An option-based view of imperfect patent protection

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Abstract

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Keywords: Capital budgeting, Intellectual property, Litigation, Real options
JEL classification: G31, O31, C73

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1 Introduction

Patents and R&D can be regarded as real options (Schwartz, 2004). In previous option-based analyses of *imperfect* patent protection, the probability of litigation typically appeared as an exogenous parameter (Dixit and Pindyck, 1994, p. 173). In reality, litigation is the result of value-maximizing behavior on the part of potential challengers. Technically speaking, the model presented endogenizes patent risk, which, as will become clear, is best treated as an option to litigate.

Generally speaking, the aim of this dicussion is to illucidate the applicability of option pricing in the wider context of uncertain property rights and flexible managerial decisions surrounding them.

Beyond its methodological appeal, the issue is of enormous practical relevance. For instance, Lemley (2001) points out a noticeable degradation of patent examination quality at the USPTO in recent years. However, because the vast majority of patents are of no appreciable business value, the incremental cost associated with marginally improving patent examination would not be justified by a substantial reduction in litigation costs.

In light of such serious deficiencies and heightened levels of competition, patenting has come to resemble the purchase of a lottery ticket, admittedly complicated by interdependencies between individual patents. Lemley and Shapiro conclude: [1]

> "Under patent law, a patent is no guarantee of exclusion but more precisely a legal right to try to exclude. ... [M]ost patents represent highly uncertain or probabilistic property rights. By this we mean that patents are a mixture of a property right and a lottery." (2004, p. 2)

As discussed in detail by Lanjouw and Schankerman (2001), the cost of engaging in litigation over intellectual property (IP) assets diminishes their value as an incentive to invest in research.

Translating the vague notion of a *lottery* into a consistent valuation approach, the author will demonstrate how patents can be described as a mixture of a property right and a short option to litigate. The option-based view (OBV) of imperfect patent protection proposed may serve as a starting point for further investigations into the impact of patent risk on firm values and innovation incentives.

The analysis of patent risk as an endogenous parameter in option-based mod-

[1] The paper was published in final form as Lemley and Shapiro (2005).

els of IP is still in its infancy. To the author's best knowledge, the only detailed discussion of the option value of litigation is due to Marco (2005). His paper, however, has a strong empirical focus. Furthermore, the approach to formalizing patent risk differs from the one adopted here. Aoki and Hu (2003) discuss time factors of patent litigation and licensing in a deterministic setting, also examining the role of settlement.

Roughly speaking, the discussion is structured as follows. The intuition behind the formal model and some basic definitions are provided in subsection 2.1, before subsection 2.2 lays out the details. Section 3 hints at a number of variations and extensions of the original setup. Section 4 concludes and contains suggestions for future research.

2 The model

Over the years, the literature on investment under uncertainty has seen a variety of generic (real) options, covering investment as well as disinvestment decisions. Since litigation, in a way, represents an investment with uncertain outcome, it seems natural to examine more closely the option value of litigation and its impact on capital budgeting decisions.

2.1 Formalization

The incumbent innovator owns a patent expiring at time T allowing him or her to commercialize some pharmaceutical product. Commercialization is associated with some expected revenue, which fluctuates randomly. This randomness is captured by specifying the revenue rate as a stochastic process.

While a variety of specifications are possible, a common choice is to let such variables evolve in analogy to the standard stock price model (Samuelson, 1965; Schwartz, 2004). Abstracting from operating costs, the dynamics of the associated profit rate or net cash flow Π_t under the martingale measure $\mathbf{P}^*$ are then described by

$$\mathrm{d}\Pi_t = \alpha^* \Pi_t \,\mathrm{d}t + \sigma \Pi_t \,\mathrm{d}W_t, \qquad 0 < \Pi_0 = \varpi, \tag{1}$$

or, in integral notation,

$$\Pi_t = \Pi_0 + \int_0^t \alpha^* \Pi_s \,\mathrm{d}s + \int_0^t \sigma \Pi_s \,\mathrm{d}W_s, \tag{2}$$

where $\alpha^* = r - \delta = r - (\mu - \alpha)$ is the risk-adjusted drift, μ the risk-adjusted rate of return, σ the corresponding volatility, that is standard deviation of returns, and $W = \{W_t\}_{t \geq 0}$ is one-dimensional Brownian motion. According to the risk-neutral pricing approach, (1) and (2) describe the profit rate process in an equivalent risk-neutral world, making it possible to discount cash flows at the risk-free rate (Harrison and Kreps, 1979; Harrison and Pliska, 1981).

Without further emphasis, risk-neutral pricing is adopted for the rest of this analysis. While, in practice, incomplete markets may pose serious valuation issues (Hubalek and Schachermayer, 2001), similar shortcomings are shared by all capital budgeting techniques. Whoever accepts the validity of the CAPM is also likely to accept the existence and uniqueness of the risk-neutral measure $\mathbf{P}^*$.

Another assumption worth pointing out is the non-negativity of net cash flows resulting from (1). It seems restrictive at first, but is sensible in many practical applications, including, in particular, pharmaceutical patents. Commercialization itself is almost always value-enhancing, because the lion's share of costs is incurred during R&D.

Due to the limited life of patents, however, cash flows will not continue indefinitely. The profit rate usually drops sharply upon expiration of the patent. In this model, the patent is taken to have a terminal value of $M\Pi_T$, where M is some multiple. The fiercer competition by imitators, or generics manufacturers, the lower M.

Let $\mathbf{E}_{\mathbf{P}^*}[\cdot]$ denote the expectation operator under the risk-neutral measure. In the absence of additional costs, the value of the project to the incumbent at time t, conditional on the information available to him or her at that time, is then given by

$$V_{\mathrm{I}}(\Pi_t, t) = \mathbf{E}_{\mathbf{P}^*}\left[\int_t^T \Pi_t \,\mathrm{e}^{-r(s-t)}\,\mathrm{d}s + M\Pi_T \,\mathrm{e}^{-r(T-t)} \,\middle|\, \mathcal{F}_t\right]. \tag{3}$$

Note that, throughout this paper and in line with the notation employed by Dixit and Pindyck (1994), V is used to refer to the project, whereas F signifies the option.

Following arguments now standard in the literature, the commercialization value must satisfy the partial differential equation (PDE)

$$\tfrac{1}{2}\sigma^2\Pi_t^2 \frac{\partial^2 V_{\mathrm{I}}(\Pi_t, t)}{\partial \Pi_t^2} + (r-\delta)\,\Pi_t \frac{\partial V_{\mathrm{I}}(\Pi_t, t)}{\partial \Pi_t} - rV_{\mathrm{I}}(\Pi_t, t) + \Pi_t + \frac{\partial V_{\mathrm{I}}(\Pi_t, t)}{\partial t} = 0 \tag{4}$$

with boundary condition

$$V_{\mathrm{I}}(\Pi_T, T) = M\Pi_T. \tag{5}$$

Ruling out speculative bubbles and assuming perfect patent protection, the complete solution to this problem can be derived as

$$V_{\mathrm{I}}(\Pi_t, t) = \left(1 - \mathrm{e}^{-\delta(T-t)}\right) \Pi_t/\delta + \mathrm{e}^{-\delta(T-t)}\, M\Pi_t. \tag{6}$$

As argued by Schwartz (2004), the associated process exhibits, in terms of risk premium and volatility, characteristics identical to those of the underlying cash flow process. It is thus possible to estimate a risk-premium $\eta = \alpha - \alpha^*$ as well as σ from data on the drift and volatility of comparable completed projects.[2]

One has also pointed out that the project value is linear in Π_t and independent of volatility. However, this conclusion hinges on the absence of flexibility once the incumbent has committed to commercialization. This not only means taking an unrealistic now-or-never view of decision-making on the side of the incumbent. It also neglects the effect of competitive action which is similarly contingent on how the revenue rate develops over time.

In the context of patent risk, which is the main focus of this analysis, it is important to note the profound impact a potential challenger has on the incumbent's optimal investment policy, as will be shown in more detail below. In the spirit of the real options paradigm, patent risk can thus be regarded as one of the many manifestation of optionality. As option value is heavily influenced by volatility, the project turns out to be sensitive to changes in this important parameter as well.

The discussion now proceeds by formalizing the above intuition. Based on the alleged infringement of a related patent, a challenger may decide to litigate at any time $\tau \in [0, T]$ and, if successful, receives a damage award, equal to a fraction $\zeta \in [0, 1]$ of the value of *past* cash flows, compounded to time τ.[3] Furthermore, the successful challenger may claim a fraction $\theta \in [0, 1]$ of *future* net cash flows. Due to improved monitoring after litigation, this fraction may very well be higher than the proportion of past net cash flows claimed.

If, on the one hand, the challenger is not willing or able to commercialize the patent, the incumbent will continue to market the product for the challenger as long as his or her participation constraint is fulfilled. Abstracting from a possible super-game, some marginally small profit is sufficient for this to be the case. Competition in other products and the threat of various forms of opportunistic behavior, however, may lead to concessions on the side of the challenger.

[2] Since $V_{\mathrm{I}}(\Pi_t, t)$ represents the value of a completed project under *perfect* patent protection, additional adjustments may become necessary in practice.

[3] For reasons of simplicity, litigation has to take place within the specified timeframe and cannot be postponed beyond patent expiration.

If the challenger, on the other hand, does not depend on the incumbent to market the product, θ becomes unity. Furthermore, legal practice prevents a patent holder from obtaining damages for a time span during which he or she was aware of the alleged infringement, but did not take action. Otherwise, the challenger would be well-advised to wait for all market uncertainty to resolve, before taking the risk of a costly patent dispute. While it might prove difficult to establish the exact point in time at which the challenger took notice, the resulting damages award, expressed as a proportion of past cash flows, should be comparatively low.

Although, in principle, it might be interesting to examine the case in which the challenger is active in the market from the outset and, together with the incumbent, forms a duopoly, the challenger is assumed to be idle at time $t = 0$. Such a variation of the model would lower the challenger's expected gain over the status quo and thereby also diminish the incentive to litigate. Moreover, an alternative scenario with mutual litigation is conceivable.

According to the so-called *American* rule, both parties have to pay their lawyers out of their own pockets. For now, the American rule is applied to calculate litigation costs incurred by the incumbent and the challenger, which are denoted by L_{I} and L_{C}, respectively. In addition, let p denote the probability of successful litigation. The expected payoff from litigation becomes

$$\begin{aligned}\mathbf{E}_{\mathbf{P}^*}[V_{\mathrm{C}}(\Pi_\tau,\tau) - L_{\mathrm{C}} \,|\, \mathcal{F}_\tau] = p\Bigg(\zeta\left(\int_0^\tau \mathrm{e}^{r(\tau-t)}\,\Pi_t\,\mathrm{d}t + \mathbf{1}_{\{\tau=T\}}M\Pi_T\right)\\ +\mathbf{E}_{\mathbf{P}^*}\left[\theta\left(\int_\tau^T \mathrm{e}^{-r(t-\tau)}\,\Pi_t\,\mathrm{d}t + \mathbf{1}_{\{\tau<T\}}\,\mathrm{e}^{-r(T-\tau)}\,M\Pi_T\right)\Bigg|\,\mathcal{F}_\tau\right]\Bigg) - L_{\mathrm{C}}. \quad (7)\end{aligned}$$

Given the information available at the time of litigation, cash flows are known for all $t \le \tau$. Cash flows beyond this point are still uncertain, making it necessary to take expectation over all possible realizations. Litigation costs are constant and known in advance.

