CAPACITY PLANNING UNDER UNCERTAINTY: AN ASIAN OPTION APPROACH

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Abstract

This short paper introduces the concept of Asian options in the capacity choice literature. We develop a simple model for optimal capacity setting under average demand uncertainty for a single firm. When the firm faces moderate or significant stochastic demands in its current product line, expanding capacity is beneficial. If the demand is extremely stochastic, a capacity lag or reduction is more profitable.

Keywords

Arithmetic Asian Options, Fixed Strike, Optimal Capacity

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Introduction

Asian options are contingent claims written on the average, arithmetic or geometric, of an underlying asset. They have been introduced in the derivatives industry to overcome price manipulation problems relative to European options (i.e. speculators were used to drive up the gains from the option near to the maturity date) (Rogers and Shi, 1995). These instruments are usually traded on foreign currency and interest rate accounts (Turnbull and Wakeman, 1991). Despite a wide coverage of the subject by mathematical finance scholars (Fu et al., 1999; Dufresne, 2000; Vecer, 2001; Henderson and Wojakowski, 2002), the number of average option applications in the real option literature is relatively scarce (Nembhard et al., 2000; Choi et al., 2002; Kleindorfer and Wu, 2003)[2]. This is perhaps due to the rare investment situations involving path dependency on the average of a real asset. In the field of capacity planning and control, however, this condition is not uncommon. Planning for capacity using the forecasts of an average expected demand constitutes one of three key strategies characterising the long-term manufacturing structure of a firm's operations (Russell and Taylor, 1995). Average capacity strategies indeed represent the set of long term decisions aiming to establish a firm's overall level of resources, based upon the movements of an average expected demand (Russell and Taylor, 1995). Real capacity options can be exercised within this framework.

This paper proceeds as follows: the second section covers some theoretical background on capacity planning and reviews a selection of real options works related to the same topic. The third section focuses on the implementation of our capacity model using Asian pricing techniques. We conclude with a discussion of results and implications.

Theory and background

In a firm manufacturing strategy, the planning of capacity is a decision of vital significance. Indeed, it is the strategic relationship between demand and capacity that establishes a firm's overall level of resources and hence determines future organisational performance (Tan, 2002). An inadequate capacity strategy might lose

[2] Choi et al. (2002) present a valuation methodology for optional calling plans on free phone calls in the telephone industry. Nembhard et al. (2000) and Kleindorfer and Wu (2003) mention the existence of such kinds of options as exotic derivatives.

customers and limit growth. A successful capacity strategy will naturally drive up profits. Capacity planning is defined as the set of long term decisions (1 to 5 years) enabling firms to expand or reduce capacity as response to stochastic demand fluctuations (Olhager et al., 2001). It determines when and (by) how much this capacity should be altered. Three different capacity strategies are listed in the elementary manufacturing literature (Hayes and Wheelwright, 1984; Russell and Taylor, 1995): lead, lag or track. Lead implies an addition of capacity in anticipation of increasing demand while lag conveys the opposite (Olhager et al., 2001). Track is a switching strategy intending to minimise differences between capacity and demand levels. Ohlager et al. (2001) argue that the mix of lead and lag is one specific case of tracking strategy. Russell and Taylor (1995) refer to average capacity planning (defined in the introduction) as another. The average capacity strategy is of particular interest to this paper. We develop a simple model for optimal capacity setting under average demand uncertainty for a single firm. Much research has implemented contingent claim principles to capacity problems in the presence of irreversibility and uncertainty[3] (Pindyck, 1988; Chung, 1990; Bean and Higle, 1992; Dangl, 1999; Birge, 2000; Pennings and Natter, 2001; Tan, 2002; Kleindorfer and Wu, 2003) but none has treated the topic using average capacity strategies or Asian type options.

Model and application

We suppose that the demand, Q, for a product is log-normally distributed with mean μ and standard deviation σ. Firm's supply is limited by capacity X. Contribution margins per unit demand are equal to m and the discount rate is fixed at r. c denotes the cost per unit capacity . We assume that there is no inventory. Thus, the present value of expected company revenues, V, can be calculated as $e^{-rT}\mathbf{E}$ [m Min (Q, X)][4]. Let A denotes the arithmetic average demand at a specific time. Under an average capacity planning strategy (Q=A), the present value of expected benefits would become $e^{-rT}\mathbf{E}$[m Min (A, X)]. Thus:

$$V = m\, e^{-rT}\, \mathrm{E}[Min(A, X)] \qquad (1.1)$$

$$V = m\, e^{-rT}\, \{X + \mathrm{E}[Min(A - X, 0)]\} \qquad (1.2)$$

[3] See Van Mieghem (2002) for a detailed review.

