Fixed Maturity Projects: Interactions between financing and investment decisions in an Agency conflicts framework

Ricardo Correia*
Manchester Business School - University of Manchester
Manchester M15 6PB
Phone: +44(0)1612756441
Email: RCorreia@dom01.mbs.ac.uk

Sydney Howell
Manchester Business School - University of Manchester
Manchester M15 6PB
Email: sydney.howell@mbs.ac.uk

Peter Duck
Department of Mathematics - University of Manchester
Manchester M13 9PL
Email: duck@ma.man.ac.uk

David Newton
Nottingham University Business School - University of Nottingham
Nottingham NG8 1BB
Email: David.Newton@nottingham.ac.uk

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Abstract

Interactions between financing and investment decisions in a context of real options represent a challenging field of research which has attracted academic interest (e.g. Trigeorgis (1993) and Mauer and Triantis (1994)). The inclusion of agency conflicts in such a framework is supported by overwhelming empirical evidence (Long and Malitz (1985), Rajan and Zingales (1995), Mackay (2003)). Although theoretically there have been recent important contributions to this area of research (Childs et al. (2005), Mauer and Sarkar (2005)) there is still a wide ground to be covered.

In this paper we analyse a model of the conflicts between equityholders and debtholders regarding the optimal exercise moment of an investment option partially financed by a commitment loan. We assume time constraints for both the investment option and the subsequent firm. For the firm we consider a fixed maturity irrelevant of the moment when the investment option is exercised, aiming at a better reflection of the reality of many real options projects (e.g. petroleum explorations, mining firms and pharmaceuticals).

Our results support the coexistence of two different incentives (overinvestment and underinvestment) in one single type of real flexibility (option to invest) evidencing why perpetual models tend to fail in capturing the true complexity of agency conflicts. Furthermore, we show how overinvestment incentives clearly dominate underinvestment incentives, in terms of their impact in the option value, and how they tend to occur at or close to maturity of the investment option. We present competing predictions for the size of the agency costs and reiterate the impact of the agency conflicts in lowering optimal debt levels. Finally we demonstrate why different measures for the agency costs must be considered (in an approach similar to Leland (1998) and Mauer and Sarkar (2005)) in order to correctly capture their full implications.
1. Introduction

This paper analyses the agency costs arising from conflicting objectives between equity and debt regarding the optimal exercise decision of an investment option partially financed with a commitment loan. We focus on determining over and under investment incentives, and the optimal amount and price of external financing. The tax advantage of interest payments, rather than any type of budgetary constraints justifies the existence of debt financing.

Both the investment option and the firm are assumed to have finite maturities. In the case of the firm we consider a fixed maturity; namely the firm will terminate at the same date, regardless of the exact moment in time that the investment option is exercised, and this will give rise to a variable life expectancy of the firm.

There is general appeal for perpetual models, where both the investment option and the subsequent firm have infinite maturities (e.g. Leland, 1998, Mauer and Ott, 2000, Jou and Lee, 2004, Mauer and Sarkar, 2005). Perpetual models allow for analytical solutions and this can be considered as an advantage for two reasons; firstly, the models are themselves more tractable and secondly their results can be replicated more easily. However, in order to fully grasp the intuition, the effects and the complexity of agency conflicts, the use of time-constrained models is in some cases almost inevitable. Although many models require the use of complex numerical solutions (making them, to say the least, more hard to replicate) they still us usually provide us with a much richer set of scenarios and results (e.g. Brennan and Schwartz, 1984, Mauer and Triantis, 1994, Childs et al., 2005)\(^1\).

When dealing with the maturity of the firm our focus is on analysing the different processes that determine the life of the firm, rather than developing any sort of argument concerning its actual duration in years\(^2\).

\(^1\) In this specific case, it might be arguable that there should not be specified a maturity for the firm if there is a going concern, however, regarding the financing, it is not feasible to assume an infinite maturity for the commitment loan contract.

\(^2\) For the cases of patent protection or exploration rights the maturity is usually consensual, however, in what concerns a ‘normal’ investment project the estimate of the life of the firm constitutes a broad area of research on its own (see Andersen and Jessen, 2003).
This framework fits those cases where there is a fixed life for the combination of investment option and subsequent project life, like for example projects in which value drivers are patent protected until a specified future date or even exploration contracts. In both cases, whether the investment is performed today or in a year’s time, it is expected that the project will be terminated on a fixed date. Within this respect we define $T_i$ as the maturity of the investment option, $T_t$ as the maturity of the firm, $T_c$ as the firm life expectancy and $t_0$ as time zero.

[Figure 1]

An illustrative example might be a pharmaceutical company, which has registered a patent on a drug and has exclusive rights of commercially exploiting the drug for $N$ years (until $T_t$). When valuing a project to built a factory to produce the drug investors have to take into account, that if the investment takes place today the company has $N$ years of exclusivity in selling the drug, whereas if it invests in a year’s time, the duration of exclusivity drops to $N - 1$ years.

The incorporation of real options within an agency theory framework can be perfectly justified due to the fact that, the additional real flexibility that real options provide by definition, widens the scope of actions among which the agent can select in order to pursue his selfish interests. Considering that traditionally the real options field developed around projects that perfectly fit this framework (petroleum explorations, mining firms, pharmaceuticals (Brennan and Schwartz, 1984, Mello and Parsons, 1992, Cortazar, Schwartz and Casassus 2001, Schwartz, 2004), the assessment of the extent to which firms are sensitive to agency conflicts between the providers of capital becomes itself an interesting field of inquiry.

In addition, different scenarios for the investment cost are being analysed. We assume a constant cost over time and examine scenarios where the cost is a function of time growing at the inflation rate. For each case we measure the size of the agency costs and the impact of the investment cost on the incentives they give equityholders to over or underinvest relatively to both, an unlevered project and a case where we disregard agency conflicts.

Regarding the study of interactions between investment and financing decisions in the context of time constrained investment decisions, we incorporate agency conflicts extending on previous work by Mauer and Triantis (1994). We also extend the study of agency
conflicts in the context of investment options by considering time constraints for both the option and the subsequent firm (incorporating in a different environment some of the features of Leland (1998), Ericsson (2000) and Mauer and Sarkar (2005)) and by embodying to this framework growing investment costs and issuance costs of debt. For all these scenarios we study the effect of the agency costs, which relate to debt financing and the optimal amounts of leverage, and the investment decisions of the firm. The objective is to provide predictions and theoretical arguments regarding the size of agency costs and debt targets in a way that they can be empirically tested.

Section 2 presents the model in detail, providing the processes governing the value of the different securities and also the constituents of the agency costs; the latter are divided into direct financing effects and operational effects. Next we analyse the incentives of equityholders in terms of under and overinvestment and detail the determinants of optimal capital structure and what follows is a sensitivity analysis of leverage ratios and agency costs to changes in some of the more influential parameters of the model. Finally in section 4 we present the main conclusions of this work.

2. The Model

Like other models incorporating agency conflicts, our model of the equity-debtholder conflict makes two basic assumptions:

(i) Having the capacity to make decisions based on his self interest, the agent will do so - in our model this translates into the fact that equityholders base their investment decisions not on the criteria of firm value maximisation, but on self profit-maximising rationales.

(ii) Being aware that the agent has the capacity to act according to his self-interest, the principal anticipates this opportunistic behaviour and demands ‘remuneration’ based on this assumption - anticipating equityholders choice of a suboptimal investment policy debtholders price the financing contract assuming that equityholders will decide to invest when it is best for them rather than when is best for the firm.

We can then define two different policies. The first-best policy is defined as pursuing the objective of maximizing the investment option expected present value and the second-best policy as reflecting the objective of maximizing equity’s expected present value. In this sense we adopt a similar approach to Mauer and Ott (2000), Titman and Tsyplakov (2002),
Childs et al. (2005) and Mauer and Sarkar (2005), other authors provide a different rational for both policies (e.g. Leland, 1998, Jou and Lee, 2004, Moyen, 2000 or Mao, 2002). The difference in option value between both policies represents the economic cost of not obtaining the maximum profitability out of a specific investment opportunity ($V^0$) in a production facility, where $P$ represents the present value of the expected after-tax cash flows generated by the assets of this investment opportunity. The market value of its assets through time follows Geometric Brownian Motion, evolving according to the following process:

$$dP = (\mu - \alpha)Pdt + \sigma Pdz$$ (1)

Where ($\mu$) represents the drift rate, ($\sigma$) represents volatility and $dz$ is the increment of a standard Wiener process. The cash-flow rate ($\alpha$) is assumed to be a constant proportion of $P$.

Investors can build this facility by paying the initial investment cost ($I$) that grows at a constant rate of $\gamma$ and this cost will be partially financed by issuing debt. Debt is represented by a commitment loan; namely a contractual promise to lend an amount $F$ (face value of debt) at a pre-determined rate during a period of time ($T_i$) for which the firm will regularly make interest payments at the rate $r_d$. At the moment debt is issued the loan will have a cost represented by a proportion $\kappa$ of the face value of the loan. The maturity of this loan will be equal to the life of the project, meaning that will have a fixed maturity of $T_t$.

The debt value is assumed to be fairly priced. In other words, the pre-committed amount of debt financing ($F$) and the coupon rate ($r_d$) are such that the present value of the future debt claims is zero ($D^0 = 0$). This does not mean that the difference between the value of debt ($D$) and the face value of debt ($F$) will be zero at all the optimal exercise points, rather it simply means that the present value of these differences is zero. The coupon rate ($r_d$) will entirely reflect the fair credit risk and any estimated agency costs and it will be equal to the risk free rate ($r$) plus the fair risk spread.