The expected payoff is maximized by choosing an optimal litigation time. At time $t = 0$, the option to litigate is worth

$$\begin{aligned}F_{\mathrm{C}}(\varpi,0) &= \sup_{\tau\in[0,T]} \mathbf{E}_{\mathbf{P}^*}\left[\mathrm{e}^{-r\tau}\Big(V_{\mathrm{C}}(\Pi_\tau,\tau) - L_{\mathrm{C}}\Big)^+\right]\\ &= \mathbf{E}_{\mathbf{P}^*}\left[\mathrm{e}^{-r\tau^*}\Big(V_{\mathrm{C}}(\Pi_{\tau^*},\tau^*) - L_{\mathrm{C}}\Big)^+\right]. \quad (8)\end{aligned}$$

All agents are assumed to follow a policy of value-maximization. Of course, the optimal litigation time τ^* cannot be specified in advance, but is chosen by the challenger in response to the resolution of uncertainty related to Π_t over time. For this reason, the stopping time τ^* is stochastic and can be described as the first time Π_t exceeds a critical level Π_t^*,

$$\tau^* = \inf\{t : \Pi_t^* < \Pi_t\}, \tag{9}$$

which is sufficiently high to justify the cost of litigation. If litigation has not become optimal by the time the patent expires, no action is taken. Intuitively, the value of the project to the incumbent, including patent risk, becomes

$$\begin{aligned}\widetilde{V}_{\mathrm{I}}(\varpi, 0) = \mathbf{E}_{\mathbf{P}^*}\Big[\Big(1 - \mathrm{e}^{-\delta T}\Big)\,\varpi/\delta + M\Pi_T\,\mathrm{e}^{-\delta T}\Big] \\ - F_{\mathrm{C}}(\varpi, 0) - \mathbf{E}_{\mathbf{P}^*}\Big[\mathbf{1}_{\{\tau^* \le T\}}\,\mathrm{e}^{-r\tau^*}\,(L_{\mathrm{I}} + L_{\mathrm{C}})\Big],\end{aligned} \tag{10}$$

that is the expected present value of cash flows from commercialization, less the option value of litigation, less the present value of additional litigation costs. In option terms, the incumbent is long the commercialization project and short an option to litigate.

2.2 Analysis

The deterministic and the stochastic case are examined in turn, presenting results for finite and infinite protection periods.

2.2.1 Deterministic payoff

As a benchmark, this subsection considers optimal litigation under certainty. In contrast to the stochastic case, an optimal litigation time τ^* is straightforward to determine.

2.2.1.1 Finite protection period Since deriving closed-form solutions in the presence of a finite patent protection period poses no difficulties under certainty, the analysis will focus on the more general model, before examining the limiting case of infinite patent duration.

With $\sigma = 0$, (2) reduces to

$$\Pi_t = \varpi + \int_0^t \alpha\,\mathrm{d}s, \tag{11}$$

which implies $\Pi_t = \mathrm{e}^{\alpha t}\,\varpi$. Assuming intense competition after patent expiration, there will be no revenue for all $t > T$, which implies $M = 0$. Further assuming $0 < \alpha < r$, the discounted payoff from litigation at some future time

τ becomes

$$
\begin{aligned}
\mathrm{e}^{-r\tau}\Big(V_{\mathrm{C}}(\varpi,\tau)-L_{\mathrm{C}}\Big) &= p\left(\zeta\int_0^{\tau}\mathrm{e}^{-(r-\alpha)t}\,\varpi\,\mathrm{d}t\right.\\
&\qquad\left.+\theta\int_{\tau}^{T}\mathrm{e}^{-(r-\alpha)t}\,\varpi\,\mathrm{d}t\right)-\mathrm{e}^{-r\tau}L_{\mathrm{C}}\\
&= p\Big(\zeta\left(1-\mathrm{e}^{-(r-\alpha)\tau}\right)\\
&\qquad+\theta\left(\mathrm{e}^{-(r-\alpha)\tau}-\mathrm{e}^{-(r-\alpha)T}\right)\Big)\frac{\varpi}{r-\alpha}-\mathrm{e}^{-r\tau}L_{\mathrm{C}}.
\end{aligned}
\tag{12}
$$

The Marshallian rule commonly used in practice neglects timing issues altogether and simply requires positive net present value, or $0<V_{\mathrm{C}}(\varpi,0)-L_{\mathrm{C}}$. If, in addition, the value of waiting is accounted for, one obtains a critical revenue rate ϖ^* that triggers litigation, provided there is a positive expected payoff. This view leads to the following proposition for the deterministic case.

Proposition 1 *A critical cash flow rate ϖ^*, above which immediate litigation becomes optimal, exists if and only if $\zeta<\theta$, and it is given by*

$$
\varpi^*=\frac{L_{\mathrm{C}}r}{p\,(\theta-\zeta)}. \tag{13}
$$

Proof of proposition 1 Consider the optimization problem

$$
F_{\mathrm{C}}(\varpi,0)=\max_{\tau\in[0,T]}\Big(G_{\mathrm{C}}(\varpi,0)\Big)^+, \tag{14}
$$

where

$$
G_{\mathrm{C}}(\varpi,0)=\mathrm{e}^{-r\tau}\Big(V_{\mathrm{C}}(\varpi,\tau)-L_{\mathrm{C}}\Big). \tag{15}
$$

A necessary condition for a maximum is

$$
\left.\frac{\partial G_{\mathrm{C}}(\varpi,0)}{\partial\tau}\right|_{\tau=\tau^*}=\mathrm{e}^{-r\tau^*}\Big(L_{\mathrm{C}}r-p\,(\theta-\zeta)\,\mathrm{e}^{\alpha\tau^*}\,\varpi\Big)=0. \tag{16}
$$

The optimal policy depends on the ratio ζ/θ. If $\theta\le\zeta$, that is the successful challenger receives a larger proportion of past than of future cash flows, (15) is strictly increasing in τ, there is no interior solution, and it is optimal to postpone litigation as long as possible. Recall that, by assumption, $0<\alpha<r$. Provided $\zeta<\theta$, (13) holds, and

$$
\tau^*=\begin{cases}0 & \text{if } \varpi^*<\varpi,\\ \frac{1}{\alpha}\ln\frac{L_{\mathrm{C}}r}{p(\theta-\zeta)\varpi} & \text{if } \mathrm{e}^{-\alpha T}\,\varpi^*<\varpi\le\varpi^*,\\ T & \text{otherwise.}\end{cases} \tag{17}
$$

It is easily verified that the sufficient condition is always fulfilled, because

$$\left.\frac{\partial^2 G_{\mathrm{C}}(\varpi,0)}{\partial\tau^2}\right|_{\tau=\tau^*} = -L_{\mathrm{C}} r\alpha \left(\frac{p\,(\theta-\zeta)\,\varpi}{L_{\mathrm{C}} r}\right)^{r/\alpha} < 0. \tag{18}$$

For the critical cash flow rate $\tau^* = 0$, so that, by (17), the deterministic trigger is in fact given by (13). $\square$

The lower the probability of success, the longer the optimal time to litigation. Increases in the fraction $L_{\mathrm{C}} r / \big(p\,(\theta-\zeta)\,\varpi\big)$ make postponing litigation more attractive. This result corresponds to the Jorgensonian investment rule,

$$p\,(\theta-\zeta)\,\varpi^* = L_{\mathrm{C}} r, \tag{19}$$

which triggers investment when the marginal revenue product equals the user cost of capital (Jorgenson, 1963). This rule applies regardless of patent duration, that is the deterministic trigger ϖ^* is independent of patent length.

Nevertheless, patent duration does have an impact on whether it will be ever optimal to litigate at all, because optimal timing alone does not automatically lead to a positive payoff. Substituting (17) into (12) yields

$$F_{\mathrm{C}}(\varpi,0) = \left(p\zeta\left(1-\mathrm{e}^{-(r-\alpha)T}\right)\frac{\varpi}{r-\alpha} - \mathrm{e}^{-rT}\,L_{\mathrm{C}}\right)^{+}, \tag{20}$$

if $\varpi < \mathrm{e}^{-\alpha T}\,\varpi^*$, that is litigation takes place at the end of the protection period ($\tau^* = T$). Immediate litigation ($\tau^* = 0$) is optimal if $\varpi^* < \varpi$, and

$$F_{\mathrm{C}}(\varpi,0) = \left(p\theta\left(1-\mathrm{e}^{-(r-\alpha)T}\right)\frac{\varpi}{r-\alpha} - L_{\mathrm{C}}\right)^{+}. \tag{21}$$

For any profit rate that does not exceed the critical level, but is greater than $\mathrm{e}^{-\alpha T}\,\varpi^*$, there is an interior solution to the optimization problem ($0 < \tau^* < T$), and

$$F_{\mathrm{C}}(\varpi,0) = \left(p\left(\zeta-\theta\,\mathrm{e}^{-(r-\alpha)T}\right)\frac{\varpi}{r-\alpha} + \frac{L_{\mathrm{C}}\alpha}{r-\alpha}\left(\frac{p\,(\theta-\zeta)\,\varpi}{L_{\mathrm{C}} r}\right)^{r/\alpha}\right)^{+}. \tag{22}$$

Intuitively, (22) decomposes the option value of litigation into two perpetuities and an option (see fig. 1). This observation comes in handy also under uncertainty (see sec. 2.2.2).

One implication of the above analysis is that, in the absence of substantial litigation costs, immediate legal action always maximizes the expected payoff from litigation.

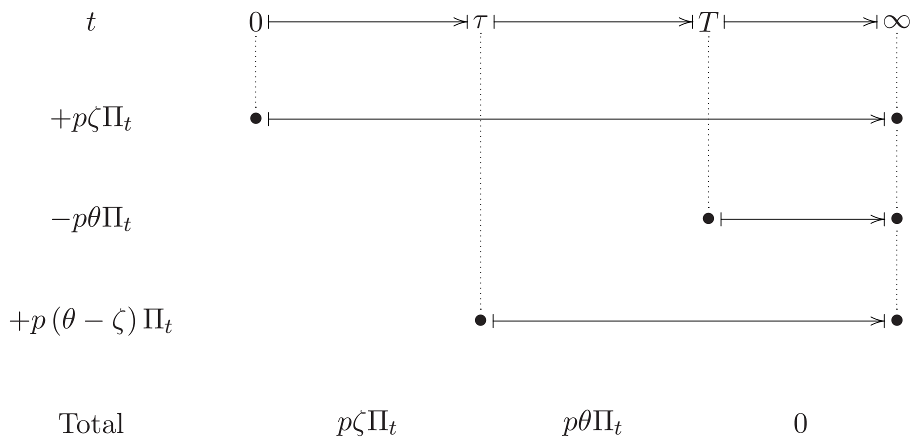


Fig. 1. Decomposing the payoff from litigation. Total payoff from litigation can be decomposed into two perpetuities and one option, creating two closed and one open interval with distinct profit rates. While T is pre-specified, the stopping time τ is chosen to maximize litigation payoff.

If litigation costs are comparatively high, however, challengers who, on the one hand, are likely to experience difficulties in claiming the full amount of their damage in court, but, on the other hand, will probably be able to negotiate participation in future cash flows benefit from immediate litigation.

Firms that aim at being compensated in full and cannot participate in future increases of commercial value should postpone litigation. Since, in reality, litigation costs are usually substantial, optimal timing becomes essential. Optimal timing is determined by the ratio ζ/θ capturing a firm's relative ability to participate in past and future profits.

