by the weak player even under a most favorable response from the uninformed player. If the strong player strictly prefers such a deviation when it is believed to be the strong type, then indeed the uninformed player puts all the probability weight on the strong type. In the one-shot signaling game, the least cost separation criterion leads to a unique least cost separating equilibrium (LCSE) in the Riley outcome. However, as shown by Kaya (2005), in dynamic settings the LCSEs may be easier to analyze and also may lead to different separating outcomes than a repeated version of the Riley outcome. We take advantage of the practical convenience of the least cost criterion in continuous time and find a unique LCSE in Section 3.3.

The intuitive criterion does not succeed in dismissing some of the less plausible pooling equilibria in the one-shot model. Neither do related refinement notions like the D1 criterion (Cho and Kreps (1987), Cho and Sobel (1990)). Of the continuum of pooling equilibria, the one at the unconstraint monopoly prices of the strong type is particularly plausible. This is a pooling outcome, which is Pareto efficient from the point of view of active players. The special interest in this equilibrium may be also justified by the fact that it is the pooling equilibrium that can arise for the largest set of model parameters. So when studying implications of the model for incidences of entry deterrence, the efficient pooling equilibrium constitutes a relevant upper bound case. This simple selection criterion for a unique pooling equilibrium is also readily applicable to the continuous-time model.

By making the choice to focus on the LCSE and the efficient pooling equilibrium, in Section 5 we will be able to identify unique equilibrium paths on X.

3.2.2 Stopping game

The game with one player taking an action at low X and the other at high X is closely related to the stochastic game of Dutta and Rustichini (1995). Two-sided triggers form a Markov perfect equilibrium of such a game. In fact, the game of Dutta and Rustichini (1995) has also more convoluted equilibria in the form of alternating intervals on X; see Murto (2004) for an application of such strategies in a model of exit in duopoly. In this paper we concentrate on the simplest strategies of two-sided triggers as defined in section 3.1. The equilibrium path of the signaling game is played undisturbedly if the market stochastic process remains within (X_D, X_E). If X reaches either of the triggers the signaling game is over.

3.3 Separation

The criterion of least cost separation states that the active player effectively separates from the other type while minimizing its incurred costs. In our model it means that the low cost type firm sets prices that yield highest profits and induce the entrant to enter at X_E^L.

The next proposition characterizes the LCSE.

Proposition 2 *In the least cost separating equilibrium, the low cost incumbent sets its unconstraint monopoly prices if X_t is below $X_E(\eta_t)$ and plays a constant fictitious cost type $\tilde{C}^{sep} = \min\{C_1^L, \{\tilde{C} : M^H(\tilde{C}) = D_1^H\}\}$ if X_t is above $X_E(\eta_t)$.*
Proof. *See the appendix.* ■

The LCSE is a continuous-time equivalent of the Riley outcome. It is important to note that in fact the LCSE in Proposition 2 is not fully separating, in the sense that the H type is not necessary completely separated. The belief η_t may be in $[0, 1]$ not only $\{0, 1\}$. But two important intuitive conditions are satisfied. First, the player which makes effort to separate, takes an action only when directly threatened by entry. An alternative behavior of the strong type engaging in costly separation from the beginning of the game, with no prospect of an entry, would be counter-intuitive in a stochastic and dynamic game. Second, the entrant (making plausible inferences in the style of the intuitive criterion) has to believe that the actions along the LCSE are credibly differentiating the strong player.

$\tilde{C}^{sep}$ defined in Proposition 2 represents the highest cost type that the weak incumbent will not want to imitate. So the incentive compatibility constraint for the H-type incumbent is satisfied by construction. For the separating equilibrium defined in Proposition 2 to occur, an additional incentive compatibility constraint for the L-type must also be satisfied. Define $T_E \equiv \inf [t \geq 0 : X_t \geq X_E(\eta_t)]$ the time the entrant would enter without a separating action of the incumbent. Then the low cost incumbent separates if

$$E\left[\int_{T_E}^{T_E^L} e^{-r(\tau - T_E)} M^L(\tilde{C}^{sep}) X_\tau d\tau\right] > E\left[\int_{T_E}^{T_E^L} e^{-r(\tau - T_E)} D_1^L X_\tau d\tau\right], \qquad (10)$$

which is clearly equivalent to $M^L(\tilde{C}^{sep}) > D_1^L$. Conversely, the separation does not occur if $M^L(\tilde{C}^{sep}) \leq D_1^L$. The last condition is satisfied if either D_1^L is high, that is if the potential entrant is weak and its entrance does not significantly affect the strong incumbent, or if $M^L(\tilde{C}^{sep})$ is low, that is if it is costly to separate or, equivalently, if the difference between the strong and the weak type is small.

3.4 Pooling

Under a pooling equilibrium the incumbent sets one price by choosing one fictitious cost process $\tilde{C}^{pool}$ irrespective of its type. No information is revealed about the incumbent cost level and such mimicry allows the weaker firm to deter entry. To conceal its type the high cost incumbent needs to decrease its price below its monopoly price. If a pooling equilibrium is possible (the necessary conditions are discussed below), then usually many $\tilde{C}$'s (including a continuum of constant $\tilde{C}$ and time varying $\tilde{C}$'s) can constitute equilibrium signals. Similarly to the two period games, the Pareto efficient pooling equilibrium from the incumbent point of view seems the most plausible outcome. Restricting our attention to this case, the pooling equilibrium prices are at the level of the low cost firm unconstraint monopoly prices, i.e. $\tilde{C}^{pool} = C_1^L$. Let $M^{H,pool} \equiv M^H(C_1^L)$.

If the entrant cannot observe the cost level of the incumbent, it will invest according to its beliefs in this respect. As we show later, the entrant's decision to enter depends also on strategic considerations and the determination of the entry trigger level of X is not as straightforward as in the full information case. However, it is clear that entry will not occur later than at T_E^L (as under the worst situation for the entrant) and not earlier than at T_E^H (the best case for the entrant). We denote the trigger level of X that induces entry in a pooling equilibrium played from the beginning of the game by $X_E^{pool} \equiv X_E(\eta_0)$ ($X_E^L \geq X_E^{pool} \geq X_E^H$) and the first passage time to X_E^{pool} by T_E^{pool}.

For the pooling price to conceal the information of the cost type, it needs to be played from the beginning of the market game, that is from $t = 0$. The pooling makes sense for the incumbent if it does not provoke immediate investment, that is $X_0 < X_E^{pool}$. Moreover, none of the incumbent types can find incentives to deviate from the equilibrium price. The incentive compatibility constraint for the high cost firm that must hold for any X_t is

$$\begin{aligned} & E\left[\int_t^{T_E^{pool}} e^{-r(\tau-t)} M^{H,pool} X_\tau d\tau + \int_{T_E^{pool}}^{\infty} e^{-r(\tau-t)} D_1^H X_\tau d\tau\right] \\ \geq \; & E\left[\int_t^{T_E^H} e^{-r(\tau-t)} M^H X_\tau d\tau + \int_{T_E^H}^{\infty} e^{-r(\tau-t)} D_1^H X_\tau d\tau\right]. \end{aligned} \tag{11}$$

In the pooling equilibrium, the high cost firm trades the monopoly profit flow $M^H X_t$ sustained over a relatively shorter time T_E^H for a lower profit flow $M^{H,pool} X_t$ but sustained over a longer time T_E^{pool}. The inequality states that if the expected present value of the former flow is higher or equal than of the latter flow, then the high cost incumbent prefers the pooling outcome over a deviation to its unconstraint monopoly prices. Constraint (11) indicates that a minimum condition for the pooling outcome is $M^{H,pool} > D_1^H$. If this is not satisfied then no pooling is possible at any level of X. If $M^{H,pool} > D_1^H$, then (11) holds for some interval on X. It can be seen from the formula that once the incumbent opted for pooling, it will not find it profitable to deviate if X is increasing. However, if X falls sufficiently, (11) may stop to hold. In other words, if the market is low enough and the distance to X_E^H is large then the incumbent may prefer to obtain its full monopoly profits over this (long) period until X_E^H is reached. Let X_D denote the level of X that triggers deviation of the high cost incumbent from the pooling prices to its unconstraint monopoly price.

The incentive compatibility constraint (11) brings clearly the idea that there exists a deviation trigger X_D, however it conveys a naïve decision rule. The decision to deviate from the pooling equilibrium is irreversible (the type is revealed) and involves an exchange of one profit flow for another, and as such it resembles an option exercise decision. Real options theory suggests that the embedded value of waiting leads to a postponement of the deviation decision compared to the rule indicated by (11). Moreover, when determining X_D, the incumbent must take into account its strategic impact on the behavior of the entrant. Timing of the decision to deviate influences chances of learning the true cost type and thus the entrant's decision to enter. Similarly, the entrant's choice of the entry trigger affects profitability of the incumbent's deviation and thus its timing. These strategic optimal timing decisions are analyzed in Section 4.