The use of debt financing can be justified by the fact that interest payments are considered to be tax deductible. In our model taxes are represented by a corporate tax rate ($\tau$) and a

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3 The differences are sometimes more of form than substance, in both cases the first-best policy acts as a benchmark and all assume the impossibility of pre-commitment regarding the investment exercise policy, thereby it is possible to relate the results and conclusions regarding the agency costs. However, in some cases (Moyen, 2000, and Mao, 2002) because the authors disregarded the financial impact of the first-best policy it becomes difficult to compare results and conclusions.
symmetrical tax system allowing for full loss offset provisions. The optimal amount of debt will be determined by the equilibrium between the tax advantage of debt financing and the additional costs of debt financing, such as issuance costs, bankruptcy costs and agency costs. The choice of the financing mix, concerning leverage ratio is expected to differ between the first and second-best policy. Each policy will be defined by the choice of the face value of debt \( F \): however, in any case can the amount of debt financing be bigger than the investment costs. Furthermore, we assume the financial markets can provide investors with a portfolio of traded securities with the same value and risk characteristics of the investment project and the derivative securities. This allows investors to construct a continuously rebalanced self-financing portfolio combining a risky and a riskless asset that yields a constant return rate of \( r \) per year, replicating the value of the project.

We now briefly analyse the cash flows generated by the firm and accruing to each of the stakeholders. Naturally, since we are dealing with an investment option, there are no cash flows generated before the exercise of the investment option. We begin with the analysis of the unlevered firm. Although we will focus on the analysis of agency conflicts due to the existence of debt financing, the analysis of this case is still important for two reasons: it clarifies the impact of debt financing and because it represents the firms’ dynamics upon exercise of the abandonment option.

For the unlevered case, the firm instantaneously (in \( dt \)) releases cash flows at the rate \( \alpha \); these cash flows are destined to reward equityholders and hence \( \alpha P \) represents the cash flows accruing for equityholders.

In the levered firm case the cash flows realised remunerate, equity and debtholders. The structure of the cash flows is different because the interest expenses, contrary to the dividend payments, are assumed to be tax deductible. The cash flows generated by the firm in \( dt \) are represented by \( \alpha P + r_i F \tau \). The interest tax shield from the interest payments is added to the operational cash flows, debt does not directly generate additional cash flows but prevents that a percentage of the previous amount of taxes be paid out. At the same time, debtholders are entitled to their fixed claims equal to the interest payment \( (r_i F) \).

When the value of the cash flows generated by the firm is higher than the interests payable to debt the difference will be distributed to equityholders in a form of a dividend equal to

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4 The model could be easily adapted to accommodate a partially symmetrical or asymmetrical tax system and also personal taxation of coupons, dividends or capital gains.
\( aP - r_d F(1 - \tau) \). In the cases whereby the cash flows available to the stakeholders are insufficient to cover interest payments (when \( aP < r_d F(1 - \tau) \)), equity will make a cash injection equal to the difference and this cash injection will be in the form of a shareholders loan (pseudo equity, in this case without commitment to receive interests). These payments will always be made unless equity decides to abandon operations, defaulting on interest payments and surrendering the firm to debtholders.

Using standard arbitrage and hedging arguments we reach the partial differential equations that govern the value of the firm, debt, equity and investment option value. The analysis starts with the unlevered firm value, representing the firm value function upon exercise of the abandonment option. Next, we analyse the value of the securities after the exercise of the investment option and finally the value of the investment options.

2.1 All equity financing

The value of the unlevered firm is governed by the following PDE,

\[
\frac{1}{2} \frac{\partial^2 V^U}{\partial P^2} - \sigma^2 P^2 + \frac{\partial V^U}{\partial t} + (r - \alpha) P \frac{\partial V^U}{\partial P} - r V^U + \alpha P = 0
\]  

(2)

This process, with proportional dividend payments, will never drop bellow zero, since we do not consider fixed costs or salvage value (which would be unusual in such a flexible organization) there will be no default condition for the unlevered firm. In the worst case scenario it will simply converge to zero. It will also make it insensitive to changes in bankruptcy costs, corporate tax rates and maturity of the project. However, since our purpose is to analyse the conflicts regarding the optimal exercise moment of an investment option partially financed by debt, to disregard bankruptcy in the all equity firm greatly simplifies our calculations without any loss of realism in the results. Ultimately we can consider this as a firm without a fixed costs structure.

The value of the investment option represents the solution to the following PDE,

\[
\frac{1}{2} \frac{\partial^2 V^0}{\partial P^2} - \sigma^2 P^2 + \frac{\partial V^0}{\partial t} + (r - \alpha) P \frac{\partial V^0}{\partial P} - r V^0 = 0
\]  

(3)

Obviously, there are no cash flows in this equation. It is an option to invest (call on underlying assets) but it does not entitle anyone to the cash flows generated by those assets;
it resembles a call on a share and the dividend on the share and only upon exercise of the
call option can one be entitled to the stream of dividends\(^5\).

The value of the investment option is subject to the following set of boundary conditions:

\[
\lim_{P \to 0} V^0(P) = 0 \quad (3.1)
\]

Condition (3.1) determines that the option becomes worthless if the expected project value
becomes zero, and it is independent of both the choice of financing (all-equity or partially
debt financing) and the exercise policy followed (first or second-best).

\[
V^0(P,t_i) = V^U(P,t_i) - Ie^{r_i} \quad (3.2)
\]

Condition (3.2) represents the value matching condition at exercise of the investment option.
Note that we define the trigger not as a point (price or time moment) but as a function. There
is no single exercise trigger defined by project value or moment in time, but a combination
of trigger points defined by project value and time to maturity of the option. The exercise
occurs when the value of the option to invest equals the value of the operating firm less the
cost of investment (which itself represents a function of time).

### 2.2 Partial debt financing

The value of equity upon exercise of the investment option is governed by the following
PDE, where henceforth the subscript \(i\) for the securities, debt face value and interest rate,
indicates differences between the investment exercise policies \(i = F, S\) (\(F\) = first-best
investment policy, \(S\) = second-best).

\[
\frac{1}{2} \frac{\partial^2 E_i}{\partial P^2} \sigma^2 P^2 + \frac{\partial E_i}{\partial t} + (r - \alpha)P \frac{\partial E_i}{\partial P} - rE_i + \alpha P - r_d F_i(1 - \tau) = 0 \quad (4)
\]

The value of equity in both investment option exercise policies must satisfy several
boundary conditions. At maturity of the project we have the following terminal condition
(for \(T = T_i\)):

\[
E_i(P, F_i, r_d, T) = \max(V_i - F_i + \alpha P - r_d F_i(1 - \tau); 0) \quad (4.1)
\]

\(^5\) If the firm did not pay the cash flows (\(\alpha P\)) there would be no optimal exercise moment before maturity, the
value of keeping the option alive would always be greater than the value of holding the underlying asset.
Because building the plant (holding the asset) generates cash flows that accrue to the securityholders, there is a
moment where the present value of these cash flows becomes bigger than the value of keeping the option alive
and thereby we have an optimal exercise moment before maturity.
The value matching condition states that at maturity of the firm (debt contract) equity must choose between paying the face value of debt and the final interest payment or surrender the firm to the bank. The maximization condition and the value of 0 for the debt claim upon abandonment ensure that both the absolute priority and limited liability rules (at default) are respected.

Analogously, at any default moment prior to the maturity of the firm we have the following default value matching condition, considering that $t^L_i \in ]t, T[$.

$$E_i(P^L_i, F_i, r_{id}, t^L_i) = 0$$

(4.2)

where $P^L_i$ and $t^L_i$ represent the combination of prices and moments in time defining the optimal default frontier. As the subscript $i$ indicates the default boundary is different for the first and second-best exercise policies and the consequent optimal financing policy chosen.

The following represents the PDE that governs the levered firm value.

$$\frac{1}{2} \frac{\partial^2 V_i}{\partial P^2} \sigma^2 P^2 + \frac{\partial V_i}{\partial t} + (r - \alpha)P \frac{\partial V_i}{\partial P} - rV_i + \alpha P + r_{id} F_i \tau = 0$$

(5)

Once again the solution is subject to a set of boundary conditions where $t^L_i \in ]t, T[$.

$$V_i(P^L_i, F_i, r_{id}, t^L_i) = (1 - \delta) V^U (P^L_i, t^L_i)$$

(5.1)

Condition (5.1) represents the abandonment option value-matching condition. It determines that after abandonment the firm no longer ‘behaves’ as levered, because the role of debtholders now changes into the role of equityholders. The firm also suffers depreciation in the proportion of the bankruptcy costs ($\delta$). Note that the boundary applies to any moment from the exercise of the investment option to the maturity of the project (included).

Alike, the following PDE, governing the value of debt, was derived following similar non-arbitrage and hedging arguments,

$$\frac{1}{2} \frac{\partial^2 D_i}{\partial P^2} \sigma^2 P^2 + \frac{\partial D_i}{\partial t} + (r - \alpha)P \frac{\partial D_i}{\partial P} - rD_i + r_{id} F_i = 0$$

(6)

The solution for the value of debt is subject to the following boundary conditions:

$$D_i(P, F_i, r_{id}, T) = F_i(1 + r_{id})$$

(6.1a)
\[ D_s(P, F, r_d, T) = (1 - \delta) V^U (P, T) \quad (6.1b) \]

For \( T = T_i \), conditions \((6.1a, b)\) represent the terminal conditions at maturity of the project according to which two situations can occur. \((6.1a)\) represents the boundary in the cases where equity chooses to fully repay the face value of debt and the last interest. \((6.1b)\) represents the situation where at the maturity date the value of the firm is insufficient to repay the face value of debt and the last interest and equity chooses to default, surrendering the firm to debtholders.

\[ D_s(P^L_A, t^L_A) = (1 - \delta) V^U (P^L_A, t^L_A) \quad (6.2) \]

Similarly to condition \((6.1b)\), \((6.2)\) represents the default value-matching boundary condition, after default debtholders take possession of the firm, but now unlevered. Additionally, the value of the firm suffers a decrease in the proportion of the bankruptcy costs.