The impact of this ratio is illustrated by figure 2, which shows the discounted expected payoff from litigation as a function of litigation time for $p = 0.5$, $\varpi = 1.0$, $r = \delta = 0.05$, $\alpha = 0.1$, $\theta = 1.0$, $T = 20.0$, and $L_{\mathrm{C}} = 10.0$. Simply inserting these parameters and $\zeta \in \{0.0, 0.5, 1.0\}$ into (13) yields the thresholds

$$\varpi_1^* = \frac{10.0 \times 0.05}{0.5\,(1.0-0.0)} = 1.0,$$
$$\varpi_2^* = \frac{10.0 \times 0.05}{0.5\,(1.0-0.5)} = 2.0.$$

Furthermore,

$$\lim_{\zeta\to 1.0} \frac{10.0 \times 0.05}{0.5\,(1.0-\zeta)} = \infty.$$

Setting $\zeta = 0.0$ or $\zeta = 1.0$ thus produces the limiting cases of immediate liti-

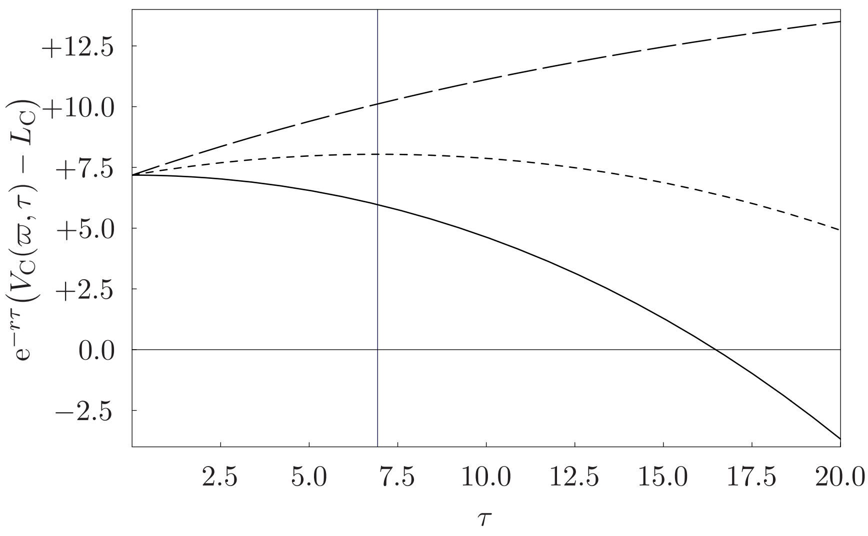


Fig. 2. Discounted expected payoff from litigation as a function of litigation time ($p = 0.5$, $\varpi = 1.0$, $r = \delta = 0.05$, $\alpha = 0.1$, $\theta = 1.0$, $T = 20.0$, and $L_{\mathrm{C}} = 10.0$). If $\zeta = \zeta/\theta = 0.0$, immediate litigation is optimal (solid line); if $\zeta = \zeta/\theta = 1.0$, postponing litigation to the end of the protection period is the value-maximizing strategy (long dashes). For $\zeta = \zeta/\theta = 0.5$, the rational investor will litigate at time $\tau = 6.93$ (short dashes and vertical line).

gation and litigation at the end of the protection period. An interior solution,

$$\tau^* = \frac{1}{0.1} \ln \frac{2.0}{1.0} = 6.93,$$

exists only for $\zeta = \zeta/\theta = 0.5$.

2.2.1.2 Infinite protection period At this point, an additional simplifying assumptions is introduced, which makes it possible to separate the effects of patent expiration and patent litigation, namely that the protection period T is infinite. This assumption also greatly facilitates the derivation of closed-form solutions for the stochastic case, analyzed in section 2.2.2. Equation (8) becomes

$$\begin{aligned} F_{\mathrm{C}}(\Pi_t) &= \max_{\tau \in [0,\infty)} p \left(\zeta + \mathrm{e}^{-(r-\alpha)\tau} \left(\theta - \zeta\right) \right) \frac{\Pi_t}{r - \alpha} - \mathrm{e}^{-r\tau} L_{\mathrm{C}} \\ &= p \left(\zeta + \mathrm{e}^{-(r-\alpha)\tau^*} \left(\theta - \zeta\right) \right) \frac{\Pi_t}{r - \alpha} - \mathrm{e}^{-r\tau^*} L_{\mathrm{C}}. \end{aligned} \tag{23}$$

Recall that the trigger deduced previously applies regardless of patent length and thus continues to hold if the protection period is infinite. In addition,

$$\widetilde{V}_{\mathrm{I}}(\Pi_t) = \frac{\Pi_t}{r - \alpha} - F_{\mathrm{C}}(\Pi_t) - \mathrm{e}^{-r\tau^*} \left(L_{\mathrm{I}} + L_{\mathrm{C}}\right). \tag{24}$$

For example, immediate litigation of a perpetual patent implies

$$F_{\mathrm{C}}(\Pi_t) = V_{\mathrm{C}}(\Pi_t) - L_{\mathrm{C}} = p\theta \frac{\Pi_t}{r-\alpha} - L_{\mathrm{C}} \tag{25}$$

and thus

$$\begin{aligned} \widetilde{V}_{\mathrm{I}}(\Pi_t) &= \frac{\Pi_t}{r-\alpha} - \left(p\theta \frac{\Pi_t}{r-\alpha} - L_{\mathrm{C}}\right) - (L_{\mathrm{I}} + L_{\mathrm{C}}) \\ &= (1-p\theta)\frac{\Pi_t}{r-\alpha} - L_{\mathrm{I}}. \end{aligned} \tag{26}$$

If litigation is successful, there is no damage award, simply a participation in future cash flows from commercialization.

Previous discussions served to highlight timing flexibility in patent litigation under certainty, that is for the special case $\sigma = 0$. Under certainty, the value of waiting is solely driven by the ratio ζ/θ. As this ratio increases, so does the critical cash flow rate. Nevertheless, this view neglects the impact of σ, which is another important value driver.

Hence, in the following section, the effect of uncertainty on the litigation decision will be considered. The case of an infinite protection period under certainty is not examined further, because it obviously represents a limiting case of the stochastic model.

2.2.2 Stochastic payoff

With the option value of litigation under certainty established, it is now possible to extend the model to a stochastic setting. The option value of litigation interacts with the option value of investing into R&D. Proceeding backwards in time, a sequential stochastic game for patent valuation will be developed.

2.2.2.1 Option to litigate The first step involves determining the payoff from commercialization, accounting for the short option to litigate held by a potential challenger.

Recall from section 2.1 that, under uncertainty,

$$\mathrm{d}\Pi_t = \alpha^* \Pi_t \, \mathrm{d}t + \sigma \Pi_t \, \mathrm{d}W_t, \qquad \Pi_0 = \varpi, \tag{27}$$

and, by assumption, $0 < \alpha^* < r$. The simplified optimization problem with no terminal value becomes

$$F_{\mathrm{C}}(\Pi_t) = \sup_{\tau\in[t,\infty)} \mathbf{E}_{\mathbf{P}^*}\left[p\left(\zeta\int_t^\tau \mathrm{e}^{-rs}\,\Pi_s\,\mathrm{d}s \right.\right.$$
$$\left.\left. +\theta\int_\tau^\infty \mathrm{e}^{-rs}\,\Pi_s\,\mathrm{d}s\right) - \mathrm{e}^{-r\tau}\,L_{\mathrm{C}}\right]. \quad (28)$$

It looks challenging at first glance, but decomposes into tractable parts just like the deterministic model. Equation (28) can be re-written as

$$F_{\mathrm{C}}(\Pi_t) = \sup_{\tau\in[t,\infty)} \mathbf{E}_{\mathbf{P}^*}\left[-p\,(\theta-\zeta)\int_t^\tau \mathrm{e}^{-rs}\,\Pi_s\,\mathrm{d}s - \mathrm{e}^{-r\tau}\,L_{\mathrm{C}}\right]$$
$$+\,\mathbf{E}_{\mathbf{P}^*}\left[p\theta\int_t^\infty \mathrm{e}^{-rs}\,\Pi_s\,\mathrm{d}s\right] \quad (29)$$

and

$$\begin{aligned}\mathbf{E}_{\mathbf{P}^*}\left[p\theta\int_t^\infty \mathrm{e}^{-rs}\,\Pi_s\,\mathrm{d}s\right] &= p\theta\int_0^\infty \mathrm{e}^{-(r-\alpha^*)s}\,\Pi_t\,\mathrm{d}s\\ &= \frac{p\theta\Pi_t}{r-\alpha^*}.\end{aligned} \quad (30)$$

The second term in (29) is thus independent of τ and can be neglected in determining an optimal stopping time.

Proposition 2 *Assuming $\zeta < \theta$, the option value of litigation is*

$$F_{\mathrm{C}}(\Pi_t) = \begin{cases} p\theta\Pi_t/\delta - L_{\mathrm{C}} & \text{if } \Pi^* < \Pi_t,\\ A^+\Pi_t^{\gamma^+} + p\zeta\Pi_t/\delta & \text{otherwise,}\end{cases} \quad (31)$$

where

$$\Pi^* = \frac{\gamma^+}{\gamma^+-1}\frac{L_{\mathrm{C}}\delta}{p\,(\theta-\zeta)} \quad (32)$$

denotes the critical cash flow rate,

$$\begin{aligned}A^+ &= \frac{L_{\mathrm{C}}}{\gamma^+-1}\left(\frac{1}{\Pi^*}\right)^{\gamma^+}\\ &= \left(\frac{p\,(\theta-\zeta)}{\gamma^+\delta}\right)^{\gamma^+}\left(\frac{\gamma^+-1}{L_{\mathrm{C}}}\right)^{\gamma^+-1},\end{aligned} \quad (33)$$

and

$$\gamma^+ = \tfrac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2}-\tfrac{1}{2}\right)^2 + 2\frac{r}{\sigma^2}}. \quad (34)$$

Proof of proposition 2 For convenience, define

$$\Psi_{\mathrm{C}}(\Pi_t) = \sup_{\tau\in[t,\infty)} \mathbf{E}_{\mathbf{P}^*}\left[-p\,(\theta-\zeta)\int_t^\tau \mathrm{e}^{-r(t-s)}\,\Pi_s\,\mathrm{d}s - \mathrm{e}^{-r\tau}\,L_{\mathrm{C}}\,\middle|\,\mathcal{F}_t\right]. \quad (35)$$

Under the abovementioned assumption that the risk in Π_t can be spanned by existing assets, it is possible to construct a risk-free portfolio consisting of one unit of the claim $\Psi_{\mathrm{C}}(\Pi_t)$ and a short position of n units of Π_t. This feat is accomplished by choosing an appropriate quantity n.

Economically speaking, the claim $\Psi_{\mathrm{C}}(\Pi_t)$ represents an abandonment (put) option on a project yielding a profit rate of $-p\,(\theta-\zeta)\,\Pi_t$. Holding the portfolio yields a "dividend" of $-\big(p\,(\theta-\zeta)+n\delta\big)\Pi_t\,\mathrm{d}t$. Expanding $\mathrm{d}\Psi_{\mathrm{C}}(\Pi_t)$ using Itô's Lemma gives the "capital gain" on the portfolio, which is

$$\mathrm{d}\Psi_{\mathrm{C}}(\Pi_t) - n\,\mathrm{d}\Pi_t = \left(\alpha\Pi_t\left(\frac{\partial\Psi_{\mathrm{C}}(\Pi_t)}{\partial\Pi_t} - n\right) + \tfrac{1}{2}\sigma^2\Pi_t^2\frac{\partial^2\Psi_{\mathrm{C}}(\Pi_t)}{\partial\Pi_t^2}\right)\mathrm{d}t + \sigma\Pi_t\left(\frac{\partial\Psi_{\mathrm{C}}(\Pi_t)}{\partial\Pi_t} - n\right)\mathrm{d}W_t. \quad (36)$$

For the portfolio to be risk-free, set $n = \partial\Psi_{\mathrm{C}}(\Pi_t)/\partial\Pi_t$ and assume continuous rebalancing. Total return equals the risk-free return:

$$\left(\tfrac{1}{2}\sigma^2\Pi_t^2\frac{\partial^2\Psi_{\mathrm{C}}(\Pi_t)}{\partial\Pi_t^2} - p\,(\theta-\zeta)\,\Pi_t - \frac{\partial\Psi_{\mathrm{C}}(\Pi_t)}{\partial\Pi_t}\delta\Pi_t\right)\mathrm{d}t = r\left(\Psi_{\mathrm{C}}(\Pi_t) - \frac{\partial\Psi_{\mathrm{C}}(\Pi_t)}{\partial\Pi_t}\Pi_t\right)\mathrm{d}t \quad (37)$$

or

$$\tfrac{1}{2}\sigma^2\Pi_t^2\frac{\partial^2\Psi_{\mathrm{C}}(\Pi_t)}{\partial\Pi_t^2} + (r-\delta)\,\Pi_t\frac{\partial\Psi_{\mathrm{C}}(\Pi_t)}{\partial\Pi_t} - r\Psi_{\mathrm{C}}(\Pi_t) - p\,(\theta-\zeta)\,\Pi_t = 0. \quad (38)$$

A general solution, which holds in the continuation region, is

$$\Psi_{\mathrm{C}}(\Pi_t) = A^{+}\Pi_t^{\gamma^{+}} + A^{-}\Pi_t^{\gamma^{-}} - \frac{p\,(\theta-\zeta)\,\Pi_t}{r-\alpha^*}, \quad (39)$$

where $\{\gamma^{+},\gamma^{-}\}$ are roots of the quadratic equation

$$\tfrac{1}{2}\sigma^2\gamma\,(\gamma-1) + (r-\delta)\,\gamma - r = 0, \quad (40)$$

sometimes referred to as the "fundamental quadratic." Given that $\alpha^* = r-\delta$,

$$\gamma^{\pm} = \tfrac{1}{2} - \frac{\alpha^*}{\sigma^2} \pm \sqrt{\left(\frac{\alpha^*}{\sigma^2} - \tfrac{1}{2}\right)^2 + 2\frac{r}{\sigma^2}}. \quad (41)$$

The constants A^{+} and A^{-} are to be determined. Setting $A^{-}=0$ ensures that $\Psi_{\mathrm{C}}(\Pi_t)$ is bounded near $\Pi_t = 0$. Since the option grants the holder the right to exchange uncertain negative profits for a negative cash flow known with certainty, it should be worthless for small Π_t.

