*A Multi-stage Investment Game in Real Option Analysis†

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Abstract

This paper investigates an interaction between the managerial flexibility and the competition in a dynamic situation. The value of the flexibility can be valued as a real option while the competition can be analyzed with the game theory. We consider a multi-stage game with two firms under demand uncertainty. In the model, One firm called firm L firstly makes an investment decision, and the other firm called firm F decides secondly after observing firm L's decision at each stage. The model developed here is an extension of the two-stage investment game of Imai and Watanabe(2005), which fully characterize the equilibrium strategies for the two competitive firms by their investment costs. We show that the project values for both firms can be considered as a special example of switching options, and hence these values can be evaluated by the extended switching option model. We apply a binomial or a trinomial lattice to the underlying demand process. Although the lattice model is discrete it is a well-known fact that the trinomial process can converge efficiently to the continuous-time process if parameter values of the lattice model are carefully chosen and the number of trading periods tends to infinity. Hence our model can be also considered as an approximation of the continuous-time model. This paper analyzes equilibrium strategies and project values of the two competitive firms quantitatively under more realistic situations. In addition, by comparing our model with the two-stage game we can investigate the effects of multiple decision opportunities under a more realistic demand process.

*This is a draft version of the research. We have changed many notations for consistency and simplicity.

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1 Introduction

Real option analysis provides a useful tool to evaluate an investment with managerial flexibility under uncertainty. In traditional NPV method, uncertainty usually depresses the value of the investment, but real option analysis shows that it may expand the opportunity of the investment when we consider the flexibility of decisions in future. However, the decision for the investment is often made under competitive situations. An early commitment of the investment may creates competitive advantages to other firms. Thus, in recent times, many studies about the relationships between the managerial flexibility and the strategies of competition, has been investigated by a real option analysis and game theory. Dixit and Pindyck (1994) is one of the earliest studies bases on the oligopoly model. Other researches are Ang and Dukas (1991), Brickley and Zimmerman (2000), Huisman and Kort (2000) , Garlappi (2000), Murto and Keppo (2002), Pawlina and Kort (2002), Weeds (2002), Thijssen, Huisman and Kort (2002), and Lambrecht and Perraudin (2003). The book written by Smit and Trigeorgis (2004) shows the advance in this field.

There are two types of models which integrate real option analysis and game theory. Some models are constructed under continuous time and diffusion processes. Grenadier (1996) considers two firms that compete in the land development business and analyzes the equilibrium price. In the model both firms can enter continuously but the model excludes the simultaneous entry. Huisman (2001) develops a rigorous model based on Fudenberg and Tirole (1985) that analyzes the optimal entry strategy under both the demand uncertainty and competition between two firms.

On the other hand, Smit and Ankum (1993) develops a simple investment game with one or two stages under uncertainty and analyzes two firms' decisions in a sub-game perfect equilibrium. Smit and Trigeorgis (2001) analyzes duopolistic competition which is a typical research in the area of industrial organization of economics and integrate useful concepts about the model. Smit and Trigeorgis (2004) is a excellent intoruduction to understand the concept. This results are implied by the model with two or three stages. They assume that only one firm can access a strategic investment. The model is an extension analyzed by **?**. In the second stages, two firms compete the project under the demand uncertainty which follows a two-period binomial process. They show in numerical examples that there could emerge the Nash equilibrium, the existence of the leader and the follower in the Stackelberg sense, and the monopoly situation, which depends on the realized value of the demand and the amount of the investment cost. They also discuss the optimal strategy for each firm. However, they do not derive conditions of these situations since their analysis is based on the numerical examples.

Imai and Watanabe (2003) consider a two-stage game with two firms and a one-period binomial process. It assumes that the future cash flow depending on the demand is uncertain and follows a one-period binomial process. Two firms are introduced to analyze the competition. Both firms consider identical projects to invest under the investment competition at each stage. The paper assumes that one firm moves first and the other firm can move after observing the first firm's decision. While both firms' managers can invest in the project at the first stage they could have flexibility to defer the investment until the next stage. This flexibility can be considered a real option to defer the project.

Imai and Watanabe (2003) is corresponding to a discrete model of Grenadier (1996), Huisman (2001) and Huisman and Kort (2002) under one-period binomial process, although in Grenadier (1996), Huisman (2001) and Huisman and Kort (2002), both firms simaltaneously decide whether they invest or not while in Imai and Watanabe (2003) they sequentially decide. The model can be understood intuitively and it provides comprehensive and analytical results. They show that the results of competitions and the value of the project depend on some factors, such as the volatility of the uncertainty for the demand and the difference of profit by first mover advantage. However, many questions remains, such as the relation between timings of investments, the linkage of the model under continuous time, and the other factors.