Next we analyse the value of the investment option under the two different policies, first-best aiming at maximizing firm value and second-best aimed at maximizing equity value. In addition to the investment option value on the firm \((V_i^0)\) we will also have to consider the present value of the future equity \((E_i^0)\) and debt claims \((D_i^0)\). The following PDE governs the value of the investment option on equity,

\[ \frac{1}{2} \frac{\partial^2 E_i^0}{\partial P^2} - \sigma^2 P^2 + \frac{\partial E_i^0}{\partial t} + (r - \alpha)P \frac{\partial E_i^0}{\partial P} - rE_i^0 = 0 \quad (7) \]

We now analyse the second-best policy value matching condition at exercise of the investment option, where \(P_{iS}\) and \(t_{iS}\), represent the combination of price and moment in time defining the exercise free boundary, where \(t_{iS} \in [t_0, T_i] \) and \( t = T_i \).

\[ E_i^0(P_{iS}, F_s, r_{dS}, t_{iS}) = E_s(P_{iS}, F_s, r_{dS}, t) + F_s (1 - \kappa) - I e^{\gamma_i} \quad (7.1) \]

This condition resembles a Net Present Value (NPV) formulation for equity, where we compare the present value of the equity claim \((E_s(P_{iS}, F_s, r_{dS}, t))\) against the cost equity bears for exercising the option \((I e^{\gamma_i} - F_s)\). In this case, the only additional element is the issuance

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6 For the sake of simplicity we shall henceforth refer to the present value of both claims as option on equity and option on debt.
costs of debt represented by \( (\kappa F) \) and this is indicative of the direct influence of the financing policy on the exercise boundary through its impact on the investment cost.

Next, we analyse the first-best case, beginning with the PDE that drives the value of the investment option \( (V^0_F) \).

\[
\frac{1}{2} \frac{\partial^2 V^0_{F}}{\partial P^2} - \sigma^2 P^2 + \frac{\partial V^0_{F}}{\partial t} + (r - \alpha)P \frac{\partial V^0_{F}}{\partial P} - rV^0_{F} = 0 \tag{8}
\]

We now elaborate on the first-best policy value matching condition at exercise of the investment option, once again \( P_{t,F} \) and \( t_{i,F} \) represent the combination of price and moment in time defining the exercise free boundary, where \( t_{i,F} \in [t_0, T_i] \) and \( t = T_F \).

\[
V^0_{F}(P_{t,F},F,F_{t,F},t) = V^0_{F}(t_{i,F},F,F_{t,F},t) - Ke^{\beta t} - F_P \tag{8.1}
\]

Condition (8.1) represents the value matching condition at exercise of the investment option for the first-best case. Once more note the resemblance with the NPV formulation; we compare the present value of the investment opportunity with the investment cost and again, we have to deduct the cost of issuing debt. In this case the exercise occurs when it is optimal for the firm. This condition governs the exercise of the investment option in the first-best policy; however, it is similar to the value matching condition for \( V^0_s \) in the second-best policy.

Finally we analyse the option on debt \( (D^0) \), similarly to the previous options its value is governed by a PDE of the type,

\[
\frac{1}{2} \frac{\partial^2 D^0_{i}}{\partial P^2} - \sigma^2 P^2 + \frac{\partial D^0_{i}}{\partial t} + (r - \alpha)P \frac{\partial D^0_{i}}{\partial P} - rD^0_{i} = 0 \tag{9}
\]

The use of the subscript \( i \) reflect the fact that the value matching exercise boundary is similar in both the first and second-best policy. At exercise of the investment option we have,

\[
D^0_{i}(P,F,F_{t,F},t) = D_{i}(P,F,F_{t,F},t) - F_i \tag{9.1}
\]

\^\footnote{For the first and second-best policies the main difference concerns the maximization objective (the boundary condition subject to maximization), thereby, we shall not make an exhaustive detail of all the boundaries (e.g. absorption or smooth pasting conditions), just keep in mind that the exercising value matching are symmetrical in both policies, having the maximization of \( E^0_s \) governing the exercise boundary on the second-best policy and the maximization of \( V^0_F \) governing the exercise boundary on the first-best policy.}
where, \( t_i \in [t_0, T_i] \) and \( t = T_i \). The value matching investment option exercise boundary condition reflects the difference between the market value of the loan and its face value on the issuance date. Recalling the assumption that we made about the fairness of the debt contract, as a reflection of the banks’ rational expectations, this does not translate directly into a difference of 0 between the market value of debt and its face value along the exercise boundary. It would if we were dealing with spot debt, however, since we are considering commitment debt it results in having an expected value of 0 for the value of \( D^0 \) at the moment the commitment loan is signed.

### 2.3 Debt financing effects on the firm value

Debt financing has several implications on the value of both the firm and the investment option. We divide these implications into five effects which are also governed by a PDE similar to the ones previously presented for the securities, firm and option values. The first effect is the interest tax shield effect of debt financing (\( Ts \) at firm level and \( Ts^0 \) at option level), representing the present value of the future tax shields earned by the firm for saving taxes with the interest payments. Second is the present value of the bankruptcy costs due to debt financing (\( Bc \) at firm level and \( Bc^0 \) at option level). Debt raises the default boundary, thereby exposing the firm to greater expected bankruptcy costs. The direct costs of issuing debt (\( Ic \), \( Ic^0 \)), representing a percentage \( \kappa \) of the amount raised externally (\( F \)) which is lost through direct administrative and transaction costs. Finally, the last two effects account for the impact that debt financing has on the investment decision. The impact occurs either by shifting the optimal exercise boundary to the left (making the investment earlier in time at the same price, or at the same moment in time at lower prices) (overinvestment effect (\( Ov^0 \))), or by shifting the exercise boundary to the right (under investment effect (\( Uv^0 \))).

We will now briefly analyse some of the boundaries at firm level (\( Ts \), \( Bc \), \( Ic \)) and option level (\( Ts^0 \), \( Bc^0 \), \( Ic^0 \)) beginning with the interest tax shields.

For \( T = T_i \), we have \( Ts(T) = r_f F \tau \)

Before maturity of the project, for \( t \in [t_i, T_i] \),

\[
Ts(t) = \begin{cases} 
  r_f F \tau \text{ for } E(t) > 0 \\
  r_f F \tau + PV(Ts(t+1)) \text{ for } E(t) \leq 0 
\end{cases}
\]

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8 Although underinvestment effects of debt financing of investment options may seem counterintuitive, they might happen for extremely large levels of debt financing when the marginal bankruptcy costs largely exceed the marginal tax shields, thereby making it less attractive to invest.
When equity does not default, the value of the tax shields equals the tax shields for the period \( r_d F \tau \), plus the present value of the future expected tax shields \( PV(Ts(t + 1)) \). When equity hands the firm over to debtholders the value of the tax shields only represent the tax shields for the period, since the firm afterwards becomes unlevered there are no more interest tax shields because debtholders become equityholders.

At the investment option exercise boundary we have,

\[
Ts^0(t_i) = PV(Ts(t_i + 1))
\]

Since the value of the tax shields accrues after debt is in place, at the moment the investment option is exercised the value of the tax shields is represented by the present value of the future expected tax shields.

For \( t < t_i \) the value of the tax shield effect is represented by,

\[
Ts^0(t) = PV(Ts^0(t + 1))
\]

At the ‘option’ level \( Ts^0 \) merely represents the stochastic discount of the expected future tax shields that firms will generate upon exercise of the investment option.

Considering the Bankruptcy costs at maturity (same definition for \( T \)),

\[
Bc(T) = \begin{cases} 0 \text{ for } E(T) > 0 \\ \delta V^U (T) \text{ for } E(T) \leq 0 \end{cases}
\]

When equity decides to exercise the abandonment option, surrendering the firm to debtholders, the bankruptcy costs amount to the proportion of firm value that is lost during the process. In this case a proportion of the unlevered firm value \( (V^U) \), since the post abandonment firm has no debt outstanding.

Before maturity of the firm \( (t < T) \), we have

\[
Bc(t) = \begin{cases} PV(Bc(t + 1)) \text{ for } E(t) > 0 \\ \delta V^U (t) \text{ for } E(t) \leq 0 \end{cases}
\]

At the investment option exercise boundary the bankruptcy costs represent the present value of the future expected bankruptcy costs. There is no possibility of simultaneous exercise of the investment option, and abandonment option by equityholders.

\[
Bc^0(t_i) = PV(Bc(t_i + 1))
\]

Similarly to \( Ts^0 \), \( Bc^0 \) at every moment in time preceding the investment option exercise boundary merely represents the stochastic discount of the expected future bankruptcy costs.

\[
Bc^0(t) = PV(Bc^0(t + 1))
\]

The analysis of the issuance costs is simpler since these costs only occur at the exercise moment of the investment option \( (t_i) \).
\[ Ic^0(t_0) = \kappa F \]

At this moment the issuance costs equal the face value of debt times the proportional costs \( \kappa^2 \),

\[ Ic^0(t) = PV(Ic^0(t + 1)) \]

Before the investment option exercise moment, \( Ic^0 \) represents the stochastic discount of the future expected issuance costs. The present value of the issuance costs \( Ic^0(t_0) \) will only vary in function of \( F \) and on the exercise moment for the investment option. Furthermore, the more delayed the exercise of the investment option is the smaller will the value of \( Ic^0(t_0) \) be, due to the greater discount that it is subject to.