In the stopping region, immediate exercise is optimal and $\Psi_{\mathrm{C}}(\Pi_t) = -L_{\mathrm{C}}$. Hence,

$$\Psi_{\mathrm{C}}(\Pi_t) = \begin{cases} -L_{\mathrm{C}} & \text{if } \Pi^* < \Pi_t, \\ A^+\Pi_t^{\gamma^+} - \frac{p(\theta-\zeta)\Pi_t}{r-\alpha^*} & \text{otherwise.} \end{cases} \tag{42}$$

Imposing C^1-continuity at $\Pi_t = \Pi^*$ as usual leads to

$$A^+(\Pi^*)^{\gamma^+} - \frac{p\,(\theta-\zeta)\,\Pi_t}{r-\alpha^*} = -L_{\mathrm{C}} \tag{43a}$$

and

$$\gamma^+ A^+(\Pi^*)^{\gamma^+-1} - \frac{p\,(\theta-\zeta)}{r-\alpha^*} = 0. \tag{43b}$$

These equations are the *value-matching* and *smooth-pasting* conditions, respectively. Solving (43b) for A^+, substituting the result in (43a) and subsequently solving for Π^* leads to

$$\Pi^* = \frac{\gamma^+}{\gamma^+-1}\frac{L_{\mathrm{C}}\,(r-\alpha^*)}{p\,(\theta-\zeta)} \tag{44}$$

$$\Leftrightarrow \frac{p\,(\theta-\zeta)\,\Pi^*}{r-\alpha^*} = \frac{\gamma^+}{\gamma^+-1}L_{\mathrm{C}} \tag{45}$$

and

$$A^+ = \frac{1}{\gamma^+}\frac{p\,(\theta-\zeta)\,(\Pi^*)^{1-\gamma^+}}{r-\alpha^*}. \tag{46}$$

Since $\alpha^* < r$ (by assumption) and $1 < \gamma^+$, Π^* will take positive values if and only if $\zeta < \theta$. Since $\Pi_t = 0$ is an absorbing barrier, litigation will never be optimal otherwise.

Summing up, provided $\zeta < \theta$, by (29), (35), and (42), the option value of litigation is given by (31). □

As expected (32) is analogous to the deterministic case from (13), but, in addition, includes the well-known “option value multiple” $\gamma^+/\left(\gamma^+-1\right)$. It is increasing in σ, which implies a higher value of waiting for higher levels of uncertainty. Also note that, compared to the Jorgensonian rule, $r-\alpha^*$ replaces r. As volatility approaches zero, the stochastic trigger converges to the deterministic trigger:

$$\lim_{\sigma\to 0}\frac{\gamma^+}{\gamma^+-1}\frac{L_{\mathrm{C}}\,(r-\alpha^*)}{p\,(\theta-\zeta)} = \frac{L_{\mathrm{C}}r}{p\,(\theta-\zeta)}. \tag{47}$$

Convergence is demonstrated by figure 3. Litigation will be postponed as long as possible if $\theta \leq \zeta$ and

$$F_{\mathrm{C}}(\Pi_t) = \frac{p\zeta\Pi_t}{r-\alpha^*}. \tag{48}$$

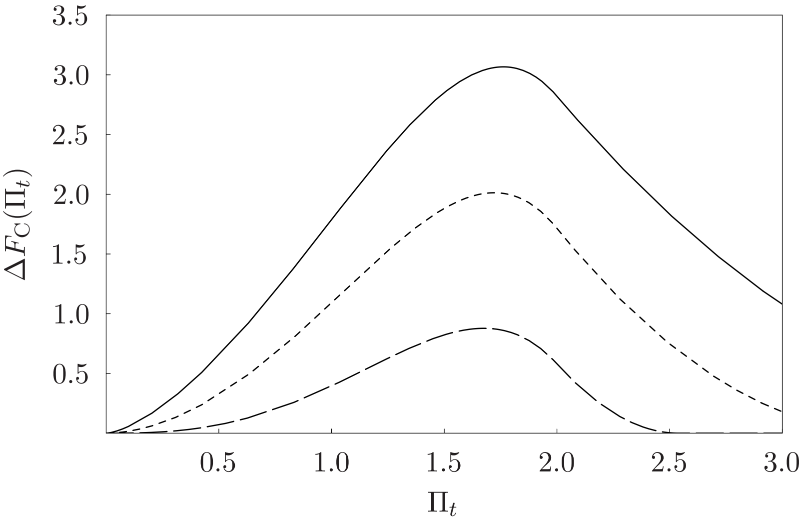


Fig. 3. Impact of uncertainty on the option value of litigation ($p = 0.5$, $r = 0.05$, $\alpha = r - \delta = 0.01$, $\theta = 1.0$, $\zeta = 0.5$, and $L_{\mathrm{C}} = 10.0$). As volatility decreases ($\sigma \in \{0.1, 0.2, 0.3\}$), option values converge to the deterministic solution.

The latter result obviously fundamentally relies on the assumption of infinite patent protection and is thus primarily of theoretical relevance.

If the revenue rate lies above the critical level, that is immediate litigation is optimal, the option value equals the expected share of future revenues, less litigation costs. Below the critical level, the option value has two components. One component is the expected payoff from litigation under the assumption of indefinite postponement. Continuation in this setting implies that the holder of the option acquires an (expected) claim on past cash flows. The other component is the value of flexibility. Under the condition that the initial revenue rate is sufficiently high, the option holder will litigate and give up this flexibility in exchange for immediate benefits.

The net payoff from commercialization corresponds to the value a rational investor would attribute to a patent if he or she were to enter the relevant market immediately. As outlined previously, it equals the net present value of expected profits, less the option value of litigation, less the expected value of additional litigation costs. The latter component deserves more detailed analysis.

With the option value of litigation known, determining the gross payoff from commercialization to the incumbent under patent risk $\widetilde{V}_{\mathrm{I}}(\Pi_t)$ seems straightforward. However, one has to account for the fact that the cost of litigation for the incumbent and the challenger might differ, that is $L_{\mathrm{I}} \neq L_{\mathrm{C}}$. It is therefore insufficient to simply subtract the "short position." Finding the appropriate discount rate for the correction introduced in (10), however, is non-trivial, because the occurrence of litigation is random. Consequently, one needs to form expectations about the "first hitting time" τ^* (Dixit and Pindyck, 1994, pp. 315–316).

Theorem 3 *If $\Pi^* \geq \Pi_t$ is a fixed upper threshold, and $\tau^* \geq t$ is the first hitting time,*

$$\mathbf{E}_{\mathbf{P}^*}\left[\mathrm{e}^{-r(\tau^*-t)}\,\middle|\,\mathcal{F}_t\right] = \left(\frac{\Pi_t}{\Pi^*}\right)^{\gamma^+}. \tag{49}$$

Similarities between (49) and the option pricing formula of proposition 2 are no coincidence. As outlined in the appendix, patent value under uncertainty can also be derived based on the first hitting time (see sec. A).

Theorem 3 holds in the continuation region of the litigation option. Immediate litigation obviously implies $\tau^* = 0$. Therefore, the following proposition can be derived.

Proposition 4 *The gross payoff from commercializing in the presence of imperfect patent protection is*

$$\widetilde{V}_{\mathrm{I}}(\Pi_t) = \begin{cases} (1-p\theta)\,\Pi_t/\delta - L_{\mathrm{I}} & \text{if } \Pi^* < \Pi_t, \\ (1-p\zeta)\,\Pi_t/\delta - B^+\Pi_t^{\gamma^+} & \text{otherwise}, \end{cases} \tag{50}$$

where

$$\Pi^* = \frac{\gamma^+}{\gamma^+-1}\frac{L_{\mathrm{C}}\delta}{p\,(\theta-\zeta)}. \tag{51}$$

is the critical profit rate, and

$$B^+ = \left(L_{\mathrm{I}} + \frac{\gamma^+}{\gamma^+-1}L_{\mathrm{C}}\right)\left(\frac{1}{\Pi^*}\right)^{\gamma^+}. \tag{52}$$

Proof of proposition 4 Since litigation risk hinges on the ratio ζ/θ, it becomes necessary to distinguish the cases $\zeta < \theta$ and $\theta \leq \zeta$.

On the one hand, provided that $\zeta < \theta$ and Π_t is in the continuation region, combining (24) and proposition 2 yields

$$\begin{aligned} \widetilde{V}_{\mathrm{I}}(\Pi_t) &= \Pi_t/\delta - F_{\mathrm{C}}(\Pi_t) - \mathbf{E}_{\mathbf{P}^*}\left[\mathrm{e}^{-r\tau^*}\,(L_{\mathrm{I}}+L_{\mathrm{C}})\right] \\ &= \Pi_t/\delta - \left(\frac{L_{\mathrm{C}}}{\gamma^+-1}\left(\frac{\Pi_t}{\Pi^*}\right)^{\gamma^+} + p\zeta\Pi_t/\delta\right) - \mathbf{E}_{\mathbf{P}^*}\left[\mathrm{e}^{-r\tau^*}\right](L_{\mathrm{I}}+L_{\mathrm{C}}) \\ &= (1-p\zeta)\,\Pi_t/\delta - \frac{L_{\mathrm{C}}}{\gamma^+-1}\left(\frac{\Pi_t}{\Pi^*}\right)^{\gamma^+} - (L_{\mathrm{I}}+L_{\mathrm{C}})\left(\frac{\Pi_t}{\Pi^*}\right)^{\gamma^+} \\ &= (1-p\zeta)\,\Pi_t/\delta - \left(L_{\mathrm{I}} + \frac{\gamma^+}{\gamma^+-1}L_{\mathrm{C}}\right)\left(\frac{\Pi_t}{\Pi^*}\right)^{\gamma^+}. \end{aligned} \tag{53}$$

If the cash flow rate exceeds the critical level, the challenger will litigate immediately, resulting in litigation costs of L_{I}. With probability p, the challenger is successful and obtains a fraction of future profits, namely $\theta\Pi_t/\delta$. The gross present value of cash flows from commercialization thus becomes

$$\begin{aligned}\widetilde{V}_{\mathrm{I}}(\Pi_t) &= \Pi_t/\delta - (p\theta\Pi_t/\delta - L_{\mathrm{C}}) - (L_{\mathrm{I}} + L_{\mathrm{C}}) \\ &= (1 - p\theta)\,\Pi_t/\delta - L_{\mathrm{I}}. \end{aligned} \tag{54}$$

If, on the other hand, $\theta \leq \zeta$ one obtains

$$\widetilde{V}_{\mathrm{I}}(\Pi_t) = (1 - p\zeta)\,\Pi_t/\delta. \tag{55}$$

The cost of litigation, which takes place in the very distant future, becomes negligible in present-value terms. □

Intuitively speaking, gross payoff equals the value of the project if the challenger were to litigate immediately, plus the value of waiting, less the expected present value of litigation costs. [4]

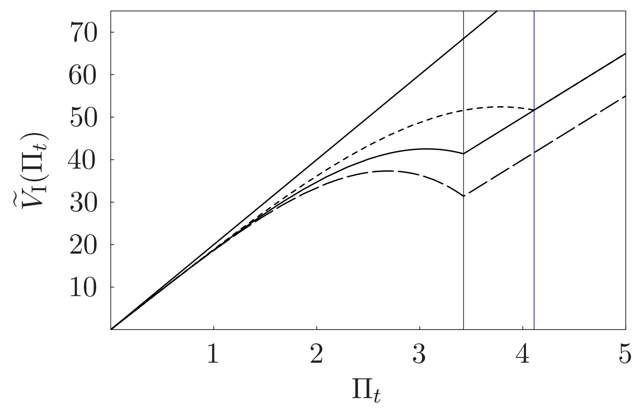
Figure 4 shows $\widetilde{V}_{\mathrm{I}}$ as a function of Π_t. Interestingly, rising profit rates under imperfect patent protection may result in declining patent value. This seemingly counter-intuitive result is due to litigation risk, which—under certain conditions—may over-compensate the positive effects of heightened profitability. The adverse effects of patent risk are particularly pronounced if the challenger's litigation costs are small compared to those incurred by the incumbent.