In the Pareto efficient pooling equilibrium, prices equal the unconstrained monopoly prices of the strong type incumbent. Therefore, if there is no immediate entry threat, the low cost firm has no incentives to deviate at any point below X_E^{pool}. At (or just before) X_E^{pool} it would consider separating from the high cost firm to signal that the entrant should not enter in the way described in Proposition 2. Its decision whether to separate or not depends on relative profitability of the alternatives.

3.5 Marginal costs and signaling outcomes

Various realizations of marginal costs C_1^H, C_1^L and C_2 may lead to different market behaviors. In this section we identify a number of distinctive cases when necessary conditions for separating and pooling outcomes are satisfied. The cases are depicted in Figure 1, with C_1^L on the horizontal axis and C_1^H on the vertical axis.

As discussed before, the necessary condition for a pooling outcome is $M^{H,pool} > D_1^H$. A weaker condition is $M^{H,pool} > 0$, which is equivalent to $C_1^H < \frac{\gamma}{\gamma-1}C_1^L$. It simply means that the difference between the incumbents costs cannot be too large (recall that $\gamma > 1$). In Figure 1, this condition is satisfied in the wedge between the two diagonals $C_1^H = \frac{\gamma}{\gamma-1}C_1^L$ and $C_1^H = C_1^L$. $M^{H,pool} > D_1^H$ is satisfied if, on the one hand, $M^{H,pool}$ is large or, equivalently, it is not too costly to pool and if, on the other side, D_1^H is low or, equivalently, it is worthwhile to deter entry by pooling. To put it yet differently, the weak firm does not pool to deter entry if the difference with the strong type is too large or if the potential entrant's cost is so high that its presence in the market is not very harmful. In Figure 1, the necessary condition for pooling holds in the region to the right of the thick dashed curve.

If the necessary conditions for pooling do not hold, the market dynamics is trivial. Both incumbent types choose their respective unconstraint monopoly prices. There are no strategic interactions involved and the entrant then enters according to the full-information entry trigger given by (9).

The possibilities arising in the pooling region in Figure 1 are more interesting. Firstly, in Case 1A the strong incumbent is a natural monopolist ($C_2 \geq \frac{\gamma}{\gamma-1}C_1^L$). In Case 1B, the low cost incumbent is not a natural monopolist, but the incentive compatibility constraint (10) is not satisfied. In Case 2A and 2B, (10) holds and the strong type separates at the threat of entry. In Case 2B the high cost incumbent is weak compared to the entrant and is out of the market after entry. Lastly, in Case 3 both incumbent types are weak compared to the entrant and the entrant is a natural monopolist.

For future reference we define the cases precisely.

Definition 3 *A pooling equilibrium may occur only if $C_1^H < \frac{\gamma}{\gamma-1}C_1^L$ and $M^{H,pool} > D_1^H$. Then under additional conditions there are the following cases possible (the conditions a's are in terms of the profit constants and b's are in terms of marginal costs; both are equivalent):*

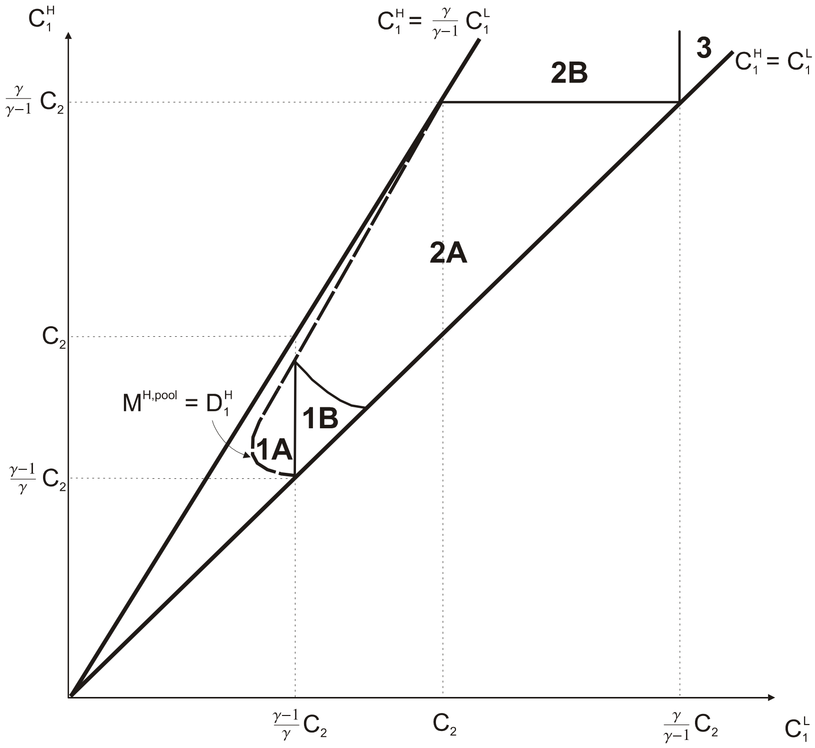

Figure 1: **Marginal costs and possible market outcomes.** The region above the 45° diagonal line represents different marginal cost of two possible incumbent types. If the cost of the high cost incumbent is above the upper thick diagonal line (that is if $C_1^H > \frac{\gamma}{\gamma-1} C_1^L$), no pooling equilibrium is feasible. Additionally, the high cost incumbent considers pooling if profit flows from pooling ($M^{H,pool}$) exceed those from allowing entry (D_1^H). The thick dashed curve represents break-even points between these flows. Pooling and limit pricing may occur if (C_1^L, C_1^H) lies between the two thick diagonal lines and to the right of the dashed curve. In region 1A, the entrant eventually enters if the incumbent is of the high cost type, but would never do so if it knew that the incumbent's cost is low. In the remaining regions 1B to 3, the entrant eventually enters under both incumbent's cost types. In region 1B, the low cost incumbent is relatively strong compared to the entrant, so that the former one would not be determined to engage in too costly entry deterrence. In region 2A, both incumbent types are seriously affected by the entry. In region 2B, the high cost incumbent is so weak, that it goes out of the market if the entrant enters. In region 3, the cost advantage of the entrant is such that it becomes a monopolist after entry irrespective of the incumbent's type.

(1) Case 1A (Low cost incumbent is a natural monopolist) if (a) $D_2^L \leq 0$, $D_2^H \geq 0$, or (b) $C_2 \geq \frac{\gamma}{\gamma-1} C_1^L$ and $C_2 \leq \frac{\gamma}{\gamma-1} C_1^H$.

(2) Case 1B (Incumbent is relatively strong) if (a) $D_2^L \geq 0$, $D_2^H \geq 0$ and there exists no such $M^{L,sep}$ and corresponding $M^{H,sep}$ that $M^{L,sep} \geq D_1^L$ and $M^{H,sep} < D_1^H$, or (b) it holds that[4]

$$\left(\tilde{C}^{\min}\right)^{-\gamma}\left[\gamma\tilde{C}^{\min}-(\gamma-1)\,C_1^H\right]-\left(\frac{2\gamma-1}{\gamma-1}\right)^{\gamma-1}\left(C_1^H+C_2\right)^{-\gamma-1}\left[\gamma C_2-(\gamma-1)\,C_1^H\right]^2<0, \tag{12}$$

where $\tilde{C}^{\min}$ is implicitly given by

$$\left(\tilde{C}^{\min}\right)^{-\gamma}\left[\gamma\tilde{C}^{\min}-(\gamma-1)\,C_1^L\right]-\left(\frac{2\gamma-1}{\gamma-1}\right)^{\gamma-1}\left(C_1^L+C_2\right)^{-\gamma-1}\left[\gamma C_2-(\gamma-1)\,C_1^L\right]^2=0. \tag{13}$$

(3) Case 2A (Both incumbent and entrant are of similar costs) (a) $D_2^L \geq 0$, $D_2^H \geq 0$ and there exists $M^{L,sep}$ and corresponding $M^{H,sep}$ such that $M^{L,sep} \geq D_1^L$ and $M^{H,sep} < D_1^H$, or (b) it holds that

$$\left(\tilde{C}^{\min}\right)^{-\gamma}\left[\gamma\tilde{C}^{\min}-(\gamma-1)\,C_1^H\right]-\left(\frac{2\gamma-1}{\gamma-1}\right)^{\gamma-1}\left(C_1^H+C_2\right)^{-\gamma-1}\left[\gamma C_2-(\gamma-1)\,C_1^H\right]^2\geq 0,$$

where $\tilde{C}^{\min}$ is as in point (2).