We now analyse the overinvestment and the underinvestment effects\(^9\). According to our definition, overinvestment and underinvestment effects arise from differences in the exercise moments of the investment option between an unlevered firm and a similar firm with a levered capital structure. These effects are always negative or zero, because they reflect a deviation from an optimal operationally exercise boundary. Overall, the changes in the exercise boundary are positive for the firm because it allows the firm to increase the amount of interest tax shields. However, they incorporate costs; a direct cost from issuing debt in terms of increased bankruptcy costs and issuance costs, and indirect costs in terms of the operationally suboptimalitity of the exercise policy. We begin with the most common effect in terms of investment options, the overinvestment effect. Overinvestment effects occur when the levered firm invests earlier (or at a lower price) relatively to the unlevered firm case. When both firms (levered and unlevered) exercise or do not exercise the investment option at the same moment there are no such effects (both in terms of overinvestment and underinvestment). If the levered firm exercises the investment option \( (V^0_i = V_i - Ie^\alpha - F_i\kappa) \) and the unlevered firm does not, and \( V^0 = 0 \) (usually at maturity of the investment option or closer to maturity of the investment option), the overinvestment costs can be expressed as:

\[ Ov^0 = Ie^\alpha - V^U \]

This difference is positive representing the operational loss the firm suffered by exercising at a lower price than would be operationally optimal. The exercise decision of the levered firm was nonetheless optimal but it suffered this decrease in value. For a better

\(^9\) The nature of these costs is detailed in the next section.

\(^{10}\) Overinvestment and underinvestment do not represent investing more or less, but a deviation from an optimal benchmark. Throughout the analysis when referring to overinvestment and underinvestment we might be assuming the unlevered case as the benchmark, when analysing the individual policies, or the first-best policy as the benchmark usually when analysing agency costs. We tried to make it always clear, however, we advise careful analysis to avoid confusion.
understanding let us now analyse the value matching exercise boundary condition for the levered firm,

\[ V_i^0 = V_i - Ie^x - F_i\kappa \]

Similarly to expression 11 for the value of the options similar expressions are used relating the value of the levered firm with the value of the unlevered firm.

\[ V_i = V_U + TS_i - BC_i - IC_i \]

If we replace this expression into the value matching exercise boundary condition for the levered firm and recalling that \( F_i\kappa = IC_i \), we must not include \( IC_i \) to avoid doubling its impact, we get,

\[ V_i^0 = (TS_i - BC_i - F_i\kappa) - (Ie^x - V_U) \]

As long as the present value of the tax shields is greater than the present value of the bankruptcy costs, debt issuance costs, and the overinvestment costs \( (Ie^x - V_U) \) the exercise decision of the levered firm is perfectly justifiable.

A similar logic applies to the case where the levered firm chooses to exercise the option and the unlevered firm does not, but its value is not 0 (usually reflecting time moments away from maturity of the option), in this case we have,

\[ Ov^0(t) = PV(V^0(t + 1)) - (V_U(t) - Ie^x) \]

The overinvestment cost in this case reflects the difference between the value of keeping the option alive and the intrinsic value of the option.

The underinvestment effect is similar but reflects the inverse situation; namely cases where the unlevered firm would choose to exercise the investment option and the levered firm decides not to. Beginning with the first case, when the unlevered firm exercises the option and the levered firm investment option is 0, we have,

\[ Uv^0 = V_U - Ie^x \]

In this case the intrinsic value of the option is positive; however, the financial impact makes the levered firm value to be negative, essentially because,

\[ TS_i - BC_i - IC_i < -(V_U - Ie^x) \]

Although it is not common, if we consider that in some projects budgetary constraints may force firms to have more debt than would be optimal it becomes realistic that the present value of the direct costs of issuing debt are greater than the present value of interest tax shields in such a proportion that distort the investment decisions.
Again, a similar logic applies to the case where the unlevered firm would exercise the option and the levered firm does not but the levered firm option value is not 0 (again, reflecting time moments away from maturity of the option), in this case we have,

\[ U^{0}(t) = V^{0}(t) - Ie^{x} - PV(V^{0}(t+1)) + PV(U^{0}(t+1)) \]

The underinvestment case is then the difference between the intrinsic value and the value of keeping the option alive plus the present value of future underinvestment costs and the logic is the opposite of the overinvestment effects. In the overinvestment case we do not add the future overinvestment costs because when we invest earlier we “kill” the investment option, whereas in the underinvestment case the option is still alive and accruing the costs of the suboptimal policy.

Generally, whenever debt has a positive impact on firm value, there is an incentive to invest earlier in order to capture interest tax shields; however, if the impact of debt is negative the firm has an incentive to delay the exercise decision, the option is worth more alive than dead.

### 2.4 Agency costs

In this section we analyse the agency costs of debt beginning with their simplest expression,

\[ Ac = V^{0}_{F} - V^{0}_{S} \quad (10) \]

It is important to notice that there is no mispricing of the debt claims, contrary to other conflicts, and there is no explicit wealth transfer effect occurring. Equityholders do not consciously try to expropriate debtholders of their wealth by making extraordinary dividend payments or by in any other way reducing the asset basis of the firm. The main cost is different, and it is “paid” by the economy at large. The difference in firm value between both policies represents the economic cost of not obtaining the maximum profitability out of a specific set of assets. It should be reminded however, that this need not involve any unforeseen or unpriced transfer of wealth from debtholders to equityholders.

In order to better understand where the loss in value comes from we begin by analysing the impact of debt financing on the value of the investment option.

\[ V^{0}_{F} = V^{0} + Ts^{0}_{F} - Be^{0}_{F} - Ic^{0}_{F} - Ov^{0}_{F} - Uv^{0}_{F} \quad (11.a) \]

\[ V^{0}_{S} = V^{0} + Ts^{0}_{S} - Be^{0}_{S} - Ic^{0}_{S} - Ov^{0}_{S} - Uv^{0}_{S} \quad (11.b) \]

The previous expressions (11.a,b) relate the value of the same investment option in a case where the investment cost and the subsequent business activity are solely equity financed.
(V^0) to the case where they are partially financed with a commitment loan \((V^0_F, V^0_S)\). The differences between them are the interest tax shield effects of debt financing \((Ts^0)\), the present value of the bankruptcy costs due to debt financing \((Bc^0)\), the direct costs of issuing debt \((Ic^0)\), the overinvestment effect \((Ov^0)\) and the underinvestment effect \((Uv^0)\). The combination of expression (10) with expressions (11.a) and (11.b) represents the agency costs as:

\[
Ac = DFC + OC
\]

where,

\[
DFC = Ts^0_F - Bc^0_F - Ic^0_F - Ts^0_S + Bc^0_S + Ic^0_S
\]

represents the direct costs of debt financing such as loss of interest tax shields, and

\[
OC = Ov^0_S - Ov^0_F + Uv^0_S - Uv^0_F
\]

represents the costs deriving from the induced shifting in the investment option policy. One further difference between both costs concerns their source; the first arises directly by effects occurring at the firm level whereas the second one arises by effects occurring at the investment option level. This degree of detail in the analysis of agency costs is essential in order to fully understand their nature in a project with life constraints. The first term represents a direct consequence of debt financing on the value of the firm (the difference between firm value without debt financing and firm value with debt financing); however, as we mentioned earlier, the existence of debt financing shifts the investment option exercise boundary. When the impact of debt financing on the cash flows of the firm is positive, it induces an early exercise of the investment option. Although this early exercise is optimal in the levered firm case, it is can be considered as sub optimal in terms of a purely operational decision (unlevered case) creating therefore over or underinvestment costs. The definition of the optimal exercise boundary reflects the balance of the positive effect of the interest tax shields, with the negative effects of increased bankruptcy costs, issuance costs and over and under investment costs.

3. Numerical Analysis

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11 Although underinvestment effects of debt financing of investment options may seem counterintuitive, they might happen for extremely large levels of debt financing when the marginal bankruptcy costs largely exceed the marginal tax shields, thereby making it less attractive to invest.

12 We shall refer to these as Operational Costs (OC). The difference between these effects is common in the agency theory literature Myers (1977, pg 149) defines OC as the “costs of the suboptimal future investment strategy.”
What follows is an analysis of a case with a fixed maturity of 10 years for the investment project. This period is slightly shorter than the average patent protection period, but still higher than some R&D stages.

As far as the bankruptcy costs are concerned, they incorporate both the direct costs of distress and costs of disrupt of the normal business activities. It is not easy to present an accurate estimate of these costs due to the wide variability of the empirical results. As Gupton et al. (2000) point out, the wide spread on the estimations of bankruptcy costs and recovery rates is a source of frustration for credit risk modellers and investors. In the base case we assume a bankruptcy cost of 40% of the unlevered firm value, as it falls in the range of recovery rates empirically observed (as high as 86% by Franks and Torous, 1994, and 90% by Andrade and Kaplan, 1998, and as low as 47% by Gupton et al., 2000). These estimates were produced for bank loans, while other types of debt the bankruptcy costs tend to be higher. According to Dahiya et al. (2003), bank loans are typically considered as senior debt and are in most cases secure. In our case they are the solely source of debt financing and are unsecured. Hence, our base-case figure of 40% is somewhat above the lower bound of empirically observed rates of recovery.