Since the payoff from commercialization includes a short position, it may also turn out to be negative.

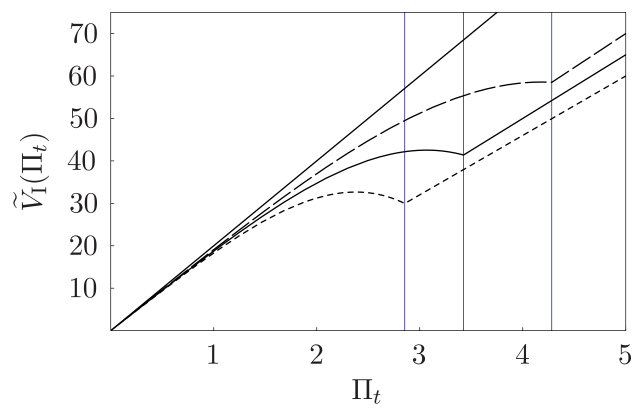
2.2.2.2 Option to commercialize The analysis can be carried one step further by examining the option to invest held by the incumbent who owns the patent, but has not yet commenced commercialization. This view implies that a patent is properly valued by pricing a (compound) call on a portfolio consisting of a project and a short option to litigate.

The extended decision problem is a sequential game in continuous time. Its discrete-time equivalent is depicted in figure 5.

[4] While this intuition served as the starting point for the proof just presented, the appendix offers an alternative derivation of proposition 4, based on the expected first hitting time (see sec. B).



(a) Litigation cost



(b) Patent quality

Fig. 4. Gross payoff from commercialization under endogenous patent risk when the protection period is infinite ($\sigma = 0.1$, $r = \delta = 0.05$, $\zeta = 0.1$, and $\theta = 0.5$). Panel (a) shows $\widetilde{V}_{\mathrm{I}}(\Pi_t)$ for $p = 0.5$. Assuming $L_{\mathrm{C}} = 10$, an increase in the incumbent's litigation cost L_{I} from 10 to 20 results in a downward shift of the corresponding graph, but does not affect the trigger (long dashes). In contrast, holding the incumbent's litigation cost constant at $L_{\mathrm{I}} = 10$, an increase in the challenger's litigation cost L_{C} from 10 to 12 leads to a higher investment threshold Π^* (vertical lines), but, for obvious reason, has no influence on $\widetilde{V}_{\mathrm{I}}(\Pi_t)$ in the stopping region (short dashes). Panel (b) illustrates the impact of patent quality, measured by the probability of litigation success p, on $\widetilde{V}_{\mathrm{I}}(\Pi_t)$. As p decreases, higher profit rates are required to trigger litigation; and $\widetilde{V}_{\mathrm{I}}(\Pi_t)$ eventually equals $V_{\mathrm{I}}(\Pi_t)$ ($L_{\mathrm{I}} = L_{\mathrm{C}} = 10$ and $p = \{0.4, 0.5, 0.6\}$).

Proposition 4 gives the value of *commercialization* under litigation risk. The value of the *patent* is the result of the nested optimization problem

$$\begin{aligned}\widetilde{F}_{\mathrm{I}}(\Pi_t) &= \sup_{\tau\in[0,\infty)} \mathbf{E}_{\mathbf{P}^*}\left[\mathrm{e}^{-r\tau}\left(\widetilde{V}_{\mathrm{I}}(\Pi_\tau) - I\right)\right] \\ &= \mathbf{E}_{\mathbf{P}^*}\left[\mathrm{e}^{-r\tau^{**}}\left(\widetilde{V}_{\mathrm{I}}(\Pi_{\tau^{**}}) - I\right)\right], \qquad (56)\end{aligned}$$

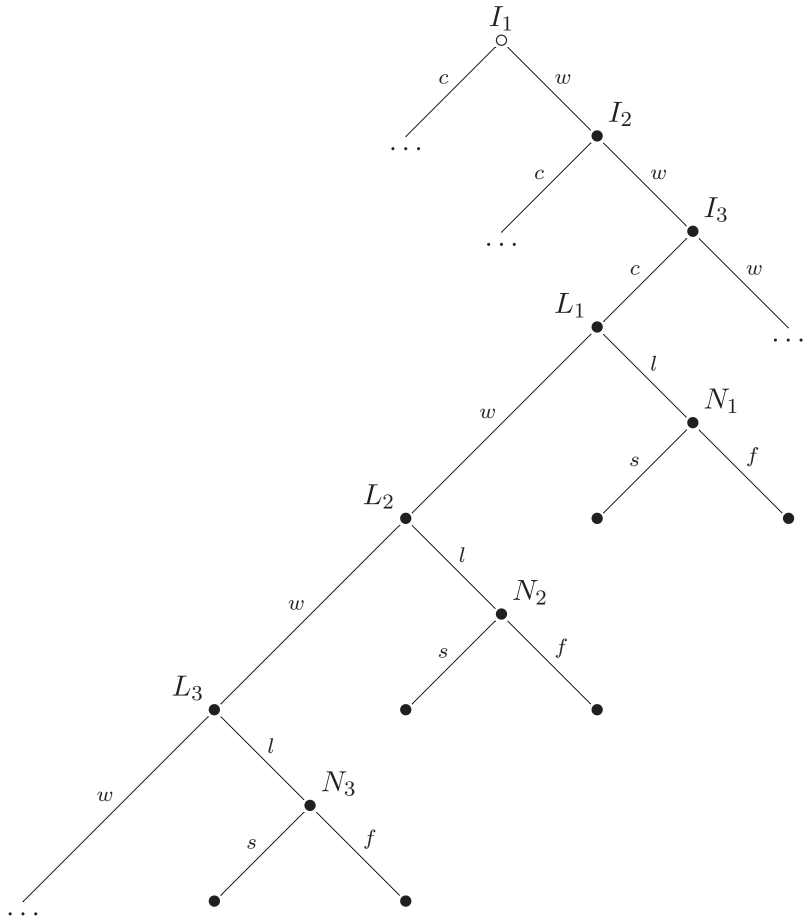


Fig. 5. Sequential game for patent valuation. At each node in $\{I_1, I_2, I_3, \dots\}$ the incumbent decides whether to commercialize the patent (c) or wait an additional period (w). Once the incumbent has decided to commercialize, the challenger faces a similar sequence of choices $\{L_1, L_2, L_3, \dots\}$. At each node he or she may either litigate (l) or postpone litigation to a later point in time (w). In the event of litigation, nature determines the outcome at $\{N_1, N_2, N_3, \dots\}$. The sub-tree starting at L_1 corresponds to the litigation option discussed in the previous subsection.

where I denotes the up-front investment required to commercialize the patent.

Due to the non-linear payoff function, deriving specific patent value requires careful analysis. Among other parameters, the ratio ζ/θ plays a key role.

Consider the case $\theta \leq \zeta$. As shown previously, litigation will be postponed as long as possible, and the underlying becomes linear in Π_t. Option exercise is only optimal above some threshold Π^{**}, making the claim quite similar to a conventional perpetual call option:

$$\widetilde{F}_{\mathrm{I}}(\Pi_t, t) = \begin{cases} \widetilde{V}_{\mathrm{I}}(\Pi_\tau) - I & \text{if } \Pi^{**} < \Pi_t, \\ C^+ \Pi_t^{\gamma+} & \text{otherwise,} \end{cases} \tag{57}$$

Substituting (55) one obtains the value-matching and smooth-pasting conditions

$$C^+ (\Pi^{**})^{\gamma^+} = (1 - p\zeta)\, \Pi^{**}/\delta - I, \tag{58a}$$
$$\gamma^+ C^+ (\Pi^{**})^{\gamma^+ - 1} = (1 - p\zeta)\, /\delta. \tag{58b}$$

Consequently,

$$\begin{aligned} C^+ &= \frac{1}{\gamma^+} (1 - p\zeta)\, (\Pi^{**})^{1-\gamma^+}/\delta \\ &= \frac{I}{\gamma^+ - 1} \left(\frac{1}{\Pi^{**}}\right)^{\gamma^+}, \end{aligned} \tag{59}$$

where

$$\Pi^{**} = \frac{\gamma^+}{\gamma^+ - 1} \frac{I\delta}{1 - p\zeta}. \tag{60}$$

These equations correspond to (44) and (46), respectively. In summary, the dynamic value of a patent under imperfect patent protection is

$$\widetilde{F}_{\mathrm{I}}(\Pi_t) = \begin{cases} (1 - p\zeta)\, \Pi_t/\delta - I & \text{if } \Pi^{**} < \Pi_t, \\ \frac{I}{\gamma^+ - 1} \left(\frac{\Pi_t}{\Pi^{**}}\right)^{\gamma^+} & \text{otherwise.} \end{cases} \tag{61}$$

For obvious reasons, patent value does not dependent on θ. Patent value is almost completely analogous to the case of perfect patent protection, with the noteworthy exception of an expected payment to the challenger litigating in the very distant future.

Consider now the case $\zeta < \theta$. If Π_t is very large, both options will end up in their respective stopping regions, so that

$$\widetilde{F}_{\mathrm{I}}(\Pi_t) = (1 - p\theta)\, \Pi_t/\delta - L_{\mathrm{I}} - I. \tag{62}$$

The incumbent commercializes, followed by immediate litigation. Nevertheless, the option value will proof to be more complicated to determine for a wide range of moderate cash flow rates. In order to provide a more comprehensive picture under various assumptions, especially with respect to patent duration, further analysis are best carried out numerically.

Hence, in the following, a numerical method for determining patent value under litigation risk is described. Taking advantage of a decomposition similar to the one depicted in figure 1 it also captures the effect of a finite protection period.

In order to improve accuracy, not a standard Cox–Ross–Rubinstein (CRR) tree, but the log-transformed variant proposed by Trigeorgis (1991) is constructed. Based on Itô's Lemma, the discrete-time equivalent of the profit

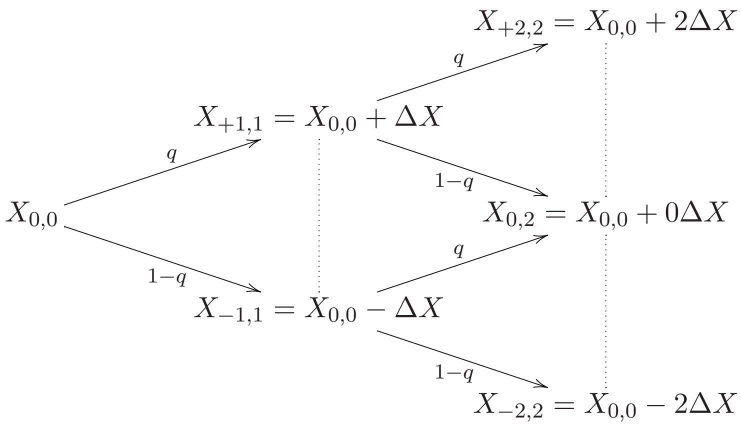


Fig. 6. Log-transformed binomial tree for the numerical valuation of patents under endogenous litigation risk.

rate process under the risk-neutral measure is

$$\Pi_{t+\Delta t} = \Pi_t \exp\!\left(\left(r - \delta - \tfrac{1}{2}\sigma^2\right)\Delta t + \sigma \Delta W_t\right). \tag{63}$$

Furthermore, consider the transformation $X_t \equiv \ln(\Pi_t)$ and $u \equiv \sigma^2 t$ (Trigeorgis, 1996, p. 321), so that $X = \{X_u\}_{u\geq 0}$ becomes arithmetic Brownian motion (ABM), and time is expressed "in units of variance." Assuming the protection period is divided into intervals of equal length $\Delta t \equiv T/n$, this choice implies $\Delta u \equiv \sigma^2 \Delta t$. Over each interval, $X_{i,j} \equiv X_{i\Delta X, j\Delta u}$ increases by

$$\begin{aligned}\Delta X &= \ln\left(\frac{\Pi_{t+\Delta t}}{\Pi_t}\right)\\ &= \left(r - \delta - \tfrac{1}{2}\sigma^2\right)\Delta t + \sigma \Delta W_t\end{aligned} \tag{64}$$

or decreases by the same amount. The probability of an upward movement is q. Figure 6 shows the binomial tree representing this discrete-time process.