(4) Case 2B (High cost incumbent is relatively weak) if (a) $D_1^H < 0$ and $D_1^L > 0$ or (b) $C_1^H > \frac{\gamma}{\gamma-1} C_2$ and $C_1^L \leq \frac{\gamma}{\gamma-1} C_2$.

(5) Case 3 (Entrant is a natural monopolist) if (a) $D_1^H < 0$ and $D_1^L < 0$ or (b) $C_1^H > \frac{\gamma}{\gamma-1} C_2$ and $C_1^L > \frac{\gamma}{\gamma-1} C_2$.

In what follows, Cases 1A and 1B are referred jointly as Case 1 and Cases 2A and 2B as Case 2.

4 Best responses

In this section we find firm i's optimal strategy given a fixed strategy of firm j in the five cases of interest described in the previous section when the necessary conditions for a pooling outcome are satisfied. In all these cases, dynamic programming arguments lead to similar differential equations to be satisfied by the value functions. However, each case has different boundary conditions which bring non-trivial differences in the resulting best response functions.

[4] These conditions may require some explanation. We use that the profit coefficient $M^\theta(\tilde{C})$ in (5) is monotonic in the imitated marginal cost $\tilde{C}$ for $\tilde{C} \leq C^\theta$. Define $C^{\min}$ as the lowest cost type the L type is willing to imitate in the separating equilibrium, i.e. $C^{\min}$ solves $M^H(C^{\min}) = D_1^L$, which is expressed in (13) where we substituted (5) and (6). Then condition $M^{H,\min} < D_1^H$ can be rephrased as (12).

In a pooling equilibrium the two firms choose their strategies as trigger barriers. The high-cost incumbent's strategy is given by a lower barrier X_D at which it reveals its type by deviating to its unconstraint monopoly price. The entrant's strategy is an upper barrier X_E^{pool} at which it decides to enter if X_D was not reached before. In Cases 1A and 1B the strong incumbent does not have incentives to signal its type, so that the entrant always enters at X_E^{pool} (and in Case 1A immediately exits if the incumbent is of the low cost). In Cases 2A and 2B the low cost incumbent separates at X_E^{pool}, so that the entrant enters at this point only if the incumbent is of the high cost and otherwise waits until X_E^L.

If the state process X first reaches X_D and the price is moved to the high cost monopoly level, then the entrant infers the true type of the incumbent, accordingly updates its beliefs and enters at X_E^H. If X_D is hit and there is no price increase then the entrant infers that the incumbent is of the low cost and enters at X_E^L.

4.1 Case 1A: Low cost incumbent is a natural monopolist

Let $V_\theta(X)$ denote a value of the incumbent firm of type θ given firm's 2 strategy in the continuation region, i.e. before either X_D or X_E^{pool} is hit. The familiar arguments (see Appendix A.1) lead to the following ordinal differential equation that must be satisfied by $V_H(X)$

$$0 = \frac{1}{2}\sigma^2 X^2 V_H'' + \mu X V_H' - rV_H + XM^{H,pool}. \tag{14}$$

Its solution is determined by appropriate boundary conditions. At X_D the incumbent deviates to its monopoly price and thus its value is

$$\begin{aligned} V_H(X_D) &= E\left[\int_t^{T_E^H} e^{-r(\tau-t)} M^H X_\tau d\tau + \int_{T_E^H}^{\infty} e^{-r(\tau-t)} D_1^H X_\tau d\tau \,\middle|\, X_t = X_D\right] && (15)\\ &= \left(\frac{X_D}{X_E^H}\right)^{\beta_1} \frac{X_E^H\left(D_1^H - M^H\right)}{r-\mu} + \frac{XDM^H}{r-\mu}. && (16)\end{aligned}$$

Note that $(X_D/X_E^H)^{\beta_1}$ can be interpreted as a stochastic discount factor; see Harrison (1985) for details of such derivations. At X_E^{pool} firm 2 enters, so the value of the incumbent is

$$V_H(X_E^{pool}) = E\left[\int_t^{\infty} e^{-r(\tau-t)} D_1^H X_\tau d\tau \,\middle|\, X_t = X_E^{pool}\right] = \frac{X_E^{pool} D_1^H}{r-\mu} \tag{17}$$

The last two equations are the value matching conditions on $V_H(X)$ at the bounds. Moreover, X_D is to be chosen optimally by firm 1 and the following smooth pasting condition ensures this

$$V_H'(X_D) = \beta_1 \left(\frac{X_D}{X_E^H}\right)^{\beta_1 - 1} \frac{D_1^H - M^H}{r-\mu} + \frac{M^H}{r-\mu}. \tag{18}$$

The solution to $V_H(X)$ in (14) is given by

$$V_H(X) = A_1 X^{\beta_1} + A_2 X^{\beta_2} + \frac{X M^{H,pool}}{r-\mu},$$

where

$$A_1 = \frac{1-\beta_2}{\beta_1-\beta_2}(X_D)^{1-\beta_1} \frac{M^H - M^{H,pool}}{r-\mu} - \left(X_E^H\right)^{1-\beta_1} \frac{M^H - D_1^H}{r-\mu},$$

and

$$A_2 = \frac{\beta_1 - 1}{\beta_1-\beta_2}(X_D)^{1-\beta_2} \frac{M^H - M^{H,pool}}{r-\mu},$$

and the best-response deviation trigger X_D is implicitly given by

$$\frac{1}{\beta_1-\beta_2} X_D \left(M^H - M^{H,pool}\right)\left[(\beta_1 - 1)\left(\frac{X_E^{pool}}{X_D}\right)^{\beta_2} + (1-\beta_2)\left(\frac{X_E^{pool}}{X_D}\right)^{\beta_1}\right]$$
$$- \left(\frac{X_E^{pool}}{X_E^H}\right)^{\beta_1} X_E^H \left(M^H - D_1^H\right) + X_E^{pool}\left(M^{H,pool} - D_1^H\right) = 0. \tag{19}$$

Next we consider the best response entry strategy X_E^{pool} of firm 2 given a fixed deviation strategy X_D of firm 1. Denote the value of firm 2 in the continuation region given X_D by $F_A(X)$ (superscript A stands for case 1A). Again dynamic programming and Itô's lemma yield a differential equation to be satisfied by $F_A(X)$.

$$0 = \frac{1}{2}\sigma^2 X^2 F_A'' + \mu X F_A' - r F_A, \tag{20}$$

subject to

$$F_A(X_E^{pool}) = \eta_0 \left[\frac{X_E^{pool} D_2^H}{r-\mu} - I\right] - (1-\eta_0) I, \tag{21}$$

$$F_A(X_D) = \eta_0 \left(\frac{X_D}{X_E^H}\right)^{\beta_1} \left[\frac{X_E^H D_2^H}{r-\mu} - I\right], \tag{22}$$

$$F_A'(X_E^{pool}) = \eta_0 \frac{D_2^H}{r-\mu}. \tag{23}$$

The boundary conditions are constructed in a similar way as above. The value matching (21) and smooth pasting (23) conditions are required at the trigger X_E^{pool} to be optimized.

At the trigger X_D set by the other player, just the value matching condition (22) is required. The solution to (20) is given by

$$F_A(X) = B_1^A X^{\beta_1} + B_2^A X^{\beta_2},$$

where

$$B_1^A = \frac{1}{\beta_1 - \beta_2} \left(X_E^{pool} \right)^{-\beta_1} \left[\eta_0 \left(1 - \beta_2\right) \frac{X_E^{pool} D_2^H}{r - \mu} + \beta_2 I \right],$$

and

$$B_2^A = \frac{1}{\beta_1 - \beta_2} \left(X_E^{pool} \right)^{-\beta_2} \left[\eta_0 \left(\beta_1 - 1\right) \frac{X_E^{pool} D_2^H}{r - \mu} - \beta_1 I \right],$$

and the best-response entry trigger X_E^{pool} is implicitly given by

$$\left(\frac{X_D}{X_E^{pool}} \right)^{\beta_1} \left[\eta_0 \left(1 - \beta_2\right) \frac{X_E^{pool} D_2^H}{r - \mu} + \beta_2 I \right] + \left(\frac{X_D}{X_E^{pool}} \right)^{\beta_2} \left[\eta_0 \left(\beta_1 - 1\right) \frac{X_E^{pool} D_2^H}{r - \mu} - \beta_1 I \right] - \frac{\beta_1 - \beta_2}{\beta_1 - 1} I \eta_0 \left(\frac{X_D}{X_E^H} \right)^{\beta_1} = 0. \quad (24)$$