The issuance costs are assumed to be 0% for the base case although later we will extend them to 5% of the face value of debt, they consist of bank fees, legal fees and other transaction costs. This figure may seem relatively high for a bank loan when compared with underwriting spreads (Drucker and Manju, 2005); nevertheless the underwriting fee does not reflect entirely the remuneration of the underwriter because part of it is reflected in the credit spread, however in our case the credit spreads only reflect the default risk of the firm and the costs arising from moral hazard. According to Krishnaswami et al. (1999) private lenders have an informational advantage over public lenders; therefore we do not consider on the credit spread any adverse selection costs. Instead, we assume that the bank has non-public information regarding the future potential of the venture and is aware of the volatility of the returns on the cash flow. As in any agency framework we also assume that equityholders will decide based on their profit-maximising rationale. However, the informed lender rationally anticipates this behaviour and forms his price of lending based on these estimates (higher credit spreads for the second-best policy case).

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13 Best et al (1993) support this hypothesis with empirical evidence.
Regarding the growth rate of the investment costs, we start off with a base-case value of 0%. In later stages we consider different scenarios where this cost evolves at the inflation rate (the rates considered will always be inferior to the risk free rate since we are dealing with ex-ante estimates (see Kandel at al, 1996). The remainder parameters were arbitrarily determined, however they are very similar to the values used in previous research for comparison purposes.

3.1 Impact of debt financing on the investment decision

We take off by analyzing the investment exercise boundaries for the unlevered and levered firms. This analysis has a dual target; to clarify the impact of debt financing in terms of the exercise decision (difference between unlevered and levered first-best case), and get a better insight of the incentives equityholders have when they invest using the criteria of equity value maximization rather than firm value maximization (difference between first-best and second-best case).

[Graph 1]

[Graph 2]

[Table 2]

By following a first-best policy of firm value maximization, investors have the incentive to move faster and invest earlier, relatively to both unlevered firm value and the second-best policy, in order to capture the cash flows and interest tax shields. However, and similarly to results reported by Mauer and Triantis (1994), the operational impact of this distortion of the investment policy is only marginal.

Graphs 1 and 2 illustrate a situation where two different types of incentives coexist during the life of the option\(^{\text{14}}\). The underinvestment incentives prior to the maturity of the option (Graph 1) are very similar to the debt overhang problem described by Myers (1977). Equity delays the exercise of the investment option in order not to share the benefits of the investment with the debtholders when equity bears a significant part of the costs. The definition of the optimal exercise boundary in the second-best policy balances several positive and negative effects. By exercising earlier (or closer to the first-best boundary) equity starts receiving dividends and interest tax shields earlier and it also partially transfers

\(^{\text{14}}\)In Graph 1, at maturity, the relative positions of the boundaries are not perfectly clear, however, as becomes clear in Graph 2 the second-best exercise boundary crosses both the unlevered and levered first-best, exercising at a significantly lower level than both the other cases.
the default risk to debtholders in the event of bankruptcy due to its limited liability (Mauer and Sarkar, 2005). On the other hand it shares part of the benefits with debtholders reducing its risk exposure and increases the probabilities of having to cover for any shortfall in the interest payments. By investing to early it also loses the value of waiting and the guarantee value of the investment option. In this case, the second effects dominate the first and we observe an underinvestment incentive prior to maturity of the investment option. However, these results cannot and should not be generalised to all the cases where investment options are partially financed with commitment loans. A mere increase in the pay-out ratios, or the possibility of financing shortfalls on interest payments with new debt or disinvestments would be sufficient to extend the overinvestment incentives from maturity to the full life of the option.

At maturity of the investment option (Graph 2) the situation is very different; in this case the decision does no longer refer to the timing of the investment, rather to whether the investment itself should take place or not. Consider a case where, at maturity, the present value of $P$ is relatively low giving rise to a negative $NPV$ of the project ($V_F < I$). The value of the debt claims in a fair contract is also negative, however, for equity it might still be beneficial to proceed with the investment as long as the loss of debt ($D_S - F_S$) more than compensates the negative intrinsic value of the exercise of the investment option. Following a first-best policy, at maturity, exercise occurs when $V_F - I > 0$. Consequently, whenever $I > V_F$ the firm chooses not to invest. Considering the same situation (negative $NPV$ to firm), according to the second-best policy exercise would still occur as long as $F - D_S > I - V_S$.

Although the analysis of Graph 1 gives us the impression of a severe underinvestment problem for the second-best policy, in reality the overinvestment effects of the second-best policy at maturity of the option more than offset the underinvestment effects along the exercise frontier. The results presented in Table 2 clearly show how marginal are the

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15 Upon default debtholders bear the full bankruptcy costs, however, prior to default, debtholders receive the interest payments they are entitled to, and equity bears the cost of covering for any shortfall. This is the reason why we mentioned that the default risk is ‘partially’ transferred.

16 For a detailed analysis of early exercise features please see Subrahmanyam (1990).

17 The effect of the guarantee value of the option is only marginal because of the aggressive overinvestment incentives equity have at the maturity of the investment option.

18 This case reflects Jensen and Meckling (1976) and Myers (1977) conclusions regarding the exercise of investment options with a negative $NPV$.

19 By underinvesting before maturity following a second-best policy, investors lose a portion of the tax shield effects of debt financing; by overinvesting at maturity they significantly expropriate debt of part of their wealth. At the investment option level, the present value of the underinvestment effect is thereby much smaller.
impacts of the overinvestment incentives for the first-best policy and the underinvestment incentives for the second-best policy. These conclusions help supporting the arguments of previous research focusing on the interactions between investment and financing decisions considering European profiles for the real options.

It is not common to find the simultaneous existence of two different incentives (overinvest and underinvest) in a single framework (investment option) analysing one single conflict (equityholders-debtholders). To our knowledge Mao (2002) is the only paper reporting similar evidence, however in his case it was built into the model by considering growing marginal volatility of the investment\textsuperscript{20}, where in our case it represents a result.

3.2 Optimal capital structure

Graph 3 presents, for different leverage ratios, the values of the option financed with equity ($V^0$), option partially financed with debt following a first-best policy ($V_F^0$) and option partially financed with debt following a second-best policy ($V_S^0$).

The value of the option, entirely financed by equity is unaffected by changes in the debt ratio. Both levered options ($V_F^0$, $V_S^0$), are sensitive to changes in leverage and have an optimal amount of debt that maximizes their present value. Although the graph does not portray this maximum very clearly, the optimal amount of debt financing is greater for the first-best policy ($F_F^{\text{max}} = 65$) than for the second-best policy ($F_S^{\text{max}} = 62$)\textsuperscript{21}.

In the first-best case, from no leverage to a 65% leverage ratio the marginal tax shield effect is greater than the combined marginal bankruptcy and operational costs ($Ov$ and $Uv$). For low levels of debt the contribution of the interest tax shields is large when compared with the low bankruptcy costs. At the same time, the level of debt is not significant to induce significant changes in the investment policy.

At 65% the marginal contribution of the tax shield effect equals the contribution of the bankruptcy and operational effects. At this point, the marginal benefits will equal the marginal costs per additional unit of debt financing. For levels of debt above 65% additional

\textsuperscript{20} It is arguable that this might simply represent a risk shifting case.

\textsuperscript{21} Please note that we consider the leverage ration as the proportion of the investment cost that is financed with debt; generally this ratio relates debt value to firm value. In the present analysis to try and present the ratio as a proportion of the firm value would involve estimating this ratio along the exercise frontier and averaging it out. See also Table 2 for more detailed results.
units of debt financing impact negatively firm value because the marginal bankruptcy and operational costs exceed the tax shield benefits. As the level of debt increases, the marginal tax shield effect does not change much while the marginal bankruptcy cost increases at a fast rate. This fast growth is explained by three effects. The first relates to the fact that increased levels of debt directly raise the default frontier. Secondly, a raise in the default frontier translates into more default occurrences and at higher firm values. Since the bankruptcy costs are assumed to be a fixed proportion of the firm value this fact generates increasing bankruptcy costs. Finally, increased debt and increased bankruptcy costs translate into higher interest payments, making it costlier for equity to cover any shortfall and faster to exhaust its market value. Thereby, bankruptcy is not only costlier but it also occurs faster.

The tax shield analysis is simpler and generally they seem to be growing in constant increments\(^2\). Although higher levels of debt and interest rates increase the level of tax shields, the resulting raised default boundary reduces the occurrences in which the interest payments are actually made, and hence it partially cancels out the previous effect.

In the second-best policy these effects are also present; however, and contrary to the first-best case the impact of the operational effects is significantly higher. As the level of debt increases, so directly does the level of overinvestment incentives (at maturity or close to maturity). Recalling the investment option exercise boundary for the second-best policy investment occurs when:

\[ E_S - (I - F_S) > 0 \]  
(a)

We also know that,

\[ E_S = V_S - D_S \]  
(b)

By replacing \( E_S \) by (b) in expression (a) and arranging the terms we have:

\[ (V_S - I) + (F_S - D_S) > 0 \]

While the marginal tax shield effect is greater than the marginal bankruptcy cost and operational effect (\( Ov \) and \( Uv \)), for higher values of debt the first difference (\( NPV \) of the project) increases. In addition, for higher values of debt, the second difference (difference between the market value and the face value of debt) always increases. In favourable movements of the underlying asset, the higher interest rate and the lower bankruptcy costs

\(^2\) This behaviour reflects reasonable amounts of debt, for extreme amounts of debt the marginal tax shield effect is expected to decrease and eventually even become negative due to the impact of exceptionally high bankruptcy probabilities.
increase the market value of debt above its face value (this is the situation that only marginally interests us). In adverse movements of the underlying asset, and despite the higher interest rate, the increased bankruptcy costs reduce the market value of debt increasing its difference with the higher face value of debt. Although at this stage the project has most likely negative \(NPV\), the negative present value of the debt claims represents ‘free’ additional financing that equity can expropriate in order to mitigate the negative \(NPV\) of the project. From then on, and due to the limited liability rules, equity may benefit from an upward movement of the market (that will only partially be shared by debtholders) and in the worst case scenario it does not lose more than \(I - F_S\).