In this paper, we extend the model Imai and Watanabe(2005) to the problem of decision making at multi-stage among competitive firms. We apply a binomial or trinomial lattice to the underlying demand process. Since continuous time models can be calculated approximately by the limitation of discrete time models as well as the financial option theory, we obtain the numerical results about the relations among the timing of investment, the difference of advantage for first-mover, volatility of the demand and the cost of the investment.

2 A Valuation Model

We consider a N-stage game with two firms denoted by firm A and firm B. The two firms consider to invest in a competing follow-up project. Both firms make decisions to maximize their project values. In each stage, both firms make their decisions for investment. This decision of each firm would be made at most once within N stages. This model is analogous to that of the real estate development studied by Grenadier (1996), and it is applicable to many projects such as R&D competition, technology adoption and a pilot plant in a new market. $(x_i(n))$ is said to be a *state* of firm i $(i = A, B)$ at time N $(n = 1, \cdots, N)$ which denotes whether firm i has already invested in a project or not. $x_i(n) = 0$ means that firm i has not invested at time n yet and $x_i(n) = 1$ means he already has. [1]

Since we assume that a firm who has already invested can not invest, alternatives for decisions of both firms at each stage n depend on their state of immediate preceding stage $n-1$. If firm i has already invested ($x_i(n-1)=1$), he does not have any alternatives to act. If firm i has not invested yet($x_i(n-1)=0$), he has an opportunity of investment. In this case, if the rival firm j has already invested, firm i can make decision of the investment personally. If the rival firm $j$also has not invested yet ($x_A(n-1)=0$ and $x_B(n-1)=0$), **we consider two situations about the timing of firms' decisions to investigate the effect of advantage for the timing of firms' decisions.**

- In scenario 1, we assume that their decisions are made sequentially at each stage. In this case, firm A makes his decisions at first and firm B does after observing the firm A's decision.
- In scenario 2, we consider the simultaneous decisions for both firms at each stage.

Imai and Watanabe (2003) is corresponding to scenario 1 and Grenadier (1996), Huisman (2001) and Huisman and Kort (2002) is scenario 2.

Let $Y(n)$ denote the demand at time $n=1,\cdots,N$. The cash flow obtained by each firm at each stage depends on the current demand and states of both firms.

In both cases, we assume that the demand does not change within each stage since decisions by the two firms are made in short time with relative to the time to change of the demand, even if the decisions are made sequentially. However, after both firms make decisions and obtain the cash flow, the demand changes following a trinomial or binomial process at the begining of the next stage. We assume that the demand $Y(n)$ ($n=1,\ldots N-1$) would change to $Y(n+1)=uY(n)$, $Y(n+1)=mY(n)$ and $Y(n+1)=uY(n)$. in trinomial model where u, m and d are rates of the demand in one period satisfying that $d<1<m<u$. In binomial model, we assume that the demand $Y(n)$ ($n=1,\ldots N-1$) would move up to $Y(n+1)=uY(n)$ and move down to $Y(n+1)=dY(n)$ satisfying

that $d < 1 < u$.

If the underlying asset of the real option can be traded in the complete market we can apply the no-arbitrage principle to value the real option[2]. It is difficult, however, to apply the principle to our model since the demand of the merchandise cannot be observed in the market.

In this paper, we take an approach that is a typical assumption for real option analyses. For example, Cox and Ross (1976), Constantinides (1978), and McDonald and Siegel (1984) propose the equilibrium approach for the real option pricing. Especially, the demand risk in this paper can be considered private risk or unsystematic risk that is independent of the market risk. Since an investor pays no risk premium with respect to the unsystematic risk in equilibrium, we can assume that the investors are risk neutral in the valuation model. Let r be the risk free rate for one period and let R define $R = 1 + r$. Then, there exists risk neutral probabilities p_u, p_m and p_d such that the conditional expected demand $Y(n+1)$ on $Y(n)$ is expressed by

$$E\left[Y(n+1)\right] = p_u uY(n) + p_m mY(n) + p_d dY(n) \equiv RY(n) \quad n = 1, \ldots N-1. \tag{1}$$

Setting the parameters properly, our model can be regarded as an approximation of the model for decisions for investment under continuous time from time 0 to time 1, in which the value of the project is followed by a geometric Brownian motion:

$$dY(t) = RY(t)dt + \sigma Y(t)dz$$

where dz is the increment of a Winner process.