4.2 Case 1B: Relatively strong incumbent

In case 1B the low cost incumbent is not a natural monopolist, however its costs are low enough to not to be greatly affected by a new entrance and thus it is not willing to engage in too costly entry deterrence. In this case the low cost incumbent does not separate at the pooling entry trigger. The best response problem of the high cost incumbent remains the same as in the previous case. The best response function is given by (19). The best response entry strategy X_E^{pool} of firm 2 differs in this case. The entrant gets a share of the market even if the incumbent is of the low type, so the payoff at X_E^{pool} is altered accordingly. Denote the value of firm 2 in the continuation region given X_D by $F_B(X)$. Again dynamic programming and Itô's lemma yield a differential equation to be satisfied by $F^B(X)$.

$$0 = \frac{1}{2} \sigma^2 X^2 F_B'' + \mu X F_B' - r F_B, \quad (25)$$

subject to

$$F_B(X_E^{pool}) = \eta_0 \left[\frac{X_E^{pool} D_2^H}{r-\mu} - I\right] + (1-\eta_0)\left[\frac{X_E^{pool} D_2^L}{r-\mu} - I\right], \quad (26)$$

$$F_B(X_D) = \eta_0 \left(\frac{X_D}{X_E^H}\right)^{\beta_1} \left[\frac{X_E^H D_2^H}{r-\mu} - I\right] + (1-\eta_0)\left(\frac{X_D}{X_E^L}\right)^{\beta_1}\left[\frac{X_E^L D_2^L}{r-\mu} - I\right], \quad (27)$$

$$F_B'(X_E^{pool}) = \eta_0 \frac{D_2^H}{r-\mu} + (1-\eta_0)\frac{D_2^L}{r-\mu}. \quad (28)$$

The solution to (25) is given by

$$F_B(X) = B_1^B X^{\beta_1} + B_2^B X^{\beta_2},$$

where

$$B_1^B = \frac{1}{\beta_1-\beta_2}\left(X_E^{pool}\right)^{-\beta_1}\left[\eta_0 (1-\beta_2)\frac{X_E^{pool} D_2^H}{r-\mu} + (1-\eta_0)(1-\beta_2)\frac{X_E^{pool} D_2^L}{r-\mu} + \beta_2 I\right],$$

and

$$B_2^B = \frac{1}{\beta_1-\beta_2}\left(X_E^{pool}\right)^{-\beta_2}\left[\eta_0 (\beta_1-1)\frac{X_E^{pool} D_2^H}{r-\mu} + (1-\eta_0)(\beta_1-1)\frac{X_E^{pool} D_2^L}{r-\mu} - \beta_1 I\right],$$

and the best-response entry trigger X_E^{pool} is implicitly given by

$$\begin{aligned}\left(\frac{X_D}{X_E^{pool}}\right)^{\beta_1}&\left[\eta_0 (1-\beta_2)\frac{X_E^{pool} D_2^H}{r-\mu} + (1-\eta_0)(1-\beta_2)\frac{X_E^{pool} D_2^L}{r-\mu} + \beta_2 I\right] \\ &+ \left(\frac{X_D}{X_E^{pool}}\right)^{\beta_2}\left[\eta_0 (\beta_1-1)\frac{X_E^{pool} D_2^H}{r-\mu} + (1-\eta_0)(\beta_1-1)\frac{X_E^{pool} D_2^L}{r-\mu} - \beta_1 I\right] \\ &- \frac{\beta_1-\beta_2}{\beta_1-1} I\left[\eta_0\left(\frac{X_D}{X_E^H}\right)^{\beta_1} + (1-\eta_0)\left(\frac{X_D}{X_E^L}\right)^{\beta_1}\right] = 0. \quad (29)\end{aligned}$$

4.3 Case 2A: Equal incumbent and entrant

We now consider the case in which the low type incumbent separates at X_E^{pool} credibly signaling its low cost and thus indicating suboptimal entry below X_E^L. The best response problem of the high cost incumbent remains the same as in the previous cases, and the best response function is given by (19). The separating strategy of the low cost incumbent affects the best response entry strategy X_E^{pool} of firm 2 with the value matching and smooth pasting conditions at X_E^{pool} adjusted accordingly. The value of firm 2 in the continuation region given X_D, denoted by $G_A(X)$, must satisfy the following differential equation

$$0 = \frac{1}{2}\sigma^2 X^2 G_A'' + \mu X G_A' - rG_A, \tag{30}$$

subject to

$$G_A(X_E^{pool}) = \eta_0 \left[\frac{X_E^{pool} D_2^H}{r-\mu} - I\right] + (1-\eta_0)\left(\frac{X_E^{pool}}{X_E^L}\right)^{\beta_1} \left[\frac{X_E^L D_2^L}{r-\mu} - I\right], \tag{31}$$

$$G_A(X_D) = \eta_0 \left(\frac{X_D}{X_E^H}\right)^{\beta_1} \left[\frac{X_E^H D_2^H}{r-\mu} - I\right] + (1-\eta_0)\left(\frac{X_D}{X_E^L}\right)^{\beta_1} \left[\frac{X_E^L D_2^L}{r-\mu} - I\right], \tag{32}$$

$$G_A'(X_E^{pool}) = \eta_0 \frac{D_2^H}{r-\mu} + (1-\eta_0)\beta_1 \left(\frac{X_E^{pool}}{X_E^L}\right)^{\beta_1 - 1} \frac{D_2^L}{r-\mu}. \tag{33}$$

The solution to (30) together with the boundary condition gives an implicit equation for the best-response entry trigger X_E^{pool}

$$\left(\frac{X_D}{X_E^{pool}}\right)^{\beta_1} \left[(1-\beta_2)\frac{X_E^{pool} D_2^H}{r-\mu} + \beta_2 I\right] + \left(\frac{X_D}{X_E^{pool}}\right)^{\beta_2} \left[(\beta_1 - 1)\frac{X_E^{pool} D_2^H}{r-\mu} - \beta_1 I\right] - \left(\frac{X_D}{X_E^H}\right)^{\beta_1} \frac{\beta_1 - \beta_2}{\beta_1 - 1} I = 0. \tag{34}$$

As can be checked, that the above equation is satisfied by $X_E^{pool} = X_E^H$.

The entrant's best response to any deviation strategy is to enter at X_E^H. The intuition for such a strategy is clear. The low cost incumbent always separates upon entry, and thus the entrant with the strategy to enter at X_E^H either observes a separation if the incumbent is of low cost or sees no action if the incumbent is of high cost. In the first case the entrant postpones entry until X_E^L and in the second enters at X_E^H; both outcomes are optimal for the entrant. In other words, by threatening to enter at X_E^H the entrant learns the true type and always enters optimally.

4.4 Case 2B: High cost incumbent is weak

In this case the high cost incumbent is out of the market if entry occurs. Some adjustments are required to the value matching and smooth pasting conditions of the high cost incumbent's and the entrant's best response problems. The high cost incumbent's gets a duopoly profit flow equal to zero. Substituting $D_1^H = 0$ in (14)-(18) yields the best-response X_D as

$$\frac{1}{\beta_1-\beta_2}X_D\left(M^H-M^{H,pool}\right)\left[(\beta_1-1)\left(\frac{X_E^{pool}}{X_D}\right)^{\beta_2}+(1-\beta_2)\left(\frac{X_E^{pool}}{X_D}\right)^{\beta_1}\right] - \left(\frac{X_E^{pool}}{X_E^H}\right)^{\beta_1}X_E^H M^H + X_E^{pool}M^{H,pool}=0. \quad (35)$$

As expected a decrease of the weak incumbent's duopoly profits to zero shifts the best-response X_D down, as the weak firm is more reluctant to reveal its true type.

A simple variant of the analysis done for Case 2A with D_2^H exchanged for entrant's monopoly profits coefficient M_2 gives the best-response X_E^{pool} as

$$\left(\frac{X_D}{X_E^{pool}}\right)^{\beta_1}\left[(1-\beta_2)\frac{X_E^{pool}M_2}{r-\mu}+\beta_2 I\right] + \left(\frac{X_D}{X_E^{pool}}\right)^{\beta_2}\left[(\beta_1-1)\frac{X_E^{pool}M_2}{r-\mu}-\beta_1 I\right] - \left(\frac{X_D}{X_E^H}\right)^{\beta_1}\frac{\beta_1-\beta_2}{\beta_1-1}I=0. \quad (36)$$

$X_E^{pool}=X_E^H$ is a solution to this equation. The meaning and intuition for this strategy is the same as in Case 2A.