[4] See Pennings and Natter (2001)

$$V = m\, e^{-rT} \{X - E[Max(X - A, 0)]\} \quad (1.3)$$

The expectation between the square brackets in (1.3) represents the pay-off of a fixed strike arithmetic put option (Turnbull and Wakeman, 1991). Let us call P_A the price of this option. V can be rewritten:

$$V = m\,(Xe^{-rT} - P_A) \quad (1.4)$$

V should be decreasing with uncertainty because min (Q, X) is a concave function of Q.

Since no general analytical solution for the price of the Asian option is known[5] (Levy, 1992; Rogers and Shi, 1995), we applied the Turnbull and Wakeman (TW) (1991) algorithm (validated by Monte Carlo simulations) to determine the price of the arithmetic average put option P_A.

The TW technique assumes that the distribution under arithmetic averaging can be approximately lognormal. It adjusts the mean and variance in order to be consistent with exact moments of the arithmetic average. The adjusted variables (b and σ_A) are used as inputs in the generalized Black and Scholes formula. The analytical approximation for a put under TW is given by[6]:

$$p_A \approx Xe^{-rT_2} N(-d_2) - Qe^{(D-r)T_2} N(-d_1) \quad (1.5)$$

$$d_1 = \frac{\ln(Q/X) + (b + 0.5\sigma^2{}_A)T_2}{\sigma_A \sqrt{T}_2}, \; d_2 = d_1 - \sigma_A \sqrt{T_2} \quad (1.6)$$

T_2 is the remaining time to maturity.

The adjusted volatility and mean are given by:

$$\sigma_A = \sqrt{\frac{\ln(M_2)}{T} - 2b} \quad (1.7)$$

$$b = \frac{\ln(M_1)}{T} \quad (1.8)$$

[5] The log-normality assumption does not always hold for the average of a set of log-normal distributions (Turnbull and Wakeman, 1991; Henderson and Wojakowski, 2002).
[6] Haug (1997).

The exact first and second moments of the arithmetic average are:

$$M_1 = \frac{e^{(r-D)T} - e^{(r-D)\tau}}{(r-D)(T-\tau)} \quad (1.9)$$

$$M_2 = \frac{2e^{(2(r-D)+\sigma^2)T}Q^2}{(r-D+\sigma^2)(2r-2q+\sigma^2)T^2} + \frac{2Q^2}{(r-D)T^2}\left(\frac{1}{2(r-D)+\sigma^2} - \frac{e^{(r-D)T}}{r-D+\sigma^2}\right) \quad (1.10)$$

τ is the time to the beginning of the average period. D is the dividend yield

Numerical Application

We assume a 5 year horizon ($T = T_2$). Let m= 1, c= 0.35, current demand Q = 100, actual capacity $\mathbf{X_0 = 100}$ and a discount rate of 10%. C= cX.

Q = 100	Capacity Levels							
Volatility	10	40	60	80	100	140	160	200
0.1	6.065307	24.26123	36.39184	48.52199	60.54688	76.65448	78.36224	78.69087
0.2	6.065307	24.2612	36.38026	48.24723	58.8874	72.47289	75.58552	78.01712
0.4	6.065299	24.09639	35.21717	44.79142	52.57995	63.40623	66.9934	71.77463
0.5	6.064619	23.61713	33.70433	42.083	48.87058	58.67059	62.16212	67.24194
0.8	5.904005	19.88654	26.71534	32.22265	36.76612	43.83607	46.64275	51.2388
Costs, c=0.35	3.5	14	21	28	35	49	56	70

Table 1.1: V and C a as function of X

Table 1.1 illustrates sensitivity results for V subject to variations in X and σ. These computations confirm that V is decreasing in uncertainty. This results from undercapacity when demand is high and overcapacity when demand is low. Figure 1.1 below graphically validates these conclusions. One can also see from the graph that optimal capacity occurs when V'= C'.