Parameters for the log-transformed tree are chosen to mirror continuous-time dynamics. Set $\mu \equiv (r - \delta)/\sigma^2 - \frac{1}{2}$.[5] Hence, one obtains

$$\begin{aligned}\mathbf{E}[\Delta X] &= \mu \Delta u\\ &= q\Delta X - (1-q)\Delta X\\ &= 2q\Delta X - \Delta X\end{aligned} \tag{65}$$

and

$$\begin{aligned}\mathbf{V}[\Delta X] &= \Delta u\\ &= \mathbf{E}\left[\Delta X^2\right] - (\mathbf{E}[\Delta X])^2\\ &= \Delta X^2 - (\mathbf{E}[\Delta X])^2.\end{aligned} \tag{66}$$

[5] Deviating from the notation employed earlier, μ here does *not* signify the risk-adjusted rate of return.

Solving for the risk-neutral probability leads to

$$q = \tfrac{1}{2}\left(1 + \mu\frac{\Delta u}{\Delta X}\right), \tag{67}$$

where

$$\Delta X = \sqrt{\Delta u + (\mu \Delta u)^2}. \tag{68}$$

Note that the procedure is unconditionally stable (Trigeorgis, 1996, p. 322).

Recall from (29) that the option to litigate decomposes into a call option and a perpetuity. However, in order to solve the optimization problem, it becomes necessary to choose a slightly different decomposition, namely

$$F_{\mathrm{C}}(\Pi_t, t) = \mathbf{E}_{\mathbf{P}^*}\left[p\zeta \int_t^T \mathrm{e}^{-r(s-t)}\, \Pi_s \,\mathrm{d}s \,\middle|\, \mathcal{F}_t\right] + \Psi_{\mathrm{C}}(\Pi_t, t), \tag{69}$$

where

$$\begin{aligned}\Psi_{\mathrm{C}}(\Pi_t, t) &= \sup_{\tau\in[t,T]} \mathbf{E}_{\mathbf{P}^*}\left[p\,(\theta-\zeta) \int_\tau^T \mathrm{e}^{-r(s-t)}\, \Pi_s \,\mathrm{d}s - \mathrm{e}^{-r(\tau-t)}\, L_{\mathrm{C}} \,\middle|\, \mathcal{F}_t\right] \\ &= \mathbf{E}_{\mathbf{P}^*}\left[p\,(\theta-\zeta) \int_{\tau^*}^T \mathrm{e}^{-r(s-t)}\, \Pi_s \,\mathrm{d}s - \mathrm{e}^{-r(\tau^*-t)}\, L_{\mathrm{C}} \,\middle|\, \mathcal{F}_t\right].\end{aligned} \tag{70}$$

The stopping times derived lead to the gross present value of commercialization under patent risk, which is

$$\begin{aligned}V_{\mathrm{I}}(\Pi_t, t) = \mathbf{E}_{\mathbf{P}^*}\Bigg[&(1-p\zeta) \int_t^{\tau^*} \mathrm{e}^{-r(s-t)}\, \Pi_s \,\mathrm{d}s \\ &- \mathrm{e}^{-r(\tau^*-t)}\, L_{\mathrm{I}} + (1-p\theta) \int_{\tau^*}^T \mathrm{e}^{-r(s-t)}\, \Pi_s \,\mathrm{d}s \,\Bigg|\, \mathcal{F}_t\Bigg]\end{aligned} \tag{71}$$

or, after rearranging,

$$V_{\mathrm{I}}(\Pi_t, t) = \mathbf{E}_{\mathbf{P}^*}\left[(1-p\zeta) \int_t^T \mathrm{e}^{-r(s-t)}\, \Pi_s \,\mathrm{d}s \,\middle|\, \mathcal{F}_t\right] + \widetilde{\Psi}_{\mathrm{I}}(\Pi_t, t), \tag{72}$$

where

$$\widetilde{\Psi}_{\mathrm{I}}(\Pi_t, t) = \mathbf{E}_{\mathbf{P}^*}\left[-p\,(\theta-\zeta) \int_{\tau^*}^T \mathrm{e}^{-r(s-t)}\, \Pi_s \,\mathrm{d}s - \mathrm{e}^{-r(\tau^*-t)}\, L_{\mathrm{I}} \,\middle|\, \mathcal{F}_t\right]. \tag{73}$$

For implementation purposes, the algorithm has to be translated into discrete-time formulae.

Employing the log-transformed model described above, a profit rate tree is constructed. Once the value of the underlying has been determined at each node, it is not difficult to calculate the present value of cash flows. Starting at the leaves of the tree, one obtains

$$V_{i,n} = \Pi_{i,n}\Delta t. \tag{74}$$

For all previous periods, the expected present value of cash flows is

$$V_{i,j} = \Pi_{i,j}\Delta t + \mathrm{e}^{-r\Delta t}\left(q\Pi_{i+1,j+1} + (1-q)\,\Pi_{i-1,j+1}\right). \tag{75}$$

Standard dynamic programming techniques lead to the flexible component of option value, namely

$$\Psi_{\mathrm{C}}(\Pi_{i,n}, n\Delta t) = \max\left\{p(\theta-\zeta)V_{i,j} - L_{\mathrm{C}}, 0\right\} \tag{76}$$

and

$$\begin{aligned}\Psi_{\mathrm{C}}(\Pi_{i,j}, j\Delta t) = \max\Big\{&p(\theta-\zeta)V_{i,j} - L_{\mathrm{C}},\\ &\mathrm{e}^{-r\Delta t}\Big(q\Psi_{\mathrm{C}}\left(\Pi_{i+1,j+1}, (j+1)\,\Delta t\right)\\ &\qquad + (1-q)\,\Psi_{\mathrm{C}}\left(\Pi_{i-1,j+1}, (j+1)\,\Delta t\right)\Big)\Big\}.\end{aligned} \tag{77}$$

The resulting policy is then used to arrive at the corresponding component of project value. At the end of the protection period,

$$\widetilde{\Psi}_{\mathrm{I}}(\Pi_{i,n}, n\Delta t) = \begin{cases} -p(\theta-\zeta)V_{i,n} - L_{\mathrm{I}} & \text{if } \Pi^* < \Pi_{i,n},\\ 0 & \text{otherwise.}\end{cases} \tag{78}$$

For all previous nodes, if option exercise is optimal ($\Pi^* < \Pi_{i,j}$),

$$\widetilde{\Psi}_{\mathrm{I}}(\Pi_{i,j}, j\Delta t) = -p(\theta-\zeta)V_{i,j} - L_{\mathrm{I}}. \tag{79}$$

If continuation is optimal ($\Pi_{i,j} \leq \Pi^*$),

$$\begin{aligned}\widetilde{\Psi}_{\mathrm{I}}(\Pi_{i,j}, j\Delta t) = \mathrm{e}^{-r\Delta t}\Big(&q\Psi_{\mathrm{I}}\left(\Pi_{i+1,j+1}, (j+1)\,\Delta t\right)\\ &+ (1-q)\,\Psi_{\mathrm{I}}\left(\Pi_{i-1,j+1}, (j+1)\,\Delta t\right)\Big).\end{aligned} \tag{80}$$

Using (69) and (72), one obtains the option value of litigation as well as the gross payoff from commercialization.

For example, consider the illustrative example shown in table 1, where $\Delta t = T/n = 20.0/2 = 10.0$. Assuming an initial profit rate of $\Pi_0 = 1.00$, cash flow volatility of $\sigma = 0.1$, $r = \delta = 0.05$, $p = 0.5$, $\zeta = 0.1$, $\theta = 0.5$, $L_{\mathrm{C}} = 2$, and $L_{\mathrm{I}} = 10$, the option value of litigation is

$$\begin{aligned}F_C(\Pi_{0,0}, 0) &= p\zeta V_{0,0} + \Psi_{\mathrm{C}}(\Pi_{0,0}, 0)\\ &= 0.5 \times 0.1 \times 19.755 + 1.951 = 2.939.\end{aligned}$$

Gross payoff from commercialization under patent risk becomes

$$\begin{aligned}\widetilde{V}_{\mathrm{I}}(\Pi_{0,0}, 0) &= (1 - p\zeta)\, V_{0,0} + \widetilde{\Psi}_{\mathrm{I}}(\Pi_{0,0}, 0) \\ &= (1 - 0.5 \times 0.1) \times 19.755 - 13.951 = 4.816.\end{aligned}$$

Accurate patent and project values, however, require significantly larger trees.

Figure 7 presents selected numerical results graphically. Although discretization brings about visible inaccuracies around the challenger's critical threshold, the overall shape of curves is in line with analytical project values provided earlier. In addition, figure 7(b) shows that longer protection periods are associated with higher project values, but also make litigation attractive at comparatively low levels of profitability.

Under finite patent protection, the potentially adverse effect of rising profit rates on gross payoff from commercialization are more pronounced. As evident from figure 7(a) and in analogy to the case of an infinite protection period, comparatively high costs of litigation for the incumbent cause project values to drop sharply as rising profit rates approach the critical threshold.

Finally, dynamic patent value can be quantified by pricing an option on the gross payoff from commercialization, that is

$$\widetilde{V}_{\mathrm{I}}(\Pi_{i,j}, j\Delta t) = (1 - p\zeta)\, V_{i,j} + \widetilde{\Psi}_{\mathrm{I}}(\Pi_{i,j}, j\Delta t). \tag{81}$$

Patent values at the leaves of the tree are given by

$$\widetilde{F}_{\mathrm{I}}(\Pi_{i,n}, n\Delta t) = \max\left\{\widetilde{V}_{\mathrm{I}}(\Pi_{i,n}, n\Delta t) - I, 0\right\}. \tag{82}$$

Proceeding backwards in time, all previous nodes are calculated as follows:

$$\begin{aligned}\widetilde{F}_{\mathrm{I}}(\Pi_{i,j}, j\Delta t) = \max\Big\{&\widetilde{V}_{\mathrm{I}}(\Pi_{i,j}, j\Delta t) - I, \\ &\mathrm{e}^{-r\Delta t}\Big(q\widetilde{F}_{\mathrm{I}}\big(\Pi_{i+1,j+1}, (j+1)\,\Delta t\big) \\ &\qquad + (1-q)\,\widetilde{F}_{\mathrm{I}}\big(\Pi_{i-1,j+1}, (j+1)\,\Delta t\big)\Big)\Big\}.\end{aligned} \tag{83}$$

Figure 8 shows dynamic patent value as a function of the initial profit rate under various assumptions concerning litigation costs and the investment required to commercialize the patent. Obviously, the resulting diagram differs substantially from the familiar "hockeystick" associated with plain-vanilla call options—real or financial. Although the drop in patent value due to rising patent risk is mitigated by the value of flexibility, it is still noticeable, in particular if commercialization is inexpensive.

Table 1
Log-transformed binomial model for patent valuation under endogenous litigation risk ($\Pi_0 = 1.00$, $\sigma = 0.1$, $r = \delta = 0.05$, $p = 0.5$, $\zeta = 0.1$, $\theta = 0.5$, $L_{\mathrm{C}} = 2$, $L_{\mathrm{I}} = 10$, $T = 20.0$, and $n = 2$). Carrying out the steps described, it is possible to determine the gross payoff from commercialization, which is indispensable for calculating dynamic patent value. Panel (d) shows the challenger's optimal policy, ones indicating nodes at which litigation is optimal.