While in the first-best policy, the impact of additional debt in terms of the operational effects is much smoother and indirect (\(V_F\) grows or decreases according to the dominance of tax shield or bankruptcy costs effects), in the second-best policy it impacts directly on the investment decision. This results into increased overinvestment costs for higher leverage ratios. Also, and contrary to the first-best policy overinvestment effects before maturity, the size of the overinvestment effect in the second-best policy is considerable. These effects occur at relatively low debt ratios, thereby precipitating the moment when the marginal tax shield effect equals the marginal bankruptcy cost and operational effect (mostly overinvestment as previously described). Hence, in the presence of agency conflicts between debtholders and equityholders, regarding the optimal exercise moment of an investment option, we predict a lower level of optimal leverage.

These arguments help to support possibly the most empirically tested hypothesis in the field of real options, commonly referred to as the ‘agency hypothesis’. It states that increased flexibility exacerbates the opportunistic behaviour of equityholders and thereby the agency costs, reducing the debt capacity of the firm. The alternative hypothesis, commonly designated as ‘value hypothesis’ states that additional real flexibility increases the value of the firm, thereby allowing for greater debt capacity.

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\(^{23}\) For a more thorough development and analysis of these arguments please see: Myers (1977) and Jensen and Meckling (1976).

\(^{24}\) Early work by Bradley, Jarell and Kim (1984) has shown low levels of leverage for firms generally associated with growth options. Long and Malitz (1985), Smith and Watts (1992) and Barclay, Smith and Watts (1995) also presented evidence of a negative relationship between market-to-book value and financial leverage. This evidence was later corroborated by Rajan and Zingales (1995), where the authors show that the relation is not only negative but significantly negative.

More recently, Barclay, Morellec and Smith (2003) and Mackay (2003) overall confirmed the agency hypothesis, although Mackay (2003) shows contexts under which the value hypothesis has some validity.
In graphs 3 and 4 we analyse the individual effects creating the differences in value between the first best and the second best policy, illustrating the previous analysis.

[Graph 4]

Confirming the results of Mauer and Triantis (1994) the small effect of debt financing in terms of its impact on the investment option exercise decision is made explicit. In the graph this impact is depicted by the Ov0 and Uv0 curves. In the worst case scenario for 99.9% debt financing, the size of the operational costs (Ov0 and Uv0) amounts to only 0.9% of the option value. Clearly the operational costs have a little impact on determining the size of optimal debt financing, at the optimal amount of debt (aprox. 62% leverage), the size of the operational costs is merely 0.03%. The optimal debt target results then from debt effects at firm value\(^{25}\). The tax shields effect represents an almost linear function of the amount of debt financing, the bankruptcy costs however, increase rapidly as the amount of debt grows. At 1% leverage ratio the bankruptcy costs represent .5% of the interest tax shields. At around 55% of leverage the marginal tax shield effect equals the marginal bankruptcy costs. For a debt ratio of 99.9% the marginal bankruptcy cost effect represents approximately three times the size of the interest tax shield effect.

[Graph 5]

The debt financing effects for the second-best policy present remarkable similarities to the first-best policy and at first sight the effects seem similar but magnified. For the same amounts of debt the effects in terms of interest tax shields and bankruptcy costs are greater in the second-best policy. The interest tax shields remain almost a linear function of the amount of debt financing, and the bankruptcy costs also exhibit a concave pattern. The higher values of interest tax shields reflect the higher interest rates paid in the second-best policy and for the same amount of debt financing the interest payments for the second-best policy are significantly higher. Higher interest payments are harder to satisfy and exhaust more rapidly the market value of equity, and raise therefore the default frontier. In this case equity decides to surrender the firm to debtholders sooner, or at higher firm values and consequently the bankruptcy costs are expected to grow faster, as displayed in graph 4. The differences between the marginal tax shields effects and bankruptcy costs are in this case more obvious and increase faster with the increase in leverage. With 1% debt financing the marginal bankruptcy cost represents, again, only .5% of the marginal interest tax shield.

\(^{25}\) Since in the base case we considered that the firm does not have issuance costs (\(k = 0\))
however at a 99.9% level of debt financing the marginal bankruptcy costs represents approximately 30 times the effect of the interest tax shields.

A more important observation is that the operational costs (over and underinvestment effects resulting from the shifting of the optimal exercise boundary induced by debt financing) are in this case much more pronounced and are actually responsible for the sudden drop in the option value for higher leverage ratios. Note also the insignificant value of the underinvestment effects in the second-best policy, similarly to the overinvestment effects in the first-best policy.

3.3 Agency costs of debt financing

The agency costs are now explained in more detail, focusing on the agency costs of debt financing as a percentage of the first-best option value ($V_F^0$).

[Graph 6]

[Graph 7]

Graph 5 presents the agency costs for different leverage ratios where $AC$ represents the total agency costs ($V_F^0 - V_S^0$), $DFC$ represents the direct financing costs and finally $OC$ represents operational costs. Direct financing costs correspond to the difference of financial synergies between the first-best and the second-best investment policies (debt effects at firm level: interest tax shields, bankruptcy costs and issuance costs). For the $OC$ let us revisit the overinvestment and underinvestment effects and clarify their economical intuition. Before maturity the overinvestment costs represent the loss of insurance value and the increase in the time value of money of the investment cost. At maturity they represent the negative net present value of the unlevered firm upon exercise of the investment option. In the case of underinvestment and before maturity, these costs reflect the loss of operational cash flow due to the delayed investment. At maturity they represent the positive NPV of the unlevered firm forgone because the investment was not realised. As it becomes clear, they all represent essentially a portion of purely operational value that will expectably be lost due to the existence of debt financing (thereby its designation as operational costs). As explained in section 3.2, these effects are more severe in the second-best case due to the incentives they create for exercising the option at negative NPV’s. As the level of debt and the probability of equity exercising the investment option at negative NPV’s increase so does the cost of debt. This creates an interesting effect (see Graph 6 for levels of debt between 19% and 46%). We observe a raise in the level of interest tax shields for the second best policy, due to the
higher interest rate, but a very marginally increase in the additional bankruptcy costs creating negative direct financing costs. In other words, for this debt levels the financial synergies for the firm in the second-best policy are higher than in the first-best policy. Graph 2 does not portray any evidence of agency costs for this levels of debt because the OC and the DFC effects actually cancel out each other; however, the analysis of Graph 6 shows a difference in credit risk spreads above 20% of debt financing. This increase of the credit spreads for small impacts of agency costs has been reported by Leland (1998) and, as the author argues, may help explain why some models of risky debt pricing tend to present predictions for the credit spreads which were proved to be too small.

In Graph 6, above the debt level of 46% we observe that the OC cost rises sufficiently to more than compensate the negative DFC, revealing for the first time the agency costs of debt at a firm level. This does not mean that they did not existed at lower levels of debt, rather that they simply had not manifested themselves in terms of firm value, despite the fact that they were already inducing suboptimal investment policies and higher credit spreads. For levels of debt above 67% we observe that the OC steadily grows and DFC grows rapidly for the reasons explained in section 3.2.

3.4 Sensitivity Analysis

This section analyses how changes in the parameters influence the choice of optimal debt levels and agency costs.

[Table 3]

From a first analysis of Table 3 results it immediate stands out that a change in parameters which increase (decrease) the optimal debt targets for the first-best policy are accompanied by increases (decreases) in the agency costs of debt. This fact is interesting since it refers to the direct positive relationship between leverage and agency costs. Several papers (e.g. Leland, 1998) have tried to analyse the existence of a positive relationship between debt levels and agency costs, something that our results seem to support.

We now analyse the sensitivity of the target debt ratios and the agency costs to variations in volatility. For projects with low volatility the debt target is high (83 – first-best; 81 – second-best), the less risky the project is the lower are the expected bankruptcy costs and the bigger is the financial effect (difference between the tax shields effect and the bankruptcy costs effect). Naturally there are high operational costs for both policies, since the financial incentives significantly lower the optimal investment exercise boundary relatively to the
unlevered firm value. As volatility increases, for the first-best case, debt becomes less attractive due to rapidly growing bankruptcy costs. However, the growing bankruptcy costs become less significant (reduced marginal bankruptcy costs) as the increase in volatility significantly increases the investment option value. The change occurs at the minimum target debt ratio (between 30% - 40% volatility). For higher values of volatility we observe that the debt target ratio rises steadily. In the second-best case beyond the minimum debt target level, increases in volatility are not accompanied by steadily increases in the debt target ratio. Contrary to the first-best case the operational costs are more pronounced and growing volatility, by increasing the value of the option, encourages equity to delay the investment exercise moment, thereby increasing the underinvestment costs and increasing the probability of exercising the investment option at negative NPV values at maturity. In other words, increases in volatility by exacerbating suboptimal investment choices by equityholders limit the growth of the target debt ratio and eliminate the effect of the higher investment option values. In our case, after the initial increase in the debt ratio we do not observe any obvious trend with the debt target ratio stabilising at around 60%. The impact of volatility on the agency costs becomes straightforward confirming the results of Leland (1998) and Childs et al. (2005). Increased volatility is accompanied by increasing agency costs. It is interesting to observe how the nature of the agency costs shifts as volatility increases. At low levels of volatility the agency costs are born essentially from operational effects, for high values of volatility, the direct financing effects represent a significant part of the agency cost in terms of the loss of interest tax shields due to the increase in the difference between the optimal debt target ratios.