4.5 Case 3: Entrant is a natural monopolist

The top-most region 3 in Figure 1 corresponds to the extreme possibility of the entrant becoming a natural monopolist after entry. Obviously, the entrant's decision to enter does not depend on the incumbent's type in this case. Hence, neither incumbent's type finds incentives to engage in either pooling or costly separation. The incumbent's best strategy is to charge its monopoly price until the entrant decides to enter. The entrant takes into account uncertainty and investment irreversibility in a similar manner as under the full information case (Subsection 2.2).

5 Equilibrium paths

5.1 Equilibrium paths in pure strategies

The implicit formulas (19), (24) and (29) conceal well-behaved best response functions as we show in the following lemma.

Lemma 4 *In Case 1, the best response functions of the incumbent and the entrant are monotone, decreasing and increasing, respectively.*
Proof. *See the appendix.* ■

Using the monotonicity and opposing slopes of the best response functions from Lemma 4, it follows that there exist a unique pair $\{\hat{X}_D, \hat{X}_E^{pool}\}$ of equilibrium triggers. We obtain the following equilibrium results.

Proposition 5 *There exists a unique equilibrium pair $\{\hat{X}_D, \hat{X}_E^{pool}\}$. In Case 1, the pooling equilibrium arises within $(\hat{X}_D, \hat{X}_E^{pool})$. In Case 2, $\hat{X}_D = \hat{X}_E^{pool} = X_E^H$ and there is no pooling.*

Figure 2 presents examples of equilibrium construction from the best response functions and relationships with some benchmark cases. In Case 1 (the left panel), the vertical dashed line of the myopic X_E^{pool} represents a decision of the entrant that does not take into account a possibility of the weak incumbent's deviation from the deterring strategy at X_D. The entrant's strategic response $R_2(X_D)$ is to further postpone entry. The dashed curve of the naive $R_1(X_E^{pool})$ represents a strategic choice of firm 1 which follows directly from constraint (11), but disregards uncertainty and irreversibility of the deviation decision (the naive entry trigger which disregards uncertainty and irreversibility is typically low and, for the given parameters, outside the plotted area). Irreversibility delays the decision as seen from the solid line $R_1(X_E^{pool})$ which depicts the fully-considerate best response function. In Case 2, the equilibrium is always at $\hat{X}_D = \hat{X}_E^{pool} = X_E^H$.

Pooling occurs in equilibrium only in region 1A and 1B. Therefore using Definition 3 it follows that

Corollary 6 *Entry deterrence occurs only if $C_2 > C_1^H$.*

It means that only if the incumbent is relatively strong (in all types), limit pricing may be used to deter entry. The one-shot models of limit pricing predict that there are more chances for pooling the higher is the entrant's marginal cost. Such a prediction is, for instance, explicitly tested in a laboratory setting by Cooper, Garvin and Kagel (1997). The entry deterring limit pricing is not sustained on the equilibrium path in the dynamic model if the entrant is relatively strong. The reason is that in such a situation the strong incumbent, when directly threatened by entry, has incentives to separate. Knowing this, the entrant can credibly threaten to enter at X_E^H. If the incumbent is of low cost, it separates, reveals its type and enables optimal entry at X_E^L. Otherwise, if the incumbent is of high cost, the entry at X_E^H is optimal. Facing such a credible threat, the high cost firm does not pool and does not deter entry.

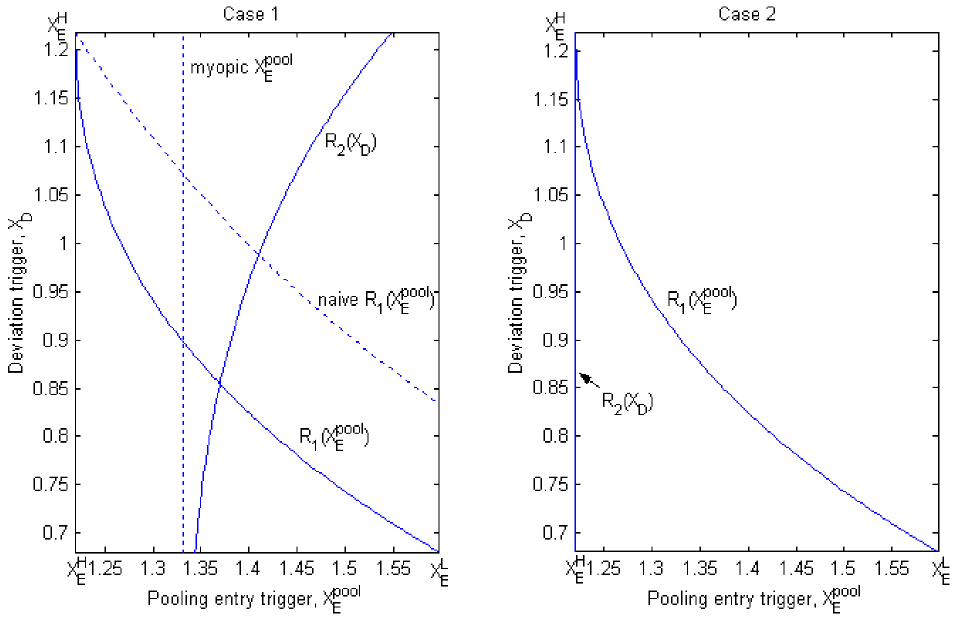

Figure 2: Best responses and the equilibrium in the timing game

To sum up, there are two equilibrium paths in pure strategies. In Case 1, within $(\hat{X}_D, \hat{X}_E^{pool})$ the weak incumbent pools with the strong incumbent and deters entry. At $\hat{X}_E^{pool}$ firm 2 enters. If $\hat{X}_D$ is reached first, the weak incumbent deviates from deterring strategy and firms set their unconstrained monopoly prices until full-information entry. In Case 2, both incumbent types set their unconstrained monopoly prices if X is below X_E^H. At X_E^H either entry or separation occurs depending on the true incumbent type. The equilibrium paths are depicted in Panel A of Figure 3.

The interesting aspect of the equilibrium path in Case 1 is that the entry decision and thus the market structure depends on the history of the market size process. For any given parameters and current level of X, the entry trigger depends on whether $\hat{X}_D$ was reached before. This history dependence is even more pronounced if mixed strategies are allowed.

5.2 Equilibrium paths with mixed strategies

Our restriction to pure strategies in the preceding sections, while convenient and not unusual in signaling and timing games, is not without consequential. The weak incumbent's behavior at X_D was referred to as a deviation. Indeed, this action is an out-of-equilibrium deviation which does not satisfy the subgame perfection criterion. Observe that if the weak incumbent's (pure) strategy is to deviate at X_D to its uncon-

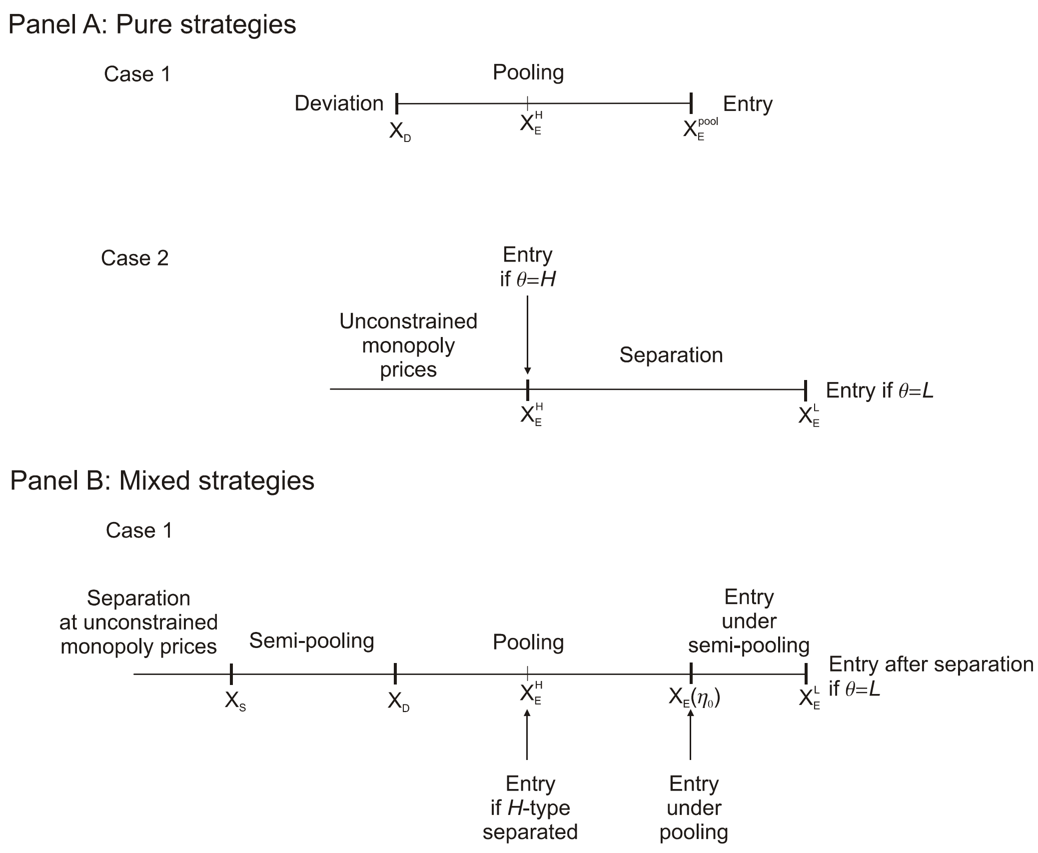

Figure 3: Equilibrium paths.