(a) Underlying

State	$\Pi_{i,j}$		
	$t = 0$	$t = 10$	$t = 20$
+2			1.897
+1		1.377	
0	1.000		1.000
−1		0.726	
−2			0.527

(b) Present value

State	$V_{i,j}$		
	$t = 0$	$t = 10$	$t = 20$
+2			18.971
+1		22.134	
0	19.755		10.000
−1		11.668	
−2			5.271

(c) Option

State	$\Psi_{\mathrm{C}}(\Pi_{i,j}, j\Delta t)$		
	$t = 0$	$t = 10$	$t = 20$
+2			1.794
+1		2.427	
0	1.951		0.000
−1		0.334	
−2			0.000

(d) Policy

State	Policy		
	$t = 0$	$t = 10$	$t = 20$
+2			1
+1		1	
0	1		0
−1		1	
−2			0

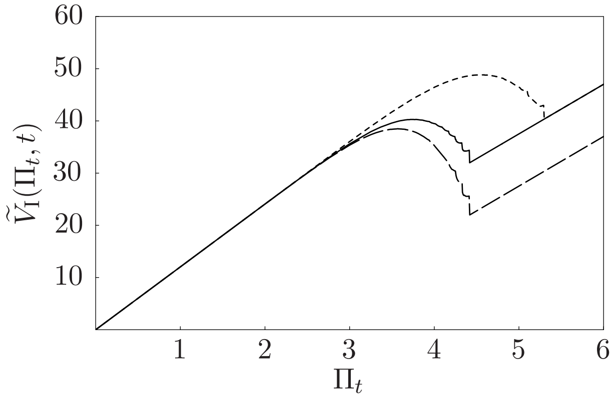
(e) Project

State	$\widetilde{\Psi}_{\mathrm{I}}(\Pi_{i,j}, j\Delta t)$		
	$t = 0$	$t = 10$	$t = 20$
+2			−13.794
+1		−14.427	
0	−13.951		0.000
−1		−12.334	
−2			0.000

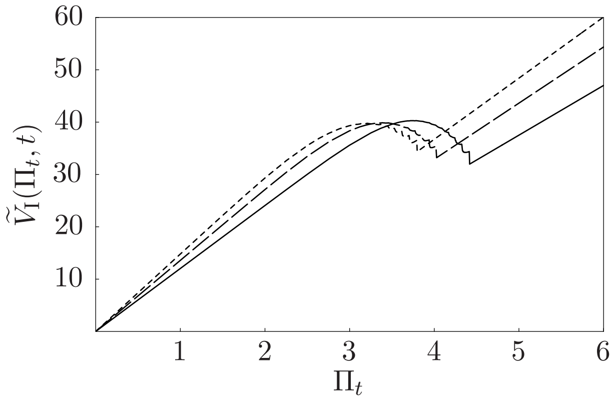
(f) Patent

State	$\widetilde{F}_{\mathrm{I}}(\Pi_{i,j}, j\Delta t)$		
	$t = 0$	$t = 10$	$t = 20$
+2			8.022
+1		11.028	
0	8.767		0.000
−1		1.084	
−2			0.000

Correspondingly, the optimal policy is far more complicated than for the fairly simple litigation option. For very low profit rates, early exercise is unattractive. As profit rates rise, early exercise becomes optimal, before increasing patent risk renders it unattractive again. Eventually, profit rates are high enough to justify early exercise despite the threat of litigation. Figure 9 shows numerical



(a) Litigation cost



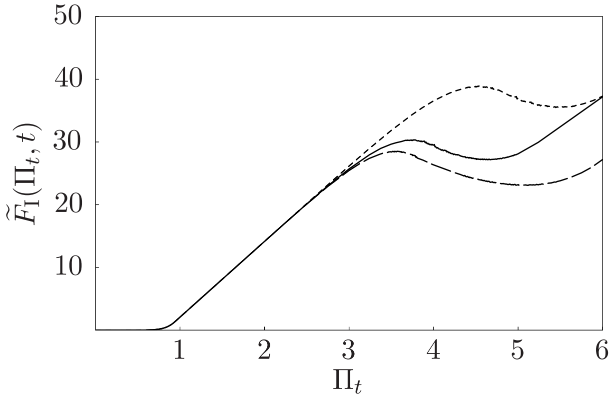
(b) Protection period

Fig. 7. Gross payoff from commercialization under endogenous patent risk when the protection period is finite ($\sigma = 0.1$, $r = \delta = 0.05$, $p = 0.5$, $\zeta = 0.1$, $\theta = 0.5$, and $n = 500$). Panel (a) shows $\widetilde{V}_{\mathrm{I}}(\Pi_t)$ for $T = 20.0$, the base case ($L_{\mathrm{C}} = L_{\mathrm{I}} = 10.0$) represented by a solid line. In analogy to previous analyses, long and short dashes illustrate results for $L_{\mathrm{I}} = 20.0$ and $L_{\mathrm{C}} = 12.0$, respectively. Panel (b) depicts gross payoff for $T = 20.0$ (solid line), $T = 25.0$ (long dashes), and $T = 30.0$ (short dashes).

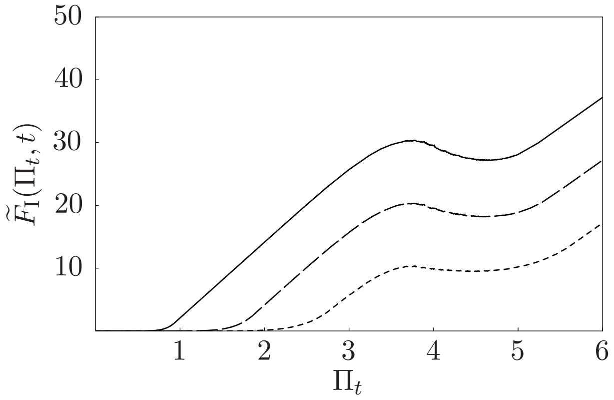
approximations of the resulting boundaries Π_t^*, Π_t^{**}, and Π_t^{***} as a function of time, assuming $L_{\mathrm{C}} = 1.0$, $L_{\mathrm{I}} = 10.0$, and $I = 1.0$.

In summary, increased litigation activity in newly-discovered growth markets calls for careful analysis. In particular, novel platform technologies may be difficult to defend using patents. Investments in promising pieces of IP, more precisely the profit opportunities associated with them, tend to attract potential challengers and increase litigation activity—to the point where lower profit rates would be preferable.

Patent risk makes the *blue oceans* turn *red* (Kim and Mauborgne, 2005). An



(a) Litigation cost



(b) Investment

Fig. 8. Dynamic patent value under endogenous patent risk when the protection period is finite ($\sigma = 0.1$, $r = \delta = 0.05$, $p = 0.5$, $\zeta = 0.1$, $\theta = 0.5$, $T = 20.0$, and $n = 500$). Panel (a) shows patent values for $I = 10.0$. Again, the base case with $L_{\mathrm{C}} = L_{\mathrm{I}} = 10.0$ is represented by a solid line, while long and short dashes serve to illustrate the sensitivity of patent value to changes in these parameters. Moreover, panel (b) depicts how increases in the investment amount required to commercialize lower patent value ($I \in \{10.0, 20.0, 30.0\}$).

option-based view of perfect patent protection is capable of capturing many of the contingencies involved in this challenging decision problem.

3 Variations and extensions

Stylized models like the one presented can always be extended in a number of ways. In the following, a selection of possible extensions will be discussed in more detail.

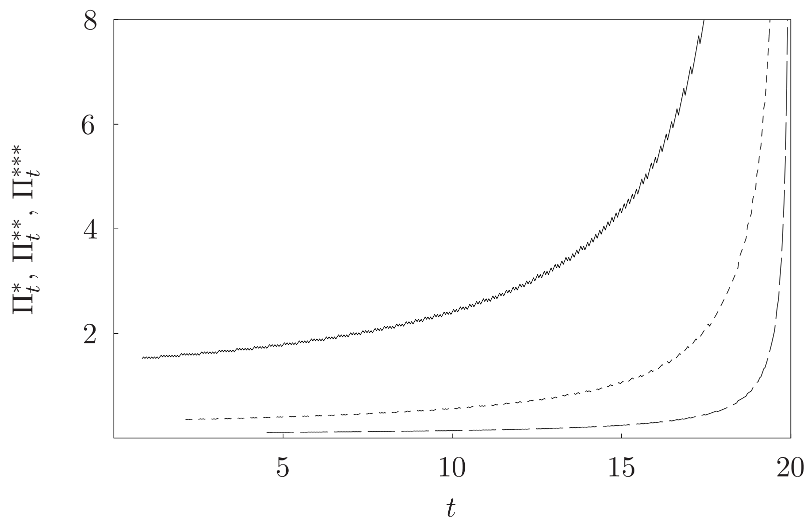


Fig. 9. Incumbent's critical thresholds under endogenous patent risk when the protection period is finite ($\Pi_0 = 1.0$, $\sigma = 0.1$, $r = \delta = 0.05$, $L_{\mathrm{C}} = 1.0$, $L_{\mathrm{I}} = 10.0$, $I = 1.0$, $p = 0.5$, $\zeta = 0.1$, $\theta = 0.5$, and $T = 20.0$). Again, approximations of the boundaries were obtained using a log-transformed binomial tree with $n = 500$ timesteps.

3.1 Alternative litigation systems

An important area of research is the design of the legal system, and the patent system in particular, addressing important issues such as the optimal length, breadth, and depth of patents. Moreover, incentives to litigate and the outcome of disputes are determined by the cost of litigation.

3.1.1 Settlement

Apart from litigation, settlement of patent disputes plays an important role in the value-based management of property rights (Bebchuk, 1984; Crampes and Langinier, 2002). Lanjouw and Schankerman (2002) find that some 95% of patent lawsuits are settled prior to a court judgment. More importantly, as argued by Shapiro (2003, p. 391), a wide range of commercial arrangements involving IP—including patent licenses, mergers, and joint ventures—can be regarded as settlements of IP disputes, effectively or even literally. Royalty rates in licensing deals, for instance, reflect the bargaining power of the contracting parties, which fundamentally depends on the likelihood of winning in court.

Technically speaking, the tradeoff between seeking a decision in court or opting to settle most likely involves the calculation of Nash bargaining solutions. The current model may be used to establish suitable threat points.

3.1.2 European rule

As mentioned before, the American rule requires both parties to bear their own legal costs. However, legal systems differ in the treatment of such expenses. If the loosing party or the state covers costs of litigation, different option values result. A thorough comparison of alternatives could provide insights into the impact on innovation incentives.

3.1.3 Variable cost of litigation

Almost needless to say, assuming a constant cost of litigation is a simplification of the actual process, because lawyers might claim a proportion of the damage award. In essence, variable costs of litigation correspond to a stochastic strike price, which, depending on the choice of parameters, could lead to a higher or lower option value of litigation. Again, an extensive sensitivity analysis would be required to draw meaningful conclusions.

3.2 Alternative underlying dynamics

Simulation results might change considerably, depending on the dynamics employed to capture the development of expected cost to completion and cash flow rates. Common variations of the standard stock price model, include mean reversion and stochastic interest rates.

3.2.1 Mean reversion

Cash flow rates usually track a product-specific lifecycle. In contrast, cash flow rates in this paper were assumed to follow geometric Brownian motion (GBM) with a positive drift, on average leading to an increase in profitability as the end of the protection period approaches. While a variety of alternative specifications are conceivable, mean-reversion processes probably better reflect the stylized facts. One example is the Ornstein–Uhlenbeck process

$$\mathrm{d}\Pi_t = \vartheta\left(\overline{\Pi} - \Pi_t\right)\mathrm{d}t + \sigma\,\mathrm{d}W_t, \tag{84}$$

where ϑ is the speed of reversion and $\overline{\Pi}$ denotes the long-run average level of profitability, to which Π tends to revert.

Similar SDEs are very popular in option-based models of natural resource investments. One way to answer the question of whether GBM indeed matches empirical data is the application of *unit root* tests (Dickey and Fuller, 1981).