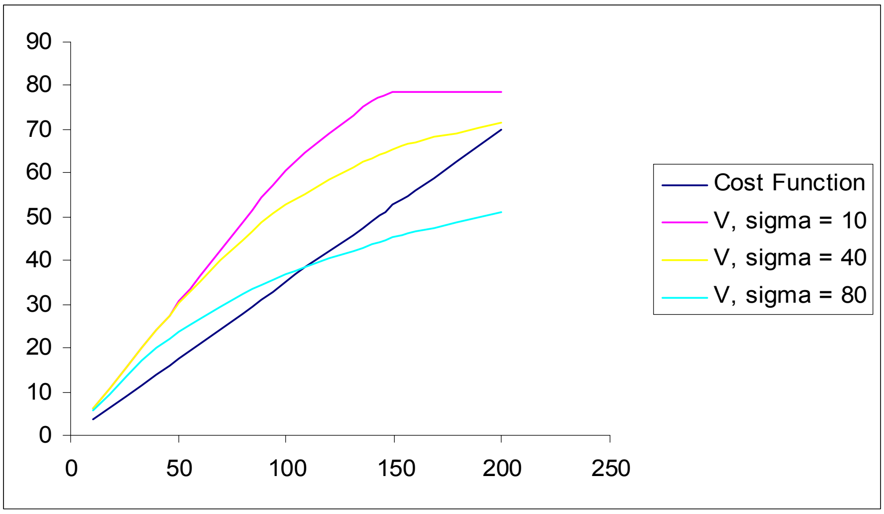

Figure 1.1: V and C as a function of X for σ = 0.1; 0.4 and 0.8

Table 2.2 highlights sensitivity analyses conducted on profits vis-à-vis changes in X and σ. It appears that optimal capacity for the various cases depends on the level of intensity of demand fluctuations. Up to a 50% volatility level, expanding capacity is optimal or profitable. Figure 1.2 reflects the maximum profits a firm may earn under each volatility case. When the demand is highly stochastic a capacity lag (σ = s = 0.5) or even reduction (s= 80%) would be optimal. Various expansion policies might be adopted as long as a firm is making profits. For the extreme volatility case (0.8), a capacity expansion would incur important losses.

Q = 100	Capacity Levels							
Volatility	10	40	60	80	100	140	160	200
0.1	2.565307	10.26123	15.39184	20.52199	25.54688	27.65448	22.36224	8.690875
0.2	2.565307	10.2612	15.38026	20.24723	23.8874	23.47289	19.58552	8.017121
0.4	2.565299	10.09639	14.21717	16.79142	17.57995	14.40623	10.9934	1.77463
0.5	2.564619	9.617132	12.70433	14.083	13.87058	9.670593	6.162117	-2.75806
0.8	2.404005	5.88654	5.715337	4.222655	1.766117	-5.16393	-9.35725	-18.7612

Table 1.2: Profit as a function of X

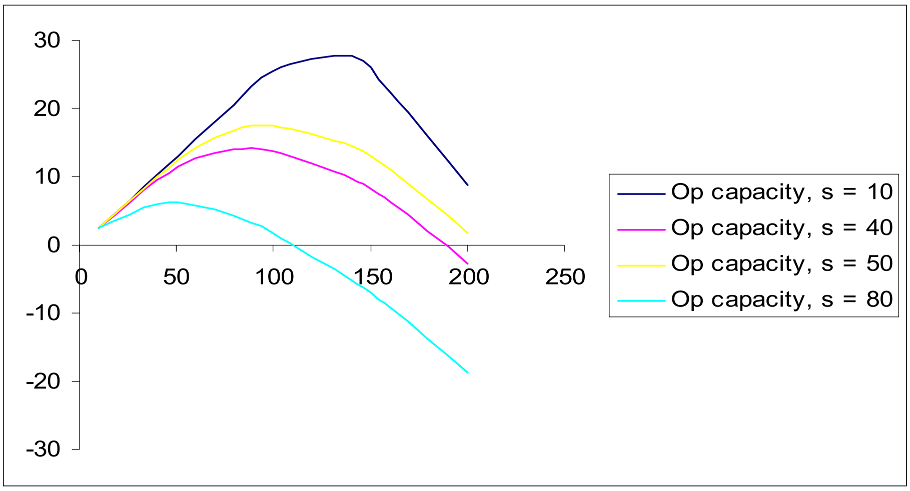

Figure 1.2: Profit as a function of X σ = 0.1; 0.4; 0.5 and 0.8

These computational results are in accord with the single firm model developed by Pennings and Natter (2001). Given that average value options are in most cases lower than European counterparts (Turnbull and Wakeman, 1991), we suggest that adoption of average capacity strategies with options to expand or reduce capacity can be more profitable. Testing this framework using American-Asian options would help determine the optimal timing of these actions.

Conclusion

This paper applies a contingent claim approach to capacity planning under uncertainty. It introduces the concept of Asian options in the literature and sheds light on the potential benefits of flexible planning in average capacity strategies. The incorporation of American-Asian options to this framework suggests that more realistic and accurate findings in terms of optimal timing can be produced. Implementing the Longstaff and Schwartz (1998) algorithm to this model is a direction for further research.

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