Their cash flow are generated after the investment decisions and their effects, hence they depends on $Y(n)$, $x_A(n)$ and $x_B(n)$. We denote cash flow per unit of demand of firm i by $D_{x_i(n)x_j(n)}$ where j is the rival firm of firm i. In other words, at time n when both firms have already invested in the project the cash flow obtained by each firms are given by $D_{11}Y(n)$ while when neither firm has not invested the cash flow is given by $D_{00}Y(n)$. When only one of the firms has invested, the firm can obtain the cash flow $D_{10}Y(n)$ and the other firm which has not invested in the project obtains $D_{01}Y(n)$. We assume that

$$D_{10} > D_{11} > D_{00} > D_{01}, \tag{2}$$

which means that a firm prefer investing in the project if the investment cost is small enough and the other firms' strategy is fixed. Furthermore, we assume that

$$D_{10} - D_{00} > D_{11} - D_{01}. \tag{3}$$

The term $D_{10} - D_{00}$ represents a marginal cash flow of the first mover, a firm that invests when the other firm does not invest, while term $D_{11} - D_{01}$ represents a marginal cash flow of the second mover, a firm that invests after the other firm has invested. Equation (3) means that the situation is preferable if the other firm do not invest.

(The definitions of the value of the firm at the equilibrium and Bellman equations are incomplete.)

3 The Case About Sequential Decisions

In this section, we investigate the case where both firms' decisions are made sequentially at each stage. In this case, firm A makes his decisions at first and firm B does after observing the firm A's decision. We examine how the advantage of firm A effects. We set the parameters as

$$\begin{array}{lllll} Y(0)=100 & D_{00}-2.0 & D_{01}=1.0 & D_{10}=5.0 & D_{11}=3.0 \\ \sigma=0.3 & r=0.05 & N=1000 & . & \end{array}$$

Figure 1 describes the value at time 1, $V_A(x_A(1), x_B(1))$ and $V_{zb}(x_A(1), x_B(1))$ means the expected value of firm A and B respectively when both firms select the action $x_A(1)$ and $x_B(1)$.

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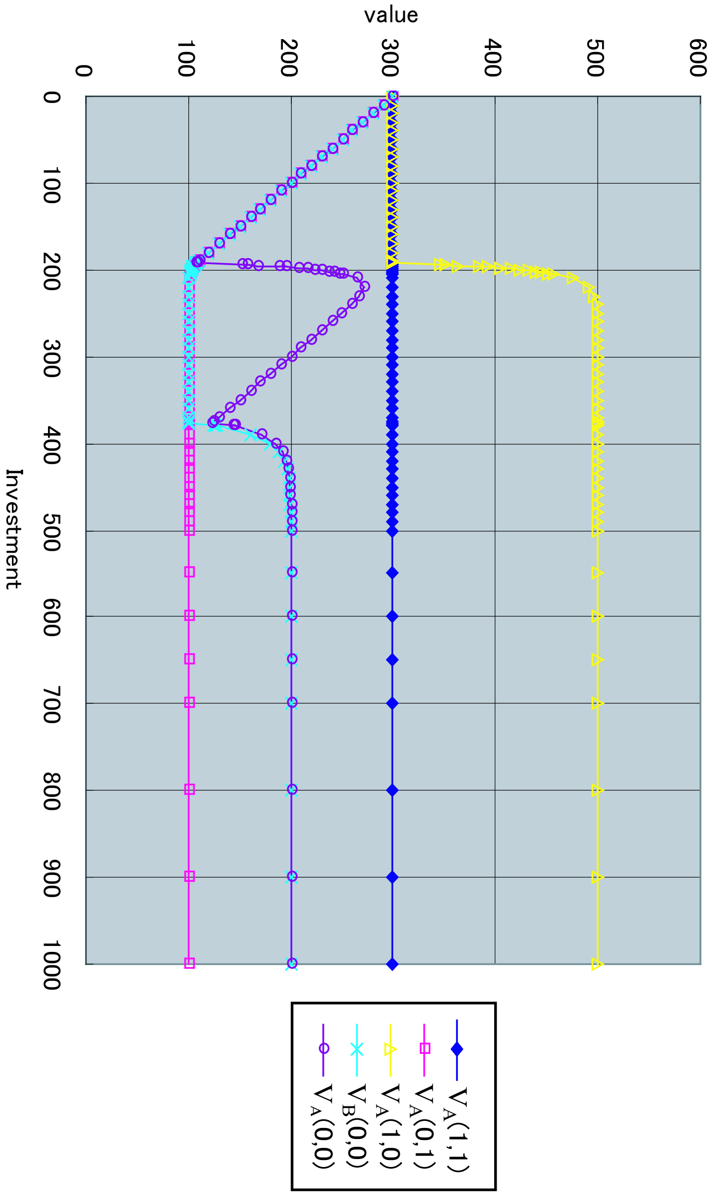

Figure 1: This figure shows the value of both firms $V_i(x_A(1), x_B(1), Y(1))$.