3.2.2 *Stochastic interest rates*

As pointed out by Schwartz (2004), analyzing patent value under stochastic interest rates is facilitated by the Monte Carlo approach. In principal, it suffices to specifiy a suitable model, generate the required number of interest rate processes and carry out all calculations employing a time-variant discount factor (Longstaff and Schwartz, 2001, pp. 131–135). Similarly, stochastic interest rates can be accounted for in tree-based option pricing, for example employing the widely-used Heath–Jarrow–Morton model of interest rates. [6]

3.3 *Exit option*

Due to the fact that the current setup abstracts from operating costs, exit options during the commercialization phase have so far been neglected. Introducing an exit option along the lines of existing analyses would complicate matters somewhat, but should not pose severe difficulties (McDonald and Siegel, 1985; Dixit and Pindyck, 1994). It is important to note, however, that—at least in the pharmaceutical industry—firms very rarely exercise the option to stop commercializing, mainly due to the paramount importance of expenditures during R&D.

3.4 *Industry equilibrium*

A closer look at industry equilibrium would call for a demand-level model. Roughly speaking, excess profits earned by commercializing certain patents are likely to attract challengers, thereby increasing patent risk. In equilibrium, these excess profits are exactly offset by the threat of litigation. Moreover, it is important to note that reputation is sure to play an important role in any type of repeated litigation game.

4 Conclusion

The aim of this paper was to develop a deeper understanding of patent risk, looking beyond the seemingly random occurrence of patent-related events, suggested by jump-diffusion models of the R&D process. Following introductory definitions and the model setup in section 2.1, section 2.2.1 served to

[6] Other possibilities include the Black–Derman–Toy model, the Hull–White model, and its Black–Karanski modification, all of which are available in commercial implementations (Heath et al., 1992; Black et al., 1990; Hull, 2000).

discuss the option value of litigation under certainty. Building on some basic insights into the composition of cash flows, section 2.2.2 outlined a sequential stochastic game for patent valuation, which was studied using both analytical and numerical methods. Finally, section 3 hinted at some variations and extensions of the basic framework presented.

Among the many noteworthy findings is the non-obvious functional relationship between cash flow rates and commercialization payoff with important implications for the option value of R&D under imperfect patent protection. As outlined in detail, higher profitability not necessarily goes along with higher patent values.

This insight highlights an important stylized fact of market entry in research intensive industries. Common sense dictates that a high probability of litigation is an indicator of attractive commercial opportunities. Not only do high profits attract potential challengers; high profits are often the result of novel products and services, which due to the limited experience of all parties involved, are typically difficult to protect through patents. This uncertainty, in turn, gives rise to increased litigation activity. Consequently, potential entrants have to trade off growth and profit potential in markets driven by innovation for a comparatively high reliability of IP protection in more mature markets.

While, as a result of various barriers to entry imposed by incumbent oligopolists, this consideration appears somewhat theoretical on the level of whole industries, similar issues arise on the project level. The formal model analyzed in this paper may be seen as a first step to more comprehensive models of R&D and commercialization, demonstrating that the impact of litigation on patent value in strategic settings can in fact be anticipated and, to some degree, even quantified.

As put forth earlier, the type of model proposed might be regarded as a suitable tool for studying the optimal level of patent protection from an option-based perspective, including, for example, not only the length of the protection period, but also other aspects, such as the reliability of patent protection, which may differ substantially across countries and industries. Using the term introduced by Lemley and Shapiro (2005), the option-based view of patent risk developed in this paper represents a formal strategic model of *probabilistic patents.*

In addition, the discussion served to present the option-based view of patent risk as a special case of a more general reconceptualization of uncertain property rights, capturing legal risk as embedded short options to litigate. It should be interesting to investigate how this idea can be developed further.

A Proof of proposition 2

The option value of litigation was introduced in section 2.2.2. The same result can be obtained in a slightly different manner, using first hitting times.

Again, the option value of litigation is decomposed into a controlled and an uncontrolled diffusion process, resulting in

$$\begin{aligned} F_{\mathrm{C}}(\Pi_t) = \max_{\Pi^* \in [0,\infty)} \mathbf{E}_{\mathbf{P}^*}\Bigg[-p\,(\theta-\zeta) \int_t^{\tau^*} \mathrm{e}^{-r(s-t)}\,\Pi_s\,\mathrm{d}s - \mathrm{e}^{-r(\tau^*-t)}\,L_{\mathrm{C}} \\ + p\theta \int_t^{\infty} \mathrm{e}^{-r(s-t)}\,\Pi_s\,\mathrm{d}s \,\Bigg|\, \mathcal{F}_t \Bigg] . \quad \text{(A.1)} \end{aligned}$$

An expression that can be employed to discount the cost of litigation, is known from previous analyses (see theorem 3) and is restated here for convenience:

$$\mathbf{E}_{\mathbf{P}^*}\left[\mathrm{e}^{-r(\tau^*-t)} \,\middle|\, \mathcal{F}_t\right] = \left(\frac{\Pi_t}{\Pi^*}\right)^{\gamma^+} . \quad \text{(A.2)}$$

The first integral remains to be evaluated. Dixit and Pindyck (1994, pp. 315–316) provide the following theorem.

Theorem 5 *If $\Pi^* \geq \Pi_t$ is a fixed upper threshold, and $\tau^* \geq t$ is the first hitting time,*

$$\mathbf{E}_{\mathbf{P}^*}\left[\int_t^{\tau^*} \mathrm{e}^{-r(s-t)}\,\Pi_s\,\mathrm{d}s \,\middle|\, \mathcal{F}_t\right] = \Pi_t/\delta - \Pi^*/\delta \left(\frac{\Pi_t}{\Pi^*}\right)^{\gamma^+} . \quad \text{(A.3)}$$

Proof of theorem 5 The present value formula follows directly from theorem 3, because

$$\begin{aligned} \mathbf{E}_{\mathbf{P}^*}\left[\int_t^{\tau^*} \mathrm{e}^{-r(s-t)}\,\Pi_s\,\mathrm{d}s \,\middle|\, \mathcal{F}_t\right] &= \mathbf{E}_{\mathbf{P}^*}\Bigg[\int_t^{\infty} \mathrm{e}^{-r(s-t)}\,\Pi_s\,\mathrm{d}s \\ &\qquad - \mathrm{e}^{-r(\tau^*-t)} \int_{\tau^*}^{\infty} \mathrm{e}^{-r(s-\tau^*)}\,\Pi_s\,\mathrm{d}s \,\Bigg|\, \mathcal{F}_t\Bigg] \\ &= \Pi_t/\delta - \mathbf{E}_{\mathbf{P}^*}\left[\mathrm{e}^{-r(\tau^*-t)} \,\middle|\, \mathcal{F}_t\right] \Pi^*/\delta, \quad \text{(A.4)} \end{aligned}$$

which is equivalent to (A.3).

It is then straightforward to deduce proposition 2 by substituting (A.2) and

(A.3) in (A.1), which yields

$$F_{\mathrm{C}}(\Pi_t) = \max_{\Pi^* \in [0,\infty)} -p\,(\theta - \zeta)\left(\Pi_t/\delta - \Pi^*/\delta\left(\frac{\Pi_t}{\Pi^*}\right)^{\gamma^+}\right) \\ - L_{\mathrm{C}}\left(\frac{\Pi_t}{\Pi^*}\right)^{\gamma^+} + p\theta\Pi_t/\delta. \quad \text{(A.5)}$$

A necessary condition for the threshold to be optimal is

$$\left(\frac{\gamma^+ L_{\mathrm{C}}}{\Pi^*} - p\,(\theta - \zeta)\,\frac{\gamma^+ - 1}{\delta}\right)\left(\frac{\Pi_t}{\Pi^*}\right)^{\gamma^+} = 0. \quad \text{(A.6)}$$

Solving for the critical profit rate yields

$$\Pi^* = \frac{\gamma^+}{\gamma^+ - 1}\frac{L_{\mathrm{C}}\delta}{p\,(\theta - \zeta)}, \quad \text{(A.7)}$$

which is the trigger deduced earlier. As is easily verified, the sufficient condition is also fulfilled. Inserting (A.7) in (A.5) leads to

$$F_{\mathrm{C}}(\Pi_t) = \frac{L_{\mathrm{C}}}{\gamma^+ - 1}\left(\frac{\Pi_t}{\Pi^*}\right)^{\gamma^+} + p\zeta\Pi/\delta \quad \text{(A.8)}$$

for dynamic patent values in the continuation region.

B Proof of proposition 4

In section 2.2.2, gross payoff from commercialization under patent risk was derived as a portfolio of claims. Alternatively, one may arrive at the same result directly, again making use of the expected first hitting time and the threshold of proposition 2 (see sec. A).

Gross payoff from commercialization is

$$\widetilde{V}_{\mathrm{I}}(\Pi_t) = \mathbf{E}_{\mathbf{P}^*}\left[\int_t^{\infty} \mathrm{e}^{-r(s-t)}\,\Pi_s\,\mathrm{d}s \right. \\ - p\left(\int_t^{\tau^*} \mathrm{e}^{-r(s-t)}\,\zeta\Pi_s\,\mathrm{d}s + \int_{\tau^*}^{\infty} \mathrm{e}^{-r(s-t)}\,\theta\Pi_s\,\mathrm{d}s\right) \\ \left. - \mathrm{e}^{-r(\tau^*-t)}\,L_{\mathrm{I}}\,\middle|\,\mathcal{F}_t\right] \quad \text{(B.1)}$$

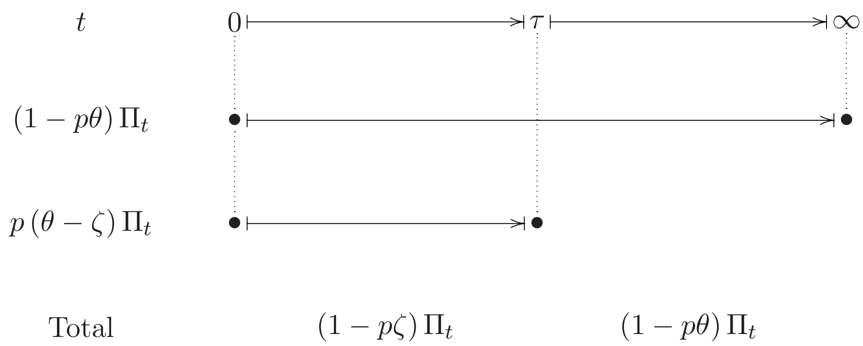


Fig. B.1. Decomposing the payoff from commercialization. Total payoff from litigation can be decomposed into one perpetuity and one option, creating one closed and one open interval with distinct profit rates. The stopping time τ is chosen by the challenger to maximize litigation payoff.

Rewrite this equation to obtain

$$\begin{aligned}\widetilde{V}_{\mathrm{I}}(\Pi_t) = (1-p\theta)\,\mathbf{E}_{\mathbf{P}^*}\left[\int_t^{\infty} \mathrm{e}^{-r(s-t)}\,\Pi_s\,\mathrm{d}s \,\middle|\, \mathcal{F}_t\right] \\ + p\,(\theta-\zeta)\,\mathbf{E}_{\mathbf{P}^*}\left[\int_t^{\tau^*} \mathrm{e}^{-r(s-t)}\,\Pi_s\,\mathrm{d}s \,\middle|\, \mathcal{F}_t\right] \\ - \mathbf{E}_{\mathbf{P}^*}\left[\mathrm{e}^{-r(\tau^*-t)} \,\middle|\, \mathcal{F}_t\right] L_{\mathrm{I}}. \quad \text{(B.2)}\end{aligned}$$

Recall that the first and second terms represent the gross payoff from commercialization under the assumption of immediate litigation and the value of waiting, respectively. Figure B.1 illustrates this decomposition graphically.

The second and the third integral follow from (A.2) and (A.3). Applying the critical profit rate and taking expectations over first hitting times leads to

$$\begin{aligned}\widetilde{V}_{\mathrm{I}}(\Pi_t) &= (1-p\theta)\,\Pi_t/\delta + p\,(\theta-\zeta)\left(\Pi_t/\delta - \Pi^*/\delta\left(\frac{\Pi_t}{\Pi^*}\right)^{\gamma^+}\right) - L_{\mathrm{I}}\left(\frac{\Pi_t}{\Pi^*}\right)^{\gamma^+} \\ &= (1-p\zeta)\,\Pi_t/\delta - \left(L_{\mathrm{I}} + \frac{\gamma^+}{\gamma^+-1}L_{\mathrm{C}}\right)\left(\frac{\Pi_t}{\Pi^*}\right)^{\gamma^+}, \quad \text{(B.3)}\end{aligned}$$

thus verifying proposition 4.

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