strained monopoly price and the entrant seeing no action at X_D updates its beliefs to zero, then the weak incumbent prefers not to deviate from the pooling price. The weak cost incumbent wants to deviate if the entry was to be at $X_E(\eta_0) = X_E^{pool}$, but not when the entry trigger moves to $X_E(0) = X_E^L$. Otherwise, if the entrant did not update its beliefs seeing no action at X_D, then it is by construction optimal for the weak incumbent to deviate at X_D from the pooling price. But the subgame perfection requires from the entrant to update beliefs if the weak incumbent was to change its price at X_D. These contradicting observations indicate that the incumbent's action (and inaction) at X_D is out of the equilibrium path in pure strategies.

The situation is similar to the problem with separating equilibria in dynamic labor market signaling models as studied by Noldeke and Van Damme (1990) and Swinkels (1999). In those models education serves solely as a signaling device. The separating equilibrium in Riley outcome typically requires some duration of education to be taken by the high-skilled worker and no education by the low-skilled worker. In a dynamic setting, the high-skilled worker enrolls in the education system and competing firms in the labor market immediately offer her a well-paid job. But if it was enough to just enroll, the low-skilled worker would imitate and disturb the equilibrium. The problem in these models is that the signal space is limited and the worker can not separate in a single period by sending a strong enough signal. In our model a similar mechanism works, but it is not a limited signal space that upsets the separation at X_D, but it is the lack of incentives of the active player to fully separate. The high cost firm moves its price only to its unconstrained monopoly level and does not have incentives for any stronger (costly) separation. The low cost firm does not separate before an immediate threat of entry (or not at all as in Case 1).

In our model, a solution to the above problem is to allow for mixed strategies in stopping times. A mixed strategy in a timing game in continuous time is a not a simple translation of mixed strategies in discrete time as time intervals converge to zero. We adopt the notions introduced by Laraki, Solan and Vieille (2005) for games of timing with complete information to our setting. A *mixed plan* of player i is a probability distribution φ^i over the set $[Z_0, Z_1]$. A *strategy* needs to be defined for all subgames starting at any point and taking into consideration any payoff relevant information. Thus a strategy is a function $\phi_t^i : \bar{Z}_t \to \varphi^i$, where $\bar{Z}_t = \min\{Z_s, s \leq t\}$. For example, a plan profile could be that a player acts at a random trigger uniformly chosen from the interval $[Z_0, Z_1]$.

To apply these notions to our limit pricing model, first note that the high cost type will always deviate from the pooling price at time t if

$$\begin{aligned} & E\left[\int_t^{T_E^L} e^{-r(\tau-t)} M^{H,pool} X_\tau d\tau + \int_{T_E^L}^{\infty} e^{-r(\tau-t)} D_1^H X_\tau d\tau\right] \\ \leq\ & E\left[\int_t^{T_E^H} e^{-r(\tau-t)} M^H X_\tau d\tau + \int_{T_E^H}^{\infty} e^{-r(\tau-t)} D_1^H X_\tau d\tau\right]. \end{aligned} \tag{37}$$

The largest X_t that satisfies this inequality defines the *separation trigger*, denoted X_S, which is the level of X at which the weak incumbent deviates with probability one. It is given by

$$X_S = \left[\frac{\left(X_E^H\right)^{1-\beta_1}\left(M^H - D_1\right) - \left(X_E^L\right)^{1-\beta_1}\left(M_{pool}^H - D_1\right)}{M^H - M_{pool}^H}\right]^{\frac{1}{1-\beta_1}}.$$

Clearly, there are no strategic considerations in the choice of X_S.

Let X_D remain to denote the deviation trigger as defined in the pure-strategy cases. Define a rescaling variable $Z(x) = \frac{X_D - x}{X_D - X_S}$ for $x \in [X_S, X_D]$, with $Z(x) \in [0,1]$, $Z(X_D) = 0$ and $Z(X_S) = 1$. For $x > X_D$, let $Z(x) = 0$ and for $x < X_S$, $Z(x) = 1$. Further define $\bar{Z}_t = \max[Z(X_s), s \leq t]$ as a new state variable. Then we can characterize a mixed strategy in the semi-pooling equilibrium.

Proposition 7 *Consider a function $\phi : Z \to [0,1]$, where ϕ is a cumulative distribution function with $\phi(0) = 0$ and $\phi(1) = 1$. Then ϕ is a mixed strategy in the deviation trigger of the weak incumbent in the semi-pooling equilibrium.*
Proof. *See the appendix.* ■

If a player follows a strategy ϕ, then $\phi(Z(x))$ is interpreted as the probability that it has chosen a deviation trigger below or equal at x. The entrant, each time seeing new lows in $[X_S, X_D]$ without separation, updates its belief taking into consideration the strategy ϕ. Precisely, on the equilibrium path its belief η_t is a mapping $\eta_t : \bar{Z}_t \times \phi \to [0,1]$, such that $\eta_t(\bar{Z}_t, \phi) = (1 - \phi(\bar{Z}_t))\eta_0$, provided no deviation occurred by t. After observing a deviation from the pooling price, the entrant correctly assigns all the probability weight to the weak incumbent and enters at X_E^H. The entrant always enters at the optimal entry point according to its information.

The equilibrium path with mixed strategies is complete. Unlike in pure strategies equilibrium, there are no out-of-equilibrium deviations and the equilibrium path satisfies Bayesian subgame perfection requirement at all levels of the market state process. Consider a game starting at $X_0 \in (X_D, X_E(\eta_0))$ under conditions of Case 1. Initially, a pooling equilibrium arises with entry at $X_E(\eta_0)$. If the market moves to $[X_S, X_D]$, the weak incumbent starts playing the mixed strategy over the deviation point. If it deviates from the pooling price, then the entrant enters at X_E^H. If no deviation is observed, η_t is updated to lower values and the entry trigger is at $X_E(\eta_t)$. When the market gets as low as X_S and the pooling price is maintained, the entrant is sure to face the strong type and enters ultimately at X_E^L. If the game starts in the semi-pooling interval $X_0 \in [X_S, X_D]$, the weak incumbent randomizes using ϕ between setting its own monopoly price and pooling with the strong firm. If the game starts in the separation interval $[0, X_S]$, both types set their unconstrained monopoly prices from the beginning. The equilibrium path with mixed strategies is presented in Panel B of Figure 3.

The entry triggers and market structure in the equilibrium path with mixed strategies exhibit strong history dependence. Past low values in X may, on the one hand, lead to a deviation from pooling and, on the other hand, postpone entry under the pooling prices (by lowering η).

The pair X_E and X_D is chosen strategically in a similar manner as in the pure-strategy cases. If we focus on Case 1A analyzed in Section 4.1, then most of the derivations done there can be adopted to the mixed strategy case. In particular, as can be checked, the incumbents problem remains essentially the same. To find X_D we use the same set of boundary conditions (16)-(18). The difficulty is that the extension to mixed strategies requires a new state variable $\bar{Z}_t$. Getting half-way around it is not difficult as $\bar{Z}_t$ can be easily incorporated in the state variable η_t. Then, however, some complicated path-dependent updating of η_t is required. To simplify the analysis but without a impact on the qualitative results, we can restrict our attention to the case when ϕ is a uniform c.d.f. Still, the entrant's problem, corresponding to the one in Section 4.1, cannot be readily solved. In particular, the value matching condition (22) at X_D must be altered and include future possibilities of learning about the incumbent type if X moves down and possibility of entering at some $X_E(\eta_t)$. These possibilities depend on particular paths taken by the market process.