Consider now that the volatility of the returns of the underlying asset represents a proxy for increased flexibility. We can then relate the first-best case with the ‘value hypothesis’ and the second-best case with the ‘agency hypothesis’ and it becomes clear how our results represent an illustration of both competing hypothesis.

We predict than, that the agency costs of debt represent a positive function of volatility. Projects with higher volatility are more prone to suffer from agency conflicts and present lower debt target ratios, thereby favouring the agency hypothesis for projects partially financed by debt.

In terms of the impact of increased volatility on the investment option exercise decision it is curious to find that in a time constrained environment and similarly to Mauer and Sarkar
we find that as volatility increases the investment is delayed by both policies. However, contrarily to our results, the authors report an inverse relationship between volatility of the underlying and agency costs. The fact that our framework collapses to a ‘now or never’ decision as maturity of the option approaches, translates into a significant increase of the overinvestment costs at maturity of the option (for which debt responds with significant increases in the credit spreads thereby reducing the appeal for debt financing) which may help explain the differences between the results.

Similarly to Leland (1998) we also find that changes in the pay-out ratios have little impact on the agency costs, for the first best case we also present a decline in the optimal debt levels and an increase in the credit spreads as the pay-out is increased. This fact relates to the decrease in the collateral value of the assets in place (higher payout ratios imply lower reinvestment on the firm). In the second best case we find no obvious trend for the debt targets.

As the investment cost growth rate \( \gamma \) increases two effects are easily predictable, the value of the option decreases in both the unlevered and levered (first and second-best) cases and the investment occurs at earlier moments in time. Because this effect impacts directly the exercise boundary and affects both the unlevered and levered firms (first and second-best) its impacts are mixed and we are unable to determine any clear patterns in terms of the agency costs. In terms of the optimal debt targets we observe an increase for the first-best case and no obvious pattern for the second-best case, however, the optimal debt levels are still always lower than for the first-best case. The impact of growing investment costs is mixed and the lack of theoretical studies of the impact of \( \gamma \) on both the agency costs and optimal leverage remains an area open for further research.

As expected, as the issuance costs of debt increase the optimal debt target decreases, higher issuance costs reduce the appeal for debt financing. In terms of agency costs, we find them to be a decreasing function of the issuance costs. Similar results were reported by Childs et al. (2005) in their static financing case. For sufficiently high issuance costs the agency costs are nonexistent. The decrease in the optimal debt target produces these results (less debt, less agency conflicts). However significant the reduction in agency costs is, issuance costs

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26. As evidenced by the lower overinvestment effects for the first-best policy and higher underinvestment effects for the second-best policy.

27. The results unfortunately do not always present monotonic patterns, whenever that is case we assume the relationship is weaker if it becomes clear the existence of a dominant effect (increase / decrease) and/or indeterminate as in the present case.
do not represent a viable moral hazard control mechanism. As we can easily observe their negative impact on the investment option value is far greater than that of the agency costs. Also, growing issuance costs do not reduce the incentives equity has to overinvest, they merely make it costlier\textsuperscript{28} to overinvest.

Bankruptcy costs have a similar impact. Increasing bankruptcy costs reduce the optimal debt target for both policies, ultimately levelling them out. The reduction in the optimal debt levels induces a reduction on the agency costs as the optimal exercise boundaries converge\textsuperscript{29}. Similarly, decreases of the corporate tax rate also diminish the agency costs, by making debt less attractive. For low levels of debt, overinvestment incentives are marginal and debt has very low risk\textsuperscript{30}. Analogous results are reported in Leland (1998), Mauer and Sarkar (2005) and Childs et al. (2005) (for the relationship between bankruptcy costs, debt levels and agency costs).

As the risk free rate increases, the value of the firm also increases thereby allowing for a greater debt capacity, however it also increases the value of the agency costs of overinvestment of the second best policy essentially due to the fact that at maturity of the investment option the value of debt in poorer states of nature decreases, thereby, increasing the subsidy it provides equity for investing at negative NPV’s of the firm. Mauer and Sarkar (2005) report similar impacts of increased risk free rates on both the optimal leverage and agency costs.

As the maturity of the investment option increases we observe an increase in both the value of the first-best and the second-best case. However, while in the first-best case this is accompanied by a steadily increase in the debt levels in the second-best case after an initially increase (until $T_i = 2.5$ years) the optimal level decreases, but seems to stabilise between 50\% - 60\% debt ratio. The agency costs reflect this pattern, giving us a strong indication of the positive relationship between both. As the maturity of the option increases equity delays the exercise of the option (due to the increase of the time-value of the investment cost). At maturity in the worst case scenario equity will exercise the investment

\textsuperscript{28} In a simple analogy we can think of the impact of issuance costs on the overinvestment problem as the impact of growing fees on a life insurance policy, they do not provide any incentives for clients to lead a healthier or life, they simply make it more expensive for clients which are reckless to take up a life insurance policy.

\textsuperscript{29} Lower overinvestment costs for the first best policy and lower underinvestment costs for the second best policy, the exception being the overinvestment costs at maturity on the second best policy.

\textsuperscript{30} Please note that the decrease the option value does not derive from the increase of the corporate tax rate only, there is a change of $P$ since it represents the present value of the after tax cash flows of the project. See notes on Table 3.
decision at lower values of the project, thereby expropriating a bigger portion of the debtholders value, anticipating this, debtholders increase the credit spread making debt less attractive. This will reduce significantly the incentives to overinvest (lower $OC$), but increases the direct financing costs (higher $DFC$ due to interest tax shields forgone), in the overall the agency costs decrease.

An increase in the maturity of the project has similar implications to an increase in the maturity of the investment option; however, in this case the main effects occur at firm level. Although the overinvestment incentives also grow$^{31}$, the main drivers are the interest tax shields and the bankruptcy costs, both positive functions of the maturity of the firm$^{32}$. Debt does not improve by the increase in the upside potential of the firm (since their claim is fixed), however, it loses significantly with the increase of the downside danger, thereby, it raises its credit spread becoming once again less attractive. The agency costs will grow due to the increasing direct financing effect as the $OC$ will tend to stabilise or even decrease.

4. Conclusions

While financing contracts remain incomplete, in the sense that they allow equityholders to adopt suboptimal investment policies while pursuing opportunistc objectives, agency costs still remains a subject of interest. Contrary to the common perception, claiming that different types of real flexibility are generally associated with different incentives$^{33}$, in our analysis we showed that different incentives (underinvestment and overinvestment) coexist in one single type of real flexibility (investment option). However, it was also made explicit that in terms of value, one type of incentive is clearly dominant. Confirming the conclusions of Mauer and Sarkar (2005), on the interactions between financing and investment decisions in the context of an option to invest, we also found preponderance of overinvestment incentives. In the model presented, we found that this dominant effect usually occurs at maturity of the option and this provides some validity for research that assumes European or Bermudian profiles for the investment options. The consideration of time constraints in a context of investment options reflects the reality of both commitment loans and many real options projects. The assumption of perpetual maturities fails then to capture the true

$^{31}$ At maturity for the worst case scenarios equity will again decide to invest at lower project values even if the maturity of the option did not change, by extending the maturity of the firm, at low levels of the underlying at maturity the impact of the growing bankruptcy costs will lower even more the value of the firm.

$^{32}$ An increase in the maturity of the firm increases both this effects: in the good states of nature the firm is worth much more than with sorter maturities, in bad states of nature it is worth much less.

$^{33}$ For the case of growth options Mauer and Ott (2000) demonstrate how equityholders have the incentive to underinvest, for the case of investment options incentives Mauer and Sarkar (2005) show how equityholders have the incentive to overinvest.
complexity of the incentives debt financing gives equityholders and may distort predictions regarding the size of the agency costs. For similar parameters we presented predictions for the relative size of the agency costs lower than previous research assuming perpetual maturities for both the investment option and subsequent firm. Nevertheless, and although we found a low impact of the agency costs, for the optimal debt target (.4%) in our base case parameters, in the presence of budgetary constrains that force investors to finance themselves above the optimal debt target we identified rapidly growing agency costs (2% - 75% debt ratio and 5% - 85% debt ratio) that should not be ignored and can hardly be considered irrelevant.

Agency costs reflect themselves in various levels and they are more complex than the mere difference of investment option values between different investment exercise policies. We provided a structure for their constitution (taking a similar approach to Leland (1998) in the DFC and Mauer and Sarkar (2005) in the separation between purely financial and operational effects) and showed its importance, in the sense that allowed us to identify situations where the agency conflicts manifest themselves only at deeper levels. Even in situations when there is no apparent difference between investment option values following different investment exercise policies, in reality, agency costs may already be inducing suboptimal investment decisions. Thereby, their impact in different aspects where they might manifest (e.g. investment option values and credit spreads) need to be further analysed.

Our results indicate that agency costs are more pronounced in high volatility markets, for high levels of interest rates and for long term projects. They are mostly acute when debt financing is particularly attractive (high corporate tax rates, low bankruptcy costs and low issuance costs). We predict a positive relationship between corporate tax rate, interest rate levels and growth rate of the investment costs with optimal debt levels. Inversely we predict low levels of debt financing for markets with higher issuance costs, volatility or bankruptcy costs.