6 Market dynamics and entry deterrence

In this section we study the impact of the parameters of the state variable X on incidence of entry deterrence.

Numerical analysis shows that the length of pooling equilibrium interval $[\hat{X}_D, \hat{X}_E^{pool}]$ increases in uncertainty. We have verified this property on a set of parameters that covers a extensive range of economic relevance. Table 1 gives one numerical example. There are a numbers of effects of uncertainty in place. As usual in with irreversible decisions under uncertainty, when isolated from strategic effects, $\hat{X}_E^{pool}$ increases in uncertainty. By the same argument, $\hat{X}_D$ decreases in σ. But X_E^H also rises with uncertainty, so

σ	$\hat{X}_D$	$\hat{X}_E^{pool}$	$\hat{X}_E^{pool} - \hat{X}_D$
0.05	0.8708	1.9481	1.0773
0.07	0.8417	2.0759	1.2342
0.09	0.8193	2.2125	1.3932
0.11	0.8028	2.3579	1.5551
0.13	0.7915	2.5124	1.7209
0.15	0.7845	2.6761	1.8916
0.17	0.7813	2.8492	2.0679
0.19	0.7815	1.7393	2.2505

Table 1: Effect of uncertainty on the pooling equilibrium interval: $\mu = 0.00$, $r = 0.05$, $\eta = 0.5$, $\gamma = 2$, $I = 5$.

deviation from pooling becomes more profitable and this may increase $\hat{X}_D$. Strategic interactions add further effects. Overall, $\hat{X}_E^{pool}$ increases in σ and $\hat{X}_D$ may decrease or increase but always less than $\hat{X}_E^{pool}$, so that the pooling interval increases in uncertainty.

This prediction differs from the result of Maskin (1999) who showed that uncertainty reduces incidence of capacity entry deterrence. Our result stems from embedded options to enter and to deviate. This aspect of the impact of uncertainty on the (partially) irreversible decision, as entry accommodation can be considerded, was not considered in Maskin (1999). The capacity deterrence as studied by Maskin (1999) differs from our model, but the insight that entry accommodation resembles an option exercise and uncertainty postpones such a decision remains valid in both frameworks.

7 Conclusion

This paper has studied limit pricing entry deterrence in a dynamic setting in continuous time under uncertainty. The strategic situation of the incumbent and entrant firms is different than in the standard two period models. There are market conditions when the incumbent may decide to deviate from the pooling equilibrium. This possibility makes entry deterrence limit pricing possible only within endogenously and strategically determined bounds on the stochastic process.

The model brings several novel insights into dynamics of limit pricing behavior. The general framework of signaling in continuous time with Brownian motion uncertainty can be also useful for analyses of other asymmetric information situations. For example, the Brownian motion environment is particularly relevant in financial applications in which it is standard to model asset prices as continuous-time processes driven by Brownian motion.

To select plausible equilibrium paths, out of many typically arising in signaling models, we applied intuitive but rather *ad hoc* criteria of least cost separation and

Pareto efficiency. A further step would be to formalize the refinement arguments by restricting out-of-equilibrium beliefs as is usually done in the simple signaling games.

A Appendix

A.1 Derivation of full-information entry triggers (9)

Denote by $F(X)$ the value function of the entrant in the continuation region before entry, that is in $\{X : X < X_E^\theta\}$. $F(X)$ must satisfy the following Bellman equation

$$rF(X) = \lim_{dt \downarrow 0} \frac{1}{dt} E\left[dF(X)\right].$$

Applying Itô's lemma we obtain the following ordinary differential equation

$$0 = \frac{1}{2}\sigma^2 X^2 F'' + \mu X F' - rF.$$

Its general solution is

$$F(X) = A_1 X^{\beta_1} + A_2 X^{\beta_2},$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are, respectively, the positive and negative roots of the characteristic equation

$$\frac{1}{2}\sigma^2 \beta(\beta - 1) + \mu\beta - r = 0.$$

A_1 and A_2 are constants to be determined by appropriate boundary conditions. Here the conditions are

$$F(X_E^\theta) = \frac{D_2^\theta X_E^\theta}{r - \mu} - I, \tag{38}$$

$$F'(X_E^\theta) = \frac{D_2^\theta}{r - \mu}, \tag{39}$$

$$F(0) = 0. \tag{40}$$

(38) is the value matching condition, which equates the continuation value at the entry moment to the net payoff the firm receives. (39) is the smooth pasting condition, which ensures continuity of the value function at X_E^θ. Condition (40) ensures that the firm will be worthless if the market is at the absorbing barrier zero. Solving this system for X_E^θ yields the investment trigger in (9).

A.2 Proof of Proposition 2

Proof of Proposition 2. The proof follows in two parts. First, we show that the LCSE in our model is of the form of repeated Riley outcome. Then we prove that the strong incumbent separates only if the market process is at and above $X_E(\eta_t)$.

The strong incumbent maximizes its expected discounted profits w.r.t. a separating signaling scheme $\tilde{C}$

$$\max_{\{\tilde{C}_\tau\}} E\left[\int_0^{T_E^L} e^{-r\tau} M^L(\tilde{C}_\tau) X_\tau d\tau + \int_{T_E^L}^{\infty} e^{-r(\tau - T_E^L)} D_1^L X_\tau d\tau\right].$$

The maximization is subject to an incentive compatibility constraint for the weak type. Define $T_E \equiv \inf[t \geq 0 : X_t \geq X_E(\eta_t)]$ the time the entrant would enter without a separating action of the incumbent. The weak incumbent does not imitate the signaling scheme $\tilde{C}$ if

$$\forall t \in [0, T_E] \quad E\left[\int_0^t e^{-r\tau} M^H(\tilde{C}_\tau) X_\tau d\tau\right] \leq E\left[\int_0^t e^{-r\tau} M^H X_\tau d\tau\right], \text{ and} \tag{41a}$$

$$\begin{aligned}\forall t \in [T_E, T_E^L] \quad & E\left[\int_0^t e^{-r\tau} M^H(\tilde{C}_\tau) X_\tau d\tau\right] \\ \leq \quad & E\left[\int_0^{T_E} e^{-r\tau} M^H X_\tau d\tau + \int_{T_E}^t e^{-r(\tau - T_E)} D_1^H X_\tau d\tau\right].\end{aligned} \tag{41b}$$

It is clear that the optimal $\tilde{C}$ will be such that the constraint will hold with equality at all t. That is $\tilde{C}$ is piecewise constant, precisely $\{\tilde{C}_t : M^H(\tilde{C}_t) = M_1^H, t \in [0, T_E]\}$ and $\{\tilde{C}_t : M^H(\tilde{C}_t) = D_1^H, t \in [T_E, T_E^L]\}$. Then strategy $\tilde{C}$ is a continuous time version of the Riley outcome.

Suppose that the optimal $\tilde{C}$ is of the form of the Riley outcome and denote it $\tilde{C}^{sep}$. The strong incumbent, which wants to ensure entry at X_E^L, will care only about the constraint in (41b) not in (41a). Note that $\tilde{C}^{sep}$ is such that (41) is binding on any time subinterval. Thus, in the LCSE the strong type separates in $t \in [T_E, T_E^L]$ and $\{\tilde{C}^{sep} : M^H(\tilde{C}) = D_1^H\}$. ■

A.3 Proof of Lemma 4

Proof of Lemma 4. The best response functions in Case 1A are very similar yet slightly simpler than those in Case 1B. Below we prove only the latter case. Very similar arguments prove the corresponding results for Case 1A.