Our overall predictions for the size of the agency costs are relatively low when compared with previous research (e.g. Leland, 1998, Ericsson, 2000, Titman and Tsyplakov, 2002, Mauer and Sarkar, 2005, Childs et al., 2005), being particularly relevant the differences with Leland (1998), since the author do not take into account the OC effects. Although we have considered a higher volatility in the base case parameters, the fact that our model is constrained in time may partially explain this fact, because as the results demonstrate there
is a positive relationship between the investment option and project maturities and the agency costs of debt (more pronounced for in the case of the maturity of the project).

Even though, this area has been subject of recent interest by academics it still needs further theoretical work to clarify the nature of some relationships (e.g. growth rate of the investment costs and agency costs) but especially empirical work that can shed some light in some of the discrepancies in the theoretical body of knowledge already developed.

References


Figures

Figure 1: Fixed maturity for the firm ($T_f$), variable firm life expectancy ($T_c$)
### Tables

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Present value of the future expected cash flows after tax of the project</td>
<td>$P$</td>
<td>95</td>
</tr>
<tr>
<td>Volatility of the returns on the cash flow</td>
<td>$\sigma$</td>
<td>40%</td>
</tr>
<tr>
<td>Cash flow rate</td>
<td>$\alpha$</td>
<td>2%</td>
</tr>
<tr>
<td>Corporate tax rate</td>
<td>$\tau$</td>
<td>20%</td>
</tr>
<tr>
<td>Debt issuance costs</td>
<td>$\kappa$</td>
<td>0% - 5%</td>
</tr>
<tr>
<td>Bankruptcy costs</td>
<td>$\delta$</td>
<td>40%</td>
</tr>
<tr>
<td>Growth rate of the investment costs</td>
<td>$\gamma$</td>
<td>0% - 5%</td>
</tr>
<tr>
<td>Investment costs</td>
<td>$I$</td>
<td>100</td>
</tr>
<tr>
<td>Risk free rate</td>
<td>$r_f$</td>
<td>6%</td>
</tr>
<tr>
<td>Maturity of the investment option</td>
<td>$T_f$</td>
<td>2 years</td>
</tr>
<tr>
<td>Fixed maturity of the project</td>
<td>$T_t$</td>
<td>10 years</td>
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**Table 1: Base case parameters**

<table>
<thead>
<tr>
<th></th>
<th>Unlevered</th>
<th>First - best</th>
<th>Second - best</th>
<th>Agency costs</th>
<th>% agency costs</th>
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<tr>
<td>$V^0_i$</td>
<td>21.37</td>
<td>22.61</td>
<td>22.53</td>
<td>0.08</td>
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<tr>
<td>$F_i$</td>
<td>-</td>
<td>64.50</td>
<td>60.00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-</td>
<td>8.9%</td>
<td>8.87%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$DFE$</td>
<td>-</td>
<td>1.25</td>
<td>1.32</td>
<td>-0.07</td>
<td>-0.31%</td>
</tr>
<tr>
<td>$OC$</td>
<td>-</td>
<td>-0.01</td>
<td>-0.16</td>
<td>0.16</td>
<td>0.69%</td>
</tr>
<tr>
<td>$T_{S_i}$</td>
<td>-</td>
<td>2.75</td>
<td>2.85</td>
<td>-0.10</td>
<td>-0.81%</td>
</tr>
<tr>
<td>$Be_i$</td>
<td>-</td>
<td>-1.50</td>
<td>-1.53</td>
<td>0.03</td>
<td>0.52%</td>
</tr>
<tr>
<td>$Ic_i$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$Ov_{ij}$</td>
<td>-</td>
<td>-0.01</td>
<td>-0.16</td>
<td>0.16</td>
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</tr>
<tr>
<td>$Uv_{ij}$</td>
<td>-</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00%</td>
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**Table 2: Investment option values, optimal debt targets and agency costs for the base case parameters: $P=95$, $I=100$, $T_i=2$, $T_t=10$, $\sigma=40\%$, $\alpha=2\%$, $\tau=20\%$, $\kappa=0$, $\delta=40\%$ and $\gamma=0$.**
<table>
<thead>
<tr>
<th>Parameter</th>
<th>$V_F^0$</th>
<th>$F_F$</th>
<th>$r_{AF}$</th>
<th>$V_S^0$</th>
<th>$F_S$</th>
<th>$r_{AS}$</th>
<th>Agency Costs (% $V_F^0$)</th>
<th>Credit spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>22.61</td>
<td>64.50</td>
<td>8.9%</td>
<td>22.53</td>
<td>60.00</td>
<td>8.8%</td>
<td>0.38%</td>
<td>-0.31%</td>
</tr>
<tr>
<td>$\alpha=10%$</td>
<td>13.64</td>
<td>69.50</td>
<td>6.7%</td>
<td>13.59</td>
<td>67.00</td>
<td>6.7%</td>
<td>0.55%</td>
<td>-0.65%</td>
</tr>
<tr>
<td>$\alpha=20%$</td>
<td>32.10</td>
<td>67.00</td>
<td>12.3%</td>
<td>31.92</td>
<td>48.50</td>
<td>11.0%</td>
<td>0.45%</td>
<td>-0.75%</td>
</tr>
<tr>
<td>$\alpha=60%$</td>
<td>288</td>
<td>67.00</td>
<td>17.2%</td>
<td>40.92</td>
<td>60.00</td>
<td>16.4%</td>
<td>0.73%</td>
<td>0.19%</td>
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<tr>
<td>$\alpha=1%$</td>
<td>23.86</td>
<td>63.00</td>
<td>8.6%</td>
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<td>$\alpha=3%$</td>
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<td>$\alpha=5%$</td>
<td>19.63</td>
<td>62.00</td>
<td>9.4%</td>
<td>19.55</td>
<td>55.50</td>
<td>9.1%</td>
<td>0.40%</td>
<td>0.01%</td>
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Table 3: Investment option values, optimal debt targets, agency costs and credit spreads for different parameter values. For the base case we have the following parameters: $p=95$, $l=100$, $T_F=2$, $T_s=10$, $\sigma=40\%$, $r_e=6\%$, $\epsilon=2\%$, $\gamma=20\%$, $\kappa=40\%$ and $\delta=0\%$. The credit spread is expressed in basis points (BP) and the agency costs are represented as a fraction of the second-best option value. Similarly, to Childs et al. (2008) in the corporate tax rate sensitivity analysis the present value of the after tax cash flows ($P$) was adjusted by $P_N = P_o \left[ 1 - \zeta_3 \right] \left[ 1 - \zeta_2 \right]$, where $P_N$ and $P_o$ represent the new values for the present value of the after tax cash flow value and new corporate tax rate and $P_o$ and $\zeta_2$ represent the base case values.
Graphs

Graph 1: Investment option exercise boundaries for the unlevered firm ($V^0 - U$) in the figure, levered firm values following a first best policy ($V^F - F$) in the figure and second best policy ($V^S - S$). The parameters are: $P=95$, $I=100$, $T_i=2$, $T_f=10$, $\sigma=40\%$, $r=6\%$, $\alpha=2\%$, $\tau=20\%$, $\kappa=0$, $\delta=40\%$ and $\rho=0$.

Graph 2: Investment option exercise boundaries for the unlevered firm ($V^0 - U$) in the figure, levered firm values following a first best policy ($V^F - F$) in the figure and second best policy ($V^S - S$) for the two days previous to maturity of the investment option. The parameters are: $P=95$, $I=100$, $T_i=2$, $T_f=10$, $\sigma=40\%$, $r=6\%$, $\alpha=2\%$, $\tau=20\%$, $\kappa=0$, $\delta=40\%$ and $\rho=0$. 
Graph 3: Value of $V^0$, $V_F^0$ and $V_S^0$ for different percentages of debt financing (0 – 100). The remainder parameters are: $P=95$, $l=100$, $T_i=2$, $T_t=10$, $\sigma=40\%$, $r=6\%$, $\alpha=2\%$, $\tau=20\%$, $\kappa=0$, $\delta=40\%$ and $\varepsilon=0$.

Graph 4: Interest tax shield effects ($T_s^0$), bankruptcy costs ($B_c^0$), over and underinvestment effects ($O_v^0$, $U_v^0$) for different percentages of leverage when option is exercised according to a first-best policy. The remainder parameters are: $P=95$, $l=100$, $T_i=2$, $T_t=10$, $\sigma=40\%$, $r=6\%$, $\alpha=2\%$, $\tau=20\%$, $\kappa=0$, $\delta=40\%$ and $\varepsilon=0$. 
Graph 5: Interest tax shield effects ($Ts^0$), bankruptcy costs ($Bc^0$), over and underinvestment effects ($Ov^0$, $Uv^0$) for different percentages of leverage when option is exercised according to a second-best policy. The remainder parameters are: $P=95$, $I=100$, $T_i=2$, $T_t=10$, $\sigma=40\%$, $r=6\%$, $\alpha=2\%$, $\pi=20\%$, $\kappa=0$, $\delta=40\%$ and $p=0$.

Graph 6: Agency costs as a percentage of the first best firm value ($V_{f1}^0$). Total agency costs ($AC$), direct financing costs ($DFC$) and operational costs ($OC$) for different percentages of leverage. The remainder parameters are: $P=95$, $I=100$, $T_i=2$, $T_t=10$, $\sigma=40\%$, $r=6\%$, $\alpha=2\%$, $\pi=20\%$, $\kappa=0$, $\delta=40\%$ and $p=0$.  

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Graph 7: Credit spread of the cost of debt in the first-best and second-best cases for different percentages of debt financing, measured in basis points (BP). The remainder parameters are: $P=95$, $I=100$, $T_i=2$, $T_t=10$, $\sigma=40\%$, $r=6\%$, $\alpha=2\%$, $\tau=20\%$, $\kappa=0$, $\delta=40\%$ and $\mu=0$. 