Denote the incumbent's best-response function given in (19) by $R_1(X_D, X_E^{pool})$. Then

$$\begin{aligned}& \frac{\partial R_1}{\partial X_S}(X_D, X_E^{pool}) \\ = \quad & \frac{(\beta_1 - 1)(1 - \beta_2)}{\beta_1 - \beta_2}\left(M^H - M^{H,pool}\right)\left[\left(\frac{X_E^{pool}}{X_D}\right)^{\beta_2} + \left(\frac{X_E^{pool}}{X_D}\right)^{\beta_1}\right] < 0,\end{aligned}$$

if $X_S < X_E^{pool}$, as $\beta_2 < 0$ and $\beta_1 > 1$. Also

$$\begin{aligned}
&\frac{\partial R_1}{\partial X_E^{pool}}(X_D, X_E^{pool}) \\
= \;& \frac{1}{\beta_1 - \beta_2}\left(M^H - M^{H,pool}\right)\left[(\beta_1 - 1)\,\beta_2\left(\frac{X_E^{pool}}{X_D}\right)^{\beta_2 - 1} + \beta_1\,(1-\beta_2)\left(\frac{X_E^{pool}}{X_D}\right)^{\beta_1 - 1}\right] \\
& -\beta_1\left(\frac{X_E^{pool}}{X_E^H}\right)^{\beta_1 - 1}\left(M^H - D_1^H\right) + M^{H,pool} - D_1^H \\
< \;& \frac{\beta_1\,(1-\beta_2)}{\beta_1 - \beta_2}\left(\frac{X_E^{pool}}{X_D}\right)^{\beta_1 - 1}\left(M^H - M^{H,pool}\right) - \beta_1\left(\frac{X_E^{pool}}{X_E^H}\right)^{\beta_1 - 1}\left(M^H - D_1^H\right) + M^{H,pool} - D_1^H \\
< \;& \frac{\beta_1\,(1-\beta_2)}{\beta_1 - \beta_2}\left(\frac{X_E^{pool}}{X_D}\right)^{\beta_1 - 1}\left(M^H - M^{H,pool}\right) \\
& -\frac{\beta_1\,(1-\beta_2)}{\beta_1 - \beta_2}\left(\frac{X_E^{pool}}{X_E^H}\right)^{\beta_1 - 1}\left(M^H - D_1^H\right) + M^{H,pool} - D_1^H \\
= \;& -\frac{\beta_1\,(1-\beta_2)}{\beta_1 - \beta_2}\left(\frac{X_E^{pool}}{X_D}\right)^{\beta_1 - 1}\left(M^{H,pool} - D_1^H\right) + M^{H,pool} - D_1^H < 0.
\end{aligned}$$

Thus from the implicit function theorem it follows that for the best response X_D it holds that $\frac{dX_D}{dX_E^{pool}} < 0$.

Denote the entrant's best response function given in (29) by $R_2(X_D, X_E^{pool})$. We prove the claimed monotonicity assuming that the best response X_E^{pool} is not smaller than the non-strategic entry trigger under pooling prices, that is $X_E^{pool} \geq \frac{\beta_1}{\beta_1 - 1}\frac{r-\mu}{\eta D_2^H + (1-\eta) D_2^L} I$.

We get

$$
\begin{aligned}
&\frac{\partial R_2}{\partial X_D}(X_D, X_E^{pool}) \\
&= \frac{1}{X_D}\left(\frac{X_D}{X_E^{pool}}\right)^{\beta_1}\left[\eta\beta_1(1-\beta_2)\frac{X_E^{pool}D_2^H}{r-\mu} + (1-\eta)\beta_1(1-\beta_2)\frac{X_E^{pool}D_2^L}{r-\mu} + \beta_1\beta_2 I\right] \\
&+ \frac{1}{X_D}\left(\frac{X_D}{X_E^{pool}}\right)^{\beta_2}\left[\eta(\beta_1-1)\beta_2\frac{X_E^{pool}D_2^H}{r-\mu} + (1-\eta)(\beta_1-1)\beta_2\frac{X_E^{pool}D_2^L}{r-\mu} - \beta_1\beta_2 I\right] \\
&\qquad - \frac{\beta_1(\beta_1-\beta_2)}{\beta_1-1}\frac{1}{X_D}I\left[\eta\left(\frac{X_D}{X_E^H}\right)^{\beta_1} + (1-\eta)\left(\frac{X_D}{X_E^L}\right)^{\beta_1}\right] \\
&< \frac{1}{X_D}\left(\frac{X_D}{X_E^{pool}}\right)^{\beta_1}\left[\eta\beta_1(1-\beta_2)\frac{X_E^{pool}D_2^H}{r-\mu} + (1-\eta)\beta_1(1-\beta_2)\frac{X_E^{pool}D_2^L}{r-\mu} + \frac{\beta_1^2(\beta_2-1)}{\beta_1-1}I\right] \\
&+ \frac{1}{X_D}\left(\frac{X_D}{X_E^{pool}}\right)^{\beta_2}\left[\eta(\beta_1-1)\beta_2\frac{X_E^{pool}D_2^H}{r-\mu} + (1-\eta)(\beta_1-1)\beta_2\frac{X_E^{pool}D_2^L}{r-\mu} - \beta_1\beta_2 I\right] \\
&\le \frac{1}{X_D}\left(\frac{X_D}{X_E^{pool}}\right)^{\beta_1}\left[\eta\beta_1(1-\beta_2)\frac{X_E^{pool}D_2^H}{r-\mu} + (1-\eta)\beta_1(1-\beta_2)\frac{X_E^{pool}D_2^L}{r-\mu} + \beta(\beta_2-1)\frac{\eta D_2^H + (1-\eta)D_2^L}{r-\mu}\right] \\
&+\frac{1}{X_D}\left(\frac{X_D}{X_E^{pool}}\right)^{\beta_2}\left[\eta\beta_1\beta_2\frac{D_2^H}{\eta D_2^H + (1-\eta)D_2^L}I + (1-\eta)\beta_1\beta_2\frac{D_2^H}{\eta D_2^H + (1-\eta)D_2^L}I - \beta_1\beta_2 I\right] \\
&= 0. \quad (42)
\end{aligned}
$$

We used Jensen's inequality and that $(\cdot)^{\beta_1}$ is convex in the first inequality and $X_E^{pool} \geq \frac{\beta_1}{\beta_1-1}\frac{r-\mu}{\eta D_2^H+(1-\eta)D_2^L}I$ in the second inequality. Define now

$$
\xi \equiv \frac{1}{X_E^{pool}}\left[\left(\frac{X_D}{X_E^{pool}}\right)^{\beta_2} - \left(\frac{X_D}{X_E^{pool}}\right)^{\beta_1}\right] > 0,
$$

then

$$
\begin{aligned}
&\frac{\partial R_2}{\partial X_E^{pool}}(X_D, X_E^{pool}) \\
&= \xi\left[\eta(\beta_1-1)(1-\beta_2)\frac{X_E^{pool}D_2^H}{r-\mu} + (1-\eta)(\beta_1-1)(1-\beta_2)\frac{X_E^{pool}D_2^L}{r-\mu} + \beta_1\beta_2 I\right] \\
&\ge \xi\left[\eta\beta_1(1-\beta_2)\frac{D_2^H}{\eta D_2^H + (1-\eta)D_2^L}I + (1-\eta)\beta_1(1-\beta_2)\frac{D_2^L}{\eta D_2^H + (1-\eta)D_2^L}I + \beta_1\beta_2 I\right] \\
&= \xi\left[\beta_1(1-\beta_2)I + \beta_1\beta_2 I\right] = \xi\beta_1 I > 0,
\end{aligned}
$$

if $X_D < X_E^{pool}$.

Thus $\frac{dX_E^{pool}}{dX_D} > 0$. ■

A.4 Proof of Proposition 7

Proof of Proposition 7. We analyze first a situation when game starts in the pooling region $(X_D, X_E(\eta_0))$ and the market process enters the semi-pooling region $[X_S, X_D]$. Seeing new lows in $[X_S, X_D]$ without separation the entrant accordingly updates its belief w.r.t. the incumbent's type taking into consideration the strategy ϕ and always enters at the optimal entry point according to its information. Precisely, on the equilibrium path its belief η_t is a mapping $\eta_t : \bar{Z}_t \times \phi \to [0,1]$, such that $\eta_t(\bar{Z}_t, \phi) = (1 - \phi(\bar{Z}_t))\eta_0$, provided no deviation occurred by t. After observing a deviation from the pooling price, the entrant correctly assigns all the probability weight to the weak incumbent and enters at X_E^H.

Randomizing over its deviation trigger, the weak incumbent needs to remain indifferent between separating and not separating at any point in $[X_S, X_D]$. Suppose that at first time X_D is reached, the probability that the incumbent deviated to its monopoly price so far is δ. The incumbent must be indifferent between deviating with an entry at X_E^H and not deviating with an entry at $X_E((1-\delta)\eta_0)$. By the definition of X_D, see (16), this implies $\delta = 0$ and agrees with the claimed strategy ϕ. A similar argument shows that if a new low in X is reached below X_D, the probability that the incumbent deviated so far must increase so that the incumbent remains indifferent between its randomized strategies. It was noted in the text preceding the proposition that the weak incumbent deviates from the pooling price with probability 1 if the market state process reaches X_S. So below X_S, it is a separating equilibrium in which both the incumbents types choose their own unconstrained monopoly prices.

If the game starts in $[X_S, X_D]$, the analysis is similar. The incumbent plays according to ϕ and from the onset of the game randomizes over playing the pooling price and its unconstrained monopoly price. ■

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